Time-Varying Formation Control Application

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***Abstract*—In this study, time-varying formation control application is re-implemented on Matlab/Simulink environment. Results are compared, discussed and potential applications of the study are investigated.**

Keywords—time-varying formation control, UAV, consensus problem, formation

I. Introduction

Formation control is one of the most considered problems in swarm system researches due to its potential applications. Basically, formation represents for swarms to form and keep geometrical shape. Time-varying formation represents the geometrical shape that swarms reach is not static but it is defined as function of time.

Formation capability of swarms are important and useful in several applications such as search and rescue missions to optimize success probability with minimum effort, reduction of drag forces to minimize disturbances in aerial vehicles [1], telecommunication to optimize coverage with available fleet of satellites or UAVs [2].

Formation control can be implemented using several methods such as leader-following, behavior-based, virtual-structure, cyclic pursuit, artificial potential function, algebraic path. Wile leader-following method is advantageous in terms of implementation, behavior-based method is advantageous because it ensures obstacle avoidance. Other methods have their own advantages and disadvantages.

There are several studies to utilize consensus problems for formation control in different formation control methods mentioned in the previous paragraph. [3] and [4] studied consensus problems for formation control of UAVs for different configuration such as adding communication delay and for time-invariant formations. In this paper, the time-varying formation control problem is studied to solve using consensus strategy.

II. Preliminaries and Problem Description

A. Basic Concepts and Results on Graph Theory

A directed graph $G = \{Q, \varepsilon, W\}$ consists of a set of nodes $Q = \{q_1, 1_2, \dots, q_N\}$, a set of edges $\varepsilon \subseteq \{(q_i, q_j): q_i, q_j \in Q\}$, and a weighted adjacency matrix $W = [w_{ij}] \in R^{NxN}$ with nonnegative elements w_{ij}. An edge of G is denoted by $e_{ij} = (q_i, q_j)$. In addition, $w_{ji} > 0$ if and only if $e_{ij} \in \varepsilon$, and $w_{ii} = 0$ for all $i \in \{1,2,\dots,N\}$. The set of neighbors of node q_i is denoted by $N_i = \{q_j \in Q: (q_i, q_j) \in \varepsilon\}$. The in-degree of node q_i is defined as $\deg_{in}(q_i) = \sum_{j=1}^{N} w_{ij}$. The degree matrix of G is denoted by $D = diag\{\deg_{in}(q_i), i = 1,2,\dots,N\}$. The Laplacian matrix of G is defined as $L = D - W$. A directed graph is said to have a spanning tree if there exists at least one node having a directed path to all other nodes. More details on graph theory can be found in [5]. The following lemma is useful in analyzing time-varying formation problems of UAV swarm systems.

Lemma 1 [6]: Let $L \in R^{NxN}$ be the Laplacian matrix of a directed graph G, then:

- L has at least one zero eigenvalue, and 1_N is the associated eigenvector, that is, $L1_N = 0$;
- If G has a spanning tree, then 0 is a simple eigenvalue of L, and all other N-1 eigenvalues have positive real parts.

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B. Problem Description

Consider a UAV swarm system with N UAVs. The interaction topology of the UAV swarm system can be described by a directed graph G. For $i, j \in \{1,2,\dots,N\}$, UAV_i can be denoted by node q_i in G and the interaction channel from UAV_i to UAV_j can be denoted by e_{ij}. It is assumed that G has a spanning tree.

For each of these UAVs, since the trajectory dynamics has much larger time constants than the attitude dynamics, if the formation is only concerned with positions and velocities, then the formation control can be implemented with an inner/outerloop structure [7], [8]. In this configuration, the outer-loop can be used to drive the UAV toward the desired position with desired velocity while the inner-loop can be used to track the attitude. This brief mainly focuses on designing the outer-loop. Therefore, on the formation control level, a UAV can be regarded as a point-mass system, and the dynamics of each UAV can be approximately described by the following double integrator:

$$\begin{cases} \dot{x}_i = v_i \\ \dot{v}_i = u_i \end{cases} \tag{1}$$

where $i = 1,2,\dots,N$, $x_i(t) \in R^n and\ v_i(t) \in R^n$ denote the position and velocity vectors of UAV i , respectively, and $u_i \in R^n$ are the control inputs. In the following, for simplicity of description, it is assumed that n = 1, if not otherwise specified. However, it should be pointed out that similar analysis can also be done for the higher dimensional case by using Kronecker product, and all the results hereafter remain valid for n > 1.

Let $\theta_i(t) = [x_i(t), v_i(t)]^T, B_1 = [0,1]^T, and$ $B_2 = [1,0]^T$. Then, UAV swarm system (1) can be written as

$$\dot{\theta}(t) = B_1 B_2^T \theta_i(t) + B_2 u_i(t) \tag{2}$$

Let $h_i(t) = [h_{ix}(t), h_{iv}(t)]^T$ $(i = 1,2,\dots,N)$ be piecewise continuously differentiable vectors and $h(t) = [h_1^T(t), h_2^T(t), \dots, h_N^T(t)]^T \in R^{2N}$.

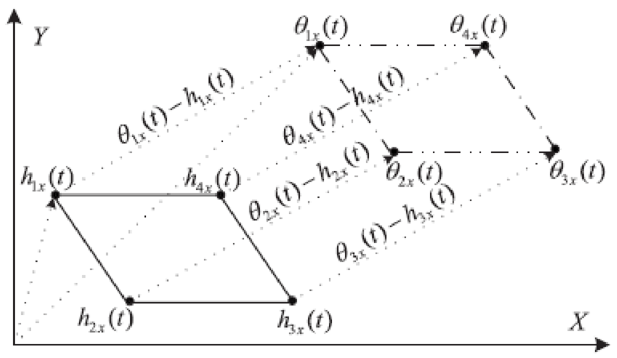

Fig. 1. Parallelogram formation in XY plane with $N = 4$.

Definition 1: A time-varying formation is specified by a vector h(t). UAV Swarm system (2) is said to achieve time-varying formation h(t) if for appropriate initial states, there exists a vector-valued function $c(t) \in R^2$ such that $li_{t\to\infty}^{nC}\big(\theta_i(t) - h_i(t) - c(t)\big) = 0$ $(i = 1,2, \dots, N)$, where c(t) is called a formation center function.

To illustrate Definition 1, consider a parallelogram formation for swarm system with four agents moving in the XY plane ($n = 2$). Let $x_{iX}(t) \in \mathbb{R}$, $h_{iXX}(t) \in \mathbb{R}$, $c_{XX}(t) \in \mathbb{R}$ and $x_{iY}(t) \in \mathbb{R}$, $h_{ixY}(t) \in \mathbb{R}$, $c_{XY}(t) \in \mathbb{R}$ denote the position, formation, formation center function of UAV i along x-axis and y-axis, respectively. Let $\theta_{ix}(t) = [x_{iX}(t), x_{iY}(t)]^T$, $h_{ix}(t) = [h_{iXX}(t), h_{iXY}(t)]^T$, and $c_x(t) = [c_{XX}(t), c_{XY}(t)]^T$. From Fig. 1, one sees that if $\theta_{ix}(t) - h_{ix}(t) - c_x(t) \to 0$ as $t \to \infty$ for all $i = 1, 2, 3, 4$, then the two parallelograms formed by $h_{ix}(t)$ and $\theta_{ix}(t)$ $(i = 1, 2, 3, 4)$ are congruent; that is, the parallelogram formation is achieved.

Definition 2: UAV swarm system (2) is said to achieve consensus if for any given bounded initial states, there exists a vector-valued function $s(t) \in \mathbb{R}^2$ such that $\lim_{t\to\infty}(\theta_i(t) - s(t)) = 0$ $(i = 1, 2, \dots, N)$, where $s(t)$ is called a consensus function.

Remark 1: From Definitions 1 and 2, one sees that when $h(t) \equiv 0$, UAV swarm system (2) achieves consensus if it achieves formation. In this case, the time-varying formation center function is equivalent to the consensus function. Therefore, for UAV swarm system described by (2), consensus problem is just a special case of formation problem.

Consider the following time-varying formation protocol:

$$u_i(t) = K_1\,(\theta_i(t) - h_i(t)) + K_2 \sum_{j \in N_i} w_{ij}((\theta_j(t) - h_j(t)) - (\theta_i(t) - h_i(t))) + \dot{h}_{iv}(t) \quad (3)$$

where $i = 1, 2, \dots, N$, $K_1 = [k_{11}, k_{12}]$ and $K_2 = [k_{21}, k_{22}]$.

Remark 2: In protocol (3), K_1 can be used to design the motion modes of the time-varying formation center, while K_2 can be designed to drive all the UAVs to achieve the desired formation. It should be noted that protocol (3) presents a general framework for consensus-based formation control protocols. Many existing protocols, such as those shown in [30], [37], and [38], and so on can be regarded as special cases of protocol (3). However, it should be pointed out that collision avoidance cannot be ensured by protocol (3) for any initial conditions.

Let $\theta(t) = [\theta_1^T(t), \theta_2^T(t), \dots, \theta_N^T(t)]^T$, $h_x(t) = [h_{1x}(t), h_{2x}(t), \dots, h_{Nx}(t)]^T$, and $h_v(t) = [h_{1v}(t), h_{2v}(t), \dots, h_{Nv}(t)]^T$. Under protocol (3), UAV swarm system (2) can be written in a compact form as follows:

$$\dot{\theta}(t) = \left(I_N \otimes \left(B_2K_1 + B_1B_2^T\right) - L \otimes (B_2K_2)\right)\theta(t) - (I_N \otimes (B_2K_1) - L \otimes (B_2K_2))\,h(t) + (I_N \otimes B_2)\,\dot{h}_v(t). \quad (4)$$

This brief mainly investigates the following three problems for UAV swarm system (4): 1) under what conditions the time-varying formation $h(t)$ can be achieved; 2) how to design protocol (3) to achieve the time-varying formation $h(t)$; and 3) how to demonstrate the theoretical results on practical quadrotor formation platform.

III. Formation Control Protocol Implementation

In this section, first, Xiwang Dong [9] transforms time-varying formation problems (4) into consensus problems. Conditions to reach formation function h(t) are presented, and formation center function c(t) is given. Finally, a procedure to determine the gain matrices in protocol (3) is proposed.

In this study, I re-implement the time-varying formation application 1 in this study to validate results on Matlab/Simulink Environment.

Application 1 is designed for time-varying formation control of 5 UAV agents. Initial values for Theta, gain matrices, radius r, angular velocity w, formation control h(t) are taken as given in Table 1.

```
Theta_1 = [-0.16 , 0.03 , -0.07, -0.01]'  ;
Theta_2 = [-4.92 , -0.08, 6.38 , -0.04]'  ;
Theta_3 = [-12.37, -0.26, 4.08 , -0.03]' ;
Theta_4 = [-12.73, 0.03 , -4.56, -0.04]' ;
Theta_5 = [-4.63 , -0.05, -6.9 , 0.02]'   ;

K1 = [-2, -1.2] ;
K2 = [0.3416, 0.7330] ;

r = 7 ; % 7 m radius of circular formation
w = 0.214 ; % angular velocity of formation

h_xX = r*(cos(w*t+2*pi*(i-1)/5)-1) * g(t,i,w)
h_vX = -w*r*sin(w*t+2*pi*(i-1)/5)*g(t,i,w)
h_xY = r*sin(w*t+2*pi*(i-1)/5)
h_vY = w*r*cos(w*t+2*pi*(i-1)/5)

g(t,i,w) = sign(sin(w*(t+0.0001)/2+pi*(i-1)/5));
```

Table 1: Application 1 Parameters

With these inputs $u_i(t)$ is calculated and applied on each agent which are designed as double integrator in Simulink. After 180 seconds of simulation, I observed results.

The first results did not match with the results given in the paper [9] because in the paper formation function was written as below.

h_xX = r ∗ cos((w ∗ t + 2 ∗ pi ∗ (i − 1)/5) − 1) ∗ g(t, i, w)

It is fixed as below.

h_xX = r ∗ (cos (w ∗ t + 2 ∗ pi ∗ (i − 1)/5) − 1) ∗ g(t, i, w)

Results given in following sections can be taken by running "Application1.m" Matlab script.

A. *UAV Trajectories*

Formation functions are showed with small circles and UAV's trajectories is drawn with points (Figure 1).

By running matlab script "Application1.m" one can observe formation is obtained drawing eight (see circles demonstration vs time). Moreover, points representing UAVs achieve such time-varying formation (see points demonstration vs time).

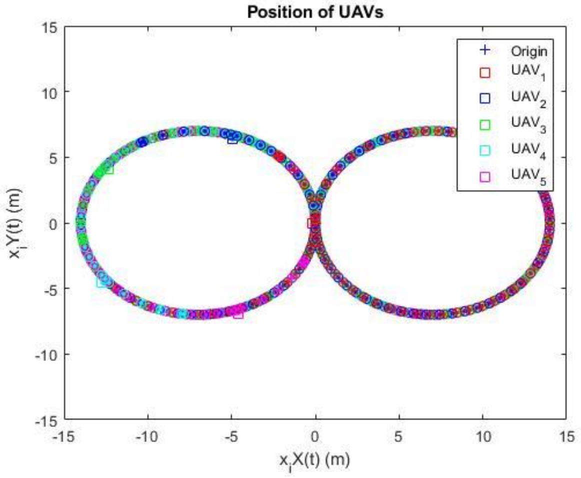

Figure 1: UAV Trajectories

B. *UAV Velocities*

Formation functions are showed with small circles and UAV's trajectories is drawn with points (Figure 2).

Velocities are obtained as given in [9].

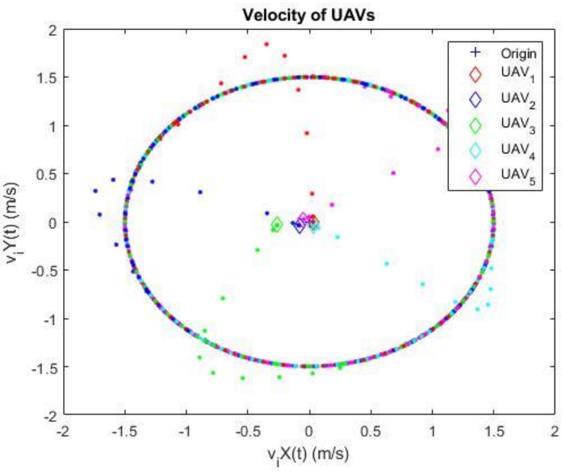

Figure 2: UAV Velocities

C. *Adjacency Matrix*

Adjacency matrix is taken as 0 and 1. It can be taken as distance to have more realistic graph G between agents in the future studies.

0	1	0	0	1
1	0	1	0	0
0	1	0	1	0
0	0	1	0	1
1	0	0	1	0

Figure 3: Adjacency Matrix of Graph

IV. CONCLUSION

Time-varying formation control is studied and implemented by several researchers using several methods and approaches. This study is one of the most generic solutions to time-varying formation control. Moreover the protocol design procedure are applicable for realistic UAV models in high level dynamic model (double integrator), indeed, real time applications are already applied with given method.

In this study, I re-implemented given method to validate results and results are satisfying comparing to results given in [9].

Formation function can be designed for specific purposes such as given potential applications of formation-control. Moreover, time-varying formation function can be designed according to mission parameters of aircraft, UAVs, satellites to maximize their mission achievements. For example, Fighter jets and rocket missiles can form according to a specific formation function which reduces drag forces so they can maximize their ranges. Or air defense systems can optimize defense coverage by forming rockets according to rockets propulsion capacities.

Matlab Scripts and Simulink models are outputs of this study and applicable for future projects. Future projects can focus on designing specific formation functions for specific purposes.

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