GEOMETRY

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PART I: BASICS

1. GEOMETRY

1.1 Lines and Angles

A. Line, Line Segment and Ray

1.1: Point
A point has no length or width. A point only has a location.

1.2: Line
A line extends in both directions till infinity.

> ➢ A line does not have a measurable length, since it extends till infinity.
> ➢ Two points define a line.

1.3: Line Segment
A line segment has a start point and an end point.

> ➢ It has a definite length, which can be measured.
> ➢ Two points define a line segment

1.4: Ray
A ray has an endpoint, and it goes indefinitely in one direction.

> ➢ A ray does not have a measurable length
> ➢ Ray is defined by its endpoint, and any other point on the ray.

A ray has properties which are similar to both line and line segment.

- ➢ Like a line segment, it has an endpoint.
- ➢ Like a line, it goes till infinity. However, it goes till infinity only in one direction.

Match the Column 1.5
For each geometrical object below, the properties of the objects have been jumbled up in the table below. They may or may not be true. Match each geometrical object with the correct properties, and rewrite the table.

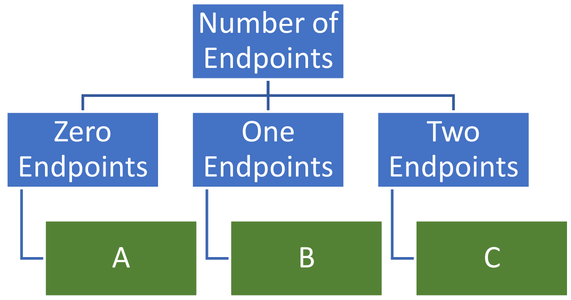
Nature of Geometrical Object	Number of Endpoints	Measurable Length	Extends till Infinity
Line	Has No Endpoint	Yes	In No Direction
Line Segment	Has One Endpoint	No	In One Direction
Ray	Has Two Endpoints		In Both Directions

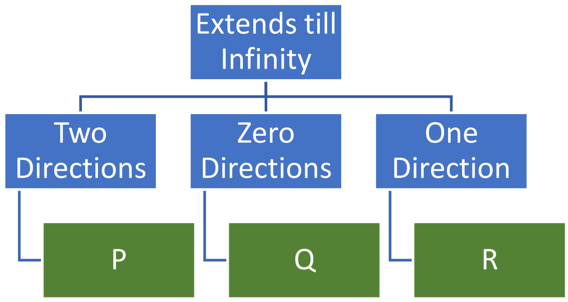
Nature of Geometrical Object	Number of Endpoints	Measurable Length	Extends till Infinity

Line	Has No Endpoints	No	In Both Directions
Line Segment	Has Two Endpoints	Yes	In No Direction
Ray	Has One Endpoint	No	In One Direction

Flowchart 1.6

Identify each object in the flowchart out of Line, Line Segment and Ray.

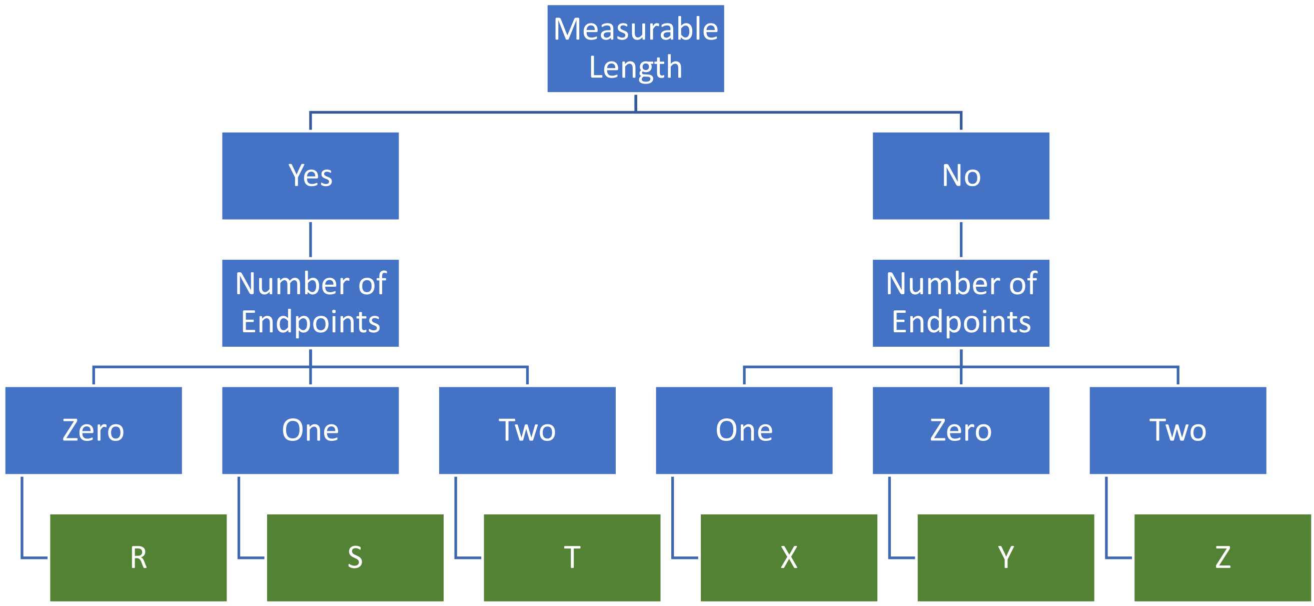




$$P = A = Line$$
$$R = B = Ray$$
$$Q = C = Line\ Segment$$

Flowchart 1.7

Identify each object in the flowchart out of Line, Line Segment and Ray. It is not necessary that the object exists. In such a case, you should mention: "The Object does not exist."

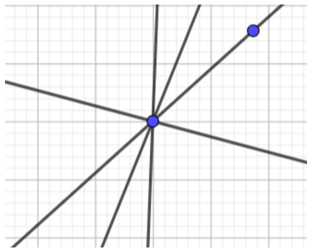


$$Does\ Not\ Exist: R, S, Z$$
$$T = Line\ Segment$$
$$X = Ray$$
$$Y = Line$$

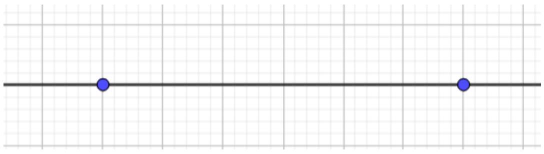
Example 1.8

From one point, how many lines can be drawn?

Infinite



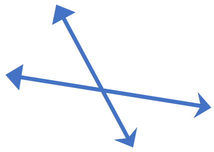
Example 1.9
How many lines can be drawn connecting two specific points A, and B?



Two points define a line.
Hence, the line which connects two points is unique.

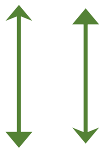
Hence, only one line can be drawn connecting two points A and B.

B. Types of Lines
The place two lines meet (or cut each other) is called an intersection point. Two distinct lines can have a maximum of one intersection point.

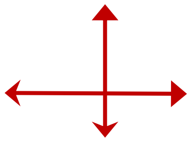


1.10: Intersecting Lines
Intersecting lines are lines which cut each other.

The blue lines in the diagram alongside are intersecting lines.
They cut each other once. Intersecting lines can cut only once. They cannot cut twice.



1.11: Parallel Lines
Parallel Lines are lines that go in the same direction. They never intersect.



1.12: Perpendicular Lines
Perpendicular lines are lines that intersect at right angles.

Match the Column 1.13
For each Type of Line, figure out how many intersection points it has and whether the lines meet. The table has the required responses, but they may be jumbled up. Fill up the table with the correct responses.

Type of Line	Intersection Points	Do they Meet
Parallel	Zero	Yes
Intersecting	One	No
Perpendicular	Two	

Type of Line	Intersection Points	Do they Meet
Parallel	Zero	No
Intersecting	One	Yes
Perpendicular	One	Yes

True or False 1.14
A. Perpendicular lines are a special case of intersecting lines.

B. If parallel lines intersect, then they are perpendicular.
C. Two lines can intersect in more than two places.
D. If a line intersects another line at an angle which is not a right angle, then the two lines are perpendicular.

Part A
Perpendicular are lines that intersect at right angles. Hence, they are intersecting lines.
In fact, they are a special case of intersecting lines.
Hence, Statement A is True.

Part B
Parallel lines cannot intersect. Hence, the statement is $self - contradictory$.
Hence, it is False.

Part C
Two lines can intersect in a maximum of one place.
Note that if the type of line is not mentioned, then it has to a straight line.

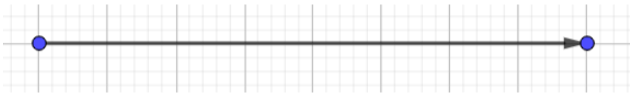
Part D
The two lines are intersecting, but not perpendicular.

C. Definition and Types of Angles

1.15: Angles: Definition
If two rays start from a vertex, they form an angle.

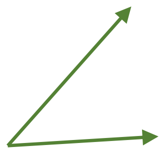
To understand the measure of an angle, we first must that
- a circle measures 360°.
- Half a circle measures 180°
- Quarter Circle measures 90°

1.16: Zero Angle
An angle which goes full circle comes back to its original position. This is called a zero angle.

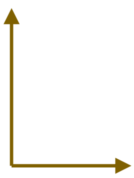


In a zero angle, the first ray, and the second ray are the same.

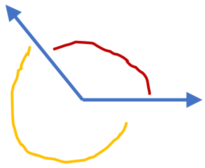
1.17: Acute Angle
Acute Angle: This is less than a quarter circle. Measure of an acute angle is between 0° and 90.



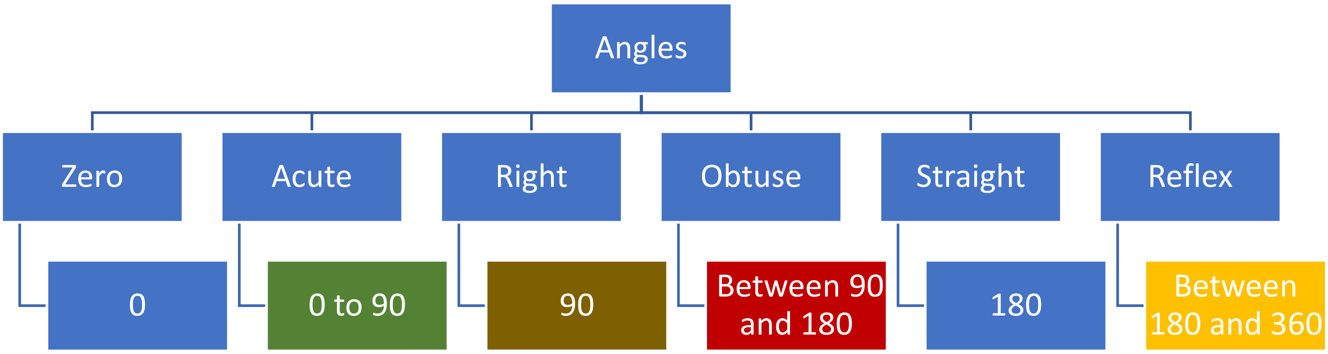
1.18: Right Angle
Right Angles: In a right angle, the measure is 90°. One ray is perpendicular to the other ray.



1.19: Obtuse Angle and Reflex Angle
Obtuse Angles: Measure is more than 90° and less than 180°
Reflex Angle: More than 180°, and less than 360°



1.20: Straight Angle
Straight Angle: Measure is 180°



Example 1.21
Classify the following angles based on whether they Acute, Right, Straight, Obtuse, or Reflex

A. 47
B. 326
C. 90
D. 123
E. 180
F. 65
G. 187

D. Supplementary Angles

1.22: Supplementary Angles
Two angles, which add up to 180°, are called supplementary.

Example 1.23
In each part below, check if the pair of angles is supplementary:

A. $70°, and\ 110°$
B. $100°, and\ 90°$

Part A

$$70 + 110 = 180$$

Since the total is 180, the two angles are supplementary.

Part B

$$100 + 90 = 190$$

Since the total is 190, the two angles are not supplementary.

Example 1.24

Find the supplementary angle to each of the following angles. Remember that two supplementary angles must add to up 180°.

A. 120
B. 90
C. 60
D. 140
E. 50
F. 34
G. 123

$$180 - 120 = 60°$$
$$180 - 90 = 90°$$
$$180 - 60 = 120°$$
$$180 - 140 = 40°$$
$$180 - 50 = 130°$$
$$180 - 34 = 146°$$
$$180 - 123 = 57°$$

E. Complementary Angles

1.25: Complementary Angles

Two angles which add up 90° are called complementary.

Example 1.26

Check if the following pair of angles is complementary:

$$40°, and\ 50°$$

$$40 + 50 = 90$$

Since the total is 90, the two angles are complementary.

Example 1.27

Check if the following pair of angles is complementary:

$$50°, and\ 60°$$

$$50 + 60 = 110$$

Since the total is 110, the two angles are not complementary.

Example 1.28

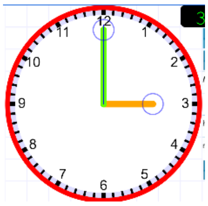
Find the complementary angle to each of the following angles. Remember that two complementary angles must add to up 90°.

A. 60
B. 50
C. 80
D. 20

E. 25
F. 34
G. 71

$$90 - 60 = 30°$$
$$90 - 50 = 40°$$
$$90 - 80 = 10°$$
$$90 - 20 = 70°$$
$$90 - 25 = 65°$$
$$90 - 34 = 56°$$
$$90 - 71 = 19°$$

F. Clock Basics

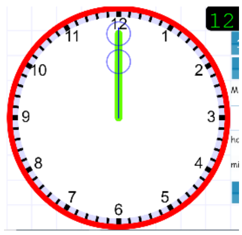


Example 1.29
When it is $3:00\ pm$, what is the angle made by the hour hand and the minute hand of a clock?

The two hands are perpendicular to each other.
Hence, the angle is 90°.

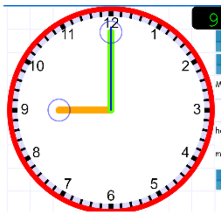
Example 1.30
When it is $6:00\ pm$, what is the angle made by the hour hand and the minute hand of a clock?

The two angles form a straight line.
Hence, the angle is 180°.



Example 1.31
When it is $12:00$ noon, what is the angle made by the hour hand and the minute hand of a clock?

The hands are exactly on top of one another.
Hence, this is a zero angle.



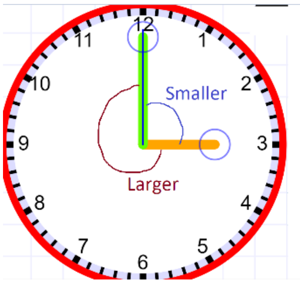
Example 1.32
When it is $9:00$, what is the angle made by the hour hand and the minute hand of a clock?

The two hands are perpendicular to each other.
Hence, the angle is 90°.

True/False 1.33
Write true or false for each of the following statements, which are about the smaller angle between the hour hand and the minute hand of an analog clock?
If the statements are false, correct them.

A. The angle made at 9.00 pm is a right angle
B. The angle made at 6.00 pm is a straight angle.
C. The angle made at 9.30 pm is acute.
D. The angle made at 3.30 pm is obtuse.

$$A: True$$
$$B: True$$
$$C: False, Obtuse\ Angle$$
$$D: False, Acute\ Angle$$



G. Smaller Versus Larger Angles

1.34: Smaller and Larger Angles

Every angle name refers to two angles: a smaller angle and a larger angle.

Example 1.35

Classify the angles at each of the below times between the hour hand and the minute hand of an analog clock as one of

$$\{Zero, Acute, Right, Obtuse, Reflex\}$$

A. Smaller angle at 9.30 pm
B. Smaller angle at 3.30 pm
C. Larger angle at 6.05 pm

$$A: Obtuse$$
$$B: Acute$$
$$C: Refkex$$

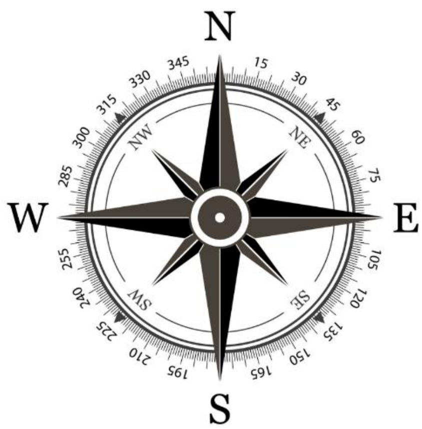
H. Angles on a Compass

A compass is a scientific instrument that tells us where North is located. Compass directions are important in navigation for planes, ships, and while travelling in locations like the Artic/Antarctic.

Example 1.36

What is the measure of the smaller angle between:

A. North and East
B. West and East
C. North and North-West
D. North and South-East



Example 1.37

Find the larger angle between:

A. West and South
B. North and South
C. West and North-East
D. North-West and North East

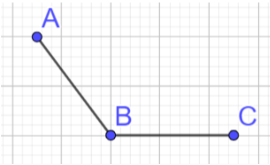
I. Naming Angles

To name an angle, we use exactly three points:

- A point on the first ray
- The vertex of the angle (*Middle Point*)

- A point on the second ray

The vertex has to come in the middle. The order of the first ray, and the second ray does not matter.



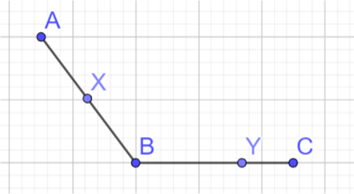
Example 1.38

Name the angle in the figure alongside.

The vertex point has to come in the middle. Hence, the two ways of naming this angle are:

$$\angle ABC$$
$$\angle CBA$$

Example 1.39

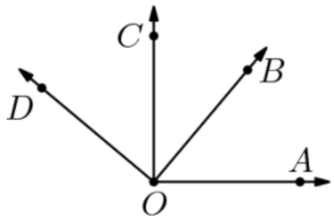


In the adjoining figure:

A. Name two angles which have different starting letters, different ending letters, but refer to the same angle.
B. Write the angle in the figure alongside in four different ways.

$$Part\ A: ABC, \quad XBY$$
$$Part\ B: ABC, \quad CBA, \quad XBY, \quad YBX$$

J. Counting Angles



Example 1.40

Which are the angles formed in the figure alongside? Name all of them. (Do not write multiple ways of referring to the same angle. Instead, write each angle only once.)

3 Angles starting from the point A:

$$\angle AOB, \quad \angle AOC, \quad \angle AOD$$

2 angles starting from point B:

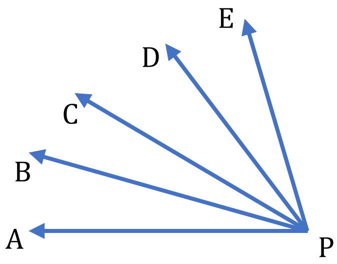
$$\angle BOC, \quad \angle BOD$$

1 Angle starting from point C:

$$\angle COD$$

Note that there is a pattern:

$$3 + 2 + 1 = 6$$

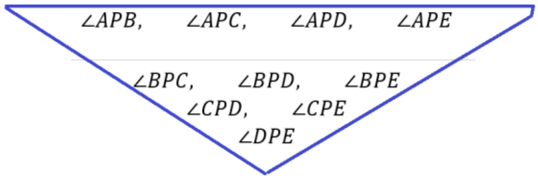


Example 1.41

Which are the angles formed in the figure alongside? Name all of them.

$$\angle APB, \quad \angle APC, \quad \angle APD, \quad \angle APE$$
$$\angle BPC, \quad \angle BPD, \quad \angle BPE$$
$$\angle CPD, \quad \angle CPE$$
$$\angle DPE$$

K. Triangular Numbers



Note the pattern which writing the angles formed. The pattern is:

$$4 + 3 + 2 + 1 = 10$$

If you write out the angles, and arrange them, you can get a triangular pattern.
Hence, such numbers are called triangular numbers.

Example 1.42

The numbers formed by adding the first few natural numbers are called triangular numbers. For example, the third triangular number is

$$1 + 2 + 3 = 6$$

A. Find the fifth triangular number.
B. Find the difference between the fourth triangular number and the second triangular number.

Part A

We can find the fifth triangular number by adding the first five counting numbers:

$$1 + 2 + 3 + 4 + 5 = 15$$

Part B

$$Fourth\ Triangular\ Number = 1 + 2 + 3 + 4 = 10$$
$$Second\ Triangular\ Number = 1 + 2 = 3$$
$$Difference = 10 - 3 = 7$$

L. Review and Challenge Questions

Example 1.43

If the sum of the measures of two angles is 90°, what kind of angles are they as a pair?

Complementary Angles

Example 1.44

If the sum of the measures of two angles is 180°, what kind of angles are they as a pair?

Supplementary Angles

Example 1.45

Match the following angles below in Column A to their definitions in Column B

Obtuse	Exactly 0
Zero	Exactly 180
Reflex	Exactly 90
Right	Between 0 and 90
Acute	Between 90 and 180

True or False 1.46

A. $\angle ABC$ is a different angle from $\angle CBA$.
B. $\angle ABC$ is a different angle from $\angle BAC$.

Part A
False. They both refer to the same angle.

Part B
True. Because the vertex in the first angle is different from the vertex in the second angle. Hence, these are two different angles

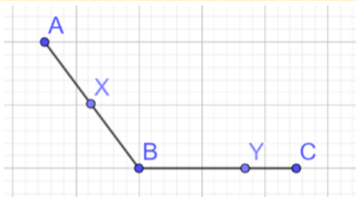
Example 1.47
What is the larger angle made by the hands of a clock at 9:00 pm?

$$270°$$

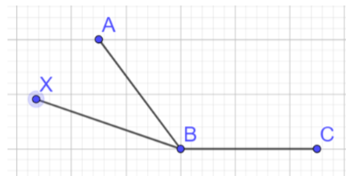
Example 1.48
Bhumika was supposed to measure the angle between the hands of a clock at 3:00 pm for a school project. But she measured the angle between the hands of the clock at 3:00 am, by mistake. What is the difference between the correct answer and the one that she got?

Example 1.49
Consider two angles, which are named $\angle ABC$ and $\angle XBC$, respectively. Then, decide which of the following statements are true or false:

A. $\angle ABC$ must be a different angle from $\angle XBC$.
B. $\angle ABC$ can be a different angle from $\angle XBC$.



Part A
False. Because since the vertex is the same, they can be the same angle.
In the diagram alongside, note that $\angle ABC$ and $\angle XBC$ are the same angle.



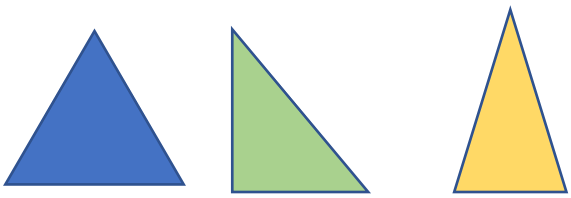
Part B
True. The two angles can be different. Consider the diagram drawn alongside, which shows the two angles as different.

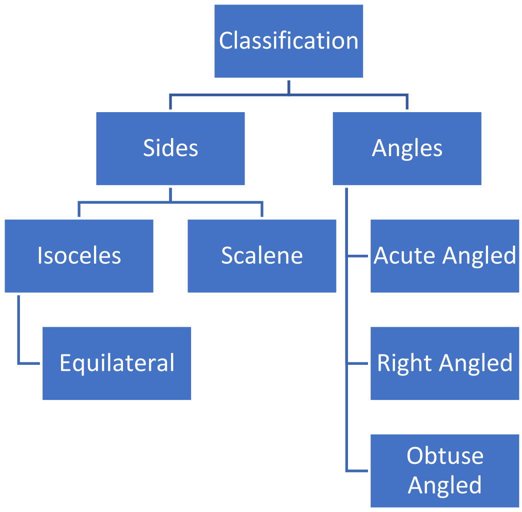
1.2 Triangles

A. Triangles

1.50: Triangle

- A three-sided closed figure is called a triangle.
- A triangle is a special case of a polygon.
- A triangle has three sides, three vertices and three angles.





1.51: Based on Sides
Equilateral: All three sides are equal
Isosceles: At least two sides are equal
Scalene: All three sides are different

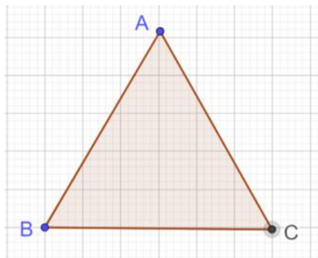
1.52: Based on Angles
Acute-Angled Triangle: All Angles are acute angles
Right-Angled Triangle: One Angle is a right angle
Obtuse-Angled Triangle: One angle is an obtuse angle

B. Equilateral Triangles

1.53: Equilateral Triangle
An Equilateral Triangle has all sides the same size.

- Each angle of an equilateral triangle is 60°.
- An equilateral has rotational symmetry of order 3. If you turn it $\frac{1}{3}rd$ of a circle, it looks the same.

Example 1.54
In the figure of an equilateral triangle, drawn alongside, name the
A. Vertices
B. Sides
C. Angles



$$Vertices: A, B \text{ and } C$$
$$Sides: AB, BC, AC$$
$$Angles: \angle A, \angle B, \angle C \text{ OR } Angles: \angle BAC, \angle ABC, \angle ACB$$

1.55: Naming Vertices, Sides and Angles
Vertices are named using a single letter.
Sides are named using two letters.
Angles are named using the angle symbol (∠) and either a single letter or three letters.

MCQ 1.56
In ΔXYZ, the length of side XY is 5 cm. And the length of side YZ is also 5 cm. Is the triangle equilateral?
A. Yes
B. No
C. Cannot be determined without further information

Because to decide if the triangle is equilateral, we need to know if all three sides are equal. Since we do not know the length of the third side, we cannot say anything about it.

Hence, we do know if the triangle is equilateral or not.

Hence, correct answer is Option C.

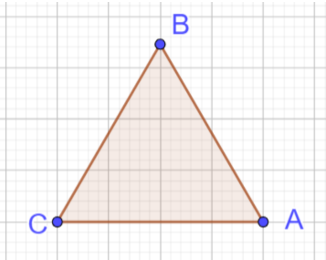
Example 1.57

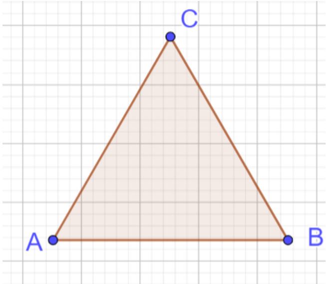
The length of the side of an equilateral triangle is 3 cm. Find the perimeter.

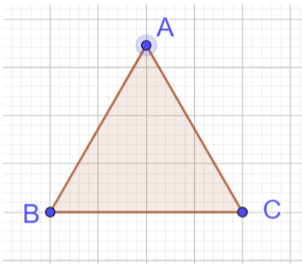
$$9\ cm$$

Example 1.58

Consider equilateral triangle ABC, drawn alongside. If you rotate the triangle around its center, two full circles, how many times will the triangle look the same?







Let's rotate the triangle till the B comes on top. See that the triangle looks the same now. Let's keep rotating till the C comes on top. Now also the triangle looks the same. And keep rotating till we get back the A in the original position, so that the triangle now looks like this:

Now we completed a full circle, and the triangle looked the same three times.

If we complete two circles, the triangle will look the same

$$3 \times 2 = 6\ times$$

And hence the final answer is

$$6$$

Example 1.59

What is the measure of $\angle P$ of equilateral triangle PQR?

$$\angle P = 60°$$

Example 1.60

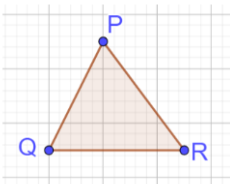
Consider equilateral ΔLMN. What is the difference in the measures of $\angle L$ and $\angle M$?

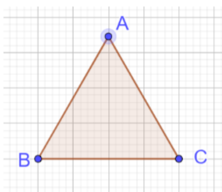
$$\angle L = \angle M = 60°$$
$$Difference = 0$$

Example 1.61

ΔABC is equilateral. ΔPQR has $\angle P$ which has measure double of $\angle A$, and $\angle Q$ which has measure half of $\angle B$.

A. Find the values of the measures of $\angle P$ and $\angle Q$.
B. Classify $\angle P$ and $\angle Q$. Are they acute, right, obtuse, reflex, or straight angles?





ΔABC is equilateral. So, each angle of the triangle is 60°.

$$\angle A = \angle B = \angle C = 60°$$

$$\angle P = 2 \times \angle A = 2 \times 60 = 120° \Rightarrow Obtuse\ Angle$$
$$\angle Q = \frac{1}{2} \times \angle B = \frac{1}{2} \times 60 = 30° \Rightarrow Acute\ Angle$$

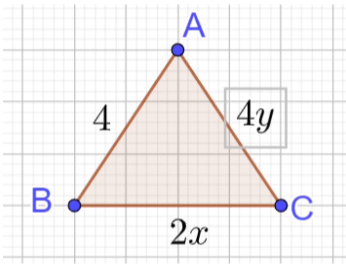
Example 1.62

ΔABC is equilateral. ΔPQR has $\angle P$ which has measure $30°$ more than $\angle A$, and $\angle Q$ which has measure $20°$ less than $\angle B$.

A. Find the values of the measures of $\angle P$ and $\angle Q$.
B. Classify $\angle P$ and $\angle Q$. Are they acute, right, obtuse, reflex, or straight angles?

$$\angle A = 60 \Rightarrow \angle P = \angle A + 30 = 60 + 30 = 90° \Rightarrow Right\ Angle$$
$$\angle B = 60 \Rightarrow \angle Q = \angle B - 20 = 60 - 20 = 40° \Rightarrow Acute\ Angle$$

C. Sides



Example 1.63

Equilateral triangle ABC has side $AB = 4$, side $BC = 2x$ and side $AC = 4y$.

A. Find the value of x.
B. Find the value of y.

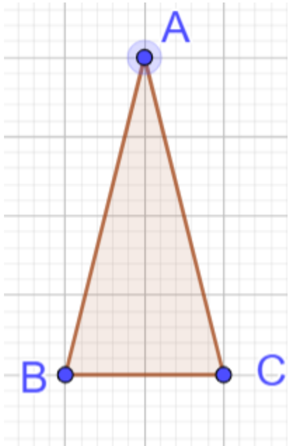
$$4 = 2x \Rightarrow x = 2$$
$$4 = 4y \Rightarrow y = 1$$

D. Isosceles Triangles

1.64: Isosceles Triangle

An Isosceles Triangle has *at least* two sides equal.

- The angles opposite the equal sides are called the base angles.
- The base angles are equal.
- The third angle is called the vertex angle. It is opposite the base.



Example 1.65

In the diagram alongside, where isosceles triangle ABC is drawn to scale, name the:

A. Equal Sides
B. Base angles
C. Base side
D. Vertex Angle

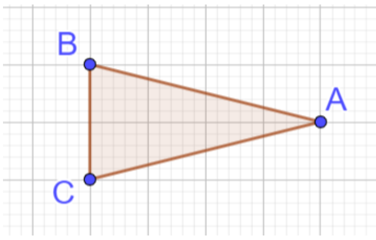
Equal Sides are

$$AB\ and\ AC$$

The base angles are the angles opposite to the equal sides.

$$Base\ Angle: \angle C\ is\ opposite\ side\ AB$$
$$Base\ Angle: \angle B\ is\ opposite\ side\ AC$$

The vertex angle is $\angle A$
The base side is side BC.
The equal sides are sides AB and AC



Look at the diagram alongside, which has the same triangle, turned on its side. Will the answers for the question above change?

The answers will not change.

$$BC\ is\ still\ the\ base$$
$$Base\ angles\ are\ \angle B\ and\ \angle C$$
$$Vertex\ angle\ is\ \angle A$$

MCQ 1.66

Triangle ABC is equilateral. Is the triangle also an Isosceles triangle?

A. Yes
B. No
C. Maybe

An equilateral triangle has three sides.
Hence, any two sides are equal.

An isosceles has two equal sides. And we just showed that two sides are equal.
Hence, the triangle is also isosceles.
Option A.

1.67: Equilateral and Isosceles Triangles

All equilateral triangles are isosceles triangles.
Some isosceles triangles are equilateral triangles.

MCQ 1.68

Triangle XYZ is Isosceles. Is the triangle also equilateral?

A. Yes
B. No
C. Cannot be determined with the given information.

Consider triangle XYZ with lengths:

$$4,4,x$$

If

$$x = 4 \Rightarrow Sides\ are\ 4,4,4 \Rightarrow Triangle\ is\ equilateral$$

If

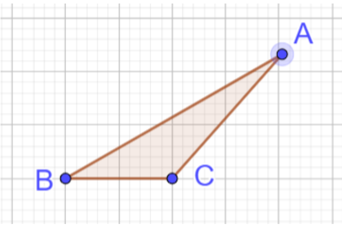
$$x = 5 \Rightarrow Sides\ are\ 4,4,5 \Rightarrow Triangle\ is\ isosceles, but\ not\ equilateral$$

Hence, yes or no cannot be determined without further information.
Option C.

E. Scalene Triangles

1.69: Scalene Triangle

A Scalene Triangle has no sides equal, and no angles equal.



Note that, in the diagram, no two sides of the triangle are equal. That is

$$AB \neq AC \neq BC$$

Example 1.70

Two sides of a scalene triangle are 4 $feet$ and 7 feet. The third side is a natural number that is more than 3, and less than 8. How many values can the length of the third side be?

The third side can take values:

$$5,6 \Rightarrow 2\ Values$$

Challenge 1.71

The longest side of a scalene triangle is 7 feet. Each side of the triangle is a natural number. If the other two sides have maximum possible length, what is the sum of the lengths of the other two sides?

$$5+6=11$$

Example 1.72

A triangle has sides 3 feet, 4 feet and 5 feet. How many feet must be reduced from the largest side, and added to the smallest side to make it an equilateral triangle?

Since you cannot change the middle value, it must be:

$$4$$

Hence, we need to make all three sides of length 4.
So, reduce the largest by 1, and increase the smallest by 1:

$$(3,4,5) \to (3+1,4,5-1) = (4,4,4) \Rightarrow Equilateral$$

MCQ 1.73

Choose All Correct Options

Classify the triangle ABC using the information given.
Side $AB = 2$ feet, Side $BC = 2$ feet, Side $AC = 24$ inches

A. Equilateral Triangle
B. Scalene Triangle
C. Isosceles Triangle

$$24\ inches = 2\ Feet$$

Hence, all three sides are equal.
Hence, the triangle is equilateral.

Also, an equilateral triangle is also an isosceles triangle.

Hence, options A and C are correct.

F. Angles in a Triangle

1.74: Sum of Angles

Sum of angles of a triangle is 180.

Example 1.75

Find the measures of the unknown angles in equilateral ΔABC.

$$\angle A = \angle B = \angle C = 60°$$

Example 1.76

Find the measures of the unknown angle in:

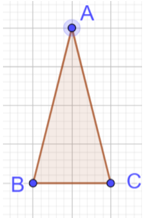
A. $\Delta LMN, \angle L = 30°, \angle M = 70°$
B. $\Delta LMN, \angle L = 40°, \angle M = 80°$

$$\angle N = 180° - (30° + 70°) = 180° - 100° = 80°$$

Example 1.77

Find the measures of the unknown angles in Isosceles ΔABC with vertex:

A. $\angle A = 70°$
B. $\angle A = 80°$
C. $\angle A = 100°$



$$\angle B = \angle C = \frac{180° - 70°}{2} = \frac{110°}{2} = 55°$$

1.78: Minimum Value of Angle of a Triangle

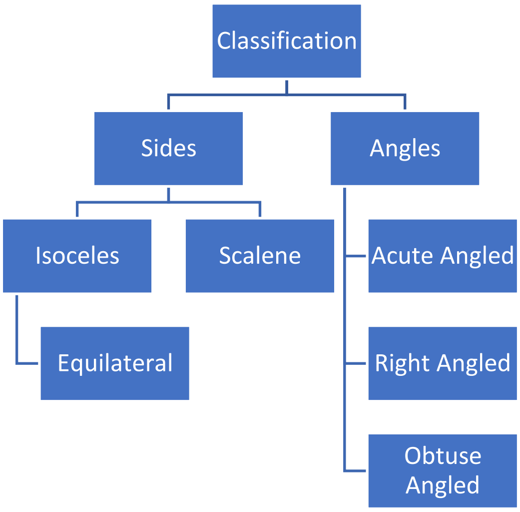
Value of angle of a triangle cannot be zero degrees.
Also, it cannot be negative.

Example 1.79

Can you have a triangle with two angles, which are 95° and 100°.

$$95 + 100 = 195° \Rightarrow Not\ Possible$$

G. Classification Based on Angles



1.80: Classification Based on Angles

- A triangle with one angle 90° is called a right-angled triangle.
- A triangle where one angle is obtuse is called an obtuse angled triangle.
- A triangle where all angles are acute is called an acute angle triangle.

Example 1.81

Is it possible to have a triangle:

A. with two right angles?
B. a right angle and an obtuse angle?
C. two obtuse angles?

$$90 + 90 = 180°$$

Third angle must be

$$0 \Rightarrow Not\ Possible$$

Example 1.82

What are the angles of an isosceles right-angled triangle?

Since the triangle is right-angled, one angle must be:

$$90°$$

Remaining two angles must be:

$$180 - 90 = 90°$$

Since the triangle is isosceles, each of the remaining angles must be equal. Hence, each angle is equal to

$$\frac{90}{2} = 45°$$

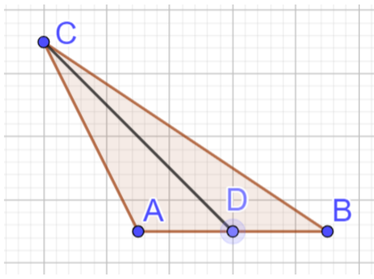
H. Special Lines in a Triangle

There are three special lines in a triangle. All three lines start from the vertex of the triangle.

- The median divides the opposite side into two.
- The angle bisector divides the angle into two.
- The altitude is perpendicular to the opposite side.

1.83: Median

- The median is the line segment joining the vertex of a triangle to the midpoint of the opposite side.
- The median will divide the opposite side into two equal parts.
- Every triangle has three medians.



Example 1.84

In the diagram alongside, CD is the median to side AB. Hence,

$$BD = DA$$

Numerical Questions

A. If $AB = 1\ m$, what is the length of AD, in cm?

B. If $AD = 3\ inches$, find the length of AB, in feet.

C. The perimeter of ΔABC is 1 foot. The lengths of AC and BC are 4 inches and 5 inches, respectively. The length from A to D is to be fenced at a cost of $Rs.$ 5 per inch. Find the cost of the fence.

Expressions

D. If $AB = 4x$, find the length of AD.

E. If $AB = x - 3$, find the length of AD.

F. If $AD = \frac{3}{4}y$, find the length of AB.

G. If $AD = \left(\frac{2}{3}y + 5\right)$ $inches$, find the length of AB. $(Challenge)$ Find the length of AB in feet.

Equations

Find the value of x, given that:

H. $AD = x + 5$, and $DB = 7$

I. $AD = 4x + 5$, and $DB = 9$

Numerical Questions

$$AD = \frac{1}{2} \times AB = \frac{1}{2} \times 1\ m = \frac{1}{2} m = 50\ cm$$

$$AB = 2 \times AB = 2 \times 3\ inches = 6\ inches = \frac{1}{2} foot$$

Part C

$$AB = 12 - 4 - 5 = 3\ inches$$

$$AD = \frac{3}{2}\ inches$$

$$Cost = 5 \times \frac{3}{2} = \frac{15}{2} = 7.5\ Rs.$$

Expressions

$$\frac{4x}{2} = 2x\ units$$

$$\frac{x-3}{2} = \left(\frac{2}{x} - \frac{3}{2}\right)\ units$$

$$\frac{3}{4}y \times 2 = \frac{3}{2}y\ units$$

Part F

$$2\left(\frac{2}{3}y + 5\right) = 2 \times \frac{2}{3}y + 2 \times 5 = \left(\frac{4}{3}y + 10\right) inches$$

To convert from inches to feet, divide by 12:

$$\frac{\frac{4}{3}y + 10}{12} = \left(\frac{4}{3}y + 10\right) \times \frac{1}{12}$$

Apply the distributive property:

$$\frac{4}{3}y \times \frac{1}{12} + 10 \times \frac{1}{12}$$

Simplify:

$$= \frac{1}{3}y \times \frac{1}{3} + \frac{5}{6}$$

Multiply:

$$= \frac{1}{9}y + \frac{5}{6}$$

Part G

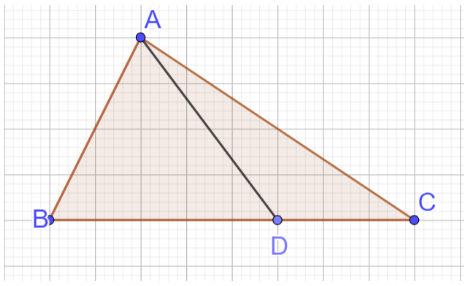
$$AD = DB \Rightarrow x + 5 = 7 \Rightarrow x = 2$$

Part H

$$AD = DB \Rightarrow 4x + 5 = 9 \Rightarrow 4x = 4 \Rightarrow x = 1$$

1.85: Angle Bisector

The line drawn from the vertex of a triangle bisecting the vertex angle is called the angle bisector.



In the diagram, AD is the angle bisector if:

$$\angle BAD = \angle CAD$$

- The angle bisector divides the vertex angle into two equal parts.
- Every triangle has three angle bisectors.
- An angle bisector never goes outside the triangle.

Example 1.86

In the diagram alongside (not drawn to scale), AD is the angle bisector.

Numerical Questions

A. If $\angle BAC = 64°$, find $\angle BAD$.

B. If $\angle ABC = 30°$, and $\angle ACD = 70°$, find $\angle BAC, \angle CAD, \angle ADB$.

C. If $\angle BAD = 15°$, and $\angle ABC$ is double of $\angle BAC$, find $\angle ACB$.

Expressions

D. If $\angle BAD = 2x + 4$ and $\angle DAC = 3x + 5$, then find the value of $\angle BAC$.

E. If $\angle ABC = 3x$ and $\angle ACB = 2x$, find the value of $\angle BAD$.

F. If $\angle ABC = 5x + 12$ and $\angle ACB = 3x - 10$, find the value of $\angle BAD$.

Equations

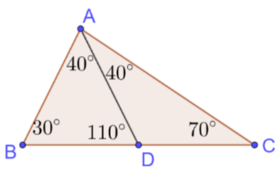
G. If $\angle BAD = 5x$ and $\angle DAC = 30$, then find the value of x.

H. If $\angle BAD = 2x + 4$ and $\angle DAC = 5x - 20$, then find the value of x. Also find the value of $\angle ABC + \angle ACD$.

Part A

$$\angle BAD = \frac{1}{2}\angle BAC = \frac{1}{2} \times 64 = 32°$$

Part B



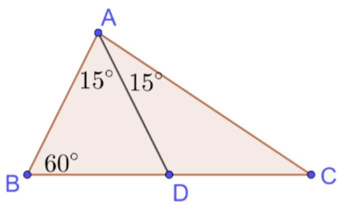
In ΔABC:

$$\angle BAC = 180 - 30 - 70 = 80°$$

$$\angle CAD = \frac{1}{2}\angle BAC = \frac{1}{2} \times 80 = 40°$$

$$\angle ADB = 180 - 30 - 40 = 110°$$

Part C



$$\angle BAC = 2 \times \angle BAD = 2 \times 15 = 30°$$

$$\angle ABC = 2\angle BAC = 2 \times 30 = 60°$$

In ΔABC:

$$\angle ACB = 180 - 30 - 60 = 90°$$

Part D

$$\angle BAC = \underbrace{(2x + 4)}_{\angle BAD} + \underbrace{(3x + 5)}_{\angle DAC} = 5x + 9$$

Part E

$$\underbrace{3x}_{\angle ABC} + \underbrace{2x}_{\angle ACB} + \angle BAC = 180$$

$$5x + \angle BAC = 180$$

Subtract $5x$ from both sides:

$$\angle BAC = 180 - 5x$$

$$\angle BAD = \frac{\angle BAC}{2} = \frac{180 - 5x}{2} = 90 - 2.5x$$

Part F

$$\underbrace{5x + 12}_{\angle ABC} + \underbrace{3x - 10}_{\angle ACB} + \angle BAC = 180$$

$$8x + 2 + \angle BAC = 180$$

Subtract $8x + 2$ from both sides:

$$\angle BAC = 180 - 8x - 2 = 178 - 8x$$

$$\angle BAD = \frac{\angle BAC}{2} = \frac{178 - 8x}{2} = 89 - 4x$$

Part G

$$\angle BAD = \angle CAD$$

$$5x = 30$$

$$x = 6$$

Part H

$$\angle BAD = \angle CAD$$

$$2x + 4 = 5x - 20$$

Subtract $2x$ from both sides:

$$4 = 3x - 20$$

Add 20 to both sides:

$$24 = 3x$$

Divide by 3 both sides:

$$x = 8$$

Now, we can find the measure of the angles

$$\angle BAD = 2x + 4 = 20$$

$$\angle CAD = \angle BAD = 20$$

$$\angle BAC = 20 + 20 = 40$$

$$40 + \angle ABC + \angle ACB = 180$$

$$\angle ABC + \angle ACB = 180 - 40 = 140$$

1.87: Altitude

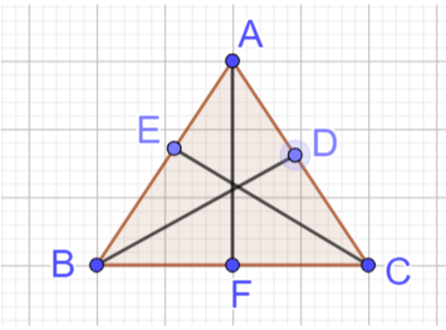
- The line segment drawn from the vertex of a triangle perpendicular to the opposite side is the altitude.
- The length of the altitude is called the height
- The side opposite the vertex to which the altitude is drawn is called the base.

There are three possible cases with respect to the altitudes of a triangle.
Case I: Acute Angled Triangle.
Case II: Right Angled
Case III: Obtuse Angled Triangle.

Example 1.88

ΔABC is an acute-angled triangle, in which the altitudes have been drawn.

A. Name the altitudes.
B.



The altitudes are:

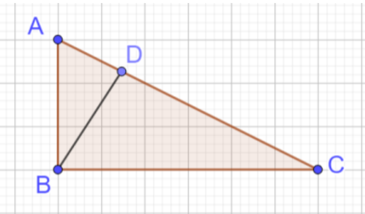
$$AF, BD, CE$$

$$\angle AFB = \angle AFC = 90°$$

$$\angle CEB = \angle CEA = 90°$$

$$\angle BDA = \angle BDC = 90°$$

Example 1.89



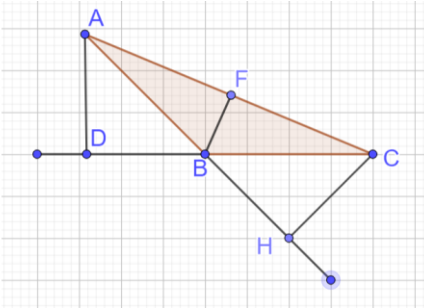
The altitudes are:

$$AB, BD, CB$$

$$\angle BDA = \angle BDC = 90°$$

$$\angle ABC = 90°$$

Example 1.90



The altitudes are:

$$AD, BF, CH$$
$$\angle BFA = \angle BFC = 90°$$
$$\angle ADC = 90°$$
$$\angle CHB = 90°$$

1.91: Properties of Special Lines

- Every triangle has three medians, three angle bisectors, and three altitudes.

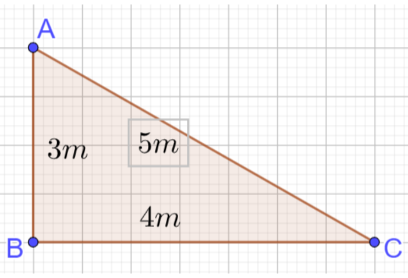
In an equilateral triangle, the median, the angle bisector and the altitude are the same.

I. Perimeter

1.92: Perimeter

Perimeter is the total length of the sides of the triangle.

$$\therefore Perimeter\ of\ \text{Triangle} = \text{Side}_1 + Side_2 + Side_3$$



Example 1.93

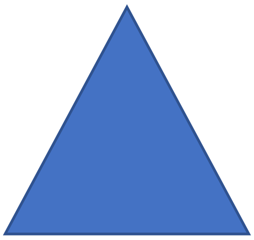
Find the perimeter of the following triangles:

A. The triangle drawn alongside
B. $5\ m, 12\ m, 13\ m$
C. $7\ m, 40\ m, 41\ m$
D. $8\ m, 15m, 17m$
E. $3\ cm, 5\ cm, and\ 7\ cm$
F. $4\ inches, 7\ inches, and\ 10\ inches$
G. $6\ feet, 7\ feet, and\ 9\ feet$
H.

$$3 + 4 + 5 = 12\ m$$
$$Part\ A:\ 5 + 12 + 13 = 30\ m$$
$$Part\ B: 7 + 40 + 41 = 88\ m$$
$$Part\ C: 8 + 15 + 17 = 40\ m$$

Example 1.94

The triangle in the diagram has all sides the same length. Find the perimeter of the triangle.



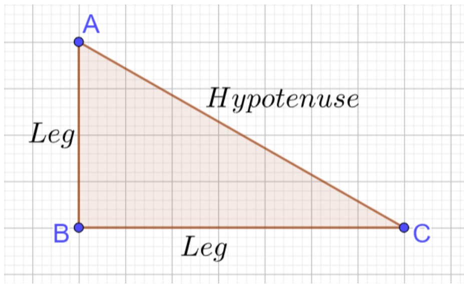
$$3m + 3m + 3m = 9m$$

J. Area of Right Triangles

If one of the angles of a triangle is a right angle, then the triangle is a right triangle.

1.95: Parts of a Right Triangle

- The side opposite the right angle in a right triangle is called the hypotenuse. It is the longest side of the triangle.
- The two sides other than the hypotenuse are called the legs of the right triangle. They are perpendicular to each other.



Example 1.96

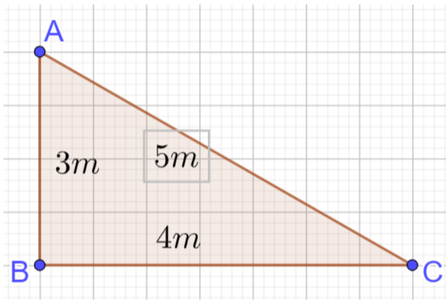
In the diagram alongside, identify the:

A. Hypotenuse
B. Legs
C. Right Angle

$$\text{Hypotenuse}: AC$$
$$\text{Legs}: AB, \& \ BC$$
$$\text{Right Angle}: \angle ABC$$

1.97: Area of Right Triangle

$$\text{Area of Right Triangle} = \frac{Leg_1 \times Leg_2}{2}$$



Example 1.98

A. Find the area of the triangle shown alongside.
B. In a right-angled triangle, the length of one leg is 4 meters, and the length of the other leg is 6 meters. Find the area of the triangle.
C. The length of one leg of a right-angled triangle is 3 meters, and the length of the other leg is two meters more than the first leg. Find the area of the triangle.
D. (*Finding the legs*) In ΔABC, $\angle B$ is a right angle, the length of AC is 5 meters, and the other two lengths are 3 meters, and 4 meters, respectively. Find the area of ΔABC.
E. (*Finding the legs*) In ΔXYZ, the sides are 5 meters, 12 meters and 13 meters respectively. Also, the triangle is a right-angled triangle. Find the area of ΔXYZ.
F. (*Unit Conversion*) A right triangle has length of one leg 350 cm, and other leg 250 cm. Find the area of the triangle in m^2.
G. (*Back Calculations*) The area of a right triangle is 30 square meters. One leg of the right triangle is 30 meters. Find the length of the other leg.
H. (*Multi Step*) A right triangle has one leg which is double of the other leg. The area of the triangle is 9 square feet. Find the sum of the lengths of the legs of the triangle.

Part A

$$\text{Area of Triangle} = \frac{3 \times 4}{2} = 6\ m^2$$

Part B

$$\text{Area} = \frac{4 \times 6}{2} = 4 \times 3 = 12\ m^2$$

Part C

$$\text{Area} = \frac{3 \times 5}{2} = \frac{15}{2} = 7\frac{1}{2}\ m^2$$

Part D

We do not need the information on the hypotenuse:

$$\frac{3 \times 4}{2} = 6\ meters^2$$

Part E

Since the hypotenuse is the longest side, the legs must be the other two sides:

$$\frac{5 \times 12}{2} = 30\ meters^2$$

Part F

$$350\ cm = 3.5\ m = 3\frac{1}{2}m = \frac{7}{2}m$$
$$250\ cm = 2.5\ m = 2\frac{1}{2}m = \frac{5}{2}m$$

$$\text{Area} = \frac{1}{2} \times \frac{7}{2}m \times \frac{5}{2}m = \frac{35}{8}m^2$$

Alternate Method
If we do not convert into meters, the calculations become quite lengthy:

$$\frac{250 \times 350}{2} = \frac{87500}{2} = 43750\ cm^2$$

To convert from cm^2 into m^2, we need to divide $100^2 = 10,000$:

$$\frac{43750}{10,000} = 4.375\ m^2$$

Part G

$$\frac{1}{2} \times Leg_1 \times Leg_2 = 30$$
$$\frac{1}{2} \times 30 \times Leg_2 = 30$$
$$15 \times Leg_2 = 30$$
$$Leg_2 = 2$$

Part H

$$\frac{1}{2} \times Leg_1 \times Leg_2 = 9 \Rightarrow Leg_1 Leg_2 = 18$$

We want values of the first and the second leg, which multiply to 18, and the first leg should be half of the second leg.
Try different values that multiply to 18:

$$18 = 1 \times 18 = 2 \times 9 = 3 \times 6$$

6 is double of 3, and hence the legs are

$$3\ and\ 6$$

And their sum is

$$3 + 6 = 9\ feet$$

Example 1.99

A. One leg of a right triangle is x feet, and the other leg is y feet. Find the area of the triangle.
B. Find the area of an isosceles right triangle with a leg of length $l\ meter$.
C. One leg of a right triangle is p feet, and the other leg is q feet. Find the area of the triangle in square inches.
D. Find, in square cm, the area of an isosceles right triangle with a leg of length $a\ meter$.

Part A

$$Area = \frac{Leg_1 \times Leg_2}{2} = \frac{x \times y}{2} = \frac{xy}{2}\ ft^2$$

Part B

$$Area = \frac{Leg_1 \times Leg_2}{2} = \frac{l \times l}{2} = \frac{l^2}{2} ft^2$$

Part C
Convert the given lengths into inches

$$p\ feet = 12p\ inches$$
$$q\ feet = 12q\ inches$$
$$Area = \frac{12p \times 12p}{2} = \frac{72pq}{2}\ inches^2$$

Part D

$$a\ meters = 100a\ cm$$
$$Area = \frac{100a \times 100a}{2} = 5000a^2 = cm^2$$

Challenge 1.100

A right triangle has legs of length 3 meters and 7 meters.
A. Find the area of the triangle.
B. If the length of one leg is doubled, and the length of the other leg is halved, what is the area of the new triangle?

Part A
Original area of the triangle

$$\frac{1}{2} \times 3 \times 7 = \frac{21}{2} = 10.5\ m$$

Part B

Find the new area of the triangle.

$$Case\ I: \left(3 \times \frac{1}{2}\right) \times (7 \times 2) \times \frac{1}{2} = 3 \times 7 \times \frac{1}{2} = \frac{21}{2}\ meters$$

And the area that we find is the same as the area of the original triangle.

Even if the smaller leg is doubled, and the other leg is halved, we still get the same answer:

$$Case\ II: (3 \times 2) \times \left(7 \times \frac{1}{2}\right) \times \frac{1}{2} = 3 \times 7 \times \frac{1}{2} = \frac{21}{2}\ meters$$

K. Area of General Triangles

1.101: Area of a Triangle

$$Area = \frac{1}{2} \times Height \times Base = \frac{1}{2}hb$$

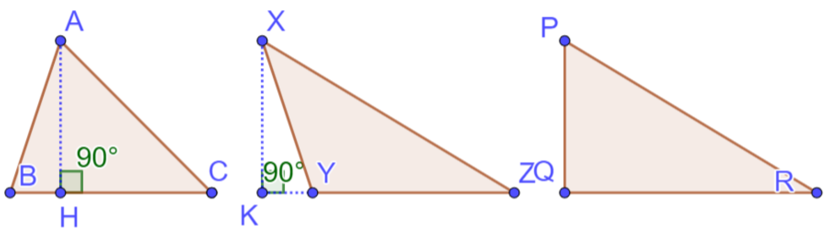
Where

$$Height = Length\ of\ Altitude$$

Example 1.102

In the diagram below, which is not drawn to scale, find the area of each triangle.

A. AH has length 2 feet, and BC has length 6 *inches*.
B. XK has length $3m$ and YZ has length 25 cm.
C. PQ, QR and PR have length 3, 4 and 5 yards.



Part A

We can calculate the area in inches:

$$\frac{hb}{2} = \frac{(AB)(BC)}{2} = \frac{(24\ in)(6\ in)}{2} = 72\ in^2$$

We can also calculate the area in feet:

$$\frac{hb}{2} = \frac{(AB)(BC)}{2} = \frac{(2\ ft)\left(\frac{1}{2}\ ft\right)}{2} = \frac{1}{2}\ ft^2$$

Part B

$$\frac{hb}{2} = \frac{(XK)(YZ)}{2} = \frac{300\ cm \times 25\ cm}{2} = \frac{7500}{2} = 3750\ cm^2$$

$$\frac{1}{2}hb = \frac{1}{2}(XK)(YZ) = \frac{1}{2} \times 3\ m \times \frac{1}{4}m = \frac{3}{8}m^2$$

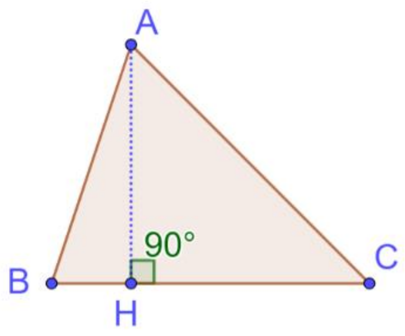
Part C

$$\frac{hb}{2} = \frac{(PQ)(QR)}{2} = \frac{(3\ yards)(4\ yards)}{2} = 6\ yards^2$$

Example 1.103

In the diagram alongside, which is not drawn to scale,

A. Find area of ΔABC if $AH = 2\ feet$, and $BC = 3\ feet$
B. Find area of ΔABH if $AH = 3\ inches$, and $BH = 6\ inches$
C. Find area of ΔABC if $AH = 5\ meters$, and $BC = 50\ cm$
D. Find area of ΔABC if $AH = \frac{1}{2} foot$, and $BC = \frac{1}{3} feet$
E. Find AH given that the area of $\Delta ABC = 6\ ft^2$, and $BC = 3\ ft$.
F. Find BC given that the area of $\Delta ABC = 1\ ft^2$, and $AH = 1\ ft$.
G. Find area of ΔABC if $AH = x\ feet$, and $BC = y\ feet$
H. Find area of ΔABC if $AH = (x + 2)\ feet$, and $BC = 2\ feet$



Part A

$$\frac{1}{2}hb = \frac{1}{2}(2\ feet)(3\ feet) = 3\ ft^2$$

Part B

$$\frac{1}{2}hb = \frac{1}{2}(3\ in)(6\ in) = 9\ in^2$$

Part C

$$\frac{1}{2}hb = \frac{1}{2}(5\ m)(50\ cm) = \frac{1}{2}(5\ m)\left(\frac{1}{2}\ m\right) = \frac{5}{4}\ m^2$$

Part D

$$\frac{1}{2}hb = \frac{1}{2}\left(\frac{1}{2}foot\right)\left(\frac{1}{3}foot\right)$$
$$= \frac{1}{2}(6\ in)(4\ in) = 12\ in^2$$

Part E

$$\frac{1}{2}hb = A(\Delta ABC)$$
$$\frac{1}{2}(AH)(3\ feet) = 6\ ft^2$$
$$AH = 4\ ft$$

Part F

$$\frac{1}{2}hb = A(\Delta ABC)$$
$$\frac{1}{2}(1\ ft)(BC) = 1\ ft^2$$
$$BC = 2\ ft$$

Part G

$$\frac{1}{2}hb = \frac{1}{2}xy = \frac{xy}{2}\ ft^2$$

Part H

$$\frac{1}{2}hb = \frac{1}{2}(x+2)(2) = x + 2\ ft^2$$

Example 1.104

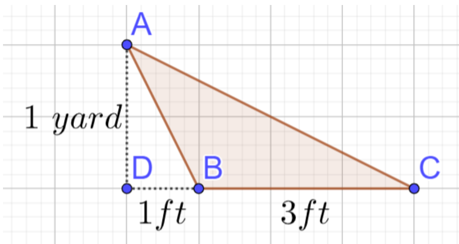
The height of a triangle is 2 $feet$. The base of the triangle is 4 $inches$. Find the area of the triangle.

$$\frac{1}{2}hb = \frac{1}{2}(2\ feet)(4\ inches) = (1\ feet)(4\ inches) = (12\ inches)(4\ inches) = 48\ in^2$$

Example 1.105

In the diagram alongside, which is not drawn to scale, $AD = 1\ yard, DB = 1\ ft, BC = 3ft$. Find the area of

A. ΔADB
B. ΔABC
C. ΔADC

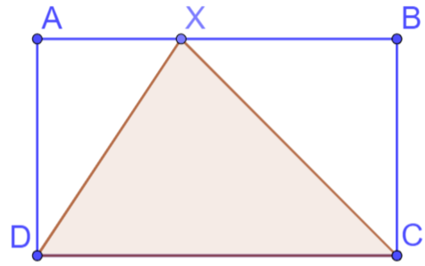


$$\Delta ADB = \frac{1}{2}hb = \frac{1}{2}(3)(1) = \frac{3}{2}\ ft^2 = 1.5\ ft^2$$
$$\Delta ADC = \frac{1}{2}hb = \frac{1}{2}(AD)(DC) = \frac{1}{2}(1\ yard)(4\ ft) = \frac{1}{2}(3\ ft)(4\ ft) = 6\ ft^2$$
$$\Delta ABC = \Delta ADC - \Delta ADB = 6 - 1.5 = 4.5\ ft^2$$
$$\Delta ABC = \frac{1}{2}hb = \frac{1}{2}(AD)(BC) = \frac{1}{2}(1\ yard)(3\ ft) = \frac{1}{2}(3\ ft)(3\ ft) = \frac{9}{2}ft^2 = 4.5\ ft^2$$

Example 1.106

Find the area of the region inside the rectangle, but outside ΔDXC (unshaded region) if:

A. Rectangle $ABCD$ has $AB = 4\ m$, and $BC = 3\ m$.
B. Rectangle $ABCD$ has $AB = \frac{2}{3}\ m$, and $BC = \frac{3}{5}\ m$.
C. Rectangle $ABCD$ has $AB = l\ m$, and $BC = w\ m$.



Part A

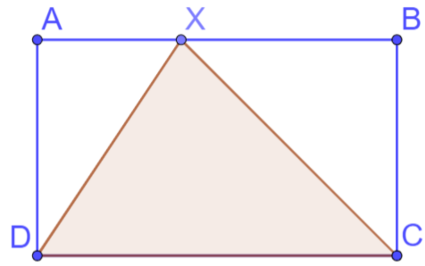
$$Area\ of\ Rectangle\ ABCD = (AB)(BC) = 4 \times 3 = 12\ m^2$$
$$Area\ of\ \Delta DXC = \frac{1}{2}hb = \frac{1}{2}(DC)(BC) = \frac{1}{2}(4)(3) = 6\ m^2$$
$$Unshaded\ Region = 12\ m^2 - 6m^2 = 6m^2$$

Part B

$$Area\ of\ Rectangle\ ABCD = (AB)(BC) = \frac{2}{3} \times \frac{3}{5} = \frac{2}{5}\ m^2$$
$$Area\ of\ \Delta DXC = \frac{1}{2}hb = \frac{1}{2}(DC)(BC) = \frac{1}{2}\left(\frac{2}{3}\right)\left(\frac{3}{5}\right) = \frac{1}{5}\ m^2$$
$$Unshaded\ Region = \frac{2}{5} - \frac{1}{5} = \frac{1}{5}\ m^2$$

Part C

$$Area\ of\ Rectangle\ ABCD = (AB)(BC) = lw\ m^2$$
$$Area\ of\ \Delta DXC = \frac{1}{2}hb = \frac{1}{2}(DC)(BC) = \frac{lw}{2}\ m^2$$
$$Unshaded\ Region = lw - \frac{lw}{2} = \frac{lw}{2}$$



Example 1.107

($Refer\ previous\ example$) What can be said about:

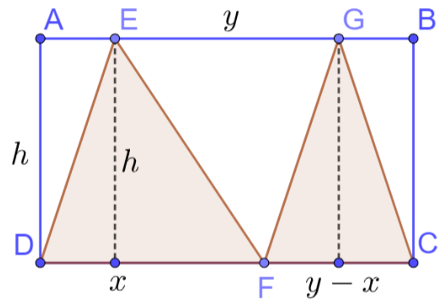
A. The area of the triangle in terms of the area of the rectangle
B. Area of the unshaded region in terms of the area of the rectangle

Area of the triangle is half of the area of the rectangle:

$$Area(Triangle) = \frac{Area(Rectangle)}{2}$$

Area of the unshaded region is half of the area of the rectangle:

$$Area(Unshaded\ Region) = \frac{Area(Rectangle)}{2}$$



Example 1.108

In rectangle $ABCD$, $AB = y, AD = h, DF = x$

$$Area\ of\ Rectangle = (AB)(AD) = hy$$

$$A(\Delta\text{DEF}) = \frac{1}{2}hb = \frac{1}{2}hx$$
$$A(\Delta\text{FGC}) = \frac{1}{2}hb = \frac{1}{2}h(y - x) = \frac{1}{2}hy - \frac{1}{2}hx$$

$$A(\Delta\text{DEF}) + A(\Delta\text{FGC}) = \frac{1}{2}hx + \frac{1}{2}hy - \frac{1}{2}hx = \frac{1}{2}hy$$

Example 1.109

A. A triangle has an area of x units. Find the area of a second triangle with same base as the first triangle, but height double of the first triangle.

$$Base = 2, Height = 2 \Rightarrow Area = \frac{1}{2}(2)(2) = 2\ units^2$$
$$Base = 2, Height = 4 \Rightarrow Area = \frac{1}{2}(2)(4) = 4\ units^2$$

$$Base = b, Height = 2 \Rightarrow Area = \frac{1}{2}(b)(2) = b\ units^2$$
$$Base = b, Height = 4 \Rightarrow Area = \frac{1}{2}(b)(4) = 2b\ units^2$$

L. Area Review

True/False 1.110

A. An isosceles triangle is a triangle with two equal sides, and a third side not equal to either of the first two.
B. An equilateral triangle is not an isosceles triangle.

Part A
False. The correct definition is:
An isosceles triangle is a triangle with at least two equal sides. The third side may or may not be equal.
Part B
False. All equilateral triangles are, by definition, isosceles triangles.

True/False 1.111

If the base angles of an isosceles triangle are equal to the vertex angle, is it a:
A. Isosceles
B. Equilateral
C. Scalene
D. Acute-Angled
E. Obtuse-Angled
(Answer True or False for each one)

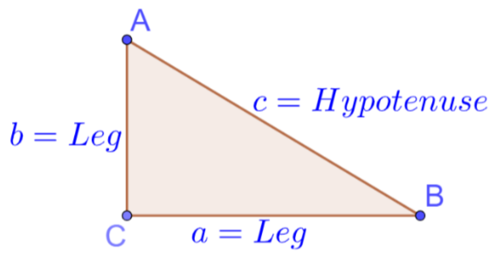
All three angles are equal.
Hence, it is an equilateral triangle.

$$A: True.$$
$$B. True$$
$$C. False$$
$$D. True$$
$$E. False$$

M. Pythagorean Theorem

1.112: Pythagorean Theorem

If a, b and c are the sides of a right-angled triangle, with c being the hypotenuse, then $a^2 + b^2 = c^2$



- The Pythagorean Theorem is named after Pythagoras, who was a Greek mathematician who lived around 2000 years.
- The longest side in a right triangle is the hypotenuse

Example 1.113: Verifying the Pythagorean Theorem

Verify that the Pythagorean Theorem holds for the following right-angled triangles.

A. 3 m, 4m, 5m
B. 5 m, 12 m, 13 m
C. 9 m, 40 m, 41 m
D. 8 m, 15m, 17m

$$\underbrace{3^2}_{a=3} + \underbrace{4^2}_{b=4} = 9 + 16 = \underbrace{25}_{c^2} = \underbrace{5^2}_{c}$$

$$5^2 + 12^2 = 25 + 144 = 169 = 13^2$$

$$9^2 + 40^2 = 81 + 1600 = 1681 = 41^2$$

$$8^2 + 15^2 = 64 + 225 = 289 = 17^2$$

1.114: Primitive Pythagorean Triplets

Sets of three natural numbers that can form the sides of a right-angled triangle are called Pythagorean Triplets.

Primitive Pythagorean Triplets			
3, 4, 5	20, 21, 29	11, 60, 61	36, 77, 85
5, 12, 13	12, 35, 37	33, 56, 65	13, 84, 85
8, 15, 17	9, 40, 41	16, 63, 65	39, 80, 89
7, 24, 25	28, 45, 53	48, 55, 73	65, 72, 97

We can make a table of the Pythagorean Triplets where the numbers are less than 100. Note that

$$HCF(3,4,5) = 1$$

In fact, this is true of any three numbers in the list above.
Numbers which are like this are called primitive Pythagorean Triplets.

1.115: Converse of Pythagorean Theorem

If the sides a, b and c of a triangle are such that $a^2 + b^2 = c^2$ then the triangle is a right-angled triangle, with c being the hypotenuse.

The converse reverses a theorem. The converse of the Pythagorean is also true.

Example 1.116

Check, using the Pythagorean Theorem, if the triangles below are right-angled:

A. 6m, 8m,10m
B. 10 m, 24m, 26 m
C. 4m, 6m, 9m
D. 9m, 12m, 15m

$$6^2+8^2=36+64=100=10^2 \Rightarrow Right-angled$$

$$10^2+24^2=100+576=676=26^2$$
$$4^2+6^2=16+36=52\neq 81=9^2 \Rightarrow Not\ Right-Angled$$
$$9^2+12^2=81+144=225=15^2$$

1.117: Non-Primitive Pythagorean Triplets

Any multiple of a Pythagorean Triplet is also a Pythagorean Triplet. This is called a non-primitive Pythagorean Triplet.

Example 1.118

Find the next few Pythagorean Triplets which are multiples of

A. (3,4,5)
B. (5,12,13)

$$(3,4,5)\rightarrow \underbrace{(6,8,10)}_{\substack{\textbf{Multiply }(3,4,5)\\ \textbf{by }2}} \rightarrow \underbrace{(9,12,15)}_{\substack{\textbf{Multiply }(3,4,5)\\ \textbf{by }3}} \rightarrow \underbrace{(12,16,20)}_{\substack{\textbf{Multiply }(3,4,5)\\ \textbf{by }4}} \rightarrow \underbrace{(15,20,25)}_{\substack{\textbf{Multiply }(3,4,5)\\ \textbf{by }5}}$$

$$(5,12,13)\rightarrow \underbrace{(10,24,26)}_{\substack{\textbf{Multiply }(5,12,26)\\ \textbf{by }2}} \rightarrow \underbrace{(15,36,39)}_{\substack{\textbf{Multiply }(5,12,13)\\ \textbf{by }3}} \rightarrow \underbrace{(20,48,52)}_{\substack{\textbf{Multiply }(5,12,13)\\ \textbf{by }4}} \rightarrow \underbrace{(25,60,65)}_{\substack{\textbf{Multiply }(5,12,13)\\ \textbf{by }5}}$$

Example 1.119

Check, using the Pythagorean Triplets, if the triangles below are right-angled:

A. 6m, 8m,10m
B. 10 m, 24m, 26 m
C. 4m, 6m, 9m
D. 9m, 12m, 15m

Part A

$$(6,8,10)$$

HCF of (6,8,10) is 2. Hence, divide each number in the triplet by 2:

$$2\left(\frac{6}{2},\frac{8}{2},\frac{10}{2}\right)=2(3,4,5)=2(Pythagorean\ Triplet)\Rightarrow Right-angled$$

Part B

$$(10,24,26)=2(5,12,13)=2(Pythagorean\ Triplet)\Rightarrow Right-angled$$

Part C

$$(4,6,9)\rightarrow HCF\ is\ 1\Rightarrow Not\ a\ Pythagorean\ Triplet$$

Part D

$$(9,12,15)=3(3,4,5)=2(Pythagorean\ Triplet)\Rightarrow Right-angled$$

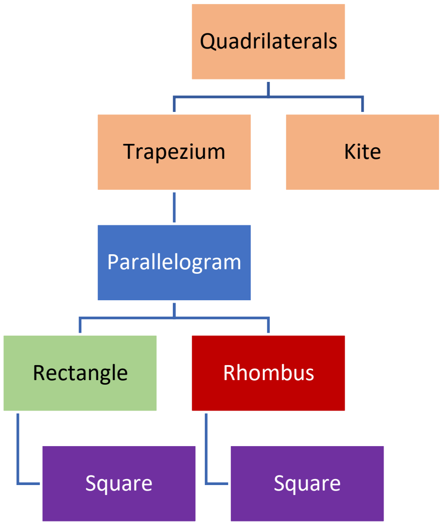
1.3 Quadrilaterals

A. Quadrilaterals

Quadrilaterals are four sided closed figures.

- They have four sides

- They have four angles
- They have four corners



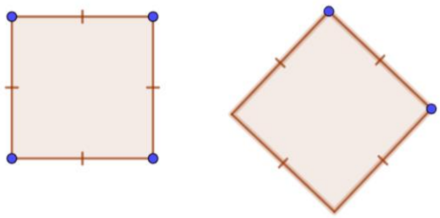
1.120: Classification of Quadrilaterals

- Trapezoid: At least one pair of opposite sides is parallel.
- Parallelogram: Two pairs of opposite sides are parallel.
- Rectangle: *All angles* $90°$
- Rhombus: *All Sides equal*
- Square: *All angles* $90°$AND *all sides equal*
- Kite: Two pairs of adjacent sides are equal.

Example 1.121

A. Is a square a kite?
B. If all four angles of a quadrilateral are equal, then what shape is it?

Part A
A square is also a kite. It can seen by rotating the square so that a corner is "up".

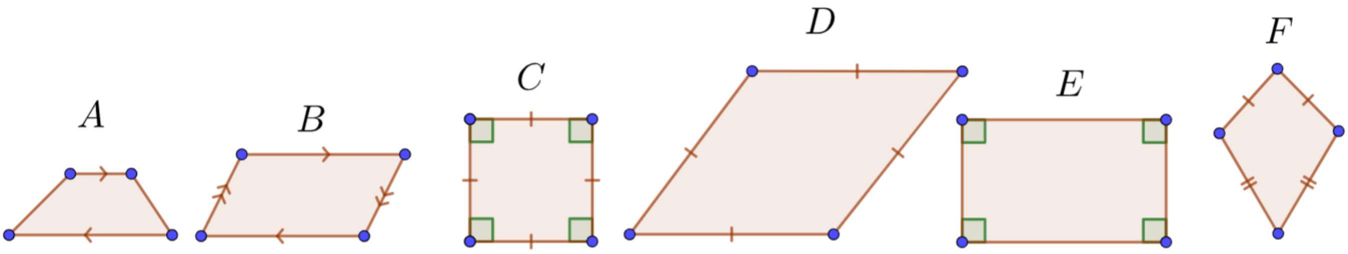


Part B
The sum of angles of a quadrilateral is 360°. If all four angles are equal, then each angle must be

$$\frac{360}{4} = 90° \Rightarrow Quadrilateral\ is\ a\ Rectangle$$

Example 1.122

Classify the quadrilaterals below. Apply all applicable terms.



A. Trapezoid
B. Parallelogram, Trapezoid
C. Square, Rectangle, Rhombus, Parallelogram, Trapezoid, Kite
D. Rhombus, Parallelogram, Trapezoid, Kite
E. Rectangle, Parallelogram, Trapezoid
F. Kite

Example 1.123

Classify the following quadrilaterals based on the information given. Apply all applicable terms.

A. Quadrilateral A has four equal sides.
B. Quadrilateral B has all four angles 90°

C. Quadrilateral C has two pairs of adjacent sides which are congruent.
D. Quadrilateral D has a pair of opposites sides which are parallel.
E. Quadrilateral E has equal sides and equal angles.

A. Rhombus, Parallelogram, Trapezoid, Kite
B. Rectangle, Parallelogram, Trapezoid
C. Kite
D. Trapezoid
E. Square, Rectangle, Rhombus, Parallelogram, Trapezoid, Kite

Challenge 1.124

For each statement below, fill in the blanks in the italicized text with *all* applicable options.

The quadrilateral cannot be ____. The quadrilateral must be _____. The quadrilateral can be _____.

Statement I: A quadrilateral has adjacent sides which are equal, but all of its angles are not equal.
Statement II: A quadrilateral has four equal sides. Its angles are not all equal.
Statement III: A quadrilateral has four equal sides and four equal angles.
Statement IV: A quadrilateral with unequal sides has one pair of parallel sides.

A. Kite
B. Trapezoid
C. Square
D. Rectangle
E. Rhombus
F. Parallelogram

Statement I

The quadrilateral cannot be $\boldsymbol{B, C, D, E, F}$. The quadrilateral must be $\boldsymbol{A},$. The quadrilateral can be ____ (No options in the third blank).

Statement II

The quadrilateral cannot be $\boldsymbol{C, D}$. The quadrilateral must be $\boldsymbol{A, B, E, F}$. The quadrilateral can be _____. (No options in the third blank).

Statement III

The quadrilateral cannot be *No Correct Options*. The quadrilateral must be $\boldsymbol{A, B, C, D, E, F}$_____. The quadrilateral can be *No Correct Options*.

Statement IV

The quadrilateral cannot be $\boldsymbol{A, C, D, E, F}$____. The quadrilateral must be $\boldsymbol{B}$,_____. The quadrilateral can be _____.

1.125: Properties of Quadrilaterals

		Rectangle	Rhombus	Square
All Sides	Congruent			
Opposite Sides	Congruent			
All Angles	Equal			
Opposite Angles	Equal			
Diagonals	Congruent			
	Bisect each other			

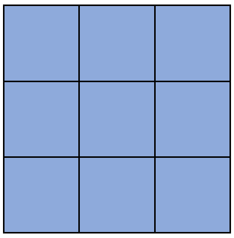
	Make right angles with other diagonals			

B. Perimeter

1.126: Perimeter of a Square

The perimeter of a square is the sum of the length of the sides of the square

$$\therefore Perimeter\ of\ Square = 4 \times Side\ Length$$



Example 1.127

Find the perimeter of the square drawn alongside.

$$Side\ Length = 3 \Rightarrow Perimeter = \underbrace{4 \times 3}_{4 \times Side} = 12\ Units$$

Example 1.128

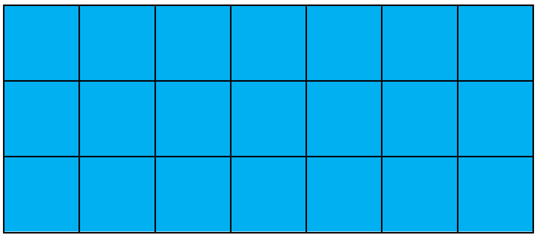
Find the perimeter (p) of a square with side length:

A. $6\ m$
B. $5\ inches$
C. $7\ cm$

$$p = 4 \times s = 4 \times 6 = 24\ m$$
$$p = 4 \times s = 4 \times 5 = 20\ inches$$

1.129: Perimeter of a Rectangle

$$\therefore Perimeter\ of\ \text{Rectangle} = P = 2 \times length + 2 \times width$$



Example 1.130

Find the perimeter of the rectangle drawn alongside.

From the diagram:

$$Length = 7\ \ Units, \qquad Width = 3\ Units$$

$$Perimeter = \underbrace{2 \times 7}_{2 \times length} + \underbrace{2 \times 3}_{2 \times width} = 14 + 6 = 20$$

Example 1.131

Find the perimeter of the rectangles below:

A. $length = 9\ m, width = 8\ m$
B. $length = 5\ cm, width = 3\ cm$
C. $length = 4\ feet, width = 4\ feet$

$$p = \underbrace{2 \times 9}_{2 \times length} + \underbrace{2 \times 8}_{2 \times width} = 18 + 16 = 34$$

Example 1.132: Fencing and Fenceposts

A fence is usually put around the boundary of a garden, or a field, or a playground, etc. The length of the fence is the same as the perimeter of the geometrical figure.

A. A square garden with roses has area 81 square feet. Find the length of fence required to fence the garden.
B. A square garden which is 12 feet long is to be fenced using a fence which costs Rs. 4 per foot. Find the cost of the fence?
C. A triangular garden has sides of length 3 meters, 4 meters and 6 meters. What is the length of fence required to fence the garden? If the cost of fence is $7 per meter. What is the cost of fencing the garden?
D. A triangular field with lengths of 7 meters, 40 meters and 41 meters is to be fenced. The cost of fence is $10 per meter, not including fenceposts. A fencepost is needed at each corner of the field, and one fencepost cost $50. Find the cost of fencing the field (including fenceposts)?
E. A triangular garden is in the shape of a right-angled triangle, and has side lengths of 3 m, 4 m and 5 m respectively. A fence is to be put up around the garden, and to support the fence, fenceposts are needed at intervals of 1 m. Fence costs $6 per meter, fenceposts cost $17 for each one. Find the total cost of fencing.

Part A

Part B

$$Length\ of\ fence = perimeter\ of\ square = 12 \times 4 = 48$$
$$Cost\ of\ fence = 48 \times 4 = 192\ Rs.$$

Part C

$$Length\ of\ Fence = 3 + 4 + 6 = 13\ meters$$
$$Cost\ of\ Fence = 13 \times 7 = \$91$$

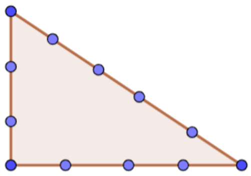
Part D

$$Cost\ of\ Fence = Length\ of\ Fence \times Cost\ per\ meter = (7 + 40 + 41) \times 10 = 88 \times 10 = \$880$$

$$Cost\ of\ Fenceposts = No.of\ Posts \times Cost\ of\ Posts = 3 \times 50 = \$150$$

$$Total\ Cost = 880 + 150 = \$1030$$

Part E



$$Cost\ of\ Fence = (3 + 4 + 5) \times 6 = 12 \times 6 = 72$$

$$Cost\ of\ Fenceposts = \$204$$

$$Total\ Cost = 276$$

Example 1.133: Laps

Laps is the number of times that you run. For example, running 2 laps means you are running twice.
A sportsman runs 4 laps around the perimeter of a square field, which has side length 400 m. How many kilometers does he run? After every 1200 meters, he changes his shoes. How many pairs of shoes (including the ones that he wears at the beginning) will he use for running?

$$s = 400\ m \Rightarrow p = 1600\ m \Rightarrow Running = 6400\ m = 6.4\ km$$

And the shoes will be needed at:

$$\underbrace{0\ m}_{\substack{1st\\Pair}},\quad \underbrace{1200\ m}_{\substack{2nd\\Pair}},\quad \underbrace{2400\ m}_{\substack{3rd\\Pair}},\quad \underbrace{3600\ m}_{\substack{4th\\Pair}},\quad \underbrace{4800\ m}_{\substack{5th\\Pair}},\quad \underbrace{6000\ m}_{\substack{6th\\Pair}} \Rightarrow 6\ Pairs$$

$$\frac{6400}{1200}=\frac{64}{12}=\frac{32}{6}=\frac{16}{3}=5\frac{1}{3}\to 5\ Shoes+1\ Beginning=6\ Pairs\ of\ Shoes$$

Example 1.134: Algebraic Applications

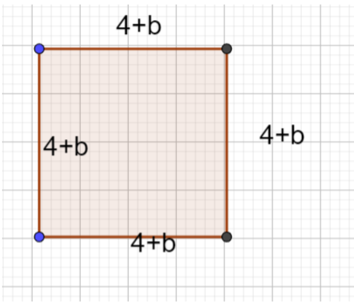
A. The numerical value of the area of a square is the same as the numerical value of its perimeter. Find the side length of the square.
B. The numerical value of the perimeter of a square is four times the numerical value of the area of the square. Find the side length of the square.
C. The side length of a square is $4+b$. Find its perimeter.
D. The side length of a square is $2b-3$. Find its perimeter.
E. The perimeter of a square is $36-24c$. Find the side length of the square.
F. The perimeter of a square is $16x+96$. Find the side length of the square.
G. The length of a rectangle is l. And the breadth is b. Find its perimeter.

Parts A and B

We can try different values

$$s=1\Rightarrow A=1,p=4$$
$$s=2\Rightarrow A=4,p=8$$
$$s=3\Rightarrow A=9,p=12$$
$$s=4\Rightarrow A=16,p=16$$

$$Part\ A: s=4\Rightarrow a=p$$
$$\boldsymbol{Part\ B: s=1\Rightarrow p=4a}$$



Part C

$$(4+b)+(4+b)+(4+b)+(4+b)$$
$$=\underbrace{4+4+4+4}_{Numbers}+\underbrace{b+b+b+b}_{Variables}=16+4b$$

Part D

$$4(2b-3)=4\times 2b-4\times 3=8b-12$$

Part E

$$\frac{36-24c}{4}=\frac{36}{4}-\frac{24c}{4}=9-6c$$

Part F

$$\frac{16x+96}{4}=\frac{16x}{4}+\frac{96}{4}=4x+24$$

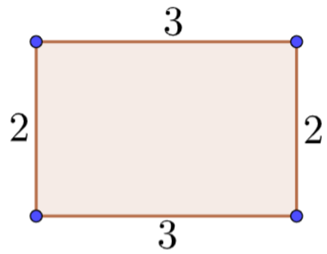
Part G

$$l+b+l+b=2l+2b=2(l+b)$$

Example 1.135: Back Calculations

A. A rectangle has length 3, and perimeter 10. Find the width.
B. A rectangle has width 2, and perimeter 12. Find the length.
C. A square has area 25 m^2. Find its perimeter.
D. Find the perimeter of a square with area 16 $inches^2$.
E. What is the perimeter of a square with an area of 36 $feet^2$
F. Jisshan walks along the sides of a square garden and measures its perimeter as 60 meters. Find its area.
G. The side of an equilateral triangle is 8 units. What is the area of a square with the same perimeter as the equilateral triangle?

$$Area = s \times s = 25 \Rightarrow s = 5 \Rightarrow p = 20$$
$$Area = Side \times Side \Rightarrow 16 = Side \times Side \Rightarrow 4 = Side$$
$$Area = Side \times Side \Rightarrow 36 = Side \times Side \Rightarrow 6 = Side$$



$$Side\ length\ of\ triangle = 8\ Units$$
$$Perimeter\ of\ Triangle = 8 \times 3 = 24\ Units$$
$$Perimeter\ of\ Square = 24\ Units$$
$$Side\ Length\ of\ Square = 6\ Units$$
$$Area\ of\ Square = 36\ Units^2$$

C. Area

Example 1.136

Find the squares below:

Numbers
A. 8
B. 4
C. 10
D. 5
E. 3
F. 1
G. 12
H. 7
I. 9
J. 6
K. 11
L. 20

Lengths
M. 5 inches
N. 6 feet
O. 9 km
P. 10 miles
Q. 11 cm
R. 20 mm

$$8^2 = 8 \times 8 = 64$$

Example 1.137

Find the square roots of the numbers below:

Numbers
A. 49
B. 36
C. 100
D. 4
E. 9
F. 64
G. 16
H. 25
I. 144
J. 400
K. 121

Lengths
L. $25\ feet^2$
M. $16\ inches^2$
N. $36\ cm^2$
O. $100\ mm^2$
P. $49\ km^2$
Q. $121\ yards^2$

D. Area Units

Area is the space occupied by a shape. We will learn how to calculate the area of some common geometrical figures.

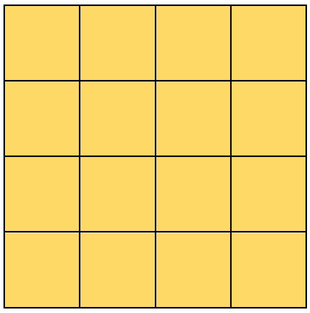
Comparing the Unit of Perimeter with Unit of Area

Perimeter is a length and hence, the unit of perimeter is same as the unit of side length.
Area is a product of two lengths, and hence the unit of area is the square of the unit of side length.

Side Length	Perimeter	Area
Meters	Meters	$Meters^2$
Inches	Inches	$Inches^2$

Feet	Feet	$Feet^2$
cm	cm	cm^2

E. Area of a Square



A square has all sides same. Each side of the square shown is 4 units long. If we draw lines, then we can see that the square is made up of $4 \times 4 = 16$ unit squares.
A unit square is a square which has sides each of length 1 unit.

$$\boldsymbol{Area\ of\ a\ Square = A_S\ = \underbrace{Side}_{s} \times \underbrace{Side}_{s} = s \times s = s^2}$$

$$\boldsymbol{Unit\ of\ Area = Units^2}$$

Example 1.138

A. A square drawn on a piece of paper has side of length 3 cm. Find the area of the square.
B. What is the writing area in a square writing pad whose one side 5 inches long?
C. A tablecloth in the shape of a square has a side which is 4 feet long. What is the area of the tablecloth?

$$\underbrace{3\ cm}_{\text{Side Length}} \times \underbrace{3\ cm}_{\text{Side Length}} = 9\ cm^2$$

$$\underbrace{5\ inches}_{\text{Side Length}} \times \underbrace{5\ inches}_{\text{Side Length}} = 25\ inches^2$$

$$\underbrace{4\ feet}_{\text{Side Length}} \times \underbrace{4\ feet}_{\text{Side Length}} = 16\ feet^2$$

F. Finding Side Length

Example 1.139

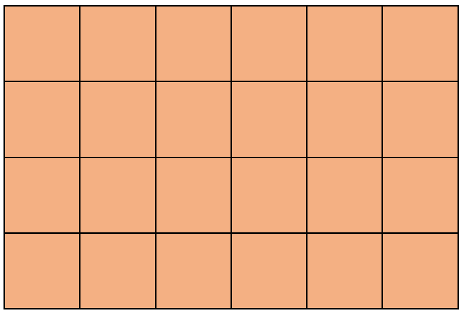
A. A square garden plot has an area of 25 $meters^2$. Find the length of the side of the garden.
B. A piece of cloth has a square shape. It has an area of 16 $inches^2$. What is the length of the side of the piece of cloth?
C. A square wall has an area of 36 $feet^2$. What is the height of the wall?

$$Area = Side \times Side \Rightarrow 25\ m^2 = Side \times Side \Rightarrow 5m = Side$$

$$Area = Side \times Side \Rightarrow 16 inches^2 = Side \times Side \Rightarrow 4\ inches = Side$$

$$Area = Side \times Side \Rightarrow 36\ feet^2 = Side \times Side \Rightarrow 6\ feet = Side$$

G. Area of a Rectangle



The rectangle below has sides of length 6 units, and width 4 units long. If we draw lines, then we can see that the square is made up of $6 \times 4 = 24$ unit squares.
A unit square is a square which has sides each of length 1 unit.

$$\textbf{Area of a Rectangle} = \boldsymbol{A_R\ = \underbrace{length}_{l} \times \underbrace{width}_{w} = lw}$$

$$\boldsymbol{Unit\ of\ Area = Units^2}$$

Example 1.140

A. A miniature painting has a length of 3 cm, and a width is 4 cm. What is the area occupied by the painting?
B. A piece of paper is 2 inches long, and has a width of 6 inches. What is the area of the paper?
C. A patch of grass is 3 feet long and 7 feet wide. What is the area of land occupied by the grass?

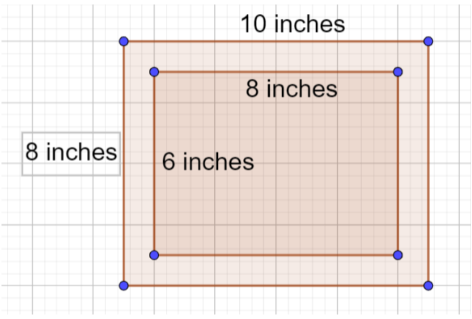
D. Sheela's room is 5 feet by 6 feet. What is the area of the room?

$$Area\ of\ Painting = \underbrace{3\ cm}_{Length} \times \underbrace{4\ cm}_{Width} = 12\ cm^2$$
$$Area\ of\ paper = \underbrace{2\ inches}_{Length} \times \underbrace{6\ inches}_{Width} = 12\ inches^2$$
$$Area\ of\ land = \underbrace{3\ feet}_{Length} \times \underbrace{7\ feet}_{Width} = 21\ feet^2$$

Example 1.141: Nested Figures

A. A canvas painting is 8 inches by 6 inches. It has a one-inch frame around all sides of it. Find the area of the frame.
B. A park has a 1 m walkway around all the sides, and the rest of the area is grass. If the grass measures 10m by 12m, find the area of the walkway.

Part A



Area of Painting
$$= 8 \times 6 = 48\ inches^2$$
Area of painting and frame
$$= 10 \times 8 = 80\ inches^2$$
Area of only frame
$$= 80 - 48 = 32$$

Part B

Area of Walkway
$$= \underbrace{(10+2)(12+2)}_{Area(Park)} - \underbrace{10 \times 12}_{Area(Grass)}$$
$$= 12 \times 14 - 120$$
$$= 168 - 120$$
$$= 48\ m$$

Example 1.142: Ratios

A. The length of a rectangle is double its width. If the width is 5 feet, find the perimeter and the area.
B. In a rectangle, which has length four times its width, the area is 36 units squared. Find the perimeter of the rectangle.
C. In a rectangle, which has length three times its width, the perimeter is 36. Find the area of the rectangle.

Part A

$$Width = 5\ feet$$
$$Length = 2 \times Width = 10\ Feet$$
$$P = 2(5+10) = 2(15) = 30$$
$$A = (5)(10) = 50$$

Part B

$$A = 36 = 2 \times 18 = \underbrace{3 \times 12}_{Works} = 4 \times 9 = 6 \times 6$$
$$P =$$

Part C

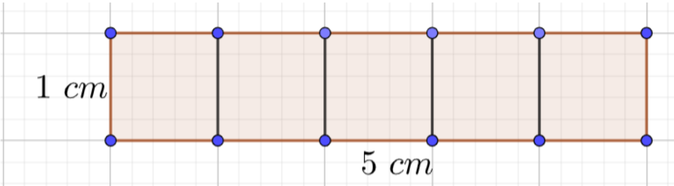
Example 1.143: Finding Possible Values

The perimeter of a rectangle is 14 feet. Find the possible values of the length and width, if they are natural numbers.

Length					
Width					

Example 1.144: Mixed Figures

A rectangle of length 5 cm, and breadth 1 cm is cut into 5 equal squares. Find the difference in perimeter of the rectangle, and the five squares.



Example 1.145: Composite Shapes

1.4 Parallelogram

A. Perimeter

Opposite sides of a parallelogram are equal.

Example 1.146

In a parallelogram, one side is 4 cm, and the other side is 7 cm.

A. Find the length of the remaining sides.
B. Find the perimeter of the parallelogram.

Example 1.147

Kirti is a making a drawing using oil on canvas. The canvas is in the shape of a parallelogram $ABCD$, with length AB 5 cm long, and length BC 3 cm. Kirti is going to make a red-coloured perimeter at the edge of the canvas to highlight her drawing. What is the length of the perimeter?

$$CD = AB = 5\ cm$$
$$DA = BC = 3\ cm$$

$$P = AB + BC + CD + DA = 5 + 3 + 5 + 3 = 16\ cm$$

B. Fence

If you fence an object, the length of fence required is the same as the perimeter of the object.
This is because the fence must run along the perimeter.

Example 1.148

Farmer Jane has an alfalfa field in the shape of a parallelogram with adjacent sides of 200 m and 300 m. It has a circular garden in the middle that has roses in it.

A. What is the perimeter of the alfalfa field?
B. Farmer Jane wants to fence the field. What is the length of the fence that Jane must purchase?
C. Find the cost of fence at $20 per meter of fence.

$$Perimeter = 200 + 300 + 200 + 300 = 1000\ m$$

The length of the fence required to fence the field is the same as the perimeter of the parallelogram.

$$\text{Length of Fence} = Perimeter = 1000$$

(Note: It is not as the same as the length of the field, because also need to fence the remaining sides).

$$Cost = Cost\ per\ meter \times Length\ of\ Fence = 20 \times 1000 = 20{,}000$$

Example 1.149

Firdous has a field in the shape of a parallelogram, next to a river. The adjacent sides of the field are 100 m and 200 m long. He wants to fence the field, but does not need to fence the side which is next to the river. Fence costs \$5 per meter. Find the cost of fence.

A. if the side which is 100 m is next to the river?
B. If the side which is 200 m is next to the river?

Let's first find the perimeter of the parallelogram

$$2(100 + 200) = 2 \times 300 = 600\ m$$

Part A

The side which is 100 m does not need to be fenced.
Hence, the length of fence is

$$600 - 100 = 500\ m$$

Cost of fence

$$= 500 \times 5 = \$2500$$

Part B

The side which is 200 m does not need to be fenced.
Hence, the length of fence is

$$600 - 200 = 400\ m$$

Cost of fence

$$= 400 \times 5 = \$2000$$

C. Different Costs

Example 1.150

A building compound in the shape of a parallelogram must be walled (see diagram for a top-down view of the compound). The longer side of the compound is fifty feet long, and the shorter side is twenty feet long. The compound is to be fenced. A straight fence will cost \$25 per foot, while a slanted fence will cost \$35 per foot. Find the cost of the fence.

Cost of the straight fence

$$= 50 \times 2 \times 25 = \$2500$$

Cost of the slanted fence

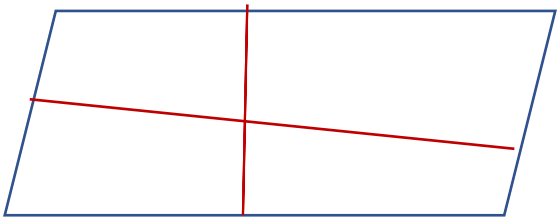
$$= 20 \times 2 \times 35 = \$1400$$

Total Cost

$$= 2500 + 1400 = \$3900$$

D. Area

The diagram shows a parallelogram. The opposite sides of the parallelogram are equal.



Each of the red lines is the height for the corresponding base.

$$\boldsymbol{Area\ of\ Parallelogram = Height \times Base}$$

Example 1.151

The height of a parallelogram is 5 units and the corresponding base is 3 units. Find the area.

We directly apply the formula

$$Area = Height \times Base = 5 \times 3 = 15\ units$$

Example 1.152

Find the area of a parallelogram with height 7 units and corresponding base 3 units. The side adjacent to the base is 12 units.

We only need the height and the base. The information on the side adjacent to the base is given just to confuse.

$$Area = Height \times Base = 7 \times 3 = 21\ units$$

Example 1.153

Parallelogram A has length of base 4 units and height 2 units. Parallelogram B has length of base 6 units and height 3 units.

A. Find the difference in the areas of the two parallelograms
B. The area of Parallelogram B is how many times the area of Parallelogram A.

$$Area(A) = 4 \times 2 = 8$$
$$Area(B) = 6 \times 3 = 18$$
$$Area(B) - Area(A) = 18 - 8 = 10$$
$$\frac{Area(B)}{Area(A)} = \frac{18}{8} = \frac{9}{4} = 2\frac{1}{4}$$

Example 1.154

A parallelogram with base 8 units and height 2 units has the same area as a square. Find the perimeter of the square.

$$Area\ of\ Parallelogram = h \times b = 8 \times 2 = 16$$
$$Area\ of\ Square = 16 \Rightarrow Side\ length\ of\ Square = 4 \Rightarrow Perimeter\ of\ Square = 16$$

Example 1.155

A parallelogram with base 12 units and height 3 units has the same area as a rectangle with one side 4 units. Find the perimeter of the rectangle.

$$Area\ of\ Parallelogram = 12 \times 3 = 36\ units^2$$
$$l \times b = 36 \Rightarrow 4b = 36 \Rightarrow b = 9 \Rightarrow P = 2(4 + 9) = 2 \times 13 = 26$$

Example 1.156

The length of base of a parallelogram is 12 meters. The area of the parallelogram is 120 meters. Find the height.

$$hb = 120 \Rightarrow 12h = 120 \Rightarrow h = 10$$

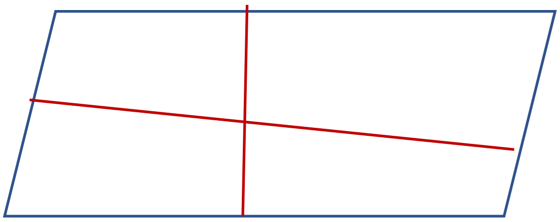
Example 1.157

The height of a parallelogram is 6 meters. The area of the parallelogram is 72 meters. Find the length of the base.

$$hb = 72 \Rightarrow 6b = 72 \Rightarrow b = 12$$

Example 1.158

The length of one side of a parallelogram is 8 meters, and the corresponding height is 6 meters. If the length of the other side is 12 meters, find the other height.



$$Area = 8 \times 6 = 48$$
$$12h = 48 \Rightarrow h = 4$$

Revenue is the money obtained from selling something.
Cost is the money spent on something

$$Profit = Revenue - Cost$$

Example 1.159

A rose garden in the shape of a parallelogram with a base of 150 meters and a height of 200 meters produces 25 roses per square meter in one year.

A. What is the area of the garden, in square meters?
B. Find the number of roses produced by the garden in one year
C. What is the revenue from selling the roses at half a pound per rose?
D. Calculate the cost of growing the roses at one-fourth of a pound per rose
E. Arrive at the profit earned, using your answers to Parts C and D.

$$Area\ of\ the\ garden = 150 \times 200 = 30{,}000\ m^2$$
$$Number\ of\ Roses = 30{,}000 \times 25 = 750{,}000$$
$$Revenue\ 750{,}000 \times \frac{1}{2} = 3{,}75{,}000$$
$$Cost = 750{,}000 \times \frac{1}{4} = 1{,}87{,}500$$
$$Profit = Revenue - Cost = 375{,}000 - 1{,}87{,}500 = 1{,}87{,}500$$

1.5 Trapezium

Example 1.160

Trapezium with one base with length 5 cm, another base with length 6 cm, and height 7 cm. Find the area.

$$Area = \frac{5+6}{2} \times 7 = \frac{11}{2} \times 7 = \frac{77}{2} cm^2$$

1.6 Intervals

A. Basics

Example 1.161

A fence is 8 meters long. Fenceposts are put at intervals of 1 meter, including the beginning and the end.

A. What is the number of fenceposts needed?
B. What is the number of intervals between fenceposts?
C. What is the relation between the number of fenceposts and the number of intervals?

Part A

Distance	0	1	2	3	4	5	6	7	8
Fencepost No.	1	2	3	4	5	6	7	8	9

Part B

The number of intervals is 8.

Part C

The number of fenceposts is 1 more than the number of intervals.

Example 1.162

A fence is 16 meters long. Fenceposts are put at intervals of 2 meters, including the beginning and the end. What is the number of fenceposts needed?

Method I: Listing It Out

0	2	4	6	8	10	12	14	16
1	2	3	4	5	6	7	8	9

Method II: Counting Intervals

The number of intervals between fenceposts

$$= \frac{Total\ Distance}{Length\ of\ each\ Interval} = \frac{16}{2} = 8$$

Number of Fenceposts

$$\underbrace{8}_{Intervals} + \underbrace{1}_{Add\ 1\ at\ the\ beginning} = 9$$

Example 1.163

A fence is 32 meters long. Fenceposts are put at intervals of 2 meters, including the beginning and the end. What is the number of fenceposts needed?

The number of intervals between fenceposts

$$= \frac{Total\ Distance}{Length\ of\ each\ Interval} = \frac{32}{2} = 16$$

Number of Fenceposts

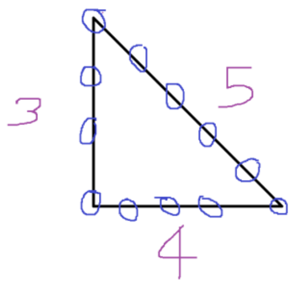
$$\underbrace{16}_{Intervals} + \underbrace{1}_{Add\ 1\ at\ the\ beginning} = 17$$

Example 1.164

A triangular garden with sides of 3 meters, 4 meters and 5 meters is to be fenced. Fenceposts are needed at intervals of 1 meters each. Find the number of fenceposts.

Method I: Draw a diagram

As we can see in the diagram alongside, the number of fenceposts needed is 12.



Method II: Logic

If the fence had been in a single line, you would need

$$12 + 1 = 13\ Fenceposts$$

Here, however, the end of the fence is connected to the beginning of the fence, and, hence, you will need one fencepost less.

$$12 + 1 - 1 = 12$$

Finally, the number of fenceposts needed

$$= \frac{Perimeter\ of\ Shape}{Distance\ between\ two\ posts} = \frac{12}{1} = 12$$

Example 1.165

Find the fenceposts needed for the following gardens, if fenceposts are to be planted at intervals of 1 meter each.

A. A square garden with side length 4 meters
B. A rectangular garden with adjacent sides of length 3 meters and 4 meters.
C. A pentagonal garden with each side of length two meters.

Square Garden

$$\frac{Perimeter\ of\ Shape}{Distance\ between\ two\ posts} = \frac{4 \times 4}{1} = \frac{16}{1} = 16$$

Rectangular Garden

$$\frac{Perimeter\ of\ Shape}{Distance\ between\ two\ posts} = \frac{2(3+4)}{1} = \frac{14}{1} = 14$$

Pentagonal Garden

$$\frac{Perimeter\ of\ Shape}{Distance\ between\ two\ posts} = \frac{2(5)}{1} = \frac{10}{1} = 10$$

Example 1.166

Farmer Jake wants to fence a rectangular garden with length 7 meters and width five meters with fenceposts at intervals of 1 meter each. What is the number of fenceposts required:

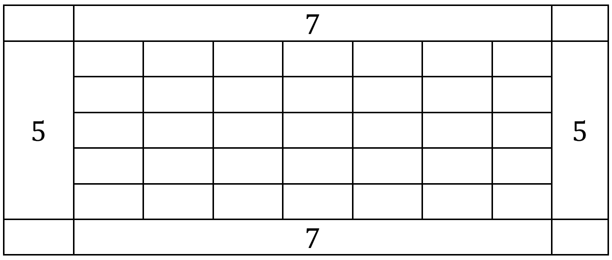
A. for the garden?
B. for the garden if a length does not need to be fenced since it is next to a mountain? (The endpoints that that connect with the mountain still need a fencepost).
C. for the garden if a width does not need to be fenced since it is next to a mountain?

Part A
First, find the perimeter

$$= 5 + 7 + 5 + 7 = 24$$

No. of Fenceposts

$$= \frac{Perimeter}{Distance\ between\ Posts} = \frac{24}{1} = 24$$

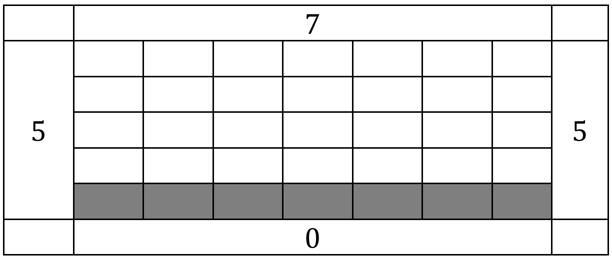


Part B
The length that needs to be fenced:

$$5 + 7 + 5 = 17$$

The number of fenceposts required is:

$$17 + 1 = 18$$

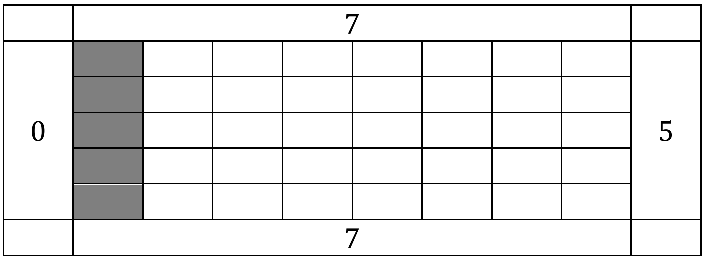


Part C
The length that needs to be fenced

$$7 + 5 + 7 = 19$$

The number of fenceposts required is

$$19 + 1 = 20$$



B. Barbed Wire

Example 1.167

A fence is 5 meters long. Fenceposts are put at intervals of 1 meter, including the beginning and the end. Barbed wire is run through the posts.

A. What is the number of fenceposts needed?
B. What is the length of barbed wire?

Part A: Number of Fenceposts

	FP 1	FP 2	FP 3	FP 4	FP 5	FP 6
Distance Between Posts		1	1	1	1	1
Total	0	1	2	3	4	5
	Beginning					End

We put the first fencepost at beginning of the fence. This at distance zero from the beginning.

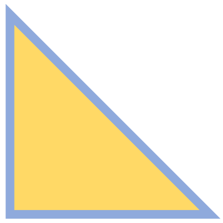
Part B: Length of barbed wire
The length of barbed wire will be the same as the length of the fence

$$Length\ of\ Barbed\ Wire = Length\ of\ Fence = 5\ m$$

C. Costs

Example 1.168

A garden is in the shape of a right triangle, with lengths of sides 5 meters, 12 meters and 13 meters. The garden is to be fenced with barbed wire running through fenceposts at intervals of 1 meter each. The barbed wire will run at three heights through the posts – low, medium and high.



A. Find the perimeter of the garden.
B. Find the length of fence
C. Find the number of posts
D. Find the length of wire
E. Find the cost of wire at $5 per meter
F. Find the cost of posts at $20 per post
G. A rare herb used for medicinal purposes grows in the garden.

$$Perimeter = 5 + 12 + 13 = 30\ meters$$

$$Length\ of\ fence = Perimeter = 30\ meters$$

D. Finding Number of Intervals

Example 169

A road has two trees – a banyan and an oak tree. How many gaps are there between the two trees?

Let a tree be $\uparrow$

Let the gap between two trees be $\leftrightarrow$

Draw two trees and the gap between them:

$$\underbrace{\uparrow}_{Tree\ 1}\ \underbrace{\leftrightarrow}_{\boldsymbol{Gap\ 1}}\ \underbrace{\uparrow}_{Tree\ 2}$$

Between two trees, there is one gap.

Example 170

Three *Ashoka* trees are on a road in a straight line. The distance between two trees is 5 m.

A. How many gaps are there between the trees?
B. What is the distance between the first tree and the third tree?

Part A

Draw three trees and the gap between them:

$$\underbrace{\uparrow}_{Tree\ 1}\ \underbrace{\leftrightarrow}_{\boldsymbol{Gap\ 1}}\ \underbrace{\uparrow}_{Tree\ 2}\ \underbrace{\leftrightarrow}_{\boldsymbol{Gap\ 2}}\ \underbrace{\uparrow}_{Tree\ 3}$$

Between three trees, there are two gaps.

Part B

$$\underbrace{\uparrow}_{Tree\ 1}\ \underbrace{\mathbf{5\ meters}}_{\mathbf{Gap\ 1}}\ \underbrace{\uparrow}_{Tree\ 2}\ \underbrace{\mathbf{5\ meters}}_{\mathbf{Gap\ 2}}\ \underbrace{\uparrow}_{Tree\ 3}$$

$$Distance = 5\ meters + 5\ meters = 10\ meters$$

Example 171

Four trees are planted on a road in a straight line. The distance between two trees is 15 feet.

A. How many gaps are there between the trees?
B. What is the distance between the first tree and the fourth tree?
C. What is the distance between the second tree and the third tree?
D. What is the distance between the second tree and the fourth tree?
E. If a fifth tree is added such that the distance between the fourth tree and the fifth tree is 15 feet, and all the trees are in a straight line, then find the distance between the first tree and the fifth tree? (The fifth tree is not planted at the location of the third tree).

Part A

Draw four trees and the gaps between them:

$$\underbrace{\uparrow}_{Tree\ 1}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 1}}\ \underbrace{\uparrow}_{Tree\ 2}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 2}}\ \underbrace{\uparrow}_{Tree\ 3}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 3}}\ \underbrace{\uparrow}_{Tree\ 4}$$

Between four trees, there are two gaps.

Parts B, C and D

$$\underbrace{\uparrow}_{Tree\ 1}\ \underbrace{\mathbf{15\ feet}}_{\mathbf{Gap\ 1}}\ \underbrace{\uparrow}_{Tree\ 2}\ \underbrace{\mathbf{15\ feet}}_{\mathbf{Gap\ 2}}\ \underbrace{\uparrow}_{Tree\ 3}\ \underbrace{\mathbf{15\ feet}}_{\mathbf{Gap\ 3}}\ \underbrace{\uparrow}_{Tree\ 4}$$

Distance between the first tree and the fourth tree

$$= 15 + 15 + 15 = 45\ feet$$

Distance between the second tree and the third tree

$$= 15\ feet$$

Distance between the second tree and the fourth tree

$$= 15 + 15 = 30\ feet$$

1.1: Number of Intervals

From the above examples, we can tabulate what we got

Trees	2	3	4
Gaps	1	2	3

We can generalise the above result.

$$\boldsymbol{No.of\ Gaps = No.of\ Objects - 1}$$

Example 172

Farmer Jones has a circular field which he has fenced using 7 fenceposts. What is the number of intervals between these fenceposts

We could draw a diagram:

$$\underbrace{\uparrow}_{FP\ 1}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 1}}\ \underbrace{\uparrow}_{FP\ 2}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 2}}\ \underbrace{\uparrow}_{FP\ 3}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 3}}\ \underbrace{\uparrow}_{FP\ 4}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 4}}\ \underbrace{\uparrow}_{FP\ 5}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 5}}\ \underbrace{\uparrow}_{FP\ 6}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 6}}\ \underbrace{\uparrow}_{FP\ 7}$$

Or, we can simply apply the formula:

$$No.of\ Gaps = No.of\ Fenceposts - 1 = 7 - 1 = 6$$

Example 173

A road has 5 telephone poles in a straight line. These telephone poles need to have wire going through them. How many gaps between the telephone poles must be connected with wires?

$$\underbrace{\uparrow}_{TP\ 1}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 1}}\ \underbrace{\uparrow}_{TP\ 2}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 2}}\ \underbrace{\uparrow}_{TP\ 3}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 3}}\ \underbrace{\uparrow}_{TP\ 4}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 4}}\ \underbrace{\uparrow}_{TP\ 5}$$

Example 174

A road going from East to West has a lamppost at its westernmost side. Starting from the westernmost side, there are 6 lampposts in a row on the road.

A. What is the number of lights needed for the lampposts, if each lamppost needs one light?
B. Every lamppost has two decorations on it. What is the number of decorations on all of the lampposts?
C. What is the number of gaps from the first to the last lamppost?
D. If there is a distance of 4 meters between two lampposts, what is the length of wire needed from the first lamppost to the last?

$$\underbrace{\uparrow}_{LP\ 1}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 1}}\ \underbrace{\uparrow}_{LP\ 2}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 2}}\ \underbrace{\uparrow}_{LP\ 3}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 3}}\ \underbrace{\uparrow}_{LP\ 4}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 4}}\ \underbrace{\uparrow}_{LP\ 5}\ \underbrace{\leftrightarrow}_{\mathbf{Gap\ 5}}\ \underbrace{\uparrow}_{LP\ 6}$$

$$\underbrace{\uparrow}_{LP\ 1}\ \underbrace{\mathbf{4}\ m}_{\mathbf{Gap\ 1}}\ \underbrace{\uparrow}_{LP\ 2}\ \underbrace{\mathbf{4\ m}}_{\mathbf{Gap\ 2}}\ \underbrace{\uparrow}_{LP\ 3}\ \underbrace{\mathbf{4}\ m}_{\mathbf{Gap\ 3}}\ \underbrace{\uparrow}_{LP\ 4}\ \underbrace{\mathbf{4m}}_{\mathbf{Gap\ 4}}\ \underbrace{\uparrow}_{LP\ 5}\ \underbrace{\mathbf{4m}}_{\mathbf{Gap\ 5}}\ \underbrace{\uparrow}_{LP\ 6}$$

Example 175

A rope is knotted every 20 cm. If there are six knots on the rope, what is the distance between the second knot and the fifth knot.

$$\underbrace{\uparrow}_{Knot\ \ 2}\ \underbrace{\mathbf{20\ cm}}_{\mathbf{Gap\ 1}}\ \underbrace{\uparrow}_{Knot\ 3}\ \underbrace{\mathbf{20}\ cm}_{\mathbf{Gap\ 3}}\ \underbrace{\uparrow}_{Knot\ 5}\ \underbrace{\mathbf{20\ cm}}_{\mathbf{Gap\ 4}}\ \underbrace{\uparrow}_{Knot\ 5}$$

Total Distance

$$20\ cm \times 3 = 60\ cm$$

Example 176

5 Coins on a Table

E. Coins in a Line

Example 177

A table has three coins in a straight line. If the distance between the first coin and the third coin is 6 cm, find the distance between the first coin and the second coin?

$$\underbrace{\uparrow}_{Coin\ 1}\ \underbrace{\leftrightarrow}_{\boldsymbol{Gap\ 1}}\ \underbrace{\uparrow}_{Coin\ 2}\ \underbrace{\leftrightarrow}_{\boldsymbol{Gap\ 2}}\ \underbrace{\uparrow}_{Coin\ 3}$$

$$No.\,of\ Gaps = No.\,of\ Coins - 1 = 3 - 1 = 2$$
$$Total\ Distance = 6\ cm$$
$$Distance\ in\ each\ gap = \frac{6}{2} = 3\ cm$$

F. Lampposts

Example 178

A road has lampposts along one side of the road, in a straight line. The distance between two lampposts is 4 meters. What is the distance between:

A. the first lamppost and the third lamppost?
B. the first lamppost and the sixth lamppost?
C. The second lamppost and the fifth lamppost?

Part A

$$\underbrace{\uparrow}_{LP\ 1}\ \underbrace{\boldsymbol{4\ m}}_{\boldsymbol{Gap\ 1}}\ \underbrace{\uparrow}_{LP\ 2}\ \underbrace{\boldsymbol{4\ m}}_{\boldsymbol{Gap\ 1}}\ \underbrace{\uparrow}_{LP\ 3}$$

Part B

$$\underbrace{\uparrow}_{LP\ 1}\ \underbrace{\boldsymbol{4\ m}}_{\boldsymbol{Gap\ 1}}\ \underbrace{\uparrow}_{LP\ 2}\ \underbrace{\boldsymbol{4\ m}}_{\boldsymbol{Gap\ 1}}\ \underbrace{\uparrow}_{LP\ 3}\ \underbrace{\boldsymbol{4\ m}}_{\boldsymbol{Gap\ 1}}\ \underbrace{\uparrow}_{LP\ 4}\ \underbrace{\boldsymbol{4\ m}}_{\boldsymbol{Gap\ 1}}\ \underbrace{\uparrow}_{LP\ 5}\ \underbrace{\boldsymbol{4\ m}}_{\boldsymbol{Gap\ 1}}\ \underbrace{\uparrow}_{LP\ 6}$$

G. Knots on a Rope

Example 179

A rope used for preparing for mountain climbing has knots at intervals of 1 foot (including the beginning and the end).

A. If the rope has five knots, what is the length of the rope?

B. If the length of the rope is five feet, what is the number of knots on the rope?

Part A

$$\underbrace{\uparrow}_{Knot\ 1}\ \underbrace{\mathbf{1}\ \boldsymbol{Foot}}_{\boldsymbol{Gap}\ \mathbf{1}}\ \underbrace{\uparrow}_{Knot\ 2}\ \underbrace{\mathbf{1}\ \boldsymbol{Foot}}_{\boldsymbol{Gap}\ \mathbf{1}}\ \underbrace{\uparrow}_{Knot\ 3}\ \underbrace{\mathbf{1}\ \boldsymbol{Foot}}_{\boldsymbol{Gap}\ \mathbf{1}}\ \underbrace{\uparrow}_{Knot\ 4}\ \underbrace{\mathbf{1}\ \boldsymbol{Foot}}_{\boldsymbol{Gap}\ \mathbf{1}}\ \underbrace{\uparrow}_{Knot\ 5}$$

Part B

$$\underbrace{\uparrow}_{Knot\ 1}\ \underbrace{\mathbf{1}\ \boldsymbol{Foot}}_{\boldsymbol{Gap}\ \mathbf{1}}\ \underbrace{\uparrow}_{Knot\ 2}\ \underbrace{\mathbf{1}\ \boldsymbol{Foot}}_{\boldsymbol{Gap}\ \mathbf{1}}\ \underbrace{\uparrow}_{Knot\ 3}\ \underbrace{\mathbf{1}\ \boldsymbol{Foot}}_{\boldsymbol{Gap}\ \mathbf{1}}\ \underbrace{\uparrow}_{Knot\ 4}\ \underbrace{\mathbf{1}\ \boldsymbol{Foot}}_{\boldsymbol{Gap}\ \mathbf{1}}\ \underbrace{\uparrow}_{Knot\ 5}\ \underbrace{\mathbf{1}\ \boldsymbol{Foot}}_{\boldsymbol{Gap}\ \mathbf{1}}\ \underbrace{\uparrow}_{Knot\ 6}$$

H. Calendars

Example 180

Every time, Alex plays football, he gets sore muscles, and he has to take a break for two days of rest (not including the day he played football). If Alex plays football on the 5th of January, when he will play football

A. For the first time after 5th of Jan

B. For the third time after 5th of Jan.

5th	6th	7th	8th	9th	10th	11th	12th	13th	14th
Football	Rest	Rest	Football	Rest	Rest	Football	Rest	Rest	Football
			1st Time			2nd Time			3rd Time

I. Applications

Example 181

In a fence where the fenceposts are equally spaced, the distance between the first and the fourth fencepost is 12 meters. What is the distance between the first and the fifth fencepost?

$$\underbrace{\uparrow}_{FP\ 1}\ \underbrace{\leftrightarrow}_{\boldsymbol{Gap}\ \mathbf{1}}\ \underbrace{\uparrow}_{FP\ 2}\ \underbrace{\leftrightarrow}_{\boldsymbol{Gap}\ \mathbf{2}}\ \underbrace{\uparrow}_{FP\ 3}\ \underbrace{\leftrightarrow}_{\boldsymbol{Gap}\ \mathbf{3}}\ \underbrace{\uparrow}_{FP\ 4}$$

The total distance between the fenceposts is 12 meters.
There are three gaps between the four fenceposts.

So, each gap will be

$$\frac{12}{3} = 4\ meters$$

$$\underbrace{\uparrow}_{FP\ 1}\ \underbrace{\mathbf{4}}_{\boldsymbol{Gap}\ \mathbf{1}}\ \underbrace{\uparrow}_{FP\ 2}\ \underbrace{\mathbf{4}}_{\boldsymbol{Gap}\ \mathbf{2}}\ \underbrace{\uparrow}_{FP\ 3}\ \underbrace{\mathbf{4}}_{\boldsymbol{Gap}\ \mathbf{3}}\ \underbrace{\uparrow}_{FP\ 4}$$

If you extend the same distance between fenceposts to five fenceposts, you get four gaps.

$$\underbrace{\uparrow}_{FP\ 1}\ \underbrace{\mathbf{4}}_{\boldsymbol{Gap\ 1}}\ \underbrace{\uparrow}_{FP\ 2}\ \underbrace{\mathbf{4}}_{\boldsymbol{Gap\ 2}}\ \underbrace{\uparrow}_{FP\ 3}\ \underbrace{\mathbf{4}}_{\boldsymbol{Gap\ 3}}\ \underbrace{\uparrow}_{FP\ 4}\ \underbrace{\mathbf{4}}_{\boldsymbol{Gap\ 4}}\ \underbrace{\uparrow}_{FP\ 5}$$

Hence, the total distance is

$$4 \times 4 = 16$$

Bogus Solution

Four fenceposts = 12 meters

$$One\ fencepost = \frac{12}{4} = 3\ meters$$

Five fenceposts = 3 * 5 = 15 meters

J. Birthday Celebrations

Example 182

The eldest child at a group birthday celebration is celebrating his tenth birthday today, while the youngest child is celebrating his second. What is the maximum number of birthdays that are being celebrated?

Youngest								Eldest
$2nd$ Birthday	$3rd$ Birthday	$4th$ Birthday	$5th$ Birthday	$6th$ Birthday	$7th$ Birthday	$8th$ Birthday	$9th$ Birthday	$10th$ Birthday
1	2	3	4	5	6	7	8	9

K. Multi-Tier Intervals

Example 183

The distance between any two lampposts on a street is 6 meters. Between every two lampposts, there are two *begonia* plants. The *begonia* plants are spaced equally between the lampposts. I start walking from a lamppost, and walk for 72 meters. I need to switch on the lampposts, and water the plants.

A. What is the distance between two *begonia* plants?
B. How many lampposts will I switch on?
C. How many *begonia* plants will I water?

Part A

$$\underbrace{\underbrace{\uparrow}_{LP}\ \underbrace{\leftrightarrow}_{\mathbf{2m}}\ \underbrace{\uparrow}_{Plant}\ \underbrace{\leftrightarrow}_{\mathbf{2m}}\ \underbrace{\uparrow}_{Plant}\ \underbrace{\leftrightarrow}_{\mathbf{2m}}\ \underbrace{\uparrow}_{LP}}_{6\ Meters}$$

Part B

The gap between two lampposts is 6 meters.
In walking 72 meters, I will cover

$$\frac{72}{6} = 12\ gaps$$

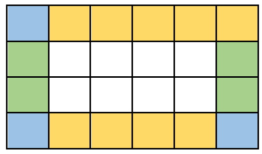
12 gaps will have

$$12 + 1 = 13\ lampposts$$

L. Fenceposts

Example 184

A farmer wants to fence a rectangular plot of land which is 40 meters wide by 60 meters long. He is going to put fenceposts at intervals of 10 meters each. Find the number of fenceposts required to fence:



A. Only the width
B. Only the length
C. The entire rectangular plot

Width

Let's number the fenceposts as $FP-1, FP-2 \ldots$

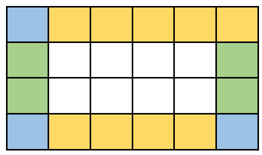
$$\underbrace{\uparrow}_{FP-1}\ \underbrace{\leftrightarrow}_{10\ Meters}\ \underbrace{\uparrow}_{FP-2}\ \underbrace{\leftrightarrow}_{10\ Meters}\ \underbrace{\uparrow}_{FP-3}\ \underbrace{\leftrightarrow}_{10\ Meters}\ \underbrace{\uparrow}_{FP-4}\ \underbrace{\leftrightarrow}_{10\ Meters}\ \underbrace{\uparrow}_{FP-5}$$

Length

$$\underbrace{\uparrow}_{FP-1}\ \underbrace{\leftrightarrow}_{10\ Meters}\ \underbrace{\uparrow}_{FP-2}\ \underbrace{\leftrightarrow}_{10\ Meters}\ \underbrace{\uparrow}_{FP-3}\ \underbrace{\leftrightarrow}_{10\ Meters}\ \underbrace{\uparrow}_{FP-4}\ \underbrace{\leftrightarrow}_{10\ Meters}\ \underbrace{\uparrow}_{FP-5}\ \underbrace{\leftrightarrow}_{10\ Meters}\ \underbrace{\uparrow}_{FP-6}\ \underbrace{\leftrightarrow}_{10\ Meters}\ \underbrace{\uparrow}_{FP-7}$$

Entire Plot

$$\underbrace{14}_{Posts\ in\ Length} + \underbrace{10}_{Posts\ in\ Width} - \underbrace{4}_{Corners}$$



M. Coins

Example 185

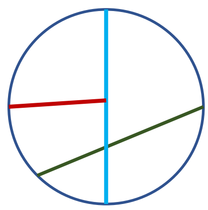
Shalu makes an equilateral triangle using $dimes(10\ cent\ US\ currency)$. One side of the triangle has 10 coins. She wants to buy candies costing 50 cents each. How many can she buy?

$$\underbrace{3}_{No.of\ Sides} \times \underbrace{10}_{Coins\ per\ Side} - \underbrace{3}_{No.of\ Corners} = 30 - 3 = 27\ coins = 270\ cents = 5\ candies$$

1.7 Circles

A. Circle: Definition

A circle is a closed figure such that the boundary of the circle is a fixed distance from the centre of the circle.



Chord is a line segment joining any two points on the circle.

Radius is the chord from the centre of the circle to the boundary. Length of radius is the distance from the centre to the boundary.

Diameter is the longest chord of the circle. Diameters pass through the centre of the circle.

Degree Measure: A full circle occupies 360°. A half circle occupies 180°. A quarter circle occupies 90°

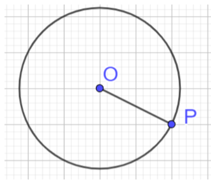
Example 1.186

Find:

A. The diameter of a circle with radius 3 inches

The radius of a circle with diameter 6 feet.

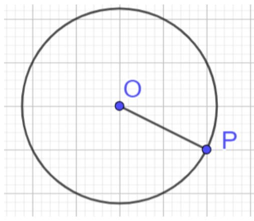
1.187: Circle
A circle is the set of points that are at the same distance from a center. In the adjoining figure.



1.188: Radius
The distance from the center of the circle to the boundary is called the radius.
This distance is the same from center to any point on the circle.

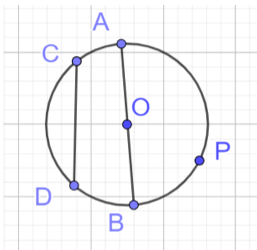
Example 1.189
In the adjoining figure:

A. What is the center?
B. What is a point on the boundary of the circle?
C. What is the radius?



$$Centre = O$$
$$Point\ on\ Boundary = P$$
$$Radius = OP$$

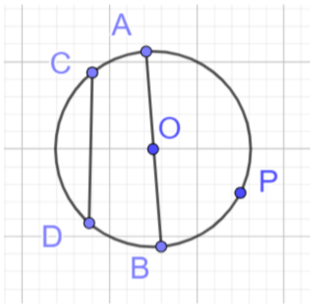
1.190: Chord
A line joining any two points on the circle is called a chord.



Example 1.191
In the diagram, identify the chords.

$$CD\ is\ a\ chord$$
$$AB\ is\ also\ a\ chord$$

1.192: Diameter
- Chord passing through the center is the diameter.
- Diameter is the longest chord.



Example 1.193
In the adjoining figure

A. Identify the diameter.
B. What can we say about the length of chord CD?

$$Diameter = AB$$
$$CD < AB$$

1.194: Diameter
Diameter divides the circle into two equal semi-circles.

Example 1.195
A circle with an area of 7 $units^2$ is divided by its diameter into parts. Find the area of each part.

$$Area\ of\ each\ part = \frac{Area\ of\ Circle}{2} = \frac{7}{2} = 3.5\ units^2$$

1.196: Calculating Diameter

Diameter is double of the radius.

$$D = 2r$$
$$D = Diameter$$
$$r = radius$$

Example 1.197

Find the diameter of a circle with given radius:

A. Radius is 5 cm.
B. Radius is 1 m. (Find in cm).
C. Radius is $\frac{1}{4}$ foot. ($Find\ in\ inches$)
D. $\frac{1}{4}m\ (in\ cm)$

$$Part\ A: D = 2r = 2 \times 5\ cm = 10\ cm$$
$$Part\ B: D = 2r = 2 \times 1m = 2m = 200\ cm$$
$$\frac{1}{4} \times 2 = \frac{1}{2} foot = \frac{1}{2} \times 12 = 6\ inches$$

1.198: Calculating Radius

Radius is half of the diameter.

$$r = \frac{D}{2}$$

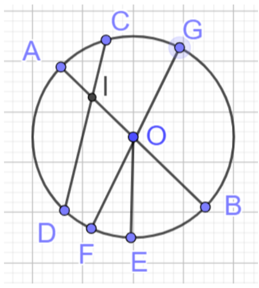
Example 1.199

Find the radius of a circle with diameter

A. 1 m (in cm)
B. 1 $foot\ (in\ inches)$
C. $\frac{1}{2}\ foot\ (in\ inches)$

$$r = \frac{D}{2} = \frac{1\ m}{2} = \frac{100\ cm}{2} = 50\ cm$$
$$r = \frac{D}{2} = \frac{1\ foot}{2} = \frac{12\ inches}{2} = 6\ inches$$

Example 1.200: Review



In the adjoining figure:

A. Classify each of CD, AB, CI, OE, GF, IO as a chord, a radius, or a diameter, or none of the previous.
B. Count the numbers of chords, diameters, and radii (plural of radius) shown in the diagram.
C. If EO is extended, till it touches the boundary of the circle at X, what will EX be?
D. If O and D are connected, what will OD be?
E. If A and D are connected, what will AD be?

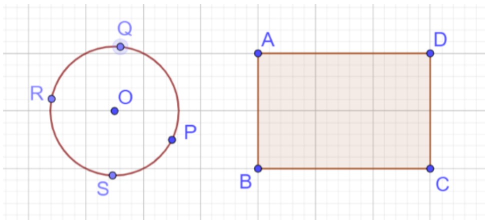
	Chords	Diameter	Radius	None of the Previous
Part A	CD AB GF	AB GF	OE	CI IO
Part B			OB OA OG OF	
	3	2	5	
Parts C,D, E	AD	EX	OD	

B. Circumference

1.201: Circumference (Perimeter of the Circle)

The length of the boundary of the circle is the circumference.

$$C = 2\pi r, \qquad \pi = \frac{22}{7}, r = radius$$



For a rectangle (or other quadrilaterals), the perimeter of rectangle $ABCD$ is the distance to travel along the boundary of the rectangle:

$$From\ A\ to\ B, B\ to\ C, C\ to\ D\ and\ D\ to\ A$$

Similarly, for a circle, the perimeter is the distance required to travel around the boundary of the circle:

$$From\ P\ to\ Q, Q\ to\ R, R\ to\ S\ and\ S\ to\ P$$

And, note that, the perimeter of a circle has a special name: circumference.

Example 1.202

Find the circumference of a circle with radius

A. $7\ feet$
B. $\frac{1}{11} feet$
C. $\frac{14}{22}\ feet$
D. $2\ meters\ (in\ cm)$

Substitute $\pi = \frac{22}{7}, r = 7$:

$$C = 2\pi r = 2 \times \frac{22}{7} \times 7 = 2 \times \frac{22}{1} \times 1 = 44\ feet$$

$$C = 2\pi r = 2 \times \frac{22}{7} \times \frac{1}{11} = 2 \times \frac{2}{7} \times \frac{1}{1} = \frac{4}{7} feet$$

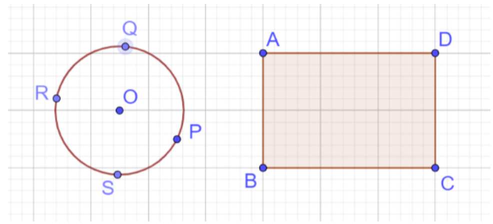
$$C = 2\pi r = 2 \times \frac{22}{7} \times \frac{14}{22} = 2 \times \frac{1}{1} \times \frac{2}{1} = 4\ feet$$

$$C = 2\pi r = 2 \times \frac{22}{7} \times 2 = \frac{88}{7} m = \frac{8800}{7} cm$$

1.203: Circumference (Perimeter of the Circle)

The length of the boundary of the circle is the circumference.

$$C = \pi D, \qquad \pi = \frac{22}{7}, D = Diameter$$



Example 1.204

Find the circumference of a circle with radius:

A. $4\ cm$
B. $7\ inches$
C. $14\ feet$
D. $\frac{21}{11}\ feet$

$$C = \pi D = \frac{22}{7} \times 4 = \frac{22}{7} \times \frac{4}{1} = \frac{88}{7} cm$$

$$C = \pi D = \frac{22}{7} \times \frac{7}{1} = \frac{22}{1} = 22\ inches$$

$$C = \pi D = \frac{22}{7} \times 14 = \frac{22}{1} \times 2 = 44\ inches$$

$$C = \pi D = \frac{22}{7} \times \frac{21}{11} = \frac{2}{1} \times \frac{3}{1} = 6\ inches$$

C. Area

1.205: Area

The area of a circle is given by:

$$A = \pi r^2 = \pi \times r \times r$$

Example 1.206

The radius of a circle is $3\ feet$. Find its area

A. in terms of π
B. as a number

$$Part\ A: Area = \pi r^2 = \pi \times 3^2 = \pi \times 9 = 9\pi\ feet^2$$

$$Part\ B: Area = 9\pi = 9 \times \frac{22}{7} = \frac{22 \times 10 - 22 \times 1}{7} = \frac{220 - 22}{7} = \frac{198}{7} feet^2$$

Example 1.207

The diameter of a circle is $2\ inches$. Find its area.

Our formula is in terms of radius. So, first, find the radius:

$$r = \frac{D}{2} = \frac{2\ inches}{2} = 1\ inches$$

Then, find the area:

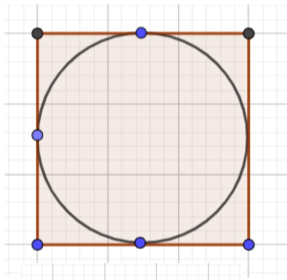
$$A = \pi r^2 = \frac{22}{7} \times 1^2 = \frac{22}{7}\ inches^2$$

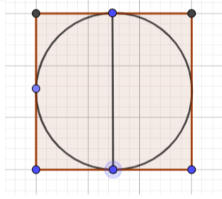
D. Inscribed Figures

Example 1.208: Circle Inscribed in a Square

In the figure alongside, a square with side 4 cm is drawn. A circle is drawn touching all the sides of the square. Find:

A. Diameter of the circle
B. Radius of the circle
C. Area of the circle

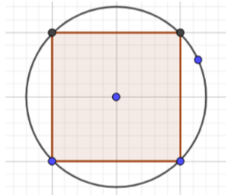




$$Diameter\ of\ Circle = 4\ cm$$
$$Radius = r = \frac{D}{2} = \frac{4}{2} = 2\ cm$$
$$A = \pi D = \frac{22}{7} \times 4 = \frac{88}{7}\ cm^2$$

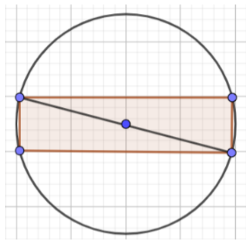
Example 1.209: Square Inscribed in a Circle

In the figure drawn alongside, the radius of the circle is 3 $feet$. Find the length of diagonal of the square.
Note: The diagonal of a square is the line segment joining one vertex to its opposite vertex.



First, calculate the diameter of the circle:
$$Diameter = 2r = 2 \times 3\ feet = 6\ feet$$

Join the opposite vertices of the square.
Note that the diagonal of the square passes through the centre of the circle.
$$Diagonal\ of\ Square = Diameter\ of\ Circle = 6\ feet$$



In the above example, if we had a rectangle instead of a square, would the diagonal still have been 6 cm

$$Yes$$

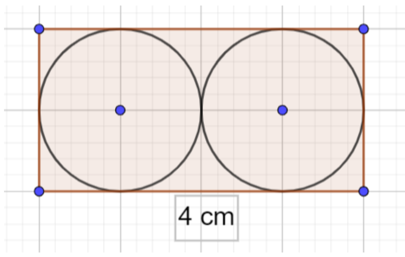
E. More Complex Figures

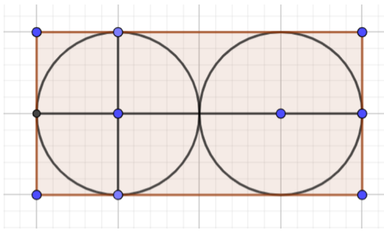
Type equation here.

Example 1.210

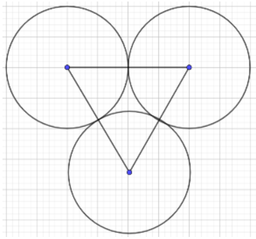
Two equal circles are drawn in a rectangle with length of side 4 cm. Find

A. Diameter of the circle
B. Width of the rectangle
C. Area of the rectangle
D. Radius of the circle
E. Area of the circle

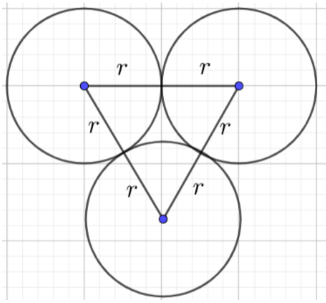




$$D = \frac{4}{2} = 2\ cm$$
$$Width\ of\ Rectangle = 2\ cm$$
$$Area\ of\ Rectangle = 4 \times 2 = 8\ cm^2$$
$$Radius\ of\ Circle = \frac{2}{2} = 1\ cm$$
$$Area\ of\ Circle = \pi r^2 = \frac{22}{7} \times 1 = \frac{22}{7} cm^2$$



Example 1.211
Three circles of equal size are placed adjacent to each other as shown. The perimeter of the equilateral triangle so formed is 1 m. Find the area of the three circles combined.



F. Back Calculations

Example 1.212

$$C = 18 \Rightarrow 2\pi r = 18 \Rightarrow r = \frac{18}{2\pi} = \frac{9}{\pi} \Rightarrow A = \pi r^2 = \pi\left(\frac{9}{\pi}\right)^2 = \frac{81\pi}{\pi^2} = \frac{81}{\pi}$$

Example 1.213
Sharon imagines walking around the earth via the equator. And her brother imagines flying over it at a height of 5 feet. Find the difference in the distance that they will travel.

$$C_1 - C_2 = 2\pi(r_1 + 5) - 2\pi r_1 = 2\pi(r_1 + 5 - r_1) = 10\pi$$

Example 1.214
Circumference of two concentric circles differs by 10π feet. Find the difference in their radii.

$$C_1 - C_2 = 10\pi \Rightarrow 2\pi r_1 - 2\pi r_2 = 10\pi \Rightarrow 2(r_1 - r_2) = 10 \Rightarrow r_1 - r_2 = 5$$

This answer is independent of the values of r_1

1.8 Polygons (2D Shapes)

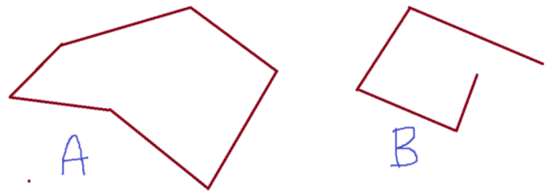
1.215: Open and Closed Figures

- A closed figure is a figure that has no gaps. The boundary of the figure separates the outside from the inside.

- An open figure is a figure that has a gap. You can use the gap to go from the inside of the figure to the outside of the figure.

Example 1.216

Consider the two figures to the right. State whether each figure is open or closed.



$$Figure\ A: Closed$$
$$Figure\ B: Open$$

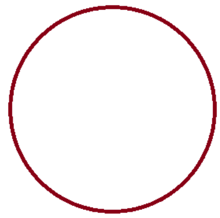
1.217: Curved Figures

Figures which are made on curves are called curved figures.

Example 1.218

Consider the circle drawn alongside.

A. Is the circle made of straight lines or curves?
B. Is the circle open or closed?

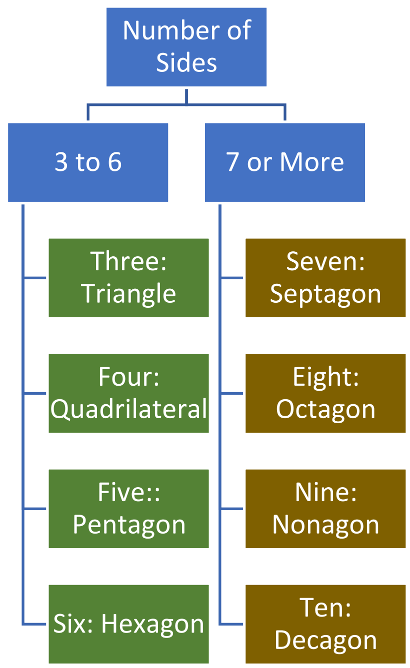


A circle is made of curves.
A circle is a closed figure.

1.219: Polygon

A polygon is a closed figure made up of line segments.

- Sides: The line segments that make up the polygon are called the sides of the polygon.
- Vertices (Corners): The place where two line segments of a polygon meet are called the vertices of the polygon.
- Diagonal: Any line joining two vertices of the polygon which is not a side is a diagonal of the polygon.



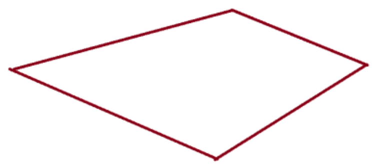
A. Classification of Polygons

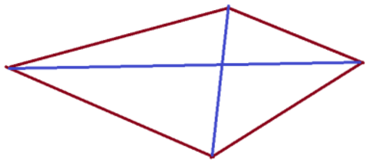
Polygons can be classified (or given different names) based on the number of sides that the polygon has.
The chart alongside shows the names of polygons with different number of sides.

Example 1.220

Look at the diagram drawn alongside.

A. Name the shape?
B. How many sides does it have?
C. How many vertices does it have?
D. How many diagonals?



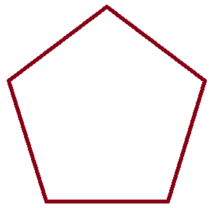


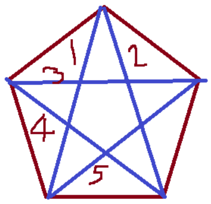
$$\begin{gathered} Shape: Quadrilateral \\ No.\,of\ Sides = 4 \\ No.\,of\ Vertices = 4 \\ No.\,of\ Diagonals = 2 \end{gathered}$$

Example 1.221

Look at the diagram drawn alongside.

E. Name the shape?
F. How many sides does it have?
G. How many vertices does it have?
H. How many diagonals?





$$\begin{gathered} Name\ of\ Shape: Pentagon\ (5\ Sides) \\ 5\ Sides \\ No.\,of\ Vertices(Corners) = No.\,of\ Sides = 5 \end{gathered}$$

Example 1.222

A nonagon is a polygon with 9 sides. How many vertices does it have?

1.223: Number of Vertices of a Polygon

The number of vertices of a polygon is the same as the number of sides of the polygon.

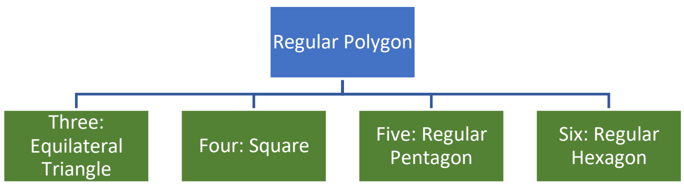
Example 1.224

A polygon is made of 112 sides. What is the number of vertices of the polygon?

$$No.\,of\ Vertices = No.\,of\ Sides = 112$$

1.225: Regular Polygon

Regular: A regular polygon is a polygon where all sides are equal, and all angles are also equal. Polygons which are not regular are called irregular.



Square is a regular four-sided polygon.

1.9 Polyhedrons (3D Shapes)

1.226: Polyhedron

A polyhedron is a three-dimensional shape with flat polygonal faces, straight edges and sharp corners or vertices

In particular curved faces are allowed in a polyhedron.

So, for example, a cylinder and a sphere are not polyhedrons.

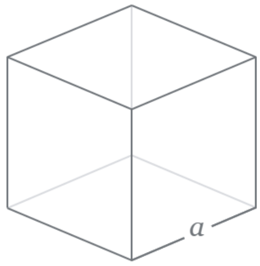
A. Pyramids

Example 1.227

You can see the diagrams in the pages ahead for examples of 3D figures. Using the diagrams (if you need), classify the figures below as polyhedrons, and non-polyhedrons:

A. Cylinder
B. Cube
C. Sphere
D. Cuboid
E. Triangular Pyramid
F. Cone
G. Triangular Prism

B. Cubes and Cuboids



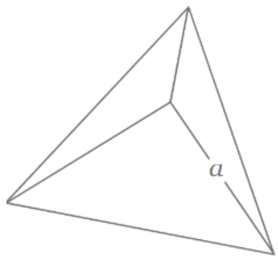
Example 1.228

The adjoining figure shows a cube. Find the

A. Number of vertices
B. Number of edges
C. Number of faces

$$8\ Vertices$$
$$12\ Edges$$
$$6\ Faces$$
$$Each\ face\ is\ a\ square$$

C. Pyramids

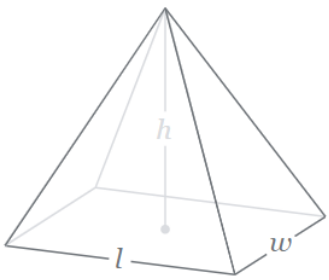


Example 1.229

The adjoining figure shows a triangular pyramid. The base is a triangle, and each of the other faces is also a triangle. Find:

A. Number of vertices
B. Number of edges
C. Number of faces

$$No.of\ Vertices = 4$$
$$No.of\ Edges = 6$$
$$No.of\ Faces = 4$$



Example 1.230

The adjoining figure shows a rectangular pyramid. The base is a rectangle, and each of the other faces is a triangle. Find:

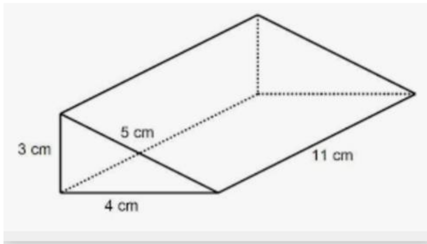
A. Number of vertices
B. Number of edges
C. Number of faces

$$No.of\ Vertices = 5$$
$$No.of Edges = 8$$
$$No.of\ Faces = 5$$

Example 1.231

The adjoining figure shows a triangular prism. The base is a rectangle, and each of the other faces is a triangle. Find:

A. Number of vertices
B. Number of edges
C. Number of faces and the shape that each face has



$$No.of\ Vertices = 6$$
$$No.of Edges = 9$$
$$No.of\ Faces = 5$$

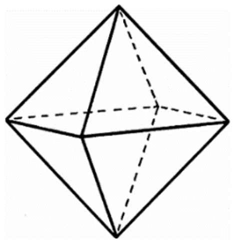
$$3\ Faces = Rectangles$$

$$2\ Faces = Triangles$$

Example 1.232

An octahedron is a convex three dimensional solid that consists of two tetrahedrons (square pyramids) joined at the base. Find:

A. Number of vertices
B. Number of edges
C. Number of faces and the shape that each face has.



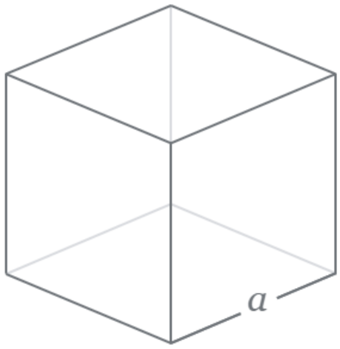
$$8\ Faces$$
$$6\ Vertices$$
$$12\ Edges$$

1.233: Euler's Formula

Euler's formula connects the number of faces, the number of vertices, and the number of edges of a $\underbrace{convex\ polygon}_{Two\ dimensions}$, or $\underbrace{convex\ polyhedron}_{Three\ Dimensions}$.

$$\underbrace{F}_{Faces} + \underbrace{V}_{Vertices} = \underbrace{E}_{Edges} + 2$$

Example 1.234
Show that a cube satisfies Euler's Formula.

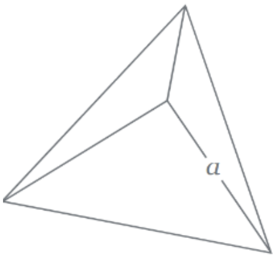


$$\underbrace{F}_{Faces} + \underbrace{V}_{Vertices} = \underbrace{E}_{Edges} + 2$$

$$6\ Faces, 8\ Vertices, 12\ Edges$$

$$6 + 8 = 12 + 2 \Rightarrow 14 = 14$$

Example 1.235
Show that a triangular pyramid satisfies Euler's Formula.

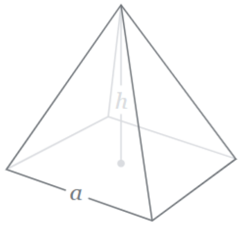


$$F + V = 4 + 4 = 8$$
$$E + 2 = 6 + 2 = 8$$

And hence

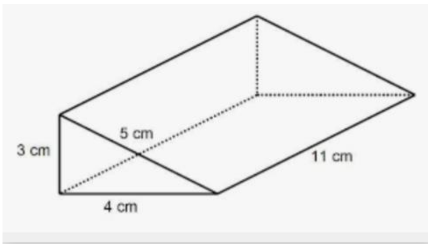
$$F + V = E + 2$$
$$4 + 4 = 6 + 2 \Rightarrow 8 = 8$$

Example 1.236
Show that a square pyramid satisfies Euler's Formula.



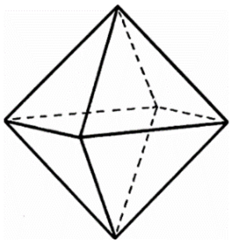
$$F + V = 5 + 5 = 10$$
$$E + 2 = 8 + 2 = 10$$

Example 1.237
Show that a triangular prism satisfies Euler's Formula.



$$F + V = 5 + 6 = 11$$
$$E + 2 = 9 + 2 = 11$$

Example 1.238
Show that an octahedron satisfies Euler's Formula.



$$F + V = 8 + 6 = 14$$
$$E + 2 = 12 + 2 = 14$$

§1.9.D Diagonals

The diagonals in a three-dimensional shape are complicated than the diagonals in a two dimensional shape.

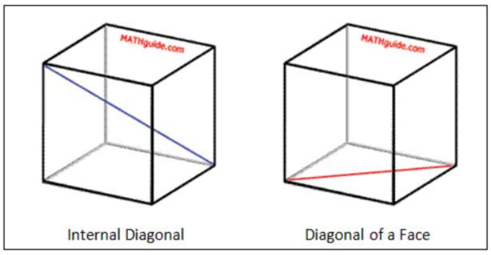
1.239: Face Diagonals
A diagonal lying completely on a face is called a face diagonal.

1.240: Space Diagonals

A diagonal which is not part of any face is called a space diagonal.

Example 1.241

Find the number of face diagonals and space diagonals of a cube.



$$Space\ Diagonals = 4$$

One each diagonal each starting from

$$Top\ Left\ Front$$
$$Top\ Right\ Front$$
$$Top\ Left\ Front$$
$$Top\ Right\ Front$$

Face Diagonals

Each face has 2 diagonals

Total Diagonals

$$2 \times 6 = 12$$

Final Answer

$$Face\ Diagonals + Space\ Diagonals = 4 + 12 = 16$$

1.10 Volume

A. Cube

$$Volume = Side \times Side \times Side = Side^3$$

Example 1.242

Find the volume of the following cubes with side length

A. $3\ m$
B. $4\ inches$
C. $2\ feet$
D. $1\ km$
E. $5\ cm$

$$A.\ Side = 3\ m \Rightarrow V = (3\ m)^3 = 3m \times 3m \times 3m = 3 \times 3 \times 3 \times m \times m \times m = 27\ m^3$$

$$B.\ Side = 4\ inches \Rightarrow V = (4\ inches)^3 = 64\ inches^3$$

$$C.\ Side = 2\ feet \Rightarrow V = (2\ feet)^3 = 8\ feet^3$$

$$D.\ Side = 1\ km \Rightarrow V = (1\ km)^3 = 1\ km^3$$

$$D.\ Side = 5\ cm \Rightarrow V = (5\ cm)^3 = 125\ cm^3$$

Example 1.243

Find the volume of the following cubes with side length

A. $\frac{1}{2}\ m$
B. $\frac{1}{5}\ inches$
C. $\frac{1}{4}\ feet$
D. $\frac{1}{3}\ km$

$$Side = \frac{1}{2}m \Rightarrow V = \left(\frac{1}{2}m\right)^3 = \frac{1}{8}\ m^3$$

$$Side = \frac{1}{5}\ inches \Rightarrow V = \left(\frac{1}{5}\ inches\right)^3 = \frac{1}{125}\ inches^3$$

$$Side = \frac{1}{4}\ feet \Rightarrow V = \left(\frac{1}{4}\ feet\right)^3 = \frac{1}{64}\ feet^3$$

Example 1.244

Find the volume of the following cubes with side length

A. $\frac{3}{4}\ m$
B. $\frac{2}{3}\ inches$
C. $\frac{4}{3}\ feet$
D. $\frac{5}{2}\ km$

$$Side = \frac{3}{4}m \Rightarrow V = \left(\frac{3}{4}m\right)^3 = \frac{27}{64}\ m^3$$

B. Cube

Example 1.245

Find the volume of the following cuboids with

A. $length = 3\ m\ , width = 5m, height = 2m$
B. $length = 4\ inches, width = 3\ inches, height = 1\ inches$
C. $length = 2\ feet, width = 4\ feet, height = 3\ feet$
D. $length = 1\ km, width = 1km, height = \frac{29}{32}km$

Example 1.246

Find the volume of the following cuboids with

A. $length = \frac{1}{3}\ m\ , width = \frac{1}{4}m, height = \frac{1}{5}m$
B. $length = \frac{1}{7}\ inches, width = \frac{1}{3}\ inches, height = \frac{1}{2}\ inches$
C. $length = \frac{1}{2}\ feet, width = \frac{1}{8}\ feet, height = \frac{1}{6}\ feet$
D. $length = \frac{1}{5}\ km, width = \frac{1}{7}km, height = 1km$

Example 1.247

Find the volume of the following cuboids with

A. $length = \frac{5}{7}\ m\ , width = \frac{2}{3} m, height = \frac{3}{8} m$

B. $length = \frac{3}{7}\ inches, width = \frac{2}{9}\ inches, height = \frac{3}{2}\ inches$

C. Problem Solving

The volume of a tank is 100 m^3. How many tanks with volume 2 m^3 can be fit into the tank.

$$\frac{Volume\ of\ Large}{Volume\ of\ Small\ Tank} = \frac{100}{2} = 50$$

I have a rectangular tank which is 2 meters long, three meters wide and four meters high. I also have a container to fill water, which is 50 cm high, 50 cm wide, and 50 cm long. How many times will I have to fill water in the tank via the container to fill it up completely?

$$Volume\ of\ tank = lwh = 2 \times 3 \times 4 = 24\ m^3$$
$$Volume\ of\ One\ Container = 50\ cm \times 50\ cm \times 50\ cm = \frac{1}{2} m \times \frac{1}{2} m \times \frac{1}{2} m = \frac{1}{8} m^3$$

Method I $V\ of\ 192\ Containers = 24 \times 1m^3 \Rightarrow V\ of\ 24\ Containers = 24 \times 1m^3 = 24\ 3m^3$

$Method\ II$:

Along the length, fit 4 containers.
Along the width, fit 6 containers
Along the height, I can fit 8 containers

$$No. of\ Containers = 4 \times 6 \times 8 = 192$$

$$Method\ III: \frac{Volume\ of\ Tank}{Volume\ of\ Container} = \frac{24}{\frac{1}{8}} = 24 \div \frac{1}{8} = 24 \times 8 = 192$$

Example 248