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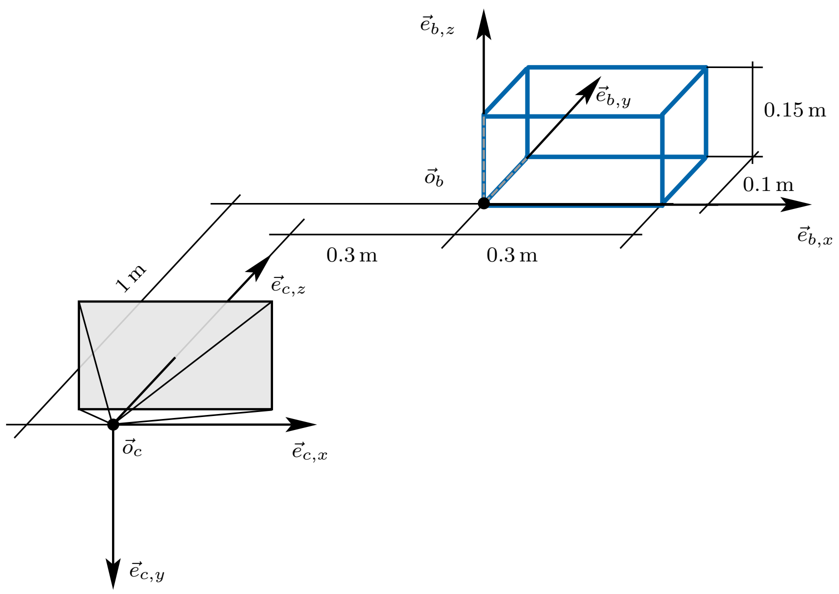
Laboratory – Camera Simulation

Learning goals

- you can properly choose the focal length of a camera
- you can intuitively express the pose between object and camera
- you can simulate the image of a 3d-object

Introduction



The sensor chip of the camera has the dimensions 8×6 mm and a resolution of 4000×3000 pixels, its center lies on the position $\vec{o}_c$ and its long edge is parallel to the x-axis

The projection of object points expressed in b-coordinates to image coordinates is described with the formula from the lecture:

$$\underbrace{\begin{bmatrix} b^c_x\, p_z \\ b^c_y\, p_z \\ p_z \end{bmatrix}}_{\check{\boldsymbol{b}}^c} = \underbrace{\begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{K}_c} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} \boldsymbol{R}^c_b & \boldsymbol{t}^c_{cb} \\ [0,\,0,\,0] & 1 \end{bmatrix}}_{\boldsymbol{T}^c_b} \cdot \underbrace{\begin{bmatrix} \boldsymbol{p}^b \\ 1 \end{bmatrix}}_{\check{\boldsymbol{p}}^b} \tag{1}$$

Exercise 1. Focal Length and Pose

a) Determine the focal length F of the camera such that the sensor chip just captures the right border of the object.

b) Determine the focal lengths f_x and f_y.

c) Determine the image center coordinates c_x and c_y for an ideal camera, i.e. with the optical axis pointing exactly through the center of the sensor chip.

d) Express the relative pose $\boldsymbol{T}^c_b \in \mathbb{R}^{4\times 4}$ in numbers.

Exercise 2. Image Simulation

a) Complete the program to compute the 8 corner points of the object. A drawing method is already prepared in the scaffold.

b) Optionally, rotate the object around its z-axis $\vec{e}_{b,z}$ by some angle α and let the image be visualized again by your script. Now we must distinguish between (body) b-coordinates and rotated body coordinates which we shortly call r-coordinates. The resulting matrix can be written as

$$\boldsymbol{R}^r_b(\alpha) = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

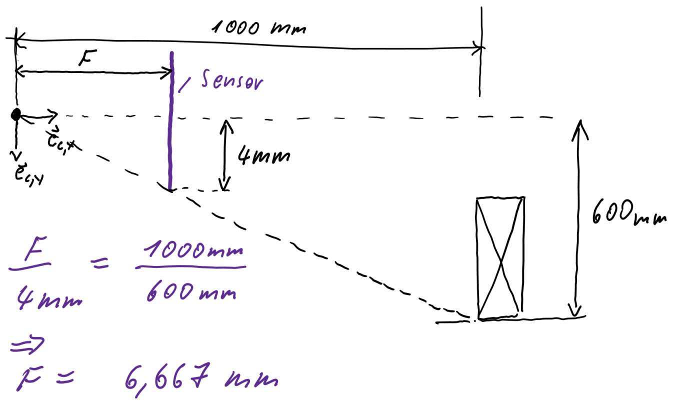
To keep our transforms layed out clearly, we rephrase (1):

$$\underbrace{\begin{bmatrix} b^c_x\, p_z \\ b^c_y\, p_z \\ p_z \end{bmatrix}}_{\check{\boldsymbol{b}}^c} = \underbrace{\begin{bmatrix} f_x & 0 & c_x \\ 0 & f_y & c_y \\ 0 & 0 & 1 \end{bmatrix}}_{\boldsymbol{K}_c} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \cdot \underbrace{\begin{bmatrix} \boldsymbol{R}^c_r & \boldsymbol{t}^c_{cr} \\ [0,\,0,\,0] & 1 \end{bmatrix}}_{\boldsymbol{T}^c_r} \cdot \underbrace{\begin{bmatrix} \boldsymbol{R}^r_b(\alpha) & \boldsymbol{t}^r_{rb} \\ [0,\,0,\,0] & 1 \end{bmatrix}}_{\boldsymbol{T}^r_b} \cdot \underbrace{\begin{bmatrix} \boldsymbol{p}^b \\ 1 \end{bmatrix}}_{\check{\boldsymbol{p}}^b},$$

with $\boldsymbol{t}^r_{rb} = [0,\,0,\,0]^T$.

Solution 1.

a)



b)

$$s_x = 4000/8\,\text{pixel/mm} = 500\,\text{pixel/mm}$$
$$s_y = 3000/6\,\text{pixel/mm} = 500\,\text{pixel/mm}$$

The Focal Length F expressed in units of pixel/mm is

$$f_x = F\,s_x = 6.667\,\text{mm} \cdot 500\,\text{pixel/mm} = 3333.33\,\text{pixel}$$
$$f_y = F\,s_y = 6.667\,\text{mm} \cdot 500\,\text{pixel/mm} = 3333.33\,\text{pixel}$$

In the standard solution the round value $f_x = f_y = 3350$ was chosen to be somewhat larger so that a small margin will be visible on left of the objects left border.

c) $c_x = 4000/2\,\text{pixel} = 2000\,\text{pixel}$,
$c_y = 3000/2\,\text{pixel} = 1500\,\text{pixel}$

d) The column vectors of the rotation matrix express the b-basis by means of c-coordinates:

$$\boldsymbol{R}_b^c = \left[\boldsymbol{e}_{b,x}^c,\ \boldsymbol{e}_{b,y}^c,\ \boldsymbol{e}_{b,z}^c\right]$$

where with $\vec{e}_{b,x} = \vec{e}_{c,x}$

$$\boldsymbol{e}_{\boldsymbol{b,x}}^{\boldsymbol{c}} = [1,\ 0,\ 0]^T$$

with $\vec{e}_{b,y} = \vec{e}_{c,z}$

$$\boldsymbol{e}_{\boldsymbol{b,y}}^{\boldsymbol{c}} = [0,\ 0,\ 1]^T$$

with $\vec{e}_{b,z} = -\vec{e}_{c,y}$

$$\boldsymbol{e}_{\boldsymbol{b,z}}^{\boldsymbol{c}} = [0,\ -1,\ 0]^T.$$

Hence,

$$\boldsymbol{R}_b^c = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}.$$

The translation $\boldsymbol{t}_{cb}^c$ expresses the vector from origin $\vec{o}_b - \vec{o}_c$ in c-coordinates, i.e. we get

$$\boldsymbol{t}_{cb}^c = \left[0.3\,\text{m},\quad 0\,\text{m},\quad 1\,\text{m}\right]^T$$

Applying the correct units we get

$$\boldsymbol{T}_b^c = \begin{bmatrix} \boldsymbol{R}_b^c & \boldsymbol{t}_{cb}^c \\ [0,\ 0,\ 0] & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0.3\,\text{m} \\ 0 & 0 & -1 & 0\,\text{m} \\ 0 & 1 & 0 & 1\,\text{m} \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$