3. HMMs for Typo Correction

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```
In [1]: import numpy

        # load the emission and transition probabilities
        transition = numpy.loadtxt("typing_transition_matrix.csv", delimiter = ",", skiprows = 1
        emission = numpy.loadtxt("typing_emission_matrix.csv", delimiter = ",", skiprows = 1)
```

1 a) Implement the forward-backward algorithm to obtain samples from the posterior distribution of the latent state sequence given the observed sequence.

```
In [2]: def filterForward(transition, emission, data):
            """ Calculates and returns the probability of each hidden state being in each locati
            sequence (translated into indices of the emission table) and probabilities of transi

            # empty forward message
            m = numpy.empty((len(data), transition.shape[0]))
            # and a variable to hold the sum of the log normalizing constants
            normc = 0
            # fill the first column
            m[0, :] = emission[:, data[0]]/transition.shape[0]
            # and normalize
            normc = numpy.log(numpy.sum(m[0, :]))
            m[0, :] = m[0, :]/numpy.sum(m[0, :])

            # loop through the message and fill each column
            for t in range(1, len(data)):
                # calculate the message
                m[t, :] = numpy.dot(numpy.transpose(transition), m[t-1, :])*emission[:, data[t]]
                # and normalize it
                normc += numpy.log(numpy.sum(m[t, :]))
                m[t, :] = m[t, :]/numpy.sum(m[t, :])

            # return the forward message and the cumulative log normalizing constant
            return m, normc
```

```
def backSample(transition, emission, data):
    """ Sample hidden states backwards. """

    # first calculate the probability of each hidden state being in each location by fil
    m, normc = filterForward(transition, emission, data)

    # create a list to hold the sampled states
    samples = numpy.empty(len(data), dtype = int)
    # initialize the last letter based on m
    samples[len(data)-1] = numpy.random.choice(27, p = m[len(data)-1])

    # loop thorugh the data backwards and sample hidden states
    for d in range(len(data)-2, -1, -1):
        # calculate the distribution for the next hidden state
        dist = m[d]*transition[:, samples[d+1]]
        # and normalize it
        dist = dist/numpy.sum(dist)
        # now sample an index from the distribution we just generated
        samples[d] = numpy.random.choice(27, p = dist)

    # return the samples
    return samples
```

2 b) Use the algorithm to sample a few thousand possible intended sequences given the observed sequence "kezrninh".

3 c) Check to see if the results are actual words and only print out the ones that are.

```
In [3]: import text_utils

        # transform the string into a sequence of indicies for convenience
        d = text_utils.encode_string_to_int_list("kezrninh")
        # save the dictionary
        engdict = text_utils.dict_from_file("brit-a-z.txt")

        # run the algorithm 1000 times, printing out each possible intended sequece that is a re
        for i in range(5000):
            r = backSample(transition, emission, d)
            result = text_utils.decode_int_list_to_string(r)
            # check to see if the word is in the dictionary
            if text_utils.check_validity(result, engdict):
                print(result)
```

```
lead inn
learning
meat inn
```

```
learning
mead inn
learning
lest inn
learning
jest inn
learning
learning
lead inn
learning
learning
```

4 d) Implement a Gibbs sampler that alternates between sampling hidden sequences conditioned on a best guess for transition and emission, and sampling transition and emission probabilities based on the current guess at the hidden sequence.

```
In [4]: def HMMGibbs(data, iters, alpha_transition, alpha_emission, K_emission):
            """ Uses Gibbs sampling to estimate the posterior on the transition and emission pro

            # array to store all of the generated sequences
            seqs = numpy.empty((iters, data.shape[0]), dtype = int)
            # with the input data as the first sequence
            seqs[0, :] = data
            # another to store the generated transition probabilities
            transitions = numpy.empty((iters, 27, 27))
            # and the generated emission probabilities
            emissions = numpy.empty((iters, 27, 27))

            # alternate updating parameters to sample from the posterior and sampling sequences
            for i in range(iters):
                # update each row individually
                for k in range(27):
                    # grab the indicies of the current best guess with value k
                    ks = numpy.where(seqs[i, :] == k)[0]

                    # calculate the number of times each transition occurs
                    nk_transition = numpy.zeros(27)
                    for kdex in ks:
                        if kdex < data.shape[0]-1:
                            nk_transition[seqs[i, kdex+1]] += 1
                    # update the posterior on the transitions
                    transition_posterior = nk_transition + alpha_transition/27
                    # sample transitions from the dirichlet
                    transitions[i, k, :] = numpy.random.dirichlet(transition_posterior)
```

```
            # calculate the number of times each emission occurs
            nk_emission = numpy.zeros(27)
            for kdex in ks:
                nk_emission[data[kdex]] += 1
            # update the posterior on the emissions
            emission_posterior = nk_emission + alpha_emission/27
            # add K to the value corresponding to the intended letter being generated
            emission_posterior[k] += K_emission
            # sample emissions from the dirichlet
            emissions[i, k, :] = numpy.random.dirichlet(emission_posterior)

        # sample a new sequence using the forward-backward algorithm unless this is the
        if i+1 < iters:
            seqs[i+1, :] = backSample(transitions[i, :, :], emissions[i, :, :], data)

    # return the results
    return seqs, transitions, emissions
```

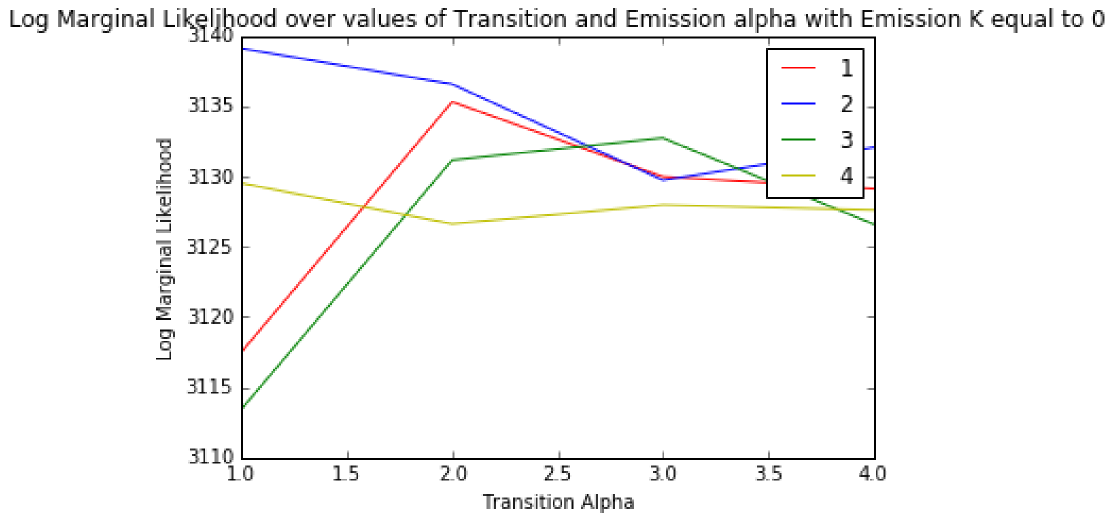
5 e) Generate three random sequences using the ground truth emission and transition probabilities and use them to train, determine the appropriate hyper-parameter values, and test the Gibbs sampler. Compare the best results to the actual probabilities.

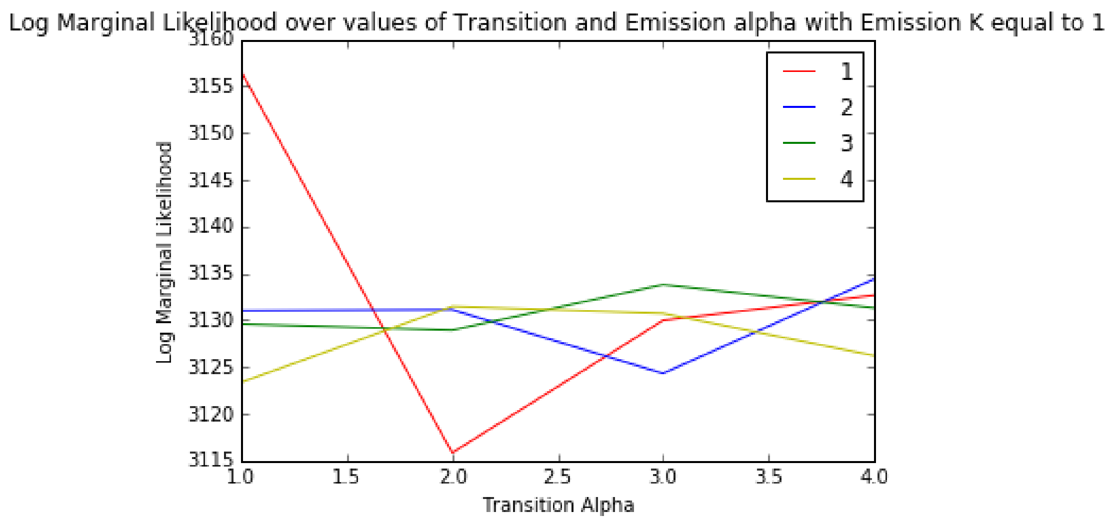
```
In [5]: def generateSeq(transition, emission, length):
            """ Generates a random sequence of the provided length using the provided emission a

            # create a list to hold the hidden states
            hiddens = numpy.empty(length, dtype = int)
            # and another to hold the resulting visible states
            visibles = numpy.empty(length, dtype = int)

            # generate the designated number of hidden and visible states
            for i in range(length):
                if i == 0:
                    # choose a random starting hidden state
                    hiddens[i] = numpy.random.choice(27)
                else:
                    # or use the previous hidden state to determine the next
                    hiddens[i] = numpy.random.choice(27, p = transition[hiddens[i-1], :]/numpy.s
                # and generate the corresponding visible state
                visibles[i] = numpy.random.choice(27, p = emission[hiddens[i], :]/numpy.sum(emis

            # return the completed sequences
            return hiddens, visibles

In [6]: def validation(trainset, validset, transition_alphas, emission_alphas, emission_Ks):
```

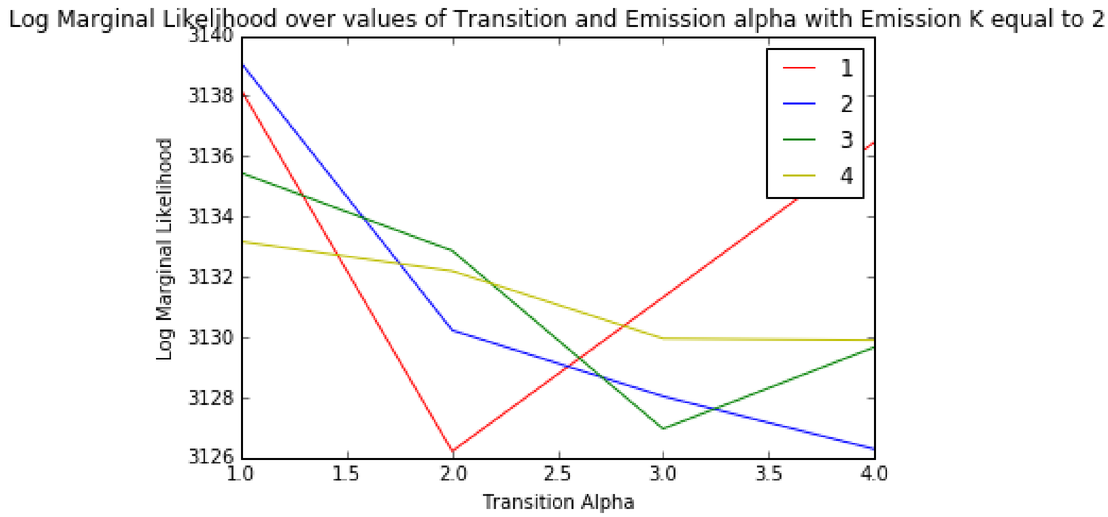
```
    """ Finds the marginal likelihood of the validation set for each combination of para

    # create an array to hold the marginal likelihood of each combination
    marginalL = numpy.empty((len(transition_alphas), len(emission_alphas), len(emission_

    for ta in range(len(transition_alphas)):
        for ea in range(len(emission_alphas)):
            for eK in range(len(emission_Ks)):
                # run Gibbs sampling for each combination of parameters
                seq, transition, emission = HMMGibbs(trainset, 5000, transition_alphas[t
                                                     emission_alphas[ea], emission_alpha
                # average all the post burn in transition and emission values
                transition = numpy.mean(transition[1000:, :, :], axis = 0)
                emission = numpy.mean(emission[1000:, :, :], axis = 0)
                # use the average parameter values to find the likelihood of the validat
                m, normc = filterForward(transition, emission, validset)
                # calculate the log likelihood of the data, save it and print it
                marginalL[ta, ea, eK] = numpy.sum(numpy.log(m[-1, :]))-normc

    # return the array of marginal likelihoods for analysis
    return marginalL
```

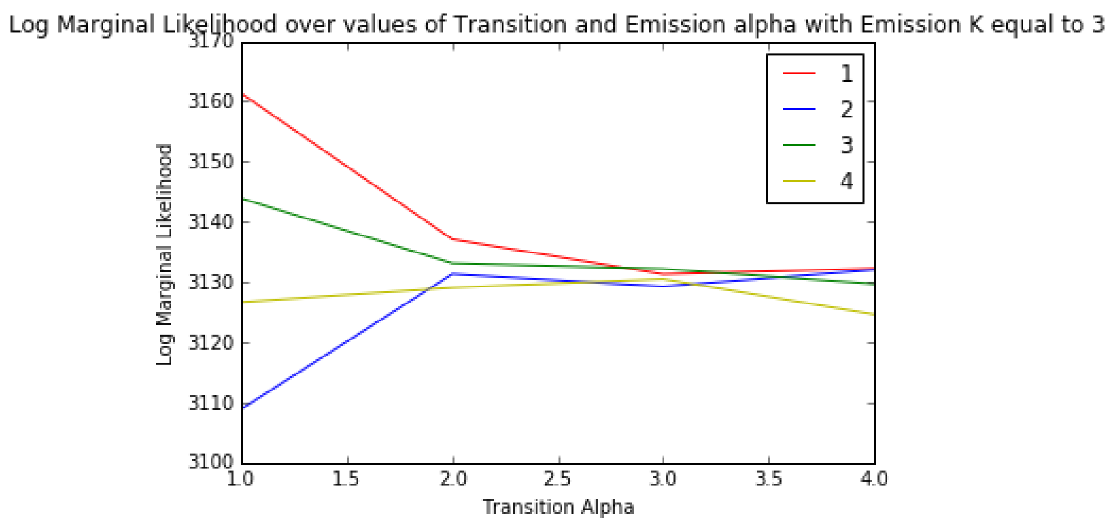
In [7]:
```
# generate the training, validation, and test sets
trainhid, trainvis = generateSeq(transition, emission, 1000)
validhid, validvis = generateSeq(transition, emission, 1000)
testhid, testvis = generateSeq(transition, emission, 1000)

marginalL = validation(trainvis, validvis, range(1, 5), range(1, 5), range(1, 5))
```

In [9]:
```
import matplotlib.pyplot as plt

for i in range(4):
    plt.title("Log Marginal Likelihood over values of Transition and Emission alpha with
    plt.plot(range(1, 5), marginalL[:, 0, i], "r", range(1, 5), marginalL[:, 1, i], "b",
         range(1, 5), marginalL[:, 2, i], "g", range(1, 5), marginalL[:, 3, i], "y")
    plt.legend(range(1, 5))
    plt.ylabel("Log Marginal Likelihood")
    plt.xlabel("Transition Alpha")
    plt.show()
```


Log Marginal Likelihood over values of Transition and Emission alpha with Emission K equal to 0
Log Marginal Likelihood
Transition Alpha
1
2
3
4
3140
3135
3130
3125
3120
3115
3110
1.0
1.5
2.0
2.5
3.0
3.5
4.0


Log Marginal Likelihood over values of Transition and Emission alpha with Emission K equal to 1
Log Marginal Likelihood
Transition Alpha
1
2
3
4
3160
3155
3150
3145
3140
3135
3130
3125
3120
3115
1.0
1.5
2.0
2.5
3.0
3.5
4.0



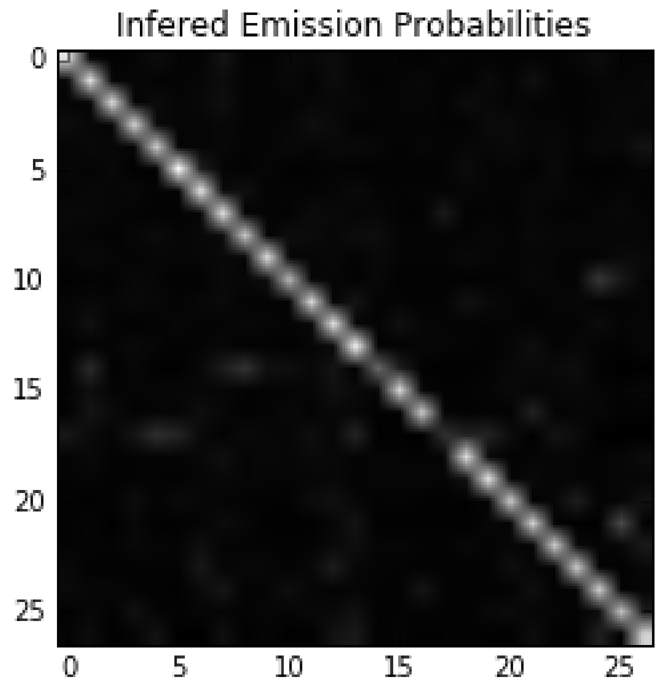


In [20]:
```
# write the Log Marginal Likelihoods to file so I don't have to do this again
for k in range(4):
    numpy.savetxt("3e_likelihood_eK" + str(k+1) + ".csv", marginalL[:, :, k], fmt = "%.
```

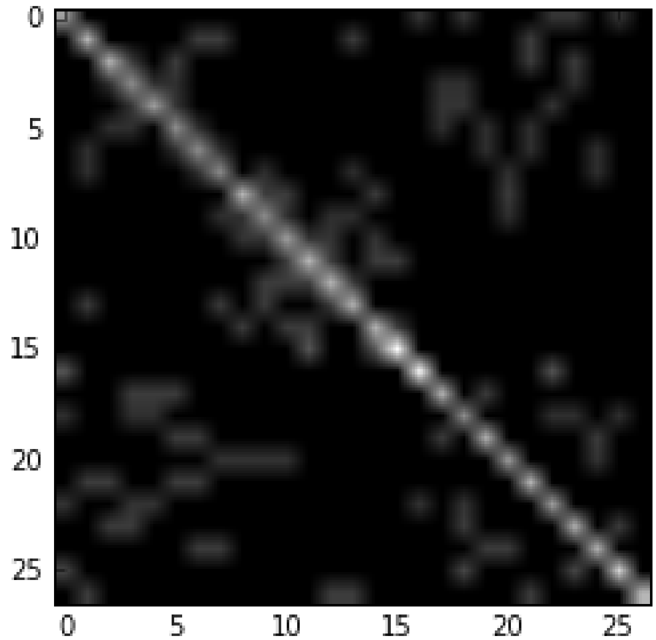
It looks like the optimal parameters are alpha of 1 for transition, an alpha of 2 for emission, and a K of 3. I can now use those to find the emission and transition probabilities for the test set.

In [8]:
```
import matplotlib.pyplot as plt

seq, esttransition, estemission = HMMGibbs(testvis, 5000, 1, 2, 3)
```

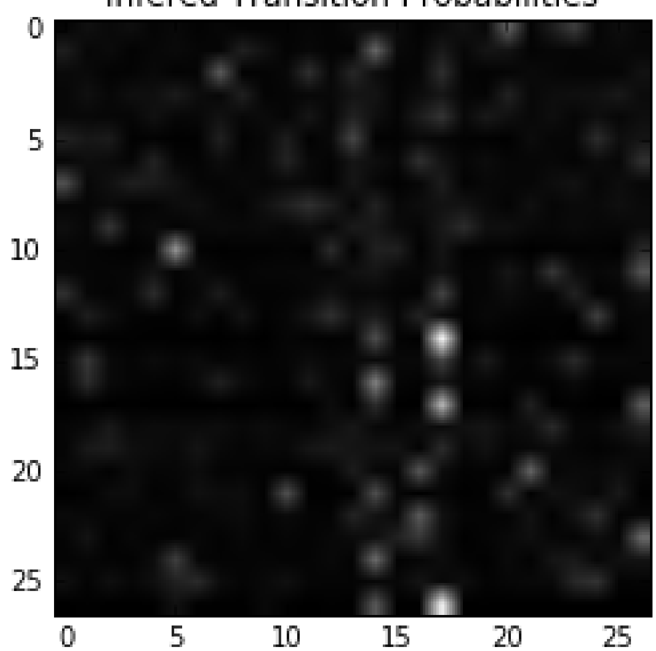
```python
# plot the emission and transition probabilities to compare to the actual values.
plt.imshow(numpy.mean(estemission[2000:, :, :], axis = 0), cmap = "gray")
plt.title("Infered Emission Probabilities")
plt.show()
plt.imshow(emission, cmap = "gray")
plt.title("Actual Emission Probabilities")
plt.show()
plt.imshow(numpy.mean(esttransition[2000:, :, :], axis = 0), cmap = "gray")
plt.title("Infered Transition Probabilities")
plt.show()
plt.imshow(transition, cmap = "gray")
plt.title("Actual Transition Probabilities")
plt.show()
```

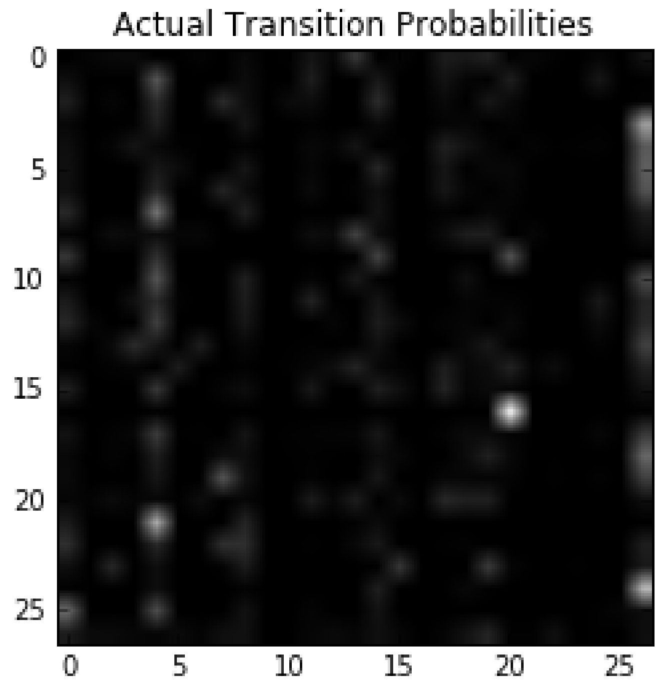


Actual Emission Probabilities



Infered Transition Probabilities





The images are rather blurry due to all the comparisons, but major trends are visible. There are some significant differences between the infered and actual transition and emission probabilities. Most clear among them is the fact that the infered probabilities tend to be lower than the actual ones. However, they exhibit a similar structure, with the diagonal of 1s in the emission probabilities and the bright vertical stripes in the transition probabilities. The stripes are in different places, but that may reflect the somewhat small sentences I used.