Time, Speed and Distance

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Part I: Time, Speed and Distance

1. Time, Speed and Distance

1.1 Summary and Formulae

A. Formulas

$$\underbrace{\underbrace{D}_{Distance} = \underbrace{S}_{Speed} \times \underbrace{T}_{Time}}_{Distance}, \qquad \underbrace{T = \frac{D}{S}}_{Time}, \qquad \underbrace{S = \frac{D}{T}}_{Speed}$$

1.2 Calculating Distance

A. Formula

$$\underbrace{D}_{Distance} = \underbrace{S}_{Speed} \times \underbrace{T}_{Time}$$

Example 1.1

Zeno can walk five miles in an hour. How many miles can he walk in the following times:

A. Two hours
B. Five hours

$$5\frac{miles}{hour} \times 2\ hours = 10\ miles$$

$$5\frac{miles}{hour} \times 5\ hours = 25\ miles$$

Example 1.2

Zeno can walk five miles in an hour. How many miles can he walk in the following times:

A. Half an hour
B. One-fourth of an hour
C. One-third of an hour

$$5\frac{miles}{hour} \times \frac{1}{2}\ hours = \frac{5}{2} miles$$
$$5\frac{miles}{hour} \times \frac{1}{4}\ hours = \frac{5}{4} miles$$
$$5\frac{miles}{hour} \times \frac{1}{3} hour = \frac{5}{3} miles$$

Example 1.3

Zeno can walk five miles in an hour. How many miles can he walk in the following times:

A. 15 minutes
B. 20 minutes
C. 30 minutes

$$5\frac{miles}{hour} \times 15\ minutes = 5\frac{miles}{hour} \times \frac{15}{60}\ hours = 5\frac{miles}{hour} \times \frac{1}{4}\ hours = \frac{5}{4} miles$$
$$5\frac{miles}{hour} \times 20\ minutes = 5\frac{miles}{hour} \times \frac{20}{60}\ hours = 5\frac{miles}{hour} \times \frac{1}{3}\ hours = \frac{5}{3} miles$$

$$5\frac{miles}{hour}\times 30\ minutes = 5\frac{miles}{hour}\times\frac{30}{60}\ hours = 5\frac{miles}{hour}\times\frac{1}{2}\ hours = \frac{5}{2}miles$$

1.3 Speed

A. Definition

Speed is distance per unit time. Speed can be expressed in different units for the time, and different units for the distance. Some standard units of speed are given below:

A. Miles per Hour $=\frac{miles}{hour}$
B. Kilometers per Hour $=\frac{km}{hour}={}^{km}/_{hr}$
C. Meters per Second $=\frac{meters}{second}={}^{m}/_{s}$

B. Properties of Speed

- Speed is never negative
- The lowest value of speed can be zero

C. Formula

$$\boldsymbol{S=\frac{D}{T}}$$

Example 1.4

Find the speed of a vehicle that travels 40 km in two hours.

$$S=\frac{D}{T}=\frac{40\ km}{2\ hours}=\frac{40}{2}\ {}^{km}/_{hr}=20\ {}^{km}/_{hr}$$

Example 1.5

A car covers 120 km in five hours. What is the speed of the car?

$$S=\frac{D}{T}=\frac{120\ km}{5\ hours}=24\ {}^{km}/_{hr}$$

Example 1.6

At what speed(in m/s) does a bicyclist who pedals 1500 meters in 300 seconds travel?

$$S=\frac{D}{T}=\frac{1500\ meters}{300\ seconds}=5\ {}^{m}/_{s}$$

Example 1.7

Find the speed of a man who walks 20 km in four hours.

$$S=\frac{D}{T}=\frac{20\ km}{4\ hours}=5\ {}^{km}/_{hr}$$

Example 1.8

Find the speed of a child who runs 300 meters in the playground in five minutes (in meters/minute)

$$S=\frac{D}{T}=\frac{300\ meters}{5\ minutes}=60\ {}^{m}/_{min}$$

D. Application

Example 1.9
A truck goes from City A to City B and back in three hours. The distance from City A to City B is 20 km. Find the speed of the truck.

$$\textit{Distance}: \underbrace{A \to B}_{20\ km}, \quad \underbrace{B \leftarrow A}_{20\ km} \Rightarrow 20 + 20 = 40\ km$$

$$\textit{Speed} = \frac{D}{T} = \frac{(20+20)\ km}{3\ hours} = \frac{40}{3} {km}/{hr}$$

Example 1.10
If a bicycle can cover 15 miles in 3 hours, find its speed. Also, find the speed of a man who can cover 10 miles less in the same time.

$$\textit{Bicycle}: S = \frac{D}{T} = \frac{15\ miles}{3\ hours} = 5\ {miles}/{hr}$$

$$\textit{Man}: S = \frac{D}{T} = \frac{15\ miles - 10\ miles}{3\ hours} = \frac{5\ miles}{3\ hours} = \frac{5}{3}\ {miles}/{hr}$$

Example 1.11
A dog can run 100 units in two hours, while a deer can run 50 units in thirty minutes. Find the difference in the speeds of the two animals.

$$\text{Required Difference} = \text{S}_{\text{Deer}} - \text{S}_{\text{Dog}} = \frac{\text{D}_{\text{Deer}}}{\text{T}_{\text{Deer}}} - \frac{\text{D}_{\text{Dog}}}{\text{T}_{\text{Dog}}}$$

Substitute the values from the question and simplify:

$$\frac{50}{1/2} - \frac{100}{2} = \left(50 \div \frac{1}{2}\right) - 50 = (50 \times 2) - 50 = 100 - 50 = 50$$

Example 1.12
A snail can cover 2.5 meters in fifty minutes. Find the speed of the snail in meters per minute.

$$S = \frac{D}{T} = \frac{2.5\ meters}{50\ minutes} = \frac{2.5}{50} {m}/{min} = \frac{5}{100} {m}/{min} = \frac{1}{20} {m}/{min}$$

Example 1.13
An earthworm can crawl past a 1-foot ruler in 2 minutes. Ignoring the length of the earthworm, find its speed in inches/minute.

$$S = \frac{D}{T} = \frac{1\text{ foot}}{2\ minutes} = \frac{12\ inches}{2\ minutes} = 6\ {inches}/{minute}$$

1.4 Calculating Time

A. Formula

$$\boldsymbol{T = \frac{D}{S}}$$

Example 1.14

A man walking at a speed of five miles per hour travels a distance of 15 miles. What is the time taken?

Substitute the values in $\frac{D}{S}$

$$\frac{D}{S} = \frac{15\ miles}{5\ {}^{miles}/_{hour}} = \underbrace{15\ miles \div \left(5\ {}^{miles}/_{hour}\right)}_{\text{Convert fractions to division}} = \underbrace{15\ miles \times \frac{1}{5}\ {}^{hours}/_{mile}}_{\substack{\text{Convert division to multiplication}\\ \text{by taking the reciprocal}}} = 3\ hrs$$

Example 1.15

A distance of 25,000 meters is covered by a car travelling at a speed of 3 km per hour. What is the time taken? Give your answer in hours and minutes.

$$T = \frac{D}{S} = \frac{25{,}000\ meters}{3\frac{km}{hr}} = \frac{25\ km}{3\ {}^{km}/_{hr}} = \frac{25}{3} hours = 8\frac{1}{3} hours = 8\ hrs\ 20\ minutes$$

Example 1.16

How many seconds does a snail inching along at a speed of 2 feet per minute take to cover a distance of 5 feet?

$$T = \frac{D}{S} = \frac{5\ feet}{2\ {}^{feet}/_{min}} = 5\ feet \div 2\frac{feet}{min} = 5\ feet \times \frac{1\ min}{2\ feet} = \frac{5}{2} min = \frac{5}{2} \times 60\ seconds = 150\ seconds$$

Example 1.17

A rectangular field has length 40 meters, and breadth 20 meters. An earthworm traverses the length at a speed of 3 meters/hour. Find the time.

$$T = \frac{D}{S} = \frac{40\ meters}{3\ {}^{meters}/_{hour}} = \frac{40}{3} hours$$

Example 1.18

Find the time in minutes, for a car travelling at a speed of 30 km per hour to cross a bridge that is 1000 meters.

$$T = \frac{D}{S} = \frac{1000\ meters}{30\ {}^{km}/_{hr}} = \frac{1\ km}{30\ {}^{km}/_{hr}} = \frac{1}{30} hrs = \frac{1}{30} \times 60\ minutes = 2\ minutes$$

2. Rates

2.1 Rates

A. Formula

Rates questions are similar to the questions asked in time, speed and distance. In fact, TSD is a special case of rates.

Example 2.1

A. Sheela can type 20 words per minute. How many words can she type in:

I. One hour

$$20 \text{ words}: 1 \text{ Minute} = 20 \times 60 \text{ words}: 1 \times 60 \text{ Minutes} = 1200 \text{ words}: 1 \text{ Hour}$$

II. Half an hour

$$\frac{1}{2} \text{an hour} = 30 \text{ Minutes} \Rightarrow 20 \times 30 \text{ words}: 1 \times 30 \text{ Minutes} = 600 \text{ Words in half an hour}$$

III. One-third of an hour

$$20 \text{ words per minute} = 20 \times 20 \text{ words per } 20 \text{ minute} = 400 \text{ words in } \frac{1}{3} \text{rd of an hour}$$

IV. One-fourth of an hour

$$\underbrace{1200}_{\text{Words per Hour}} \times \frac{1}{4} = 300 \text{ Words}$$

V. One-fifth of an hour

$$\frac{1}{5} \text{th of an hour} = 60 \times \frac{1}{5} = 12 \text{ minutes} \Rightarrow \underbrace{20}_{Words\ per\ Minute} \times \underbrace{12}_{Minutes} = 240 \text{ Words}$$

VI. Half a minute

$$20 \times \frac{1}{2} = 10 \text{ Words}$$

VII. One-fourth of a minute

$$20 \times \frac{1}{4} = 5 \text{ Words}$$

Example 2.2

Nitesh is able to type 300 words in 15 minutes. How many words can he type in one hour?

$$300 \text{ words in } 15 \text{ minutes} \Rightarrow 300 \times 4 = 1200 \text{ words in } (15 \times 4 = 60 \text{ minutes} = 1 \text{ hour})$$

Example 2.3

Vanita is able to type 30 words in two minutes. How long will she take to type a document that has 275 words.

$$\textbf{Words per minute} = \frac{30}{2} = 15$$

$$\textbf{Time}_{\textbf{Vanita}} = \frac{\text{No. of Words}}{\text{Words per minute}} = \frac{275}{15} = 18\frac{5}{15} = 18\frac{1}{3} = 18 \text{ Minutes } 20 \text{ Seconds}$$

Example 2.4

Raju typed a five-page document, with each page having 20 words, in ten minutes.

VIII. How long will he take to type two documents, if the first document has five pages (with 35 words per page), and the second document has three pages (with 50 words per page).

IX. If he starts typing the documents at 12:24 pm, and takes a three-minute coffee break in the middle of typing, what is the time that he will complete typing both the documents.

X. If Raju's deadline to complete typing the document is 1:00 pm, how much time will he have left to spare when he completes typing.

$$\mathbf{Speed}_{\mathbf{Raju}} = \frac{\text{No. of Words}}{No.of\ Minutes} = \frac{5 \times 20}{10} = 10 \text{ words per minute}$$

$$\boldsymbol{Time}_{\boldsymbol{Raju}} = \frac{No.of\ Words}{Words\ per\ minute} = \frac{5 \times 35 + 3 \times 50}{10} = \frac{175 + 150}{10} = \frac{325}{10} = 32 \text{ Minutes } 30 \text{ Seconds}$$

$$\boldsymbol{End\ Time} = \underbrace{12{:}\,24\ pm}_{\textbf{Start Time}} + \underbrace{32\ Minutes\ 30\ Seconds}_{\textbf{Typing Time}} + \underbrace{3\ Minutes}_{\textbf{Coffee Break}} = 12{:}\,59{:}\,30$$

$$Time\ Left = 1{:}\,00{:}\,00\ pm - 12{:}\,59{:}\,30 \text{ pm} = 30 \text{ Seconds}$$

Example 2.5

A. Cost of a Single Item

I. If nine apples cost three dollars what is the cost of one apple?

$$Cost\ per\ Apple = \frac{Cost\ of\ All\ Apples}{No.of\ Apples} = \frac{9}{3} = 3 \text{ dollars}$$

II. If three pineapples cost 9 francs, what is the cost of one apple?

$$Cost\ per\ pineapple = \frac{Cost\ of\ All\ Pineapples}{No.of\ Pineapples} = \frac{3}{9} = \frac{1}{3} \text{ francs}$$

III. If three watermelons costs twelve rupees, what is the cost of one watermelon?

$$Cost\ of\ one\ watermelon = \frac{Cost\ of\ All\ watermelons}{No.of\ Watermelons} = \frac{12}{3} = 4\ Rupees$$

B. Cost of More than One Item

Example 2.6

Three cats can catch three rats in three minutes. How long will nine cats take to catch nine rats?

Example 2.7

I can drink half a glass of milk in one-fourth of a minute. If I want to drink one-fourth of a glass of milk in one-half of a minute, how many times faster or slower must I drink the milk?

3. Time and Work-I

3.1 Basics

A. Introduction

Example 3.1

Clara is building a house, which will take her 180 days to build.

1. Her two sisters, Jane and Jenna, are also building another house of the same type. In how many days can Jane and Jenna build the house?
2. If Jane and Jenna join Clara in building her house, in how many days can they build the house?

Part A:

Jane and Jenna are two people working on the same amount of work. So, they will get done twice as fast:

$$\underbrace{Jane + Jenna}_{Two\ People} \Rightarrow Time\ taken = \frac{180}{2} = 90$$

Part B:

Here we have three people working on the project. It will get done three times as fast:

$$Part\ B: \underbrace{Jane + Jenna + Clara}_{Three\ People} \Rightarrow Time\ taken = \frac{180}{3} = 60$$

B. Work = Man-Days

The work required to build the house can be measured. In the previous example, we can calculate the number of days that one person took to build the house. This concept is very useful.

$$Man - Days = No.of\ People \times No.of\ Days\ Worked\ by\ Each$$

Example 3.2

Calculate the number of Man-Days in the previous example for all three parts.

$$Man - Days = No.of\ People \times No.of\ Days\ Worked\ by\ Each$$

$$Part\ A: \underbrace{2}_{People} \times \underbrace{90}_{Days} = \underbrace{180}_{Man-Days}$$

$$Part\ B: \underbrace{3}_{People} \times \underbrace{60}_{Days} = \underbrace{180}_{Man-Days}$$

C. Calculating Man-Days

Example 3.3

Calculate the number of man-days in each part below.

A. Five people working for seven days
B. Seven people working for three days
C. Twelve people working for one day
D. Six working for half a day
E. Three people working for two days each, and four people working for five days each

$$5 \times 7 = 35$$
$$7 \times 3 = 21$$
$$12 \times 1 = 12$$
$$6 \times \frac{1}{2} = 3$$
$$3 \times 2 + 4 \times 5 = 6 + 20 = 26$$

D. Measuring Work: Different Units of Time, and Worker

It is not necessary that the unit of time be only days. It can be weeks, months, years, or any other unit specified in the question.

In such a case, we will still calculate the effort, but **the effort will not be measured in man-days**. Instead, it will be measured in the units given in the question.

Example 3.4

Calculate the work in each part below.

A. Five engineers working on designing a bridge for three weeks
B. Seven doctors working on researching a cure for two months each
C. Three Bulldozers clearing roadblocks working for 7 hours each
D. Three Beavers repairing a dam for twenty minutes each

$$Part\ A: \underbrace{5}_{Engineers} \times \underbrace{3}_{Weeks} = \underbrace{15}_{Engineer-Weeks}$$
$$Part\ B: \underbrace{5}_{Doctors} \times \underbrace{3}_{Months} = \underbrace{15}_{Doctor-Months}$$
$$Part\ C: \underbrace{3}_{Bulldozer} \times \underbrace{7}_{Hours} = \underbrace{21}_{Bulldozer-Hours}$$
$$Part\ C: \underbrace{3}_{Beaver} \times \underbrace{20}_{Minutes} = \underbrace{60}_{Beaver-Minutes} = \underbrace{1}_{Beaver-Hour}$$

E. Conversions in Effort

Example 3.5

Parineeti is planning her study of Biology. She has ten chapters to study, and each chapter will take her one hour. Every day she will study for two hours. Find Parineeti's effort in terms of

A. Hours
B. Days

$$10 \times 1 = 10\ Study - Hours$$
$$10 \div 2 = 5\ Study - Days$$

Example 3.6

Two weaver birds working for 7 hours are able to complete weaving a nest. If scientists have noted that one weaver works for 30 minutes each day, find the number of weaver-days needed to complete a nest.

$$\underbrace{2}_{Weavers} \times \underbrace{7}_{Hours} = \underbrace{14}_{Weaver-Hours} = \underbrace{28}_{Weaver\ Half-Hours} = \underbrace{28}_{Weaver-Days}$$

F. Finding Number of Workers

Example 3.7

A county is vaccinating its people against a newly discovered disease, for which one doctor working for 12 months is sufficient. How many doctors would be needed, if the vaccination is to be completed in:

A. Six Months
B. Three Months

$$Required\ Work: \underbrace{1}_{Doctors} \times \underbrace{12}_{Months} = \underbrace{12}_{Doctor-Months}$$

Method I

$$Part\ A: \underbrace{2}_{Doctors} \times \underbrace{6}_{Months} = \underbrace{12}_{Doctor-Months}$$
$$Part\ B: \underbrace{4}_{Doctors} \times \underbrace{3}_{Months} = \underbrace{12}_{Doctor-Months}$$

Method II

$$Part\ A: No.of\ Doctors = \frac{12\ Doctor-\mathbf{Months}}{6\ \mathbf{Months}} = \frac{12}{6} Doctors = 2\ Doctors$$
$$Part\ B: No.of\ Doctors = \frac{12\ Doctor-\mathbf{Months}}{3\ \mathbf{Months}} = \frac{12}{3} Doctors = 4\ Doctors$$

Example 3.8

Shubhangi is making a study plan for her exam. She knows she has 6 Units to study, and each Unit will take her 7 hours. She also needs 10 hours for Revision, and 8 hours for practicing mocks. Find the number of days she will need, if every day she studies for:

A. 1 hour
B. 2 hours
C. 3 hours
D. 4 Hours
E. 5 Hours
F. 6 Hours
G. 10 Hours
H. 12 Hours

$$Total\ Study\ Time = \underbrace{6}_{\substack{No.of\\Units}} \times \underbrace{7}_{Time\ per\ Unit} + \underbrace{10}_{Revision} + \underbrace{8}_{Mocks} = 42 + 10 + 8 = 60$$

G. Two Types of Workers

If there are two types of workers, then it is often necessary to calculate the work for each separately.

Example 3.9

Calculate the work in each part below:

A. Running a vaccination campaign requires three doctors and twelve nurses for four weeks
B. Building a house requires five laborers and two masons seven months

$$Part\ A: \underbrace{3}_{Doctors} \times \underbrace{4}_{Weeks} = \underbrace{12}_{Doctor-Weeks}, \quad \underbrace{12}_{Nurses} \times \underbrace{4}_{Weeks} = \underbrace{48}_{Nurse-Weeks}$$

$$Part\ B: \underbrace{5}_{Labourers} \times \underbrace{7}_{Months} = \underbrace{35}_{Labourer-Months}, \quad \underbrace{2}_{Masons} \times \underbrace{7}_{Months} = \underbrace{14}_{Mason-Months}$$

H. Finding Number of People

Example 3.10

It will take twelve software engineers six months to complete a project. If the project must be completed in four months, what is the number of people that must be put on the project.

$$Work = Engineer - Months = 12 \times 6$$

$$To\ complete\ in\ 4\ Months = \frac{12 \times 6\ Engineer - Months}{4\ Months} = 18\ Engineers$$

Example 3.11

It will take seven teachers five weeks to correct answer papers. If the work available for the correction increases to eight weeks, what is the minimum of number of teachers required?

$$Work = Teacher - Weeks = 7 \times 5 = 35$$

$$To\ complete\ in\ 8\ Weeks = \frac{35\ Teacher - Weeks}{8\ Weeks} = \frac{35}{8} Teachers = 4\frac{3}{8}\ Teachers \to 5\ Teachers$$

I. Reduction in Workforce

Example 3.12

Three workers working for four days each can finish a project. If, at the beginning of the project, one worker falls ill, then how long will it take the remaining workers to complete the project?

$$Work = Workers - Days = 3 \times 4$$

$$New\ Time\ Taken\ = \frac{3 \times 4\ Workers - Days}{2\ Workers} = 6\ Days$$

3.2 Calculating People

A. Calculating People

Example 3.13

A construction facility requires 500 man-days of labour to be completed. If it must be completed in 20 days, find the number of labourers required?

Example 3.14

A dog walking business has seven poodles that need to be walked for 20 minutes each, and eight Great Danes that need to be walked for 30 minutes each. If each dog-walker is available for one hour in the day, what is the number of dog-walkers needed?

$$Total\ Walking\ Time = \underbrace{7 \times 20}_{Poodles} + \underbrace{8 \times 30}_{Great\ Danes} = 140 + 240 = 380$$

$$No.of\ Dog\ Walkers = \frac{380}{60} = \frac{19}{3} = 6\frac{1}{3} \to 7\ Dog\ Walkers$$

3.3 Different Rates

Example 3.15

Holly's Horse Transport provides horses and ponies for transporting equipment in mountainous locations. One horse can carry as much weight as two ponies. She agreed to do a project that required three horses and four ponies. But the ponies fell ill. How many total horses will she need?

Example 3.16

On a construction project, a junior worker can do half the work that a senior worker can. The project needs 12 junior workers working for 7 days each, and 5 senior workers working for 3 days each to complete the project. If three senior workers working for 10 days each are available, what is the number of junior workers required to complete the project while working for nine days each?

3.4 Variables

Example 3.17

If m men can do a job in d days, then m+r men can do the job in:

A. $d + r \; days$
B. $d - r \; days$
C. $\frac{md}{m+r} \; days$
D. $\frac{d}{m+r} \; days$
E. None of these (AHSME 1950/19)

$$\frac{md}{m+r}$$

4. Time and Work-II

4.1 Doing Work Together

A. Two Workers

Example 4.1

Soham can paint a room in ten days. Rohan can paint the same room in six days. Both Soham and Rohan work eight hours every day. If they paint the room together how many

A. days will they take?
B. hours will they work on the last day of the painting.

Part A

Soham can paint the room in 10 days. Therefore, in 1 day, he can paint

$$\frac{1}{10} \text{ of the room}$$

Rohan can paint the room in 6 days. Therefore, in 1 day, he can paint

$$\frac{1}{6} \text{ of the room}$$

They are going to paint the room together, which means everyday they will paint

$$\underbrace{\frac{1}{10}}_{Soham} + \underbrace{\frac{1}{6}}_{Rohan} = \frac{8}{30} = \underbrace{\frac{4}{15}}_{Total} \text{ of the room}$$

Number of Days needed to paint the room

$$= 1 \div \frac{4}{15} = 1 \times \frac{15}{4} = \frac{15}{4} = 3\frac{3}{4}$$

Part B

No. of Hours worked on last day

$$= 8 \times \frac{3}{4} = 6 \text{ Hours}$$

Example 4.2

Pritam can dig a ditch in five hours. Ritam can dig a ditch in six hours. If they dig the ditch together, how many hours will they take?

$$\underbrace{\frac{1}{5}}_{Pritam} + \underbrace{\frac{1}{6}}_{Ritam} = \underbrace{\frac{11}{30}}_{Total} \Rightarrow \frac{30}{11} Hours = 2\frac{8}{11} Hours$$

Example 4.3

Jay can complete a project in seven weeks. Vijay can complete a project in ten weeks. If they work together, in how many weeks will they complete the project?

$$\frac{1}{7} + \frac{1}{10} = \frac{17}{70} \Rightarrow \frac{70}{17} = 4\frac{2}{17} Weeks$$

Example 4.4

Charlie can build a house in four months, working alone. William can build the same house, in seven months, also working alone. If they work together, how many months will they take to build the house?

$$\frac{1}{7}+\frac{1}{4}=\frac{11}{28}\Rightarrow\frac{28}{11}=2\frac{6}{11}\ Months$$

Example 4.5

Carpenter A can make five chairs in 1 hour. Carpenter B can make four chairs in 80 minutes. In how much time can they make a chair together?

Carpenter A:

$$5\ Chairs\ in\ 1\ Hour\Rightarrow 1\ Chair\ in\ 12\ Minutes\Rightarrow\frac{1}{12}\ of\ a\ Chair\ per\ minute$$

Carpenter B:

$$4\ Chairs\ in\ 80\ Minutes\Rightarrow 1\ Chair\ in\ 20\ Minutes\Rightarrow\frac{1}{20}\ of\ a\ Chair\ per\ minute$$

Together:

$$\frac{1}{12}+\frac{1}{20}=\frac{5+3}{60}=\frac{8}{60}=\frac{2}{30}of\ a\ chair\ per\ minute\Rightarrow 7.5\ Minutes$$

Example 4.6

Tap A can fill a swimming pool in three hours. Tap B can fill the swimming pool in four hours. If they are both opened at the same time, how long will they take to fill the swimming pool?

B. Three Workers

Example 4.7

Three different sheep shearing machines can shear a flock of sheep in three, four, and five hours respectively when working independently. Find the number of hours required to shear the flock of sheep if all three machines work together.

$$\frac{1}{3}+\frac{1}{4}+\frac{1}{5}=\frac{20+15+12}{60}=\frac{47}{60}\Rightarrow\frac{60}{47}\ of\ an\ hour$$

4.2 Doing Work Alone

A. Two Workers

Example 4.8

Priyesh and Ritesh can dig a ditch in five hours. Ritesh can dig a ditch in six hours. If Priyesh digs the ditch, how long will he take?

$$\underbrace{\frac{1}{5}}_{Together}-\underbrace{\frac{1}{6}}_{Ritesh}=\underbrace{\frac{1}{30}}_{Total}\Rightarrow 30\ Hours$$

Example 4.9

Carpenter A and B working together can make five chairs in 1 hour. Carpenter A can make four chairs in 80 minutes. In how much time can Carpenter B make a chair?

Carpenter A and B:

$$5\ Chairs\ in\ 1\ Hour \Rightarrow 1\ Chair\ in\ 12\ Minutes \Rightarrow \frac{1}{12}\ of\ a\ Chair\ per\ minute$$

Carpenter B:

$$4\ Chairs\ in\ 80\ Minutes \Rightarrow 1\ Chair\ in\ 20\ Minutes \Rightarrow \frac{1}{20}\ of\ a\ Chair\ per\ minute$$

Together:

$$\frac{1}{12} - \frac{1}{20} = \frac{5-3}{60} = \frac{2}{60} = \frac{1}{30} of\ a\ chair\ per\ minute \Rightarrow 30\ Minutes$$

Example 4.10

A and B together can build a house in 30 days. A works at twice the speed of B. In how many days will A alone complete the work.

Every day

$$B\ does\ 1\ unit\ of\ work \Rightarrow A\ does\ 2\ Units\ of\ Work$$

The total work done

$$= 1 + 2 = 3\ Units\ of\ Work$$

Out of the 3 units, A does 2 Units

$$= \frac{2}{3} rd\ of\ the\ Work$$

Also, every day, A and B together do

$$\frac{1}{30}\ of\ the\ work$$

Of this above work, A does

$$\frac{2}{3} \times \frac{1}{30} = \frac{1}{45} of\ the\ work$$

A along will take

$$45\ days\ to\ complete\ the\ work.$$

4.3 Doing Work Alone

Example 4.11

Three cats can catch three rats in three minutes. How much time will five cats take to catch five rats?

$$Three\ Cats\ can\ catch\ three\ rats\ in\ three\ minutes$$

Put one cat to catch one rat:

$$One\ Cat\ can\ catch\ one\ rat\ in\ three\ minutes$$

And if you have five cats to catch five rats, then

$$Five\ Cats\ can\ catch\ five\ rat\ in\ three\ minutes$$

Example 4.12

Five cubes take five minutes to melt when taken out of the fridge, and put into five different plates. How long will ten cubes take to melt?