Neural Networks Reading Project Report

1st Ahmet Burak KOÇ
Dept. Computer Eng.
Hacettepe University
Ankara, Turkey
koc.ab@hotmail.com

***Abstract*— In this project, "Active attitude fault-tolerant tracking control of flexible spacecraft via the Chebyshev neural Network" article, which was written by Kunfeng Lu, Tianya Li and Lijun Zhang in 2018, is read, and reimplemented with own codes and Simulink models to confirm the findings of the paper. The article describes a novel finite-time attitude tracking control approach for flexible spacecraft. This is achieved by integrating sliding-mode control and the active real-time fault-tolerant reconfiguration method. In this approach, the attitude dynamics and kinematics of the flexible spacecraft are established. Then, external disturbances, and uncertain inertia parameters are approximated and estimated. The nominal control law and the compensation control law to obtain the active reconfiguration fault-tolerant controller are finally developed in normal and fault conditions, respectively.[1]**

Keywords*— Flexible spacecraft, attitude control, Chebyshev neural network, terminal sliding mode, actuator fault

I. INTRODUCTION

Development and widening of space programs and missions brought development of huge and complex structures. This development lets spacecraft to be more flexible.

The challenges in attitude control of flexible spacecraft are actually due to complicated and serious coupling effects between flexible and rigid modes. The environmental disturbances (such as drag force, gravitational torques, magnetic torques) may induce elastic vibration on spacecraft. These vibrations decrease attitude control accuracy.

In this study, Simulink model is developed in order Chebyshev Polynomial Basis Neural Network to estimate all external disturbances, uncertain dynamics and actuator faults simultaneously. If such uncertainties can be estimated correctly, Chebyshev neural network based adaptive terminal sliding mode controller can be designed to compensate total perturbation and to achieve satisfying attitude control.

II. PRELIMINARIES AND PROBLEM FORMULATION

A. Mathematical model of flexible spacecraft

Attitude dynamics of a flexible spacecraft is described in two equations.

$$J\dot{\Omega} + \delta^{T}\ddot{\eta} = -\Omega^{x}(J\Omega + \delta^{T}\dot{\eta}) + (Dsat(u) + G_{\delta}) + d \tag{1}$$

$$\ddot{\eta} + L\dot{\eta} + K\eta + \delta\dot{\Omega} = 0 \tag{2}$$

Where a^{x} is defined as below skew-symmetric matrix.

$$a^{\times} = \begin{bmatrix} 0 & -a_3 & a_2 \\ a_3 & 0 & -a_1 \\ -a_2 & a_1 & 0 \end{bmatrix}$$

And where J is 3x3 spacecraft moment of inertia matrix, and it is sum of nominal inertia J_0 and uncertain inertia matrix ΔJ. η represents flexible mode and Nx1 matrix with modal order N. $K = diag\{\omega_{ni}^2, i = 1,2, \ldots, N\}$ and $L = diag\{2 * \zeta_i \omega_{ni}, i = 1,2, \ldots, N\}$ are damping matrix and stiffness matrices. ω_{ni} is the vibration-mode frequency and ζ_i is the ratio of vibration-mode damping. d is 3x1 external disturbance torque matrix. $sat(u)$ is actual control vector generated by actuator which represents saturation. D and G_{δ} are 3x3 partial loss factor matrix and 3x1 additive fault matrix which represents actuator failures together.

Kinematic equations defined in the article was a bit confusing so below kinematic equations are used.[2]

$$\dot{q} = \frac{1}{2} * \begin{bmatrix} 0 & \Omega(3) & -\Omega(2) & \Omega(1) \\ -\Omega(3) & 0 & \Omega(1) & \Omega(2) \\ \Omega(2) & -\Omega(1) & 0 & \Omega(3) \\ -\Omega(1) & -\Omega(2) & -\Omega(3) & 0 \end{bmatrix} * q \tag{3}$$

Same kinematics are used for both actual quaternion values of spacecraft and desired quaternion values.

B. Open Loop Attitude Tracking Error

Attitude error is calculated by quaternion multiplication rule as

$$e_v = q_{d4}q_v - q_{dv}^{x}q_v - q_4 q_{dv} \tag{4}$$

$$e_4 = q_{dv}^{T}q_v + q_4 q_{d4} \tag{5}$$

Where e_4 converges to 1 and e_v converge to 3x1 zero matrix while attitude error approaches to 0.

Error in angular velocity vector is calculated as $\Omega_e = \Omega - C\Omega_d$ where C is 3x3 rotation matrix given by $C = (e_4^2 - e_v^T e_v)I_3 + 2e_v e_v^T - 2e_4 e_v^x$.

Applying equations (1), (4) and (5) attitude tracking error system of the flexible spacecraft is established as

$$\dot{e}_v = \frac{1}{2}(e_4 I_3 + e_v^{x})\Omega_e \tag{6}$$

$$\dot{e}_4 = -\frac{1}{2}e_v^{T}\Omega_e \tag{7}$$

$$J\dot{\Omega}_e = -(\Omega_e + C\Omega_d)^{x}J(\Omega_e + C\Omega_d) + J(\Omega_e^{x}C\Omega_d - C\dot{\Omega}_d) + (Dsat(u) + G_{\delta}) + \tilde{d} \tag{8}$$

where $\tilde{d} = d - \delta^{T}\ddot{\eta} - \Omega^{x}\delta^{T}\dot{\eta}$.

C. Chebyshev Neural Network

Orthogonal polynomials can be used to develop function approximations. As a kind of orthogonal polynomials, Chebyshev polynomials are used in several studies to estimate nonlinear uncertainties such as [3] and [4].

Chebyshev polynomials can be obtained by using recursive formula of $T_{i+1}(x) = 2xT_i(x) - T_{i-1}(x), T_0(x) = 1$ where x is reel and $T_1(x)$ can vary (x, 2x, …, 2x-1, 2x+1). In the article authors chose $T_1(x) = x$.

XXX-X-XXXX-XXXX-X/XX/$XX.00 ©20XX IEEE

Enhanced pattern using Chebyshev polynomials for a vector $X = [x_1, \dots, x_m]^T$ is given by $\xi(X) = [1, T_1(x_1), \dots, T_n(x_1), \dots, T_1(x_m), \dots, T_n(x_m)]$ where n is order of Chebyshev polynomial order.

Based on the approximation property of Chebyshev neural network, any continuous nonlinear function $f(x)$ can be approximated by the Chebyshev neural network as $f(x) = W^*\xi(\mathrm{X}) + \varepsilon$ where W^* and ε represents optimal weight matrix and error boundary respectively.

III. Attitude Finite-Time Tracking Controller Design

A. Development of Sliding Mode

In this study, the sliding mode surface is developed as

$$S = \Omega_e + K_1 e_v + K_2 S_c \tag{9}$$

$$S_c = \begin{cases} e_i^{\frac{q}{p}}, & if\ \bar{S}_i = 0\ or\ \bar{S}_i \neq 0, |e_i| \geq \epsilon_i \\ \iota_1 e_1 + \iota_2 sign(e_i) e_i^2, & \bar{S}_i \neq 0, |e_i| < \epsilon_i \end{cases} \tag{10}$$

where $\bar{S}_i = \Omega_{ei} + k_{1i} e_i + k_{2i} e_i^{\frac{q}{p}}, i = 1,2,3$. Other parameters are positive scalars and defined later. If an appropriate control input is designed such that the systems states can reach $S_i = \bar{S}_i = 0$ and stay on the surface thereafter, then $\{e_v(t) = 0, e_4(t) = 1, \Omega_e(t) = 0\}$, after a finite time.

B. Applying the Chebyshev Neural Network to Approximate Uncertain Dynamics

Since inertia matrix is denoted by nominal inertia matrix and uncertain inertia matrix, sliding mode derivation can be established as (11) and uncertainties can be summed in one term (12).

$$\begin{aligned} J_0\dot{S} = &-(\Omega_e + C\Omega_d)^\times J_0(\Omega_e + C\Omega_d) + J_0 K_1 Q(e)(\Omega_e + C\Omega_d) \\ &+ J_0 K_2 E_e Q(e)(\Omega_e + C\Omega_d) - (\Omega_e + C\Omega_d)^\times \Delta J(\Omega_e + C\Omega_d) \\ &+ (J_0 + \Delta J)(\Omega_e^\times C\Omega_d - C\dot{\Omega}_d) + D\mathrm{sat}(u) + G_\delta + \tilde{d} - \Delta J\dot{\Omega}_e \\ = &-M + M_1 + M_2 + N + Du \end{aligned} \tag{11}$$

$$\begin{aligned} N = &-(\Omega_e + C\Omega_d)^\times \Delta J(\Omega_e + C\Omega_d) + (I_3 - \Delta J J^{-1})\tilde{d} \\ &+ \Delta J(\Omega_e^\times C\Omega_d - C\dot{\Omega}_d) - \Delta J J^{-1} J(\Omega_e^\times C\Omega_d - C\dot{\Omega}_d) \\ &+ \Delta J J^{-1}(\Omega_e + C\Omega_d)^\times J(\Omega_e + C\Omega_d) + D\theta_o \\ &- \Delta J J^{-1} D\mathrm{sat}(u) + (I_3 - \Delta J J^{-1}) G_\delta \end{aligned} \tag{12}$$

$$N_1 = J_0^{-1}\mathrm{N} \tag{13}$$

Chebyshev neural network with vector input $[\Omega, \dot{\Omega}]$ is to approximate uncertain dynamics N_1 within approximation error boundaries (14).

$$N_1 = W^*\xi(\Omega, \dot{\Omega}) + \varepsilon \tag{14}$$

C. Chebyshev Neural Network Based Adaptive Terminal Sliding-mode Controller

Controller designed as,

$$u = J_0\left[-\left(\tau + \frac{1}{\mathrm{X}}\right)S - \gamma sig^\lambda(S) - W\xi\right] \tag{14}$$

Where function (15) is defined as in [5],

$$\mathrm{sig}^\beta(x) \qquad \begin{aligned} &\mathrm{sig}^\beta(x) = |x|^\beta \mathrm{sgn}(x) \text{ for } \beta > 0, \\ &\mathrm{sig}^\beta(x) = [\mathrm{sig}^\beta(x_1), \mathrm{sig}^\beta(x_2), \dots, \mathrm{sig}^\beta(x_n)]^T \end{aligned} \tag{15}$$

For the controller law (14), $W\xi$ is applied to compensate for the total perturbation of the dynamical system. The nonlinear feedback control power $-\left(\tau + \frac{1}{\mathrm{X}}\right)S - \gamma sig^\lambda(S)$ is employed to guarantee that the systems states can reach the terminal sliding mode after a finite time.

IV. Simulation Results

In simulation moment of inertias are selected as

$$J_0 = \begin{bmatrix} 800.027 & 0 & 0 \\ 0 & 839.93 & 0 \\ 0 & 0 & 289.93 \end{bmatrix} kg.m^2 \tag{16}$$

$$\Delta J = \begin{bmatrix} 50 & 0 & 0 \\ 0 & 30 & 0 \\ 0 & 0 & 20 \end{bmatrix} kg.m^2 \tag{17}$$

External disturbance is assumed to be

$$\begin{aligned} &d(t) \\ &= [10\sin(0.1t) \quad 15\sin(0.2t) \quad 20\sin(0.2t)]^T \end{aligned} \tag{18}$$

Flexible parameters of the spacecraft are taken as N=4, $\omega_{n1} = 1.0973$, $\omega_{n2} = 1.2761, \omega_{n3} = 1.6538$, $\omega_{n4} = 2.2893, \zeta_1 = 0.05, \zeta_2 = 0.06, \zeta_3 = 0.08, \zeta_4 = 0.025$ and

$$\delta = \begin{bmatrix} 6.45637 & 1.27814 & 2.15629 \\ -1.25619 & 0.91756 & -1.67264 \\ 1.11687 & 2.48901 & -0.83674 \\ 1.23637 & -2.65810 & -1.12503 \end{bmatrix} \tag{19}$$

Initial attitude is $q(0) = [0.3 \quad -0.2 \quad -0.3 \quad 0.8832]^T$, initial angular velocity is $\Omega(0) = [0 \quad 0 \quad 0]^T$, desired ang. velocity is $\Omega_d = 0.05\left[\sin\left(\frac{\pi t}{100}\right) \quad \sin\left(\frac{2\pi t}{100}\right) \quad \sin\left(\frac{3\pi t}{100}\right)\right]^T$ and initial flexible vibration is $\eta(0) = [0.01242 \quad 0.01584 \quad -0.01749 \quad 0.01125]^T$.

Actuator fault is assumed to be

$$\delta_{oi} = \begin{cases} 1, & t < 10\mathrm{s} \\ 0.75 + 0.1\sin\left(0.5t + \frac{\pi}{3}\right), & t \geqslant 10\mathrm{s} \end{cases} \tag{20}$$

$$G_{\delta i} = \begin{cases} 0, & t < 15\mathrm{s} \\ 0.5 + 0.25\sin\left(0.2t + \frac{\pi}{4}\right), & t \geqslant 15\mathrm{s} \end{cases} \tag{21}$$

Moreover, control gains are chosen as $\iota_1 = 3, \iota_2 = 5, \mathrm{X} = 25, \gamma = 0.35, \sigma_1 = 100, \sigma_2 = 0.01, q = 3, p = 5, \bar{\epsilon}_i = 0.01, k_{11} = k_{12} = k_{13} = 0.5, k_{21} = k_{22} = k_{23} = 0.5, \tau = 10\ and\ \lambda = 0.6$

A. Figures

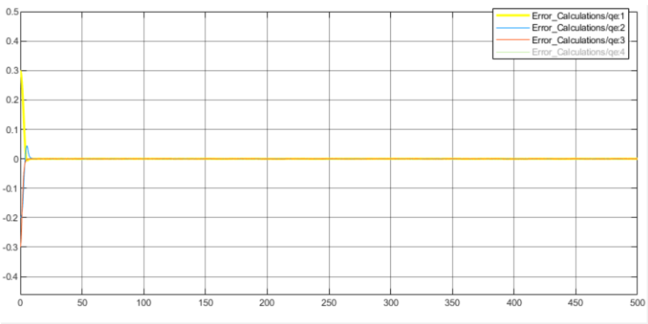

Fig. 1: Attitude tracking error e_v

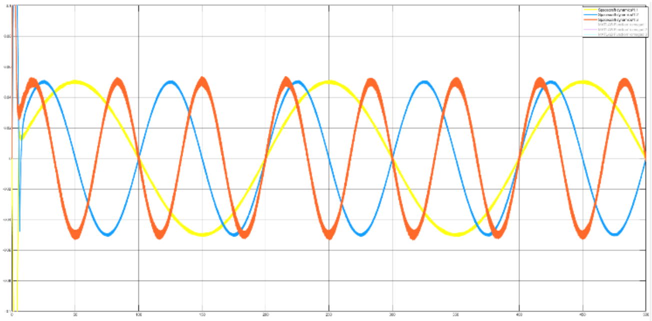

Fig. 2: Angular velocity tracking error Ω_e

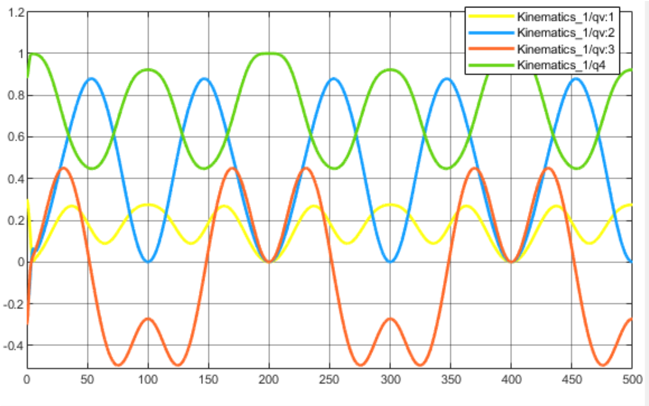

Fig. 3: Attitude q of the spacecraft

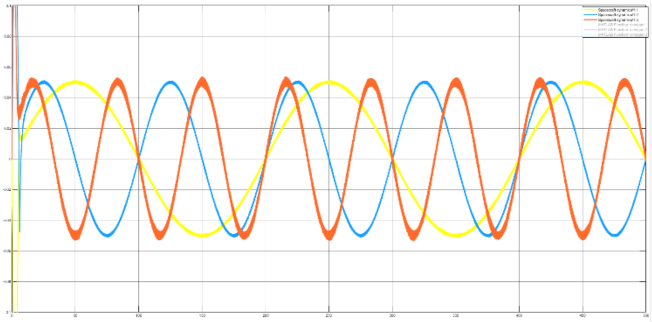

Fig. 4: Angular velocity Ω of the spacecraft

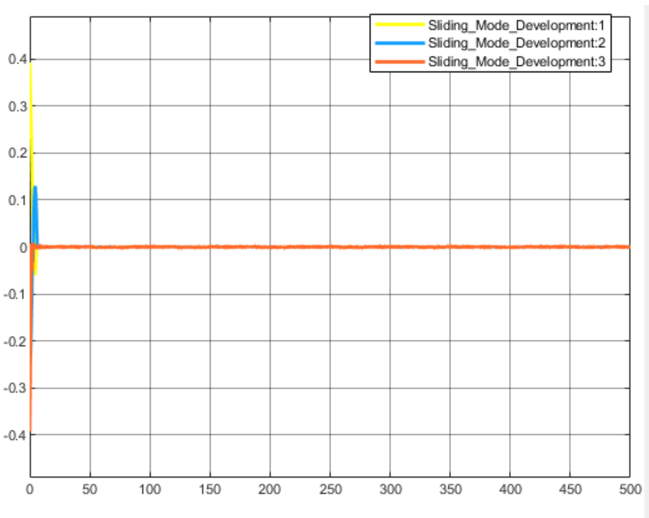

Fig. 5: Sliding-mode surface S

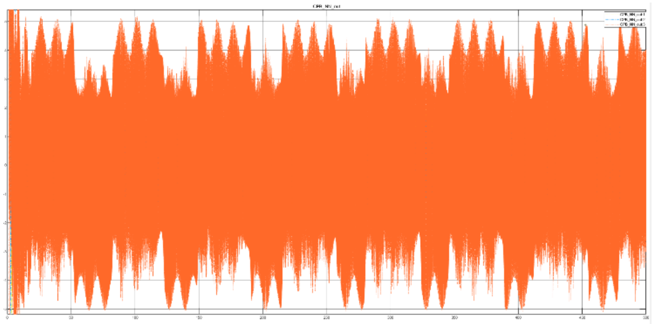

Fig. 6: Output $W\xi$ of the Chebyshev neural network (CNN)

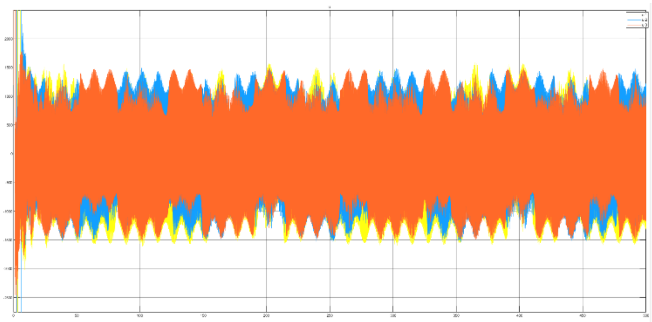

Fig. 7: Control input u

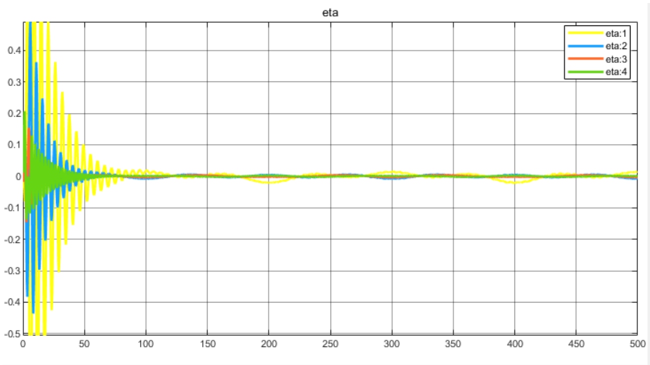

Fig. 8: Flexible vibration η

References

[1] Lu, K., Li, T., & Zhang, L. (2019). Active attitude fault-tolerant tracking control of flexible spacecraft via the Chebyshev neural network. Transactions of the Institute of Measurement and Control, 41(4), 925–933.

[2] Spacecraft Attitude Determination and Control. (1978). In J. R. Wertz (Ed.), Astrophysics and Space Science Library. Springer Netherlands.

[3] Nguyen N.T. (2018) Function Approximation and Adaptive Control with Unstructured Uncertainty. In: Model-Reference Adaptive Control. Advanced Textbooks in Control and Signal Processing. Springer, Cham.

[4] Tsu-Tian Lee and Jin-Tsong Jeng, "The Chebyshev-polynomials-based unified model neural networks for function approximation," in IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), vol. 28, no. 6, pp. 925-935, Dec. 1998.

[5] Xia, Y., Zhang, J., Lu, K., & Zhou, N. (2019). Finite Time and Cooperative Control of Flight Vehicles. In Advances in Industrial Control. Springer Singapore.