

## Lab 4 – Frequency Domain Operations

### 1 Introduction

This lab also deals with the topic of digital image improvement. This time, we will work in frequency domain.

### 2 Learning Objectives

- You know how to filter an image in frequency domain.
- You know how to employ a Wiener filter to remove motion blur.

### 3 Tasks

#### 3.1 Filtering in Frequency Domain

In the following we want to apply a simple averaging filter  $h$  (with varying filter size) to a grayscale image (`MenInDesert.jpg`). This image is shown in Figure 1.

1. Load and display the image (`MenInDesert.jpg`). What are the image dimensions?
2. Compute the image's two-dimensional Fourier transform  $F$ . The easiest way to do so is employing `scipy.fft.fft2(...)`. Which data type does the transformed image have? Read the corresponding documentation, in particular the part about zero-padding.
3. Display both the original image and the *modulus* of it's Fourier transform.
4. Transform the image back into real space using the `scipy.fft.ifft2(...)`-command. What size does the image have after back-transformation? Why? Compare the data types of the original image and the back-transformed image.
5. Create and apply a simple  $9 \times 9$  averaging filter and apply it to the original image using the `convolve2d(...)`-command. This carries out the convolution in real space.



Figure 1: Men in desert—the reference image

6. Now, compute the filter spectrum  $H$ . Apply it to the transformed image by multiplying both spectra ( $G = F * H$ ) and convert the product back into real space.
7. Make sure that both computations (in real and in frequency space) feature the same result.
8. Compare the computation times for both approaches. Then, increase the size of both image and filter mask by a factor of 2 and carry out the same comparison. What happens if you again increase both sizes by the same factor?

### 3.2 Image Reconstruction Using Wiener Filter

In this exercise we intend to reconstruct the blurred image  $g(x, y)$ —shown in Figure 2—by means of an optimal Wiener filter. The reconstructed image is denoted as  $f$ . In frequency domain, the Wiener filter can be written as

$$\hat{F}(u, v) = \frac{H^*(u, v)}{|H(u, v)|^2 + K} G(u, v)$$

Here, the asterisk denotes the complex conjugate and the parameter  $K$  is a real-valued constant.

1. The blurred image was computed from the original image by applying a motion blur filter mask  $h$ . This filter mask is already provided in the file `exercise_2.py`. It contains two parameters:  $\alpha$  indicates the direction of the motion, and  $L$  denotes the shift.



Figure 2: Blurred image with hidden text

2. Extend the given script by implementing the above equation. Try to reconstruct the image with your estimated values for  $L$  and  $\alpha$ . Which parameter value  $K$  yields the best results? What does the original text say?