Modeling CaSSIS lens distortions

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Distortion/Undistortion Models

Lets denote distorted image coordinate as $\mathbf{d} = (x_d, y_d)$ and undistorted image coordinate $\mathbf{u} = (x, y)$. There are several models that relate these coordinate, such as

Brown-Conrandy [1]

$$\begin{aligned} x_d &= x + \delta x_r + \delta x_t \\ y_d &= y + \delta y_r + \delta y_t \end{aligned} \tag{1}$$

This model has two components. The first one is symmetric, radial components δx_r:

$$\begin{aligned} \delta x_r &= x * (k_1 * r^2 + k_2 * r^4 + k_3 * r^6) \\ \delta y_r &= y * (k_1 * r^2 + k_2 * r^4 + k_3 * r^6) \end{aligned} \tag{2}$$

The second is tangential, decentering component δx_t:

$$\begin{aligned} \delta x_t &= p_1 * (r^2 + 2 * x^2) + 2 * p_2 * x * y \\ \delta y_t &= p_2 * (r^2 + 2 * y^2) + 2 * p_1 * x * y, \end{aligned} \tag{3}$$

where $r = \sqrt{(x^2 + y^2)}$ is distance from undistorted point to the center of distortions (distortion center coincides with origin). We fit the model and distortion center position (c_x, c_y) using nonlinear LSE formulation and direct Nelder-Mead method (fminsearch in Matlab).

MARCI [2]

$$\begin{aligned} x &= x_d * r / r_d \\ y &= y_d * r / r_d \end{aligned} \tag{4}$$

$$r = c_0 + c_1 * r_d^2 + c_2 * r_d^4 + c_3 * r_d^2, \tag{5}$$

where $r_d = \sqrt{(x_d^2 + y_d^2)}$ is distance from distorted point to the center of distortion (distortion center coincides with origin). We fit the model and distortion center position (c_x, c_y) using nonlinear LSE formulation and direct Nelder-Mead method (fminsearch in Matlab).

Rational Function (RF) [3]

Lets denote $\chi_6(x_d, y_d)$ as "lifting" of image coordinates to 6D space:

$$\chi_6(x_d, y_d) = \left[x_d^2 \; x_d y_d \; y_d^2 \; x_d \; y_d \; 1\right]^T \tag{6}$$

In this case, undistorted coordinates is computed as

$$(x, y) = \left(\frac{A_1^T \chi_6(x_d, y_d)}{A_3^T \chi_6(x_d, y_d)}, \frac{A_2^T \chi_6(x_d, y_d)}{A_3^T \chi_6(x_d, y_d)}\right) \tag{7}$$

where $A_{1\ldots3}^T$ are rows of 6x3 matrix A. We fit the model using linear LSE formulation and SVD trick [4] (svd in Matlab).

Bi-cubic [5]

Lets denote $\chi_{10}(x_d, y_d)$ as "lifting" of image coordinates to 10D space:

$$\chi_{10}(x_d, y_d) = \left[x_d^3 \; x_d^2 y_d \; x_d y_d^2 \; y_d^3 \; x_d^2 \; x_d y_d \; y_d^2 \; x_d \; y_d \; 1\right]^T \tag{8}$$

In this case, undistorted coordinates is computed as

$$\begin{bmatrix} x \\ y \end{bmatrix} = A * \chi_{10}(x_d, y_d), \tag{9}$$

where A is 10x2 matrix. We fit the model using linear LSE formulation and normal equation (mvregress in Matlab).

CAHVOR [6]

This model was shown to be equivalent to the model [6]

$$\begin{aligned} x &= x_d + x_d \delta r / r_d \\ y &= y_d + y_d \delta r / r_d \end{aligned} \tag{10}$$

$$\delta r = (k_0 * r_d + k_1 * r_d^3 + k_2 * r_d^5), \tag{11}$$

where $r_d = \sqrt{(x_d^2 + y_d^2)}$ is distance form distorted point to the distortion center (distortion center coincides with origin). We fit the model and distortion center position (c_x, c_y) using nonlinear LSE formulation and direct Nelder-Mead method (fminsearch in Matlab).

All above models can be divided into two classes:

- Undistortion model $\mathbf{u} = f_u(\mathbf{d})$ (MARCI, RF, Bi-cubic, CAHVOR)
- Distortion model $\mathbf{d} = f_d(\mathbf{u})$ (Brown-Conrady)

Often these functions can not be analytically inverted. To get undistorted image we should probably use distortion model.

Result and discussions

We fitted all above models to CaSSIS data and compared goodness of each model using leave-one-out cross validation. The table below shows mean square error (MSE) of each model. The figures below present distortion/undistortion fields produced by each model.

Table: Models cross validation.

Method	MSE (pix)	No of parameters
Brown-Conrandy	0.4873	7
MARCI	1e5	6
Rational Function	0.0024	18
Bi-cubic	0.00018	20
CAHVOR	10.82	5

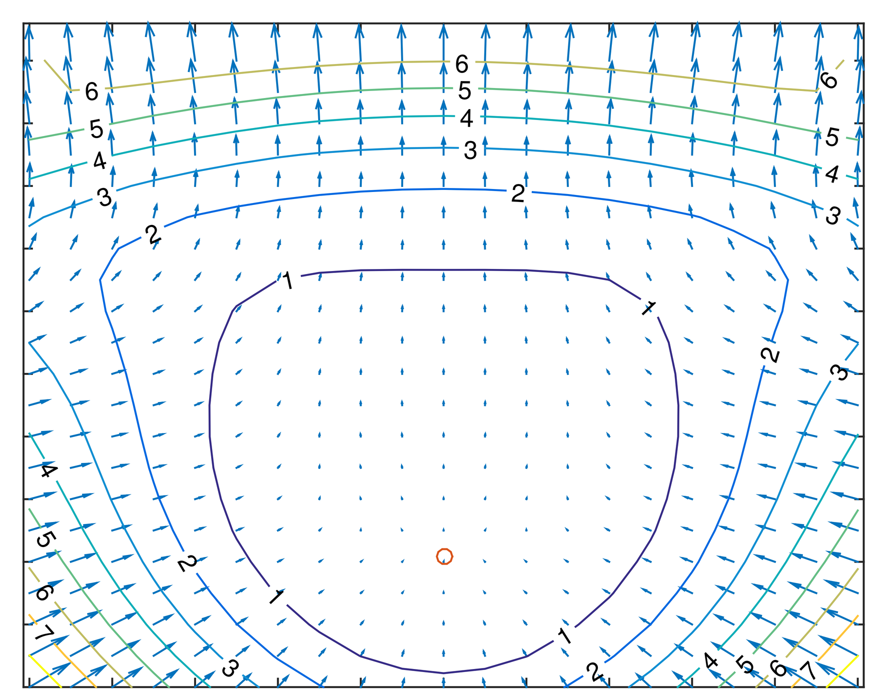

Figure: Brown-Conrady Distortion Field

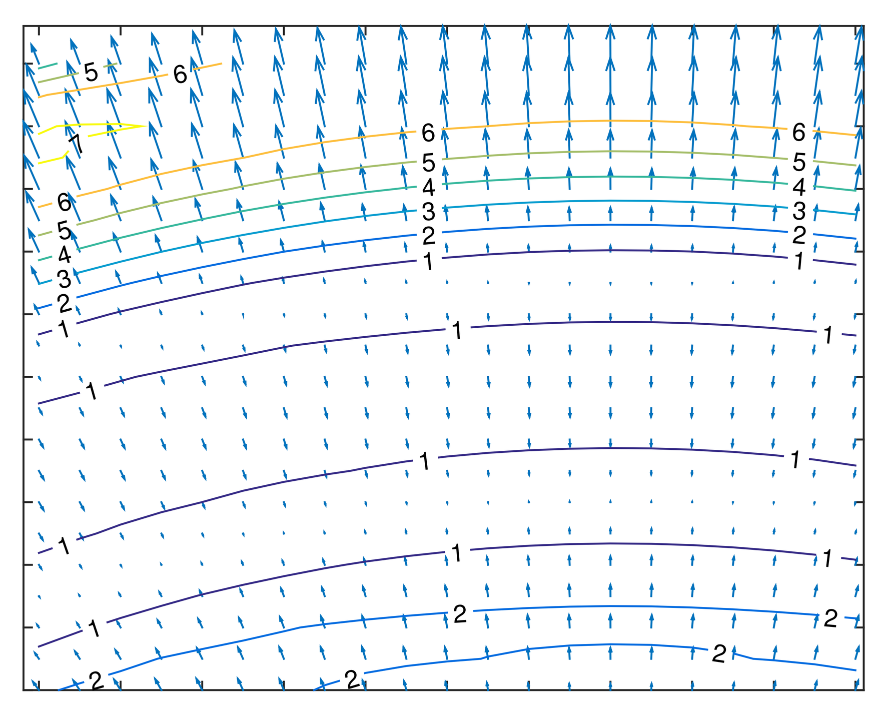

Figure: MARCI Undistortion Field (FAILED)

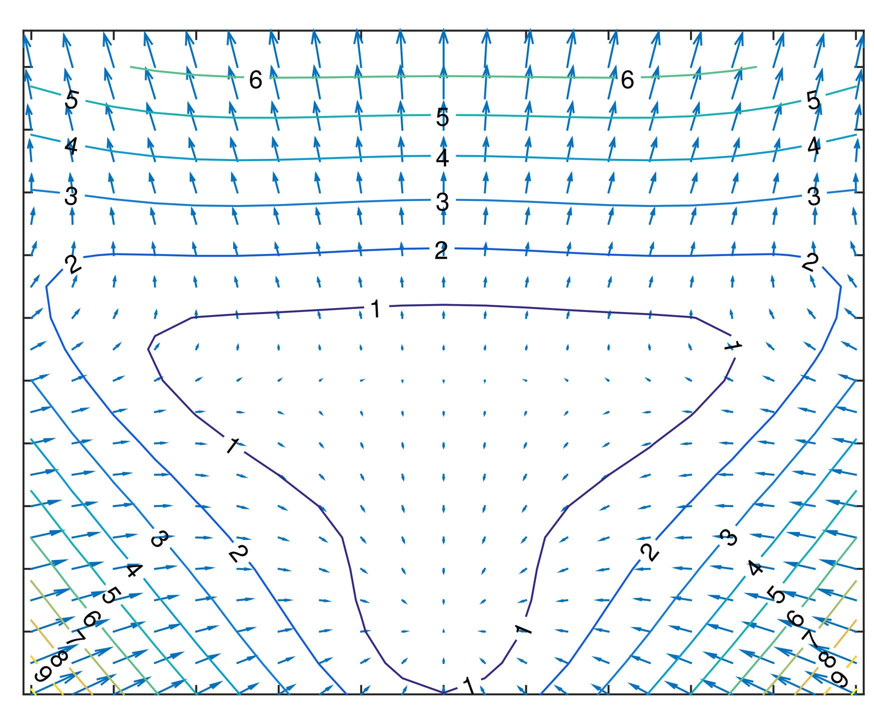

Figure: Rational Function Undistortion Field

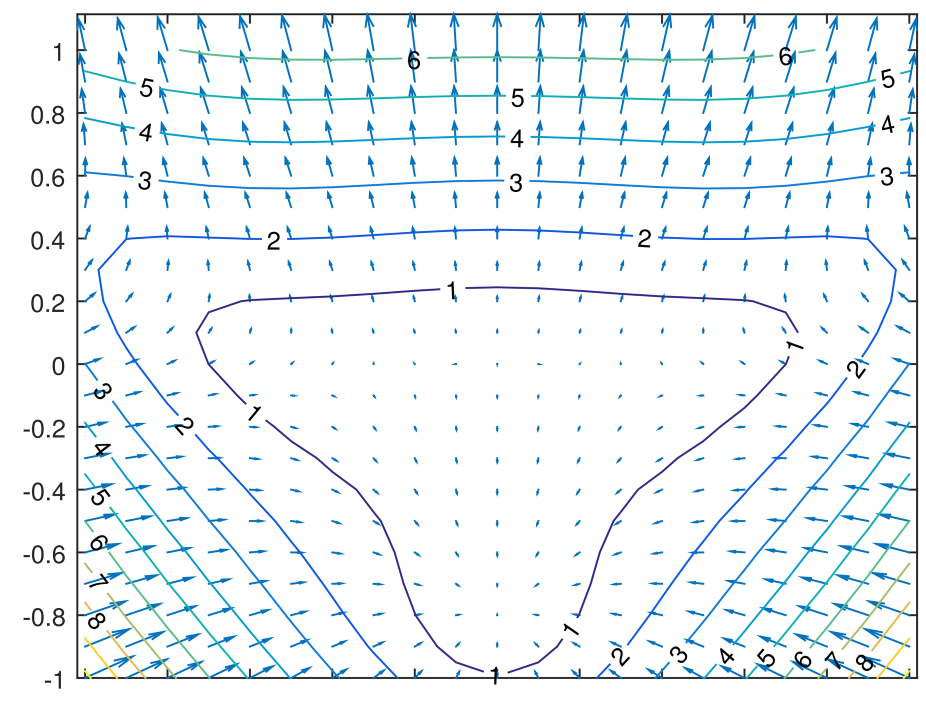

Figure: Bicubic Undistortion Field

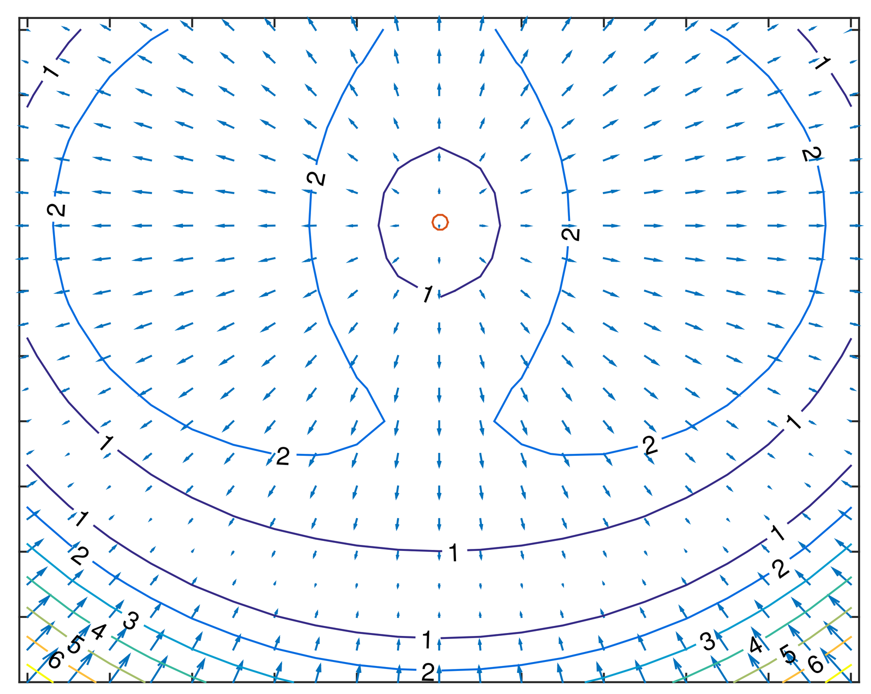

Figure: CAHVOR Undistortion Field (FAILED)

Summary and conclusions

- The MARCI and the CAHVOR models fail to fit CaSSIS distortions. I think, it is because the models are symmetric.
- The Brown-Conrandy model perform fairly well, but it seems it is not flexible enough. Maybe, the model can show better results with more efficient optimization method.
- The Rational Function and (surprisingly) the Bicubic models fit CaSSIS very well. Moreover, efficient optimization methods can be used to fit these models. However, more data point is required to insure that the models are not over-fitted.

References

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[3] D. Claus and A. Fitzgibbon, "A rational function lens distortion model for general cameras," in *IEEE Computer Society Conference on Computer Vision and Pattern Recognition, 2005. CVPR 2005*, vol. 1, pp. 213–219 vol. 1, June 2005.

[4] K. Inkilä, "Homogeneous least-squares problem," *The Photogrammetric Journal of Finland*, vol. 19, no. 2, pp. 34–42, 2005.

[5] E. Kilpelä, "Compensation of systematic errors of image and model coordinates," *Photogrammetria*, vol. 37, pp. 15–44, Nov. 1981.

[6] K. Di and R. Li, "CAHVOR camera model and its photogrammetric conversion for planetary applications," *J. Geophys. Res.*, vol. 109, p. E04004, Apr. 2004.