ARITHMETIC

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1. Time and Work (& Pipes and Cisterns)

1.1 Teams of People Working

Man-Days

We being with the simplest variety of questions.
If some people are working on a project, the time taken for the project is inversely proportional to the number of workers working on it.

$$\text{Time} \propto \frac{1}{\text{Workers}}$$

We can write this in an equation as:

$$\text{Time} = \frac{k}{\text{Workers}}$$

Or:

$$\text{Time} \times \text{Workers} = k$$

where k is a constant.

Instead of k, we generally use the terms Man-Days and write:

Man-days = No. of People × No. of Days Worked

Days Required
= Man-Days / Number of People Working
People Required
= Man-Days / No. of Days Worked

The interpretation of People and Days is flexible:

- Instead of Man, we can have Women, Children, Machines, Computers, etc
- Instead of Days, we can have weeks, hours, minutes, years, etc

Example 1.1

Answer the questions using the information given below
A monument is to be built by x machines in y days. Unless otherwise mentioned, these are the only parameters important in completing the monument.
Q1: If the number of machines increases, the time taken to finish the monument will:
A. increase
B. decrease
C. remain constant
D. either increase or decrease
S1: Option B
Q2: If the number of days available to build the monument decreases, the number of machines required will:
A. increase
B. decrease
C. remain constant
D. either increase or decrease
S2: Option A
Q3 : State True or False for the following statements:
A. If the number of machines doubles, the number of days will halve.
B. If the number of machines increases by two, the number of days will decrease by two.
C. If the number of machines becomes one-third, the number of days will increase by two times the

current number of days.

S3: T, F, T

A. Calculating Time Required

Example 1.2

If seven people working together can finish a project in five days, how long will it take a single person to finish the work?

Number of Days = No. of Man-Days = No. of People × No. of Days Worked = 7 × 5 = 35

Example 1.3

If twelve men working together can finish a project in seven days, how long will it take three men to finish the work?

(12 × 7) / 3 = 28

Alternate:

People ÷ 4 ⇒ Days × 4 = 7 × 4 = 28

$$No.of\ Man - Days = 12\ Men \times 7\ Days = 84\ Man - Days$$

$$\frac{84\ \boldsymbol{Man} - Days}{3\ \boldsymbol{Man}} = 28\ Days$$

B. Calculating People Required

Example 1.4

If eighteen people can complete a project in 12 days, how many people will be required to complete the project in two days?

$$Effort = 18\ People \times 12\ Days = (18 \times 12) People - Days$$

$$No.of\ People = \frac{(18 \times 12) People - Days}{2\ Days} = 108\ People$$

Days ÷ (12/2) = 6

⇒ People × 6 = 18 × 6 = 108

Example 1.5

If sixteen people can complete a project in 9 days, how many fewer people will be required if the project can be completed in thirty-six days?

Days × (36/9) = 4 ⇒ People ÷ 4 = 16 ÷ 4 = 4 Fewer people = 16 – 4 = 12

C. Proportion of Work Completed

Example 1.6

If six masons working for fifteen weeks can build a wall, then what proportion of the wall will be completed if one mason does not join the work, and the masons leave work one week early?

Proportion of Work = Man-Days Available / Man-Days Required = $(5 \times 14) / (6 \times 15) = 7/9$

D. Change in Number of Workers

Days Required = Remaining Man-Days / Remaining Workers

Example 1.7

A team of twenty-one people started work on a project that was to last twenty-five days. After working for three days, nine of them fell ill. How long will the remaining workers take to finish the project?

Number of Days: $[21 \times (25 - 3)] / [21 - 9] = (21 \times 22) / 12 = (7 \times 11) / 2 = 77/2 = 38\frac{1}{2}$

E. Increase in Workers

Example 1.8

A group of twenty-five archaeologists are sifting through the remains of sculptures found in the Fertile Crescent. They have targeted finishing the project in seven months (out of which three have been completed). In order to finish faster, they are joined by another fifteen archaeologists. In how much time from the start of work will they finish work?

Number of Months: $[25 \times (7 - 3)] / [25 + 15] = (25 \times 4) / 40$ = 2 Months 15 Days

F. Special Conditions

Each question below introduces an additional element that requires special attention and treatment

(Workers on Last Day of Work) Example 1.9

A team of eighteen people targeted completing a project in twenty-one days. After working for eight days, six of them were asked to start working on another project. The entire remaining team worked on the project for X days. On the $X + 1^{st}$ day of the project, only Y people were required.

$X + Y =$

Number of Days:

$$\frac{18 \times (21 - 8)}{18 - 6} = \frac{18 \times 13}{12} = \frac{3 \times 13}{2} = \frac{39}{2} = 19\frac{1}{2} \Rightarrow X = 19$$

Number of Workers

$$Y = 12 \times \frac{1}{2} = 6 \Rightarrow X + Y = 19 + 6 = 25$$

(Stopping work at intervals) Example 1.10

A team of nine workers works Monday to Friday on a project that takes seventeen days to complete for seven people. If they begin their project on Tuesday, 22nd January, which day will they complete their work? (26th Jan is a holiday)

26th Jan (Sat), is already a holiday.

Days Required = $(7 \times 17) / 9 = 119 / 9 = 13\ 2/9$
22 Jan (Tue) to 25 Jan (Fri) = 4 days
28 Jan (Mon) to 1 Feb (Fri) = 5 days (Total: 9 days)
4 Feb (Mon) to 8 Feb (Fri) = 5 days (Total: 14 days)

(Holding the Fort) Example 1.11

On the morning of March 2nd, an outpost with twenty-four soldiers, and water stocks for sixty days, is attacked by the enemy. On the evening of March 8th, six soldiers sneak out to get help. It is a travelling distance of three days from the outpost to the base fort. If the soldiers can hold out until they go completely for an entire day without water, then the last day on which the base fort must send reinforcements so that the soldiers survive is: May

$[24 \times (60 - 7)] / 18$
$= (24 \times 53) / 18 = 4/3 \times 53 = 70\ 2/3$
= 72 complete days from March 9 onwards

Help has to be sent three days before:
March 9 + 72 days – 3 days
= March 9 + 69 days
= March 1 + 77 days
= April 1 + 46 days (31 days in March)
= May 1 + 16 days (30 days in April)
= May 17
Hence, option C.

G. More than two Parameters

The underlying concepts remain the same. Additional parameters can be introduced as multiplying factors to make the question lengthier and more complex, such as:

- Number of hours worked per day
- Efficiency factor

Example 1.12

Five software programmers working five days a week were able to complete a software project in five weeks while allocating five hours a day to the project. If three software programmers work three days a week, three hours each day, then how many weeks will be required?

$(5 \times 5 \times 5 \times 5) / (3 \times 3 \times 3) = 625 / 27 = 23\ 5/27$

Example 1.13

Three stonemasons working at five days a week, seven hours a day, at an efficiency factor of 60% will leave pending what percent of work that can be done by four stonemasons working six days a week, eight hours a day at an efficiency factor of 70%.

$1 - (3 \times 5 \times 7 \times 0.6)/(4 \times 6 \times 8 \times 0.7) = 1 - 15/32 = 17/32 = 53.125\%$

H. Categorization of Workers

Different types of workers (who can work at different rates) introduce an efficiency factor into the question, directly or indirectly.

Replacements

Men and Women

Example 1.14

Without fractions

Example 1.15

Three men can do the same work in the same time as two women. If a team of seven men working for twelve days needs to be replaced by a team of women who will complete the work in ten days, then the number of women required is:

Efficiency factor of Men : Women

3M = 2W
(3/2) M = 1W

Convert the data on men to women

7M = 7 ÷ (3/2) W = 7 × (2/3) W = 14/3W

Women Required = Women-Days / Days Available

= (14/3 × 12) / 10 = 56/10 = 5.6

Shortcut Method

(7 × 2/3 × 12)/10 = 56/10 = 5.6

(Men, Women and Children) Example 1.16

Four women can do work at the same rate as three men. Five children can do work at the same rate as seven women.

(Mixed Teams) Example 1.17

A team of three experts and six trainees can finish a project in 17 1/7 hours. A trainee can complete 3 units of work in the same time that 6 units of work are done by an expert.
What will be the time taken by a team of nine experts and eighteen trainees to complete the project?
What will be the time taken by a team of six experts and three trainees to complete the project?

E & T increased in the ratio (3:1).
Time taken will decrease in the inverse ratio (1:3).
Time taken = 17 1/7 * (1/3) = 120/7 * 1/3 = 40/7

Ratio:
3E = 6T
E = 2T
Current Team:
3E + 6T = 6T + 6T = 12T
New Team:
6E + 3T = 12T + 3T = 15T
Time Taken:
= Inverse Ratio of Manpower

$= 120/7 * (12/15) = 96/7 = 13\ 5/7$

1.2 Workers and Pipes

A. Rate of filling a tank

Check the section on rates

B. Time to complete

If a man can do a work in n units of time, then he will do $1/n$ of the work in one unit of time
If a pipe can fill a tank in n units of time, then it will fill $1/n$ of the tank in one unit of time

Example 1.18

Mohan can complete a project in 12 days, and Sohan can complete the same project in 15 days. If they start the project together in how many days will the project be completed?

Work done in one day by:
Mohan $= 1/12$, Sohan $= 1/15$
Total work in one day:
$= 1/12 + 1/15 = (5 + 4)/60 = 9/60 = 3/20$
Days required $= 20/3 = 6\ 2/3$

1.1: Combined Speed of Two Workers

If worker A can complete a task in a hours, and worker B can complete a task in b hours, then the two of them can complete the task in

$$\frac{1}{\frac{1}{a}+\frac{1}{b}} = \frac{ab}{a+b}\ hours$$

Work done every hour of the entire task is:

$$\frac{1}{a}\ for\ A, \qquad \frac{1}{b}\ for\ B$$

Work done by both together every hour is:

$$\frac{1}{a}+\frac{1}{b}$$

Time taken by them to complete is:

$$\frac{1}{\frac{1}{a}+\frac{1}{b}} = \frac{1}{\frac{a+b}{ab}} = \frac{ab}{a+b}$$

(Alternate Solution) Example 1.19

Mohan can complete a project in 12 days, and Sohan can complete the same project in 15 days. If they start the project together in how many days will the project be completed?

$$\frac{(12)(15)}{12+15} = \frac{(12)(15)}{27} = \frac{(4)(5)}{3} = \frac{20}{3}$$

Example 1.20

A green pipe can fill a tank in three hours, while a red pipe can fill a tank in two hours. How long will both of them combined take to fill the tank, in minutes?

Work = 1/3 + ½ = (3 + 2)/6 = 5/6
Time = 6/5 hr = 6/5 * 60 = 72 minutes

The prior two questions use exactly the same logic.

Don't think of these as separate questions / types / chapters.

Example 1.21

Pipes A, B, C and D fill tank X in 3, 4, 5, and 6 hours respectively. Ramesh got into the lift to start three pipes (A, B and C) at 3:50 pm. If it takes him seven minutes to go up and start the pipes, by when will they fill tank Y (which has twice the capacity of tank X)?

$$\frac{\text{Work}}{\text{Hour}} = \frac{1}{3} + \frac{1}{4} + \frac{1}{5} = \frac{20 + 15 + 12}{60} = \frac{47}{60}$$

$$\text{Hours to fill} = \frac{60}{47} \times 2 = \frac{120}{47} = 2\frac{26}{47} = 2\frac{23.5}{47} + \frac{2.5}{47} \approx 2.5 + \frac{2.5}{50} = 2.5 + \frac{3}{60}$$

Total time = 3:50 pm + 7 minutes + 2.5 hours + 3 minutes = 4:00 pm + 2.5 hours = 6:30 pm

C. Time for individual worker

These questions require reverse calculations.

(Individual Worker) Example 1.22

Answer the next three questions based on the information below.

A and B together can complete a task in 5 five days. A alone can complete the task in seven days.

A. How long will B alone take to complete the task?
B. What is the rate of work of B compared to the rate of work of A?
C. How many times faster is A compared to B?

Part A

Work done by B= 1/5 – 1/7 = (7 – 5)/35 = 2/35
Time taken by B = 35/2 = 17.5 days

Part B

2/35 ÷ 1/7 = 2/35 * 7 = 2/5 = Two-fifths

Part C

1/7 ÷ 2/35 = 1/7 * 35/2 = 5/2 = Two and a half times

Shortcut: Take the reciprocal of the previous answer

Example 1.23

A manufacturer built a machine which will address 500 envelopes in 8 minutes. He wishes to build another machine so that when both are operating together they will address 500 envelopes in 2 minutes. The equation used to find how many minutes x it would require the second machine to address 500 envelopes alone is:

A. $8 - x = 2$
B. $\frac{1}{8} + \frac{1}{x} = \frac{1}{2}$
C. $\frac{500}{8} + \frac{500}{x} = 500$
D. $\frac{x}{2} + \frac{x}{8} = 1$
E. None of these answers (AHSME 1950/)

Method I

Substitute $a = 8, b = x$ in $\frac{1}{\frac{1}{a}+\frac{1}{b}}$ to get:

$$\frac{1}{\frac{1}{8}+\frac{1}{x}} = 2 \Rightarrow \frac{1}{8}+\frac{1}{x} = \frac{1}{2} \Rightarrow Option\ B$$

Method II

$$\frac{8x}{8+x} = 2 \Rightarrow 8x = 16 + 2x \Rightarrow x = \frac{8}{3}$$

Substitute into each of the options.
Option B is correct.

(Individual Worker) Example 1.24

Given that A, B and C can finish a task in X, Y and Z hours, and that A, B, C and D together can finish the same task in XX hours, what is the time that D alone will take to finish the task?

Workers in pairs

These questions make use of simultaneous linear equations.

(Workers in Pairs) Example 1.25

A software company uses a partial Follow The Sun approach with teams from US, India and Europe working for eight hours each on the project. The US and the Indian team together can complete the project in eight months. The Indian and the European team together can complete the project in nine months. The US and the European team together can complete the project in ten months. Find the time taken if the teams worked together, and for each team individually.

Rate of Work:
1: U + I = 1/8
2: I + E = 1/9
3: U + E = 1/10

Add **1:** , **2:** , **3:**
2(U + I + E)
= 1/8 + 1/9 + 1/10
= (90 + 80 + 72)/720
= 242/720 = 121/360

4: U + I + E = 121/720

Subtract 1: from 4:
E = 121/720 – 1/8 = 121/720 – 90/720 = 31/720

Subtract 2: from 4:
U = 121/720 – 1/9 = 121/720 – 80/720 = 41/720

(Workers in Pairs) Example 1.26

A and B together can build a mausoleum in six months, while B and C together can do the same in nine months. D can work at twice the speed that B does. If A, C and D work on the mausoleum together, how many months will they take?

Rate of Work:
1: $A + B = 1/6$
2: $B + C = 1/9$
Add 1: and 2:
$A + 2B + C = 1/6 + 1/9 = 5/18$
D = 2B
$A + D + C = 5/18$
Months
$= 18/5$

(Negative Work) Example 1.27
A pipe can fill a cistern in six hours, while a second pipe can empty it in nine hours. If both of them are opened simultaneously when the cistern is completely empty, how long will it take for the cistern to be filled completely?

Work done $= 1/6 - 1/9 = (3 - 2)/18 = 1/18$
Time taken = 18 hours

(Child Playing) Example 1.28

Example 1.29
Changes to Work Conditions
Finding Days left
Finding number of days food will last
Worker Added
Worker Removed
Alternating Work

D. Geometry-Based Questions

Example 1.30
Diameter of Pipe
Volume of Tank
Diameter and Volume

1.3 Summary and Review

Example 1.31
A son, working at three-fourth of the speed of his father, can build a tank in 17 1/7 hours when working alongside his father. The son's mother works at five-fourths the speed that the son does.

A. What is the time that each will take to build a tank (individually)?
B. If the son, father and mother, each working separately build one, two and three tanks (not necessarily respectively), what is the minimum average time to build a tank?
C. The son, father and mother start building tanks separately. When one of them finishes a tank, they go and chat with the person who is about to finish their tank next. Once that person's tank is finished, they go back to their work. How many hours will it take till the seventh tank is made?

Part A
S and F = Work
Replace S with (3/4)F
F at 7/4 effort takes 120/7 hours
F takes $(120/7) \times (7/4) = 30$ hours
$S = D\ (3/4) = 30 \times 4/3 = 40$ hours
$M = S\ (5/4) = 40 \times (4/5) = 32$ hours

Part B
Average time= Total time/3= $(3 \times 30 + 2 \times 32 + 1 \times 40)/3$= 194/3= 64 hours 40 minutes

Part C
$F \rightarrow 30 \rightarrow 32$
$M \rightarrow 32 \rightarrow 40$
$S \rightarrow 40 \rightarrow 62$
$F \rightarrow 62 \rightarrow 72$
$M \rightarrow 62 \rightarrow 72$
$S \rightarrow 102$
$F \rightarrow 102$

2. Rates

2.1 Basic Rates

Unit Rates

Unit Rates are an application of the Unitary Method.

Cost - Buying Apples

Example 2.1

If twelve apples cost Rs. 3, what is the cost of one apple?

Twelve apples cost Rs. 3
One Apple costs 3/12 = 0.25 Rs. = 25 Paise

Making Items

Example 2.2

If a carpenter takes 12 hours to make 15 chairs, how many minutes does it take him to make a chair?

15 chairs in 12 hours=1 chair in 12/15 hours= $(12 \times 60)/15$ minutes= 48 minutes

Area – Sowing Fields

Example 2.3

If a farmer can sow 120 square meters of land in 3 days while working six hours every day, how many square centimeters can he sow in a minute?

$(120 \times 100 \times 100)$ / $(3 \times 6 \times 60)$= 10,000 / 9= 1111.11……

Conversions - Filling a tank with a pipe

Example 2.4

If a pipe fills a tank of capacity 3 meters by 2 meters by 5 meters in 3 hours, then much will it fill in a second?

$$\frac{300 \text{ cm} \times 200 \text{ cm} \times 500 \text{ cm}}{3 \text{ hours} \times 3600 \frac{\text{seconds}}{\text{hour}} \times 1000 \frac{\text{cm}^3}{\text{l}}}$$

You do not need to write the units if you are confident of the calculation

$$\frac{3 \times 2 \times 5 \times 100 \times 100 \times 100}{3 \times 3600 \times 1000} = \frac{2 \times 50}{36} = \frac{25}{9} = 2\frac{7}{9}$$

Conversions – Speed

Example 2.5

A soldier has a marching speed of 6 km/hr, an official marching time in the day of 8 hours, and six days to be marched in a week. What is the official distance to be covered in a:

Q1: Day
Q2: Week
Q3: Minute (in cm)

Part A
6 km/hr
* 8 Both sides
48 km/ (8 hrs = 1 Day)

Part B
48 km/ (1 Day) * 6 Both sides
288 km/ (6 Days)

Part C
6 km/hr= 6000 m/hr = (600000/3600) cm/min= (6000/36) = (500/3) = 167

Example 2.6
What is the degree measure that the:
A. Second hand of a clock covers in a second
B. Minute hand of a clock covers in a minute?
C. Minute hand of a clock covers in a second?
D. Hour hand of a clock covers in an hour?
E. Hour hand of a clock covers in a minute?

360 degrees in 60 seconds
One second = 360 / 60 = 6 degrees

360 degrees in 60 minutes
One minute = 360 / 60 = 6 degrees

One minute = 360 / 60 = 6 degrees
One Second = 6/60 = 0.1 Degrees

360 degrees in 12 hours
One hour = 360 / 12 = 30 degrees

One hour = 360 / 12 = 30 degrees
One minute = 30/60 =0.5 degrees

Results above are used in questions involving clocks and should be committed to memory

Rates with Costs
Example 2.7
Mayank has twelve Rupees, and can buy sixteen kiwis with them. If he instead wanted to buy as many kiwis as the number of Rupees that he currently has, how many Rupees would be left over?

Cost of Twelve Kiwis = 12/16 × 12 = 9 Rs.
Left Over = 12 – 9 = 3

Rates with TDS
Example 2.8
How many feet will a car that travels $b/3$ inches every x seconds cover in y minutes?

State the information as a ratio:
$b/3$ inches : x seconds

Divide the LHS by 12 (to convert to feet), and the RHS by 60 (to convert to minutes):
$(b/36)$ feet : $x/60$ minutes

Multiply both sides by 60/x to find the distance travelled in one minute:
$(b/36) * (60/x)$ feet : 1 minute
$(5b/3x)$ feet : 1 minute

Multiply both sides by y to find the distance travelled in y minutes:
$(5by/3x)$ feet : y minutes

Rates with Clocks

Example 2.9

If a clock gains 4 minutes every half an hour, how many hours will it gain in five weeks?

4 minutes per 30 minutes= 8 Minutes per hour= (8 * 24 * 7 * 5)/60 Hours in 7 weeks= 112 Hours

Example 2.10

If a clock gains 4 minutes every three hours, while another loses 1 minutes every two hours, and both are set to the right time at 2:15 pm on Monday, 2nd January, what will be the difference in time after exactly one week?

Gaining Clock:
4 Minutes every 3 hours= 4×8 Minutes every day= 32×7 Minutes in a week
Losing Clock:
1 Minute every 2 hours= 1×12 Minutes every day= 12×7 Minutes in a week
Total Difference:
$32 \times 7 + 12 \times 7 = 44 \times 7$ Minutes= 308 Minutes= 5 Hours 8 Minutes

Little's Law (Optional)

$$Avg.\,Flow\ Time = \frac{Inventory}{Flow\ Rate}$$

Example 2.11

$$Inventory = 20\ Jobs$$
$$Flow\ Rate = 8\ Jobs\ per\ week$$

A. Current flow time?
B. Analyze impact of reducing flow time to 1 week

Part A

$$Current\ flow\ time = \frac{Inventory}{Flow\ Rate} = \frac{20}{8} = 2.5\ Weeks$$

Part B

$$New\ Flow\ Time = 1\ Week$$

$$Flow\ Rate = 8$$

$$Avg.Flow\ Time = \frac{Inventory}{Flow\ Rate} \Rightarrow 1 = \frac{Inventory}{Flow\ Rate}$$

There are two ways to achieve this:

$$Inventory = 8$$
$$Flow\ Rate = 20$$

2.2 Rates with Time and Work

Rates with 2 parameters

Example 2.12
If three cats can catch three mice in three minutes, how many mice can nine cats catch in nine minutes?

Three cats & Three M = Three Mice
One cat & Three M = One Mice
One cat & One M = 1/3 of a Mice
Nine cats & Nine M = $1/3 \times 9 \times 9$ Mice = 27 Mice
Alternate:
Cats are thrice, Minutes are also thrice
Mice increases to nine times = $3 \times 9 = 27$

Rates with >2 parameters

Example 2.13
If eight octopuses, using their eight appendages, can wrap eight gifts in eight minutes, how many gifts can ten squid wrap in ten minutes using their ten appendages?
80 * 8A * 8M = 8
Gifts = 8 * ($10^3/8^3$)

3. Time, Speed and Distance

3.1 Conversions

A. Proportional Relationships

Conversions	From	To	
	Km/hr	m/s	5/18
	m/s	Km/hr	18/5
D = Distance	D = Speed × Time	Speed = Distance/Time	Time = Distance/Speed
S = Speed T = Time	Speed ∝ Distance	Time ∝ Distance	Time ∝ 1/Speed

B. Conversions

3.1: Conversion Ratio

Speed in metric units is often mentioned in either kilometers per hour, or meters per second.

The conversion ratio between $\frac{km}{hr}$ and $\frac{m}{s}$ is:

$$\frac{5}{18}$$

The given unit is:

$$\frac{km}{hr}$$

Substitute $km = 1000\ m, hr = 3600\ seconds$

$$\frac{km}{hr} = \frac{1000\ m}{3600\ s} = \frac{5}{18}\frac{m}{s}$$

3.2: Reverse Conversion Ratio

The conversion ratio between $\frac{m}{s}$ and $\frac{km}{hr}$ is the reciprocal of the conversion ratio the other way around:

The given unit is:

$$\frac{m}{s}$$

Substitute $m = \frac{1}{1000}km, s = \frac{1}{3600}hr$

$$= \frac{\frac{km}{1000}}{\frac{hr}{3600}} = \frac{km}{1000} \div \frac{hr}{3600} = \frac{km}{1000} \cdot \frac{3600}{hr} = \frac{18}{5}\frac{km}{hr}$$

Example 3.3

Convert the following given in $\frac{km}{hr}$ to $\frac{m}{s}$:

A.

Convert the following given in $\frac{m}{s}$ to $\frac{km}{hr}$:

B.

Example 3.4

A peregrine falcon has diving speed $320\frac{km}{hr}$. Find the speed in $\frac{m}{s}$.

Substitute the conversion factor $\frac{km}{hr}=\frac{5}{18}\frac{m}{s}$:

$$320\times\frac{5}{18}\frac{m}{s}=\frac{800}{9}\frac{m}{s}$$

(Calc) Example 3.5

Olympic sprinter Usain Bolt's speed is $12.2\frac{m}{s}$. If his speed were stated in $\frac{km}{hr}$, what would it be?

$$12.2\cdot\frac{18}{5}\frac{m}{s}=43.92\frac{km}{hr}$$

(Calc) Example 3.6

Sound travels at a speed of $332\,{}^{m}/{}_{s}$ in air. Find the speed of sound, in air, in km/hr.

$$332\cdot\frac{18}{5}\frac{km}{hr}=1195.2\frac{km}{hr}$$

C. Other Conversions

It is not necessary that only the standard units

$$\frac{km}{hr},\frac{m}{s}$$

Are used in a question.

ake the question more difficult, the units asked are non-standard, such as:

$$\frac{m}{min},\frac{km}{min}$$

Hence, it is important to understand the process by which a conversion factor has been arrived at (rather than memorizing the conversion factor).

Example 3.7

The Taipei 101 observatory elevator travels at a speed of $16.7\,{}^{m}/{}_{s}$. Find the speed in ${}^{m}/{}_{min}$.

The elevator will travel more in a minute than in a second.

One minute is sixty seconds, and the elevator will travel:

$$16.7\times 60=1002\frac{m}{min}$$

D. *Large Numbers

Example 3.8

The speed of light is approximately 300,000 kilometers per second. The speed of light can be written as $a\times 10^b$ meters per hour, where a and b are integers, and a is as small as possible. Find $a+b$.

$$300{,}000\times 1{,}000\times 3{,}600\frac{m}{hr}=108\times 10^{10}\frac{m}{hr}$$

Then:

$$a + b = 108 + 10 = 118$$

Example 3.9

A light year is a unit of distance that measures how much light travels in one year. It is approximately 9.46 trillion kilometers. The speed of a snail is 1 mm per second. For coprime positive integers a, b, with $1 \le \frac{a}{b} < 10$, and positive integer c, the speed of a snail can be written as $\frac{a}{b} \times 10^{-c}$ light years per second. Find $a + b + c$.

Hints:

1. A trillion is one thousand billion
2. A billion is one thousand million

$$1 \frac{mm}{s}$$

Convert to $\frac{km}{s}$ by substituting $1\ mm = \frac{1}{1{,}000} \cdot \frac{1}{1{,}000}\ km$

$$\frac{1}{1{,}000{,}000} \frac{km}{s} = \frac{1}{10^6} \frac{km}{s} = 1 \cdot 10^{-6} \frac{km}{s}$$

Find the conversion factor between km and lightyears:

$$1\ lightyear = 9.46\ trillion\ km = 9.46 \cdot 10^{12} = 946 \cdot 10^{10}\ km$$

$$1\ km = \frac{1}{946 \cdot 10^{10}}\ lightyears$$

Hence:

$$1 \cdot 10^{-6} \cdot \frac{1}{946 \cdot 10^{10}} \frac{lightyears}{s}$$

$$= \frac{1}{946} \cdot 10^{-16}$$

Substitute $1 = \frac{1000}{1000} = \frac{1000}{10^3} = 1000 \cdot 10^{-3}$:

$$= \frac{1000}{946} \cdot 10^{-3} \cdot 10^{-16} = \frac{500}{473} \cdot 10^{-19}$$

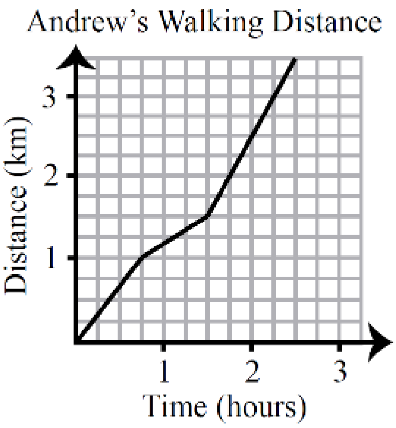
$$a + b + c = 500 + 473 + 19 = 992$$

3.2 Proportional Relationships

A. Graphs

Example 3.10

The line graph shows the distance that Andrew walked over time. How long did it take Andrew to walk the first 2 km? (CEMC Gauss 7 2020/8)



Part A

The time taken is

$$1\frac{3}{4}\ hours = 1\ hour\ 45\ Minutes$$

Example 3.11

A. What is Andrew's average speed over the first two kilometers in $\frac{m}{s}$?
B. What is Andrew's average speed over the first kilometer in $\frac{km}{hr}$?
C. What is Andrew's average speed over the second kilometer in $\frac{km}{hr}$?
D. Is the average of the values from Parts B and C equal to the answer from Part A?

Part A

$$Average\ Speed = \frac{Total\ Distance}{Total\ Time} = \frac{2\ km}{\frac{7}{4}\ hr} = \frac{8}{7}\frac{km}{hr} = \frac{8}{7}\cdot\frac{5}{18}\frac{m}{s} = \frac{20}{63}\frac{m}{s}$$

Part B

$$\frac{Distance}{Time} = \frac{1\ km}{\frac{3}{4}\ hr} = \frac{4}{3}\frac{km}{hr}$$

Part C

$$\frac{Distance}{Time} = \frac{1\ km}{1\ hr} = 1\frac{km}{hr}$$

Part D

$$\frac{\frac{4}{3}+1}{2} = \frac{\frac{7}{3}}{2} = \frac{7}{6}$$

B. Calculating Distance

Distance is directly proportional to both speed and time.

- As the speed increases, distance increases
- As the time increases, distance increases

3.12: TSD Equation

$$D = S \times T$$

Example 3.13

A. How many kilometers can a man walking at a speed of $5\frac{km}{hr}$ travel in 160 minutes?
B. How many miles can a camel-rider travelling at a speed of $12\frac{miles}{hour}$ cover in 570 minutes?

$$D_A = S \times T = 5\frac{km}{hr} \times \frac{160}{60}hr = \frac{160}{12}km = \frac{40}{3}km$$

$$D_B = S \times T = 12\frac{miles}{hr} \times \frac{570}{60}hr = \frac{570}{5}miles = 114\ miles$$

Example 3.14

If you walk for 45 minutes at a rate of 4 mph and then run for 30 minutes at a rate of 10 mph, how many miles will you have gone at the end of one hour and 15 minutes? (AMC 8 1985/13)

$$D = 4 \times \frac{3}{4} + 10 \times \frac{1}{2} = 3 + 5 = 8\, miles$$

Example 3.15

Sounds travels at about 330 meters per second; light travels so fast that it arrives almost instantaneously. If the gap between a flash of lightning and the clap of thunder is 6 seconds, roughly how far away is the storm. (NMTC Primary/Screening 2005/19)

Since light travels so fast, we can ignore the time taken by light.
(In case you are interested, speed of light is around 300,000 km per second).

$$D = S \times T = 330 \frac{m}{s} \times 6\, s = 1980\, m$$

C. Calculating Speed

3.16: Speed

$$S = \frac{D}{T}$$

Example 3.17

A. What is the speed of a horse rider who can cover 36 km in five hours?
B. An airplane covers a distance of 2000 miles in 360 minutes. Find the number of miles that it can cover in an hour.

$$S = \frac{D}{T} = \frac{36}{5} \frac{km}{hr}$$

$$S = \frac{D}{T} = \frac{2000}{6} = \frac{1000}{3} \frac{miles}{hr}$$

D. Calculating Time

3.18: Time

$$T = \frac{D}{S}$$

Example 3.19

What is the time taken by an airplane that has a constant speed of 240 miles / hour to travel 2500 miles?

$$T = \frac{D}{S} = \frac{2500}{240} = \frac{250}{24} = \frac{125}{12}\, hrs$$

Example 3.20

What is the time taken, in hours, by a horse rider who covers 1 mile in ten minutes to travel 60 miles.

Method I

$$1\, mile \rightarrow 10\, minutes$$

Multiply both sides by 6:

$$6 \, miles \rightarrow 1 \, hour$$

$$T = \frac{D}{S} = \frac{60}{6} = 10 \, hours$$

Method II

$$T = \frac{60 \, miles}{\frac{1}{10} \frac{mile}{minute}} = 60 \, miles \times 10 \frac{minutes}{mile} = 600 \, minutes = 10 \, hours$$

Example 3.21

What is the time taken, in hours, by a worm which covers one and a half feet in seven and a half minutes to travel 100 feet.

$$1.5 \, feet \text{ in } 7.5 \, minutes$$

Multiply both sides by 8:

$$12 \, feet \text{ in } 60 \, minutes = 1 \, hour$$

$$T = \frac{D}{S} = \frac{100 \, feet}{12 \frac{feet}{hour}} = \frac{100}{12} hours = \frac{25}{3} hours$$

Example 3.22

What is the time taken, in hours, by a snail which covers five feet in 17 minutes to travel seven feet.

$$5 \, feet \text{ in } 17 \, minutes$$

$$\frac{5}{17} feet \text{ in } 1 \, minute$$

$$\frac{300}{17} feet \text{ in } 60 \, minutes = 1 \, hour$$

$$T = \frac{D}{S} = \frac{7 \, feet}{\frac{300}{17} \frac{feet}{hour}} = 7 \times \frac{17}{300} hrs = \frac{119}{300} hrs$$

E. Average Speed

3.23: Average Speed

$$Average \, Speed = \frac{Total \, Distance}{Total \, Time}$$

Example 3.24

The artic tern has the longest migratory journey of a bird, travelling from one pole to the other: around 20,000 km. What is the average speed (in km/hr) of an artic tern that completes its migration in twenty-five days?

$$Average \, Speed = \frac{Total \, Distance}{Total \, Time} = \frac{20{,}000}{25 \times 24} = \frac{100}{3} \frac{km}{hr}$$

Example 3.25

A car travels from Mumbai to Pune and back at an average speed of 40 m/s. If the distance from Mumbai to Pune by road is 175 km, what is the time taken?

$$T = \frac{D}{S} = \frac{175 \times 2}{40 \times \frac{18}{5}} = \frac{175 \times 2 \times 5}{40 \times 18} = \frac{175}{72}\ hr$$

(Important) Example 3.26

A car travels 120 miles from A to B at 30 miles per hour but returns the same distance at 40 miles per hour. The average speed for the round trip is closest to which integer? **(AHSME 1950/27)**

$$S = \frac{Total\ Distance}{Total\ Time} = \frac{120 \times 2}{\frac{120}{30} + \frac{120}{40}} = \frac{240}{3+4} = \frac{240}{7} \approx 34\ hr$$

Note that in the above scenario, 120 cancels "nicely" with 30 and 40, but this is not required. If you consider 120 as a number, and carry out the calculations, it cancels:

$$S = \frac{Total\ Distance}{Total\ Time} = \frac{120 \times 2}{\frac{120}{30} + \frac{120}{40}} = \frac{120 \times 2}{\frac{120(40+30)}{30 \times 40}} = \frac{2 \times 30 \times 40}{70} = \frac{240}{7}$$

Not substituting the 120 still let us calculate the same answer:

$$S = \frac{Total\ Distance}{Total\ Time} = \frac{D \times 2}{\frac{D}{30} + \frac{D}{40}} = \frac{D \times 2}{\frac{D(40+30)}{30 \times 40}} = \frac{2 \times 30 \times 40}{70} = \frac{240}{7}$$

Example 3.27

A car travels 62.5 miles from A to B at 30 miles per hour but returns the same distance at 40 miles per hour. The average speed for the round trip is closest to which integer?

$$S = \frac{Total\ Distance}{Total\ Time} = \frac{D \times 2}{\frac{D}{30} + \frac{D}{40}} = \frac{D \times 2}{\frac{D(40+30)}{30 \times 40}} = \frac{2 \times 30 \times 40}{70} = \frac{240}{7}$$

3.28: Average Speed for Going and Coming Back

If the speed for going from point A to point B is a, and the speed for coming back is b, then the average speed over the entire trip is:

$$Average\ Speed = \frac{2ab}{a+b}$$

Let the distance between points A and B be D:

$$S = \frac{Total\ Distance}{Total\ Time} = \frac{D \times 2}{\frac{D}{a} + \frac{D}{b}} = \frac{2D}{\frac{bD + aD}{ab}} = \frac{2D}{\frac{D(a+b)}{ab}} = 2D \cdot \frac{ab}{D(a+b)} = \frac{2ab}{a+b}$$

This is the expression for the Harmonic Mean of two numbers.
(You can see that Note for more detail).

F. Idle Time

3.29: Idle Time

$$Total\ Time = Travel\ Time + Idle\ Time$$

When calculating distance from speed – use Travel Time.
When calculating distance from average speed – use Total Time.

Example 3.30

A journey involves driving a car for five hours at a speed of 30 $\frac{km}{hr}$, with a one-hour lunch break.

A. Find the total travel time
B. Find the distance travelled
C. Find the average speed over duration of the journey.

Total Time

$$Total\ Time = \underbrace{5\ Hours}_{Travel\ Time} + \underbrace{1\ Hour}_{Idle\ Time} = 6\ Hours$$

Distance Travelled

$$D = S \times T = 30 \times 5 = 150\ km$$

Average Speed

While the speed of the car when travelling is

$$30\frac{km}{hr}$$

This is not the average speed of the car. Why?
In all, the car has travelled 150 km in a time of 6 hours, and hence the average speed is

$$S = \frac{D}{T} = \frac{150}{6} = 25\frac{km}{hr}$$

Example 3.31

A car starts at 10 am and moves at a constant speed of 30 km/hr. It reached its destination 225 minutes later, while travelling at a constant speed of 30 km/hr (other than a 25-minute stop to refuel). What is the distance travelled?

$$Travel\ time = \underbrace{225}_{Total\ Time} - \underbrace{25}_{Idle\ Time} = 200\ min = \frac{200}{60} = \frac{10}{3}\ hrs$$

$$D = ST = 30 \times \frac{10}{3} = 100\ km$$

Example 3.32

The Shatabdi travels between Mumbai and Ahmedabad, which is a distance of 600 km. It starts at 7.00 am in the morning and makes scheduled stops of 15 minutes each at Wapi, Surat, Baroda, and Nadiad. It reaches Ahmedabad at 2.00 pm.

A. What is the train's speed?
B. What is the train's average speed?
C. What is the difference in the two speeds?

Part A

$$Idle\ Time = 15\ min \times 4 = 60\ min = 1\ hr$$

$$Travel\ time = \underbrace{2.00\ pm}_{End\ Time} - \underbrace{7.00\ am}_{Start\ Time} - \underbrace{Idle\ Time}_{1\ hr} = 7 - 1 = 6\ hr$$

$$S = \frac{D}{T} = \frac{600}{6} = 100\frac{km}{hr}$$

Part B

$$Average\ Speed = \frac{Total\ Distance}{Total\ Time} = \frac{600}{7}\frac{km}{hr}$$

Part C

$$100 - \frac{600}{7} = \frac{700}{7} - \frac{600}{7} = \frac{100}{7}\frac{km}{hr}$$

Example 3.33

A horse rider travelling at a speed of 7 miles per hour, starts from Fort A at 7 am to reach Fort B, 77 miles away. The rider stops after every three hours of riding to feed and water the horse (which takes thirty minutes each time). What time will the rider reach Fort B?

Travel Time

$$D = 77\ miles, \qquad S = 7\frac{m}{hr} \Rightarrow Travel\ Time = \frac{D}{S} = \frac{77}{7} = 11\ hours$$

Idle Time

The horse has to stop every three hours to be fed and watered. In other words, the horse must be fed and watered at the following time intervals of riding

$$\underbrace{3}_{1st\ Stop}, \underbrace{6}_{2nd\ Stop}, \underbrace{9}_{3rd\ Stop}, \underbrace{12}_{\substack{4th\ Stop \\ Not\ Needed}}$$

Time Required

$$Idle\ Time = \underbrace{3}_{Stops} \times \underbrace{0.5}_{Hours\ per\ Stop} = 1.5\ Hours$$

Total Time

$$Total\ Time = 11\ Hrs + 1.5\ Hrs = 12.5\ Hours$$
$$Reaching\ Time = Start\ Time + Total\ Time = 7.00\ am + 12.5\ Hours = 7.30\ pm$$

Example 3.34

A horseback rider covers 140 km in two days, while riding $\frac{2.5}{8}$ of a 24-hour day. What is the:

A. travel time for the rider
B. speed of the rider (in km/hr)
C. average speed of the rider?
D. idle time for the rider

Part A

$$Travel\ Time = \underbrace{2}_{No.of\ Days} \times \underbrace{\frac{2.5}{8}}_{\substack{Riding \\ Fraction}} \times \underbrace{24}_{\substack{Hours \\ in\ a\ Day}} = 2.5 \times 6 = 15\ hours$$

Part B

$$Speed = \frac{D}{T} = \frac{140}{15} = \frac{28}{3}\frac{km}{hr}$$

Part C

$$Average\ Speed = \frac{Total\ Distance}{Total\ Time} = \frac{140}{48} = \frac{35}{12}\frac{km}{hr}$$

Part D

$$Idle\ Time = Total\ Time - Travel\ Time = 24 \times 2 - 15 = 48 - 15 = 33$$

G. Speedometer and Odometer

3.35: Speedometer

The speedometer is an instrument on the dashboard of a car that tells you the speed at any point in time.

Example 3.36

Read the information, and answer each part separately. Do not use information from one part in another part. Anshul travelled from X to Y. The minimum speed on the speedometer was $40\frac{km}{hr}$, and the maximum was $60\frac{km}{hr}$.

A. If the distance from X to Y is 240 km, determine the minimum and maximum time that Anshul could have taken.
B. If the time that Anshul took was 6 hours, determine the minimum and maximum distance between X and Y.

Part A

For the time to be minimum, the speed should be maximum:

$$Min(T) = \frac{D}{Max(S)} = \frac{240}{60} = 4\ hours$$

For the time to be maximum, the speed should be minimum:

$$Max(T) = \frac{D}{Min(S)} = \frac{240}{40} = 6\ hours$$

Part B

For the distance to be minimum, the speed should be minimum:

$$Min(D) = Min(S) \times T = 40 \times 6 = 240\ km$$

For the distance to be maximum, the speed should be maximum:

$$Max(D) = Max(S) \times T = 60 \times 6 = 360\ km$$

3.37: Odometer

The odometer is an instrument on the dashboard of a car that tells you the total distance travelled at any point in time.

Example 3.38

Pratik travelled from A to B. When he started the odometer showed 234 km. When he ended, the odometer had the same digits, but in reverse order. How many kilometers did he travel?

$$432 - 234 = 198\ km$$

H. Fuel Efficiency

3.39: Mileage

The mileage of a car tells us how efficient the car is in using its fuel. It is given by:

$$\frac{Distance}{Fuel\ Consumed}$$

Example 3.40

A car has a fuel tank that has a maximum capacity of 40 liters. It is filled, and then the distance on the odometer is noted to be 332 km. When the odometer reaches 400 km, the fuel tank is empty. Determine the mileage of the car.

$$\frac{Distance}{Fuel\ Consumed} = \frac{400 - 332}{40} = \frac{68}{40} = \frac{17}{10} = 1.7\frac{km}{liters}$$

3.3 Algebra

A. Linear Equations

Example 3.41

Jessica has calculated the perfect speed that optimizes her fuel efficiency. Once, she took two hours to her destination at a speed $1\frac{mile}{hr}$ more than the perfect speed. On the way home, she travelled at a speed $10\frac{mile}{hr}$ less than the perfect speed, and it took her three hours. What is Jessica's perfect speed?

Let

$$Distance\ to\ the\ destination = D$$
$$Jessica's\ Perfect\ Speed = x$$

Use the formula $\frac{D}{S} = T$:

$$Going: \frac{D}{x+1} = 2 \Rightarrow D = 2x + 2$$
$$Coming\ back: \frac{D}{x-10} = 3 \Rightarrow D = 3x - 30$$

But the distance in both cases is the same, and hence:

$$2x + 2 = 3x - 30$$
$$x = 32\ \frac{miles}{hr}$$

Example 3.42

Two boys A and B start at the same time to ride from Port Jervis to Poughkeepsie, 60 miles away. A travels 4 miles an hour slower than B. B reaches Poughkeepsie and at once turns back meeting A 12 miles from Poughkeepsie. The rate of A was: **(AHSME 1950/28)**

The time taken for both boys is the same:

$$T_A = T_B$$

Use the formula for time:

$$\frac{D_A}{S_A} = \frac{D_B}{S_B}$$

Substitute $D_A = 60 - 12 = 48, D_B = 60 + 12 = 72, S_A = a, S_B = a + 4$:

$$\frac{48}{a} = \frac{72}{a+4}$$
$$2a + 8 = 3a$$
$$a = 8\frac{miles}{hour}$$

Example 3.43

A car travels certain number of kilometers on Monday. On Tuesday, the average speed is increased by 5 km per hour, and the time is increased by 1 hour. The distance travelled on Tuesday is 70 km more than Monday. On Wednesday, the speed is increased by 5 km per hour, and the time is increased by 1 hour (both compared to Tuesday). Determine the increase in the distance travelled (compared to Monday).

On Monday:

$$Speed = s, time = t \Rightarrow D_{Mon} = st$$

On Tuesday:

If speed is increased by 5, and time is increased by 1, the distance increases by 70:

$$D_{Tue} - D_{Mon} = 70$$
$$(s+5)(t+1) - st = 70$$
$$st + 5t + s + 5 - st = 70$$
$$\underbrace{5t + s = 65}_{Equation\ I}$$

On Wednesday:

Speed is increased by 10, and time is increased by 2. The increase in distance compared to Monday is:

$$D_{Wed} - D_{Monday}$$
$$= (s+10)(t+2) - st$$
$$= st + 10t + 2s + 20 - st$$
$$= 10t + 2s + 20$$
$$= 2(5t + s) + 20$$

Substitute the value of $5t + s = 65$ from Equation I:

$$2(65) + 20 = 150$$

Example 3.44

In some countries, automobile fuel efficiency is measured in liters per 100 kilometers while other countries use miles per gallon. Suppose that 1 kilometer equals m miles, and 1 gallon equals l liters. Find the fuel efficiency in liters per 100 kilometers for a car that gets x miles per gallon. **(AMC 10A 2022/4)**

Substitute $\frac{1}{m}\ km = 1\ mile, l\ liters = 1\ gallon$, and rearrange:

$$x\frac{miles}{gallon} = x\frac{\frac{1}{m}km}{l\ liters} = \frac{x}{ml}\frac{km}{liters}$$

Multiply and divide by 100:

$$= \frac{x}{100ml} \frac{100\ km}{liters}$$

Since we want liters per 100 km, move the expression $\frac{x}{100ml}$ to the denominator:

$$= \frac{100\ km}{\frac{100ml}{x} liters}$$

And the final answer is:

$$\frac{100ml}{x} liters\ per\ 100\ km$$

3.4 Trains

A. Passing point objects

When an object has a length (e.g. a train), the length of the object must be added to determine the time taken to cross.

Example 3.45

A train with length 112 meters travelling at a speed of $30\frac{m}{s}$ passes a man standing next to a stationary pole. Find the time taken.

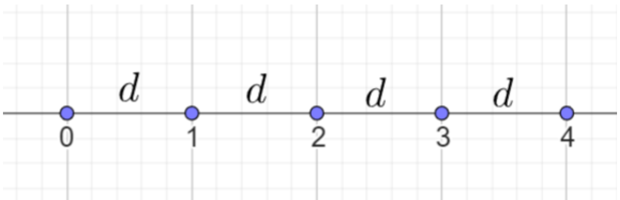
Substitute $D = 112\ meter, S = 30\frac{m}{s}$:

$$T = \frac{D}{S} = \frac{112}{30} = \frac{56}{15}\ s$$

B. Intervals with Poles

Example 3.46

A car running at a speed of $90\frac{km}{hr}$ runs parallel to a set of electric poles that are equally spaced. It crosses the first pole at $10:00\ am$. By $10:01\ am$ it has crossed 5 poles. Find the distance between two consecutive poles.



Speed of the car

Convert the speed of the car from $\frac{km}{hr}$ into $\frac{m}{s}$:

$$90 \cdot \frac{5}{18} = 25\frac{m}{s}$$

Time to cross distance between two poles

In one minute, it crosses five poles, which is 4 times the consecutive distance between poles.
The time to cross the distance between two poles is:

$$T = \frac{60}{4} s = 15\ s$$

Final Answer

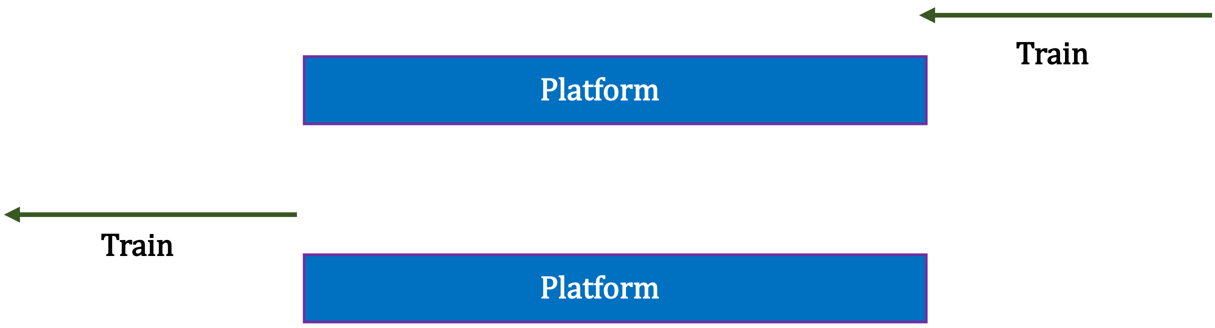
Distance travelled divided by number of consecutive distances travelled

$$D = S \times T = 25\frac{m}{s} \times 15\ s = 375\ m$$

C. Passing long objects

Example 3.47
How will a 225m-long train travelling at speed $40\frac{m}{s}$ take to cross a 300m-long platform?

Consider the situation below
In the first diagram, the train is just starting to cross the platform.
In the second diagram, the train has finally crossed the platform. Note that it only had to cover the distance of the platform, it also had to cover its own length.



Information Given

$$D = 225 + 300 = 525\,meters, \qquad S = 40\frac{m}{s}, \qquad T = ?$$

Use the Formula and Substitute

$$T = \frac{D}{S} = \frac{525}{40} = \frac{105}{8}\ s$$

Example 3.48
A one-kilometer-long train travels at a speed of one kilometer every five minutes. How long, in hours, does this train take to pass completely through a tunnel 2 kilometers long?

The distance to covered by the train is

$$2\ km + 1\ km = 3\ km$$

Time taken by the train

$$= 5 \min for\ 1\ km = 15 \min for\ 3\ km$$

$$15\ min = \frac{15}{60} hours = \frac{1}{4} hours = 0.25\ hours$$

Example 3.49
Find the time taken if a train with length 125 meters travelling at a speed of $25\frac{m}{s}$ goes through a tunnel of length 200 metres.

Information Given

$$D = 125 + 200 = 325\,meters, \qquad S = 25\frac{m}{s}, \qquad T = ?$$

Use the Formula and Substitute

$$T = \frac{D}{S} = \frac{325}{25} = 13\ sec$$

Example 3.50

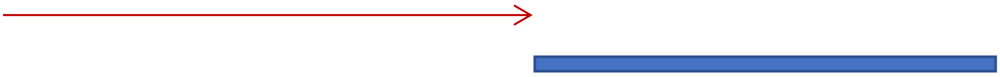
Train I: A to B

Fruit Stall:

$$D = L(Train)$$

Bridge

$$D = L(Train) + L(Bridge)$$



We have time, not distance.
The additional time taken to cover the bridge, comes from the additional distance of the bridge.

$$\therefore T(Only\ Bridge) = T(Bridge) - T(Fruit\ Stall) = 75s - 45s = 30s$$

$$S(Train) = \frac{D}{T} = \frac{L(Bridge)}{T(Only\ Bridge)} = \frac{400}{30} = \frac{40}{3}\,{}^{m}/_{s} = \frac{40}{3} \times \frac{18}{5} = 48\,{}^{km}/_{hr}$$

Train II: A to B

$$S = 50\,{}^{km}/_{hr}, D(A\ to\ B) = 232\ km$$

Train III: A to B

$$S = 25\,{}^{m}/_{s} = 25 \times \frac{18}{5} = 90\ {}^{km}/_{hr}$$

$$\underbrace{T(Train\ I) = 30s, T(Train\ II) = 27s}_{\substack{Both\ from\ same\ direction \\ Add\ the\ speeds}}$$

Question 1: Train I starts from A. Train II starts 45 minutes later from B. Find the distance travelled by each.

$$D(Train\ I)_{45\ minutes} = S \times T = 48 \times \frac{45}{60} = 48 \times \frac{3}{4} = 36\ km$$

For Remaining Distance:

$$D(TrainI): D(TrainII) = 48: 50 = 96: 100$$

Total Distance:

$$96 + 36: 100 = 132: 100$$

Question 2: Train III starts from A at 6.00 am. Stoppage = 5.2 minutes. When does it reach B?

$$T = \frac{D}{S} = \frac{232}{90} \times 60 + 5.2 = \frac{232}{3} \times 2 + 5\frac{1}{5} = \frac{464}{3} + 5\frac{1}{5} = 154\frac{2}{3} + 5\frac{1}{5} = 159 + \frac{2}{3} + \frac{1}{5} \approx 2h\ 40m$$

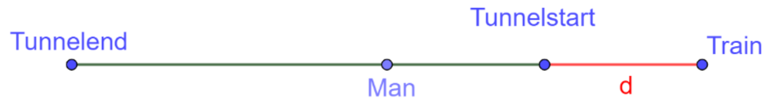
Question 3: What are the lengths of Trains I, II and III

$$L(Train\ I) = S \times T = \frac{40}{3} \times 45 = 600m$$

D. Man in a Tunnel

Example 3.51

A man jogging inside a railway tunnel at a constant speed hears a train approaching the tunnel from behind at a speed of 30 km per hour, when he is one third of the way inside the tunnel. Whether he keeps running forward or turns back, he will reach the end of the tunnel at the same time the train reaches that end. The speed at which the man is running is: **(JMET 2010/73)**



Let
- the train be at a distance d from the start of the tunnel
- the tunnel have length l
- the man have speed s

Since time taken to reach start of tunnel is same for both man and train:

$$\frac{\frac{l}{3}}{s}=\frac{d}{30}\Rightarrow\boldsymbol{\frac{l}{3s}=\frac{d}{30}}$$

Since time taken to reach end of tunnel is same for both man and train:

$$\frac{\frac{2l}{3}}{s}=\frac{d+l}{30}\Rightarrow\frac{2l}{3s}=\frac{d+l}{30}\Rightarrow\boldsymbol{\frac{l}{3s}}+\frac{l}{3s}=\boldsymbol{\frac{d}{30}}+\frac{l}{30}\Rightarrow\frac{l}{3s}=\frac{l}{30}\Rightarrow s=10$$

3.5 Relative Speed

A. Point Objects

3.52: Relative Speed

If two objects are both travelling, their relative speeds will be different from their actual speeds with reference to the ground.
- Add the speeds if travel is in opposite directions
- Subtract the speeds if travel is in the same direction

Example 3.53

You are travelling at $3\,\frac{km}{hr}$. The car ahead of you is travelling at $2\frac{km}{hr}$ in the same direction. At what speed will you overtake the other car?

Currently, suppose you are at 0 on a number line, and the car in front is at 2 on the number line. Both cars are moving to the right.

Current distance between the two cars

$$=D_1=2$$

After one hour, you will be at:

$$You: Position\ 3$$
$$Car\ in\ front: Position\ 4$$

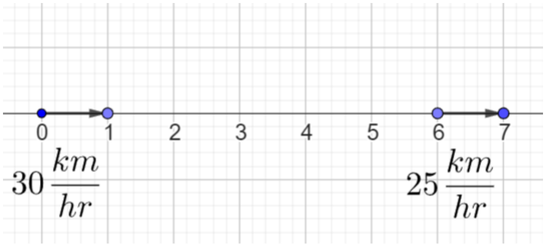
After one hour, the distance is:

$$=D_2=1$$

$$S=\frac{D_1-D_2}{T}=\frac{2-1}{1}=1\frac{km}{hr}$$

Example 3.54

You are travelling at $30\frac{km}{hr}$. The car ahead of you is travelling at $25\frac{km}{hr}$ in the same direction. At what speed will you overtake the other car?



Currently, suppose you are at 0 on a number line, and the car in front is at 6 on the number line. Both cars are moving to the right.

Current distance between the two cars

$$= D_1 = 6$$

After one hour, you will be at:

$$You: Position\ 30$$
$$Car\ in\ front: Position\ 31$$

After one hour, the distance is:

$$= D_2 = 1$$

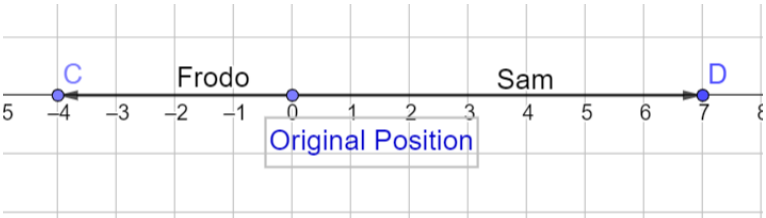
$$S = \frac{D_1 - D_2}{T} = \frac{6-1}{1} = 5\frac{km}{hr}$$

Example 3.55

Sam and Frodo are at Mount Doom. After the One Ring has been destroyed, they get frightened and go in diametrically opposite directions. Sam runs at a speed of 7 km/hour. Frodo is injured (has just lost a finger), and hobbles away at 4 km/hour. What is the distance between the two of them after:

A. One hour
B. Three Hours
C. Five Hours

Part A



Since Sam and Frodo are running in diametrically opposite directions, the angle between them is 180°. The total distance between them is:

$$\underbrace{7}_{Sam} + \underbrace{4}_{Frodo} = 11\ km$$

Part B

The distance between them after three hours is:

$$(7+4) \times 3 = 11 \times 3 = 33\ km$$

Part C

The distance between them after five hours is:

$$(7+4) \times 5 = 11 \times 5 = 55\ km$$

Example 3.56

Fred and George are both running towards each other. The current distance between them is 48 m. Fred is running at a speed of $5\frac{m}{s}$, while George is running at a speed of $3\frac{m}{s}$.

A. What is the distance between them after one second?
B. What is the distance between them after three seconds?
C. When will they meet each other?

Example 3.57

A car and a jeep are at a distance of 7 km from each other, travelling towards each other. The car is travelling at a speed of 23 km/hr, while the jeep is travelling at a speed of 37 km/hr. In how much time (rounded to the nearest minute) will they first meet?

$$Relative\ Speed = S(Car) + S(Jeep) = 23 + 37 = 60\frac{km}{hr} = 1\frac{km}{min}$$
$$Time\ taken\ =\ 7\ minutes$$

B. Thieves and Police

Example 3.58

A thief breaks into a museum at 3.00 in the night, and runs away with a jewelled case at a speed of 10 $\frac{km}{hr}$. The theft is found, and a policeman runs after the thief at 4.00 in the night at a speed of 15 $\frac{km}{hr}$.

A. What time will the policeman catch the thief?
B. What time will he hand over the jewelled case to the museum?

Part A

The policeman will be able to overtake the thief at a speed of:

$$15 - 10 = 5\frac{km}{hr}$$

At 4.00 am, the thief will be 10 km from the museum. Hence:

$$T = \frac{D}{S} = \frac{10}{5} = 2\ Hr \Rightarrow 4.00\ am + 2\ hrs = 6.00\ am$$

Part B

The policeman ran for two hours away from the museum. He will take the same time to run back.

$$Handover\ Time = 6.00\ am + 2hrs = 8.00\ am$$

Example 3.59

A thief steals a jewelled case from a museum at 2.00 AM, and goes due north at a speed of 25 km/hr. At 3.00 AM, the police reach the museum, and pursue the thief, going due north, at a speed of 30 km/hr. What time will they catch the thief?

Headstart = 3.00 AM – 2.00 AM = 1 hours = 25 km
Time to catch up= (25)/(30-25)= 5 hours
Time when they catch the thief = 3.00 AM + 5 hours = 8.00

Example 3.60

A father runs after his son, who is 1000 meters ahead. The father runs at a speed of 1 km every 8 minutes, and the son runs at a speed of 1 km every 12 minutes. How much distance has the son covered at the point when the father overtakes him? (JMET 2010/70)

$$1\ km\ per\ 8\ min = 1.5\ km\ per\ 12\ min = 0.5\ km\ more\ than\ the\ son\ in\ 12\ min$$

Hence, the father will overtake the son

$$12 \times 2 = 24\ min$$

In that time the son will travel

$$2\ km$$

Example 3.61

Mohan, cycling at a speed of $20\frac{km}{h}$, is 500 metres ahead of a Sohan, who is in a car, travelling at a speed of $50\frac{km}{hr}$. They are both headed for a picnic spot, which is 7 km away from Sohan.

A. How much time will it take for Sohan to catch up with Mohan?
B. In how many seconds will Sohan have a distance of 300 meters with Mohan?
C. On one of them reaching the picnic spot, what will be the distance between Mohan and Sohan?
D. What is the difference in time between the two of them reaching the picnic spot?

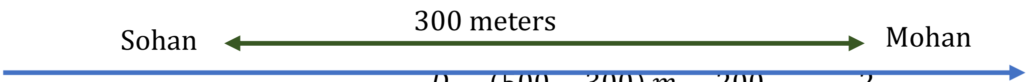
Part A

$$Relative\ Speed\ = 50-20=30\frac{km}{hr}=\frac{30}{60}\frac{km}{min}=\frac{\frac{1}{2}km}{min}=\frac{500\ m}{min}$$

Time taken = 1 min

Part B

This will happen twice, first when Sohan is catching up with Mohan.



$$\text{Catching up: } T=\frac{D}{S}=\frac{(500-300)\ m}{500\frac{m}{min}}=\frac{200}{500}min=\frac{2}{5}min=24\ Sec$$

It will happen next when Sohan is ahead of Mohan.

$$\text{Going Ahead: } T=\frac{D}{S}=\frac{(500+300)\ m}{500\frac{m}{min}}=\frac{800}{500}min=\frac{8}{5}min=96\ Sec$$

Part C

Who will reach first

From Part A, time to catch up = 24 sec.
Time to reach picnic spot with distance 7 km @ 50 km/hr ≈ 7 minutes
7 min is much greater than 24 sec, so the person to reach first is Sohan.

Sohan

$$S(Sohan)=50\frac{km}{hr}=\frac{50}{60}\frac{km}{min}=\frac{5}{6}\frac{km}{min}$$
$$T(Sohan)=\frac{D}{S}=\frac{7\ km}{\frac{5}{6}\ km/min}=\frac{7\times 6}{5}\ min=\frac{42}{5}min=8.4\ min$$

Mohan

$$S(Mohan)=20\frac{km}{hr}=\frac{20}{60}\frac{km}{min}=\frac{1}{3}\frac{km}{min}$$

$$D(Mohan) = S \times T = \frac{1}{3} \times 8.4 = \frac{84}{30} = 2\frac{24}{30} = 2\frac{4}{5} = 2.8\,km = 2800\,meters$$

Mohan was 6500 meters away from the picnic spot.
When Sohan reaches, he will have another

$$6500 - 2800 = 3700\,meters\ to\ travel$$

Part D

$$T(Mohan) = \frac{D}{S} = \frac{6.5}{\frac{1}{3}\frac{km}{min}} = 6.5 \times 3 = 19.5\,min$$

$$Difference\ in\ Time = 19.5 - 8.4 = 11.1$$

C. Mirrors

Example 3.62

As Emily is riding her bicycle on a long straight road, she spots Emerson skating in the same direction $1/2$ mile in front of her. After she passes him, she can see him in her rear mirror until he is $1/2$ mile behind her. Emily rides at a constant rate of 12 miles per hour, and Emerson skates at a constant rate of 8 miles per hour. For how many minutes can Emily see Emerson? (AMC 2010 8/8)

D. Long Objects

Example 3.63

A train of length 600 meters is travelling at a speed of 32 m/s. A car is 12 km ahead of it, and travelling at a speed of 17 m/s on a road parallel to the railway tracks. Find the time taken by the train to overtake the car.

Relative Speed = S(Train) – S(Car) = 32 – 17 = 15 m/s
Distance = Gap + Train Length = 12000 + 600 = 12600
Time = D/S = (12600)/15 = 840 seconds = 840/60 minutes = 14

Example 3.64

A train of length 350 m is travelling at a speed of 30 m/s is such that its engine is immediately next to the end of the last coach of a second train with a length 200 meters travelling at a speed of 22 m/s. Find the time taken by the first train to overtake the second train.

$$\text{Time} = \frac{D}{S} = \frac{\text{Gap} + \text{Length}(T_1 + T_2)}{\text{Speed}(S_1 - S_2)} = \frac{0 + 200 + 350}{30 - 22} = 56.25\text{ sec}$$

Example 3.65

42 m/s passes by a train of length 300 meter going in the opposite direction travelling at a speed of 18 m/s (first train length: 300 meter). The two trains currently have a distance of 400 meters between them

$$\text{Time} = \frac{D}{S} = \frac{\text{Gap} + \text{Length}(T_1 + T_2)}{\text{Speed}(S_1 + S_2)} = \frac{400+300+300}{42+18} = = \frac{1000}{60} = 16\frac{2}{3}$$

3.6 Media: Rivers and Mountains

A. Boats, Mountains and Escalators

The direction of travel in boats, mountains and escalators impacts the speed of movement.

Default Speed	Speed of stream Difference in speed due to mountain/escalator	Travelling downstream Going down a mountain, or escalator	Travelling upstream Going up a mountain, or escalator
X	Y	$x+y$	$x-y$
$(½)(a+b)$	$(½)(a-b)$	a	b

B. Basics

Example 3.66

The speed of a boat when travelling downstream is 7 km/hr, and 2.5 km/hr when travelling upstream.

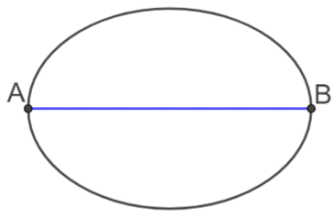
A. What is the speed of the boat in still water?
B. What is the speed of the current?

Speed in still water = (Speed Downstream + Speed Upstream)/2 = (7 + 2.5) / 2 = 9.5/2 = 4.25 km/hr
Speed of current = (Speed Downstream – Speed Upstream)/2 = (7 - 2.5) / 2 = 4.5/2 = 2.25 km/hr

Example 3.67

A rowboat with a sail attached covers the distance of 8 km from point A to point B on an elliptical lake (see diagram) in an hour. The return journey takes one and a half hour. Given that the wind direction is due east, determine:

A. The speed of the rowboat in still water with no wind
B. The speed of the wind



b=speed of boat, $w = speed\ of\ wind$:

$$b + w = 8$$
$$b - w = \frac{8}{\frac{3}{2}} = \frac{16}{3}$$
$$b = \frac{20}{3}, w = \frac{4}{3}$$

Example 3.68

Shalu can climb a mountain at a speed of 2 km/hour, while she can go downhill at double that speed. How long will she take to travel 18 km on level ground?

Speed on level ground = (Speed-Uphill + Speed-Downhill) / 2 = (2 + 4) / 2 = 6/2 = 3
Time = D/S = 18/3 = 6 hours

C. Level I

Example 3.69

The round-trip time on a river from A to B and back is 3 hours and 30 minutes. The ratio of upstream speed to downstream speed is 3: 4. The distance from A to B is 36 miles along the river. Determine the speed of the river, and the boat.

Let $u = upstream\ speed, d = downstream\ speed$. Using $T = \frac{D}{S}$:

$$\frac{36}{u}+\frac{36}{d}=\frac{7}{2}\Rightarrow 36d+36u=\frac{7}{2}ud$$

Substitute $d=\frac{4}{3}u$:

$$36\left(\frac{4}{3}u\right)+36u=\frac{7}{2}u\left(\frac{4}{3}u\right)$$
$$84u=\frac{14}{3}u^2$$
$$u=18$$
$$d=\frac{4}{3}u=24$$

Now, $b=boat\ speed, r=river\ speed$:

$$d=b+r=24$$
$$u=b-r=18$$

$$2b=42$$
$$b=21$$
$$r=3$$

3.7 Races, Headstarts and Handicaps

A. Relative Advantage

Example 3.70

A can beat B in a race of 1 km by 50 meters. If A continues running after crossing the finish line, how many meters further will he be when B crosses the finish line?

Ratio of distance for A and B = 1000 : 950
When B runs 1000 meters, A will run: (1000/950) * 1000 meters = (20/19) * 1000 meters = 1053 meters

Example 3.71

In a race of 200 meters A can beat B by 20 m, and B can beat C by 20 meters. By how meters can A beat C?

Ratio of distances:
Equate the ratios using LCM(200,180) = 1800
A:B = 200: 180= 2000 : 1800
B:C = 200: 180 = 1800 : 1620
A:B:C = 2000: 1800: 1620 = 200 : 180: 162
When A runs 200 meters, A will beat C by 200 – 162 = 38 m

When A runs 200 m, B will run 180, and C will run:

= (180/200) * 180 = (9/10) * 180 = 162

Example 3.72

Greg, Charlize, and Azarah run at different but constant speeds. Each pair ran a race on a track that measured 100 m from start to finish. In the first race, when Azarah crossed the finish line, Charlize was 20 m behind. In the second race, when Charlize crossed the finish line, Greg was 10 m behind. In the third race, when Azarah crossed the finish line, how many meters was Greg behind? **(Gauss 2013/23)**

$$C = \frac{80}{100} = \frac{4}{5}A$$
$$G = \frac{90}{100} = \frac{9}{10}C$$
$$G = \frac{9}{10}C = \frac{9}{10}\left(\frac{4}{5}A\right) = \frac{9}{10}\left(\frac{4}{5} \times 100\right) = 9 \times 4 \times 2 = 72 \Rightarrow 28\ Meters\ Behind$$

B. Headstarts and Handicaps

3.8 Ratios: Steps and Escalators

A. Steps

Example 3.73

Two jumps of a male kangaroo cover the same distance as seven jumps of a baby kangaroo. A female kangaroo has the same jump distance as a male kangaroo, but jumps only half the distance on every third jump. If a male kangaroo jumps 42 times, how many times must a baby kangaroo jump to cover the same distance.

$2:7 \times 21 = 42:147$
Hence, 147 steps.

How many jumps must a baby kangaroo make to catch up to 32 jumps by a female kangaroo?

3 steps (female = F) = 2.5 steps (male = M)

$$32f = 30f + 2f = 30 \times \frac{2.5}{3}m + 2m = 25m + 2m = 27m$$

Male steps : Baby Steps = 2:7
$2:7 \times 13.5 = 27:94.5$
Since there is no mention of a half a jump, we round upwards.

Number of steps required = 95

If a baby kangaroo takes half the time to jump that an adult kangaroo takes, what is the ratio of speeds of all three types of kangaroos (in decreasing order of speed)?

I. Ratio of jumps
2.5 jumps of male = 3 jumps of female
2 jumps of male= 7 jumps of baby

Therefore, distance ratios:
Male : Female = 3: 2.5
Male : Baby: = 7:2

II. Assume each takes LCM $(3, 7) = 21$ jumps
Substitute I. above to get Male jumps for everyone, and multiply by 2:
$21:2.5 \times 7:2 \times 3 = 21: 17.5:6 = 42:35:12$

III. To get speed, for the baby taking twice the number of jumps in the same time
$42:35:12 \times 2 = 42:35:24$

B. Escalators

3.9 Geometry: Circles and Cricket Pitches

A. Basics

Example 3.74
Starting at a vertex, an insect crawls in the direction that it is facing along the fence of a rectangular garden with length 3 meters and breadth 7 meters at a speed of 40 cm / minute. It requires an additional minute to re-orient itself every time it takes a turn. How long will it take reach its starting spot?

Perimeter of Garden
$= 2\ (L + B)\ = 2\ (3 + 7) = 20 \text{ m} = 2000 \text{ cm}$
Total Time
$=$ Travel Time $+$ Re-orientation Time
$=$ D/S $+$ No. of Turns
$= 2000/40 + 3 = 50 + 3 = 53$

B. Circular Tracks

Example 3.75
Rohan and Sohan are at the entrance of a circular track with a circumference of 300 meters. Rohan jogs clockwise $2\frac{m}{s}$, while Sohan jogs anti-clockwise $3\frac{m}{s}$. When and where will they meet the 1^{st} and the 2^{nd} time?

$$\text{Time} = \frac{\text{D}}{\text{RS}} = \frac{300}{2+3} = 60 \text{ seconds} \Rightarrow \underbrace{\text{Rohan} = 60 \times 2 = 120 \text{ m}}_{\textit{Clockwise}}, \quad \underbrace{\text{Sohan} = 60 \times 3 = 180 \text{ m}}_{\textit{Anti-clockwise}}$$
$$Rohan = 120 \times 2 = \underbrace{240\ m}_{\textit{Clockwise}} = \underbrace{60m}_{\textit{Anti-Clockwise}}$$

What is the difference in the position each time they meet?
120m further (clock-wise)

When will they meet at the entrance to the gate again?

Method I
LCM (Change in meeting point, Circumference) $=$ LCM $(120, 300) = 600$ meters
Number of meetings $= 600/120 = 5$
Each meeting happens after 60 seconds $= 1$ minute
5 meetings in 5 minutes

Method II

$$LCM(Time_{Rohan}, Time_{Sohan}) = \left(\frac{300}{2}, \frac{300}{3}\right) = (150,100) = 300\ seconds = 5\ Minutes$$

C. Meeting at any point: Applications

Example 3.76

What are the angles made (in degrees) when we consider the centre of the circular park as the vertex, the entrance gate as a point on one arm, and the point where Rohan and Sohan first meet on the second arm?

Smaller Angle = 20/300 = 24/60 = 144/360 =144 degrees, Reflex angle = 360 – 144 = 216 degrees

Example 3.77

If we superimpose a large clock over the garden, with 12'O Clock at the entrance of the park and the hour hand is pointing towards exactly the point where Rohan and Sohan meet for the first time, what is the time on the clock?

144 degrees = 120 degrees + 24 degrees = 4 hours and 48 minutes

D. Meeting at the Starting Point

Example 3.78

Teenu(4 m/s) and Meenu(3 m/s) are running laps with length 1200 meters on an elliptical track. When will Teenu and Meenu meet next at the starting point?

$$LCM(\text{Teenu}, \text{Meenu}) = LCM\left(\frac{1200}{4}, \frac{1200}{3}\right) = LCM\ (300, 400) = \ 1200\ \text{Seconds}$$

Teenu carries an extra weight of 5 kg, while Meenu carries an extra weight of 2 kg. If the energy expended in carrying a one-kg weight over a distance of 1 km is x calories, then how much energy (for carrying only the weights) will be expended by Teenu and Meenu from the time they start to the time they first meet?

$$Teenu\ = 5\ kg \times 4\frac{m}{s} \times 1200s = \ 24000\ kg - meters = \ 24\ kg - km = 24x\ calories$$

Meenu
= 2 kg * 3 m/s * 1200 Seconds
= 7200 kg-meters
= 7.2 kg-km
= 7.2x calories

Total energy
= Teenu + Meenu
= 24x + 7.2x
= 31.2x

E. Different Starting Points, Overtaking

Example 3.79

A format of bicycle racing at an international level has two bicyclists diametrically opposite each other on a circular track. The race is won when

F. Cricket Pitches

G. Triangles

Example 3.80

A man at A on the north bank of a sixty-unit-wide river spots his cat at B, exactly opposite him. At the same instant, the cat spots a dog at D (due west of B), and runs at a speed of 40 $\frac{units}{min}$ on the riverbank, away from the dog. The man rows his boat in a straight line towards the cat at a speed of 50 $\frac{units}{min}$ and meets it at Point C.

A. In how much time will the man meet his cat?
B. If the cat is twenty units away from Point C when the dog reaches Point B, and the dog reaches Point C at the same time as the cat and the man, then the speed of the dog is:
C. What is the distance AD?
D. If the dog had swum (at a speed two-thirds of his speed on land) to Point A, and then Point C instead of the path that he took, how much longer would he have taken to reach Point C?

Applying Pythagoras in right-angled triangle △ABC, and using $D = S \times T$ to calculate the distance:

$$AB^2 + BC^2 = AC^2 \Rightarrow (60)^2 + \left(\frac{4}{5} \times 50x\right)^2 = (50x)^2 \Rightarrow 900x^2 = 3600 \Rightarrow x^2 = 4 \Rightarrow x = 2 \text{ minutes}$$

BC = 40 * 2 = 80
Dog covers 80 units in the time cat covers 20 units.
S(dog)
= 4 * S(cat)
= 4 * 40
= 160 units/min
= 2 2/3 units/seconds

3.10 Further Topics

81 Examples