Given Dataset

F1	F2	F3
1	5	3
4	2	6
1	4	3
4	4	1

Mean $= \dfrac{\text{Sum of the terms}}{\text{total numbers}(n)}$

$n=4,\ F_1 = 2.5,\ F_2 = 3.75,\ F_3 = 3.25$

Standard Deviation (SD): $= \sqrt{\dfrac{\Sigma (x-\text{mean})^2}{n-1}}$

$F_1 = 1.73,\ F_2 = 1.25,\ F_3 = 2.06$

Standardization: $z = \dfrac{x - \text{mean}}{SD}$

F1	F2	F3
-0.87	1	-0.12
0.87	-1.4	1.33
-0.87	0.2	-0.12
0.87	0.2	-1.09

Covariance: $\frac{\Sigma(x-\bar{x}_i)(y-\bar{y}_i)}{n-1}$

[যেহেতু already Standardization করা আছে (x_i, y_i) অর্থাৎ mean $= 0$ হবে]

	F1	F2	F3
F1	1.0 VAR(F1)	−0.696 COV(F1, F2)	0.139 (F1, F3)
F2	−0.696 (F2, F1)	1.0 VAR(F2)	−0.741 (F2, F3)
F3	0.139 (F3, F1)	−0.741 (F3, F2)	0.99 ≈ 1 VAR(F3)

$\boxed{n-1=4-1=3}$

Feature Vector: (Eigenvalue & Eigenvectors)

$$Cv - \lambda v = 0$$

$$\Rightarrow (C-\lambda)v = 0$$

$$\Rightarrow (C-\lambda I)v = 0$$

$$\Rightarrow C - \lambda I = 0 \quad [I = \text{Identity Matrix})$$

$$C - \lambda I = C - \lambda \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

$$= C - \begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix}$$

$$= \begin{vmatrix} 1-\lambda & -0.696 & 0.139 \\ -0.696 & 1-\lambda & -0.741 \\ 0.139 & -0.741 & 1-\lambda \end{vmatrix}$$

$$\therefore \underbrace{a}_{1}\lambda^3 - \underbrace{b}_{\text{Sum of diagonal (only value)}}\lambda^2 + \underbrace{c}_{\text{Sum of diagonal minors (2x2) (only value)}}\lambda - \underbrace{|c|}_{3\times3} = 0$$

for eigen value

$$\therefore \lambda^3 - 3\lambda^2 + 1.948\lambda - 0.090 = 0$$

for c,
0.451
0.981
+0.516
= 1.948

Eigen values are:

$\lambda_1 = 0.05$

$\lambda_2 = 2.09$

$\lambda_3 = 0.86$

Eigen vectors:

For, $\lambda_1 = 0.05$

$$\therefore C-\lambda_1 \begin{vmatrix} 1-0.05 & -0.696 & 0.139 \\ -0.696 & 1-0.05 & -0.741 \\ 0.139 & -0.741 & 1-0.05 \end{vmatrix}$$

$$\therefore (C-\lambda_1) V = 0$$

$$\Rightarrow \begin{bmatrix} 0.95 & -0.696 & 0.139 \\ -0.696 & 0.95 & -0.741 \\ 0.139 & -0.741 & 0.95 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

Using Crammer's rule:

$$0.95 v_1 - 0.696 v_2 + 0.139 v_3 = 0 \quad \text{--- (I)}$$

$$-0.696 v_1 + 0.95 v_2 - 0.741 v_3 = 0 \quad \text{--- (II)}$$

$$0.139 v_1 - 0.741 v_2 + 0.95 v_3 = 0 \quad \text{--- (III)}$$

using equation (I) & (II), (যে কোনো দুটো equation use করা যাবে)

$$\frac{v_1}{\begin{vmatrix} -0.696 & 0.139 \\ 0.95 & -0.741 \end{vmatrix}} = -\frac{v_2}{\begin{vmatrix} 0.95 & 0.139 \\ -0.696 & -0.741 \end{vmatrix}} = \frac{v_3}{\begin{vmatrix} 0.95 & -0.696 \\ -0.696 & 0.95 \end{vmatrix}}$$

$$\Rightarrow \frac{v_1}{0.383} = -\frac{v_2}{(-0.607)} = \frac{v_3}{0.418}$$

$$\Rightarrow \frac{v_1}{0.383} = \frac{v_2}{0.607} = \frac{v_3}{0.418}$$

$$\therefore E_1 = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.383 \\ 0.607 \\ 0.418 \end{bmatrix}$$

For $\lambda_2 = 2.09$

$$\therefore (C - \lambda_2) v = \begin{bmatrix} -1.09 & -0.696 & 0.139 \\ -0.696 & -1.09 & -0.741 \\ 0.139 & -0.741 & -1.09 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

For $\lambda_3 = 0.86$,

$$\therefore (C - \lambda_3) v = \begin{bmatrix} 0.14 & -0.696 & 0.139 \\ -0.696 & 0.14 & -0.741 \\ 0.139 & -0.741 & 0.14 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$0.14 v_1 - 0.696 v_2 + 0.139 v_3 = 0$ —— (I)

$-0.696 + 0.14 v_2 - 0.741 v_3 = 0$ —— (II)

$0.139 v_1 - 0.741 v_2 + 0.14 v_3 = 0$ —— (III)

using eqation (I) & (II),

$$\frac{v_1}{\begin{vmatrix} -0.696 & 0.139 \\ 0.14 & -0.741 \end{vmatrix}} = \frac{-v_2}{\begin{vmatrix} 0.14 & 0.139 \\ -0.696 & -0.741 \end{vmatrix}} = \frac{v_3}{\begin{vmatrix} 0.14 & -0.696 \\ -0.696 & 0.14 \end{vmatrix}}$$

$$\Rightarrow \frac{v_1}{0.496} = \frac{-v_2}{(-0.0069)} = \frac{v_3}{-0.465}$$

$$-1.09v_1 - 0.696v_2 + 0.139v_3 = 0 \quad \text{——— (I)}$$

$$-0.696v_1 - 1.09v_2 - 0.741v_3 = 0 \quad \text{——— (II)}$$

$$0.139v_1 - 0.741v_2 - 109v_3 = 0 \quad \text{——— (III)}$$

using equation (I) & (II),

$$\frac{v_1}{\begin{vmatrix} -0.696 & 0.139 \\ -1.09 & -0.741 \end{vmatrix}} = -\frac{v_2}{\begin{vmatrix} -1.09 & 0.139 \\ -0.696 & -0.741 \end{vmatrix}} = \frac{v_3}{\begin{vmatrix} -1.09 & -0.696 \\ -0.696 & -1.09 \end{vmatrix}}$$

$$\Rightarrow \frac{v_1}{0.667} = \frac{-v_2}{0.904} = \frac{v_3}{0.703}$$

$$\therefore E_2 = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.667 \\ -0.904 \\ 0.703 \end{bmatrix}$$

$$\therefore E_3 = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = \begin{bmatrix} 0.496 \\ 0.0069 \\ -0.465 \end{bmatrix}$$

E_1	E_2	E_3
0.383	0.667	0.496
0.607	−0.904	0.006
0.418	0.703	−0.465

As the eigenvectors with the highest eigenvalues represent the direct with the highest variance. So, the top 2 (highest) eigenvalues are λ_2 & λ_3 for eigenvectors v_2 & v_3 (E2&E3)

So the principle components:

E_2	E_3
0.667	0.496
−0.904	0.006
0.703	−0.465

Now, Final data set = Standardized original data set × Feature Vector

F_1	F_2	F_3
−0.87	1	−0.12
0.87	−1.4	1.33
−0.87	0.2	−0.12
0.87	0.2	−1.09

×

E_2	E_3
0.667	0.496
−0.904	0.006
0.703	−0.465

=

F_2 $\boxed{(E_2)}$	F_3 $\boxed{(E_3)}$
−1.569	−0.369
2.780	−0.195
−0.845	−0.375
−0.367	0.939

Ans.