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CMP 755
Robotics HW2

Q1)

DH Parameters

i	α_{i-1}	a_{i-1}	d_i	θ_i
1	0	0	0	θ_1
2	-90°	0	d_2	θ_2
3	90°	0	d_3	0
4	0	a_3	d_4	θ_4
5	90°	0	0	θ_5
6	-90°	0	0	θ_6

$i-1 \rightarrow R$: rotation α_{i-1} around X_{i-1}

$R \rightarrow Q$: translation a_{i-1} along X_{i-1}

$Q \rightarrow P$: rotation θ_i around Z_i

$P \rightarrow i$: translation d_i along Z_i

Note;
Kinematics will change due to 3rd joint DH parameters.

Forward kinematics: Same kinematics with PUMA 560 where $\theta_3 = 0^\circ$ constant
$a_2 = 0$ constant

$$
{}^0_1T = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^1_2T = \begin{bmatrix} c_2 & -s_2 & 0 & 0 \\ 0 & 0 & 1 & d_2 \\ -s_2 & -c_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2_3T = \begin{bmatrix} c_i & -s_i & 0 & 0 \\ 0 & 0 & -1 & -d_3 \\ s_i & c_i & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^3_4T = \begin{bmatrix} c_4 & -s_4 & 0 & a_3 \\ s_i & c_i & 0 & 0 \\ 0 & 0 & 1 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^4_5T = \begin{bmatrix} c_5 & -s_5 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\qquad
{}^5_6T = \begin{bmatrix} c_6 & -s_6 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -s_6 & -c_6 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0_6T = {}^0_1T\,{}^1_2T\,{}^2_3T\,{}^3_4T\,{}^4_5T\,{}^5_6T = \begin{bmatrix} r_{11} & r_{12} & r_{13} & P_x \\ r_{21} & r_{22} & r_{23} & P_y \\ r_{31} & r_{32} & r_{33} & P_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$r_{11} = \dots$

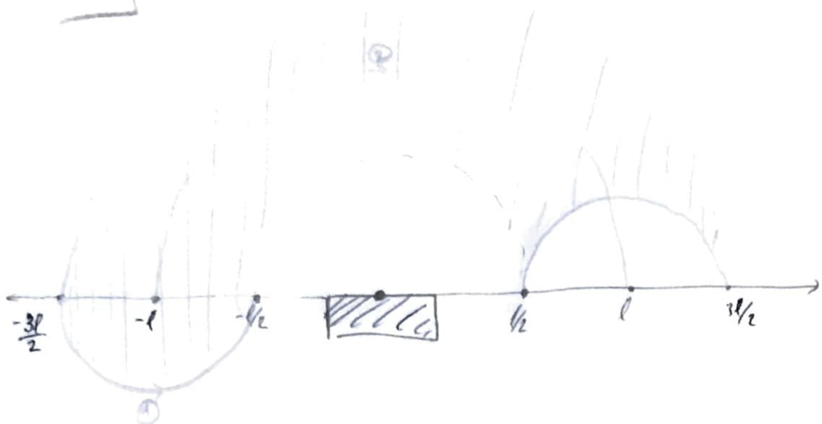
$P_x =$

$P_y = \dots$

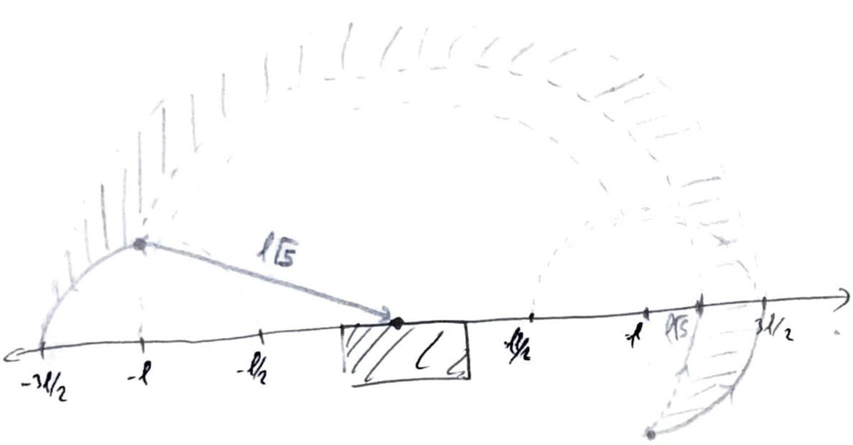
$P_z =$

it can be written [illegible]

Q2

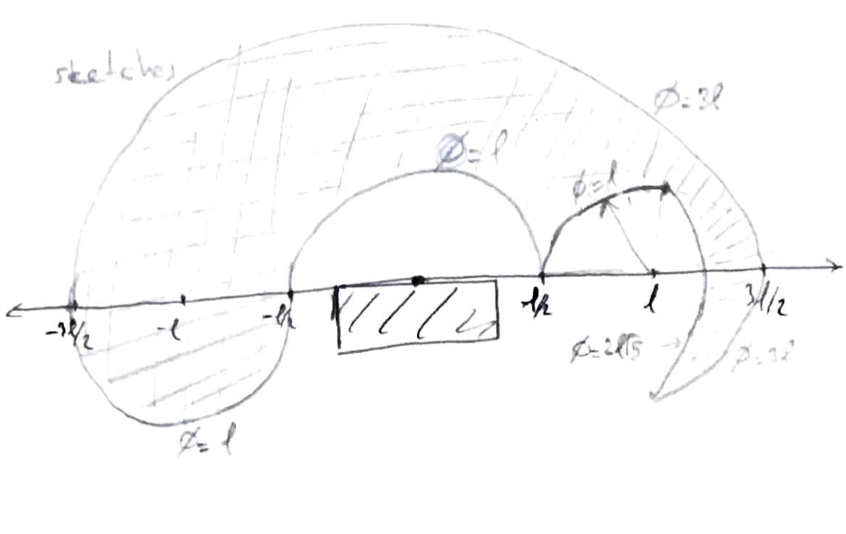


① $\theta_1 = 180°$
$\theta_2 = [0, 180°]$ → ② rotate θ_1 for all points in ①



③ $\theta_1 = 0°$
$\theta_2 = [-90, 0]$ → ④ rotate θ_1 for all points in ③

combine 2 sketches



Q3

Goal is defined as orientation so we will we only rotation matrix.

$$ {}^{0}_{2}T = \begin{bmatrix} c_1 s_2 & -c_1 s_2 & s_1 & l_2 s_1 + l_2 \\ s_2 & c_2 & 0 & 0 \\ -s_1 c_2 & s_1 s_2 & c_1 & l_2 c_1 + h_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

first 3 row and 3 column forms rotation matrix R.

$$ {}^{0}_{2}R = \begin{bmatrix} c_1 c_2 & -c_1 s_2 & s_1 \\ s_2 & c_2 & 0 \\ -s_1 c_2 & s_1 s_2 & c_1 \end{bmatrix} $$

$$ {}^{0}\hat{z} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad {}^{2}\hat{v} = \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} $$

and

$$ {}^{0}\hat{z} = {}^{0}_{2}R\, {}^{2}\hat{v} \Rightarrow \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = {}^{0}_{2}R \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} $$

eqn.s

$$ \begin{cases} (1)\ c_1 c_2 V_x - c_1 s_2 V_y + s_1 V_z = 0 \\ (2)\ s_2 V_x + c_2 V_y = 0 \\ (3)\ -s_1 c_2 V_x + s_1 s_2 V_y + c_1 V_z = 1 \end{cases} $$

note that there is 3 more equations

(4) $s_1^2 + c_1^2 = 1$

(5) $s_2^2 + c_2^2 = 1$

(6) $V_x^2 + V_y^2 + V_z^2 = 1$

from eq (2), $\theta_2 = \arctan\left(-\frac{V_y}{V_x}\right)$ or $180^\circ + \arctan\left(-\frac{V_y}{V_x}\right)$

from eq (1), (3)

(7) $$ \begin{cases} (c_1 c_2 V_x - c_1 s_2 V_y)^2 + s_1^2 V_z^2 + 2(c_1 c_2 V_x - c_1 s_2 V_y) . s_1 V_z = \\ = c_1^2 c_2^2 V_x^2 + c_1^2 s_2^2 V_y^2 - 2 c_1^2 c_2 s_2 V_x V_y + s_1^2 V_z^2 + 2 c_1 c_2 s_1 V_x V_z - 2 c_1 s_2 s_1 V_y V_z = 0 \end{cases} $$

(8) $$ \begin{cases} (-s_1 c_2 V_x + s_1 s_2 V_y)^2 + c_1^2 V_z^2 + 2(-s_1 c_2 V_x + s_1 s_2 V_y) . c_1 V_z = \\ = s_1^2 c_2^2 V_x^2 + s_1^2 s_2^2 V_y^2 - 2 s_1^2 c_2 s_2 V_x V_y + c_1^2 V_z^2 - 2 s_1 c_2 c_1 V_x V_z + 2 s_1 s_2 c_1 V_y V_z = 1 \end{cases} $$

by (7) and (8) (7) + (8) is performed

$$= c_2^2 V_x^2 + s_2^2 V_y - 2 c_2 s_2 V_x V_y \cancel{(s_1^2 + c_1^2)}^{1} + V_z \cancel{(s_1^2 + c_1^2)}^{1}$$

$$+ 2 c_1 c_2 s_1 V_x V_z - 2 c_1 s_2 s_1 V_y V_z - 2 s_1 c_2 c_1 V_x V_z + 2 s_1 s_2 c_1 V_y V_z = 1$$

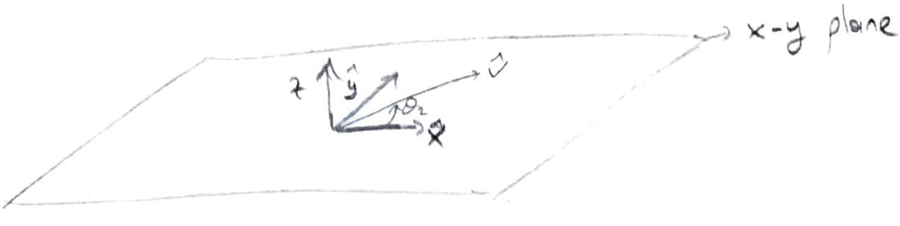
$$= V_z \left(\cancel{2 c_1 s_1 c_2 V_x} \cancel{- 2 c_1 s_1 s_2 V_y} \cancel{- 2 c_1 s_1 c_2 V_x} \cancel{+ 2 c_1 s_1 s_2 V_y} + 1 \right)$$

$$+ c_2^2 V_x^2 + s_2^2 V_y^2 - 2 s_2 c_2 V_x V_y = 1$$

$$= V_z + (c_2 V_x - s_2 V_y)^2 = 1$$

I could not derive it.

Instinctive result:



θ_2 is all about $\hat{x}$ and $\hat{y}$ so for $\hat{z}$ orientation θ_2 does not have any effect.

θ_1 is the angle between $\hat{v}$ and x-y plane.

Q4

$$ {}^{i-1}_{i}T = \begin{bmatrix} c\theta_i & -s\theta_i & 0 & a_{i-1} \\ s\theta_i \cdot c\alpha_{i-1} & c\theta_i \cdot c\alpha_{i-1} & -s\alpha_{i-1} & -s\alpha_{i-1} d_i \\ s\theta_i \cdot s\alpha_{i-1} & c\theta_i \cdot s\alpha_{i-1} & c\alpha_{i-1} & c\alpha_{i-1} \cdot d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

$$ \arccos T_{33} = \begin{cases} \alpha_{i-1} & \text{if} \quad \alpha_{i-1} \in [0, 180°] \\ 360° - \alpha_{i-1} & \text{if} \quad \alpha_{i-1} \in (180°, 360°) \end{cases} $$

$$ \Rightarrow \quad \arccos T_{33} = \begin{cases} \alpha_{i-1} & \text{if} \quad T_{23} = -s\alpha_{i-1} \leq 0 \\ 360° - \alpha_{i-1} & \text{if} \quad T_{23} = -s\alpha_{i-1} > 0 \end{cases} $$

$$ \Rightarrow \quad \alpha_{i-1} = \begin{cases} \arccos T_{33} & \text{if} \quad T_{23} \geq 0 \\ 360° - \arccos T_{33} & \text{if} \quad T_{23} < 0 \end{cases} $$

with the same derivation way,

$$ \theta_i = \begin{cases} \arccos T_{11} & \text{if} \quad T_{12} \geq 0 \\ 360° - \arccos T_{11} & \text{if} \quad T_{12} < 0 \end{cases} $$

$$ a_{i-1} = T_{14} $$

$$ T_{24}^2 + T_{34}^2 = (s^2\alpha_{i-1} + c^2\alpha_{i-1}) d_i = d_i $$

$$ d_i = T_{24}^2 + T_{34}^2 $$