Zürcher Hochschule
für Angewandte Wissenschaften

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Digital Image Processing
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Exercise Sheet 4

1 Convolution in Frequency Domain

Let x be an image with size 1024×768 pixels. We want to apply a 37×37 filter h to this image. In order to make this computationally efficient the convolution is carried out in frequency domain.

1. Briefly list the most important steps to carry out the convolution in frequency domain.
2. What is the minimum padding size?
3. What would be the minimum FFT-friendly padding size?

2 Spatial Derivatives in Frequency Domain

Given an image $f[x, y]$ the first order derivative in x-direction can be written as

$$g_x[x, y] = f[x + 1, y] - f[x, y].$$

Similarly, the first order derivative in y-direction can be written as

$$g_y[x, y] = f[x, y + 1] - f[x, y].$$

Both operations can be written as convlutions. **Hint:** Recall that

$$f[x, y] = F[u, v] \Longrightarrow f[x + 1, y] = F[u, v] \cdot \exp\left(-j2\pi \frac{u}{M}\right)$$

1. Write down the convolution kernel h_x that computes g_x.
2. Write down the convolution kernel h_y that computes g_y.
3. Write down the filter functions H_x and H_y in frequency domain (i.e., the Fourier transforms of h_x and h_y).

3 Sketch Spectra

Please sketch die spectra of the following functions (you do not need a computer or calculator for that).

1. $$g_1[x,y] = \sin\left(2\pi\ \frac{10}{M}x\right) + \sin\left(2\pi\ \frac{10}{N}y\right)$$

2. $$g_2[x,y] = \sin\left(2\pi\ \frac{10}{M}x\right) + \sin\left(2\pi\ \frac{5}{N}y\right)$$

3. $$g_3[x,y] = \sin\left(2\pi\ \frac{10}{M}x + 2\pi\ \frac{10}{N}y\right)$$

4. $$g_4[x,y] = \sin\left(2\pi\ \frac{10}{M}x + 2\pi\ \frac{10}{N}y\right)$$

Solutions

Exercise 1 – Convolution in Frequency Domain

1.
   - Zero-padding (image and filter)
   - 2-D FFT of zero-padded image and filter
   - Element-wise multiplication of the spectra
   - 2D iFFT
2. 1061×805
3. 2048×1024

Exercise 2 – Spatial Derivatives in Frequency Domain

Assume in the following that the filter length is N.

1. $$h_x = \begin{bmatrix} 1 & -1 \end{bmatrix}$$
2. $$h_y = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$
3. $$H_x(u,v) = \left[1 - \exp\left(-2\pi i \frac{u}{N}\right)\right]$$
4. $$H_y(u,v) = \left[1 - \exp\left(-2\pi i \frac{v}{N}\right)\right]$$

Exercise 3 – Sketch Spectra

The spectrum of a sine wave is a set of two δ-distributions.

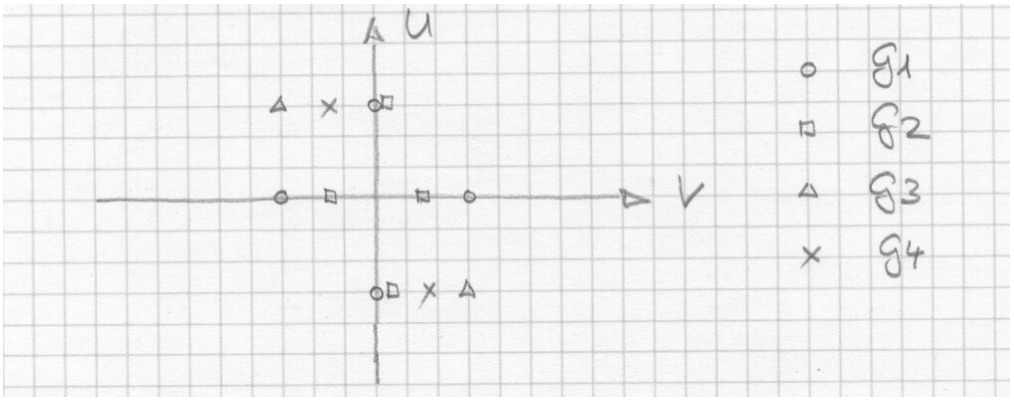

Figure 1: Spectra of different functions.