NLM

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1. Newton's 1st Law

1.1 NLM – 1:Static Equilibrium

A. Newton's First Law

In the Kinematics chapter, we considered motion without taking into account the forces that caused motion. In this chapter, we will begin to look at those forces.

1.1: Newton's First Law: Inertia
A body remains at rest, on in motion at constant velocity in a straight line unless acted upon by a force.

- Velocity is constant
- Motion is in a straight line

Example 1.2
Decide whether the following are an example of Newton's First Law. If not an example, explain why.
A. A book lies on a table in your room.
B. The earth orbits around the Sun.

Part A
If you introduce the earth as a frame of reference, the book is at rest.

$$Valid$$

Part B
Earth moves in a circle around the sun (not a straight line)[1].
The force causing the orbit is gravity.

$$Not\ Valid$$

1.3: Newton's First Law: Restatement
If the net force acting on a body is zero, it has zero acceleration.

$$\vec{\boldsymbol{F}}_{Net}=\sum\vec{\boldsymbol{F}}=0$$

- $\vec{\boldsymbol{F}}_{Net}=\sum\vec{\boldsymbol{F}}$ has a direction and hence is a vector quantity.

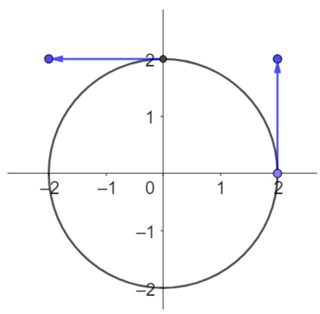
Example 1.4

$$Spot\ the\ mistake, if\ any, in\ the\ reasoning$$

The earth orbits around the Sun. Gravity acts on the Earth. But the earth continues to go around the Sun in a circular orbit. Its velocity remains same. This is a violation of Newton's First Law.

Velocity is a vector quantity.
When an object moves in a circle, its direction is constantly changing.

Hence, the velocity does not remain the same.
Hence, there is no violation of Newton's First Law.



1.5: Unit of Mass
Mass is a scalar quantity.

[1] To be precise, an ellipse, but for now we approximate it as a circle.

$$SI\ unit\ of\ mass = kg = kilogram$$

1.6: Unit of Force: Newton

One Newton is the force which gives a mass of 1 kilogram an acceleration of 1 meter per second per second.

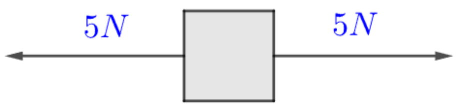
$$1\ N = 1\frac{kg \cdot m}{s^2}$$

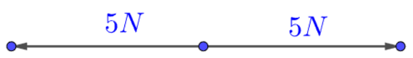
1.7: Free Body Diagram

A free body diagram shows the forces acting on a single body.

Example 1.8

An object of mass m lies at position (x, y) on the coordinate plane. It has a force acting rightwards on it with a force of $5N$. It has another force acting on it leftwards with a force of $5N$. What is the position of the object after five minutes?





We can represent this using a diagram ($left\ diagram$), where the box represents the object. The net force is

$$-5N + 5N = 0$$

We can also use just a point for the object. ($Right\ diagram$)

B. Gravity and Weight

1.9: Gravity

Gravity acts in the direction of Earth.

$$g \approx 9.8\frac{m}{s^2}\ OR\ g \approx 10\frac{m}{s^2}$$

1.10: Weight

Weight is the gravitational force acting on a body.

$$|\vec{W}| = W = mg$$

- Unit of weight is Newton.
- $W = (kg)\left(\frac{m}{s^2}\right)$
- Weight and mass are not the same thing. If you go to the moon, weight will change, but mass will remain same.

Example 1.11

Calculate the weight of a mass of 2 kg

A. On the earth (take $g = 10\frac{m}{s^2}$)
B. On the moon (take $g = 1.6\frac{m}{s^2}$)

Mass of 2 kg on the Earth

$$= mg = (2\ kg)\left(10\frac{m}{s^2}\right) = 20\frac{kg \cdot m}{s^2} = 20N$$

Mass of 2 kg on the Earth

$$= mg = (2\ kg)\left(1.6\frac{m}{s^2}\right) = 3.2\frac{kg \cdot m}{s^2} = 3.2N$$

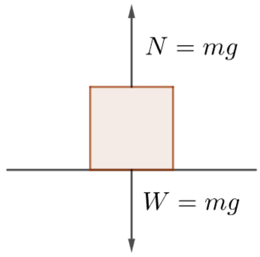
C. Normal Force

1.12: Normal Force

Normal force is the component of a contact force that is perpendicular to the surface that an object contacts,

Example 1.13

A. Draw the free body diagram of an object with mass m on a table.



D. Tension

1.14: String/Cord/Rope

Many objects in physics will be attached to a rope. If the string is:

- massless, then we do not need to take the mass of the string into account.
- inextensible, then the length of the string is fixed.

The force on a string is called the tension.

Example 1.15

Mark all correct options

From which of the following phrases can you approximate using a string of zero mass:

A. String of negligible mass
B. Light string
C. String of mass $20g$ attached to an object of mass $20g$
D. String of mass $20g$ attached to an object of mass $20kg$
E. Massless string

$$A, B, D, E$$

Ratio of mass of string to mass of object:

$$= 20g : 20\ kg = \frac{1}{1000} = 0.001 \approx 0$$

Example 1.16

A mass of m is suspended from the ceiling (but above the ground) using a massless, inextensible cord attached to a ring. By drawing a free body diagram for each, determine:

A. the tension in the rope at the point where the mass is suspended by drawing a free body diagram for mass

B. the tension in the rope at the point where the cord is attached to the ceiling by drawing a free body diagram for the point where the cord is attached to the ceiling.

C. the tension in the rope

Part A

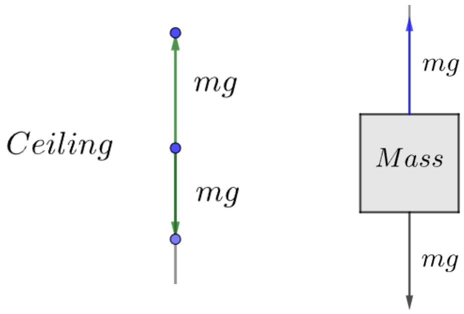
mg

Part B

mg

Part C

mg



Example 1.17

Buddy the dog has a leash of negligible mass attached to his collar. The other end of the leash is fixed to his kennel (at the same height as the collar). Buddy pulls with a force of $5N$ on the leash, but it does not budge. If all relevant forces are in the same dimension, draw a free body diagram for each, and calculate:

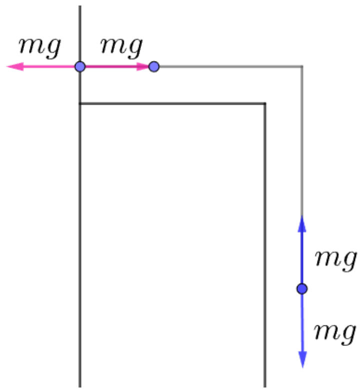
A. the tension in the leash at the collar
B. the tension in the leash at the place where the leash is fixed to the kennel.
C. the tension in the leash

$$Tension\ at\ collar = 5N$$
$$Tension\ at\ kennel = 5N$$
$$Tension = 5N$$

Example 1.18

An object of mass m hangs over a table via a massless, frictionless cord attached to a wall. Determine:

A. The tension in the cord at the point where the mass is attached
B. The tension in the cord at the point where the cord is attached to the wall
C. The tension in the cord



Part A
The mass has the force of gravity pulling it down, which the cord must balance to maintain equilibrium:

$$T = mg$$

Part B
The cord pulls at the wall with force mg, while the walls pulls back.

$$T = mg$$

Part C
The overall tension is:

$$T = mg$$

Example 1.19
Team A, playing a game of $tug - of - war$, pulls on a rope to the left, with a force of $20N$. Team B pulls the rope with just enough force to maintain their position.

A. How much force, and in which direction, does Team B apply?
B. Determine the tension in the rope at the point where Team A pulls
C. Determine the tension in the rope at the point where Team B pulls
D. Determine the tension in the rope

$$A: 20N$$
$$B, C, D: 20N$$

1.20: Breaking Strength
The maximum weight that can be supported by a string without breaking is called its breaking strength.

Example 1.21
A string can have an object of mass maximum 10 kg attached to it. Calculate the maximum tension in the string.

$$T = W = mg = 10(10) = 100N$$

E. Masses in a Row

1.22: Spring Balance
A spring balance will have a spring with a hook to which a mass can be attached.
Putting a mass on the spring will lengthen the spring, and the force on the spring can be measured.

- The force on the spring balance will be the force due to gravity.

Example 1.23
A mass of $3kg$ hangs from the hook of a spring balance. Determine the force with which the mass pulls the spring down.

$$F = mg = 3(10) = 30N$$

Example 1.24
A spring balance of mass m_1 kg hangs from the hook of the other spring balance (which has mass m_2 kg) and a block of mass M kg hangs from the former one. Determine the reading for each spring balance (in terms of kg). (JEE Main 2003, Adapted)

Spring balance with mass m_1 has an object of mass M hanging from it:

$$Reading = M\ kg$$

Spring balance with mass m_2 has the spring balance of mass m_1 hanging from it, and that spring balance has an object of mass M hanging from it:

$$Reading = (m_1 + M)\ kg$$

Example 1.25

Object A of mass 3 kg hangs from a hook in the ceiling via a cord. Object B of mass 5 kg hangs from Object A via a second cord. Object C of mass 8 kg hangs from object B via a third cord. If the cords are massless and inelastic, determine the tension in the cords. (Take $g = 10\frac{m}{s^2}$).

$$C3 = 160N$$
$$C2 = 130N$$
$$C1 = 80N$$

(Extension) Example 1.26

In the previous example we assumed the cords were massless and inelastic. Take the same data as before but assume the cords each have a mass of 100 grams. Determine the tension in each cord:

A. At the upper end
B. At the lower end

	Cord 1	Cord 2	Cord 3
Upper	$81N$	$132N$	$163N$
Lower	$80N$	$131N$	$162N$

Example 1.27

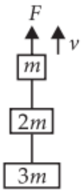
A hangs from the ceiling (via cord 1 with tension $100N$), B hangs from A (via cord 2 with tension $60N$), C hangs from B (via cord 3 with tension $50N$), and D hangs from C (via cord 4 with tension $20N$). If the cords are massless and inelastic, determine the masses of A, B, C and D.

$$D = \frac{20}{10} = 2\ kg$$
$$C = \frac{50-20}{10} = 3\ kg$$
$$B = \frac{60-50}{10} = 1\ kg$$
$$A = \frac{100-60}{10} = 4\ kg$$

Example 1.28

Three blocks with masses m, $2m$ and $3m$ are connected by strings, as shown in the figure. After an upward F is applied on block m, the masses move upward at constant speed v. What is the net force on the block of mass $2m$ in terms of g, where g is the acceleration due to gravity. (NEET 2013)

67. Three blocks with masses m, $2m$ and $3m$ are connected by strings, as shown in the figure. After an upward force F is applied on block m, the masses move upward at constant speed v.



What is the net force on the block of mass $2m$? (g is the acceleration due to gravity)

$$Velocity = v = constant$$
$$Acceleration = a = \frac{dv}{dt} = 0$$
$$Net\ Force = F_{Net} = 0$$

(Continuation) Example 1.29

The masses move upward at constant speed v from $t = 2\ seconds$ to $t = 4\ seconds$. At $t = 3$, what is the

A. Magnitude of upward force on each of the masses.
B. Magnitude of F

Part A

Using $F_{Net} = 0$

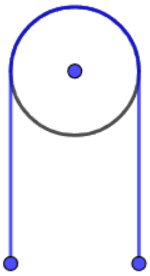
$$For\ 3m: 3mg$$
$$For\ 2m: 3mg + 2mg = 5mg$$
$$For\ m: 3mg + 2mg = 6mg$$

Part B

$$6mg$$

F. Pulleys

1.30: Pulley



Assumptions:

- If the pulley has mass, then turning the pulley requires force. If we assume that the pulley has no mass then we do not need to take the mass of the pulley into account.
- If friction is not there, then force gets transmitted throughout the rope.

Example 1.31

Two objects, each of mass m are hanging from a massless rope at a height of 3 feet from the ground. The rope passes over a massless, frictionless pulley suspended from the ceiling.

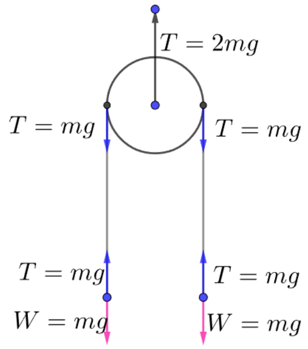
A. Calculate the tension in the rope.
B. Draw a free body diagram of the pulley and the masses.

Tension in the rope keeping the pulley attached to the ceiling will be:

$$2mg$$

Tension in the rope attached to the masses will be

$$mg$$



1.32: Atwood Machine

1.2 Static Friction

A. Friction

Definition

When you push an object, frictional force is the force that resists the movement.

- Friction is caused by attractive forces between molecules of two surfaces in contact.
- Rough surfaces have greater friction. Smooth surfaces have less friction. This is measured using coefficient of friction (discussed later).

Types of Friction

Force due to friction has two different behaviors. If an object is

- at rest, the frictional force is higher ($static\ friction$)
- moving, the frictional force is less ($kinetic\ friction$)

1.33: Friction is a Variable Force

$$Frictional\ force = f_s, 0 \le f_s \le f_{s,Max}$$

If an object

- rests on a table $f_s = 0$
- is pushed with a force $F \le f_{s,Max}$, frictional force $f_s = F$ will be in the direction opposite to the push and resist the movement
- is pushed with a force $F > f_{s,Max}$ the object will start moving.

Example 1.34

A. You push a heavy truck with a force of $F_1 = 5N$ in the rightward direction. The object does not move. Draw a free body diagram of the truck.

B. You become tired, and the force F that you apply reduces to $1N$. What is the value of f_s.

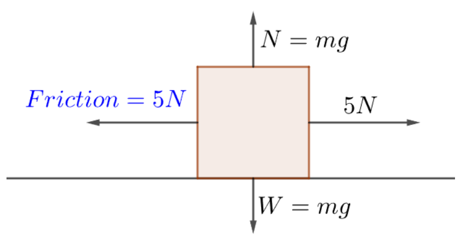
Part A

$$f_s = F_1 = 5N$$

Part B

$$f_s = F_2 = 1N$$

(Diagram not shown)



1.35: Coefficient of Friction

The coefficient of friction is used to numerically compare the friction between two surfaces

$$Coefficient\ of\ friction = \mu_s, \mu_s > 0$$

Example 1.36

Mark all correct options

From which of the following can you approximate using zero friction:

A. Smooth pulley
B. Frictionless table
C. Surface with negligible friction
D. Surface where coefficient of friction is close to zero

$$A, B, C, D$$

1.37: Max Value of Frictional Force

The maximum value of friction is proportional to the normal force. The constant of proportionality is the coefficient of friction

$$f_{s,Max} \propto F_n \Leftrightarrow f_{s,Max} = \mu_s F_N$$

$$f_{s,max} = \mu_s F_N$$

is not a vector equation.

- f_s is parallel to the surface
- F_N is normal to the surface

$$Frictional\ force = f_s, 0 \le f_s \le f_{s,Max} = \mu_s F_N$$

Example 1.38

Mark the correct option

Which one of the following statements is incorrect?

A. Rolling friction is smaller than sliding friction.
B. Limiting value of static friction is directly proportional to normal reaction.
C. Frictional force opposes the relative motion.
D. Coefficient of sliding friction has dimensions of length.

$$f_{s,Max} = \mu_s F_N \Rightarrow \mu_s = \frac{F_N}{f_{s,Max}}$$

Which means

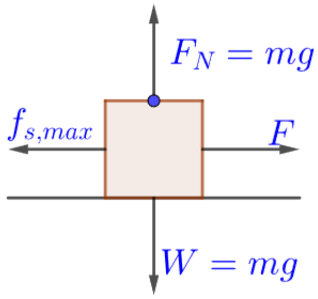
$$\mu_s \text{ is dimensionless} \Rightarrow Option\ D$$

Example 1.39

A body of mass 1 kg lies on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force F N in the horizontal direction. The value of F will be (Round to the nearest integer) (Take $g = 10\frac{m}{s^2}$). **(JEE Main, March 17, 2021, Shift-II, Adapted)**

For the body to move, the force applied must be equal to the maximum frictional force, which is:

$$f_{s,max} = \mu_s F_N = \mu_s mg$$



Substitute values:

$$\frac{1}{\sqrt{3}}(1)(10) = \frac{10\sqrt{3}}{3}$$

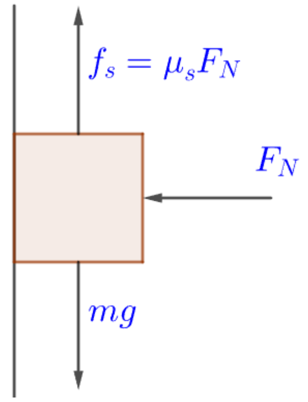
Approximate:

$$\approx \frac{17.1}{3} \approx 6$$

B. Weights against a Wall

1.40: Object pushed against a wall

An object can be held in place against a wall through a combination of horizontal force and friction.



The force in the upward direction ($Friction = f_s = \mu_s F_N$) must balance the force in the downward direction ($gravity = W = mg$)

$$\mu_s F_N = mg \Rightarrow F_N = \frac{mg}{\mu_s}$$

Example 1.41

The coefficient of static friction between a wooden block of mass $0.5kg$ and a vertical rough wall is 0.2. The magnitude of horizontal force that should be applied on the block to keep it adhered to the wall will be ____ N. ($g = 10\frac{m}{s^2}$) (JEE Main Feb 24, 2021, Shift-I; JEE Main 2003)

$$F_N = \frac{mg}{\mu_s} = \frac{(0.5)(10)}{0.2} = \frac{5}{0.2} = 25N$$

1.42: Holding an object against a Wall

If you want to hold an object fixed in position against a vertical wall:

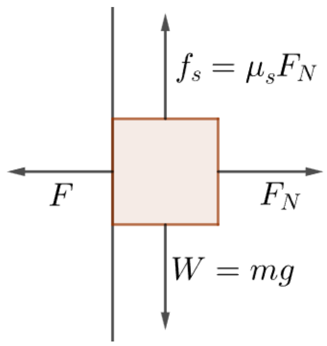
- the force of gravity $= W = mg$ will pull the object down
  - ✓ An object of larger mass will require greater force to keep it in position
- The frictional force will resist the pull of gravity
  - ✓ Greater coefficient of friction between the wall and the object will require less force to keep it in position.

Example 1.43

$\left(\text{Use } g = 10\frac{m}{s^2}\right)$ An object of mass 1 kg is held in position against a vertical wall by a horizontal force F. The coefficient of friction between the object and the wall is 0.25. Find the

A. Frictional force
B. minimum value of F. Call it F_{min}.
C. maximum static frictional force when $(a) F = F_{min}$, $(b)\ F = 2 \cdot F_{min}$
D. actual static frictional force when $(a) F = F_{min}$, $(b)\ F = 2 \cdot F_{min}$
E. reason the answers to Part B and C differ.

Part A



If the object is held in position, then the frictional force in upward direction must be just enough to balance the pull of gravity in downward direction.

$$f_s = W = mg = 10N$$

Part B

$$\mu_s F_N = mg$$

Solve for F_N:

$$F_N = \frac{mg}{\mu_s} = \frac{(1)(10)}{0.25} = 40N$$

Logical Method

Out of the force F applied, because $\mu_s = 0.25 = \frac{1}{4}$, only $\frac{1}{4}$ of the force can be converted into friction. Hence, the minimum force required is

$$4 \times mg = 40N$$

Part C

$$f_s = \mu_s F_N = \left(\frac{1}{4}\right)(40) = 10N$$
$$f_s = \mu_s F_N = \left(\frac{1}{4}\right)(80) = 20N$$

Part D

For both cases:

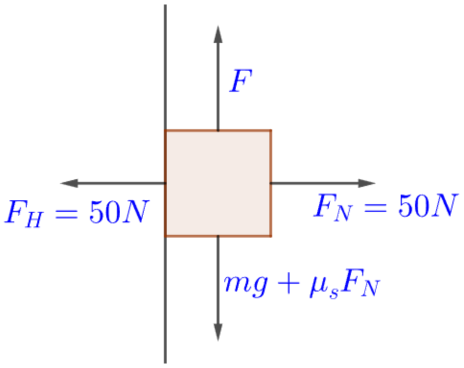
$$Actual\ Friction = f_s = mg = 10N$$

But the maximum possible value differs.

Example 1.44

A 2 kg block is pushed against a vertical wall by applying a horizontal force of $50N$. The coefficient of static friction between the block and the wall is 0.5. A force F is also applied on the block vertically upward. The maximum value of F applied so that the block does not move upward will be: $(g = 10\frac{m}{s^2})$ **(JEE Main, June 30, 2022, Shift-II)**

Draw a diagram.



Horizontal Direction[2]

Let

$$Horizontal\ Force = F_H = F_N = 50N$$

There is equilibrium in the horizontal direction

$$F_H = F_N$$

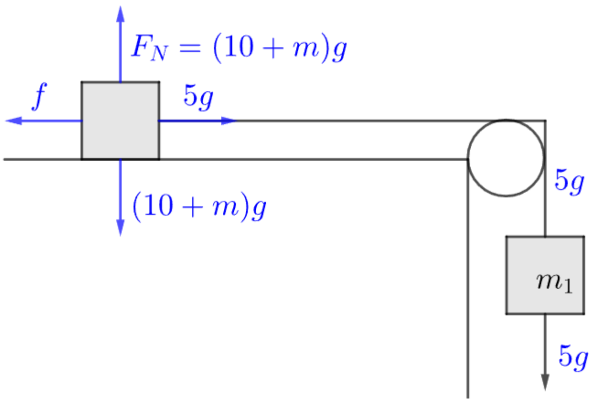
Vertical Direction

The force is applied in the upward direction. Friction will be applied in the downward direction, as will gravity.

In equilibrium, we must have:

$$F = mg + \mu_s F_N = 2(10) + (0.5)(50) = 20 + 25 = 45N$$

C. Weights overhanging a Table



Example 1.45

A mass $m_1 = 5\ kg$ hanging from a table, and $m_2 = 10\ kg$ on the table connected by an inextensible string over a frictionless pulley are moving. The coefficient of friction of the table is 0.15. The minimum weight m, measured in kg, that should be put on top of m_2 to stop the motion is: **(JEE Main 2018, Adapted)**

In the absence of information, assume that the coefficient of static and kinetic friction is the same:

[2] This step is useful for the concept. It is not needed for calculation of answer. You can bypass, if you are comfortable.

$$\mu = \mu_s = \mu_k = 0.15 = \frac{15}{100} = \frac{3}{20}$$

The equilibrium condition is:

$$f = \mu F_N = \mu(10 + m)g = 5g$$

Cancel g and divide by μ:

$$10 + m = \frac{5}{\mu} = \frac{5}{\frac{3}{20}} = 5 \times \frac{20}{3} = \frac{100}{3} = 33\frac{1}{3}\ kg$$

Solving for m gives us:

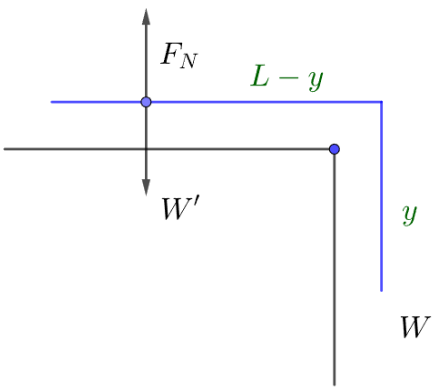
$$m = 33\frac{1}{3}\ kg - 10kg = 23\frac{1}{3}\ kg$$

Example 1.46

A. A heavy uniform chain lies on a horizontal table top. If the coefficient of friction between the chain and the table surface is 0.25, then the maximum fraction of the length of the chain that can hang over one edge of the table is: **(NEET 1991)**

B. Explain why the answer does not depend on either the mass of the chain, or the value of g. Suggest a way to use this property.

Part A



Let M be the mass of the chain.
If L is the length of the chain, and y is the length overhanging, then the length of the chain on the table

$$= L - y$$

The maximum length is obtained when frictional force and weight of the hanging chain are equal:

$$\mu_s F_N = W$$

Substitute $W = mg, \mu_s = \frac{1}{4}$:

$$\frac{1}{4}F_N = mg \Rightarrow F_N = 4mg$$

Substitute $m = M\left(\frac{y}{L}\right),\ F_N = mg = M\left(\frac{L-y}{L}\right)g$:

$$M\left(\frac{L-y}{L}\right)g = 4M\left(\frac{y}{L}\right)g$$
$$L - y = 4y$$
$$y = \frac{L}{5}$$

Part B

The chain is of uniform mass. Hence, we only need the proportion.
Similarly, gravity is the same both for the hanging chain and for the chain on the table.

$$\frac{L-y}{4} = y$$

1.3 Components; Walls and Tables

A. Working in Multiple Dimensions

1.47: Newton's First Law: Restatement

If the net force acting on a body is zero, it has zero acceleration.

$$\vec{F}_{Net} = \sum \vec{F} = 0$$

- $\vec{F}_{Net} = \sum \vec{F}$ has a direction and hence is a vector quantity.

Example 1.48

Three forces $F_1 = 10N$, $F_2 = 8N$,$F_3 /= 6N$ are acting on a particle of mass 5 kg. The forces F_2 and F_3 are applied perpendicular so that the mass remains at rest. If the force F_1 is removed, then the magnitude of acceleration of the particle is: **(JEE Main, April 12, 2023, Shift-I)**

Since F_1 is removed, the remaining sum of the remaining forces is:

$$\boldsymbol{F_2} + \boldsymbol{F_3}$$

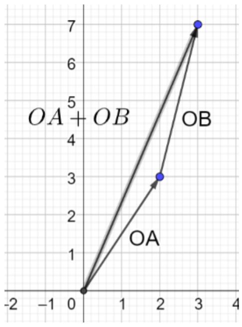
But since the mass is at rest.

$$\sum \vec{F} = 0 \Rightarrow \boldsymbol{F_1} + \boldsymbol{F_2} + \boldsymbol{F_3} = 0 \Rightarrow \boldsymbol{F_1} = -(\boldsymbol{F_2} + \boldsymbol{F_3})$$

$$F_1 = |\boldsymbol{F_2} + \boldsymbol{F_3}| = 10N$$

$$F = ma \Rightarrow a = \frac{F}{m} = \frac{10}{5} = 2\frac{m}{s}$$

1.49: Geometrical Addition: Triangle Approach



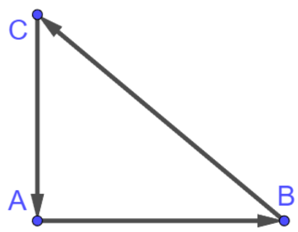
To add vectors $\vec{a}$ and $\vec{b}$:

- Place the tail of the second vector where the tip of the first vector ends.
- The sum of the two vectors is the vector starting from the tail of the first vector, and ending at the tip of the second vector

This approach is also the triangle approach because it creates a triangle.

1.50: Sum of a Vector Polygon

The sum of the vectors in a vector polygon is zero.



Example 1.51

Three forces start acting simultaneously on a particle moving with velocity $\vec{v}$. These forces are represented in magnitude and direction by the three sides of a triangle ABC. The particle will now move with velocity: **(JEE Main 2003; NEET 2019)**

Sum of vectors that form a vector polygon is zero.
Hence, sum of these vectors is zero.

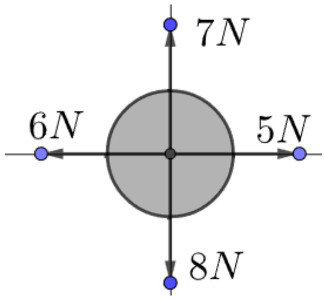
Hence, the velocity of the particle remains unchanged at:

$\vec{v}$

B. Resolving into Components

1.52: Components

$$\vec{F}_{Net} = \sum \vec{F} = \vec{F}_x + \vec{F}_y + \vec{F}_z$$
$$\vec{F}_x = Force\ in\ x\ direction$$
$$\vec{F}_y = Force\ in\ y\ direction$$
$$\vec{F}_z = Force\ in\ z\ direction$$



Example 1.53

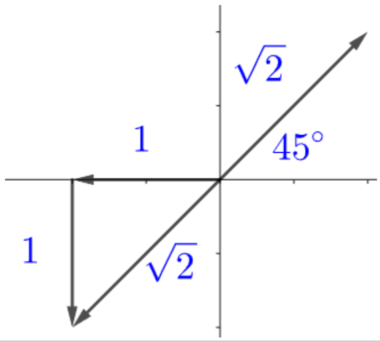
For a free body diagram shown in the figure, the four forces are applied in x and y direction. What additional force must be applied and at what angle with positive x axis so that the net acceleration of the body is zero. **(JEE Main, July 25, 2022, Shift-II)**

Net force

$$F_x = -6 + 5 = -1N, F_y = -8 + 7 = -1N$$

Combined net force

$$= (-1, -1) = (\sqrt{2}, 225°)$$



This will be balanced by:

$$(\sqrt{2}, 45°)$$

1.54: Vector Equation

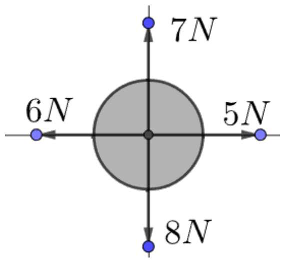
We can also form (and solve) a vector equation to get our answer.

- Both methods have similar calculations (and same final answer)
- The vector equation method is more abstract, and hence potentially more confusing.

Example 1.55

Solve using Vector Equations

A body has the following forces applied on it: $F_1 = 6N$ West, $F_2 = 5N$ East, $F_3 = 7N$ North, $F_4 = 8N$ North. What additional force must be applied and at what angle with positive x axis so that the net acceleration of the body is zero.

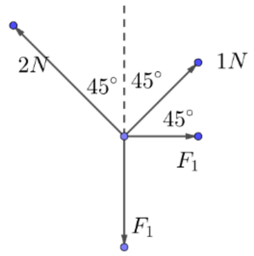


Write each force as a vector in vector notation.

$$F_1 = (-6,0), \quad F_2 = (5,0), \quad F_3 = (0,7), \quad F_4 = (0,8)$$

Use the property that the vector sum of the forces must be zero for the body to be in equilibrium:

$$(-6,0) + (5,0) + (0,7) + (0,8) + \vec{F} = \vec{0}$$
$$\vec{F} = (1,1) = (\sqrt{2}, 225°)$$



Example 1.56

Four forces are acting at a point P in equilibrium as shown in figure. The ratio of forces of F_1 to F_2 is $1: x$ where x is: (JEE Main July 25, 2022, Shift-I)

Forces in the x direction must be in equilibrium:

$$F_1 + (1)\cos 45° = 2\cos 45° \Rightarrow F_1 = \frac{2}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$$

Forces in the y direction must be in equilibrium:

$$F_2 = 1\sin 45° + 2\sin 45° = \frac{1}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{3}{\sqrt{2}}$$

The required ratio is then:

$$F_1 : F_2 = \frac{1}{\sqrt{2}} : \frac{3}{\sqrt{2}} = 1:3 \Rightarrow x = 3$$

C. Walls

1.57: Object held away from a Wall

When an object attached to a wall with a string is pulled away from the way, the tension in the string can be resolved into horizontal and vertical components.

(Important) Example 1.58

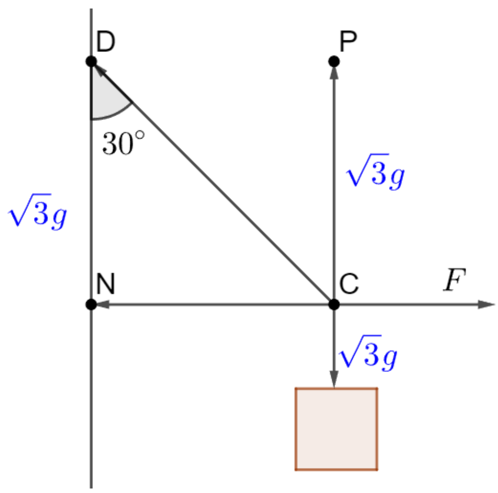
A block of $\sqrt{3}kg$ is attached to a string whose other end is attached to the wall. An unknown force F is applied so that the string makes an angle of 30° with the wall. $\left(\text{Given } g = 10\frac{m}{s^2}\right)$.

A. The tension T is: (JEE Main, Jan 30, 2023, Shift-II)[3]
B. The force F is:

Draw a diagram. The string is

$$DC, \text{and then going on to the object}$$

Point C must be in equilibrium. Hence, the net force on it must be zero.



Part A

Consider forces in the y direction. The downward force due to gravity is also the magnitude of the upward force keeping it in equilibrium:

$$CP = W = mg = \sqrt{3}g$$

Translate CP to the left to get:

$$ND = CP = \sqrt{3}g$$

In right ΔDNC:

$$\cos 30° = \frac{ND}{CD}$$

[3] The original question had a diagram, which is not given here in the question.

Solve for CD and substitute $ND = \sqrt{3}g$:

$$CD = \frac{ND}{\cos 30°} = \frac{\sqrt{3}g}{\frac{\sqrt{3}}{2}} = \sqrt{3}g \cdot \frac{2}{\sqrt{3}} = 2g = 20N$$

The force DC is also the tension on the string.

$$T = 20N$$

Part B

$$\sin 30° = \frac{CN}{CD} \Rightarrow CN = CD \sin 30° = 20 \cdot \frac{1}{2} = 10N$$
$$F = NC = 10N$$

Example 1.59

A. A mass of 10 kg is suspended vertically by a rope of length $5m$ from the roof. A force of $30N$ is applied at the middle point of the rope in the horizontal direction. The angle made by upper half of the rope with vertical is $\theta = \tan^{-1}(x \times 10^{-1})$. The value of x is: (Given $g = 10\frac{m}{s^2}$) (JEE Main, June 27, 2022, Shift-II)

B. Is the length of the rope used in arriving at the answer. Why is it given?

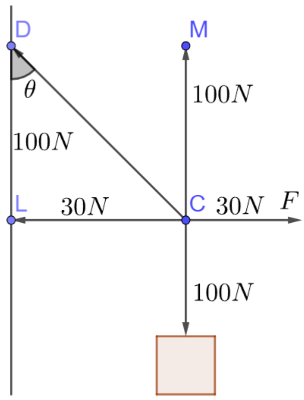
Part A

The forces in the y direction must be balanced:

$$DL = CM = W = mg = 10(10) = 100N$$

The forces in the x direction must be balanced

$$CL = F = 30N$$



In right ΔDLC

$$\tan\theta = \frac{CL}{LD} = \frac{30N}{100N} = \frac{3}{10} = 3 \times 10^{-1}$$
$$\theta = \tan^{-1}(3 \times 10^{-1})$$
$$x = 3$$

Part B

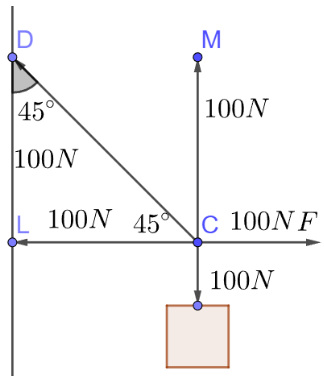
$$\begin{gathered} \textit{Not used} \\ \textit{It is extra information} \end{gathered}$$

Example 1.60

A mass of $10kg$ is suspended by a rope of length $4m$ from the ceiling. A force F is applied horizontally at the midpoint of the rope such that the top half of the rope makes an angle of $45°$ with the vertical. Then F equals: (Take $g = 10\frac{m}{s^2}$ and the rope to be massless). (JEE Main, 9 Jan 2019, Shift-II; JEE Main 7 Jan 2020, Shift-II)

Draw a diagram. The forces in the y direction must balance and hence:

$$DL = MC = W = mg = 100N$$



Since ΔDLC is a $45 - 45 - 90$ triangle:

$$CL = DL = 100N$$

The forces in the x direction must balance and hence:

$$F = CL = 100N$$

D. Tables

1.61: Normal Force

The normal force on an object that is being by a force F that acts at an angle θ to the horizontal is:

$$Push: mg + F\sin\theta$$
$$Pull: mg - F\sin\theta$$

Consider an object on a plane surface with mass m. The normal force on the object from the plane surface will be:

$$N = W = mg$$

The push or pull has force F. The vertical component of F will be:

$$\sin\theta = \frac{y}{F} \Rightarrow y = F\sin\theta$$

If the object is pushed, the vertical component is downward.

$$N = mg + F\sin\theta$$

If the object is pulled, the vertical component is upward. The object has an upward force acting on it. The normal force on the object from the plane surface is:

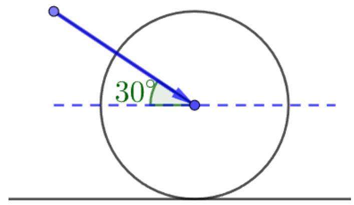
$$N = mg - F\sin\theta$$

Example 1.62

A. As shown in figure, a 70 kg garden roller is pushed with a force of $\vec{\boldsymbol{F}} = 200N$ at an angle of $30°$ with the horizontal. The normal reaction on the roller is: $\left(g = 10\frac{m}{s^2}\right)$ **(JEE Main, Jan 31, 2023, Shift-I)**

B. If other things remain the same, but the roller is pulled instead of pushed, calculate the normal reaction on the roller.

C. Explain the difference in the answers to part A and part B.



Part A

Normal Reaction from roller mass

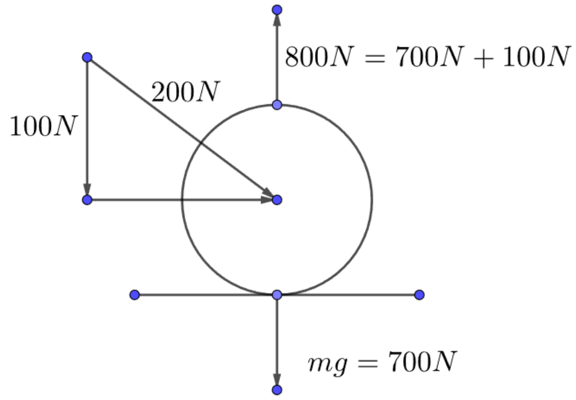
$$= mg = (70)(10) = 700$$

Normal Reaction from pushing force

$$= F_y(200) = 200\sin 30° = 200 \cdot \frac{1}{2} = 100$$

Total

$$= 700 + 100 = 800N$$



Part B

The normal reaction from roller mass remains the same:

$$= mg = 700$$

Normal Reaction from pushing force

$$= F_y(200) = 200\sin 30° = 200 \cdot \frac{1}{2} = 100$$

Total

$$= 700 - 100 = 600N$$

Part C

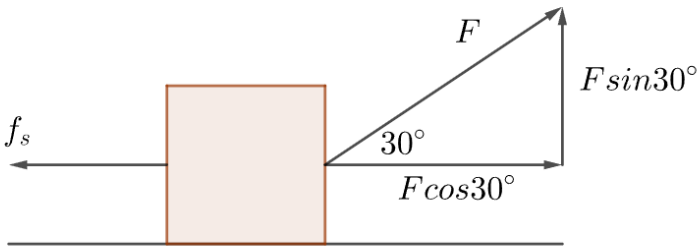
In pushing, the roller is being pushed down, while in pulling the roller is being pulled up.
This causes the difference.

Example 1.63

A block of mass 10 kg lying on a horizontal surface is pulled by a force F acting an angle 30° with horizontal. For $\mu_s = 0.25$, the block will just start to move for the value of $F =$ _____. (Given $g = 10\frac{m}{s^2}$) **(JEE Main, Feb 1, 2023, Shift-II)**

The force moving the block is the horizontal component of the force F given by:

$$F_x = F\cos 30° = \frac{\sqrt{3}F}{2}$$



The maximum value of frictional force

$$= f_{s,max} = \mu_s F_n = \mu_s(mg - F\sin 30°)$$

$$= 0.25\left(10 \cdot 10 - \frac{F}{2}\right) = \frac{50 - 0.25F}{2}$$

In order for the block to just move the frictional force must be equal to the force moving the block:

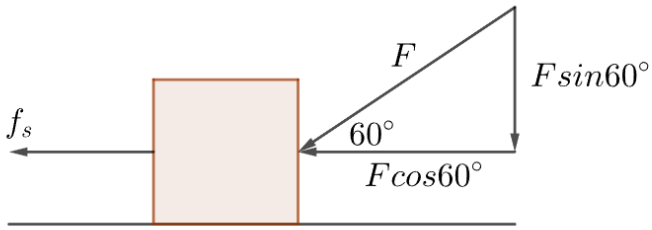
$$F_x = f_{s,max}$$
$$\frac{\sqrt{3}F}{2} = \frac{50 - 0.25F}{2}$$
$$\sqrt{3}F + 0.25F = 50$$
$$F = \frac{50}{\sqrt{3} + 0.25} \approx \frac{50}{1.71 + 0.25} \approx \frac{50}{1.96} \approx 2$$

Example 1.64

A block of mass $\sqrt{3}kg$ is kept on a horizontal rough surface of coefficient of friction $\frac{1}{3\sqrt{3}}$. The critical pushing force to be applied on object at an angle 60° with the horizontal such that the object does not move is $3x$. The value of x will be _____. ($g = 10\frac{m}{s^2}$) **(JEE Main, Feb 26, 2021, Shift-I)**

The force moving the block is the horizontal component of the force F given by:

$$F_x = F\cos 60° = \frac{F}{2} = \frac{3F}{6}$$



The maximum value of frictional force

$$= f_{s,max} = \mu_s F_n = \mu_s(mg + F\sin 60°)$$

$$= \frac{1}{3\sqrt{3}}\left(10\sqrt{3} + \frac{\sqrt{3}F}{2}\right) = \frac{10}{3} + \frac{F}{6} = \frac{20 + F}{6}$$

In order for the block to just move the frictional force must be equal to the force moving the block:

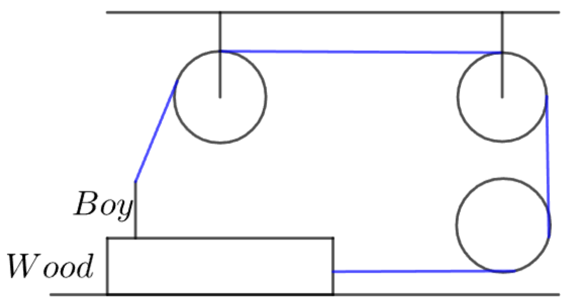
$$F_x = f_{s,max}$$
$$\frac{3F}{6} = \frac{20 + F}{6}$$

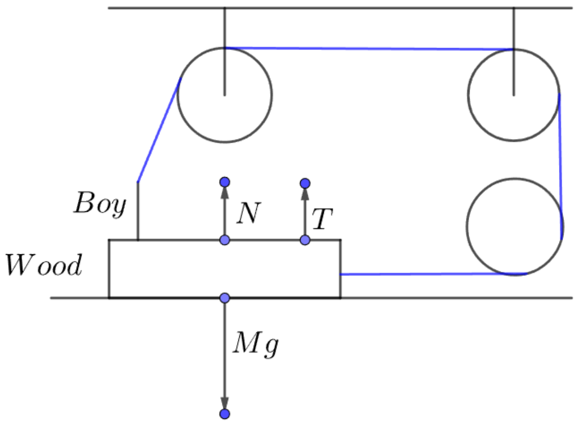
$$F = 3x = 10 \Rightarrow x = \frac{10}{3}$$

E. Objects of Variable Mass

(Important) Example 1.65

A boy of mass 4 kg is standing on a piece of wood having mass 5 kg. If the coefficient of friction between the wood and the floor is 0.5, the maximum force that the boy can exert on the rope so that the piece of wood does not move from its place is _____ N. (Round off to the nearest integer) (Take $g = 10\frac{m}{s^2}$). (JEE Main March 17, 2021, Shift-II)





Consider the boy and the piece of wood as a single object. The mass of the object

$$= M = 4 + 5 = 9$$

Suppose the boy pulls on the rope with a force F. This force is also the tension in the rope:

$$Force = F = T$$

When the boy pulls the rope down, the rope will pull the boy up. Hence, the object is pulled up by

$$T$$

Let the normal force on the object be

$$F_N$$

Vertical Direction

Since the system is in equilibrium, the forces in the vertical direction are in equilibrium:

$$F_N + T = Mg \Rightarrow \underbrace{F_N = Mg - T}_{Equation\ I}$$

Horizontal Direction

The tension in the rope pulls the piece of wood in the rightward direction, and is:

$$T_{Max}$$

The frictional force resists the movement and is:

$$f_{s,Max} = \mu_s F_N = \mu_s(Mg - T) = \mu_s Mg - \mu_s T_{Max}$$

Forces in horizontal direction are also in equilibrium (and we want maximum value):

$$T_{Max} = \mu_s Mg - \mu_s T_{Max}$$

Collate all terms with T_{Max} on the LHS:

$$T_{Max} + \mu_s T_{Max} = \mu_s Mg$$

Factor T_{Max} and divide both sides by $1 + \mu_s$:

$$T_{Max} = \frac{\mu_s Mg}{1 + \mu_s} = \frac{(0.5)(9)(10)}{1 + 0.5} = \frac{45}{1.5} = \frac{90}{3} = 30N$$

Example 1.66

In the above question, if the mass of boy were to be 2 kg, and the mass of wood were to be 7 kg, then answer the question.

If the mass of the boy is 2 kg, then the maximum force that can be applied is:

$$mg = (2)(10) = 20N$$

If a force greater than 20N is applied, the boy will move up the rope.

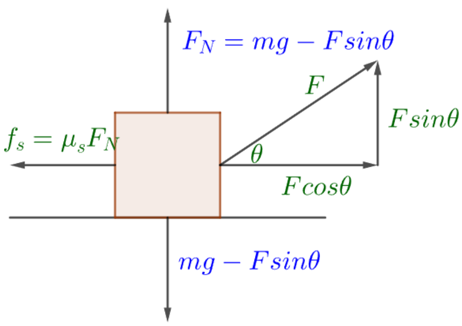
F. Using Calculus

1.67: Comparing Pushes versus Pulls

If a body of mass $m\ kg$ lies on a horizontal floor with coefficient of static friction μ_S, the minimum possible force $F\ N$ to make the body move is:

$$\frac{\mu_s mg}{\sqrt{1+\mu_s^2}}$$

$$\underbrace{F\cos\theta}_{\substack{\text{Rightward}\\ \text{Force}}} = \underbrace{\mu_s F_N}_{\substack{\text{Leftward}\\ \text{(Frictional) Force}}}$$



$$F\cos\theta = \mu_s(mg - F\sin\theta)$$

$$F\cos\theta + \mu_s F\sin\theta = \mu_s mg$$

Solving for F, we obtain the function we would like to minimize:

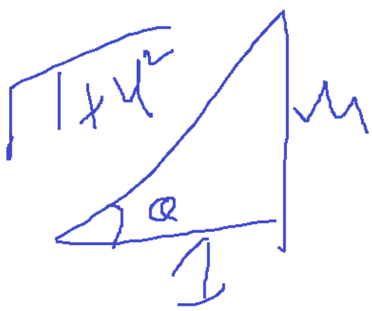
$$F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

Since μ_s, m and g are constants, we can instead maximize the denominator:

$$\frac{d}{d\theta}(\cos\theta + \mu_s\sin\theta) = -\sin\theta + \mu_s\cos\theta =$$

$$\mu_s = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

$$\sin\theta = \frac{\mu_s}{\sqrt{1+\mu_s^2}}, \cos\theta = \frac{1}{\sqrt{1+\mu_s^2}}$$



$$\frac{\mu_s mg}{\frac{1}{\sqrt{1+\mu_s^2}} + \mu_s\left(\frac{\mu_s}{\sqrt{1+\mu_s^2}}\right)} = \frac{\mu_s mg}{\frac{1+\mu_s^2}{\sqrt{1+\mu_s^2}}} = \frac{\mu_s mg}{\sqrt{1+\mu_s^2}}$$

Example 1.68: Optimization

A body of mass 1kg lies on a horizontal floor with which it has a coefficient of static friction $\frac{1}{\sqrt{3}}$. It is desired to make the body move by applying the minimum possible force $F\ N$. The value of F will be (Round to the nearest integer) (Take $g = 10\frac{m}{s^2}$). **(JEE Main, March 17, 2021, Shift-II)**

Note: This question is similar to one done earlier. What is the $difference$?

$$= \frac{\mu_s mg}{\sqrt{1+\mu_s^2}} = \frac{\left(\frac{1}{\sqrt{3}}\right)(1)(10)}{\sqrt{1+\left(\frac{1}{\sqrt{3}}\right)^2}} = \frac{\frac{10}{\sqrt{3}}}{\sqrt{1+\frac{1}{3}}} = \frac{\frac{10}{\sqrt{3}}}{\frac{2}{\sqrt{3}}} = 5$$

1.4 Ramps

A. Resolving Components

Just as a table is an example of a horizontal plane, a ramp is an example of an inclined plane. The calculations are more difficult because the orientation of the forces is different from the weight orientation.

1.69: Ramp

In a ramp, weight (direction of force due to gravity) is downward. It can be resolved into components:

- Magnitude of force parallel to ramp is $mg \sin\theta$
- Magnitude of force perpendicular to ramp is $mg \cos\theta$

Ramp AC makes an angle θ with the horizontal ground AE. Gravity will be perpendicular to the ground, and hence:

$$\underbrace{\angle CEA = 90°}_{Equation\ I}$$

In ΔACE, using the sum of angles of a triangle, and Equation I:

$$\angle ACE = 180 - \angle CEA - \angle CAE = 180 - 90 - \theta = 90 - \theta$$

Components parallel and perpendicular to the ramp must be mutually perpendicular:

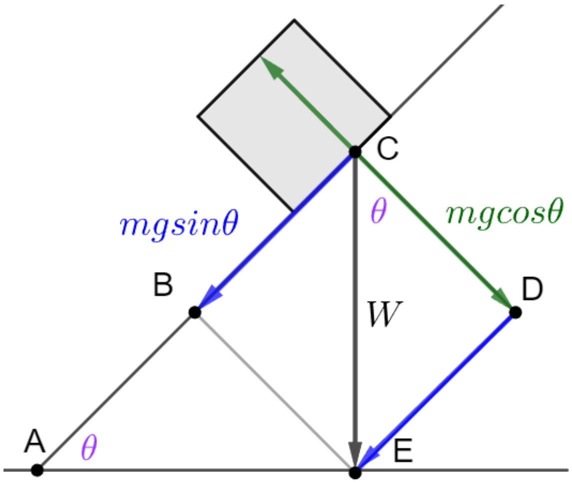
$$CD \perp BC \Rightarrow \underbrace{\angle BCD = 90°}_{Equation\ II}$$

In ΔDCE, using Equation II:

$$\angle DCE = \angle BCD - \angle BCE = 90 - (90 - \theta) = \theta$$

$\angle CDE$ is a right angle because by the Parallelogram law of vector addition

$$BCDE\ is\ a\ paralellogram$$
$$\angle BCD = 90° \Rightarrow BCDE\ is\ a\ rectangle$$



Apply right triangle trigonometry to ΔCDE. The components are:

$$Perpendicular\ to\ the\ ramp\!: \cos\theta = \frac{CD}{W} \Rightarrow CD = W\sin\theta = mg\cos\theta$$
$$Parallel\ to\ the\ ramp\!: \sin\theta = \frac{DE}{W} \Rightarrow DE = W\sin\theta = mg\sin\theta$$

By the parallelogram law of vector addition

$$CD = DE = mg\sin\theta$$

Example 1.70

What is the significance of the component of force:

A. Parallel to ramp

B. Perpendicular to ramp

Part A
This is the force oriented in the direction of the ramp.
We can apply $1D$ Kinematics once we have this component.
Part B
The force perpendicular to ramp will give us the magnitude of normal force.
The magnitude of normal force is needed in the formula for friction.

Example 1.71
A ramp inclined at an angle of 30° to the horizontal ground is next to a wall. An object of mass 5 kg is connected by a massless cord to the wall. Ignore friction. Use $g = 10\frac{m}{s^2}$. Determine the:

A. normal force on the object.
B. tension in the cord.

Part A
The normal force is:

$$mg\cos\theta = (5)(10)(\cos 30°) = 50\left(\frac{\sqrt{3}}{2}\right) = 25\sqrt{3}N$$

Part B
Tension in cord is the force required to hold the object in place

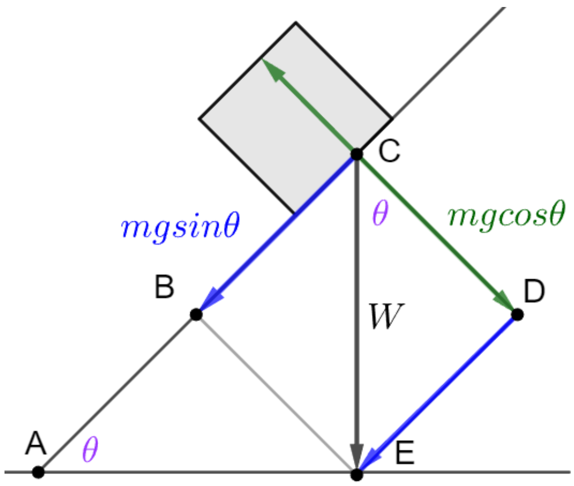
$$= mg\sin\theta = (5)(10)\left(\frac{1}{2}\right) = 25N$$

B. Frictional Force

1.72: Magnitude of Frictional Force
If an object of mass m lies in equilibrium, on a ramp that makes an angle θ with the horizontal, the maximum value of the magnitude of the frictional force is:

$$f_{s,max} = \mu_s mg\cos\theta$$

$$f_{s,max} = \mu_s F_N = \mu_s mg\cos\theta$$



1.73: Equilibrium Condition
If an object of mass m lies in equilibrium, on a ramp that makes an angle θ with the horizontal, the *actual* value of the magnitude of the frictional force is:

$$f_s = mg\sin\theta$$

Example 1.74
A block rests on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of static friction between the block between the block and the plane is 0.8. If the frictional force on the block is $10N$, the mass of the block (in kg) will be: (Given $g = 10\frac{m}{s^2}$) (JEE Main 2004)

Since the object is in equilibrium, the leftward force from the component due to gravity must balance the frictional force

$$f_s = mg\sin\theta$$
$$10 = m(10)\left(\frac{1}{2}\right)$$
$$m = 2\ kg$$

C. Working with Angles

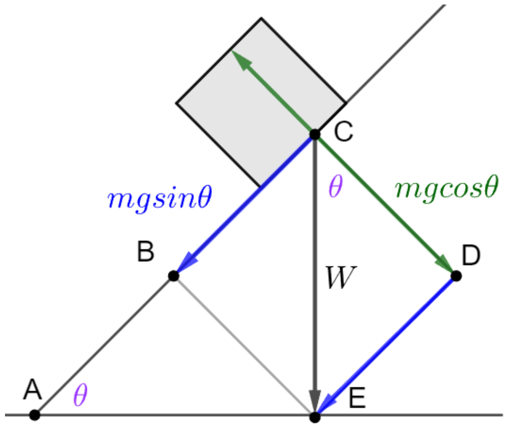
1.75: Maximum Angle for Static Equilibrium

Taking friction into account as an opposing force, an object on a ramp will remain in position if the coefficient of friction is greater than the tangent of the angle that the ramp makes with the horizontal ground[4]

$$\mu_s \geq \tan\theta$$

Alternatively, it will remain in position if the angle is less the tan inverse of the coefficient of friction:

$$\theta \leq \tan^{-1}\mu_s$$



The frictional force must be greater than or equal to the force moving the object on the ramp:

$$\mu_s F_N \geq mg\sin\theta$$
$$\mu_s(mg\cos\theta) \geq mg\sin\theta$$

Divide by $mg\cos\theta$ both sides:

$$\mu_s \geq \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Taking the tan inverse of both sides gives us:

$$\tan^{-1}\mu_s \geq \theta$$
$$\theta \leq \tan^{-1}\mu_s$$

Example 1.76

Interpret $\mu_s \geq \tan\theta$ in terms of the diagram on the right.

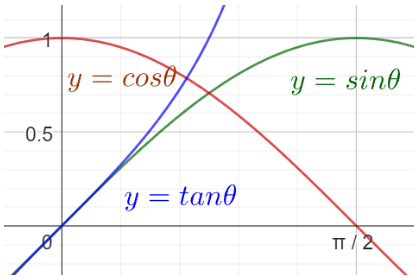
$\tan\theta$ is the ratio of the blue vector to the green vector.
As θ increases, the blue vector increases relative to the green vector, and the coefficient of friction must increase correspondingly.

[4] This assumes that the object does not "topple" or "roll". A tall object would fall over (topple). A cylinder would roll.

When

$$\theta = 0 \Rightarrow \sin\theta = 0, \cos\theta = 1, \tan\theta = 0$$

$$\theta = 45° \Rightarrow \sin\theta = \frac{1}{\sqrt{2}}, \cos\theta = \frac{1}{\sqrt{2}}, \tan\theta = 1$$



Example 1.77
Interpret the graphs of the given trigonometric functions in terms of the forces on an object on a ramp[5].

$$Component\ perpendicular\ to\ ramp = \cos\theta\ \ decreases\ from\ 0\ to\ 90°$$
$$Component\ parallel\ to\ ramp = \sin\theta\ \ increases\ from\ 0\ to\ 90°$$
$$\tan\theta\ \ is\ the\ ratio\ of\ the\ two$$

Example 1.78
A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30°, the box starts to slip and slides 4.0 m down the plank in 4.0s. The coefficient of static friction between the box and the plank (written to one significant digit as a decimal) will be **(NEET 2015, Adapted)**

$$\mu_s = \tan\theta = \tan 30° = \frac{\sqrt{3}}{3} \approx \frac{1.71}{3} \approx 0.6$$

Example 1.79
A ramp that makes an angle θ with the horizontal ground has an object of mass m on it. The ramp is next to a a wall. The object is attached to a wall at a point by a cord with a breaking strength of B Newtons. The cord is parallel to the ramp, but not touching it. Determine the minimum value of the coefficient of friction so that the object does not move.

The forces that move the object leftward on the ramp are:

$$Component\ parallel\ to\ ramp = mg\sin\theta$$

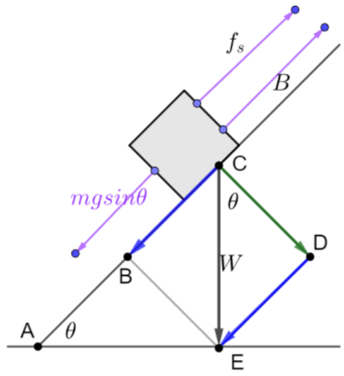
The forces that keep the object in place are:

$$Static\ Frictional\ Force = f_s = \mu_s F_N = \mu_s mg\cos\theta$$
$$Tension\ on\ the\ cord = B$$

The condition to be met is:

$$Rightward\ Forces \geq Leftward\ Forces$$
$$B + f_s \geq mg\sin\theta$$

[5] This is a graph of the trigonometric functions, *and also* the graph of the relative forces on an object on a ramp.



Substitute $f_s = \mu_s F_N = \mu_s mg\cos\theta$:

$$B + \mu_s mg\cos\theta \geq mg\sin\theta$$

Subtract B from both sides:

$$\mu_s mg\cos\theta \geq mg\sin\theta - B$$

Divide both sides $mg\cos\theta$

$$\mu_s \geq \frac{mg\sin\theta}{mg\cos\theta} - \frac{B}{mg\cos\theta}$$

Substitute $\frac{\sin\theta}{\cos\theta} = \tan\theta$:

$$\mu_s \geq \tan\theta - \frac{B}{mg\cos\theta}$$

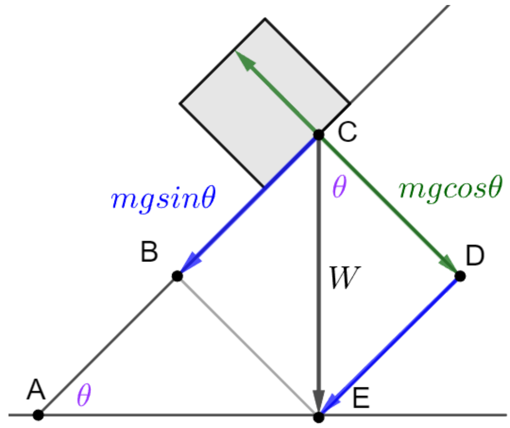
If there is no cord, this reduces to:

$$\mu_s \geq \tan\theta$$

1.80: Angle for Static Equilibrium is independent of value of g

$$\mu_s \geq \tan\theta$$

The condition for static equilibrium of an object on a ramp does not depend on the value of g.



As we saw, an object will remain in static equilibrium on a ramp if:

$$\mu_s \geq \tan\theta$$

This expression does not have g in it.

$$\mu_s(mg\cos\theta) \geq mg\sin\theta$$

This is because when we derived the expression, g was on both sides
As the force due to gravity changes, so does the frictional force.

Hence, the angle remains the same independent of the value of g.

Example 1.81

An astronaut's supply pack is placed on the ramp connecting the moon lander to the horizontal surface. The value of the gravitational constant for the moon is

$$g_{moon} \approx 1.625\frac{m}{s^2} = \frac{13}{8}\frac{m}{s^2} \approx \frac{1}{6}g_{earth}$$

What is the minimum value of the angle between the ramp and the horizontal so that the supply pack will slide down the ramp?

For static equilibrium:

$$\mu_s \geq \tan\theta \Rightarrow \tan\theta \leq \mu_s$$

Since we want the object to slide, take:

$$\tan\theta \geq \mu_s$$

D. Weights overhanging a Ramp

(Important) Example 1.82

Two bodies of masses 5 kg and 3 kg are connected by a light string going over a light pulley on a smooth inclined plane. One mass is on the inclined plane, and the other hangs over the plane. The system is at rest. $\left(Given\ g = 10\frac{m}{s^2}\right)$

A. When solving questions based on above information, what will you consider as coefficient of friction μ_s.
B. Which mass is on the plane, and which one hanging?
C. What is the angle made by the plane with the horizontal ground?
D. What is the force exerted by the inclined plane on the body which is on the plane. **(JEE Main, July 29, 2022, Shift-II, Adapted)**

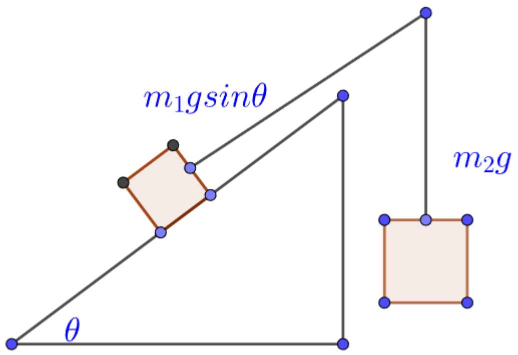
Part A

$$0$$

Part B

Let the masses of the

$$Hanging\ Object = m_2$$
$$Object\ on\ the\ ramp = m_1$$



The force due to gravity on the hanging object

$$= m_2 g$$

The force on the string pulling the hanging object up will be the force on the object on the ramp parallel to the plane

$$m_1 g \sin\theta$$

The force at both ends must be equal since the system is at rest:

$$m_2 g = m_1 g \sin\theta$$

Solve for $\sin\theta$:

$$\sin\theta = \frac{m_2}{m_1} < 1 \Rightarrow m_2 < m_1 \Rightarrow m_2 = 3, m_1 = 5$$

Part C

$$\sin\theta = \frac{m_2}{m_1} = \frac{3}{5} \Rightarrow \theta = \sin^{-1}\left(\frac{3}{5}\right)$$

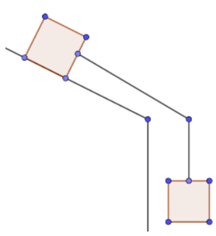
Part D

The force exerted by the inclined plane on the body which is on the plane

$$m_1 g \cos\theta = 5(10)\left(\frac{4}{5}\right) = 40N$$

Example 1.83

What is wrong with drawing the system as shown to the right?



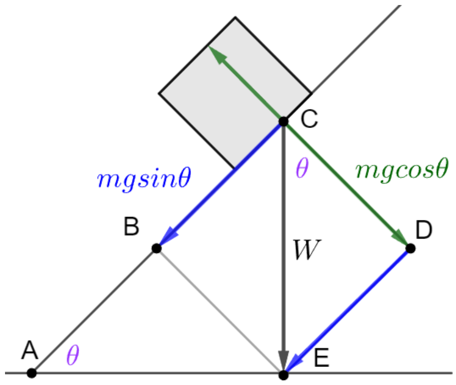
If you draw the system like this, the object will move, and the system is not at rest.

E. Horizontal Forces to Maintain Equilibrium

1.84: Using a Horizontal Force to maintain Equilibrium

An object of mass $m\ kg$ lies on a smooth ramp inclined at an angle θ to the horizontal. If the object is to be held in position by a $horizontal\ force$ of $F\ N$, then the value of F is

$$F = mg\tan\theta$$



Object on a Ramp (1st Diagram)

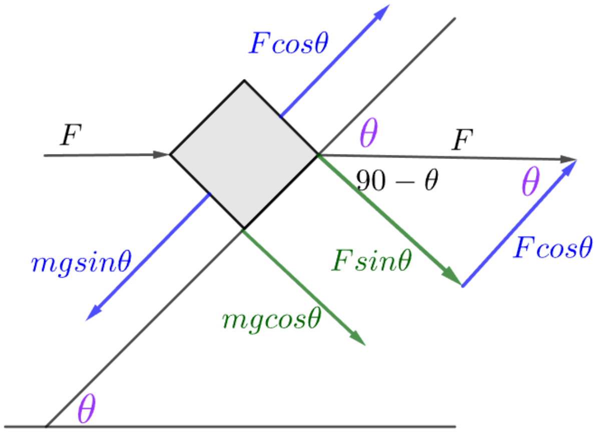
As we have seen before, for an object on a ramp, the force due to gravity can be resolved into components

$$Parallel\ to\ the\ ramp = mg\sin\theta$$
$$Perpendicular\ to\ the\ ramp = mg\cos\theta$$

Horizontal Force applied to object on a Ramp (2^{nd} Diagram)

In the situation that we have, an additional force, parallel to the ground is being applied to the object. Resolving this force into components gives us:

$$Parallel\ to\ the\ ramp = F\cos\theta$$
$$Perpendicular\ to\ the\ ramp = F\sin\theta$$



Analyzing the Situation

Force moving the object leftward in the direction of the plane

$$= mg\sin\theta$$

Force moving the object rightward in the direction of the plane

$$= F\cos\theta$$

In equilibrium, the leftward and rightward forces must be equal:

$$mg\sin\theta = F\cos\theta$$
$$F = mg\frac{\sin\theta}{\cos\theta} = mg\tan\theta$$

Example 1.85

A block of mass $200g$ is kept stationary on a smooth inclined plane by applying a minimum horizontal force $F = \sqrt{x}N$. The plane makes an angle of 60° with the horizontal. The value of $x =$ ______. $\left(Given\ g = 10\frac{m}{s^2}\right)$ **(JEE Main, June 25, 2022, Shift-II)**

$$F = mg\tan\theta = (0.2)(10)(\tan 60°) = 2\sqrt{3} = \sqrt{12}$$

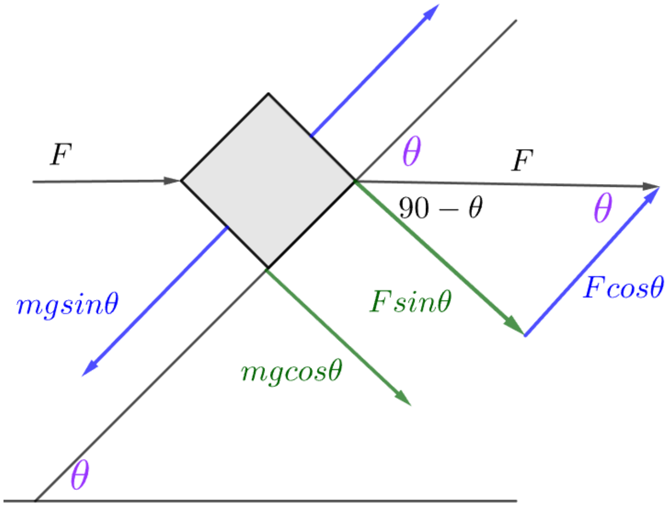
1.86: Using a Horizontal Force to maintain Equilibrium

If an object of mass $m\ kg$ is held in position by a horizontal force $F\ N$ on a ramp inclined at an angle θ to the horizontal, and the coefficient of static friction between the ramp and the object is μ_s, the minimum value of F is

$$F_{min} = \frac{mg(\sin\theta - \mu_s\cos\theta)}{\cos\theta + \mu_s\sin\theta}$$

Force moving the object leftward in the direction of the plane

$$= mg\sin\theta$$



The frictional force is:

$$f_s = \mu_s F_N = \mu_s(F\sin\theta + mg\cos\theta)$$

Force moving the object rightward in the direction of the plane

$$= F\cos\theta + \mu_s(F\sin\theta + mg\cos\theta)$$
$$= F(\cos\theta + \mu_s\sin\theta) + \mu_s mg\cos\theta$$

For the equilibrium, the two forces must be equal:

$$mg\sin\theta = F(\cos\theta + \mu_s\sin\theta) + \mu_s mg\cos\theta$$

$$F = \frac{mg(\sin\theta - \mu_s\cos\theta)}{\cos\theta + \mu_s\sin\theta}$$

F. Comparing Opposite Forces

Example 1.87

Consider a block kept on an inclined plane (inclined at 45°). If the force required to just push it up the incline is 2 times the force required to just prevent it from sliding down, the coefficient of friction between the block and the inclined plane (μ) is equal to: (JEE Main, Jan 25, 2023, Shift-II)

Component of force due to gravity which slides the object on the ramp

$$= mg\sin\theta = mg\left(\frac{1}{\sqrt{2}}\right)$$

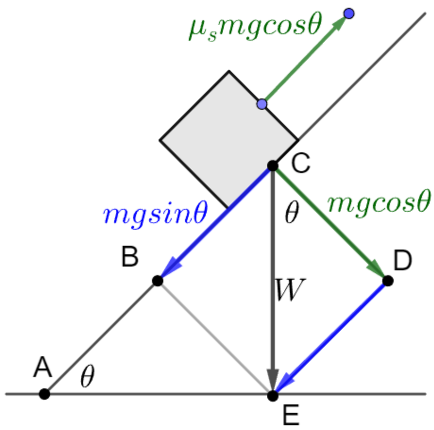
Maximum value of frictional force

$$= \mu_s mg\cos\theta = \mu_s mg\left(\frac{1}{\sqrt{2}}\right)$$

Case I: Applying force to prevent the object from sliding down

In equilibrium, total rightward force must equal total leftward force:

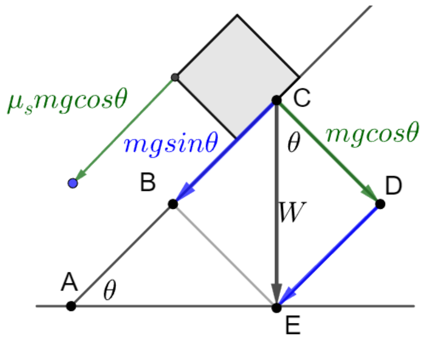
$$F + \underbrace{\mu_s mg\left(\frac{1}{\sqrt{2}}\right)}_{\substack{Frictional\\ Force-Rightward}} = \underbrace{mg\left(\frac{1}{\sqrt{2}}\right)}_{Gravity}$$



Solving for $2F$:

$$\underbrace{2F = \left(\frac{2}{\sqrt{2}}\right)mg(1-\mu_s)}_{Equation\ I}$$

Case II: Applying just enough force to push the object up



The force required to just push the object up

$$= 2 \times F = 2F$$

The object is being moved rightward. Frictional force applies leftward.

In equilibrium, total rightward force must equal total leftward force:

$$\underbrace{2F = mg\left(\frac{1}{\sqrt{2}}\right) + \mu_s mg\left(\frac{1}{\sqrt{2}}\right) = \left(\frac{1}{\sqrt{2}}\right) mg(1 + \mu_s)}_{Equation\ II}$$

From Equations I and II:

$$\left(\frac{2}{\sqrt{2}}\right) mg(1 - \mu_s) = \left(\frac{1}{\sqrt{2}}\right) mg(1 + \mu_s)$$

Solving for μ_s:

$$2(1 - \mu_s) = 1 + \mu_s \Rightarrow \mu_s = \frac{1}{3}$$

Pending NLM-1

Example 1.88

104: A block kept on a rough inclined plane remains at rest up to a maximum force $2N$ down the inclined plane (parallel to the plane). The maximum external force up the inclined plane that does not move the block is $10N$. The coefficient of static friction between the block and the plane is $\left(Given\ g = 10\frac{m}{s^2}\right)$ **(JEE Main, 12 Jan, 2019, Shift-II; JEE Main, 9 Jan, 2019, Shift-I)**

2. Newton's 2nd Law

2.1 *NLM* – 2: Net Force; Tables

A. Newton's Second Law

2.1: Newton's Second Law

The net force on an object is the product of its mass and its acceleration.

$$\sum \vec{F} = \vec{F}_{Net} = m\vec{a}$$

- $\vec{F}$ is a vector quantity obtained by calculating the vector sum of the forces acting on the body.
- This is called principle of superposition of forces.
- m here is a constant. It represents the mass of the object under consideration.[6]

Example 2.2

If an object falls towards the Earth due to gravity, the Earth should also be attracted to the object. Why then, do we neglect it. To see why this is so, calculate the ratio of acceleration of a ball with respect to the acceleration of the Earth if the ball falls from the first floor of a building. Use 6×10^{24} kg for the mass of the Earth, and 150 g for the mass of the ball.

$$m_1 a_1 = m_2 a_2$$
$$(1.5 \times 10^{-1})a_1 = (6 \times 10\text{^}24)a\_2$$
$$\frac{a_2}{a_1} = 1.5 \times \frac{10^{-1}}{6 \times 10^{24}} = \frac{10^{-25}}{4} = 0.25 \times 10^{-25} = 2.5 \times 10^{-26}$$

B. Calculating Time

Example 2.3

A force of 6 N acts on a body at rest and of mass 1 kg. During this time, the body attains a velocity of 30 m/s. The time for which the force acts on the body is **(NEET 1997)**

$$\vec{a} = \frac{\vec{F}}{m} = \frac{6N}{1} = 6\frac{m}{s^2}$$
$$v = u + at = 0 + 6(t)$$
$$t = \frac{v}{6} = \frac{30}{6} = 5s$$

C. Calculating Force

Example 2.4: Formula for Force

Explain what is wrong with the "derivation" below, and correct it:

$$\vec{F} = m\vec{a} \Rightarrow m = \frac{\vec{F}}{\vec{a}}$$

Vector division is not defined.

---

[6] We will consider variable mass later when we consider momentum.

The correct method is to take the magnitude both sides:

$$|\vec{\boldsymbol{F}}| = m|\vec{\boldsymbol{a}}|$$

Solve for m:

$$m = \frac{|\vec{\boldsymbol{F}}|}{|\vec{\boldsymbol{a}}|} = \frac{F}{a}$$

Example 2.5

A. A $10N$ force is applied on a body producing in it an acceleration of $1\frac{m}{s^2}$. The mass of the body is **(NEET 1996)**

B. A body, under the action of a force $\vec{\boldsymbol{F}} = 6\hat{\boldsymbol{\imath}} - 8\hat{\boldsymbol{\jmath}} + 10\hat{\boldsymbol{k}}$, acquires an acceleration of $1\frac{m}{s^2}$. The mass of this body must be **(NEET 1996, 2009)**

Part A

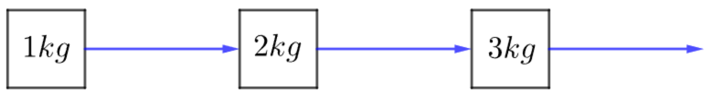
$$m = \frac{F}{a} = \frac{10N}{1} = 10\ kg$$

Part B

$$m = \frac{F}{m} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{1} = \sqrt{200} = 10\sqrt{2}\ kg$$

Example 2.6

Three objects of mass $1\ kg$, $2kg$ and $3\ kg$ as shown in the diagram are connected by ropes and pulled rightwards with an acceleration of $1\frac{m}{s^2}$ over a smooth surface. Determine the



A. tension in the rope at the point of connection of each object.

B. Force with which each object is pulled

Move from left to right:

$$Mass\ 1kg: T = F = ma = (1)(1) = 1N$$
$$Mass\ 2kg: T = F = ma = (1+2)(1) = 3N$$
$$Mass\ 3kg: T = F = ma = (1+2+3)(1) = 6N$$

Example 2.7

Press F11 to exit full screen

I'm not sure if this warrants a completely new thread, but here goes:

Three dynamics carts have force sensors placed on top of them. Each force sensor is tied to a string that connects all three carts together (**Figure 10**). You use a sixth force sensor to pull the three dynamics carts forward. The reading on force sensor 2 is 3.3 N. Assume that the force sensors are light and that there is negligible friction acting on the carts. T/I

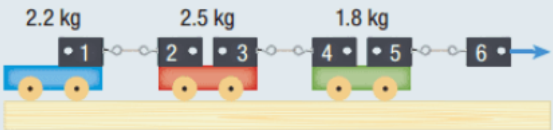


Figure 10

(a) What is the acceleration of all the carts?
(b) What is the reading on each force sensor?
(c) What force are you applying to force sensor 6?

I got all the answers correct for a and b.

I determined (a) from knowing that if force sensor two is having a "force" of tension pulling it back 3.3N, that is the force required to pull the very first cart (blue), so I solved for acceleration using FNET = ma, and got 1.5m/s/s.

Then, knowing acceleration of all the carts (they must accelerate at the same rate because they are all connected), and, knowing the masses of the carts, I solved for the forces on the other sensors. Also knowing that force sensors that are connected must be having the force of tension (S1/S2: 3.3N, S3/S4: 7.05N, S5/S6: 9.78N).

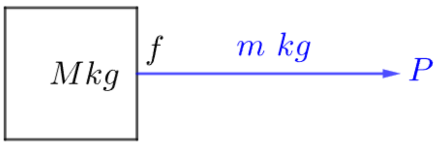
For c, I thought it would be 9.75N, considering that's the force detected on sensor 6, but it's not. It's ~20N, which is the sum of all forces required to pull the carts. Why is it the sum of the forces rather than the force detected by sensor 6?

Reply

Example 2.8

A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m. If a force P is applied at the free end of the rope, the force exerted by the rope on the block in terms of m and P is: **(JEE Main 2003)**

Method I



For the block and the rope:

$$a = \frac{P}{M+m}$$

For the block:

$$a = \frac{f}{m}$$

But the acceleration of both is the same since they are a connected system:

$$\frac{P}{M+m} = \frac{f}{m} \Rightarrow f = \frac{PM}{M+m}$$

Shortcut

The force gets divided in the ratio of the masses:

$$P \cdot \frac{Mass_{Block}}{Mass_{System}} = P \cdot \frac{M}{M+m}$$

D. Kinematics

2.9: Constant Force

Unless otherwise told, assume that constant force is applied in a straight line with zero friction. Hence,

$$Distance = Displacement$$

$$v = Velocity = Speed = \frac{Distance}{Time} = \frac{D}{T}$$

Example 2.10

A. A body of mass 5 kg under the action of constant force $\vec{\boldsymbol{F}} = F_x\hat{\boldsymbol{\imath}} + F_y\hat{\boldsymbol{\jmath}}$ has velocity $t = 0$ as $\vec{\boldsymbol{v}} = 6\hat{\boldsymbol{\imath}} - 2\hat{\boldsymbol{\jmath}}\frac{m}{s}$ and at $t = 10$ as $\vec{\boldsymbol{v}} = 6\hat{\boldsymbol{\jmath}}\frac{m}{s}$. The force $\vec{\boldsymbol{F}}$ is: (JEE Main, April 11, 2014)

B. A force acts for 20s on a body of mass 20 kg, starting from rest, after which the force ceases, and then the body describes 50m in the next 10s. The value of force will be: (JEE Main, Jan 29, 2023-II)

Part A

Rearrange $v = u + at$ to solve for acceleration in the time interval (0,10) when force was applied:

$$a = \frac{v-u}{t} = \frac{(0,6)-(6,-2)}{10} = \frac{(-6,8)}{10} = \frac{(-3,4)}{5}\frac{m}{s^2}$$

$$F = ma = 5\left[\frac{(-3,4)}{5}\right] = (-3,4)N$$

Part B

Since there is no friction, the velocity in the time interval (20,30) after the force stops is constant:

$$v = \frac{D}{T} = \frac{50}{10} = 5\frac{m}{s}$$

Rearrange $v = u + at$ to solve for acceleration in the time interval (0,20) when force was applied:

$$a = \frac{v-u}{t} = \frac{5-0}{t} = \frac{5-0}{20} = \frac{1}{4}\frac{m}{s^2}$$

The force is then:

$$F = ma = (20)\left(\frac{1}{4}\right) = 5N$$

Example 2.11: Position

A. A boy pushes a box of mass 2kg with a force $F = (20\hat{\imath} + 10\hat{\jmath})N$ on a frictionless surface. If the box was initially at rest, then ______m is the displacement along the x axis after 10s. (JEE Main, Feb 26, 2021-I)

B. A force $F = (40\hat{\imath} + 10\hat{\jmath})N$ acts upon a body of mass 5 kg. If the body starts from rest, its position vector at time $t = 10s$, will be: (JEE Main, July 25, 2021-II)

Part A

The displacement in the x direction is:

$$s_x = u_x t + \frac{1}{2}a_x t^2 = 0(t) + \frac{1}{2}\frac{F_x}{m}t^2 = \frac{1}{2}\cdot\frac{20}{2}\cdot 10^2 = 500\,m$$

Part B

$$\vec{\boldsymbol{a}} = \frac{\vec{\boldsymbol{F}}}{m} = \frac{(40,10)}{5} = (8,2)\frac{m}{s}$$

$$\vec{s} = \vec{u} + \frac{1}{2}\vec{a}t^2 = (0,0) + \frac{1}{2}(8,2)(100) = (400,100)\ m$$

E. Overhanging Masses: Acceleration

2.12: System connected by inextensible cords

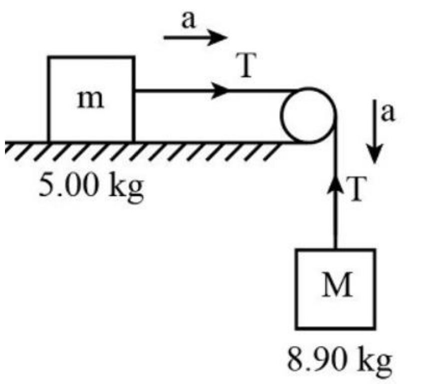
When a system is connected by inextensible cords, the entire system moves together or not at all.

Example 2.13

Choose all correct options

An object of mass m overhangs a table via a cord attached to an object of mass M. Ignore friction. Assume the cord is massless. Then:

A. The object of mass m will move.
B. The object of mass M will move.
C. The object of mass m will have different acceleration from the object of mass M.
D. The cord will be slack.
E. The cord will be taut, but with zero tension.
F. The cord will move



Consider the object of mass M. The downward force acting on it

$$= W = Mg$$

This downward force is transmitted via the cord to the object with mass m, which experiences rightward force.

$$A, B\ and\ F$$

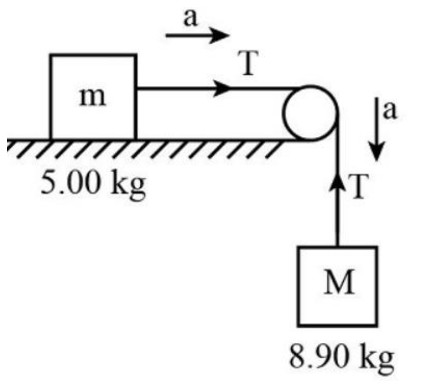
Option C is not correct since the system is connected via an inextensible cord, and hence both masses will move at same acceleration.

Example 2.14

Calculate acceleration for each part (separately)

Assume the cords are massless and inextensible. Ignore friction. An object of mass m overhangs a table via a cord attached to an object of mass

A. $2m$.
B. pm.
C. pm, which is attached to an object of mass qm, which is attached to an object of mass rm, in that order.
D. $(p + q + r)m$



Follow-up Question: Explain why the answers to Part C and D are the same. What shortcut does this give you?

Part A

The force of gravity on the object with mass $2m$ can be ignored since it is counteracted by the normal force on the object.

We only need to take into account the force of gravity on the object hanging over the table, which has mass m:

$$a = \frac{Force}{Mass} = \frac{mg}{m + 2m} = \frac{g}{3}$$

Part B

$$a = \frac{Force}{Mass} = \frac{mg}{m + pm} = \frac{g}{p + 1}$$

Part C

$$a = \frac{Force}{Mass} = \frac{mg}{m + pm + qm + rm} = \frac{g}{p + q + r + 1}$$

Part D

$$a = \frac{Force}{Mass} = \frac{mg}{m + (p + q + r)m} = \frac{g}{p + q + r + 1}$$

Follow-Up

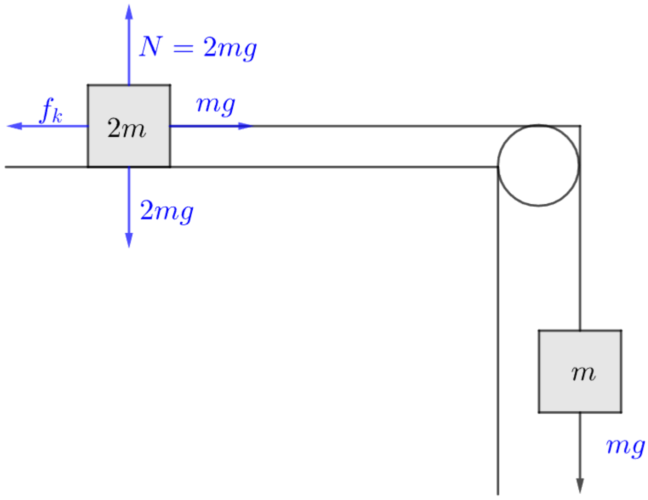
The cord is inextensible. Hence, the system will accelerate at the same rate.

Example 2.15

Calculate the acceleration of the system of masses in each part below. Assume the cords are massless and inextensible. The coefficient of kinetic friction is μ_k

A. An object of mass m overhangs a table via a cord attached to an object of mass $2m$.
B. An object of mass m overhangs a table via a cord attached to an object of mass pm.
C. An object of mass m overhangs a table via a cord attached to an object of mass pm, which is attached to an object of mass qm, which is attached to an object of mass rm, in that order.
D. Explain why in Part C, you can "convert" the three objects with masses pm, qm and rm into a single object with mass $(p + q + r)m$

Part A



As before, the force of gravity on the object with mass $2m$ can be ignored.

The forces acting on the system:

$$Weight\ of\ object\ with\ mass\ m = mg$$
$$Kinetic\ Friction = f_k = u_k F_N = u_k(2mg)$$

Net force on the object with mass $2m$ is

$$mg - u_k(2mg)$$

Finally, the acceleration for the system:

$$= a = \frac{Force}{Mass} = \frac{mg - u_k(2mg)}{m + 2m} = \frac{g - u_k(2g)}{3}$$
$$= \frac{g(1 - 2\mu_k)}{3}$$

Part B

$$= a = \frac{Force}{Mass} = \frac{mg - u_k(pmg)}{m + pm} = \frac{g - u_k(pg)}{p + 1} = \frac{g(1 - p\mu_k)}{p + 1}$$

Part C

$$= a = \frac{Force}{Mass} = \frac{mg - u_k(pmg) - u_k(qmg) - u_k(rmg)}{m + pm + qm + rm} = \frac{g[(1 - (p + q + r)\mu_k)]}{p + q + r + 1}$$

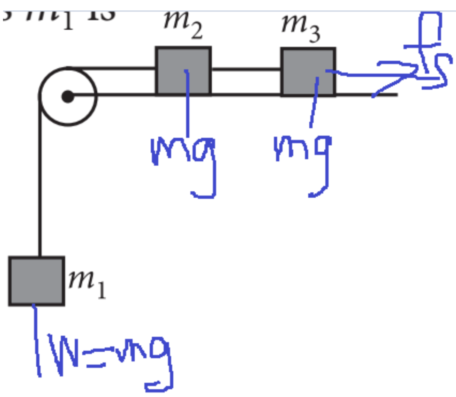
Part D

Because of the simplifying assumptions.

Example 2.16

A system consists of three masses m_1, m_2 and m_3 connected by a string passing over a pulley P. The mass m_1

hangs freely and m_2 and m_3 are on a rough horizontal table (the coefficient of friction $= \mu$). A pulley is attached at the corner of the table. The pulley is frictionless and of negligible mass. If $m_1 = m_2 = m_3 = m$, the downward acceleration of mass m_1 is: **(NEET 2014)**



The mass of the system

$$= M = m_1 + m_2 + m_3 = m + m + m = 3m$$

Force accelerating the system is weight of the overhanging mass

$$= W = mg$$

Frictional force resisting the movement is:

$$f_k = \mu F_N = \mu(2mg)$$

The net force on the system is:

$$F = W - f_k = mg - \mu(2mg)$$

The acceleration of the system is:

$$a = \frac{F}{M} = \frac{mg - \mu(2mg)}{3m} = \frac{g(1 - 2\mu)}{3}$$

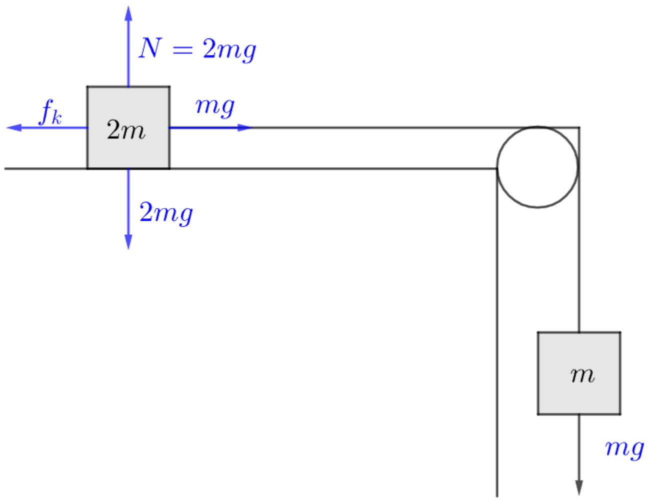
F. Overhanging Masses: Tension

Example 2.17

An object of mass m overhangs a table via a cord attached to an object of mass $2m$. Ignore friction. Assume the cord is massless. Calculate the tension in the cord by

A. first calculating the acceleration of the system
B. setting up a system of equations

Part A



$$a = \frac{Force}{mass} = \frac{mg}{m + 2m} = \frac{mg}{3m} = \frac{g}{3}$$

The net force on the object which is overhanging is:

$$\underbrace{F_{Net} = ma}_{Equation\ I}$$

The net force is also

$$\underbrace{F_{Net} = mg - T}_{Equation\ II}$$

$$ma = mg - T$$

$$T = m(g - a) = m\left(g - \frac{g}{3}\right) = \frac{2}{3}mg$$

Part B

For the overhanging weight, the net force is:

$$\underbrace{ma = mg - T}_{} \Rightarrow \underbrace{2ma = 2mg - 2T}_{Equation\ I}$$

For the weight on the table, the net force is:

$$\underbrace{2ma = T}_{Equation\ II}$$

From Equations I and II:

$$2mg - 2T = T \Rightarrow T = \frac{2}{3}mg$$

Example 2.18

An object of mass m overhangs a table via a massless cord attached to an object of mass M. Define the harmonic mean(H) of m and M to be the reciprocal of the arithmetic mean of m and M. Ignore friction. Calculate the tension in the cord:

A. in terms of H.
B. if M is n times m

Part A

$$a = \frac{Force}{Mass} = \frac{mg}{m+M}$$
$$T = Ma = M\left(\frac{mg}{m+M}\right) = \left(\frac{Mm}{m+M}\right)g = \frac{1}{2}\left(\frac{2Mm}{m+M}\right)g = \frac{H}{2}g$$

Part B

$$T = \left(\frac{Mm}{m+M}\right)g = \left(\frac{(nm)m}{m+nm}\right)g = \left(\frac{nm^2}{(n+1)m}\right)g = \left(\frac{n}{n+1}\right)mg$$

Example 2.19

A block A of mass m_1 rests on a horizontal table. A light string connected to it passes over a frictionless pulley at the edge of table and from its other end another block B of mass m_2 is suspended. The coefficient of kinetic friction between the block and the table is μ_k. When the block A is sliding on the table, the tension in the string is **(NEET 2015)**

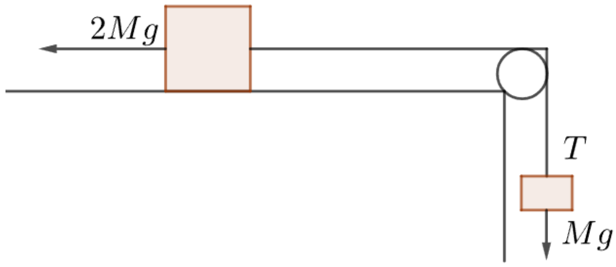
$$m_2 a = m_2 g - T$$
$$m_1 a = T - \mu_k m_1 g$$

$$m_1 m_2 a = m_1 m_2 g - m_1 T$$
$$m_2 m_1 a = m_2 T - \mu_k m_1 m_2 g$$

$$m_1 m_2 g - m_1 T = m_2 T - \mu_k m_1 m_2 g$$
$$m_1 m_2 g + \mu_k m_1 m_2 g = m_2 T + m_1 T$$
$$m_1 m_2 g(1+\mu_k) = T(m_1 + m_2)$$
$$T = \frac{m_1 m_2 g(1+\mu_k)}{m_1 + m_2}$$

Example 2.20

An object of mass $4M\ kg$ lies on an ice block. It is pulled to the left by a force of $2Mg$. It is connected to a mass of $M\ kg$ by using a string-pulley arrangement such that the mass of $M\ kg$ is suspended to the right. In this situation, the tension in the string is $\frac{x}{5}Mg$ for $x =$ _____. Neglect mass of the string and friction of the larger mass with the ice block. (where g =acceleration due to gravity). **(JEE Main, June 28, 2022-I)**



For the suspended object:

$$Net\ Force = T - Mg$$
$$Ma = T - Mg$$
$$T = Ma + Mg$$

Since

$$a = \frac{F_{Net}}{Mass} = \frac{2Mg - Mg}{4M + M} = \frac{Mg}{5M} = \frac{g}{5} \Rightarrow Ma = \frac{Mg}{5}$$

Hence:

$$T = Ma + Mg = \frac{Mg}{5} + Mg = \left(\frac{6}{5}\right)Mg \Rightarrow x = 6$$

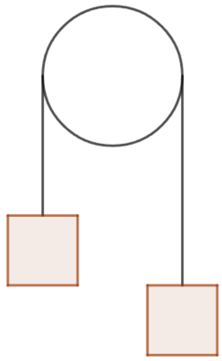
2.2 Kinetic Friction

2.21: Tension in Moving System of Blocks with Kinetic Friction

The tension in the cord in the system shown when the blocks are moving is:

$$T = \frac{m_1 m_2 g(1 + \mu_k)}{m_1 + m_2}$$

Assumptions:
- Cord is massless
- Pulley is frictionless



A. Kinetic Friction

2.22: Kinetic Friction

The magnitude of kinetic friction is:

$$f_k = u_k F_N$$

Note:
1. μ_k is coefficient of kinetic friction
2. $f_k = u_k F_N$ has the same formula as static friction ($f_s = u_s F_N$), except that μ_k is replaced by μ_s
3. Coefficient of kinetic friction is always less than coefficient of static friction: $\mu_k < \mu_s$

Example 2.23

A block of mass 10 kg placed on rough horizontal surface having coefficient of friction $\mu = 0.5$. If a horizontal force of $100N$ is acting on it, then acceleration of the block will be **(NEET 2002)**

$$f_s = \mu_k F_N = \mu_k mg = (0.5)(10)(10) = 50N$$
$$a = \frac{F}{m} = \frac{100N - 50N}{10\ kg} = \frac{50N}{10kg} = 5\frac{m}{s^2}$$

Example 2.24

A block B is pushed momentarily along a horizontal surface with an initial velocity V. If μ is the coefficient of sliding friction between B and the surface, block B will come to rest after a time: **(NEET 2007)**

$$a = \frac{F}{m} = -\frac{f_k}{m} = -\frac{\mu F_N}{m} = -\frac{\mu mg}{m} = -\mu g$$

Substitute $v = 0, u = V, a = -\mu g$ in $v = u + at$:

$$0 = V - \mu gt \Rightarrow t = \frac{V}{\mu g}$$

Example 2.25

A body of mass 10 kg is moving with an initial speed of $20\frac{m}{s}$. The body stops after $5s$ due to friction between the body and the floor. The value of the coefficient of friction is: $\left(g = 10\frac{m}{s^2}\right)$. **(JEE Main, Jan 31, 2023-II)**

$$v = u + at \Rightarrow a = \frac{v-u}{t} = \frac{0-20}{5} = -4\frac{m}{s^2} \Rightarrow |a| = 4$$

$$F = ma \Rightarrow a = \frac{F}{m} = \frac{\mu_k mg}{m} = \mu_k g$$

$$\mu_k g = |a| = 4 \Rightarrow \mu_k = \frac{4}{g} = 0.4$$

2.26: Minimum Stopping Distance

Given a car travelling at a speed of u $units$, and a coefficient of kinetic friction between the car and the road of μ_k, the minimum stopping distance for the car is

$$s = \frac{u^2}{2\mu g}$$

Let

$$m = mass\ of\ the\ car$$

The acceleration that the car will achieve is:

$$a = -\frac{F}{m} = -\frac{f_k}{m} = -\frac{\mu_k F_N}{m} = -\frac{\mu mg}{m} = -\mu g, \qquad a < 0$$

Solve the equation of motion below for s:

$$v^2 = u^2 + 2as \Rightarrow s = \frac{v^2 - u^2}{2a}$$

Substitute $a = -\mu g, Final\ Velocity = v^2 = 0$ in the above equation of motion:

$$s = \frac{0 - u^2}{2a} = \frac{-u^2}{2(-\mu g)} = \frac{u^2}{2\mu g}$$

Example 2.27

A. A block of mass $10kg$ starts sliding on a surface with an initial velocity of $9.8\frac{m}{s^2}$. The coefficient of friction between the surface and the block is 0.5. The distance covered by the block before coming to rest is: (Given $g = 9.8\frac{m}{s^2}$) **(JEE Main, June 24, 2022-I)**

B. Consider a car moving along a straight horizontal road with a speed of $72\frac{km}{h}$. If the coefficient of friction between the tires and the road is 0.5, the shortest distance, in meters, which the car can be stopped is: **(NEET 1992)**

Part A

$$s = \frac{u^2}{2\mu g} = \frac{g^2}{2\left(\frac{1}{2}\right)g} = g = 9.8\ m$$

Part B

Substitute $u = 72\frac{km}{h} = 72 \cdot \frac{5}{18}\frac{m}{s} = 20\frac{m}{s}$ in the formula for minimum stopping distance:

$$s = \frac{u^2}{2\mu g} = \frac{20^2}{2\left(\frac{1}{2}\right)(10)} = 40\ m$$

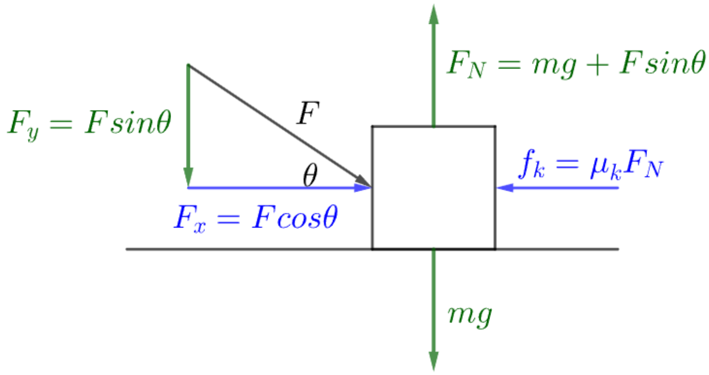
B. Pushes versus Pulls

2.28: Acceleration from a Push

An object with mass m lies on a plane surface with coefficient of friction μ_k. It is pushed with a force F that makes an angle θ to the horizontal.

The acceleration of the object is:

$$a = \frac{F\cos\theta - \mu_k(mg + F\sin\theta)}{m}$$



Using Newton's Second Law:

$$a = \frac{F_{Net}}{m}$$

The net force is the force in the horizontal direction (F_x) counteracted by the frictional force $f_k = \mu_k F_N$:

$$= \frac{F_x - \mu_k F_N}{m}$$

The normal force comprises gravity (mg) and the vertical component of the force (F_y) applied to the object:

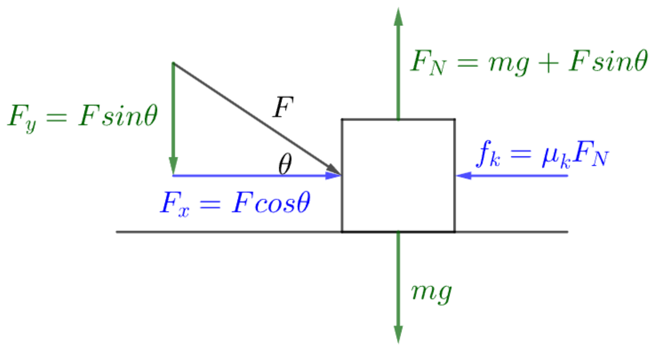
$$= \frac{F_x - \mu_k(mg + F_y)}{m}$$

Resolving F into its vertical and horizontal components gives $F_x = F\cos\theta, F_y = F\sin\theta$

$$= \frac{F\cos\theta - \mu_k(mg + F\sin\theta)}{m}$$

Example 2.29

An object moving on a plane surface with velocity $v, v \in \mathbb{R}$ is pushed by a constant force $F, F \in \mathbb{R}^+$ at an angle θ to the horizontal, $0° < \theta < 90°$. The angle θ is increased. Four items are mentioned below. For each item, determine whether it will increase, decrease, remain unchanged, or cannot be determined:



1. F_x
2. F_y
3. Acceleration
4. Velocity

Items I and II: F_x and F_y

Increasing the angle will decrease the magnitude of the horizontal component, and increase the magnitude of the vertical component. Hence:

$$F_x: Decrease$$
$$F_y: Increase$$

Item III

$F_x \downarrow$ and $F_y \uparrow$. Hence:

$$a = \frac{F_x - \mu_k(mg + F_y)}{m} \quad decreases$$

Item IV

If the acceleration and magnitude are

- in opposite direction, the velocity will decrease.
- in the same direction, the velocity will increase

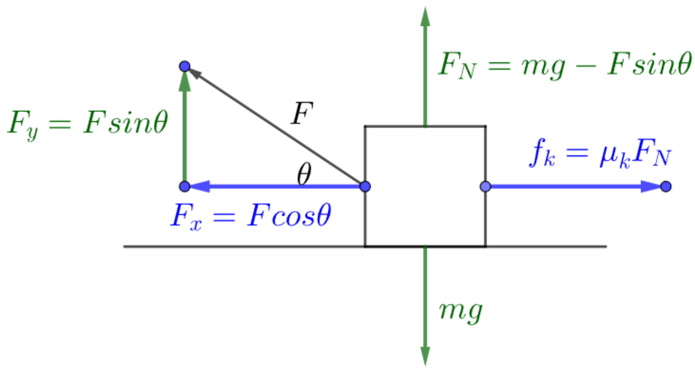
$$Cannot\ be\ determined$$

2.30: Acceleration from a Pull

An object with mass m lies on a plane surface with coefficient of friction μ_k. It is pulled with a force F that makes an angle θ to the horizontal.

The acceleration of the object is:

$$a = \frac{F\cos\theta - \mu_k(mg - F\sin\theta)}{m}$$



The analysis proceeds as in the case of a push, except that the vertical component of the pull is directed upwards (instead of downwards) for a pull.

Hence, the sign of the vertical component changes (and all else remains the same):

Example 2.31

A push and pull on an object with mass m have the same force, and are oriented at the same angle θ. Will the magnitude of acceleration be greater for the push or the pull?

$$a_{Pull} = \frac{F\cos\theta - \mu_k(mg - F\sin\theta)}{m}$$
$$a_{Push} = \frac{F\cos\theta - \mu_k(mg + F\sin\theta)}{m}$$

The two expressions are the same except for the term due to kinetic friction.
The kinetic friction term itself differs only due to the normal force.

The expression for pull has

$$\underbrace{mg + F\sin\theta}_{\substack{Normal\ Force \\ for\ Push}} > \underbrace{mg - F\sin\theta}_{\substack{Normal\ Force \\ for\ pull}}$$

Since the normal force is greater for the push, the kinetic friction is also greater for the push.

Hence, the acceleration is greater for the pull.
Because pull decreases the normal force while push increases the normal force.

2.32: Difference in Acceleration

An object lies on a plane surface. The coefficient of friction is μ_k.
Case I: The object is pushed by a force F that make an angle θ with the horizontal.
Case II: The object is pulled by a force F that make an angle θ with the horizontal.

The difference in acceleration between the two cases is:

$$\frac{2\mu_k F\sin\theta}{m}$$

As before the difference only comes from the $F\sin\theta$ term. Hence:

$$a_{Pull} - a_{Push}$$
$$= \frac{F\cos\theta - \mu_k mg + \mu_k F\sin\theta}{m} - \frac{F\cos\theta - \mu_k mg - \mu_k F\sin\theta}{m}$$

Except for $\mu_k F\sin\theta$ everything cancels, and we are left with:

$$\frac{2\mu_k F\sin\theta}{m}$$

Example 2.33

A block of mass 5 kg is (i)pushed in case (A), and (ii) pulled in case (B), by a force $F = 20N$, making an angle 30° with the horizontal. The coefficient of friction between the block and the floor is $\mu = 0.2$. The difference between the accelerations of the block in case (B) and case (A) will be: **(JEE Main, 12 April 2019, Shift II)**

The difference is:

$$\frac{2\mu_k F\sin\theta}{m} = \frac{2(0.2)(20)\left(\frac{1}{2}\right)}{5} = \frac{4}{5} = 0.8\frac{m}{s^2}$$

C. Overhanging Weights

Example 2.34

A block of mass 40 kg slides over a surface when a mass of 4 kg is suspended through an inextensible massless string passing over a frictionless pulley. The coefficient of kinetic friction between the surface and the block is 0.02. The acceleration of the block is: (Given $g = 10\frac{m}{s^2}$) **(JEE Main, June 29, 2022-II)**

Note: Standard diagram of a mass overhanging a table is applicable.

$$a = \frac{F_{Net}}{Mass} = \frac{4g - (0.02)(40)g}{44} = \frac{3.2g}{44} = \frac{32}{44} = \frac{8}{11}\frac{m}{s^2}$$

D. Air Resistance

2.35: Air Resistance

- Air resistance is a specific case of kinetic friction. It is caused by friction of a moving object with air.
- Air resistance can be modelled in a number of different ways.

Example 2.36: Air Resistance

An object of mass 5 kg is thrown vertically upwards from the ground. The air resistance produces a constant retarding force of $10N$ throughout the motion. The ratio of time of ascent to time to descent will be equal to: $\left(Use\ g = 10\frac{m}{s^2}\right)$**(JEE Main, June 24, 2022-II)**

The acceleration due to air resistance

$$a = \frac{F}{m} = \frac{10}{5} = 2\frac{m}{s^2}$$

Displacement when travelling upwards

When the object is moving upwards, the acceleration is:

$$a = -g - 2 = -10 - 2 = -12\frac{m}{s^2}$$

Consider the point where the object reaches its maximum height. At maximum height, $final\ velocity = v = 0$

$$v = u + at_A \Rightarrow u = v - at_A = -at_A = -(-12)t_A = 12t_A\frac{m}{s^2}$$

$$S = ut_A + \frac{1}{2}at_A^2 = (12t_A)t_A + \frac{1}{2}(-12)t_A^2 = 12t_A^2 - 6t_A^2 = 6t_A^2$$

$$t_A^2 = \frac{S}{6}$$

Displacement when travelling downwards

When the object is moving upwards, the acceleration is:

$$a = -g + 2 = -10 + 2 = -8\frac{m}{s^2}$$

Consider the interval from where the object reached its maximum height to the point where it hit the ground.
$Initial\ velocity = u = 0$

$$S = ut_D + \frac{1}{2}at_D^2 = (0)t_D + \frac{1}{2}(-8)t_D^2 = -4t_D^2$$

$$t_D^2 = \frac{S}{-4}$$

Since we want the time ratio, we consider the displacement to be positive in both directions:

$$t_A^2 : t_D^2 = \frac{S}{6} : \frac{S}{4} = 4:6 = 2:3$$

Take the square root:

$$t_A : t_D = \sqrt{2} : \sqrt{3}$$

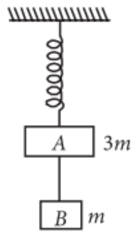
2.3 Suspended Objects; Pulleys; Elevators

A. Suspended Objects

Example 2.37

Two blocks A and B of masses $3m$ and m respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in the figure. The magnitudes of acceleration of A and B immediately after the string is cut, are respectively (write your answer in terms of g)
(NEET 2017)

65. Two blocks A and B of masses $3m$ and m respectively are connected by a massless and inextensible string. The whole system is suspended by a massless spring as shown in figure. The magnitudes of acceleration of A and B immediately after the string is cut, are respectively



Object B

Object B will fall due to gravity. This is the free fall that we encountered in the chapter on Kinematics. Hence, magnitude of acceleration is:

$$g$$

Object A

Object A just before cutting has:

$$\underbrace{4mg}_{\text{Upward Force}} = \underbrace{4mg}_{\text{Downward Force}}$$

After cutting, the net force is:

$$4mg - 3mg = mg$$

Substitute $Mass\ of\, a = 3m$ in Newton's Second Law to find acceleration:

$$a = \frac{F}{3m} = \frac{mg}{3m} = \frac{g}{3}$$

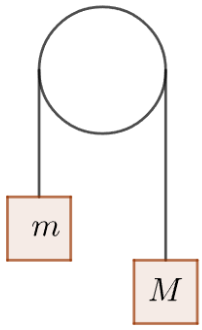
$$(a_A, a_B) = \left(\frac{g}{3}, g\right)$$

B. Atwood Machine

2.38: Atwood Machine

An Atwood machine consists of two objects of masses m and M connected by an inextensible, massless string going over an ideal, massless pulley.

$$M > m$$



2.39: Acceleration in an Atwood Machine

$$a = \frac{(M-m)g}{M+m}$$

Consider the two masses as a system. The net force on the system is

$$Mg - mg = (M-m)g$$

The mass of the system is:

$$M + m$$

The acceleration of the system

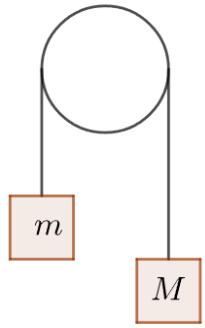
$$a = \frac{F_{Net}}{Mass} = \frac{(M-m)g}{M+m}$$

Example 2.40

A. Two bodies of mass 4 kg and 6 kg are tied to the ends of a massless string. The string passes over a pulley which is frictionless. The acceleration of the system in terms of acceleration due to gravity (g) is: **(NEET 2020)**

B. Two objects of masses of $5m$ and $10m$ are suspended from a massless pulley (one on each side). The acceleration of the system, in terms of g, when the masses are left free is: **(NEET 2000, Adapted)**

C. Two masses $m_1 = 5\ kg$ and $m_2 = 4.8\ kg$ tied to a string are hanging over a light frictionless pulley, with one mass on the left of the pulley, and the other on the right of the pulley. What is the acceleration of the masses when left free to move. (Take $g = 9.8\frac{m}{s^2}$) **(JEE Main 2002S, 2004)**



Part A

$$a = \frac{F_{Net}}{m} = \frac{6g-4g}{6+4} = \frac{2g}{10} = \frac{g}{5}\frac{m}{s^2}$$

Part B

$$a = \frac{F_{Net}}{m} = \frac{10mg-5mg}{15m} = \frac{5mg}{15m} = \frac{g}{3}$$

Part C

$$a = \frac{F_{Net}}{Mass} = \frac{5g-4.8g}{5+4.8} = \frac{0.2(9.8)}{9.8} = 0.2\frac{m}{s}$$

C. Tension in String

Example 2.41

Spot the mistake

Question: Two objects of masses m and M, with $M > m$ are attached to a cord going over a pulley, and the system is released. Ignore friction and mass of cord and pulley. Calculate the tension in the cord in terms of m, M and g

Solution: Set up the following set of equations:

$$Mass\ M\colon \underbrace{Ma = Mg - T}_{\textbf{Equation I}}, \qquad Mass\ m\colon \underbrace{ma = mg - T}_{\textbf{Equation II}}$$

Add equations I and II and solve for T:

$$ma + Ma = Mg + mg - 2T$$
$$a(m+M) = g(M+m) - 2T$$
$$2T = g(M+m) - a(m+M)$$
$$T = \frac{g(M+m) - a(m+M)}{2}$$

The equations refer to object with mass M and m respectively.
The magnitude of acceleration is the same for both objects, but the direction is different. This has to be taken into account.

$$Mass\ M\colon \underbrace{Ma = Mg - T}_{\textbf{Equation I}}$$
$$Mass\ m\colon \underbrace{m(-a) = mg - T}_{\textbf{Equation II}}$$

And this is now correct since the acceleration in the second equation has opposite sign to that in the first.

Example 2.42

Spot the mistake

Question: Two objects of masses m and M, with $M > m$ are attached to a cord going over a pulley, and the system is released. Ignore friction and mass of cord and pulley. Calculate the tension in the cord in terms of m, M and g

Solution: Set up the following set of equations:

$$Mass\ M:\ \underbrace{Ma = Mg - T}_{Equation\ I},\qquad Mass\ m:\ \underbrace{ma = \mathrm{T} - mg}_{Equation\ II}$$

Since the forces are equal, we must have:

$$Ma = ma$$
$$Mg - T = \mathrm{T} - mg$$
$$2T = g(m + M)$$

Acceleration is equal:

$$a = a$$

This means that forces are not equal:

$$Ma \neq ma$$

Example 2.43

Two objects of masses m and M, with $M > m$ are attached to a cord going over a pulley, and the system is released. Ignore friction and mass of cord and pulley. Calculate the

A. Tension in the cord in terms of m, M and g
B. Acceleration of the system if $m = 40N, M = 60N, g = 10\frac{m}{s^2}$.

$$Ma = Mg - T \Rightarrow a = \frac{Mg - T}{M}$$
$$ma = \mathrm{T} - mg \Rightarrow a = \frac{T - mg}{m}$$

$$\frac{Mg - T}{M} = \frac{T - mg}{m}$$
$$mMg - MT = mT - mMg$$
$$2mMg = mT + MT$$
$$T = \frac{2mMg}{m + M}$$

D. Elevator

2.44: Tension in String

When an object is being

- pulled against gravity, the tension in the string will be more than the force due to gravity.
- let down against gravity, the tension in the string will be less than the force due to gravity

Example 2.45

A person of mass 60 kg is inside a lift of mass 940 kg and presses the button on control panel. The lift starts moving upward with an acceleration $1.0\frac{m}{s^2}$. If $g = 10\frac{m}{s^2}$, the tension in the supporting cable is: **(NEET 2011; NEET 2002)**

Let the force in the upward direction be
$$F = xm, m = 940 + 60 = 1000\ kg = mass\ of\ lift + person$$

The force in the downward direction
$$= mg = 10m$$

The net force is
$$F_{Net} = ma \Rightarrow m(x - 10) = ma \Rightarrow x = 11$$

Then, the force pulling the lift up (which is also the tension in the cable)
$$= F = xm = 11(1000) = 11{,}000N$$

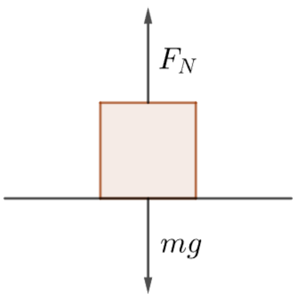
Example 2.46

The mass of a lift is 2000 kg. When the tension in the supporting cable is $28{,}000N$, then its acceleration (give magnitude and direction) is: **(NEET 2011)**

$$F_{Net} = 28000N - 10m = 14m - 10m = 4m$$
$$a = \frac{F_{Net}}{m} = \frac{4m}{m} = 4\frac{m}{s^2}\ upward$$

2.47: Elevator Movement

$$F_N > mg \Rightarrow Upward\ Acceleration$$
$$mg > F_N \Rightarrow Downward\ Acceleration$$
$$mg = F_N \Rightarrow Zero\ Acceleration$$



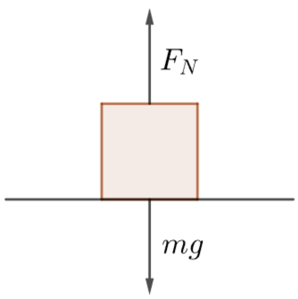
(Important) Example 2.48

Mark all correct options

A person is standing in an elevator. In which situation does he experience weight loss. When the elevator moves

A. upward with constant acceleration
B. downward with constant acceleration
C. downward with increasing acceleration
D. downward with uniform velocity **(JEE Main, June 26, 2022-I, Adapted)**

If the normal force is equal to the gravitational force then
$$F_{Net} = F_N - mg = ma = 0$$
Hence,
$$Option\ D\ is\ incorrect.$$



The person will experience weight gain, if the normal force is greater than the gravitational force
$$F_{Net} = F_N - mg = ma > 0 \Rightarrow Upward\ Acceleration$$
$$Option\ A\ is\ incorrect$$

The person will experience weight loss, if the normal force is less than the gravitational force

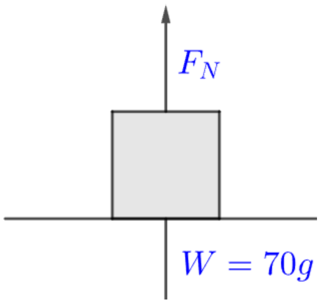
$$F_{Net} = F_N - mg = ma < 0 \Rightarrow Downward\ Acceleration$$

$$Options\ B, C$$

E. Elevator-Like System

Example 2.49

A block of 10 kg rests on a horizontal floor. When three iron cylinders are placed on it, the block and the cylinders go down with an acceleration $0.2\frac{m}{s^2}$. The normal reaction R_2 by the floor if mass of the iron cylinders are equal and of 20 kg each is _____ N. (Take $g = 10\frac{m}{s^2}$) **(JEE Main, July 20, 2021, Shift-I)**



The mass of the system is:

$$m = 10 + 3(20) = 70\ kg$$

From the diagram, the net force is:

$$mg - F_N = ma$$

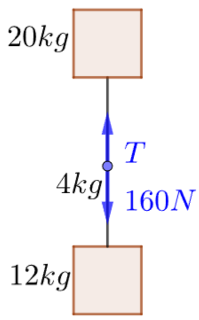
Solving for F_N:

$$F_N = mg - ma = m(g - a) = m(10 - 0.2) = 70(9.8) = 686N$$

Example 2.50

Two blocks of mass $M_1 = 20\ kg$ and $M_2 = 12\ kg$ are connected by a metal rod of mass 8 kg. The mass of 12 kg is exactly below the mass of 20 kg. The system is pulled vertically up by applying a force of 480N. Calculate the:

A. acceleration of the system.
B. tension at the midpoint of the rod **(JEE Main, April 22, 2013)**
C. tension where the rod connects with the mass of 20 kg.
D. tension where the rod connects with the mass of 12 kg.



Part A

$$a = \frac{F}{m} = \frac{480 - 400}{40} = \frac{80}{40} = 2\frac{m}{s^2}$$

Part B

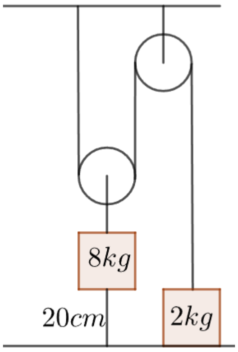
$$T - mg = ma \Rightarrow T = mg + ma = m(g + a)$$

Substitute $g = 10\frac{m}{s^2}, a = 2\frac{m}{s^2}$

$$T = 16(10 + 2) = 16(12) = 192N$$

F. Mechanical Advantage

2.51: Acceleration under Constant Force



Under constant force/acceleration:

$$s = \frac{1}{2}at^2 \Rightarrow a = \frac{2s}{t^2}$$

Let the displacement of the left pulley in time t be s_L. The acceleration in the left pulley is:

$$a_L = \frac{2s}{t^2} = \frac{2s_L}{t^2}$$

The right pulley moves twice the distance that the left pulley does in time t. Hence, $s_R = 2s_L$:

$$a_R = \frac{2s}{t^2} = \frac{2s_R}{t^2} = \frac{2(2s_L)}{t^2} = \frac{4s_L}{t^2} = 2\left(\frac{2s_L}{t^2}\right) = 2a_L$$

Example 2.52

Two boxes of masses $2kg$ and $8kg$ are connected by a massless string passing over smooth pulleys. Calculate the time taken by box of mass $8kg$ to strike the ground starting rest. $\left(g = 10\frac{m}{s^2}\right)$.

a

2.4 Ramps-I

A. Frictionless Ramps

2.53: Components

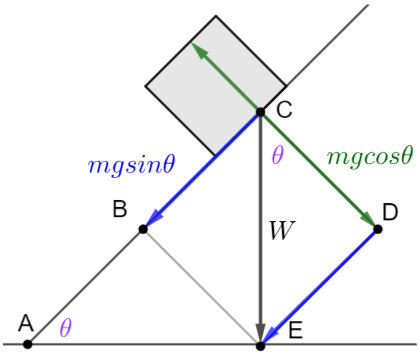
$$\boldsymbol{F}_{Net} = m\vec{\boldsymbol{a}}$$

$$F_x = ma_x, \qquad F_y = ma_y, \qquad F_z = ma_z$$

$$F_x, F_y, F_z = Force\ in\ x, y\ and\ z\ directions$$
$$a_x, a_y, a_z = acceleration\ in\ x, y\ and\ z\ directions$$
$$F_y = Force\ in\ y\ direction$$
$$F_z = Force\ in\ z\ direction$$

- Two vectors are equal if and only if their individual components are equal.
- The component form is useful in solving questions.

2.54: Forces & Acceleration on a Frictionless Ramp



In the absence of friction, a ramp makes an angle θ with the horizontal. Then, the force due to gravity:

$$In\ the\ direction\ of\ the\ ramp = mg\sin\theta$$
$$Normal\ to\ the\ ramp = mg\cos\theta$$

Also, the acceleration due to gravity:

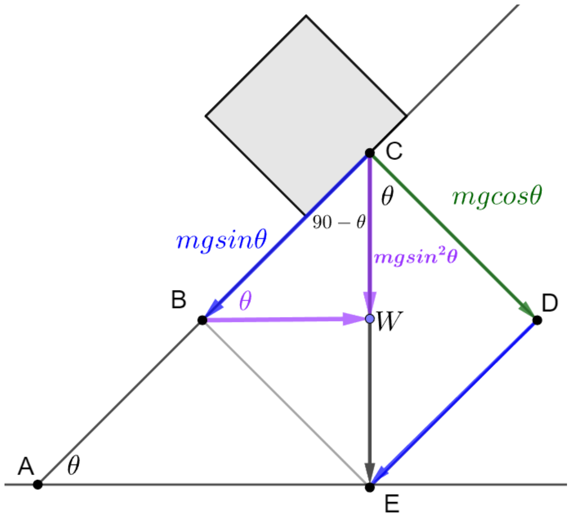
$$In\ the\ direction\ of\ the\ ramp = g\sin\theta$$
$$Normal\ to\ the\ ramp = g\cos\theta$$

$$a_R = \frac{F}{m} = \frac{mg\sin\theta}{m} = g\sin\theta$$
$$a_N = \frac{F}{m} = \frac{mg\cos\theta}{m} = g\cos\theta$$

2.55: Downward acceleration for an object on a ramp

Ignoring friction, the downward acceleration for an object on a ramp that makes an angle θ with the horizontal is:

$$g\sin^2\theta$$



Consider gravity, which will have a downward force of:

$$mg$$

The force in the direction of the ramp is:

$$mg\sin\theta$$

If the force downward is x, then

$$\sin\theta = \frac{x}{mg\sin\theta} \Rightarrow x = mg\sin^2\theta$$

And the acceleration in the downward direction is:

$$a = \frac{F}{m} = \frac{mg\sin^2\theta}{m} = g\sin^2\theta$$

Example 2.56

Mark all correct options

A solid sphere, a hollow sphere, a cuboid, and a ring are released from top of an inclined plane (frictionless) so that they slide down the plane. Then, maximum acceleration down the plane is for (no rolling):

A. Solid Sphere
B. Hollow sphere
C. Cuboid
D. Ring (JEE Main 2002, Adapted)

The acceleration is independent of the mass and the shape.

$$Option\ A, B, C, D$$

Example 2.57

Two fixed frictionless inclined planes make an angle 30° and 60° with the vertical. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B? (JEE Main 2010)

Since we want relative acceleration, we need to need the difference between the two values of acceleration:

$$a_{30°} - a_{60°}$$

Using the formula $a = g\sin^2\theta$:

$$g\sin^2 30 - g\sin^2 60 = g\left(\frac{1}{4} - \frac{3}{4}\right) = g\left(-\frac{2}{4}\right) = -\frac{g}{2}$$

B. Kinematics with Smooth Ramps

Example 2.58

A block A takes $2s$ to slide down a frictionless incline of $30°$ and length l, kept inside a lift with uniform velocity v. If the incline is changed to $45°$, the time taken by the block to slide down the incline will be $a^{\frac{b}{c}}$ where a, b, c are integers and $HCF(b, c) = 1$. Find $a + b + c$. **(JEE Main, July 27, 2022-II, Adapted)**

In the equation of motion $s = ut + \frac{1}{2}at^2$, we know that

$$Initial\ Velocity = u = 0$$

Further, the displacement along the length of the ramp is the same in both the cases:

$$s = \frac{1}{2}a_{30°}t^2 = \frac{1}{2}a_{45°}T^2$$
$$a_{30°}t^2 = a_{45°}T^2$$

Substitute $a_{30°} = g\sin\theta = \frac{g}{2}, a_{45°} = g\sin\theta = \frac{g}{\sqrt{2}}, t = 2$:

$$\left(\frac{g}{2}\right)(2^2) = \frac{g}{\sqrt{2}}T^2$$
$$2\sqrt{2} = T^2$$
$$T = \sqrt{2\sqrt{2}} = \left(2 \cdot 2^{\frac{1}{2}}\right)^{\frac{1}{2}} = \left(2^{\frac{3}{2}}\right)^{\frac{1}{2}} = 2^{\frac{3}{2}\times\frac{1}{2}} = 2^{\frac{3}{4}}$$

$$a + b + c = 2 + 3 + 4 = 9$$

Example 2.59: Ratio of Time

A small block slides on a smooth inclined plane, starting from rest at time $t = 0$. Let S_n be the distance travelled by the block in the interval $t = n - 1$ to $t = n$. Then, the ratio $\frac{S_n}{S_{n+1}}$, in terms of n, is **(NEET 2021)**

S_n is the distance travelled in the n^{th} second, which is given by

$$Distance\ in\ n\ seconds - Distance\ in\ 1\ to\ n-1\ seconds$$

Hence, we get:

$$S_n = \underbrace{\frac{1}{2}an^2}_{\substack{Distance\\ in\ n\ seconds}} - \underbrace{\frac{1}{2}a(n-1)^2}_{\substack{Distance\ in\\ 1\ to\ n-1\ seconds}} = \frac{1}{2}a(2n-1)$$

Substituting $n + 1$ in the above formula for S_n gives:

$$S_{n+1} = \frac{1}{2}a[2(n+1) - 1] = \frac{1}{2}a[2n+1]$$

The required ratio is:

$$\frac{S_n}{S_{n+1}} = \frac{\frac{1}{2}a(2n-1)}{\frac{1}{2}a[2n+1]} = \frac{2n-1}{2n+1}$$

C. Kinetic Friction

2.60: Force due to Kinetic Friction on a Ramp

The kinetic force due to friction for an object of mass m on a ramp that makes angle θ with the ground

$$f_k = u_k F_N = \mu_k mg\cos\theta$$

Where

$$\mu_k = Coefficient\ of\ kinetic\ friction$$

The force due to friction is the product of coefficient of friction and normal force:

$$f_k = u_k F_N$$

Substitute normal force for an object on a ramp $= F_N mg\cos\theta$

$$f_k = \mu_k mg\cos\theta$$

2.61: Solving for Coefficient of Kinetic Friction

If an object slides with acceleration a down a ramp inclined with angle θ to the horizontal, the coefficient of kinetic friction is:

$$\mu_k = \frac{\sin\theta - \frac{a}{g}}{\cos\theta}$$

The force that moves the moves the object on the ramp is the component of gravity parallel to the ramp:

$$mg\sin\theta$$

The force that holds the object back is the force of friction:

$$\mu_k mg\cos\theta$$

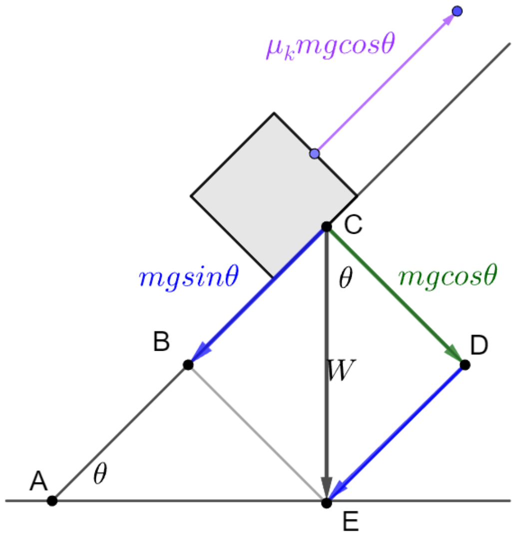
The net acceleration is determined by the net force:

$$ma = mg\sin\theta - \mu_k mg\cos\theta$$

Divide both sides by mg:

$$\frac{a}{g} = \sin\theta - \mu_k\cos\theta$$

$$\mu_k = \frac{\sin\theta - \frac{a}{g}}{\cos\theta}$$



Example 2.62

A block of mass m slides down the plane inclined at angle 30° with an acceleration $\frac{g}{4}$. The value of coefficient of kinetic friction will be: **(JEE Main, Jan 29, 2023-I)**

$$\mu_k = \frac{\sin\theta - \frac{a}{g}}{\cos\theta} = \frac{\frac{1}{2} - \frac{\frac{g}{4}}{g}}{\frac{\sqrt{3}}{2}} = \left(\frac{1}{2} - \frac{1}{4}\right) \div \frac{\sqrt{3}}{2} = \left(\frac{1}{4}\right) \times \frac{2}{\sqrt{3}} = \frac{1}{2\sqrt{3}} = \frac{\sqrt{3}}{6}$$

Example 2.63

A plank with a box on it at one end is gradually raised about the other end. As the angle of inclination with the horizontal reaches 30°, the box starts to slip and slides 4.0 m down the plank in 4.0 s. The coefficient of kinetic friction between the box and the plank (written to one significant digit) will be **(NEET 2015, Adapted)**

Find the acceleration. Substitute $u = 0, t = 4, s = 4$ in

$$s = ut + \frac{1}{2}at^2 \Rightarrow 4 = \frac{1}{2}a(4^2) \Rightarrow a = \frac{1}{2}$$

Substitute $a = \frac{1}{2}$ in the formula for coefficient of kinetic friction:

$$\mu_k = \frac{\sin\theta - \frac{a}{g}}{\cos\theta} = \frac{0.5 - \frac{0.5}{10}}{\frac{\sqrt{3}}{2}} = 0.45 \times \frac{2}{\sqrt{3}} = \frac{9}{10} \times \frac{\sqrt{3}}{3} = \frac{3\sqrt{3}}{10} \approx \frac{3 \times 1.71}{10} \approx 0.5$$

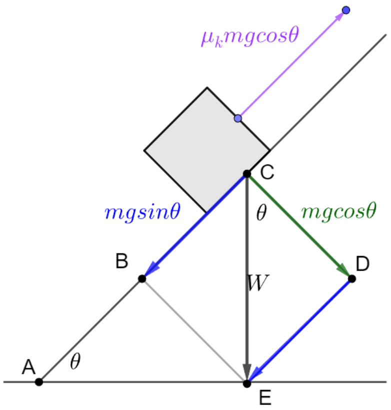
Example 2.64

A block has been placed on an inclined plane with the slope angle θ. The block slides down the plane at constant speed. The coefficient of kinetic friction is equal to: **(NEET 1993)**

$$Acceleration = a = 0$$
$$Net\ Force = 0$$

Refer diagram on the right.

$$\underbrace{mg\sin\theta}_{Leftward\ Force} = \underbrace{\mu_k mg\cos\theta}_{Rightward\ Force}$$



Solve the above for μ_k:

$$\mu_k = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

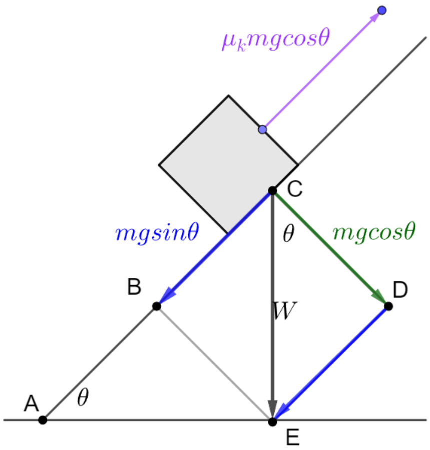
D. Kinematics: Calculating coefficient of Kinetic Friction

2.65: Coefficient of Kinetic Friction

An object slides down a ramp, inclined at angle θ, in time t if the ramp is smooth, and time nt if the ramp is rough. The coefficient of kinetic friction is:

$$\mu_k = \tan\theta\left(\frac{n^2 - 1}{n^2}\right)$$

$$Smooth\ plane: Time = t, \quad Acceleration = g\sin\theta$$
$$With\ friction: Time = nt, \quad Acceleration = g\sin\theta - \mu_k g\cos\theta$$



$$s = \frac{1}{2}at^2$$

But the distance travelled is the same in both cases. Hence, we must have:

$$\frac{1}{2}(g\sin\theta)t^2 = \frac{1}{2}(g\sin\theta - \mu_k g\cos\theta)(nt)^2$$

Divide both sides by $\frac{1}{2}gt^2$:

$$\sin\theta = n^2\sin\theta - n^2\,\mu_k\cos\theta$$
$$n^2\,\mu_k\cos\theta = n^2\sin\theta - \sin\theta$$

Divide both sides by $n^2\cos\theta$:

$$\mu_k = \left(\frac{\sin\theta}{\cos\theta}\right)\left(\frac{n^2 - 1}{n^2}\right) = \tan\theta\left(\frac{n^2 - 1}{n^2}\right)$$

Example 2.66

A. Starting from rest, a body slides down a 45° inclined plane in twice the time it takes to slide down the same distance in the absence of friction. The coefficient of friction between the body and the inclined plane is: **(NEET 1988)**

B. The time taken by an object to slide down a 45° rough inclined plane is n times what it takes to slide down a perfectly smooth 45° incline plane. The coefficient of kinetic friction between the object and the plane is: **(JEE Main, Jan 29, 2023-II)**

E. When a body slides down from rest along a smooth plane making an angle of 30° with the horizontal, it takes time T. When the same body slides down from rest along a rough inclined plane making the same angle and through the same distance, it takes time αT, where α is a constant greater than 1. The coefficient of friction between the body and the rough plane is $\frac{1}{\sqrt{x}}\left(\frac{\alpha^2-1}{\alpha^2}\right)$ where $x =$ _____ **(JEE Main Sep 1, 2021-II; JEE Main April 15, 2018)**

Part A

$$\mu_k = \tan\theta\left(\frac{n^2-1}{n^2}\right) = (1)\left(\frac{2^2-1}{2^2}\right) = \frac{3}{4}$$

Part B

Substituting $\tan\theta = 1$:

$$\mu_k = \tan\theta\left(\frac{n^2-1}{n^2}\right) = \frac{n^2-1}{n^2}$$

Part C

Substituting $\tan\theta = \tan 30° = \frac{1}{\sqrt{3}}, n = \alpha$

$$\mu_k = \tan\theta\left(\frac{n^2-1}{n^2}\right) = \frac{1}{\sqrt{3}}\left(\frac{\alpha^2-1}{\alpha^2}\right) \Rightarrow x = 3$$

E. Smooth and Rough Parts

Example 2.67

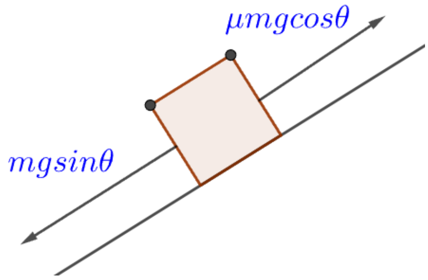
The upper half of an inclined plane of inclination θ is perfectly smooth while the lower half is rough. A block starting from rest at the top of the plane will again come to rest at the bottom, if the coefficient of friction between the block and the lower half of the plane is given by: **(NEET 2013; JEE Main 2005)**

Newton's Second Law

The acceleration of the block will be:

$$Smooth\ Part: a = \frac{F}{m} = \frac{mg\sin\theta}{m} = g\sin\theta$$

$$Rough\ Part: a = \frac{F}{m} = \frac{mg\sin\theta - \mu mg\cos\theta}{m} = g\sin\theta - \mu g\cos\theta$$



Equations of Motion

To apply equations of motion, we need constant acceleration. Hence, we consider movement in smooth and rough parts separately.

Smooth Part: Substitute $u = 0, a = g\sin\theta$ in $v^2 = u^2 + 2as$:

$$v^2 = u^2 + 2as = 0 + 2(g\sin\theta)s$$

Rough Part: The final velocity for the smooth part is the initial velocity for the rough part.
Substitute $v^2 = 0, u^2 = 2(g\sin\theta)s, a = g\sin\theta - \mu g\cos\theta$ in $v^2 = u^2 + 2as$:

$$0 = 2(g\sin\theta)s + 2(g\sin\theta - \mu g\cos\theta)s$$

Divide both sides by $2gs$:

$$0 = \sin\theta + \sin\theta - \mu\cos\theta$$

Solve for μ:

$$\mu = \frac{2\sin\theta}{\cos\theta} = 2\tan\theta$$

Example 2.68

In the above example, let the time travelled on the smooth part be t, and the acceleration on the smooth part be a_s.

A. Determine the acceleration in the rough part in terms of A.
B. Determine the time travelled in the rough part in terms of t.

Part A

$$v^2 = u^2 + 2as \Rightarrow A = a_s = \frac{v^2 - u^2}{2s}$$

If the speed at the transition point from smooth to rough is V

$$Smooth\ Part: \frac{V^2}{2s} = \frac{V^2}{2s}$$
$$Rough\ Part: a_r = \frac{0 - V^2}{2s} = -\frac{V}{2s} = -a_s$$

Part B

Then:

$$v = u + at \Rightarrow t = \frac{v - u}{a}$$

$$t_s = \frac{v - u}{a_s} = \frac{V - 0}{a_s} = \frac{V}{a_s}$$
$$t_r = \frac{v - u}{a} = \frac{0 - V}{-a_s} = \frac{-V}{-a_s} = \frac{V}{a_s}$$

Example 2.69

Use your answer from Part A to solve the question using the equation of motion $v = u + at$

$$Smooth\ part: v = u + at = 0 + (g\sin\theta)t$$
$$Rough\ Part: 0 = (g\sin\theta)t + (g\sin\theta - \mu g\cos\theta)t$$

$$0 = \sin\theta + \sin\theta - \mu\cos\theta$$

Solve for μ:

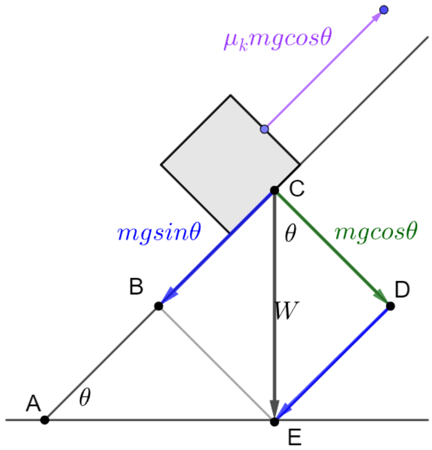
$$\mu = \frac{2\sin\theta}{\cos\theta} = 2\tan\theta$$

F. Acceleration

2.70: Acceleration of Descent

If a body initially at rest descends a ramp, the acceleration is given by:

$$a = g\sin\theta - \mu_k g\cos\theta$$



Leftward force due to gravity is

$$mg\sin\theta$$

Rightward force due to friction is:

$$f_k = \mu_k F_N = \mu_k mg\cos\theta$$

The acceleration is

$$a = \frac{F_{Net}}{m} = \frac{mg\sin\theta - \mu_k mg\cos\theta}{m} = g\sin\theta - \mu_k g\cos\theta$$

Example 2.71

$$a_d = g\sin\theta - \mu_k g\cos\theta$$

The above expression gives the acceleration of a body descending a ramp with no external forces (other than gravity and friction). Explain why the expression has a opposite signs for the two terms.

$g\sin\theta$ is the component of gravity in the direction of the ramp (and is downward).

$\mu_k g\cos\theta$ is the component of gravity that is the normal force, multiplied by the coefficient of friction. Since the movement of the body is downward, the frictional force resists the movement and has opposite sign.

Example 2.72

A body starts from rest on a long, inclined plane of slope 45°. The coefficient of friction between the body and the plane varies as $\mu = 0.3x$, where x is the distance travelled down the plane. The body will have maximum speed when $x =$ _____ (Given $g = 10\frac{m}{s^2}$) (JEE Main, April 22, 2013)

The downward acceleration is:

$$a_d = g\sin\theta - \mu_k g\cos\theta$$

Substitute $\sin\theta = \cos\theta = \frac{\sqrt{2}}{2}, \mu_k = 0.3x, g = 10$:

$$= (10)\left(\frac{\sqrt{2}}{2}\right) - (0.3x)(10)\left(\frac{\sqrt{2}}{2}\right) = 5\sqrt{2} - 1.5\sqrt{2}x$$

Velocity will increase so long as acceleration is positive.
Hence, velocity will be maximum when acceleration is zero:

$$5\sqrt{2} - 1.5\sqrt{2}x = 0$$
$$5\sqrt{2} = 1.5\sqrt{2}x$$
$$x = \frac{5}{1.5} = \frac{10}{3}\ m$$

2.73: Acceleration of Ascent

A body is launched up a ramp with initial velocity v. The acceleration is given by:

$$a = g\sin\theta + \mu_k g\cos\theta$$

Leftward force due to gravity is

$$mg\sin\theta$$

Left force due to friction is:

$$f_k = \mu_k F_N = \mu_k mg\cos\theta$$

The acceleration is

$$a = \frac{F_{Net}}{m} = \frac{mg\sin\theta + \mu_k mg\cos\theta}{m} = g\sin\theta + \mu_k g\cos\theta$$

Example 2.74

The acceleration of a body ascending a ramp with no external forces (other than gravity and friction) is given by $g\sin\theta - \mu_k g\cos\theta$. Explain why the expression has same signs for the two terms.

$g\sin\theta$ is the component of gravity in the direction of the ramp (and is downward).

$\mu_k g\cos\theta$ is the component of gravity that is the normal force, multiplied by the coefficient of friction. Since the movement of the body is upward the frictional force resists the movement and has same sign as gravity.

Example 2.75

A body of mass 2 kg slides down with an acceleration of $3\frac{m}{s^2}$ on a rough inclined plane having a slope of 30°. The external force required to take the same body up the plane with the same acceleration will be: (Given $g = 10\frac{m}{s^2}$)
(JEE Main April 15, 2018)

When sliding down:

$$F_{Net} = ma$$
$$mg\sin\theta - f_k = ma$$

Substitute $mg\sin\theta = (2)(10)\left(\frac{1}{2}\right) = 10N, ma = (2)(3) = 6N$

$$10 - f_k = 6$$
$$f_k = 4N$$

When going up:

$$F_{Net} = ma$$
$$F - mg\sin\theta - f_k = ma$$

$$F - 10 - 4 = 6$$
$$F = 20N$$

G. Time of Ascent and Descent

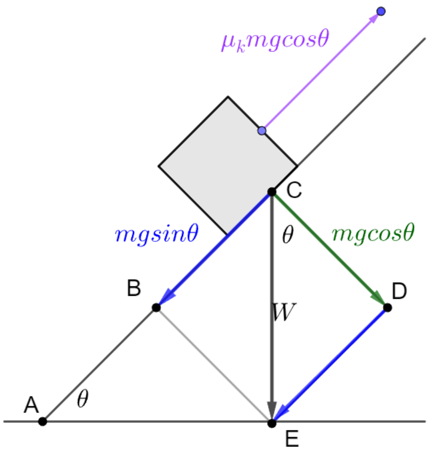
2.76: Time of Descent

If a body initially at rest descends a ramp, the time taken to reach a position on the ramp is:

$$t_d = \sqrt{\frac{2s}{a_d}}$$

Where

$$a_d = acceleration = g\sin\theta - \mu_k g\cos\theta$$
$$s = length\ of\ ramp\ travelled$$
$$\theta = angle\ of\ inclination\ of\ the\ ramp$$



Recall the equation of motion:

$$s = ut + \frac{1}{2}a_d t^2$$

Substitute $Initial\ Velocity = u = 0$ in the above:

$$s = \frac{1}{2}a_d t^2$$

Solve for t:

$$t = \sqrt{\frac{2s}{a_d}}$$

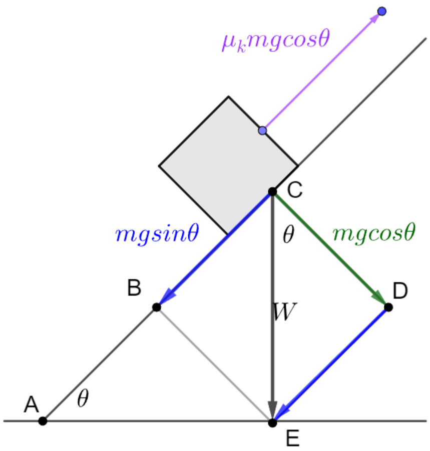
2.77: Time of Ascent

If a body is launched up a ramp with initial velocity u, the time taken to reach a position up the ramp is:

$$t_a = \sqrt{\frac{2s}{a_a}}$$

Where:

$$a_a = acceleration = g\sin\theta + \mu_k g\cos\theta$$
$$s = length\ of\ ramp\ travelled$$
$$\theta = angle\ of\ inclination\ of\ the\ ramp$$



Recall the equation of motion:

$$s = vt - \frac{1}{2}at^2$$

Substitute $final\ velocity = v = 0, acceleration = (-a_a)$ in

$$s = -\frac{1}{2}(-a_a)t^2 = \frac{1}{2}a_a t^2$$

Solve for t:

$$t = \sqrt{\frac{2s}{a_a}}$$

Example 2.78

$$t = \sqrt{\frac{2s}{a_a}}$$

The formula for time of ascent up a ramp, given above, does not include the initial velocity u. Does this mean that the time taken is the same irrespective of the initial velocity?

No, because

$$v^2 = u^2 + 2as$$
$$0 = u^2 - 2as$$
$$2as = u^2$$

Hence, as

$$u \text{ increases}, s \text{ increases}$$

Hence the time taken is not the same.

Example 2.79

Mark all correct options

An object is launched up a ramp. It reaches a point, and then slides back down the ramp that makes an angle $\theta, 0 < \theta < \frac{\pi}{2}$. The time of ascent will be equal to the time of descent when the coefficient of friction is:

A. 1
B. 0
C. $\sin\theta$
D. $\cos\theta$
E. $\tan\theta$

$$t_a = \sqrt{\frac{2s}{a_a}}, \qquad t_d = \sqrt{\frac{2s}{a_d}}$$

$$a_a = a_d$$

$$g\sin\theta + \mu_k g\cos\theta = g\sin\theta - \mu_k g\cos\theta$$
$$2\mu_k g\cos\theta = 0$$
$$\mu_k g\cos\theta = 0$$

Since $g \neq 0, \cos\theta \neq 0$:

$$\mu_k = 0 \Rightarrow \text{Option } B$$

2.80: Coefficient of Friction by Comparison of ascent/descent

An object is launched up a ramp that makes an angle θ with the ground.
Case I: The time of descent is n times the time of ascent
Case II: The return velocity is $\frac{1}{n}$ times the original velocity

In both cases, the coefficient of friction is:

$$\mu_k = \left(\frac{n^2 - 1}{n^2 + 1}\right)\tan\theta$$

Case I: Time of ascent versus time of descent

The time of descent is n times the time of ascent:

$$nt_a = t_d$$

Substitute the formula for time of ascent and time of descent:

$$n\sqrt{\frac{2s}{a_a}} = \sqrt{\frac{2s}{a_d}}$$

Square both sides, cancel $2s$ on both sides and rearrange:

$$\frac{n^2}{a_a} = \frac{1}{a_d}$$

Eliminate fractions:

$$\underbrace{a_a = n^2 a_d}_{\textit{Equation I}}$$

Case II: Upward velocity versus return velocity

$$v^2 = u^2 + 2as$$

When going up, $initial\ velocity = v_0, final\ velocity = 0, acceleration = a_a$. Substitute into the equation of motion and solve for displacement:

$$v_0^2 = 0^2 + 2a_a s \Rightarrow s = \frac{v_0^2}{2a_a}$$

When going down, $final\ velocity = \frac{v_0}{n}, initial\ velocity = 0, acceleration = a_d$. Substitute into the equation of motion and again solve for displacement:

$$0 = \left(\frac{v_0}{n}\right)^2 + 2a_d s \Rightarrow s = -\frac{v_0^2}{2n^2 a_d}$$

The signs of the displacements are opposite since they are in different directions. But their magnitude will be the same:

$$\frac{v_0^2}{2a_a} = \frac{v_0^2}{2n^2 a_d}$$

Solve for the relation between the two accelerations:

$$\underbrace{a_a = n^2 a_d}_{\textit{Equation I}}$$

Final Result

Both the cases result in the same equation. Substitute the values of acceleration to get:

$$g\sin\theta + \mu_k g\cos\theta = n^2 g\sin\theta - n^2\mu_k g\cos\theta$$

Collate terms:

$$(n^2+1)\mu_k g\cos\theta = (n^2-1)g\sin\theta$$

Solve for μ_k:

$$\mu_k = \left(\frac{n^2-1}{n^2+1}\right)\tan\theta$$

Example 2.81

A. A body of mass m is launched up on a rough inclined plane making an angle of 30° with the horizontal. The coefficient of friction between the body and the plane is $\frac{\sqrt{x}}{5}$ if the time of ascent is half the time of descent. The value of x is: **(JEE Main, July 20, 2021-II)**
B. A block starts moving up an inclined plane of inclination 30° with an initial velocity of v_0. It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient of kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$. The nearest integer to I is ____ **(JEE Main, Sep 03, 2020-II)**

Part A

$$\mu_k = \left(\frac{n^2-1}{n^2+1}\right)\tan\theta = \frac{3}{5}\cdot\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{5}$$

Part B

$$\mu_k = \left(\frac{n^2-1}{n^2+1}\right)\tan\theta = \frac{3}{5}\cdot\frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{5} = \frac{2\sqrt{3}}{10} = \frac{200\sqrt{3}}{1000} \approx \frac{346}{1000} \Rightarrow I = 346$$

2.82: Ratio of Time of Ascent/Descent

$$t_a < t_d$$

Method I

$$a_a > a_d$$
$$\frac{1}{a_a} < \frac{1}{a_d}$$
$$\frac{2s}{a_a} < \frac{2s}{a_d}$$

$$\sqrt{\frac{2s}{a_a}} < \sqrt{\frac{2s}{a_d}}$$
$$t_a < t_d$$

Method II

We wish to compare time of ascent with time of descent.
The direction of acceleration of ascent is opposite to direction of acceleration of descent.

Consider a though experiment where:
- The object moves up
- Reaches point P
- Moves down

At the point where the object reaches P, run the time dimension backwards.

Example 2.83

A body is launched up a ramp making an angle of 30° with the horizontal. The time of descent is half the time of ascent.

A. Calculate the value of the coefficient of friction
B. Does the "answer" make sense. Explain.

$$\mu_k = \left(\frac{n^2-1}{n^2+1}\right)\tan\theta = \left(\frac{\left(\frac{1}{2}\right)^2-1}{\left(\frac{1}{2}\right)^2+1}\right)\cdot\frac{1}{\sqrt{3}} = \left(\frac{-\frac{3}{4}}{\frac{5}{4}}\right)\cdot\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{5}$$

The coefficient of friction is negative, which is not meaningful.
This happened because the time of descent is less than the time of ascent (which will *never* happen).

2.5 Relative Motion; Pseudoforces

A. Law of Inertia and Pseudoforces

2.84: Newton's First Law
Every object will remain at rest or in uniform motion in a straight line unless compelled to change by an external force.

2.85: Pseudoforce
- A pseudoforce is not a real force. Rather, it is a convenience introduced to let us model real-life scenarios.

Example 2.86
A car accelerates from 0 to 100 $\frac{km}{hr}$ over a minute. Describe what the driver in the car feels with respect to the acceleration in terms of:

A. Relative Motion
B. Pseudoforces

Part A
The car is accelerating. But the driver is not.
Hence, in the frame of reference of the car, the driver experiences negative acceleration.

Part B
The driver experiences the feeling of being pushed back in the car.
In reality, there is no force pushing him back. Rather, the car is moving forward.

Hence, we call the "force" experienced by the driver a pseudoforce.

Example 2.87
A car accelerates at $x\frac{m}{s^2}$. Consider the frame of reference of the driver. Determine the acceleration for the driver.

$$Acceleration_{Car} = x\frac{m}{s^2}$$
$$Acceleration_{Driver} = -x\frac{m}{s^2}$$

Example 2.88
An object lies on a frictionless, horizontal table. The system is initially at rest. The table is accelerated $x\frac{m}{s^2}$. Determine the acceleration of the object in the frame of reference:

A. With respect to the table
B. With respect to an external observer

Part A
Since there is no friction, the object remains where it is. With reference to table, the object has:

$$a = -x\frac{m}{s^2}$$

Part B
Since there is no friction, the object remains where it is. With respect to external observer

$$a = 0$$

B. Rockets

2.89: Scale Reading for Weight
If a is the magnitude of acceleration of a rocket/elevator, then the reading on a weighing scale/spring balance will be:

$$Reading = m(g + a), a\ in\ upward\ direction$$
$$Reading = m(g - a), a\ in\ downward\ direction$$

Due to inertia, if an elevator moves up, an object (or a person) in the elevator experiences a pseudoforce. This pseudoforce is equal to the acceleration of the elevator.

Since weighing scales are meant to measure force of acceleration due to gravity, they will give a reading in an accelerating elevator different from the true reading.

Example 2.90
An object is in a rocket that is fired up vertically from the earth with an acceleration of $3g$. 1 second after liftoff, what is the scale reading, in Newtons, on a weighing scale show for an object that was measured to weigh $4N$ 20 minutes before liftoff. (Given $g = 9.8\frac{m}{s^2}$)

$$Original\ weight = mg$$
$$New\ Weight = m(g + a) = m(g + 3g) = 4mg$$

Hence, it is four times the original reading

$$= 4 \cdot 4N = 16N$$

Example 2.91
A rocket is fired vertically from the earth with an acceleration of $2g$, where g is the gravitational acceleration. On an inclined plane inside the rocket making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is: (JEE Main, April 9, 2016)

The force on the object due to gravity and acceleration is:

$$m(g + a) = m(g + 2g) = 3mg$$

We can consider this as equivalent to being in a planet with gravitational constant

$$g_{planet} = 3g$$

However, the minimum value of coefficient of friction for a mass to not move is independent of the value of the gravitational constant[7]:

$$\mu_s = \tan\theta$$

C. Elevators

Example 2.92

If a rocket is launched to go to space, does an astronaut sitting in the rocket experience force greater than gravity or less than gravity?

$$Force\ greater\ than\ gravity$$

Example 2.93

A. A man weights 80 kg. He stands on a weighing scale in a lift which is moving upwards with a uniform acceleration of $5\frac{m}{s^2}$. What would be the reading on the scale. (Given $g = 10\frac{m}{s^2}$) (Answer in Newtons) **(NEET 2003)**

B. A spring balance is attached to the ceiling of a lift. A man hangs his bag on the spring and the spring reads $49N$, when the lift is stationary. If the lift moves downward with an acceleration of $5\frac{m}{s^2}$, the reading of the spring balance will be: (Given $g = 10\frac{m}{s^2}$) **(JEE Main 2003)**

Part A

$$R = m(g + a) = 80(10 + 5) = 80(15) = 1200N$$

Part B

$$R = m(g - a) = \frac{49}{10}(10 - 5) = \frac{49}{2}N$$

Example 2.94

A mass of 1 kg is suspended by a thread. It is:

A. Lifted up with an acceleration $4.9\frac{m}{s^2}$

B. Lowered with an acceleration $4.9\frac{m}{s^2}$

The ratio of the tensions ($g = 9.8\frac{m}{s^2}$) is **(NEET 1998)**

$$\frac{m(g+a)}{m(g-a)} = \frac{g+a}{g-a} = \frac{9.8+4.9}{9.8-4.9} = \frac{3}{1} = 3:1$$

D. Balloons

[7] Refer the section on Ramps in the chapter on NLM-1 to see the property and its derivation.

Example 2.95
What causes a balloon to rise? Explain in terms of the gas inside the balloon.

The gas in a balloon is less dense, and hence has less mass, then the air around it. This can be because the:
- gas is different (*say a helium balloon*)
- it is regular air heated to a different temperature (*a hot air balloon*)

2.96: Drag
Drag is a force acting opposite to the relative motion of any object moving with respect to a surrounding fluid.

- In the context of balloons, we can consider the air to be a fluid.
- Drag is caused by friction.

Example 2.97
A large balloon in the atmosphere when descending does not descend at an acceleration of g. Explain the possible causes of this.

A. Drag/Upthrust
B. Gas in the balloon could be less dense reducing overall density of balloon (compared to a solid object like machinery or a person).

Example 2.98
A balloon with mass m is descending down with acceleration a (where $a < g$). How much mass should be removed from it so that it starts moving up with acceleration a? **(NEET 2014)**

Assume a constant upthrust F on the balloon. The net force is:

$$\underbrace{ma = mg - F}_{Equation\ I}, \qquad mg > F$$

Let the mass to be removed from the balloon be

$$m_0$$

The acceleration has the same magnitude but the opposite direction.

$$(m - m_0)a = F - (m - m_0)g, \qquad F > (m - m_0)g$$
$$ma - m_0a = F - mg + m_0g$$

Solve for ma:

$$\underbrace{ma = F - mg + m_0(g + a)}_{Equation\ II}$$

Add Equations I and II:

$$2ma = m_0(g + a) \Rightarrow m_0 = \frac{2ma}{g + a}$$

E. Horizontal Surfaces

Example 2.99

A. On the horizontal surface of a truck, a block of mass 1 kg is placed ($\mu = 0.6$), and truck is moving with acceleration $5\frac{m}{s^2}$, then the frictional force on the block will be: ($g = 10\frac{m}{s^2}$) **(NEET 2001)**

B. In the above question, answer if other information is the same, and truck is moving with acceleration $7\frac{m}{s^2}$.

C. Based on the answer to Parts A and B, what advice would you like to give the truck driver.

Part A

The maximum friction is:

$$f_{max} = \mu F_N = \mu mg = (0.6)(1)(10) = 6N$$

Due to acceleration of the truck, $a = 5\frac{m}{s^2}$

$$Rightward\ Pseudoforce = F = ma = (1)(5) = 5N$$
$$Leftward\ friction = f_s = 5N$$

Part B

$$f = f_{max} = 6N$$

Hence, block will move.

Part C

Acceleration should be less than $6\frac{m}{s^2}$ if the block is to remain in position without restraints.

Example 2.100

On the horizontal surface of a truck, a block of mass 1 kg is placed ($\mu_k = 0.3, \mu_s = 0.5$). What is the maximum acceleration at which the truck can be driven?

$$f_s \le 5N$$
$$F \le 5N$$

Divide by m both sides:

$$a < 5\frac{m}{s^2}$$

1 Pending NLM-2

2.101: Maximum Acceleration

The maximum value of static friction between the two blocks is:

$$F_{Pseudo} = f_{s,max} = \mu_s F_N = \mu_s mg$$

This is also the maximum value of the pseudo force that can be applied to the upper block. The acceleration of the upper block is then:

$$a = \frac{F_{Pseudo}}{m} = \frac{\mu_s mg}{m} = \mu_s g$$

The acceleration of the two blocks must remain same, so, for the entire system:

$$F = M_{System}a = (m + M)\mu_s g$$

2 Pending NLM-2

Example 2.102

A. A block of $M = 8\ kg$ is placed on a smooth table. A block of mass $m = 2\ kg$ is placed on the block of 8 kg. The coefficient of static friction between the two blocks is 0.5. The maximum horizontal force that can be applied to the block of mass M so that the blocks move together will be: (Take $g = 9.8\frac{m}{s^2}$) **(JEE Main, June 27, 2022-I, Adapted)**

B. A block with mass 2kg is placed on a table. A block with mass 1 kg is placed on the block with mass 2 kg. The block with mass 2 kg is placed on a smooth table. If the coefficient of static friction between the two blocks is 0.5, calculate the maximum horizontal force that can be applied to move the blocks together (Take $g = 15\frac{m}{s^2}$) **(JEE Main, Aug 26, 2021-II; JEE Main, Mar 17, 2021-I)**

Part A

$$F = (m + M)(\mu_S g) = (2 + 8)(0.5)(9.8) = 49N$$

Part B

$$F = (m + M)(\mu_S g) = (1 + 2)(0.5)(9.8) = 49N$$

3 Pending NLM-2

(Important) Example 2.103

A block of mass 4 kg is placed on another block B of mass 5 kg, and the block B rests on a smooth horizontal table. If the maximum force that can be applied on A so that both the blocks move together is 12N, the maximum force that can be applied to B for the blocks to move together is: (JEE Main, April 9, 2014)

Maximum force for A

$$= f_s = 12N$$

Maximum acceleration of A

$$= a = \frac{F}{m} = \frac{12N}{4\ kg} = 3\frac{m}{s^2}$$

Maximum acceleration of system of A and B

$$= 3\frac{m}{s^2}$$

Maximum force applied at B

$$= F = ma = (4 + 5)(3) = 9(3) = 27N$$

F. Tables and Conveyor Belts

Example 2.104

A. An object of mass 2 kg is placed on a table of mass 8 kg. The object is pushed with a force of $5N$ due east. Calculate the displacement of the object and the table after 5 seconds. Ignore friction.

B. An object of mass 2 kg is placed on a table of mass 8 kg. The table is pushed with a force of $5N$ due east. Calculate the displacement of the object and the table after 5 seconds. Ignore friction.

Part A

$$a = \frac{F}{m} = \frac{5N}{2\,kg} = \frac{5}{2}\frac{m}{s^2}$$

The displacement of the object is:

$$s = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{5}{2}\right)(5^2) = \frac{125}{4}\ m$$

Since there is no friction, the displacement of the table is:

$$Zero$$

Part B

$$a = \frac{F}{m} = \frac{5N}{8\,kg} = \frac{5}{8}\frac{m}{s^2}$$

The displacement of the table is:

$$s = \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{5}{8}\right)(5^2) = \frac{125}{16}\ m$$

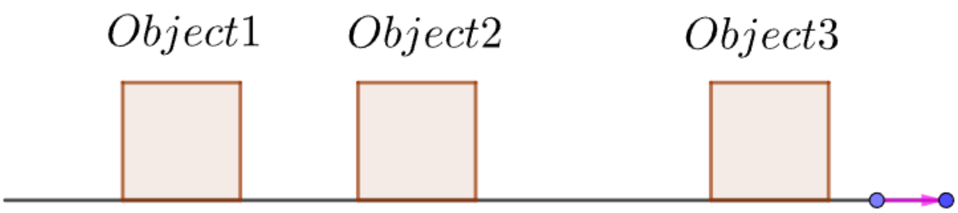
Since there is no friction, the displacement of the object is

$$Zero$$

2.105: Conveyor Belt

You can think of a conveyor belt as a horizontal surface that moves, or a table that is being pulled. The coefficient of friction between the belts and objects on the belt will determine the behaviour:

- If the belt and objects are smooth (which means there is no friction), the belt will move but the objects will not move. This is an ideal case, and not a real-world case.



Example 2.106

A frictionless bag is dropped on a smooth conveyor belt that is moving rightwards at 2 m/s.

A. How much will the bag move in 5 seconds.
B. What is the velocity of the belt with respect to the bag.
C. What is the velocity of the bag with respect to the belt.

Note: Assume, as is usual convention, that rightwards is positive and leftwards is negative.

Part A

Bag will not move. Since there is no force being applied on it.

Parts B and C

We make use of the concept of relative motion:

$$v_{Belt} = 2\frac{m}{s}$$

$$v_{Bag} = -2\frac{m}{s}$$

Hence, the belt moving rightward by $2\frac{m}{s}$ is the same as bag moving leftward by $2\frac{m}{s}$.

Example 2.107

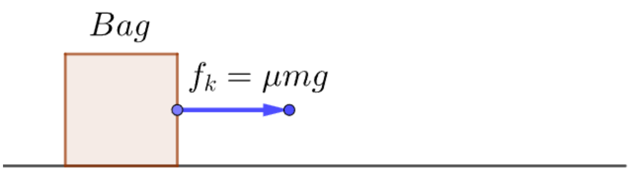
A bag is gently dropped on a conveyor belt moving at a speed of 1 m/s. The coefficient of friction between the conveyor belt and the bag is 0.4. Initially, the bag slips on the conveyor belt before it stops due to friction. Calculate the displacement between the dropping point and the final point from the point of view of:

A. A mouse which is on the belt
B. A man standing 10m away from the belt **(JEE Main, July 27, 2022-I, Adapted)**

Part A

Velocity of bag with respect to belt

$$V_{Bag} = -V_{Belt} = -1\frac{m}{s}$$



The bag moves leftward with respect to the belt and hence the frictional force acts rightward. Hence, the sign of acceleration will be positive.

The velocity will be reduced by friction, which will make the bag move rightward.

$$a = \frac{F}{m} = \frac{Friction}{m} = \frac{\mu mg}{m} = \mu g = 0.4(10) = 4\frac{m}{s^2}$$

Rearrange an equation of motion for displacement:

$$u = Initial\ Velocity\ of\ bag = -1\frac{m}{s}$$

$$v = Final\ Velocity\ of\ bag = 0\frac{m}{s}$$

$$v^2 = u^2 + 2as \Rightarrow s = \frac{v^2 - u^2}{2a} = \frac{0^2 - 1^2}{2(4)} = -\frac{1}{8}\ m$$

Part B

From part A, displacement of the bag with respect to the belt

$$= -\frac{1}{8}\ m$$

Calculate the time period taken for the bag to stop movement:

$$v = u + at \Rightarrow 0 = -1 + 4t \Rightarrow t = \frac{1}{4}$$

Since the belt moves at a constant speed, displacement of the belt:

$$Distance = Speed \times Time = 1\frac{m}{s} \times \frac{1}{4}s = \frac{1}{4}\ m$$

Net displacement of the bag:

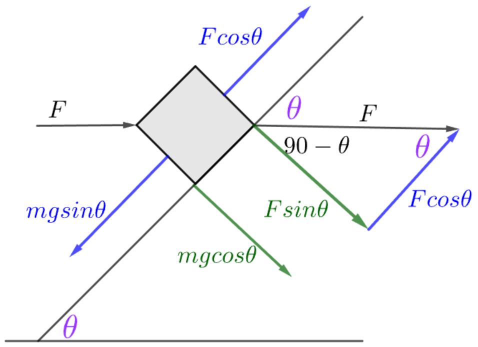
$$= \frac{1}{4} - \frac{1}{8} = \frac{2}{8} - \frac{1}{8} = \frac{1}{8}\ m$$

G. Ramps

2.108: Acceleration to maintain Equilibrium

An object of mass $m\ kg$ lies on a smooth ramp inclined at an angle θ to the horizontal. If the object is to be held in position by a horizontal acceleration, then the magnitude of acceleration is:

$$a = g\tan\theta$$



The leftward acceleration of the ramp creates a rightward (horizontal) pseudoforce.

For a horizontal force to maintain equilibrium, the leftward and rightward forces must be equal.

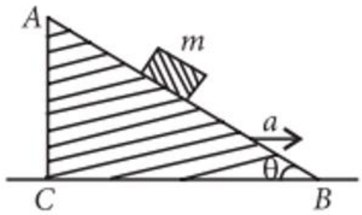
$$mg\sin\theta = F\cos\theta \Rightarrow F = mg\frac{\sin\theta}{\cos\theta}$$

Use the formula for acceleration:

$$a = \frac{F}{m} = \frac{mg\tan\theta}{m} = g\tan\theta$$

Example 2.109

A block of mass m is placed on a smooth inclined wedge ABC of inclination θ. The wedge is given an acceleration a. Determine the magnitude and direction of a. **(NEET 1999, 2018; JEE Main 2005)**



From the property

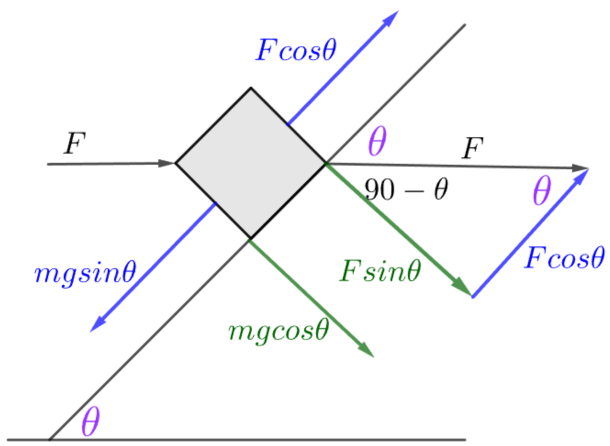
$$a = g\tan\theta$$

Example 2.110

A block of mass m is placed on a smooth wedge of inclination θ. The whole system is accelerated horizontally so that the block does not slip on the wedge. The force exerted by the wedge on the block, in terms of g, where g is acceleration due to gravity, is: **(NEET 2004)**

The force exerted by the wedge on the block is nothing but the

$$Normal\ Force$$



Total normal force

$$= mg\cos\theta + F\sin\theta$$

Substitute pseudoforce in horizontal direction $F = \frac{mg\sin\theta}{\cos\theta}$

$$= mg\cos\theta + \left(\frac{mg\sin\theta}{\cos\theta}\right)\sin\theta$$

Add the fractions:

$$= \frac{mg(\cos^2\theta + \sin^2\theta)}{\cos\theta}$$

Substitute $\cos^2\theta + \sin^2\theta = 1$:

$$= \frac{mg}{\cos\theta} = mg\sec\theta$$

2.6 Contact Force

A. Types of Forces

There are different kinds of forces in physics. Some of these forces, we will study in much greater detail in later chapters (electricity, etc).

2.111: Contact Force

A contact force is any force caused by two forces coming in contact with each other.
A force which is not a contact force is called a non-contact force.

2.112: Body Force

A force which acts throughout the volume of a body is a body force.

Example 2.113

Classify the following forces as contact forces or body forces:

A. Normal force
B. Gravitational Force
C. Force caused by electrical field
D. Frictional force
E. Force caused by magnetic field

$$Contact: A, D$$
$$Body\ Forces: B, C, E$$

B. Contact Forces

2.114: Contact Force

The contact force on an object is the resultant of the normal force and the frictional force.
Equivalently, the contact force on an object can be decomposed into a normal force and a frictional force.

$$\overrightarrow{\boldsymbol{F_C}} = \overrightarrow{\boldsymbol{F_N}} + \vec{\boldsymbol{f}}$$

Normal force and frictional force are vector quantities. Contact force is the *vector sum* of normal force and frictional force.

Example 2.115

A table and an object have coefficient of friction 0.3. An object with mass 3 kg lies on a table. Calculate the contact force.

The normal force

$$Normal\ Force = F_N = mg = 30N$$

Since the object is not moving

$$Frictional\ force = f = 0$$

The contact force

$$= F_N + f = F_N = 30N\ (directed\ downward)$$

Example 2.116

A table and an object have coefficient of friction 0.3. An object of mass 3 kg is being pushed rightward with a force of 1N. Calculate the contact force vector in component form. Also, calculate its magnitude.

The normal force

$$Normal\ Force = F_N = mg = 30N$$

Since the object is not moving

$$f_{s,max} = \mu_s F_N = (0.3)(30N) = 9N$$
$$f_s = 1N$$

The contact force

$$Contact\ Force\ Vector = \vec{C} = \overrightarrow{F_N} + \overrightarrow{f_s} = (0,30) + (-1,0) = (-1,30)$$
$$|\vec{C}| = \sqrt{901}$$

2.117: Magnitude of contact force

The magnitude of contact force is the square root of the square of the magnitudes of the normal force and the frictional force.

$$|\overrightarrow{F_C}| = \sqrt{|\overrightarrow{F_N}|^2 + |\vec{f}|^2}$$

We know that:

$$\overrightarrow{F_C} = \overrightarrow{F_N} + \vec{f}$$

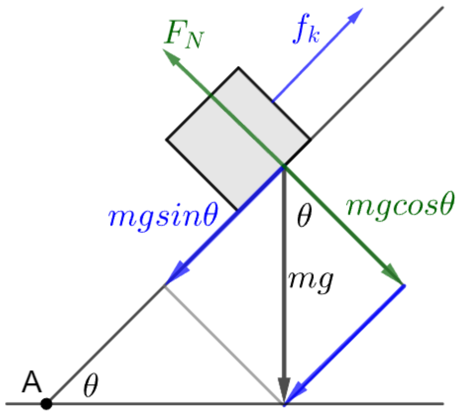
Since the normal force and the frictional force are perpendicular to each other, we can introduce a coordinate system parallel to the direction of the plane, and then the frictional force and normal force are each components in a single direction.

The above result follows using the formula for magnitude.

C. Ramp

Example 2.118

A block of mass M slides down on a rough inclined plane with constant velocity. The angle made by the inclined plane with horizontal is θ. The magnitude of the contact force, in terms of M and g will be: **(JEE Main, July 27, 2022-II)**



Draw a diagram. Using the standard property, the decomposition of the force due to gravity (Mg) gives

$$F_N = Normal\ Force = Mg\cos\theta$$
$$Force\ in\ direction\ of\ plane = Mg\sin\theta$$

We know that

$$Velocity = Constant \Rightarrow Acceleration = 0$$

If the acceleration is zero, the net force on the body is zero:

$$a = 0 \Rightarrow ma = 0 \Rightarrow F_{Net} = 0$$

If the net force is zero, the kinetic friction (resisting the movement of the body), and force moving the body in the direction of the plane must be opposite in direction and equal in magnitude:

$$f_k = Mg\sin\theta$$

$$|\overrightarrow{F_C}| = \sqrt{(F_N)^2 + f^2}$$

$$= \sqrt{(Mg\cos\theta)^2 + (Mg\sin\theta)^2}$$
$$= \sqrt{(Mg)^2(\cos^2\theta + \sin^2\theta)}$$
$$= \sqrt{(Mg)^2} = Mg$$

2.7 Calculus-Based Problems

A. Differentiation

Example 2.119: Calculating Acceleration

A. At any instant, the velocity of a particle of mass $500g$ is $(2t\hat{\boldsymbol{\imath}} + 3t^2\hat{\boldsymbol{\jmath}})\frac{m}{s}$. If the force acting on the particle at $t = 1s$ is $(\hat{\boldsymbol{\imath}} + x\hat{\boldsymbol{\jmath}})N$, then the value of x will be: (JEE Main, April 8, 2023, Shift-I)

B. A body of mass $500g$ moves along x axis such that its velocity varies with displacement x according to the relation $v = 10\sqrt{x}\ \frac{m}{s}$ the force acting on the body is: (JEE Main, April 11 2023, Shift-II)

Part A

$$\boldsymbol{F} = m\boldsymbol{a} = \boldsymbol{m}\left(\frac{d\boldsymbol{v}}{dt}\right) = \boldsymbol{0.5}(2\hat{\boldsymbol{\imath}} + 6t\hat{\boldsymbol{\jmath}}) = (\hat{\boldsymbol{\imath}} + 3t\hat{\boldsymbol{\jmath}})$$

At $t = 1$

$$\boldsymbol{F} = \hat{\boldsymbol{\imath}} + 3\hat{\boldsymbol{\jmath}} \Rightarrow x = 3$$

Part B

Differentiate both sides of $v^2 = 100x$ to get:

$$2v \cdot \frac{dv}{dx} = 100$$

Substitute $a = \frac{dv}{dt} = \frac{dx}{dt} \cdot \frac{dv}{dx} = v \cdot \frac{dv}{dx}$:

$$a = v \cdot \frac{dv}{dx} = 50\frac{m^2}{s}$$
$$F = ma = \left(\frac{1}{2}\right)(50) = 25N$$

B. Integration Basics

Example 2.120

An object of mass 3 kg is at rest. Now a force of $\vec{\boldsymbol{F}} = 6t^2\hat{\boldsymbol{\imath}} + 4t\hat{\boldsymbol{\jmath}}$ is applied on the object then the velocity of the object at $t = 3s$ is (NEET 2002)

A. Calculate the answer using integration.

B. Calculate the answer using the equation of motion $\vec{\boldsymbol{v}} = \vec{\boldsymbol{u}} + \vec{\boldsymbol{a}}t$

C. Compare the answers to Part A and B. Which method is correct? Why?

Part A

$$\vec{\boldsymbol{a}} = \frac{\vec{\boldsymbol{F}}}{m} = \frac{6t^2\hat{\boldsymbol{\imath}} + 4t\hat{\boldsymbol{\jmath}}}{3} = 6t^2\hat{\boldsymbol{\imath}} + \frac{4}{3}t\hat{\boldsymbol{\jmath}}$$

Integrate acceleration to get velocity:

$$\hat{\boldsymbol{v}} = \int \vec{\boldsymbol{a}}\ dt = \int \left(6t^2\hat{\boldsymbol{\imath}} + \frac{4}{3}t\hat{\boldsymbol{\jmath}}\right)\ dt = \frac{2t^3}{3}\hat{\boldsymbol{\imath}} + \frac{2}{3}t^2\hat{\boldsymbol{\jmath}} + \boldsymbol{C}$$

Since the object is initially at rest

$$\hat{\boldsymbol{v}}(0) = 0 \Rightarrow \boldsymbol{C} = 0 \Rightarrow \hat{\boldsymbol{v}} = \frac{2t^3}{3}\hat{\boldsymbol{\imath}} + \frac{2}{3}t^2\hat{\boldsymbol{\jmath}}$$

Finally,

$$\hat{\boldsymbol{v}}_{t=3} = \frac{2t^3}{3}\hat{\boldsymbol{\imath}} + \frac{2}{3}t\hat{\boldsymbol{\jmath}} = \frac{2(3)^3}{3}\hat{\boldsymbol{\imath}} + \frac{2}{3}3^2\hat{\boldsymbol{\jmath}} = 18\hat{\boldsymbol{\imath}} + 6\hat{\boldsymbol{\jmath}}$$

Part B

$$\vec{\boldsymbol{v}} = \vec{\boldsymbol{u}} + \vec{\boldsymbol{a}}t = 0 + \left(6t^2\hat{\boldsymbol{\imath}} + \frac{4}{3}t\hat{\boldsymbol{\jmath}}\right)t = 6t^3\hat{\boldsymbol{\imath}} + \frac{4}{3}t^2\hat{\boldsymbol{\jmath}}$$

Part C

Equation of motion assumes constant acceleration.
Since acceleration is not constant (it is a function of time), equation of motion is not valid.
Integration is the correct method to apply here.

C. Calculating Velocity

Example 2.121

A block of mass 2 kg moving on a horizontal surface with speed of $4\frac{m}{s}$ enters a rough surface ranging $x = 0.5$ to $x = 1.5\ m$. The retarding force in this range of rough surface is related to distance by $F = -kx$ where $k = 12\frac{N}{m}$. The speed of the block as it just crosses the rough surface will be: **(JEE Main, June 28, 2022-II)**

$$F = -kx$$

Substitute $F = ma = 2a, k = 12$:

$$2a = -12x \Rightarrow a = -6x$$

Rearrange $a = \frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\cdot\frac{dv}{dx}$ and integrate:

$$a\,dx = v\,dv \Rightarrow \int a\,dx = \int v\,dv$$

Substitute $a = -6x$ and also the limits of integration:

$$Retardation\ takes\ place\ from\ x = 0.5\ to\ x = 1.5$$

$$Velocity\ decreases\ from\ v = 4\ to\ v = V$$

$$-6\int_{0.5}^{1.5} x\,dx = \int_4^V v\,dv$$

In the above, we wish to find the value of V. Carry out the integration:

$$(-6)\left[\frac{x^2}{2}\right]_{0.5}^{1.5} = \left[\frac{v^2}{2}\right]_4^V$$

Substitute the limits of integration:

$$(-6)\left(\frac{1.5^2 - 0.5^2}{2}\right) = \frac{V^2 - 4}{2}$$

Solve for V:

$$12 = V^2 - 4 \Rightarrow V = 4$$

Example 2.122

A particle of mass M originally at rest is subjected to a force whose direction is constant, but magnitude varies with time according to the relation

$$F = F_0\left[1 - \left(\frac{t-T}{T}\right)^2\right], F_0\ and\ T\ are\ constants$$

The force acts only for the time interval $2T$. The velocity of the particle after time $2T$ is:

$$v(t) = \int a\,dt = \int \frac{1}{M} \cdot F\,dt = \int \frac{1}{M} \cdot F_0 \left[1 - \left(\frac{t-T}{T}\right)^2\right] dt$$

$$1 - \left(\frac{t-T}{T}\right)^2 = 1 - \frac{(t-T)^2}{T^2} = \frac{T^2 - (t^2 - 2tT + T^2)}{T^2} = \frac{2tT - t^2}{T^2}$$

Since F_0 and m are constants, they can be moved outside the integration:

$$= \frac{F_0}{MT^2} \int (2tT - t^2)\,dt$$

Carry out the integration, and note that the constant of integration$= C = 0$ because the object was initially at rest:

$$v(t) = \frac{F_0}{MT^2}\left(t^2 T - \frac{t^3}{3}\right)$$

To find the velocity,

$$v(2T) = \frac{F_0}{MT^2}\left((2T)^2 T - \frac{(2T)^3}{3}\right) = \frac{F_0}{MT^2}\left(4T^3 - \frac{8T^3}{3}\right) = \frac{F_0}{MT^2}\left(\frac{4T^3}{3}\right) = \frac{F_0}{M}\left(\frac{4T}{3}\right) = \frac{4TF_0}{3M}$$

Example 2.123

A small ball of mass m is thrown upward with velocity u from the ground. The ball experiences a resistive force mkv^2 where v is its speed. The maximum height attained by the ball is:

$$a = \frac{F}{m} = -\frac{mkv^2 + mg}{m} = -(kv^2 + g)$$

Substitute $a = v \cdot \frac{dv}{dh}$

$$v \cdot \frac{dv}{dh} = -(kv^2 + g)$$

Rearrange and integrate:

$$\int \frac{v}{kv^2 + g} dv = \int -dh$$

Use a change of variable. Substitute $V = kv^2 + g \Rightarrow dV = 2kv\,dv$

$$\frac{1}{2k}\int \frac{1}{V} dV = [-h]_0^h$$

$$\frac{1}{2k} \ln V = -h$$

Change back to the original variable:

$$\frac{1}{2k}[\ln(kv^2 + g)]_u^0 = -h$$

$$-\frac{1}{2k}(\ln(ku^2 + g)) = -h$$

$$h = \frac{1}{2k}\ln(ku^2 + g)$$

3. NLM-3 & Momentum

3.1 *NLM* – 3: Action-Reaction

A. Newton's Third Law

3.1: Newton's Third Law

- For every action (force) in nature, there is an equal and opposite reaction.
- If two bodies exert forces on each other, these forces have the same magnitude but opposite directions.

If object A exerts a force on object B, then object B also exerts an equal and opposite force on object A.

Example 3.2

A. You are in a chair with wheels, next to a table. You push the table rightward. What happens to your chairs?
B. You are on roller skates, next to a wall. You push against the wall, leftward. What happens to you?
C. You are in a boat in still water. You use a rope to pull another boat. Both boats have equal mass. Which boat(s) will move and why?
D. You are ice skating at a rink, near the entry point. You "pull" on the gate pole. What happens?

Part A
Moves leftward
Part B
You move rightward
Part C
Both boats
Part D
You move closer to the gate pole.

Example 3.3

A. When you walk forward on sand (such as sand at a beach), does the sand move backward?
B. When you walk forward on a hard surface, does the surface move backward?

Part A
Yes
Part B
Yes, but by a very small, negligible amount

B. Action Reaction Pairs

3.4: Action Reaction Pairs
Action reaction pairs consist of two objects on which equal and opposite forces act.

Example 3.5
Identify the action reaction pair in each case:

A. A gun is fired.
B. A man at an ice rink wearing skates throws a heavy object north.

Part A
Bullet moves forward. Gun (and person firing the gun moves backward). The movement of the person is called "recoil".

Part B
Action: Throwing object north
Reaction: Man moving south

3.6: Forces on the same object
Forces on the same object do not action reaction pair.

- Forces on the same object can result in equilibrium, but that equilibrium does not make the forces an action reaction pair.

Example 3.7
Explain why Force A and Force B are not examples of action-reaction pairs:

A. An object moves leftward(down) a ramp with constant velocity. The leftward force A due to gravity is balanced by the rightward force B due to friction.
B. Two children are fighting over a toy. One pulls the toy rightward (Force A). The other pulls the toy leftward (Force B).

Parts A, B
Not an action-reaction pair since both rightward and leftward forces act on the same object.

Example 3.8
$$True\ or\ False$$
Decide whether each statement below is true or false.
Statement 1: If you push on a cart being pulled by a horse so that it does not move, the cart pushes you back with an equal and opposite force.
Statement 2: The cart does not move because the forces described in statement 1 cancel each other. **(JEE Main, May 26, 2012)**

$$Statement\ 1\ is\ true$$

The forces in Statement 1 are an action reaction pair. They act on two different objects.
The cart is in equilibrium because the forces on the cart:
$$Push\ (from\ you), and\ pull(\ from\ the\ horse)$$
Are equal and opposite.

Hence:
$$Statement\ 2\ is\ false$$

C. Gravity

3.9: Gravity with Newton's Third Law:
Earth's gravity pulls an object. The object's gravity pulls on Earth.

Example 3.10
When a ball falls from a building, the action is that the Earth attracts the ball. What is the reaction?

When a ball falls from a building, the earth attracts the ball, and the ball attracts the earth.
But, the ball is much lighter, and hence the movement of the earth is negligible.

Example 3.11

Out of the two parts below, one pair forms an action reaction pair. The other pair does not. Identify which is which and explain why.

A. A bag lies on the floor. Gravity pulls the object down against the floor (force A). The normal force from the floor pushes the bag up (Force B).
B. Bag pushes down on the surface (Force A). Surface pushes bag (Force B)

Part A
Not an action-reaction pair since both upward and downward forces act on the same object (bag)

Part B
Action reaction pair. Two forces on two difference objects.

D. Strings, Ropes and Cords

3.12: Reaction on a Rope

A person climbing a rope has upward displacement.
The rope experiences downward force equivalent to the acceleration of the person.

3.13: Breaking Strength

Breaking strength is the maximum tension that can a rope can sustain without breaking.

3.14: Climbing Upward

When climbing upward, the tension on the rope is:

$$T = mg + ma$$

Where

$$g = acceleration\ due\ to\ gravity$$
$$m = mass\ of\ person$$
$$a = acceleration\ of\ person$$

Example 3.15

A monkey of mass 20 kg is holding a vertical rope. The rope will not break when a mass of 25 kg is suspended from it but will break if the mass exceeds 25 kg. What is the maximum acceleration with which the monkey can climb up along the rope? $\left(g = 10\frac{m}{s^2}\right)$ **(NEET 2003)**

Solve $ma + mg = T$ for a to get:

$$a = \frac{T - mg}{m}$$

Acceleration will be maximum when T is maximum. Substitute $T = 25g$:

$$= \frac{25g - 20g}{20} = \frac{5}{20}g = \frac{1}{4}(10) = 2.5\frac{m}{s}$$

3.16: Descending downward

When descending a rope, the tension on the rope is:

$$T = mg - ma$$

Applying Newton's Third Law tells us that in the process of moving downward, we are pushing the rope upward.
This reduces the tension on the rope.

Example 3.17

A monkey is descending from the branch of a tree with constant acceleration. If the breaking strength of the branch is 75% of the weight of the monkey, the minimum acceleration with which monkey can slide down without breaking the branch, in terms of g, is: **(NEET 1993)**

Note: Read "rope" instead of a branch

$$T = mg - ma$$

At the minimum acceleration, the tension must equal the breaking strength:

$$\frac{3}{4}mg = mg - ma$$

Cancelling m on both sides and solving for a:

$$a = \frac{g}{4}$$

Example 3.18

A monkey of mass 50 kg climbs on a rope which can withstand tension of $350N$. Monkey initially climbs down with an acceleration of $4\frac{m}{s^2}$ and then climbs up with an acceleration of $5\frac{m}{s^2}$. Considering $g = 10\frac{m}{s^2}$, calculate the tension when climbing down, and when climbing up. Will the rope withstand the tension or break? If so, when? **(JEE Main, July 26, 2022-I, Adapted)**

$$T = mg - ma = 50(10) - 50(4) = 50(6) = 300N$$

Rope will withstand the tension.

But when climbing up:

$$T = mg + ma = 50(10) + 50(5) = 750N > 350N$$

Rope will break.

3.2 Impulse and Momentum

A. Momentum

3.19: Momentum

Momentum is the product of the mass and the velocity of an object

$$\vec{p} = m\vec{v}$$

- Momentum is a vector quantity since velocity is a vector quantity and mass is a scalar quantity.
- $m\vec{v}$ is a scalar product
- Momentum has the same direction as velocity

3.20: Assumptions

When working with momentum, you can make certain assumptions which simplify formulas.
Recognizing which assumptions is crucial in determining which formulas are valid.

Assumption of constant mass:

- Valid if a ball dropped from a tower
- Not valid if a rocket is accelerated in space because the rocket loses mass as it ejects fuel
- Not valid if a body is moving at speeds which are close to speed of light since the mass of the body will be different from its rest mass.

Assumption of constant force:

- Valid if a uniform force is applied to an object
- Valid if average force is being calculated
- Not Valid if force applied is a function of time

B. Impulse at Constant Mass

3.21: Impulse

Impulse is the change in momentum:

$$Impulse = \Delta\vec{p}$$

3.22: Impulse at constant mass

At constant mass, impulse scales linearly with velocity:

$$Impulse = \Delta\vec{p} = m\Delta\vec{v}$$
$$Impulse \propto m\Delta\vec{v}$$

$$Impulse = \Delta\vec{p}$$

Substitute the definition of momentum:

$$= \Delta(m\vec{v})$$

Since the mass is constant, it does not change. Move it out of the Δ operator:

$$= m\Delta\vec{v}$$

Example 3.23

A rocket is fired into the air vertically upward. Is the formula below applicable:

$$Impulse = m\Delta\vec{v}$$

No. Because as the fuel burns, it is ejected from the rocket, and the mass reduces.

Example 3.24

A particle of mass m is moving with uniform velocity v_1 in a particular direction. It is given an impulse such that its velocity becomes v_2 in the same or opposite direction. The magnitude of impulse, in terms of m, v_1 and v_2 is equal to: **(NEET 1990, Adapted)**

$$Impulse = m\Delta v = m(v_2 - v_1)$$

3.25: Normal Collisions with a Wall

In normal collisions, an object hits a wall, and bounces back along the same path.

- Normal is a technical word that means perpendicular.
- For example, the normal to a curve is perpendicular to the curve at that point.

Example 3.26

A stone is dropped from a height h. It hits the ground with a certain momentum P. If the same stone is dropped from a height 100% more than the previous height, the momentum when it hits the ground will change by: **(NEET 2012)**

For the ball dropped from height h:

$$v^2 = u^2 + 2as \Rightarrow v^2 = 2gh \Rightarrow v = \sqrt{2gh}$$

For the ball dropped from height $2h$:

$$V = \sqrt{2g(2h)} = \sqrt{2gh}\sqrt{2} = \sqrt{2}v$$

Since mass of both the balls is the same, the percentage change depends only on the velocity:

$$\%\ change = \frac{V - v}{v} = \frac{\sqrt{2}v - v}{v} = \frac{\sqrt{2} - 1}{1} \approx 1.41 - 1 = 0.41 = 41\%$$

Example 3.27

A. A body of mass M hits normally a rigid wall with velocity V and bounces back with the same velocity. The impulse experience by the body is: **(NEET 2011)**

B. A ball of mass 0.15 kg is dropped 10m, strikes the ground and rebounds to the same height. The magnitude of impulse imparted to the ball is nearly (take $g = 10\frac{m}{s^2}$) **(NEET 2021)**

Part A

$$Impulse = m\Delta v = m(v_2 - v_1) = m[V - (V)] = 2MV$$

Part B

Substitute $m = 0.15, v = \sqrt{2gh}$ in $Impulse = 2mV$

$$= 2(0.15)\sqrt{2(10)(10)} = 3 \times 1.4 = 4.2N$$

C. Impulse at Constant Mass and Constant Force

3.28: Impulse at Constant Mass and Constant Force

If a constant force is applied to a constant mass for a time Δt, then:

$$Impulse = \vec{\boldsymbol{F}}\Delta t$$

$$Impulse = \Delta\vec{\boldsymbol{p}} = \Delta m\vec{\boldsymbol{v}} = m\Delta\vec{\boldsymbol{v}} = m\frac{\Delta\vec{\boldsymbol{v}}}{\Delta t}\cdot\Delta t = m\vec{\boldsymbol{a}}\cdot\Delta t = \vec{\boldsymbol{F}}\Delta t$$

Example 3.29

Statement I: If a force is applied to a constant mass, then $\underbrace{Impulse = m\Delta\vec{\boldsymbol{v}}}_{Equation\ I}$

Statement II: If a constant force is applied to a constant mass, then $\underbrace{Impulse = \vec{\boldsymbol{F}}\Delta t}_{Equation\ II}$

A. Use Equations I and II to solve for $\vec{\boldsymbol{F}}$ in terms of m, Δt and $\Delta\vec{\boldsymbol{v}}$.
B. Do the dimensions on the equation that you arrived in Part A validate?

Part A

$$\vec{\boldsymbol{F}}\Delta t = m\Delta\vec{\boldsymbol{v}} \Rightarrow \vec{\boldsymbol{F}} = \frac{m\Delta\vec{\boldsymbol{v}}}{\Delta t}$$

Part B

$$LHS = Force = kg\frac{m}{s^2}$$

$$RHS = \frac{m\Delta\vec{\boldsymbol{v}}}{\Delta t} = \frac{(\boldsymbol{kg})\left(\frac{\boldsymbol{m}}{\boldsymbol{s}}\right)}{s} = kg\frac{m}{s^2} = RHS$$

3.30: Average Force

If a body with constant mass experiences acceleration for a time Δt, then the average force applied is:

$$\vec{\boldsymbol{F}} = \frac{m\Delta\vec{\boldsymbol{v}}}{\Delta t}$$

Example 3.31

A cricketer catches a ball of mass 150 gm in 0.1 sec moving with speed $20\frac{m}{s}$. He experiences a force with a magnitude of: (NEET 2001)

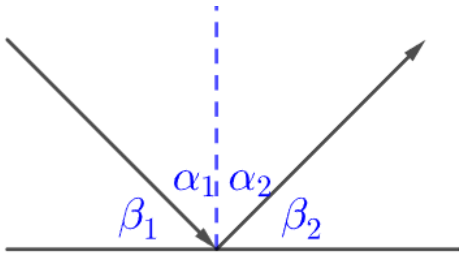
$$\boldsymbol{F} = \frac{m\Delta\vec{\boldsymbol{v}}}{\Delta t} = \frac{(0.15)(20)}{0.1} = 30N$$

D. Collisions at an Angle

3.32: Symmetry in collisions at an angle

Exam questions often assume symmetry in collisions.
So, in the diagram alongside

$$\alpha_1 = \alpha_2$$
$$\beta_1 = \beta_2$$



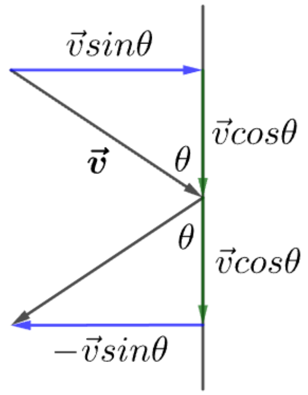
3.33: Impulse in collisions at an angle

When an object strikes a wall at an angle, and gets reflected symmetrically without loss of speed:

- Component of impulse parallel to the wall is zero.

- Magnitude of impulse perpendicular to the wall is double the horizontal component of the velocity.
- Direction of impulse perpendicular to the wall is opposite to the direction of initial velocity.

Consider an object which makes an angle θ with a wall, and collides with it. With no loss of speed, it bounces at angle θ to the wall.
$(Diagram\ alongside)$



Impulse is:

$$\Delta\vec{\boldsymbol{p}} = m\boldsymbol{\Delta}\vec{\boldsymbol{v}}$$

Write the velocity vector in component form:

$$m\Delta v_x + m\Delta v_y$$

Using trigonometry, write the components of the velocity vector:

$$v_{x_1} = \vec{v}\sin\theta\,, \qquad v_{x_2} = -\vec{v}\sin\theta\,, \qquad v_{y_1} = v_{y_2} = \vec{v}\cos\theta$$

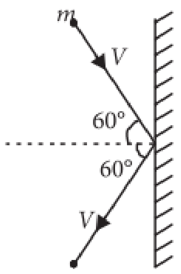
$$\Delta v_y = v_{y_2} - v_{y_1} = 0$$
$$\Delta v_x = -\vec{v}\sin\theta - \vec{v}\sin\theta = -2\vec{v}\sin\theta$$

Calculate momentum:

$$= m\Delta v_x + m\Delta v_y = -2m\vec{\boldsymbol{v}}\sin\theta$$

Example 3.34

A rigid ball of mass m strikes a rigid wall at $60°$ and gets reflected without loss of speed as shown in the figure. The absolute value of impulse imparted by the wall on the ball will be: **(NEET 2016)**



The angle is $60°$ with the normal. The angle with the wall is:

$$90° - 60° = 30°$$

Substitute $velocity = V\ (from\ the\ diagram), \theta = 30°$ in:

$$Impulse = -2m\vec{\boldsymbol{v}}\sin\theta = m\left(2V \cdot \frac{1}{2}\right) = mV\ N$$

Example 3.35

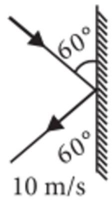
A 0.5 kg ball moving with a speed of 12 $\frac{m}{s}$ strikes a hard wall at an angle of $30°$ with the wall. It is reflected with the same speed at the same angle. If the ball is in contact with the wall for 0.25 seconds, the average force acting on the wall is: **(NEET 2006)**

$$\Delta\vec{\boldsymbol{v}}_{\boldsymbol{x}} = 2v\sin\theta = 2(12)\left(\frac{1}{2}\right) = 12$$

Substitute $\Delta t = 0.25, m = 0.5,\ \Delta\vec{\boldsymbol{v}}_{\boldsymbol{x}} = 12$

$$\vec{\boldsymbol{F}} = \frac{m\Delta\vec{\boldsymbol{v}}}{\Delta t} = \frac{0.5 \times 12}{0.25} = 24N$$

Example 3.36
A body of mass 3 kg hits a wall at an angle of 60° and returns at the same angle. The impact time was $0.2s$. The force exerted on the wall is: **(NEET 2000)**



Substitute $v = 10\frac{m}{s}$ from the diagram into:

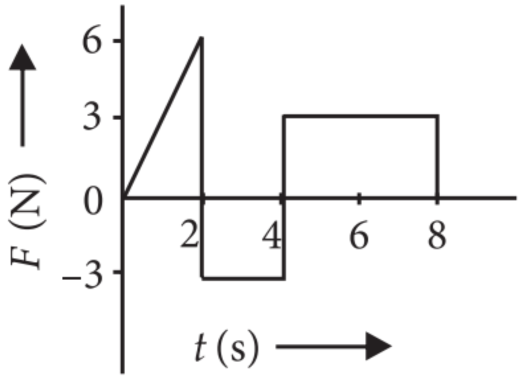
$$\Delta v_x = 2v\sin\theta = 2(10)\left(\frac{\sqrt{3}}{2}\right) = 10\sqrt{3}$$

Substitute $\Delta t = 0.2, m = 3, \Delta v_x = 10\sqrt{3}$

$$\boldsymbol{F} = \frac{m\Delta\vec{\boldsymbol{v}}}{\Delta t} = \frac{3\times 10\sqrt{3}}{0.2} = 150\sqrt{3}N$$

E. Non-Constant Force

3.37: Impulse from Graphical Integration
Impulse is the area under the curve of a force-time graph.
- Area above the area is positive.
- Area above the area is negative.



Example 3.38
The force F acting on a particle of mass m is indicated by the force-time graph as shown. The change in momentum of the particle over the time interval from zero to $8s$ is: **(NEET 2014)**

$$(t_0, t_2) = \frac{1}{2}\cdot 2\cdot 6 = 6$$
$$(t_2, t_4) = (-3)\cdot 2 = -6$$
$$(t_4, t_8) = 3\cdot 4 = 12$$

Total

$$= 12$$

3.3 Conservation of Momentum

A. One Dimension

3.39: Law of conservation of energy
Energy can be neither be created nor be destroyed. It can only be converted from one form into another.

3.40: Conservation of momentum
The law of conservation of momentum says that the momentum of a system is conserved during a collision:

$$\underbrace{\vec{\boldsymbol{p}}_i}_{\text{Initial Momentum}} = \underbrace{\vec{\boldsymbol{p}}_f}_{\text{Final Momentum}}$$

- This assumes that no kinetic energy is converted into other forms of energy.
- This is not a formula. Rather, this is an idea from which formulas that are useful will be derived.

3.41: Object initially at rest

$$m_1v_1 = m_2v_2$$

If a system is at rest

$$Initial\ Momentum = 0$$

(For example, a system could consist of $gun + bullet$)

If a bullet is fired from the gun, the bullet has a certain velocity, the gun has a certain velocity, but the momentum of the system is conserved:

$$m_{Gun}V_{Gun} + m_{Bullet}V_{Bullet} = 0$$
$$m_{Gun}V_{Gun} = -m_{Bullet}V_{Bullet}$$

Use a change of variable. $m_{Gun} = m_1, V_{Gun} = v_1, m_2 = m_{Bullet}, v_2 = -V_{Bullet}$

$$m_1v_1 = m_2v_2$$

- v_1 and v_2 are in one dimension. Direction is indicated using positive and negative.
- m is always positive. Hence, v_1 and v_2 must be opposite in sign.

Example 3.42

A man fires a bullet of mass $200g$ at a speed of $5\frac{m}{s}$. The gun is of one kg mass. By what velocity the gun rebounds backwards? (NEET 1996)

$$m_1v_1 = m_2v_2$$
$$(1)v_1 = (0.2)(5)$$
$$(1)v_1 = (0.2)(5) = 1\frac{m}{s}\ backwards$$

3.43: Muzzle Velocity

Muzzle velocity is the velocity with which the bullet leaves the gun.

Example 3.44

A person holding a rifle (mass of person together with rifle is 100 kg) stands on a smooth surface and fires 10 shots horizontally, in $5s$. Each bullet has a mass of $10g$ with muzzle velocity of $800\frac{m}{s}$. The final velocity acquired by the person and the average force exerted on the person are: (NEET 2013)

$$m_1v_1 = m_2v_2$$

Substitute $m_1 = 100\ kg, m_2 = 10 \times 10g = 100g = 01.kg$:

$$(100)v_1 = (0.1)(800)$$
$$v_1 = 0.8\frac{m}{s^2}$$

$$\boldsymbol{F} = \frac{m\Delta\vec{\boldsymbol{v}}}{\Delta t} = \frac{100 \times 0.8}{5} = 16N$$

3.45: Object not initially at rest

$$m_1v_1 + m_2v_2 = mv$$

$$Initial\ Momentum = mv$$
$$Final\ Momentum = m_1v_1 + m_2v_2$$

$$Momentum\ of\ First\ Part = m_1 v_1$$
$$Momentum\ of\ Second\ Part = m_2 v_2$$

- v_1 and v_2 are in one dimension. Direction is indicated using positive and negative.

Example 3.46
A mass of 1 kg is thrown up with a velocity of $100\,\frac{m}{s}$. After 5 seconds, it explodes into two parts. One part of mass $400g$ comes down with a velocity of $25\,\frac{m}{s}$. The velocity of other part is: (take $g = 10\,\frac{m}{s^2}$). **(NEET 2000)**

The velocity of the body at $time\ t = 5s$:
$$v = u + at = 100 + (-10)(5) = 50\,\frac{m}{s}$$

Using the law of conservation of momentum:
$$m_1 v_1 + m_2 v_2 = mv$$

Substitute $m_1 = 0.4\ kg, v_1 = -25, m_2 = 0.6\ kg, m = 1\ kg, v = 50\,\frac{m}{s}$:
$$(0.4)(-25) + (0.6)v_2 = (1)(50)$$
$$-100 + 6v_2 = 500$$
$$v_2 = 100\,\frac{m}{s}$$
The other part is going upwards.

B. Two Dimensions

3.47: Conservation of Linear Momentum
Two bodies A and B with mass m_A and m_B collide. They have initial velocity $\vec{\boldsymbol{u}}_A$ and $\vec{\boldsymbol{u}}_B$. Their velocity after collision is $\vec{\boldsymbol{v}}_A$ and $\vec{\boldsymbol{v}}_B$

Then, by law of conservation of momentum:
$$m_A\vec{\boldsymbol{u}}_A + m_B\vec{\boldsymbol{u}}_B = m_A\vec{\boldsymbol{v}}_A + m_B\vec{\boldsymbol{v}}_B$$

- Conservation of momentum applies in multiple dimensions, as in one dimension.
- In one dimension, we indicated direction using sign, and hence a scalar equation was sufficient.
- In two dimensions, we need a vector equation.
- Equivalently, we can consider components, and equate the components.

Example 3.48
An explosion breaks a rock into three parts. Two of them go off at right angles to each other. The first part of mass 1 kg moves with a speed of $12\,\frac{m}{s}$ and the second part of mass 2 kg moves with $8\,\frac{m}{s}$ speed. If the third part flies off with $4\,\frac{m}{s}$ speed, then its mass is: **(NEET 2009; NEET 2013)**

Method I
The object is initially at rest. Hence, the final momentum must also sum to zero.
Draw a diagram and use Pythagoras.
$$p_1 = (1)(12) = 12$$
$$p_2 = (2)(8) = 16$$

By Pythagorean Triplet $4(3,4,5) = (12,16,20)$:

$$Combined\ momentum = m_3 v_3 = 20$$
$$m = \frac{20}{4} = 5\ kg$$

Method II

$$Initial\ Momentum = Final\ Momentum = 0$$
$$\vec{\boldsymbol{p}}_1 + \vec{\boldsymbol{p}}_2 + \vec{\boldsymbol{p}}_3 = 0$$
$$\vec{\boldsymbol{p}}_3 = -(\vec{\boldsymbol{p}}_1 + \vec{\boldsymbol{p}}_2) = -[(12,0) + (0,16)] = -[12,16]$$
$$|\vec{\boldsymbol{p}}_3| = \sqrt{12^2 + 16^2} = 20$$
$$m_3 v_3 = 20$$
$$v_3 = \frac{20}{4} = 5\ kg$$

Example 3.49

A 1 kg stationary bomb is exploded in three parts having mass 1: 1: 3 respectively. Parts having same mass move in perpendicular direction with velocity $30\frac{m}{s}$. The magnitude of velocity of bigger part will be: **(NEET 2001; Similar: NEET 1989)**

Draw a diagram and use Pythagoras.

$$p_1 = p_2 = (1)(30) = 30$$

By $45 - 45 - 90$ triangle:

$$p_3 = m_3 v_3 = 30\sqrt{2}$$
$$v_3 = \frac{30\sqrt{2}}{3} = 10\sqrt{2}\frac{m}{s}$$

C. Three Dimensions

Example 3.50

An object flying in air with velocity $(20\hat{\boldsymbol{\imath}} + 25\hat{\boldsymbol{\jmath}} - 12\hat{\boldsymbol{k}})$ suddenly breaks in two pieces whose masses are in the ratio 1: 5. The smaller mass flies off with a velocity $(100\hat{\boldsymbol{\imath}} + 35\hat{\boldsymbol{\jmath}} + \boldsymbol{8}\hat{\boldsymbol{k}})$. The velocity of the larger piece will be: **(NEET 2019)**

Without generality, let the masses be

$$1\ and\ 5$$

By law of conservation of momentum:

$$(1)(100,35,8) + 5\vec{\boldsymbol{v}}_2 = 6(20,25,-12)$$
$$5\vec{\boldsymbol{v}}_2 = (120 - 100, 150 - 35, -72 - 8)$$

Simplify and divide by 5 both sides:

$$\vec{\boldsymbol{v}}_2 = \left(\frac{20}{5}, \frac{115}{5}, -\frac{80}{5}\right) = (4,23,-16)$$

D. Kinetic Energy

3.51: Kinetic Energy

The kinetic energy of a moving body with mass m and velocity $\vec{\boldsymbol{v}}$ is

$$\frac{1}{2}mv^2$$

Example 3.52

A body A with mass m_A moving with velocity $\vec{u}_A$ strikes a motionless body B with mass m_B. If the velocity after the collision of body A is $\vec{v}_A$, find the velocity vector $\vec{v}_B$ for body B after the collision.

Using the law of conservation of momentum:

$$m_A\vec{u}_A + m_B\vec{u}_B = m_A\vec{v}_A + m_B\vec{v}_B$$

Since body B is initially at rest, $m_B\vec{u}_B = 0$, and the equation simplifies to:

$$m_A\vec{u}_A = m_A\vec{v}_A + m_B\vec{v}_B$$

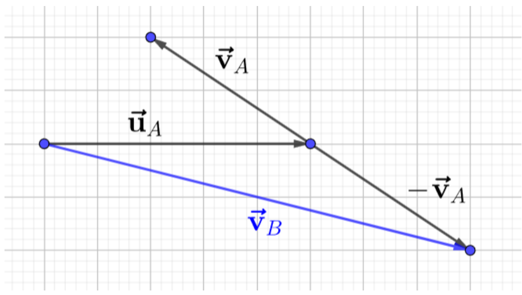
Solve for $\vec{v}_B$:

$$\vec{v}_B = \left(\frac{m_A}{m_B}\right)(\vec{u}_A - \vec{v}_A)$$

(Calc) Example 3.53

Body A moving due East at $32\frac{m}{s}$ on a frictionless, horizontal surface strikes body B, and rebounds at a speed of $8.0\frac{m}{s}$ at an angle $29°$ north of west. If the masses of A and B are 2.1 kg and 3.6 kg respectively, find the percentage of the original kinetic energy of the system that is converted into other forms of energy during the collision.

$$\vec{v}_B = \left(\frac{m_A}{m_B}\right)(\vec{u}_A - \vec{v}_A)$$



Substitute $m_A = 2.1, m_B = 3.6, \vec{u}_A = (32,0), \vec{v}_A = (-8\cos 29°, 8\sin 29°)$:

$$\vec{v}_B = \left(\frac{2.1}{3.6}\right)(32 + 8\cos 29°, -8\sin 29°)$$

Take the magnitude on both sides:

$$|\vec{v}_B| = \left(\frac{2.1}{3.6}\right)\sqrt{(32 + 8\cos 29°)^2 + (8\sin 29°)^2} \approx 22.860$$

Initial Kinetic Energy

$$= \frac{1}{2}mu_A^2 + \frac{1}{2}mu_B^2 = \left(\frac{1}{2}\right)(2.1)(32^2) + 0 = 1075.5$$

Final Kinetic Energy

$$= \frac{1}{2}mv_A^2 + \frac{1}{2}mv_B^2 = \left(\frac{1}{2}\right)(2.1)(8^2) + \left(\frac{1}{2}\right)(3.6)(22.860^2) \approx 1007.84$$

Percentage of kinetic energy converted into other forms:

$$= \frac{1075.5 - 1007.84}{1075.5} = \frac{67.66}{1075.5} \approx 6.29\%$$

3.4 Calculus

A. Rockets

3.54: Rockets with constant velocity

Mass will vary in a rocket with working engine which uses combustion engine.

Rocket will work on:

- Newton's Third Law of Motion
- Principle of conservation of motion

3.55: Momentum and Net Force-I

Net force is the derivative of momentum.
That is, the rate of change of momentum of a body is equal to the net force on it.

$$F_{Net}=\frac{d}{dt}\vec{\boldsymbol{p}}$$

$$F_{Net}=ma=m\left(\frac{d}{dt}\boldsymbol{v}\right)=\left(\frac{d}{dt}m\boldsymbol{v}\right)=\frac{d}{dt}\vec{\boldsymbol{p}}$$

3.56: Constant Velocity

$$F_{Net}=v\frac{dm}{dt}$$

$$F_{Net}=\frac{d}{dt}\vec{\boldsymbol{p}}=\frac{d}{dt}(mv)=v\frac{dm}{dt}$$

Example 3.57

If the force on a rocket, moving with a velocity of $300\frac{m}{s}$ is $210N$, then the rate of combustion of the fuel is: **(NEET 1999)**

$$F_{Net}=v\frac{dm}{dt}$$
$$210=(300)\frac{dm}{dt}$$
$$\frac{dm}{dt}=\frac{210}{300}=0.7\frac{kg}{s}$$

3.58: Momentum and Net Force-II

Impulse (change of momentum) is the integral of net force.

$$p=\int F\,dt$$

$$F_{Net}=\frac{d}{dt}p$$

Integrate both sides:

$$\int F\,dt=p$$

Example 3.59

A bullet is fired from a gun. The force on the bullet is given by $F=600-2\times10^5t$ where F is in Newton and t in seconds. The force on the bullet becomes zero as soon as it leaves the barrel. What is the average impulse imparted to the bullet? **(NEET 1998)**

Determine the time when the force is zero:

$$0 = 600 - 2 \times 10^5 t$$
$$2 \times 10^5 t = 600$$
$$t = 300 \times 10^{-5} = 3 \times 10^{-3}\ seconds$$

The impulse is then

$$\int F\,dt = \int (600 - 2 \times 10^5 t)\,dt = 600t - 10^5 t^2 + C$$

Since initial momentum is 0,

$$C = 0$$

Substitute $t = 3 \times 10^{-3}$:

$$600(3 \times 10^{-3}) - 10^5(3 \times 10^{-3})^2$$
$$= 1800 \times 10^{-3} - 10^5(9 \times 10^{-6})$$
$$= 1.8 - 0.9$$
$$= 0.9N$$

B. Thrust

Example 3.60

11: A 5000kg rocket is set for vertical firing. The exhaust speed is $800\frac{m}{s}$. To give an initial upward acceleration of $200\frac{m}{s^2}$, the amount of gas ejected per second to supply the needed thrust will be (take $g = 10\frac{m}{s^2}$). (NEET 1998)

Example 3.61

15: In a rocket, fuel burns at the rate of $1\frac{kg}{s}$. This fuel is ejected from the rocket with a velocity of $60\frac{km}{s}$. This exerts a force on the rocket equal to: (NEET 1994)

Example 3.62

18: A 600 kg rocket is set for vertical firing. If the exhaust speed is $1000\frac{m}{s}$, the mass of the gas ejected per second to supply the thrust needed to over the weight of the rocket is: (NEET 1990)

3.5 Further Topics

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