Project Report

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Introduction

For our project, a manager is interested in purchasing a new cooling system for a reactor if it can meet the reliability requirement. The system information is as follows: A cooling system for a reactor has five identical cooling loops. Each cooling loop has two identical pumps connected in parallel. The cooling system requires that at least 3 of the 5 cooling loops operate successfully. A two-component parallel system is defined as if both components are working then the system is working. If either component 1 or 2 fails, the system is still working. The system will fail if and only if both components fail. We were then given 24 values representing the lifetimes (in years) of the pump based on a sample of historical data from the manufacturer:

15.4	23.6	23.7	18.5	13.6	18.7	14.7	24.3	18.2	13.2	19.6	22.5
17.8	12.3	15.8	16.7	15.3	19.7	13.9	21.6	16.3	19.8	13.8	15.8

Objectives

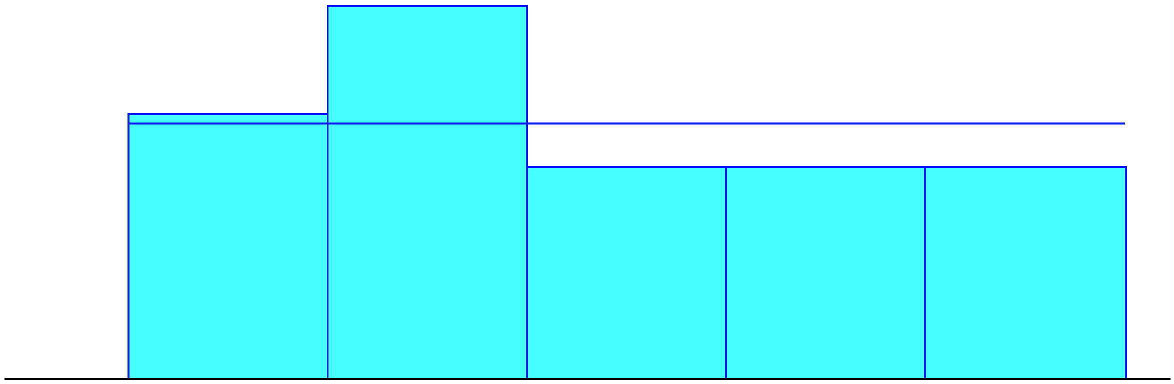
Our objectives for this project were to very carefully study and solve the following problems consisting of two parts based on the mean squared error, Akai information criterion (AIC), Bayesian information criterion (BIC), and Pham criterion.

Problem Statements

The problem stated a manager is interested in purchasing a new cooling system for a reactor if it can meet the reliability requirement. We had to compute the reliability of the cooling system for a mission of 5 years and obtain the expected lifetime of the cooling system through both Arena simulation and analytical methods.

Data Collection

Using input analyzer in Arena, we were able to fit the model with the different distributions and use the fit all function. In the arena, the highest value of time is days. Therefore, we had to convert the data from years into days.The data file also had to be converted into a .txt file for it to work in the input analyzer. Then, with the fit all function, the input analyzer will fit all the distributions against the data and find the most optimal distribution to use for our data. Below is the most optimal distribution that the input analyzer found for the data.



```
        Distribution Summary

Distribution:   Uniform
Expression:     UNIF(4.49e+003, 8.87e+003)
Square Error:   0.011806
```

Using matlab we were able to obtain the data of the log likelihoods and the aic/bic criterion using the functions that they have built into matlab. In the image below are the values that matlab printed out. The first values are the likelihoods, the second is the aic, and the third column of values is the bic.

```
Lbeta          >> code_for_log_likelihood
 -196.2613     Lbeta aic/bic:
                 396.5227   398.8788
Lerlang
 -208.5027     Lerlang aic/bic:
                 421.0053   423.3614
Lexponential
 -260.7655     Lexponential aic/bic:
                 523.5311   524.7091
Lgamma
 -204.4274     Lgamma aic/bic:
                 412.8548   415.2109
Llogn
 -476.6907     Llogn aic/bic:
                 957.3814   959.7376
Lnormal
 -205.6630     Lnormal aic/bic:
                 415.3261   417.6822
Ltriangular    Ltriangular aic/bic:
 -214.2296       434.4592   437.9933

Luniform       Luniform aic/bic:
 -201.2900       406.5801   408.9362

LWeibull       Lweibull aic/bic:
 -269.9503       543.9006   546.2567
```

Model Discussion

Arena Models:

In our model, we represented the system using five pairs (loops) of two pumps. Each pump is represented by a create module (with uniform distribution from input analyzer). Each create module went to a decide module, this decide determines if TNOW=0 and discarded the first failure that Arena automatically creates. If this decide module was false, then it proceeds into a record, and then into another decide. This decide checked if this was the first failure, if not, go into the match module. Each of the 10 pumps had this structure.

The five matches each (for each loop) connect to a record module, which counts the number of failures. If the number of failures was equal to 6 pumps, then it proceeds to a record module to calculate TNOW and the last connection is to a dispose which ends the model. We also created a terminating condition to determine if 6/8 pumps failed so the model does not run infinitely.

In part 1, set the replication length to 1825 days (5 years) and found that the number of times the system failed. In part 2, we set the model to run infinitely, and we determined the expected lifetime of the cooling system for each of the 25 replications.

Analytical Models:

For finding the reliability of the cooling system in five years we first had to convert years to days to match our data. Then using the cdf of our distribution plug in the time and subtract that value from one to get the reliability of one pump. The next step is using the equations covered in class for parallel systems to find the reliability of one loop. Final use k out of n to solve for the reliability of the system for 3 loops out of 5.

Uniform Pdf = 1/b-a ---> cdf = (x-a/b-a)   5 yrs==1825days   min=4490days max=8870days

Since x=1825 and x<a cdf value is equal to 0

$$R_{pump} = 1 - 0 = 1$$

$$R_{loop} = 1 - (1 - 1) * (1 - 1) = 1 - 0 * 0 = 1$$

$$R_{sys} = 6 * 1^5 - 15 * 1^4 + 10 * 1^3 = 16 - 15 = 1$$

$$MTTF = (a + b)/2 = (8870 + 4490)/2 = 6680\text{days or 18.3 years}$$

For the Mean time to Failure, we normally have to integrate to infinity, but we can’t since we have a uniform distribution. Therefore, we used the mean formula to get the expected life.

Methodologies

For the parallel system loops, the methodology we use to create the arena model was creating one loop out of the whole system and then copying it five times since the system is composed of five identical loops. In each loop, there are two identical pumps in parallel where both pumps have to file another for that loop to fail. In an arena, the first random value will also be a failure therefore we have to create a decision model that will throw away the first value which is the failure. In the arena it also produces an abundance of failure when we only care about one so another decided module is created to throw away the excess failures. Then all five loops are connected into a match, record, and decide module that will determine if the system has failed. The random generated values that are formed by the loops are generated from the uniform distribution that we obtained from the input analyzer.

To solve for the system analytically, we used the equations for a uniform cdf distribution to find the reliability of 1 pump which is 1-(x/b-a) where x is the value of 5 years (1825 days), b is the maximum value of the data, and a is the minimum value of the data. Then to find the reliability of the loop it would be 1-(1-Rpump)^2 because the system is in parallel. Then the equation we used to find the reliability of the whole system would be $R_{sys} = 6 * 1^5 - 15 * 1^4 + 10 * 1^3$ because for the system to work it needs 3 out of 5 to function. In a uniform distribution, the expected lifetime is found by taking the average of the maximum and minimum value.

To find the aic and bic we can use the matlab functions to find the values, the aic/bic function first needs the value of the log likelihood, numparam, and numobs. To get the log likelihoods, we plug in the function and input the distribution and data. Then to get the aic/bic, you input the log likelihood, number of parameters for each distribution, and the number of observations in the data set. Then the n and k values can be found analytically using the aic/bic formulas and the Pahm criterion can be found analytically. MSE can be obtained by dividing the square error by n-k.

Modeling Analysis

To determine which distribution is the best for our reliability project, we tested the data against different distributions for mean square error(MSE), Akai information criterion(AIC), Bayesian information criterion(BIC), and Pham criterion. Using the program matlab, we were able to get the log of the likelihoods and calculate the aic/bic using the matlab function which resulted in the values below.

Fit	AIC=-2log(L)+2k	BIC=-2log(L)+kLog(n)	PC=(n-k/2)log(SSE/n)+k(n-1/ n-k)	MSE=SSE/(n-k)
Beta	396.5227	398.8788	-3816.922848	0.000008404454261

Erlang	421.0053	423.3614	-3415.725727	0.00002873379969
Exponential	523.5311	524.7091	-3417.803659	0.00002872387547
Gamma	412.8548	415.2109	-3492.222156	0.00002272025674
Lognormal	957.3814	959.7376	-3134.095792	0.00006805789344
Normal	415.3261	417.6822	-3589.568911	0.00001685410091
Triangular	434.4592	437.9933	-3476.445331	0.00002360940607
Uniform	406.5801	408.9362	-3839.380471	0.000007846195398
Weibull	543.9006	546.2567	-3652.210561	0.00001392428967

The Pham criterion cannot be found using matlab and has to be found analytically using the equation above. We find the Pham criterion by finding n and k from the aic and bic. For the criterions the most optimal distribution will be the smallest value. From the table above we can see that for the aic/bic criterion, the most optimal distribution to be used is the beta model and for the MSE and PC, the most optimal distribution to use is the uniform distribution. The optimal distribution for each criterion is highlighted in yellow. Therefore, the aic/bic shows that we should use the beta distribution instead of the uniform distribution that was found using an input analyzer and the PC and MSE shows that we should use uniform,the distribution that input analyzer gave us. In appendix 1, we can find the table that shows how the value n and k are found from the aic and bic equations respectively.

Results

In part 1, we found that the number of times the system failed in 25 replications was 0 for 5 years. Thus, 0/25 means 0% probability of failure or 100% reliability for the system in 5 years. For the analytical model, it was also 100% reliable for 5 years, which is the same result as the simulation model.

In part 2, we set an infinite replication length for 25 replications. We found that the expected lifetime of the cooling system for 25 replications was 7,521.10 days on average through Arena. The analytical model had the expected lifetime as 6,680 days.

Method	Reliability of Cooling System for a Mission of 5 Years	Expected Lifetime of Cooling System
Arena	100%	7,521.10 days (20.606 years)
Analytical	100%	6,680 days (18.301 years)

Conclusions and Findings

The results for part 1 in the simulation and analytical model are the same value of 100% reliability for 5 years. This makes sense because based on the given lifetimes the lowest lifespan of a pump is 12 years so for the pump and the system to fail in five years is relatively low to impossible. Additionally, we have a uniform distribution so the CDF states that the probability of getting a value less than the minimum range in 0.

The expected life of the system is normally calculated by the MTTF which is integrated from 0 to infinity but since we have bounds from our distribution and cannot integrate to infinity, we use the expected value equation to calculate the mean lifetime of the system is approximately 6680 days which is different then the 7,521 days calculated by Arena. The difference between the two could be caused by the randomness in the model and that each time it was run it was independent of the previous.

Appendix

1. Work shown to solve Pham's Criterion

https://docs.google.com/spreadsheets/d/1kpUDPDss76jLu7YEbRQWzGfoHqavQyYJsB9-eLBChMA/edit?usp=sharing

Fit	**SSE**	**logL**	**k=(AIC+2log(L))/2**	**n=10^((BIC+2log(L))/k)**
Beta	0.012646	-196.2613	2.00005	1506.678356
Erlang	0.043241	-208.5027	1.99995	1506.882718
Exponential	0.043241	-260.7655	1.00005	1506.402761
Gamma	0.034189	-204.4274	2	1506.780531
Lognormal	0.102424	-476.6907	**2**	1506.954015
Normal	0.02536	-205.663	2.00005	1506.678356
Triangular	0.035502	-214.2296	3	1506.722707
Uniform	0.011806	-201.29	2.00005	1506.678356
Weibull	0.020953	-269.9503	2	1506.780531

2. Analytical Model Work

$$CDF = \begin{cases} 0 & x<a \\ \frac{x-a}{b-a} & a<x<b \\ 1 & x>b \end{cases}$$

$x<a$

$1825 < 4{,}490$

$R_{pump} = 1-0 = 1$

$R_{loop} = 1-(1-1)(1-1) = 1-(0.0) = 1$

$R_{sys} = \binom{5}{3}[R(t)]^i[1-R(t)]^{n-i}$

$$= \binom{5}{3}R^3(t)[1-R(t)]^2 + \binom{5}{4}R^4(t)[1-R(t)] + R^5(t)$$
$$= 10R^3(t)[R^2(t)-2R(t)+1] + 5R^4(t)[1-R(t)] + R^5(t)$$
$$= 10R^5(t) - 20R^4(t) + 10R^3(t) + 5R^4(t) - 5R^5(t) + R^5(t)$$
$$= 6R^5(t) - 15R^4(t) + 10R^3(t)$$
$$= 6(1^5) - 15(1^4) + 10(1^3) = 16-15 = 1$$

System Reliability for 5 yrs = 1

$$E[X] = \frac{a+b}{2} = \frac{8{,}870+4{,}490}{2} = 6{,}680 \text{ days}$$
$$\approx 18.3 \text{ yrs}$$