The Pitfalls of Simplicity Bias in Neural Networks

Shah *et al*, NeurIPS 2020

Presented by: {Richard Zhu, Abishek Sridhar, Simon Seo, Rebecca Yu, Selina Carter}
(Group 1)
ML-715 (Fall 2022)

Carnegie Mellon University

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- The reason: NNs lead to "**extreme** simplicity bias," i.e., reliance *exclusively* on simple features, even when more complex features are better predictors.

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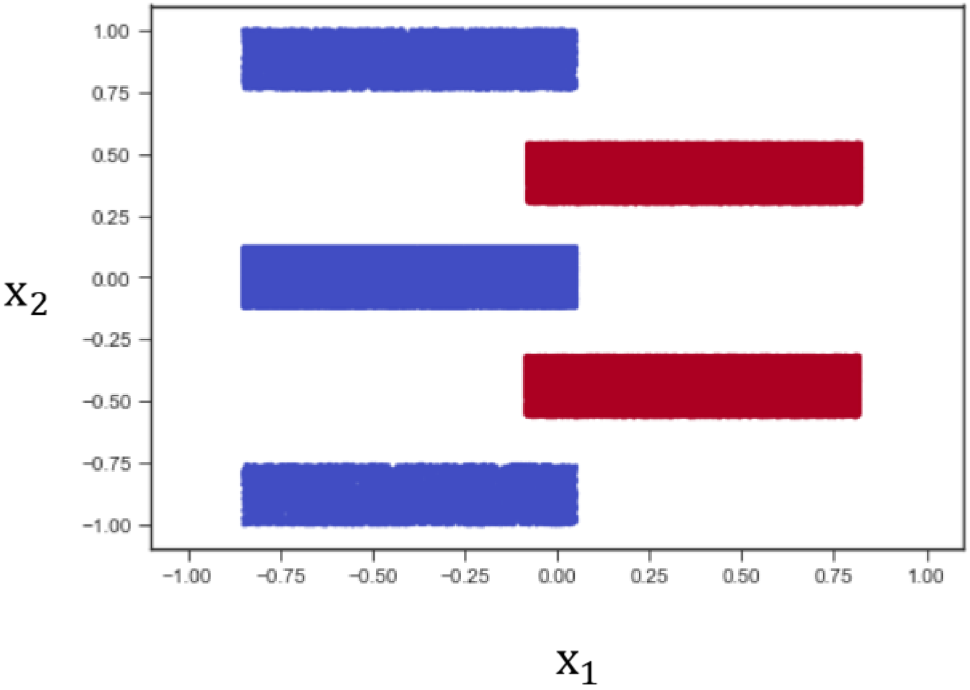

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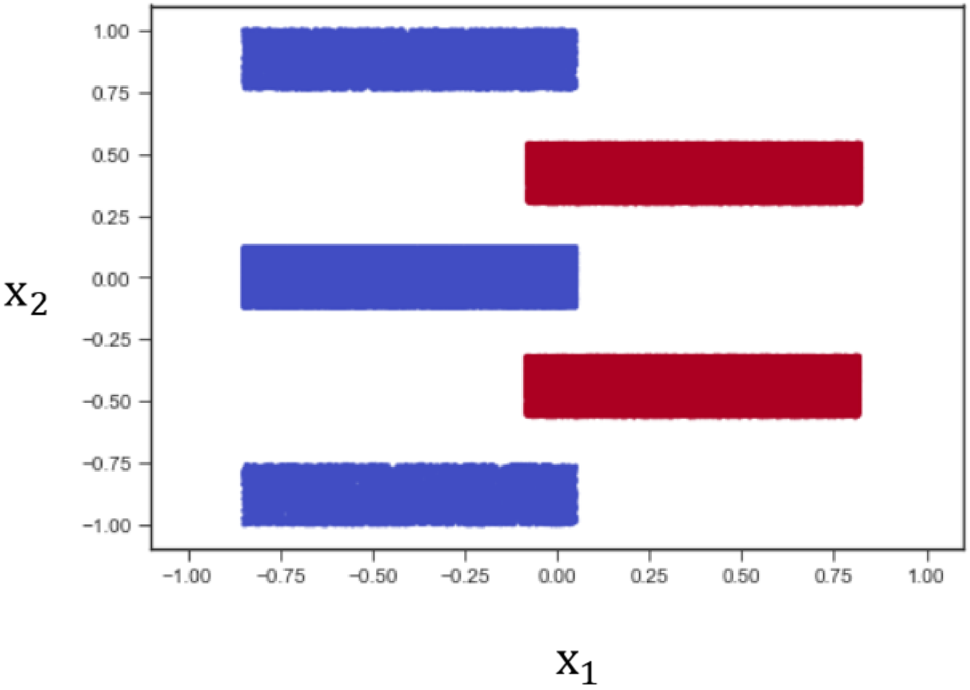

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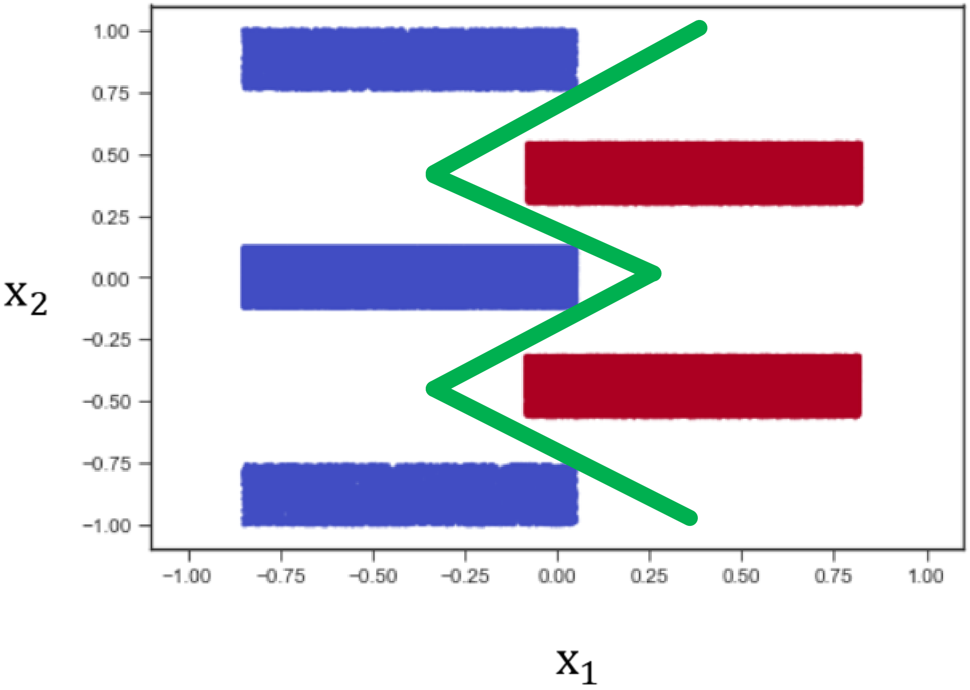

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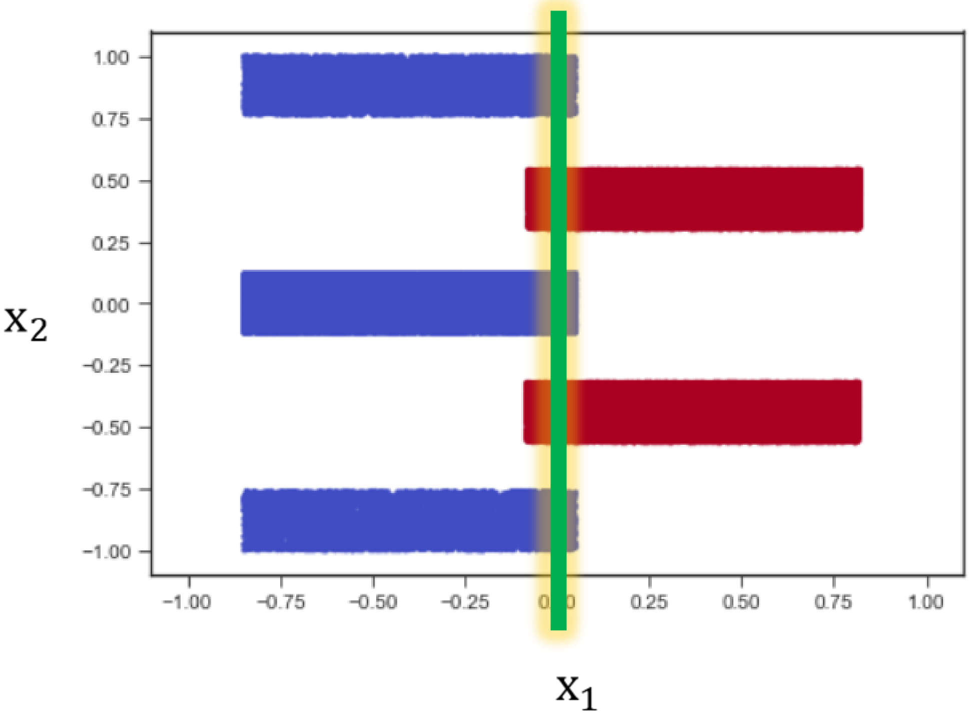

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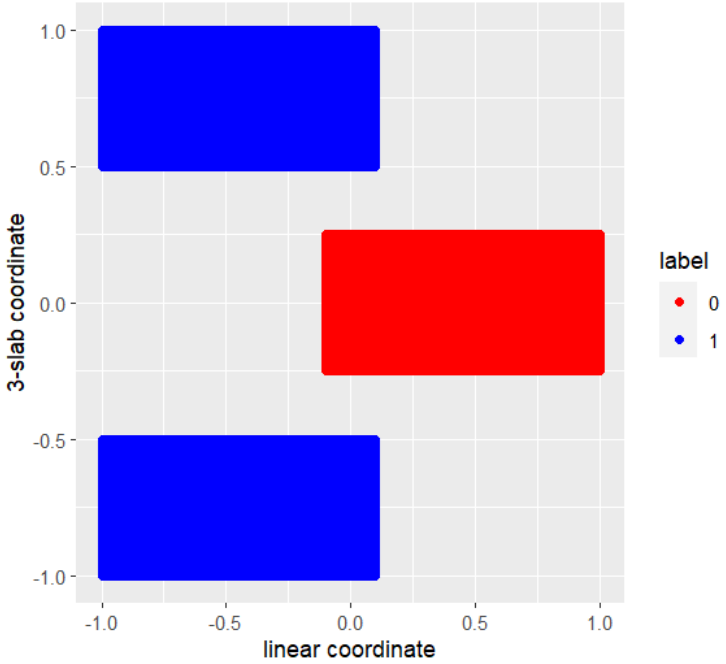

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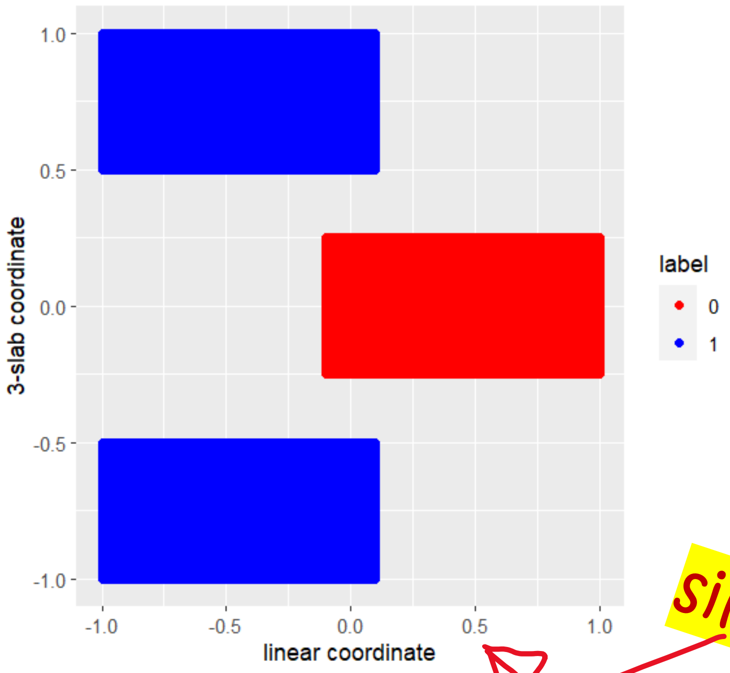

simple

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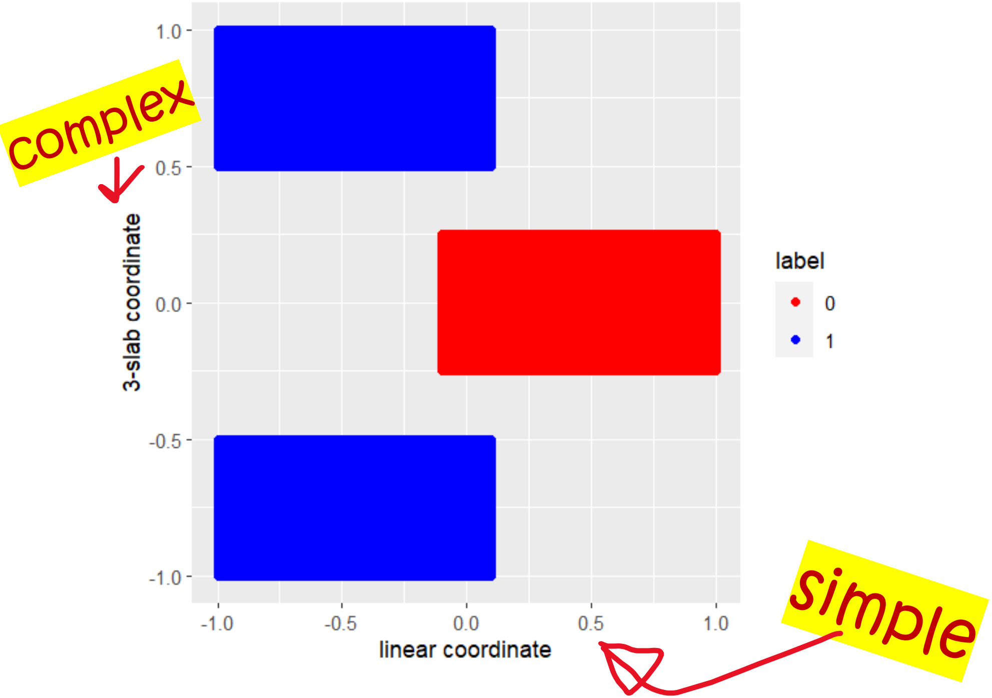

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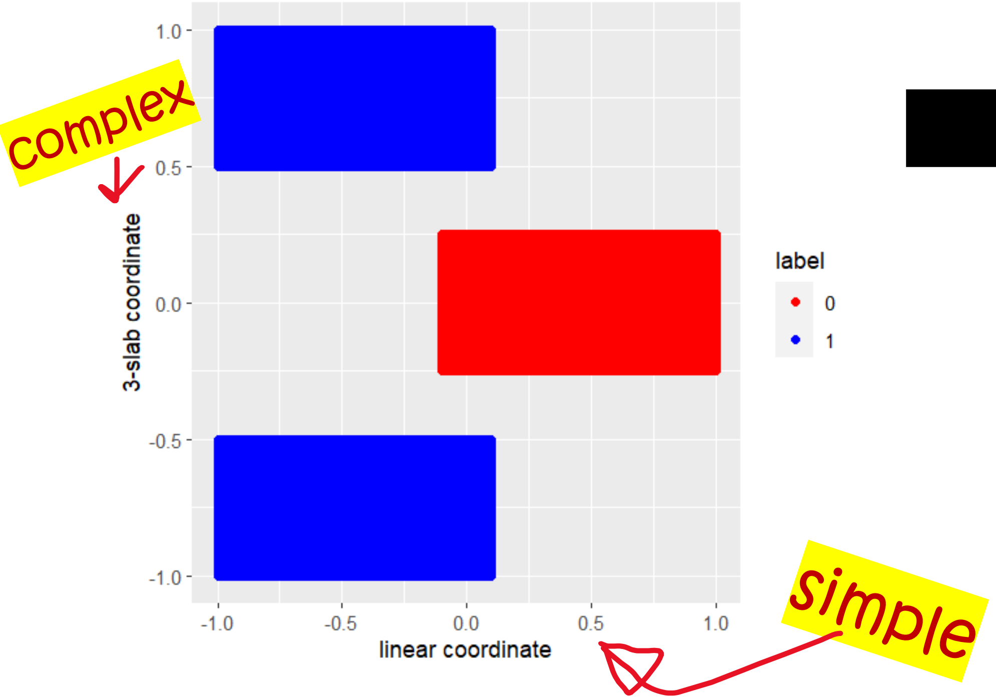

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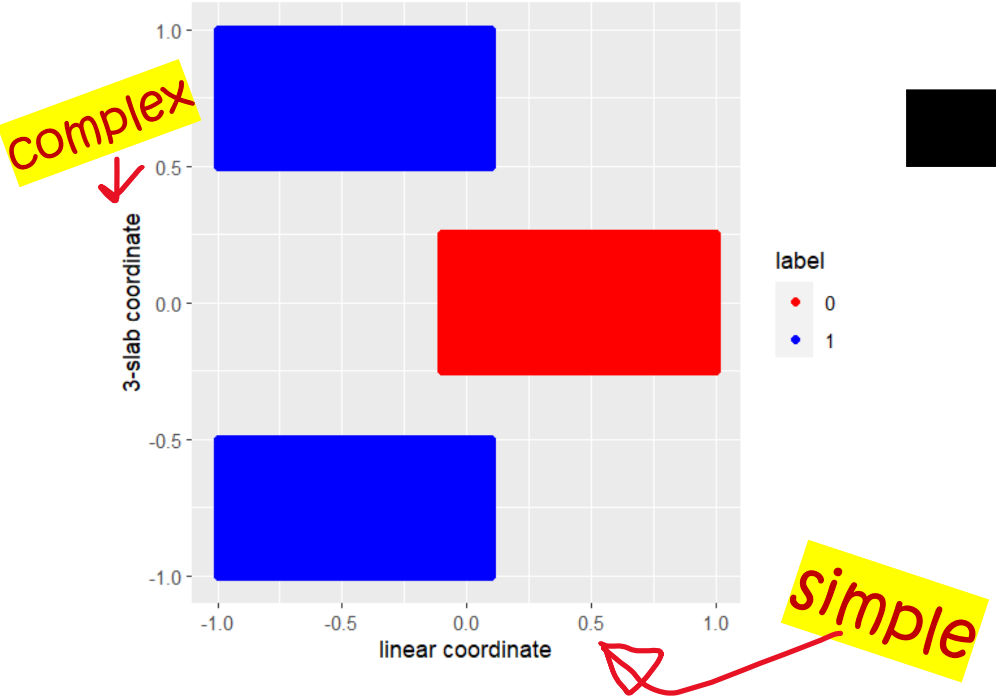
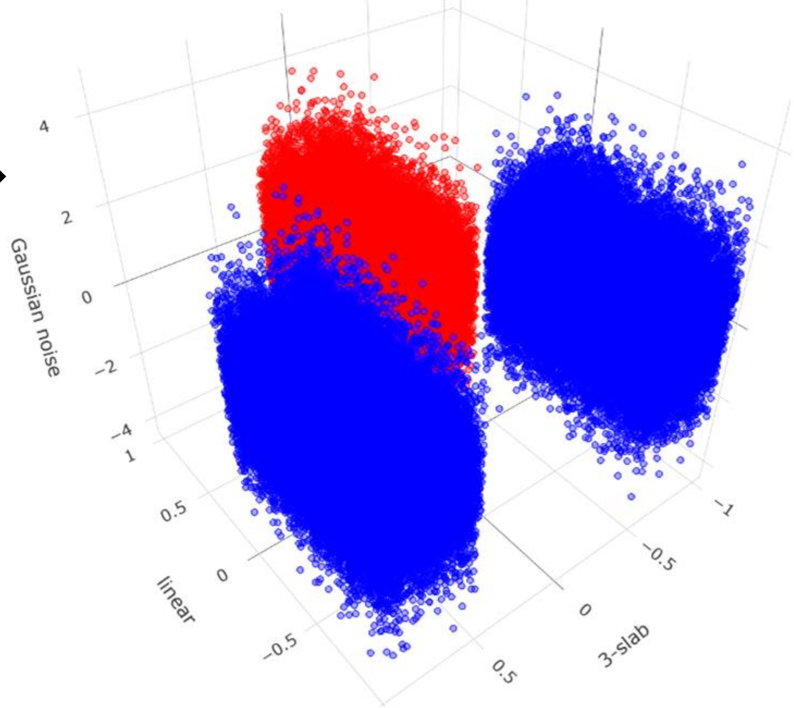


LSN: Gaussian noise in remaining d-2 dimensions.



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Assumptions

- One hidden-layer NN, k hidden neurons
- ReLU activation
- Dimension $d > \Omega(\sqrt{k} \log k)$ (dimension is **big**)
- Hinge loss
- SGD
- LSN data
- $O(1)$ iterations: single pass over training data

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$\Longrightarrow$ the learned weights are:

$$\underbrace{|w_{1j}| = \frac{2}{\sqrt{k}}\left(1 - \frac{c}{\sqrt{\log d}}\right) + O\left(\frac{1}{\sqrt{dk}\log d}\right)}_{\textit{Linear Coordinate}}, \quad \underbrace{|w_{2,j}| = O\left(\frac{1}{\sqrt{dk}\log d}\right)}_{\textit{Slab Coordinate}}, \quad \underbrace{\|w_{3:d,j}\| = O\left(\frac{1}{\sqrt{k}\log d}\right)}_{d-2\ \textit{Noise Coordinates}}$$

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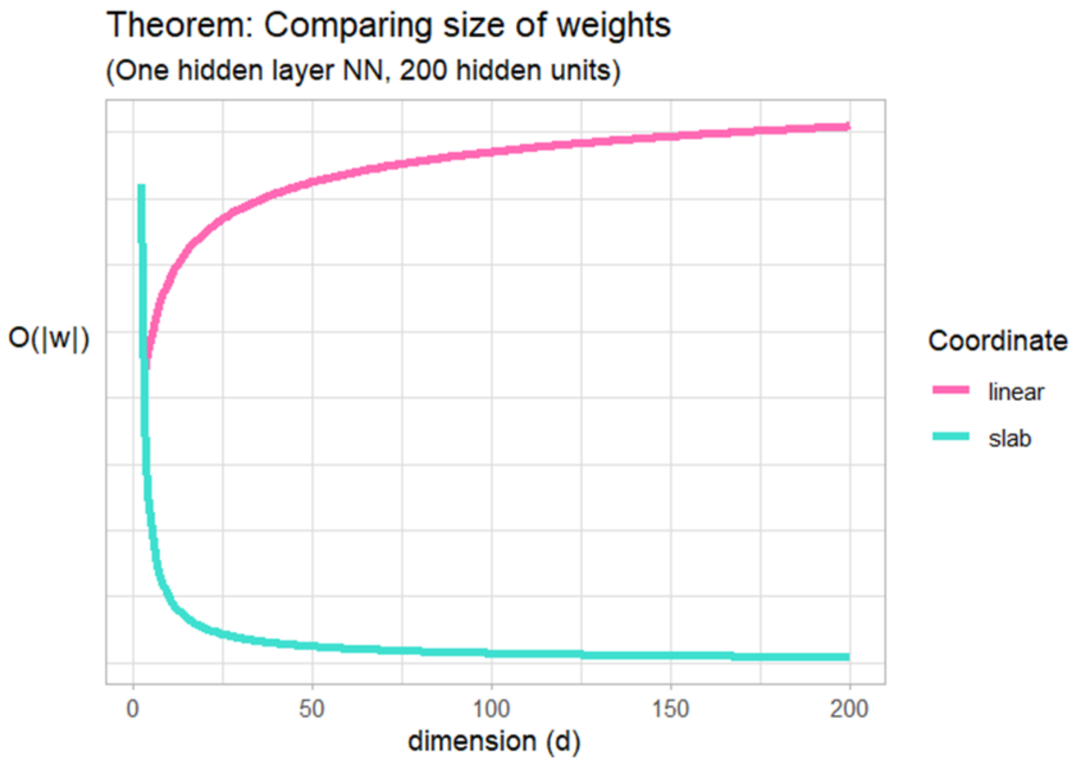

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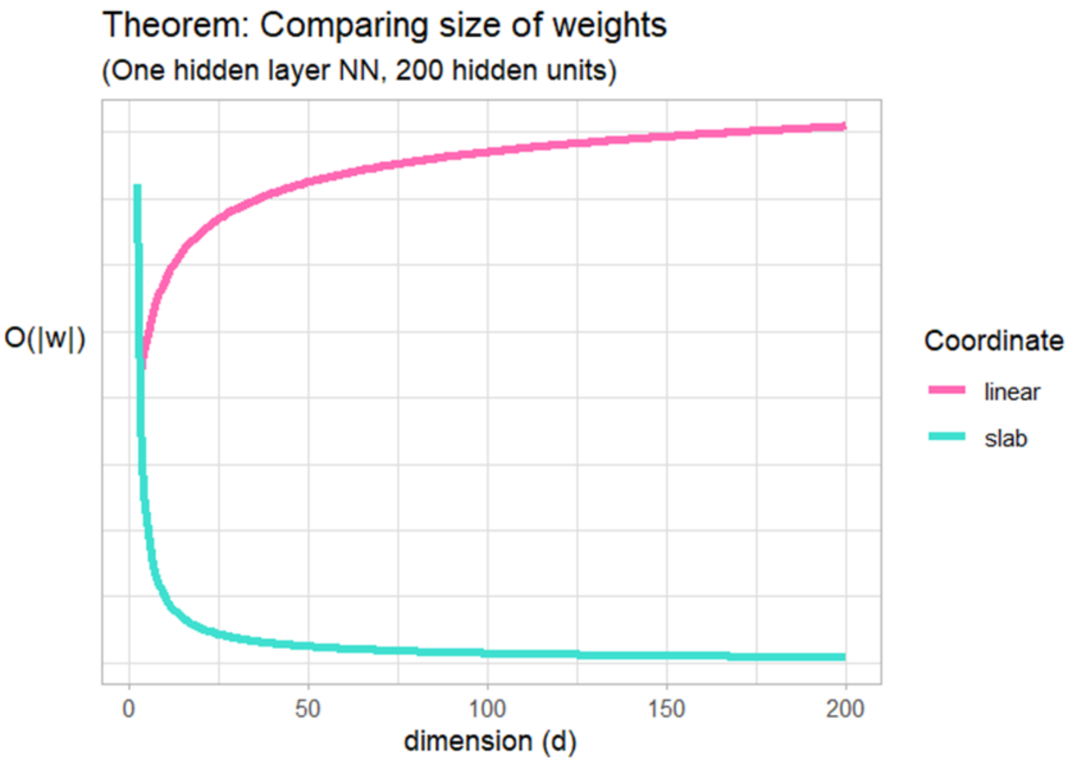

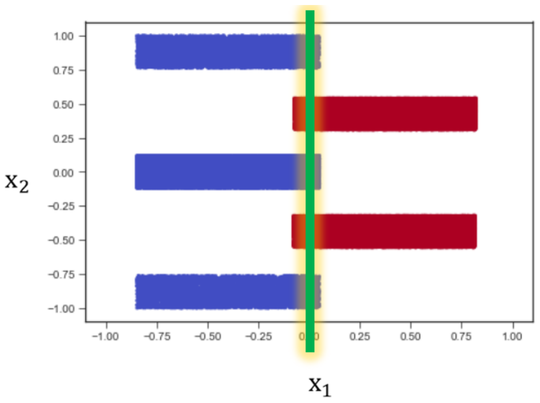

Dissection

❶ **Missing**: *No* experiments using LSN and real-world datasets

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- But, the results might not hold for real-world datasets.
  - Real-world data have highly **correlated** features and noise.[1]
  - But datasets in the paper have many **independent** feature coordinates.
- Their theorem uses LSN, but their experiments do not.
  - Paper leaves out certain failing cases.

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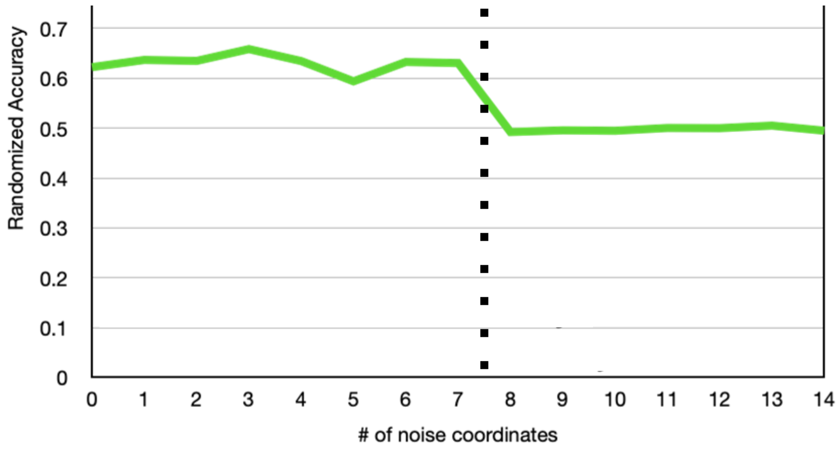

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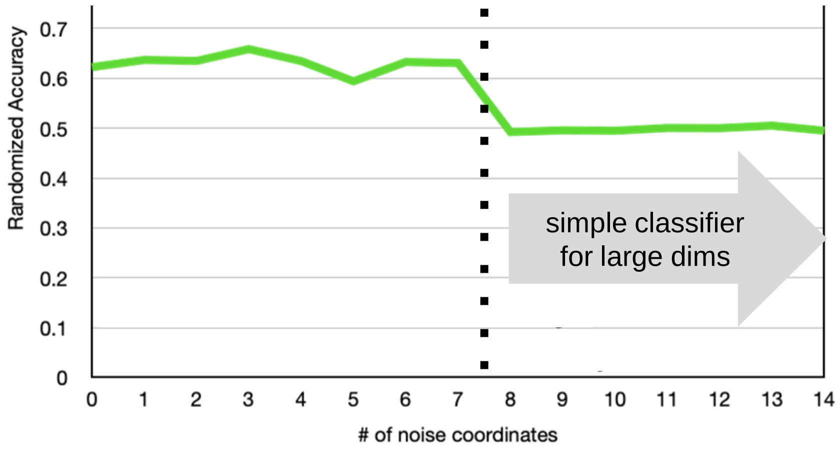

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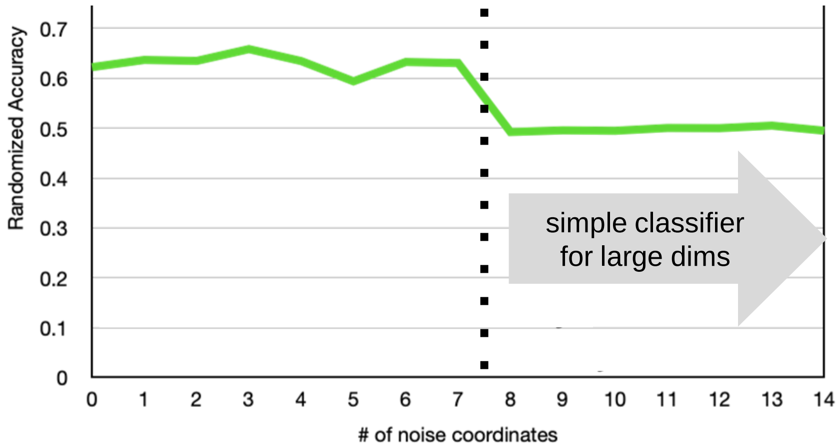

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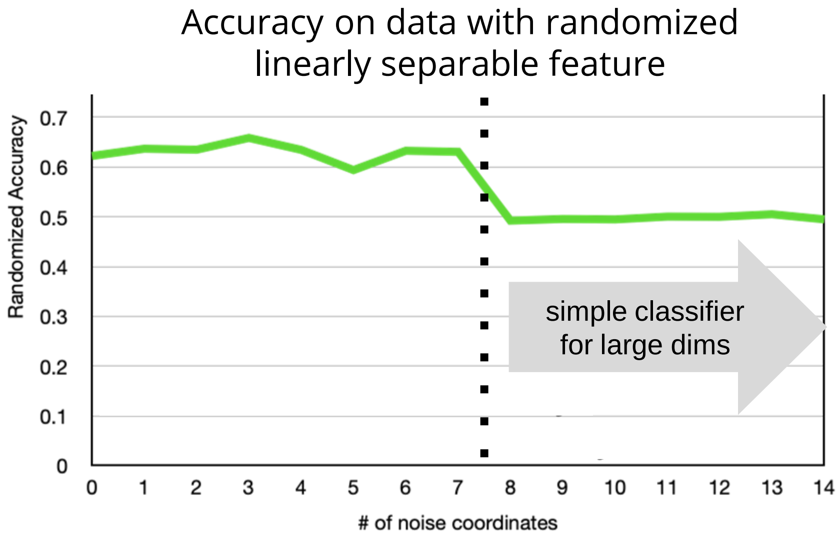

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Going forward… ➡ Explore theorems explaining extreme SB for smaller dimensions.

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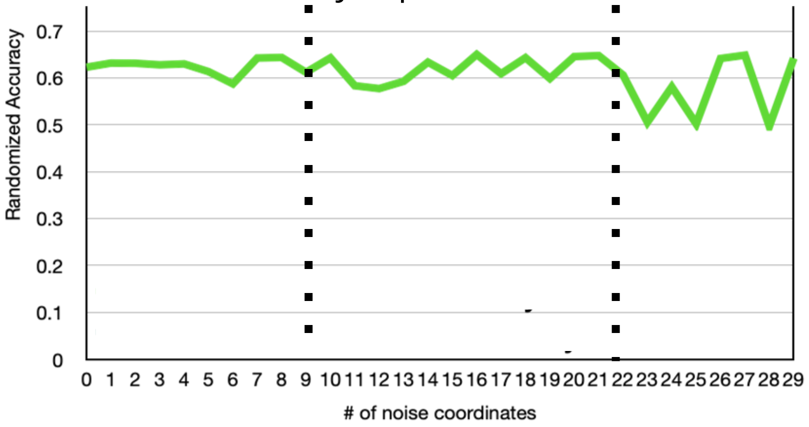

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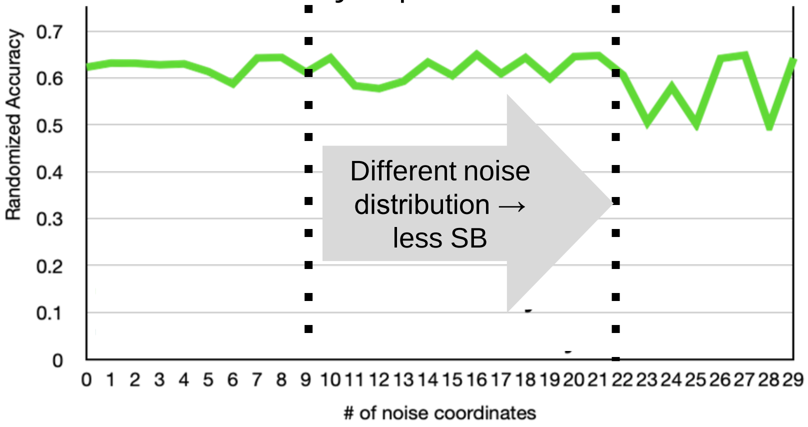

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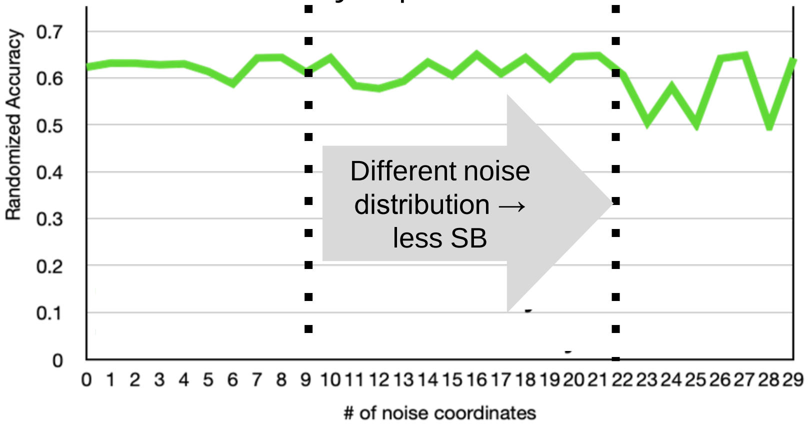

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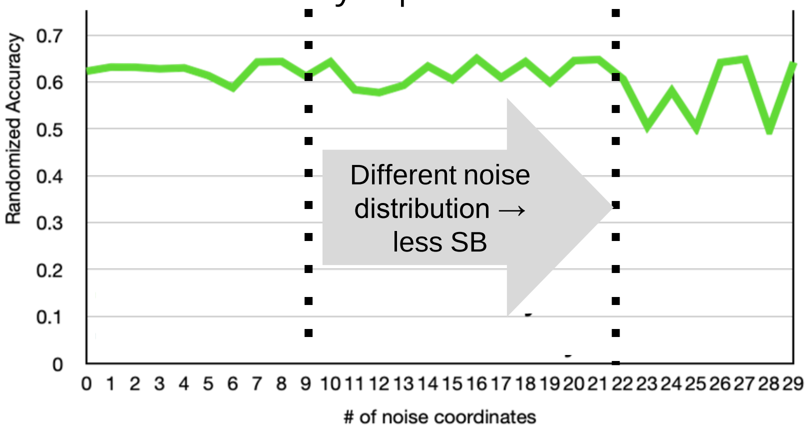

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- ➡ Explore effect of real-world noise distribution on extreme SB.
- ➡ Explore Gaussian noise removal techniques (like smoothening).

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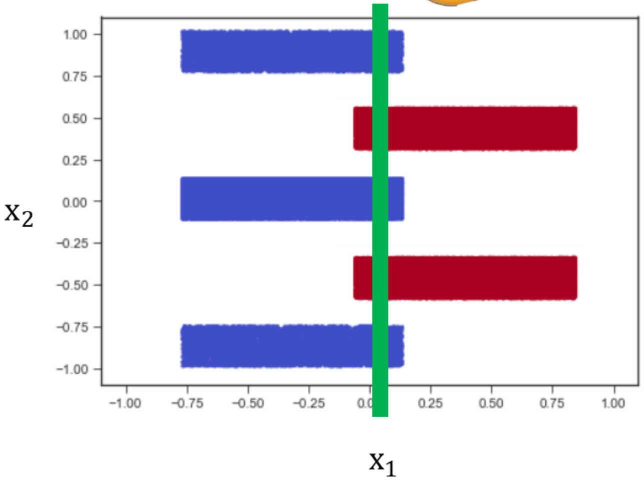

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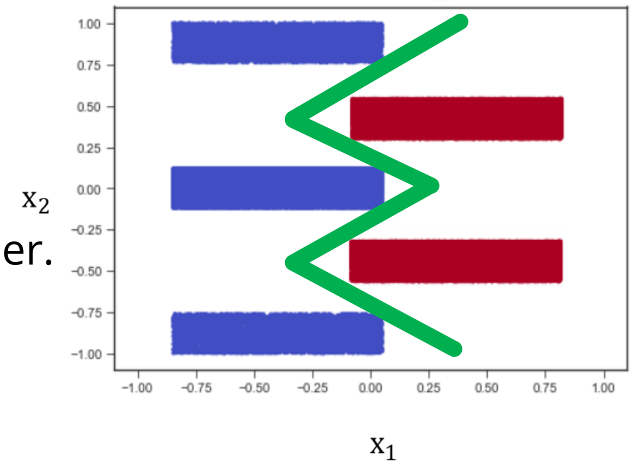

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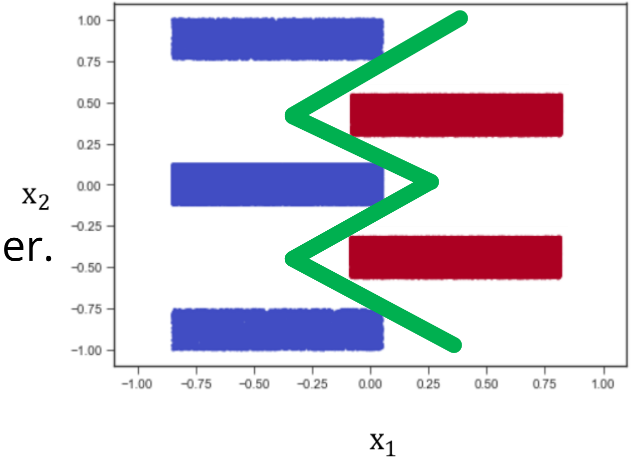

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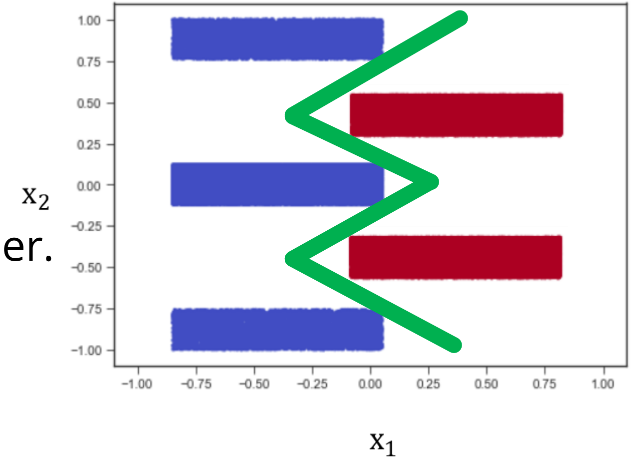

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Going forward…

➡ Analyze what happens when NN is trained until convergence.

➡ Prove validity of theorem to subsequent stages of training.

Summary

	Critique	How and why this might happen	Future work and how to pursue this
1	They don't experiment with LSN data	✶ Mysterious	➡ Do experiment with LSN data
2	Result requires Gaussian noise	✶ High variance of noise ✶ GD fails to find optimal path	➡ Explore real-world noise distribution on extreme SB ➡ Gaussian noise removal techniques
3	Result requires large dimension	✶ Small model used in paper ✶ Toy dataset had uncorrelated noise	➡ Theorems explaining extreme SB for smaller dimensions
4	They assume a small NN	✶ "Complex" features become simpler to classify in deeper layers	➡ Explore theorems explaining extreme SB for larger NNs
5	Theorem assumed a single epoch	✶ Updates to certain feature weights can take precedence after several epochs	➡ Further training of NN ➡ Prove validity of theorem for more epochs