Project Report

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Introduction

Arrivals

- Customer arrivals follow Poisson distributions with the rates given in Table 1.

Table 1(a): Arrival rate of customers per hour during the weekdays

Time	Arrival rate (λ) per hour
06:00 am ~ 07:00 am	15
07:00 am ~ 09:00 am	22
09:00 am ~ 01:00 pm	25
01:00 pm ~ 06:00 pm	30
06:00 pm ~ 08:00 pm	24
08:00 pm ~ 09:00 pm	16

Table 1(b): Arrival rate of customers per hour during the weekend (Saturday & Sunday)

Time	Arrival rate (λ) per hour
06:00 am ~ 07:00 am	12
07:00 am ~ 09:00 am	22
09:00 am ~ 01:00 pm	32
01:00 pm ~ 06:00 pm	35
06:00 pm ~ 08:00 pm	24
08:00 pm ~ 09:00 pm	10

Resources

- There are 4 gas pump machines in the gas station. Both sides of a machine are available to provide service to a customer. The two machines are serially placed.

- Total number of workers at this station is 12. Each works 9 hours a day, and all have a working capability of 1. 3 workers are responsible for one machine. Three work schedules in a team are shown in Table 2.

Table 2: Work schedules for a team

	Work Schedule 1	Work Schedule 2	Work Schedule 3
	6 am ~ 11 am	8 am ~ 1 pm	11 am ~ 2 pm
Break time	--	--	--
	12 pm ~ 4 pm	2 pm ~ 6 pm	3 pm ~ 9 pm

Processing Times

- Taking an order: Triangular distribution with minimum 0.7, the most likely value 1.0 and maximum 1.4 minutes
- Fueling: *Gal* x 0.30 minutes, where *Gal* is the amount of gas that a customer orders as shown in Table 3.
- Paying: Gamma distribution with scale parameter of 1.2 and shape parameter of 1.3.

Queues

- With the probability of 0.80, a new arrival leaves the system without service if there are more than 16 waiting customers in the system.
    - First, once an arrival comes in the system, it goes to the “Entering” queue.

- Then, if any of four waiting lines which are for four service lines is available, the customer moves on the waiting line. Each of the waiting lines for a service line is capable to let 5 customers wait on the line
- When a customer gets released from "Entering" queue, the customer goes to a "Waiting line" of the corresponding service line according to the following rules:
  - The customer goes to the service line whose machine is available. If there are more than one of the service lines available, a customer occupies the place whose index number of the available service lines is the smallest.
  - The customer goes to the service line whose waiting line is shorter than others
  - If the numbers of customers in "Waiting Line" of some service lines are the same and the smallest, the customer goes to the service line whose index is smaller.

Constraints on Gas

- The max storage tank of the gas station is 25,000 gallons. If the storage tank < 500 gallons, the gas station does not receive any more customers and cars in the "Entering" queue leave without service, but cars in the "Waiting" queues do not leave the gas station and will get service. Until the storage tank is supplied, the gas station service is not resumed.
- The gas truck refills up to 25,000 at once. Refill trucks arrive according to a uniform distribution of UNIF(5,7) days.

Cost and Profit

- Consumption of gas by customers follows a discrete distribution in Table 3. Once an arrival is created, the amount of fueling gas is determined. The sale price for 1 gallon is $2.50.
- Gas supply costs: Fixed cost for an order = $300. Supply price for 1 gallon = $0.40.
- Hiring cost for a worker is $15 per hour.
- Operating cost for a machine is $200 per day.

Simulation Requirements

- A day consists of 15 hours (6am - 9pm).
- Run for 7 days.
- Time unit for the simulation is minutes.
- Schedule rule: "Ignore"
- Seize rule: "Random"
- Release rule: "Last seized member"

Problem Statements

In Part 1, there are four problem statements:

1. Build an Arena model to simulate the operation of the pump and estimate the daily profit.
2. Simulate the operation of the gas station for one week, and create an animated plot of the cumulative profit as a function of time.
3. Assume that a complaint from the customer (only cars in the "Entering" queue) who leaves the system without a service occurs, and the complaint is regarded as potential cost. If the cost is $10 per complaining customer, estimate the weekly profit.
4. Make a suggestion to improve the profit for the gas station, justify your suggestion, and show the modification and the corresponding result of simulation based on the suggestion.

In part 2, there are five problem statements. The average inter-arrival times (in seconds) of cars in the month of March 2019 of a gas station (same as the layout in Figure 1) for a period of 30 minutes during the weekdays and the weekend are given, respectively, as follows:

Weekday (in seconds):
215.2 112.4 154.7 89.2 179.3 145.4 91.2 210.4 76.5 62.3 65.1 140.3 89.4

Weekend (in seconds):
163.8 90.3 183.6 135.8 96.2 85.5 197.7 95.4 84.3 164.5 157.7 82.1
110.2 76.3 124.7

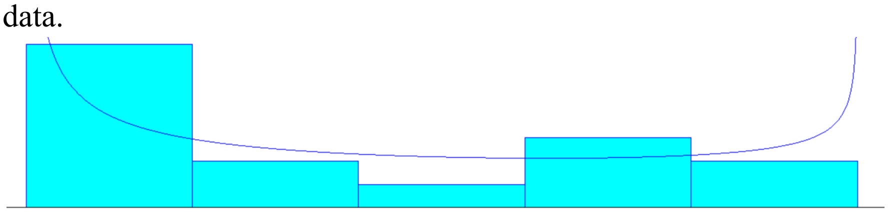
1. Establish the observed distribution of inter-arrival times of cars based on the square errors, Akai information criterion (AIC), Bayesian information criterion (BIC), and Pham criterion.
2. Using the tests for independence, plot and analyze the inter-arrival times. Do they appear to be independent during the weekday and the weekend?
3. Build an Arena model to simulate the operation of the pump and estimate the weekly profit.
4. Simulate the operation of the gas station for one week (i.e., 7 days). Create an animated plot of the cumulative profit as a function of time.
5. Study the Arena report, and compare the results above with an assumption that the cars arrive at the station with inter-arrival time that is exponentially distributed with a mean value of 2.75 minutes.

Objectives

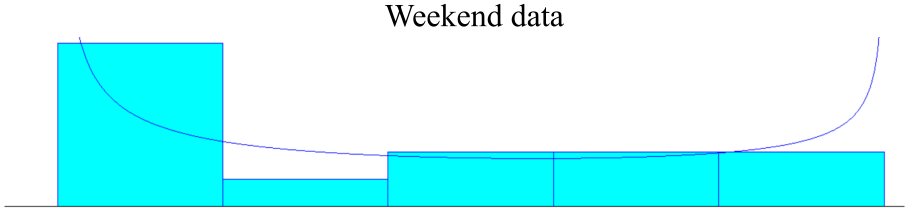
Our objectives for this project were to simulate a model in an arena for 4 double sided gas machines with the given resources, costs, profits, constraints, and arrival rates. Using the same model we are then inputting the best distribution for the given interarrival times of cars for the weekend and weekday and comparing it with another model using a distribution of EXPO(2.75) minutes. We are then finding the profits that each of the models produces and comparing them.

Data Collection

Using input analyzer in Arena, we were able to fit the model with the different distributions and use the fit all function. From the independence test, we get that the data for the inter arrival time of cars is independent from each other. This means that the data cannot be analyzed together and must be done separately. Using an input analyzer, we get that the best distribution for the weekend data and the weekday is the beta distribution. Below is the weekend data and weekday data.



Weekend data



Weekday data

We see that the data variability is very similar between the two data sets and it is likely that they would recommend the same distribution for the data sets.

Using matlab we were able to obtain the data of the log likelihoods and the aic/bic criterion using the functions that they have built into matlab. In the table below are the values that matlab printed out. The values are the log likelihoods, aic, and bic that matlab gave us.

Log_likelyhood	**AIC**	**BIC**	**Log_likelyhood**	**AIC=-2log(L)+2**	**BIC=-2log(L)+kLog(n)**
-54.6944	117.3889	120.2211	-45.3067	98.6134	100.8732
-552.7245	1.11E+03	1.11E+03	-622.8492	1.25E+03	1.25E+03
-72.818	147.6361	148.3441	-66.9622	135.9244	136.4893
-5.5579E+02	1.12E+03	1.12E+03	-749.5687	1.50E+03	1.50E+03
-167.7949	113.3889	114.805	-155.7622	94.6134	95.7433
-76.4533	156.9066	158.3227	-69.525	143.05	144.1799
-79.3457	164.6914	166.8156	-74.434	154.8681	156.5629
-72.0603	148.1206	149.5367	-65.4804	134.9608	136.0907
-96.9101	197.8202	199.2363	-79.9762	163.9523	165.0822

Weekend Data Weekday Data

Methodologies

Arena Models:

In part 1 for our model, we built an arena model that represented a gas station with x amount of workers and y amount of workers scheduled, in a double sided four machine gas system. For the schedule we input it as ignore.

Each side of the pump has its own waiting system and is put in series. The arena model will decide if it will take another customer only if there is a certain amount of gas available. If it's not available, all customers will be denied and exit the system until the gas is refilled. If there are more than 16 cars waiting overall, then there is a chance that they will leave or stay which is modeled by a decision variable. Each of the machines are next to each other so each side of the gas machine is in series and if the gas pump ahead of the one behind is occupied then the car cannot leave until the car in front finishes their task. There are four individual waiting lines that the customer could be put entered in and is modeled by an n way by condition module. Then the customer waits for the task until it is their turn and leaves when they're task is finished.

In the arena model, we also tracked the amount of gas in the system and when the gas truck arrived to refill the tank. They are modeled by a create module and a process module which is determined by the condition we are given. The gas levels were also modeled in the arena visually but creating an interactive graph that shows the gas levels during the simulations run time.

Profit is also modeled in the simulation and can be calculated by taking the total revenue and subtracting it by the supply cost and hiring cost and operating cost. Additionally, we created an arena model for profit where the cost is $10 per complaining customer.

Our suggestion to improve profit for the gas station was to increase the sale price for 1 gallon from $2.50 to $3.00. Since the price of gas near our homes is around $3.00 we thought it was a reasonable solution to increase the profits at the gas station.

In part 2 we fit the arrival times for weekend and weekday using the lognormal distribution. The model was then simulated using the same method as part 1 to find the weekly profit. We also compare this model with the lognormal value for the create with the model with an EXPO(2.75) minutes arrival time for all customers.

Modeling Analysis

To determine which distribution is the best for our reliability project, we tested the data against different distributions for , Akai information criterion(AIC), Bayesian information criterion(BIC), and Pham criterion. Using the program matlab, we were able to get the log of the likelihoods and calculate the aic/bic using the matlab function which resulted in the values below.

Weekday data	inputs:13	input analyzer: beta					
Fit	**SSE**	**Log_likelyhood**	**k=(AIC+2log(L))/2**	**n=10^((BIC+2log(L))/k)**	**AIC=-2log(L)+2k**	**BIC=-2log(L)+kLog(n)**	**PC=(n-k/2)log(SSE/n)+k(n-1/n-k)**
Beta	0.016043	-45.3067	4	13	98.6134	100.8732	-7.755626626
Erlang	0.045374	-622.8492	2	13	1.25E+03	1.25E+03	-11.33243139
Exponential	0.045374	-66.9622	1	13	135.9244	136.4893	-13.74281771
Gamma	0.037902	-749.5687	2	13	1.50E+03	1.50E+03	-11.76222856
Lognormal	0.037088	-155.7622	2	13	94.6134	95.7433	-11.81408648
Normal	0.127337	-69.525	2	13	143.05	144.1799	-8.86761988
Triangular	0.101717	-74.434	3	13	154.8681	156.5629	-6.932749048
Uniform	0.089941	-65.4804	2	13	134.9608	136.0907	-9.69810284
Weibull	0.041103	-79.9762	2	13	163.9523	165.0822	-11.56856589

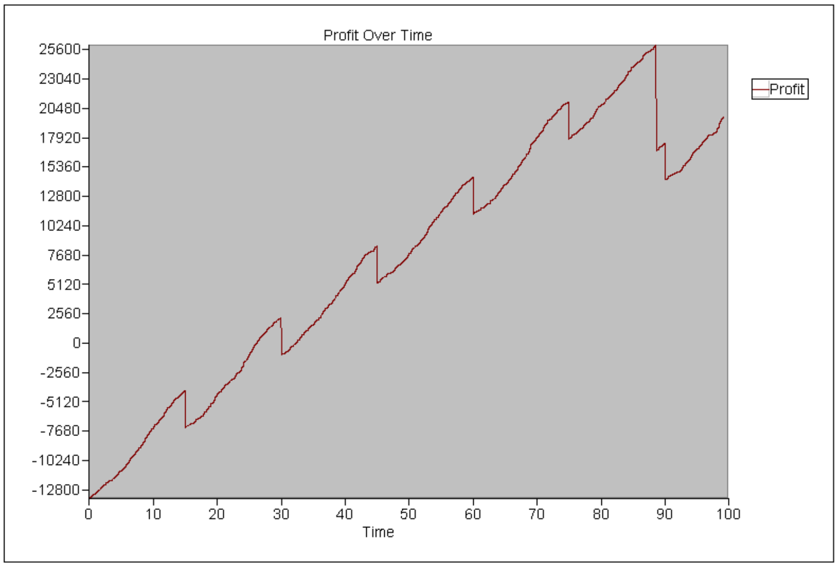
Weekend data	inputs:15	Input analyzer: beta					
Fit	**SSE**	**Log_likelyhood**	**k=(AIC+2log(L))/2**	**n=10^((BIC+2log(L))/k)**	**AIC=-2log(L)+2k**	**BIC=-2log(L)+kLog(n)**	**PC=(n-k/2)log(SSE/n)+k(n-1/n-k)**
Beta	0.017514	-54.6944	4	15	117.3889	120.2211	-11.03897343
Erlang	0.04127	-552.7245	2	15	1.11E+03	1.11E+03	-14.48912298
Exponential	0.04127	-72.818	1	15	147.6361	148.3441	-16.92319753
Gamma	0.037157	-5.5579E+02	2	15	1.12E+03	1.12E+03	-14.78548285
Lognormal	0.034087	-167.7949	2	15	113.3889	114.805	-15.02891996
Normal	0.132656	-76.4533	2	15	156.9066	158.3227	-11.19302219
Triangular	0.099753	-79.3457	3	15	164.6914	166.8156	-9.56299176
Uniform	0.097778	-72.0603	2	15	148.1206	149.5367	-12.05417956
Weibull	0.039261	-96.9101	2	15	197.8202	199.2363	-14.62999821

The Pham criterion cannot be found using matlab and has to be found analytically using the equation above. We find the Pham criterion by using the equation that was given to us and plugging in the values that we found. For the criterions the most optimal distribution will be the smallest value. From the table above we can see that overall, the most optimal distribution to be used in the weekend and weekday data is the lognormal distribution. The optimal distribution for each criterion is highlighted in yellow. We chose lognormal because for both aic/bic lognormal is the best distribution and for the PC, is it the second optimal distribution which further proves that lognormal distribution is the best distribution to use.

Results

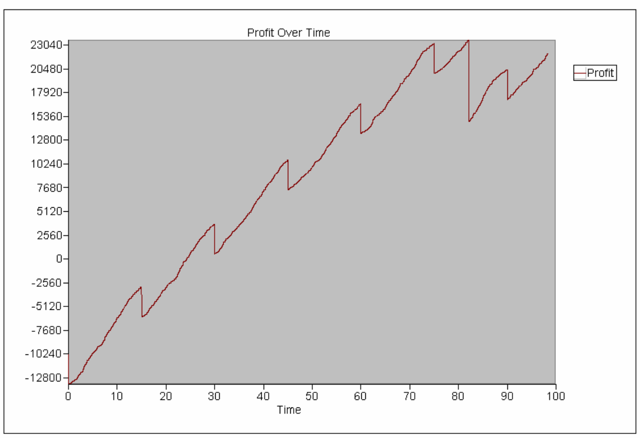
Part 1

Number 2:



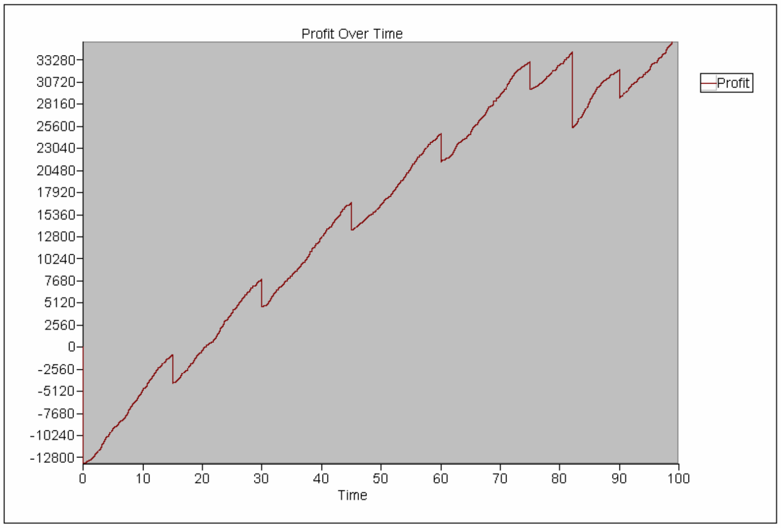
We averaged the following values for all 25 replications. The Average Weekly Profit was $9029.0352, the average Daily Weekly Profit: $1289.862171, the average Customer Complaints: 0.0048686852, the average complaint costs: $0.048686852

Number 3:



We averaged the following values for all 25 replications. The Average Weekly Profit was $9029.0352, the average Customer Complaints: 0.0048686852, the average complaint costs: $0.048686852, and average Weekly Profit with Complaints: $9028.9860.

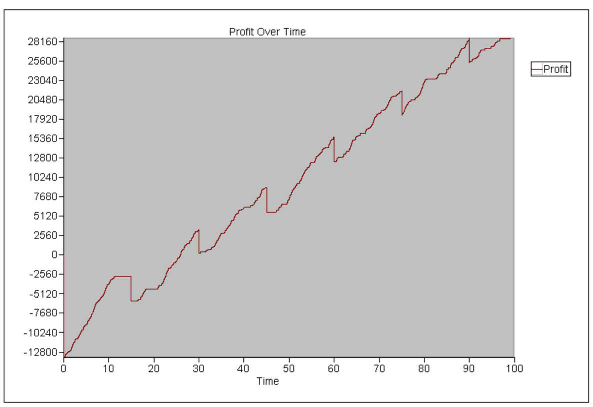
Number 4:



We averaged the following values for all 25 replications. The Average Weekly Profit was $15710.0112, the average Daily Weekly Profit: $2244.2873, the average Customer Complaints: 0.0049

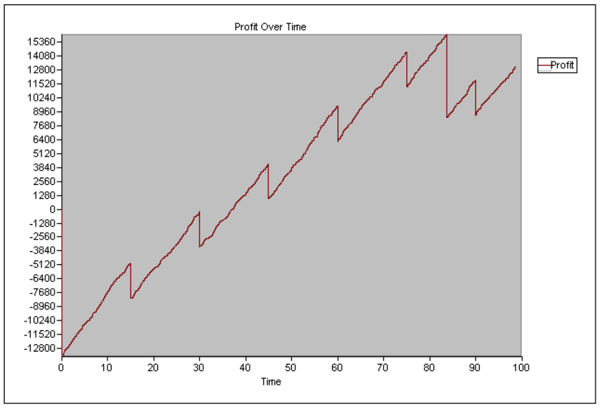
Part 2

Number 4:



We averaged the following values for all 25 replications. The Average Weekly Profit was $12,592.57, the average Daily Weekly Profit: $1798.938914, the average Customer Complaints: 0.0256260476

Number 5:



We averaged the following values for all 25 replications. The Average Weekly Profit was \$5030.25, the average Daily Weekly Profit: \$718.61, the average Customer Complaints: 0.00

Conclusions and Findings

In summation, after comparing the simulations in part 1 we found that increasing the price of gas to \$3 would result in a \$6680.976 increase in revenue over the initial distribution. Then running part 2 simulations to determine if a different distribution would work better and we found that lognormal distribution is more profitable than an exponential distribution by \$7562.32. Finally we compare the best methods from both parts and find that increasing the price of gas to \$3.00 has the best impact on profit.