Machine Learning

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2. Supervised Learning
   - Linear Regression, Logistic Regression, Support Vector Machines, Trees, Random Forests, Boosting, Artificial Neural Networks
3. Unsupervised Learning
   - Principal Component Analysis, K-means, Mean Shift

Supervised Learning

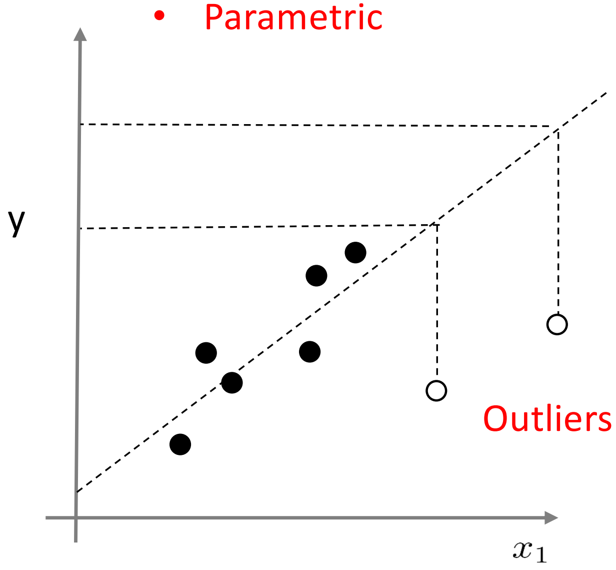
- Linear Regression
- Logistic Regression
- Support Vector Machines
- **Trees (Decision and Regression)**
- **Random Forests**
- **Boosting**
- Artificial Neural Networks

Classification and Regression Trees (CART)

Regression

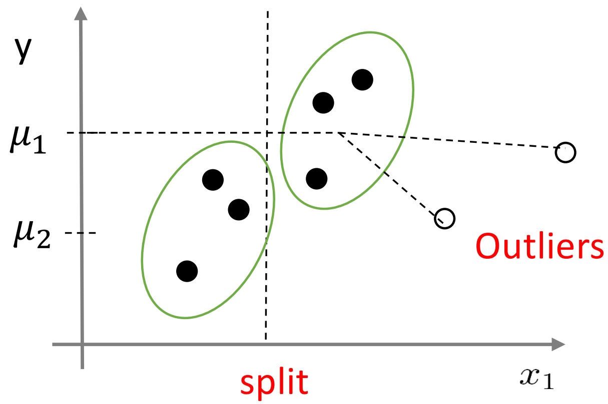
Linear regression

- Linear models
- Parametric



Regression trees

- Non Linear model
- Non-Parametric

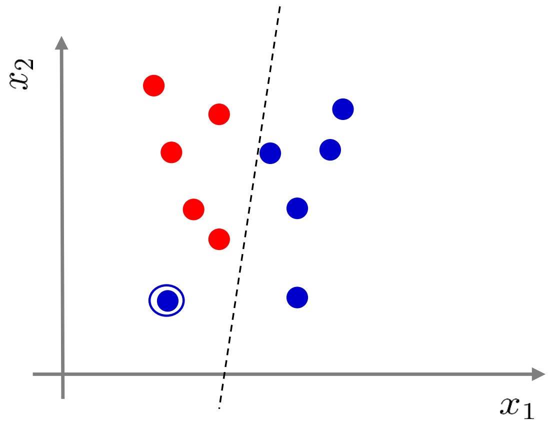


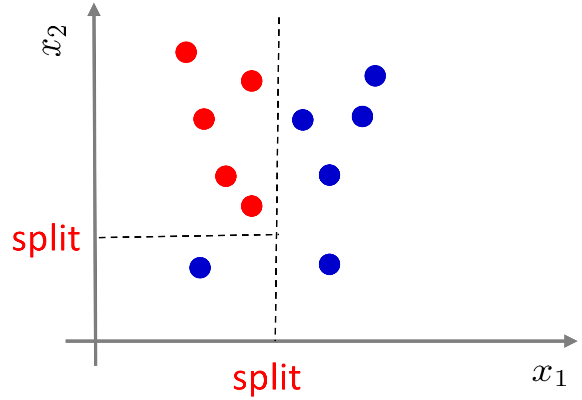
Classification and Regression Trees (CART)

Logistic regression
- Linear models
- Parametric

Classification trees
- Non Linear model
- Non-Parametric

Classification





Classification Trees (aka Decision Trees)

Example of Restaurant Data

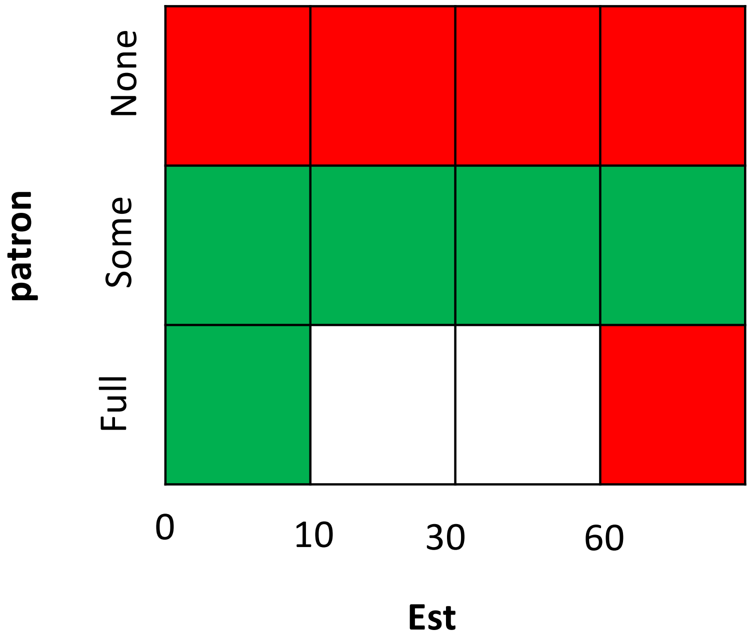
X	F										Y
Client	**Alt**	**Tea**	**Fri**	**Hun**	**Patron**	**Price**	**Rain**	**Res**	**Type**	**Est**	**Wait**
1	T	F	F	T	Some	$$$	F	T	Moroccan	0-10	T
2	T	F	F	T	Full	$	F	F	Chinese	30-60	F
3	F	T	F	F	Some	$	F	F	Burger	0-10	T
4	T	F	T	T	Full	$	F	F	Chinese	10-30	T
5	T	F	T	F	Full	$$$	F	T	Moroccan	>60	F
6	F	T	F	T	Some	$$	T	T	Italian	0-10	T
7	F	T	F	F	None	$	T	F	Burger	0-10	F
8	F	F	F	T	Some	$$	T	T	Chinese	0-10	T
9	F	T	T	F	Full	$	T	F	Burger	>60	F
10	T	T	T	T	Full	$$$	F	T	Italian	10-30	F
11	F	F	F	F	None	$	F	F	Chinese	0-10	F
12	T	T	T	T	Full	$	F	F	Burger	30-60	T

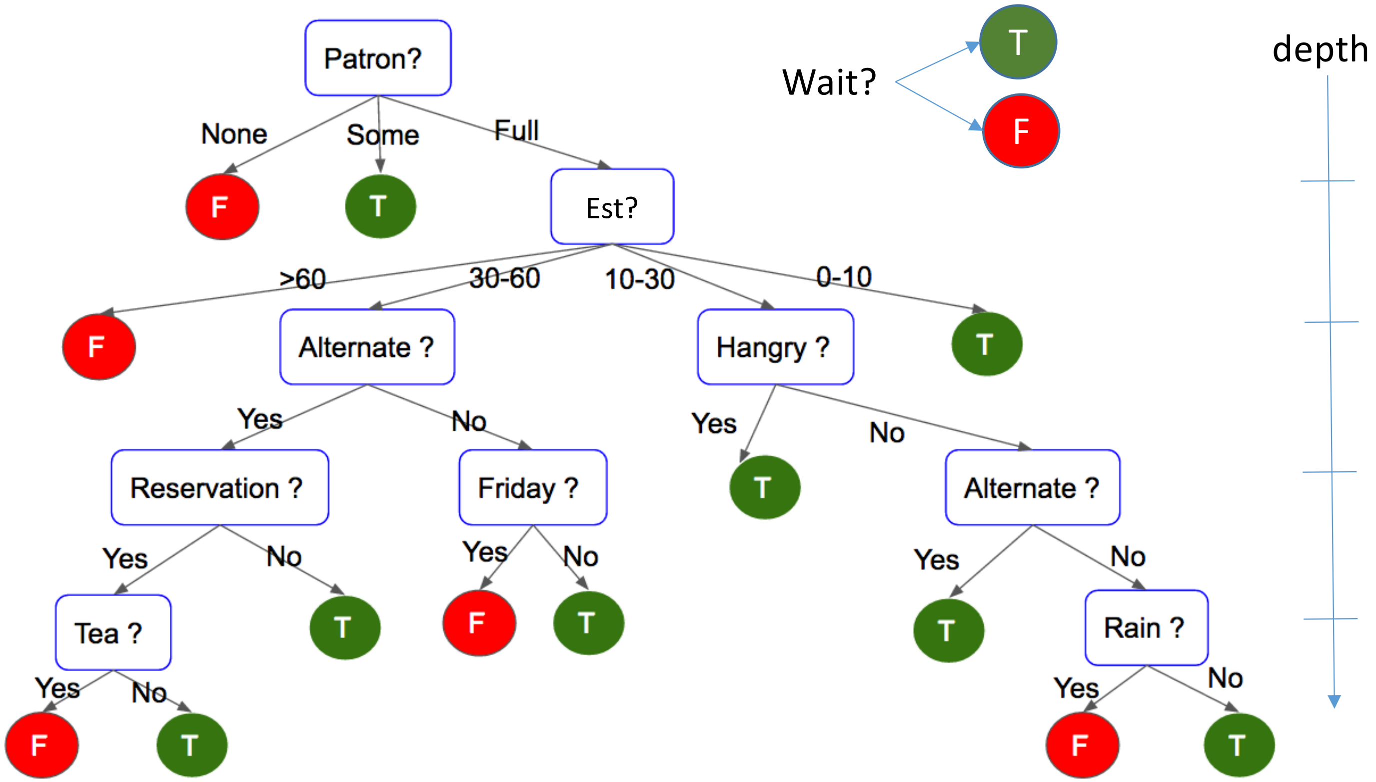
- Alt: is there any other alternative?
- Fri: is it Friday?
- Hun: is the client hungry?
- Patron: how many people are in the restaurant?
- Res: Restaurant
- Est: wait estimate

Most of the features are
Discrete (=categorical, =qualitative)

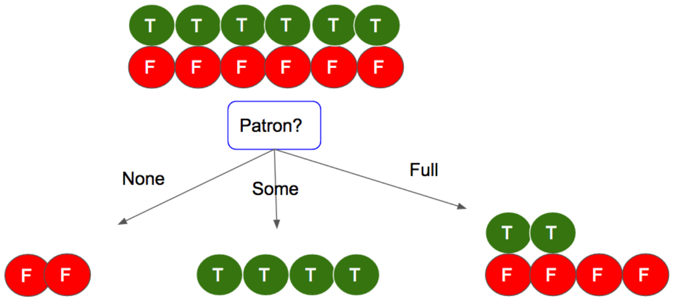
Classification Trees (aka Decision Trees)

Pick one feature at a time,
and make a binary decision

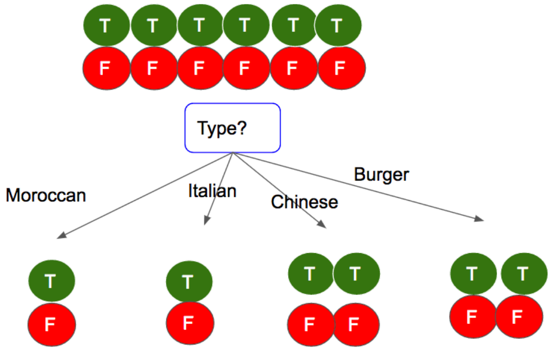




DT: Which feature to split with first?



More informative
Less impurity



Less informative
More impurity

Split with the feature F that maximizes the Information Gain

$$I(F) = H(S) - EH(F)$$

Information Gain — Parent entropy — Expected entropy

S: subset = {p positives and n negatives}
F: Feature

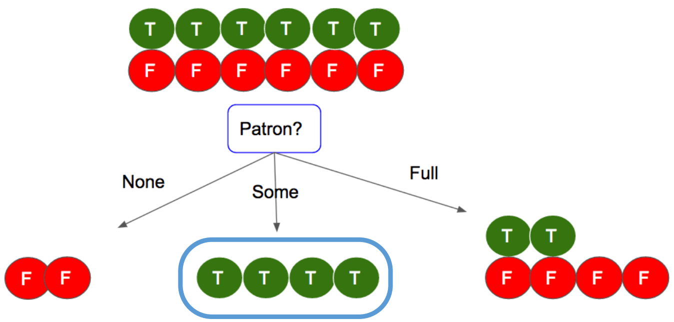
Entropy

$$H(S) = -\sum_{c} p_c \log_2(p_c)$$

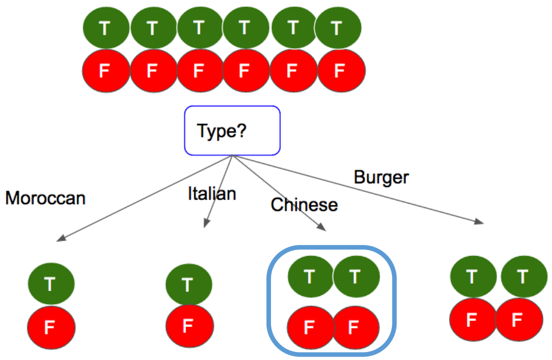
Python: np.log2()

p_c: probability of examples in class c
S : subset of data examples

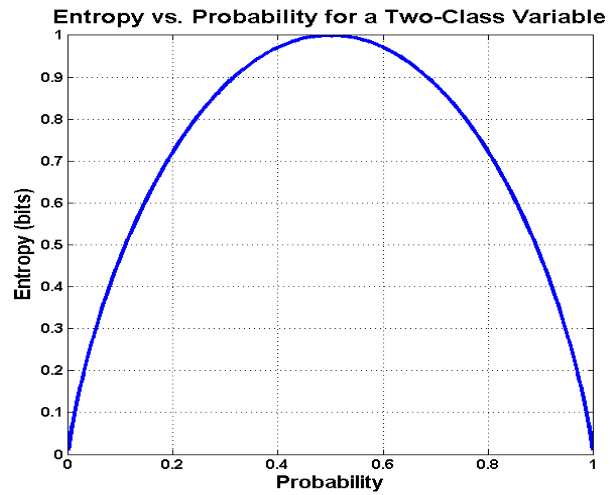
Interpretation: Measure of the impurity in a subset of examples



All examples in the same class
Entropy = 0



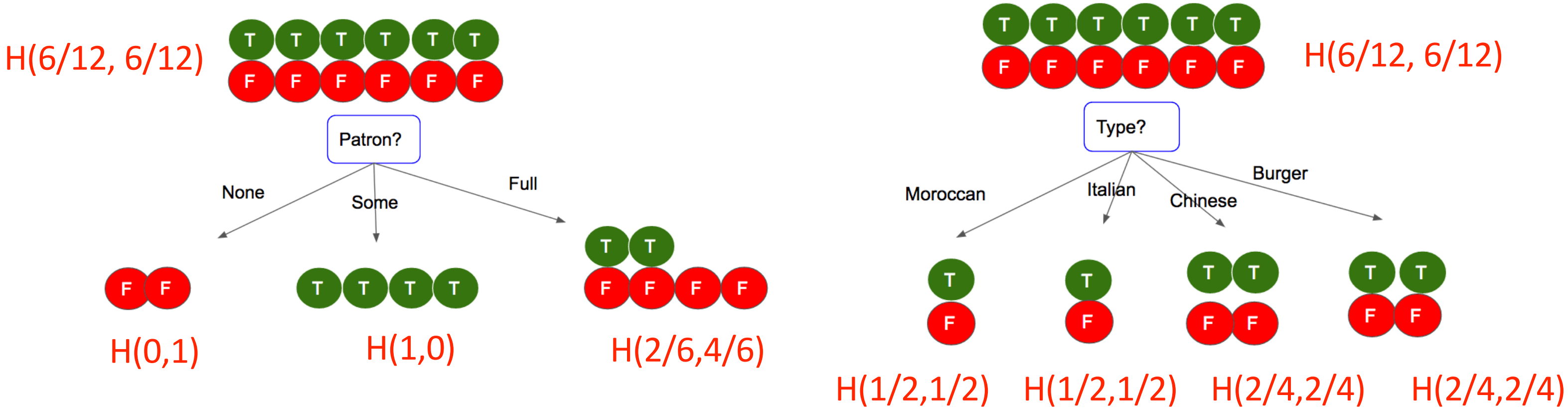
All examples evenly split between classes
Entropy = 1



Entropy (binary classification)

$$H\left(\frac{p}{p+n},\frac{n}{p+n}\right) = -\frac{p}{p+n}\log\left(\frac{p}{p+n}\right) - \frac{n}{p+n}\log\left(\frac{n}{p+n}\right)$$

p: positive
n: negative



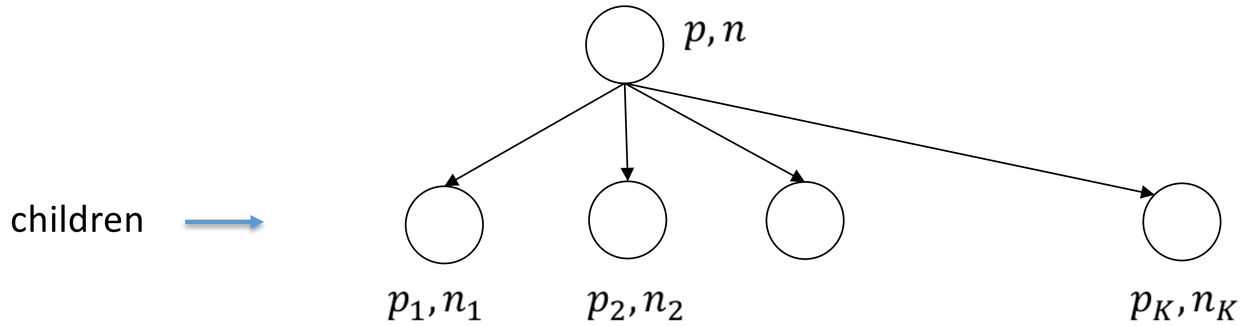
Expected Entropy

$$EH(F) = \sum_{i=1}^{K} \frac{p_i + n_i}{p + n} H\left(\frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i}\right)$$

K = number of splits (regions) with Feature F

= number of children nodes

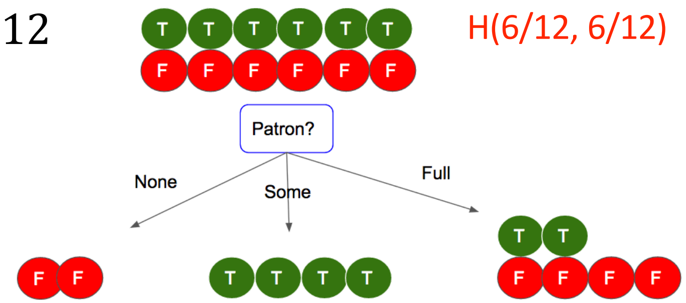
Expectation Entropy = weighted average of children entropy



Expected Entropy (Example)

$$EH(F) = \sum_{i=1}^{K} \frac{p_i + n_i}{p + n} H\left(\frac{p_i}{p_i + n_i}, \frac{n_i}{p_i + n_i}\right)$$

$p + n = 12$



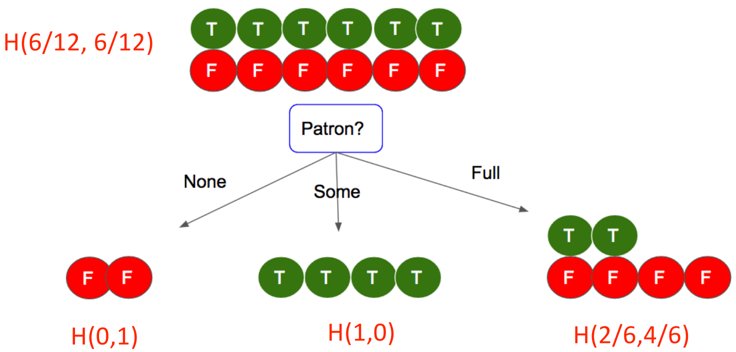
$K = 3$

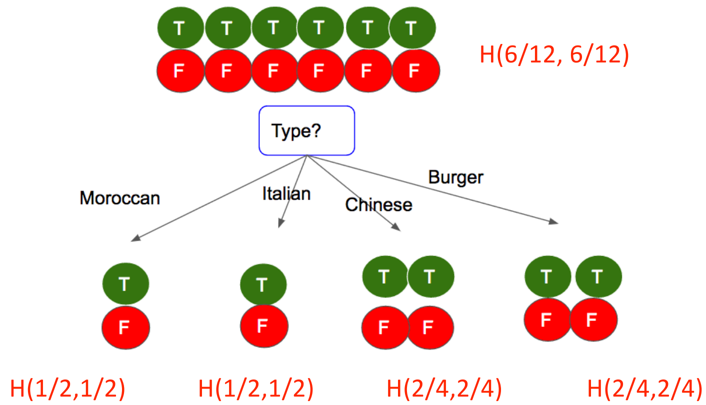
H(0,1) = 0 | H(1,0) = 0 | H(2/6,4/6) = .92

$p_i + n_i$ | 2 | 4 | 6

Information Gain

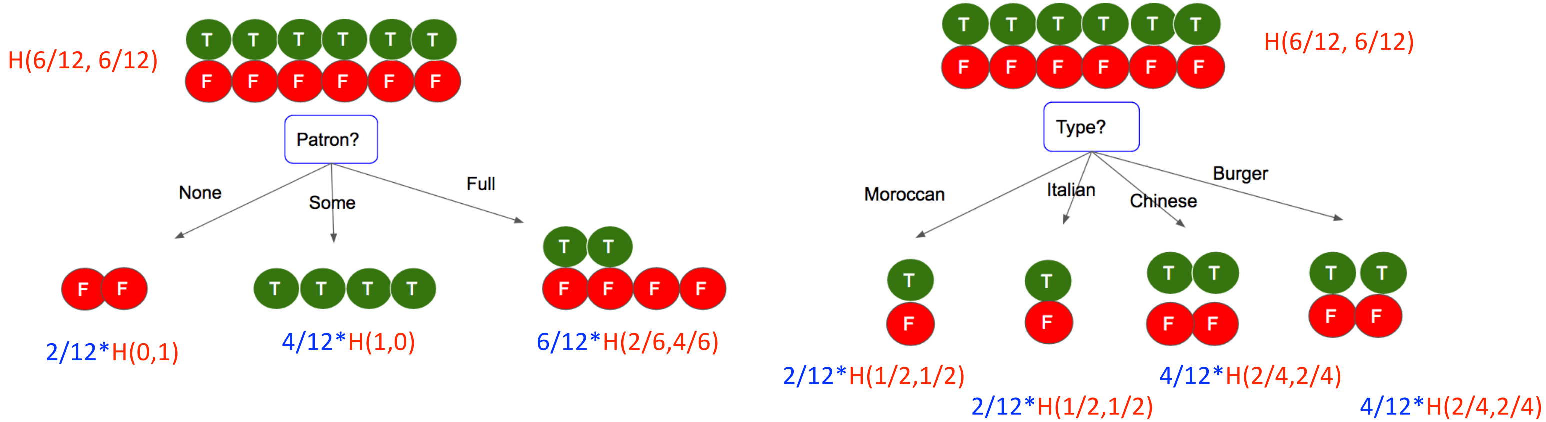
$$I(F) = H\left(\frac{p}{p+n}, \frac{n}{p+n}\right) - EH(F)$$





Information Gain

$$I(F) = H\left(\frac{p}{p+n}, \frac{n}{p+n}\right) - EH(F)$$



I(Patron) = 0.54

I(Type) = 0.0

Other impurity measures

- p_c: probability of examples in class c
- S : subset of data examples

- CART algorithm uses the Entropy

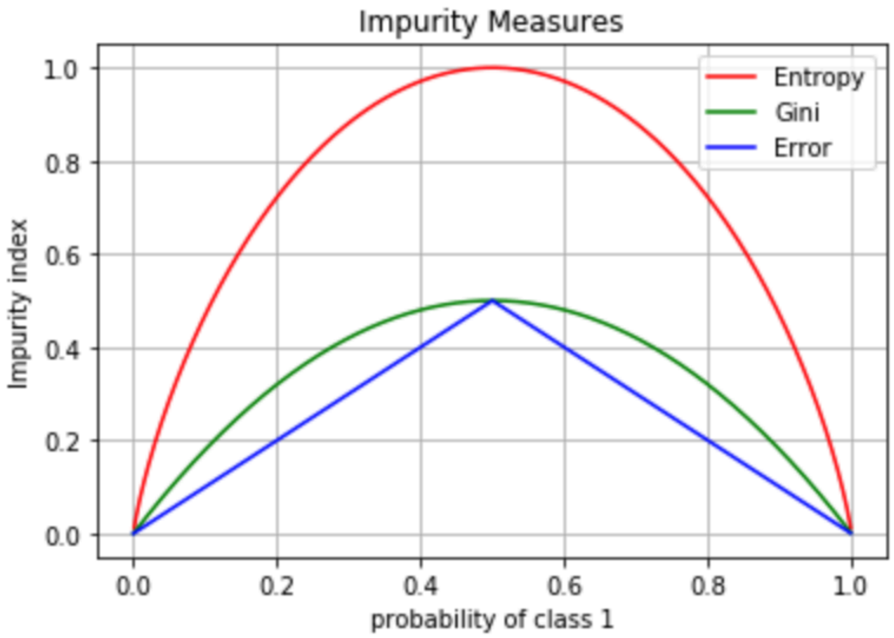
$$H(S) = -\sum_c p_c \log_2(p_c)$$

- Iterative Dichotomiser ID3 and C4.5 algorithms use Gini Index

$$G(S) = 1 - \sum_c p_c^2$$

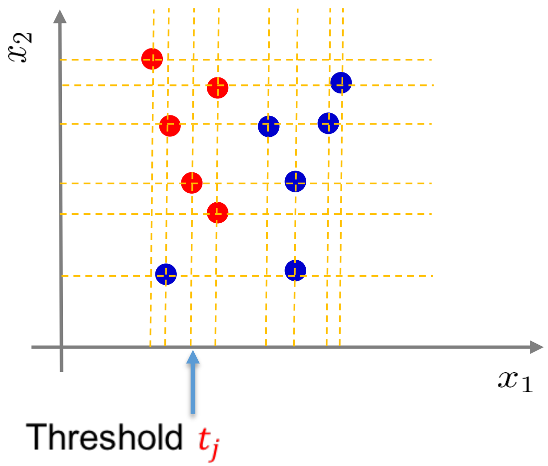
- One could also use the Class Error

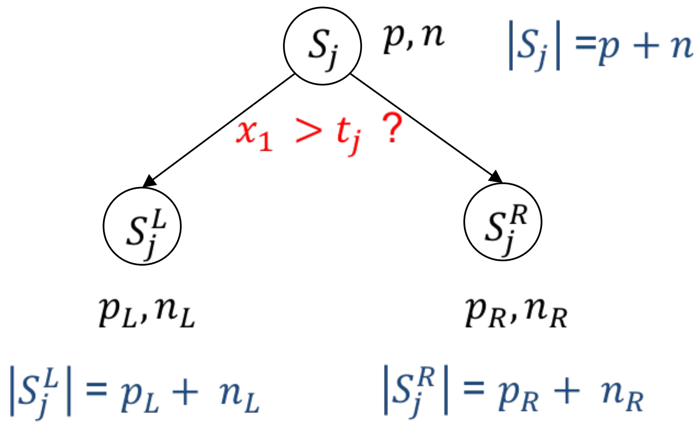
$$\mathrm{E}(S) = 1 - \max_c(p_c)$$



What if a feature is continuous (quantitative)?

$$I_j = H(S_j) - \sum_{i \epsilon \{L,R\}} \frac{|S_j^i|}{|S_j|} H(S_j^i)$$





Note: Doing so, trees are almost Binary! Even if a feature is categorical (qualitative)

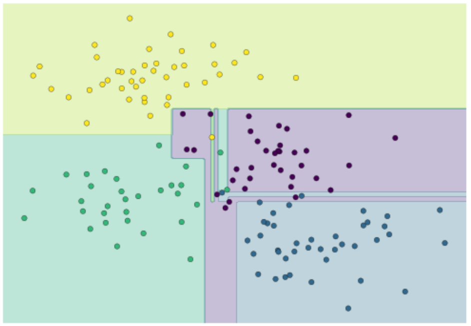
Decision Trees

- **Advantages**
  - Easy to interpret
  - Deals with non linearity
  - Handle qualitative features without the need to create fictive ones (one hot vector)
  - Provide most important features (in terms of information gain)

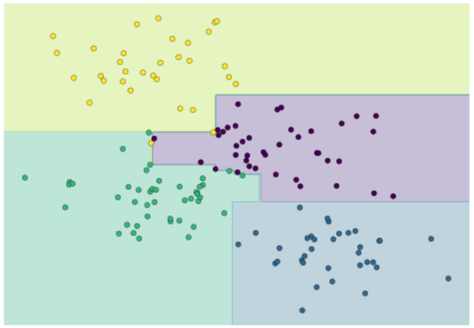
Decision Trees

- **Disadvantages**
  - Trees leads to overfitting (high variance): little change in little number of examples affect the whole tree.

DT on Data1



DT on Data2 = half of Data1

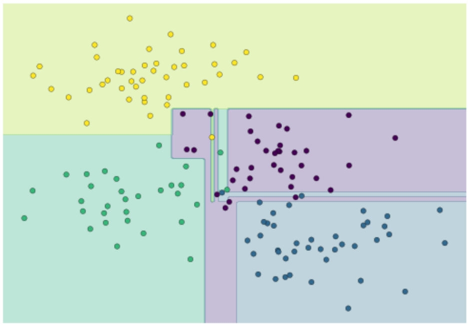


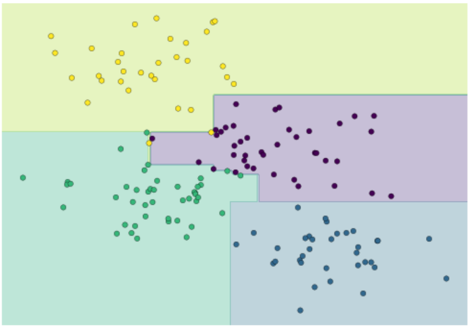
Decision Trees

- **Disadvantages**
  - Trees leads to overfitting (high variance): little change in little number of examples affect the whole tree.
- **Solution: Ensemble Methods**
  - Bagging: Random Forest (Leo Breiman 2001) consists of combining multiple independent weak trees to reduce variance.
  - Boosting to reduce bias

DT on Data1

DT on Data2 = half of Data1



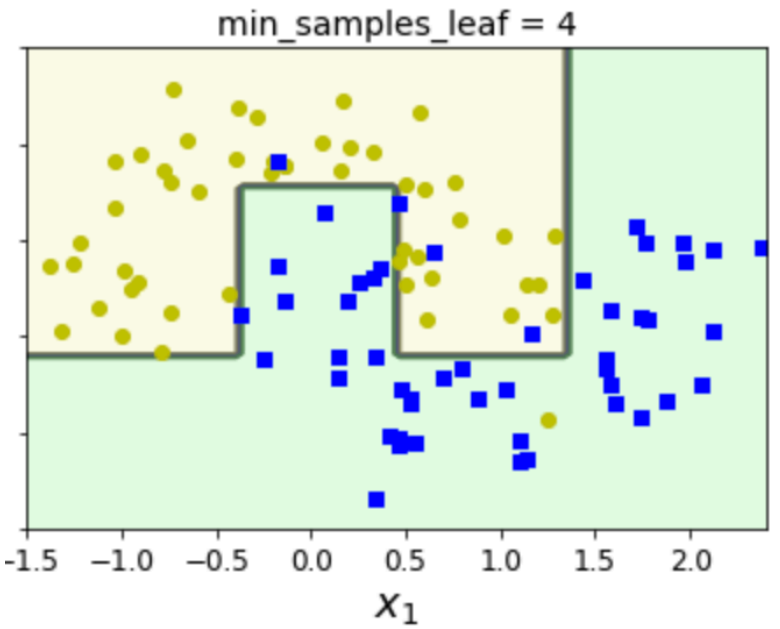


A tree alone will overfit.
However, it is clear that in some places, the two trees together produce consistent results

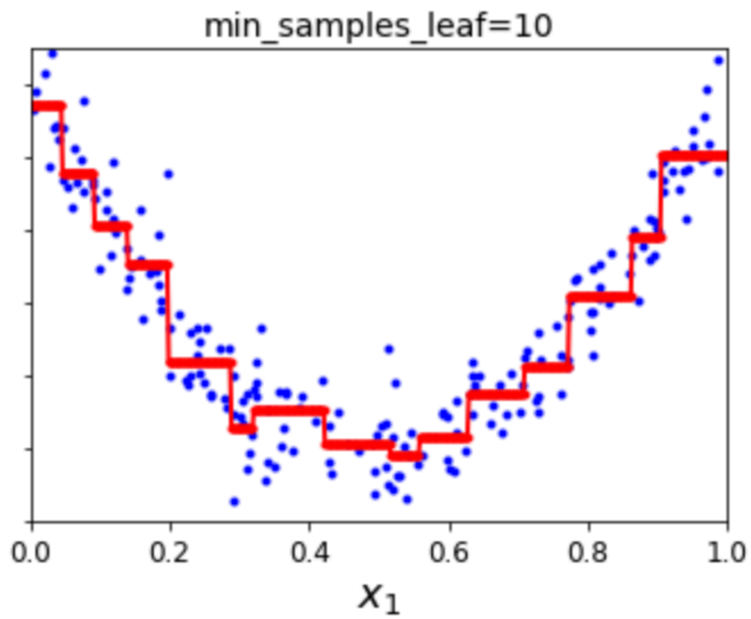
This idea comes from Bootstrapping (Brad Efron 1979):
Given a set of m independent observations o_1.., o_m, each with variance σ^2, the variance of the mean o of the observations is given by σ^2/m.

Classification And Regression Trees

Classification (Decision) Trees



Regression Trees



Supervised Learning

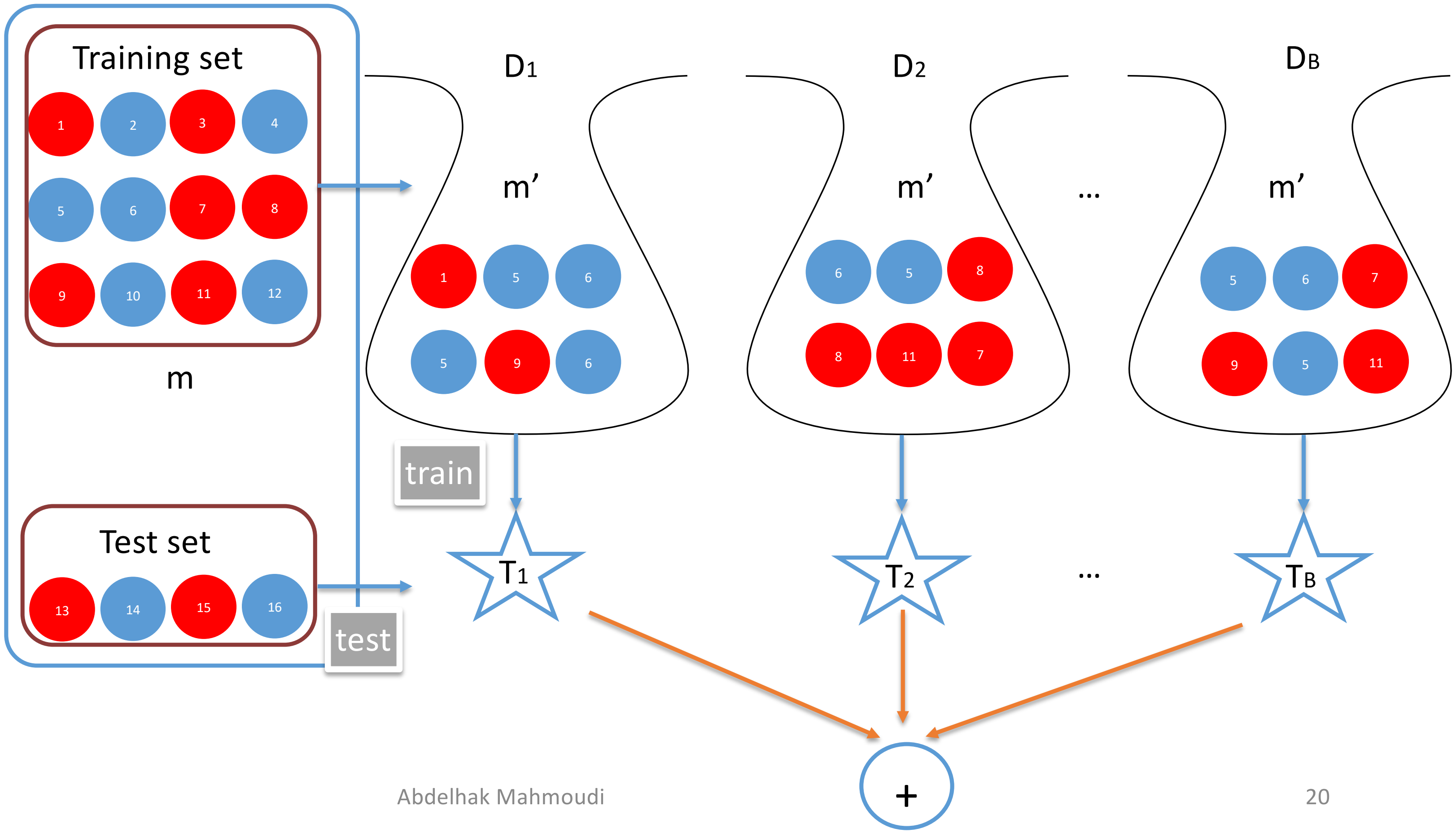
- Linear Regression
- Logistic Regression
- Support Vector Machines
- Trees (Decision and Regression)
- **Random Forests**
- Boosting
- Artificial Neural Networks

Random Forest

Bagging
Bootstrap Aggregating

Training
Pick m' examples with replacement and train B trees

Testing
Regression: mean errors of all the B trees
Classification: vote

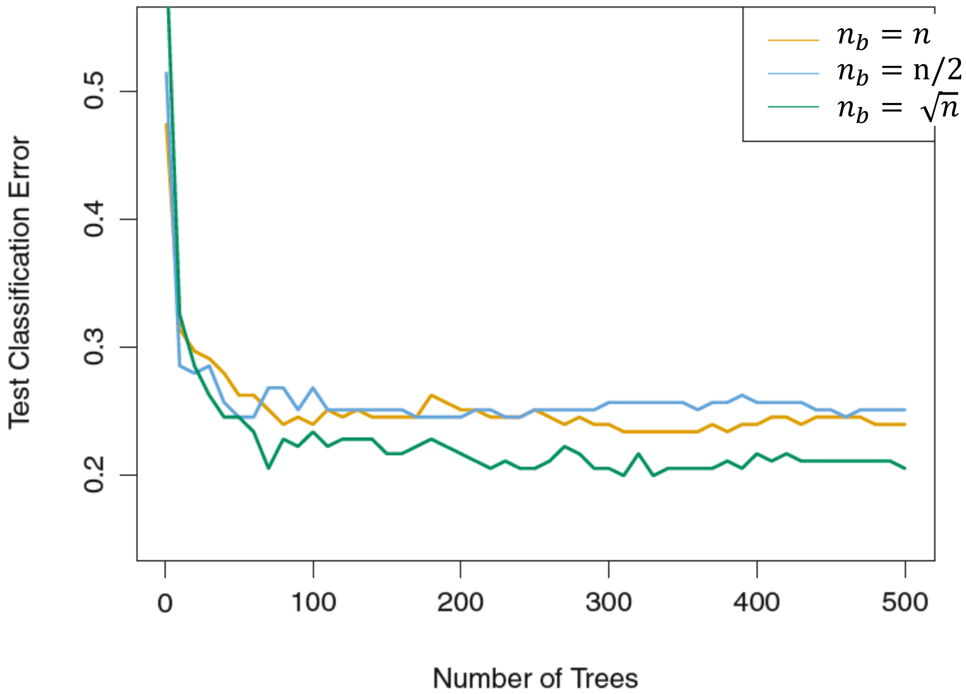


Random Forest

- **Problem:** Bagged trees will look quite similar to each other, so averaging them will not led to much reduction of variance!

- **Solution:** Random Forest constructs multiple trees where each tree uses n_b random features from the n initial features (generally $n_b = \sqrt{n}$)

- $n_b = n$-> Bagging case



Random Forest

- Both training and prediction are very fast, because of the simplicity of the underlying decision trees.
- Tasks can be straightforwardly parallelized, because the individual trees are entirely independent entities.
- The multiple trees allow for a probabilistic classification: a majority vote among estimators gives an estimate of the probability
- RF is a Nonparametric model, extremely flexible, and can thus perform well on tasks that are under-fit by other models.

Random Forest

- Hyper-Parameters Tuning
  - d: Depth of the trees
  - B: number of Bags (Estimators)

Supervised Learning

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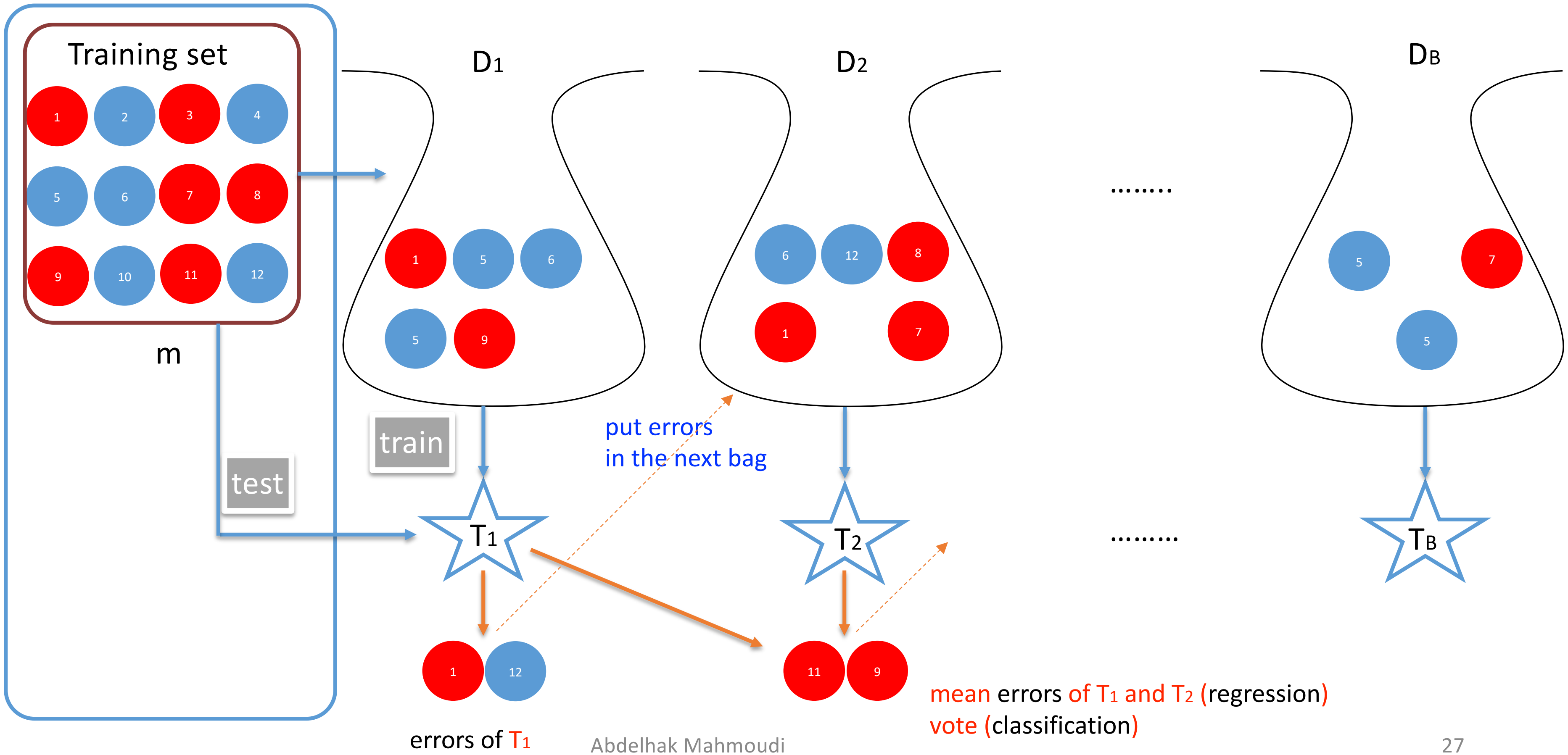
Boosting


Training set
1 2 3 4
5 6 7 8
9 10 11 12
m
test
train
D1
1 5 6
5 9
D2
6 12 8
1 7
....
DB
5 7
5
T1
put errors
in the next bag
T2
....
TB
1 12
errors of T1
11 9
mean errors of T1 and T2 (regression)
vote (classification)

Boosting

- Outperforms RF
- Smaller Trees (depth = 1) are sufficient because the growth of a particular tree takes into account preceding trees.
- Smaller trees can aid in interpretability.
- Boosting (Freund & Schapire 1990)
- Adaboost (Adaptive Boosting), 1996