Arithmetic

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PART I: ADDITION AND SUBTRACTION

1. SHORTCUTS

1.1 Basics

A. Expanded Notation

Example 1

We can write a number in expanded notation. In expanded notation, we break the number into tens and ones:

$$22 = 20 + 2$$
$$32 = 30 + 2$$
$$43 = 40 + 3$$
$$57 = 50 + 7$$

$$30 + 7 = 37$$
$$20 + 4 = 24$$
$$60 + 5 = 65$$
$$70 + 2 = 72$$
$$80 + 9 = 89$$
$$50 + 1 = 51$$
$$40 + 0 = 40$$
$$60 + 8 = 68$$

B. Adding Using Expanded Notation

Example 2

$$22 + 4 = \underbrace{20 + 2}_{\textit{Breaking up 22}} + 4 = 20 + \underbrace{2 + 4}_{\textit{Add 2 and 4}} = 20 + 6 = 26$$

$$23 + 5 = \underbrace{20 + 3} + 5 = 20 + 8 = 28$$

$$54 + 3 = \underbrace{50 + 4}_{\textit{Expanded 54}} + 3 = 50 + \underbrace{4 + 3}_{\textit{Add 4 and 3}} = 50 + 7 = 57$$

$$62 + 6 = 60 + 2 + 6 = 60 + 8 = 68$$
$$43 + 6 = 40 + 3 + 6 = 40 + 9 = 49$$
$$81 + 7 = 80 + 1 + 7 = 80 + 8 = 88$$

$36 + 20 = 56$
$42 + 30 = 72$
$68 + 10 = 78$

1.2 Bridging-Two Digit Addition

A. Addition

Example 3

$45 + 23 = 45 + 20 + 3 = 55 + 3 = 58$

B. Creating a Bridge

When you want to create a bridge, you connect the number with a round figure, like

$$10,20,30 \ldots.$$

Or

$$100,200,300, \ldots$$

Example 4

Rewrite the numbers below in a form suitable for bridging:

A. 69
B. 48
C. 37
D. 19
E. 98
F. 97
G. 99

$$69 = 70 - 1$$
$$48 = 50 - 2$$
$$37 = 40 - 3$$
$$19 = 20 - 1$$

$$98 = 100 - 2$$
$$97 = 100 - 3$$
$$99 = 100 - 1$$

C. Adding Two Digits

Example 5

Use bridging to do the following additions without using pen and paper.

A. $36 + 29$
B. $23 + 18$
C. $74 + 39$
D. $26 + 19$
E. $58 + 49$

$$36 + 29 = 35 + 30 = 65$$
$$23 + 18 = 21 + 20 = 41$$
$$74 + 39 = 73 + 40 = 113$$
$$26 + 19 = 25 + 20 = 45$$
$$58 + 49 = 57 + 50 = 107$$

Example 6

A. $29 + 29$
B. $38 + 38$
C. $39 + 59$

$$29 + 29 = 30 - 1 + 30 - 1 = 60 - 2 = 58$$
$$38 + 38 = 40 + 36 = 76$$
$$39 + 59 = 38 + 60 = 98$$

Example 7

A. $29 + 49 + 59$

$$29 + 49 + 59 = 30 + 50 + 57 = 137$$
$$29 + 49 + 59 = (30 - 1) + (50 - 1) + (60 - 1) = 140 - 3 = 137$$

Example 8

A. $84 + 99$
B. $45 + 98$
C. $23 + 98$
D. $31 + 98$
E. $63 + 99$
F. $78 + 98$

$$84 + 99 = 83 + 100 = 183$$
$$45 + 98 = 43 + 100 = 143$$
$$23 + 98 = 21 + 100 = 121$$
$$31 + 98 = 29 + 100 = 129$$
$$63 + 99 = 62 + 100 = 162$$
$$78 + 98 = 76 + 100 = 176$$

1.3 Bridging-Three Digit Addition

A. Two Numbers

Example 9

A. $199 + 199$
B. $601 + 599$

$$199 + 199 = 200 + 198 = 398$$
$$601 + 599 = 600 + 600 = 1200$$

B. Three Numbers

Example 10

A. $199 + 599 + 399$

$$199 + 599 + 399 = 200 + 600 + 397 = 1197$$

1.4 Bridging-Subtraction

A. Two Digit Subtraction

Example 11

A. $92 - 19$
B. $45 - 19$
C. $56 - 18$
D. $86 - 38$

$$92 - 19 = 93 - 20 = 73$$
$$45 - 19 = 46 - 20 = 26$$
$$56 - 18 = 58 - 20 = 38$$
$$86 - 38 = 88 - 40 = 48$$

Example 12

A. $812 - 99$
B. $745 - 99$
C. $653 - 99$
D. $319 - 99$

$$812 - 99 = 813 - 100 = 713$$
$$745 - 99 = 746 - 100 = 646$$
$$653 - 99 = 654 - 100 = 554$$
$$319 - 99 = 320 - 100 = 220$$

B. Three Digit Subtraction

Example 13

A. $453 - 199$
B. $867 - 298$

$$53 - 199 = 454 - 200 = 254$$
$$867 - 298 = 869 - 300 = 569$$

2. Algorithm

2.1 Standard Algorithm for Calculations

A. Addition: No Regrouping
B. Addition: Regrouping
C. Subtraction: No Regrouping
D. Subtraction: Regrouping

3. Word Problems

3.1 Word Problems

A. Basics

Example 1

A. There were five birds. Two flew away. How many were left?
B. I had four apples. One was spoilt and became rotten. How many were good?
C. There were seven girls and two boys sitting in a row. How many children were in the row?
D. I had two chocolate muffins during the week, and three banana muffins. How many muffins did I have in all?
E. I have three brothers and two sisters. How many siblings do I have?
F. I like to eat oranges, apples and pineapples. But I don't like to eat grapes. How many fruits do I like to eat?
G. I visited three cities in Europe and four cities in Asia. How many cities in all did I visit?

B. Difference

Example 2

I have five red cars and eight blue cars.

A. What is the $\underbrace{total}_{add}$ number of cars I have?
B. How many more blue cars do I have compared to red cars?
C. What is the $\underbrace{difference}_{subtract}$ between the number of blue cars and the number of red cars?

Red	Red	Red	Red	Red			
Blue	Blue	Blue	Blue	Blue	Blue	Blue	Blue

C. Age

Example 3

I am five years old.

A. My brother is two years elder. How old is he?
B. My sister is one year younger. How old is she?
C. What is the difference in the ages of my brother and my sister?

D. Changes

Example 4

I collect stamps. I have five stamps from India, and seven stamps from Brazil.

A. What is the total number of stamps I have?
B. I got three more stamps from India. What is the number of stamps from Brazil that I have?
C. After including the new stamps from India, what is the number of stamps from India that I have?

E. Grouped Questions

Example 5

I have five tangerines, four pineapples, and three apples. I also have three aubergines and six bitter gourds.

A. What is the number of the fruits that I have?
B. What is the number of vegetables that I have?
C. Do I have more fruits, or more vegetables? How many more?

If one aubergine is spoilt, how many aubergines can I use to make dinner?

F. Mixed

Example 6: Ages

A. Disha is three years elder than her younger sister, and two years younger than her elder brother. Find the difference in the age of her brother and her sister.

$$\underbrace{7}_{Saturday} + \underbrace{7+2}_{Sunday} = 7 + 9 = 16$$

Example 7: Positions

A. I walk three feet forward and five feet backward. How far am I from my starting position?

		Starting Position	Forward 1	Forward 2	Forward 3
Back 5	Back 4	Back 3	Back 2	Back 1	

Example 8: Families

Ishita has two brothers, called Rohan and Sohan. Each of her brothers has two sisters. Find the total number of siblings in the family. (Note: Siblings mean the number of children in family.)

Boys	Girls
Rohan	Ishita
Sohan	*Ishita's Sister*

G. Multi-Step

Example 9

A. Sheela likes to eat chocolates. She eats one chocolate on Monday, two chocolates on Tuesday, three chocolates on Wednesday, *and so on*, increasing by one chocolate every day. If she follows this pattern, 2how many chocolates will she eat in one week?
B. Hitesh's weight was 75 kg. He lost 1 kg in March. In April, he lost 2 kg more than what he lost in March. In May, he lost as much weight as he lost in March and April combined. In June, he gained 2 kg. What is his weight at the end of June?

$$\underbrace{1}_{Monday} + \underbrace{2}_{Tuesday} + \underbrace{3}_{Wednesday} + \underbrace{4}_{Thursday} + \underbrace{5}_{Friday} + \underbrace{6}_{Saturday} + \underbrace{7}_{Sunday} = 28$$

$$75 - \underbrace{1}_{March} - \underbrace{3}_{April} - \underbrace{4}_{May} + \underbrace{2}_{June} = 75 - \underbrace{8}_{Weight\ Lost} + \underbrace{2}_{Weight\ Gained} = 69$$

H. More and Less

Example 10

I did three homework problems today. Yesterday, I did two more homework problems compared to today.

I. How many homework problems did I do yesterday?

How many less homework problems did I do today compared to yesterday?

Example 11: More

I ate three mangoes during the week. My sister ate two more mangoes than I did.

A. How many mangoes did my sister eat?
B. How many mangoes did we eat in all?
C. How many mangoes less than my sister did I eat?
D. Next week, I plan to eat one less mango, and my sister plans to eat two more mangoes compared to this week. How many mangoes do we plan to eat next week?

Example 12: Less

There are seven girls in a school. There are four less boys than girls in the school.

A. How many boys are there in the school?
B. How many children are there in the school in all?
C. If one girl leaves school, and two boys join school, what is the total number of children in the school?

Example 13: How many more

I have five red cars, seven blue cars, and eight yellow cars.

A. How many more blue cars do I have compared to red cars?
B. How many more yellow cars do I have compared to red cars?
C. How many more yellow cars do I have compared to blue cars?

Example 14

A. Pritam loves cars. He has seventeen toy cars, which is thirteen more cars than his brother has. Find the number of cars his brother has.
B. Shaurya likes to go fishing. He caught seven fish on Saturday, and two more fish on Sunday, as compared to Monday. What is the total number of fish he caught over the weekend?

Example 15: Comparing

I have seventeen dollars in my pocket. My friend has nineteen dollars in his pocket. Who has more money, and by how much?

I. More and Less-II

Example 16

The cost of a dollhouse is five more than the cost of a doll. If the dollhouse and the doll together cost Rs. 45, then find the cost of each.

Logic

Put Rs. 5 aside for the dollhouse, leaving us with

$$45 - 5 = 40$$

Divide the Rs. 40 equally among the doll and the dollhouse

$$\frac{40}{2} = 20$$

Hence, the costs are

$$20 + 25 = 45$$

Algebra-Style

$$Doll + \underbrace{Doll + 5}_{\textbf{\textit{Dollhouse}}} = 45 \Rightarrow 2\ Dolls + 5 = 45 \Rightarrow 2\ Dolls = 40 \Rightarrow 1\ Doll = 20$$

Example 17

A school has a total of 300 students, with the number of boys more than the number of girls by 20.

A. Find the number of girls.
B. Classes are separated by gender, with each class being either all girls or all boys. If there are four classes for the girls, find the number of girls in each class.

First, divide the students equally:

$$\frac{300}{2} = 150\ Girls + 150\ Boys$$

Girls	Boys	Difference
150	150	
$150 - 1 = 149$	$150 + 1 = 151$	2
$150 - 2 = 148$	$150 + 150 = 152$	4
$150 - 3 = 147$	153	6
$150 - 10 = 140$	$150 + 1 = 160$	20

PART II: MULTIPLICATION AND DIVISION

4. MULTIPLICATION

4.1 Calculations

A. Single Digit Multiplication

B. Multiplication: No Regrouping

Carryover									
	3	4							
	×	2							
	6	8							

C. Multiplication: Regrouping

Carryover									
	2	8							
	×	2							

4.2 Word Problems: Introduction

A. Single Step

Example 1

A. A rose flower has seven petals. What is the number of petals in five such flowers?
B. A bench in a park can seat four people. The park has five benches. How many people can sit on these benches?
C. A notebook costs three rupees. What is the cost of eight notebooks?
D. A taxi can seat four passengers? The taxi makes twelve trips from Churchgate Station to the Gateway of India. How many passengers has it transported?
E. Rivaan has three pencils. He has two pens for every pencil that he has. How many pens does he have?
F. An orchard has thirteen mango trees. Each tree has eight mangoes on it. How many mangoes are there on the trees in all?

$$5 \times 7 = 35$$
$$4 \times 5 = 20$$
$$3 \times 8 = 24$$
$$4 \times 12 = 48$$
$$3 \times 2 = 6$$
$$13 \times 8 = 104$$

Example 2

Roshan eats two apples every day of the week. The days from Monday to Friday in the week are called weekdays. And Saturday and Sunday are called weekends. A month has four weeks.

A. How many apples will he eat in a week?
B. How many apples will he eat in one week on weekdays?
C. How many apples will he eat in one week on weekends?
D. How many apples will he eat in a month?

Parts A, B and C

$$In\ One\ Week: 2 \times 7 = 14$$
$$In\ One\ Week\ on\ Weekdays: 2 \times 5 = 10$$
$$In\ One\ Week\ on\ Weekends: 2 \times 2 = 4$$

Part D

Method I

No. of Days in a Month

$$= No.of\ Weeks \times No.of\ Days\ in\ a\ Week = 7 \times 4 = 28$$

Apples eaten in a Month

$$= Apples\ per\ Day \times No.of\ Days = 28 \times 2 = 56$$

Method II

Apples eaten in a Month

$$= Apples\ eaten\ per\ Week \times No.of\ Weeks = 14 \times 4 = 56$$

4.3 Word Problems: Multi-Step

A. Multi-Step

Example 3

An apple orchard has eight trees, each with twenty apples on it. The apples are harvested, and five apples are found to be rotten. How many apples are good?

No. of Apples

$$8 \times 20 = 160$$

Good Apples

$$= Total\ Apples - Rotten\ Apples = 160 - 5 = 155$$

Example 4

A school has four classrooms. Each class has twenty students, a teacher, and a co-teacher. Apart from the people in the classrooms, the school also has the Principal, and five support staff.

A. What is the number of students in the school?
B. What is the number of teachers in the school (count co-teachers as teachers, but not the Principal or the support staff)?
C. What is the number of support staff in the school?

D. What is the number of the people in the school?

No. of Students

$$= \underbrace{20}_{\substack{Students \\ per\ Classroom}} \times 4 = 80$$

No. of Teachers in the school

$$= 2 \times 4 = 8$$

No. of Support Staff

$$= 5$$

Principal

$$= 1$$

Total

$$= \underbrace{80}_{Students} + \underbrace{8}_{Teachers} + \underbrace{5}_{\substack{Support \\ Staff}} + \underbrace{1}_{Principal} = 88 + 6 = 94$$

Example 5

Amol has four boxes of white chocolates, each with seven chocolates, and five boxes of brown chocolates, each with six chocolates. What is the total number of chocolates he has?

$$White\ Chocolates = 4 \times 7 = 28$$
$$Brown\ Chocolates = 5 \times 6 = 30$$
$$Total = 28 + 30 = 58$$

Example 6

Rishima is planning a trip around the world. She will visit three cities in Europe, four cities in the US, and five cities in Africa. Each European city has five attractions to visit; each US city has four malls to go to, and each African city has three special places to travel to. How many places is Rishima planning to visit in all?

$$European\ Attractions = 3 \times 5 = 15$$
$$US\ Malls = 4 \times 4 = 16$$
$$African\ Special\ Places = 5 \times 3 = 15$$

$$Total\ Number\ of\ Places = 15 + 16 + 15 = 46$$

4.4 Word Problems: Money and Change

A. Money

Example 7

A doll costs five dollars, and a dollhouse costs twelve dollars. Sheetal goes to the market, and buys two dolls and a dollhouse. What is the amount she must pay?

$$\underbrace{5 \times 2}_{Dolls} + \underbrace{12}_{Dollhouse} = 10 + 12 = 22$$

Example 8

A pencil costs two rupees, and a pen costs three rupees. What is the cost of seven pencils and nine pens?

$$\underbrace{2 \times 7}_{Pencils} + \underbrace{3 \times 9}_{Pens} = 14 + 27 = 41$$

Example 9
A pencil costs three rupees. A box of pencils has eight pencils in it. What is the cost of five boxes of pencils?

$$\underbrace{3}_{\substack{Cost\ of \\ a\ Pencil}} \times \underbrace{8}_{\substack{Pencils \\ per\ box}} \times \underbrace{5}_{\substack{No.of \\ Boxes}} = 3 \times 40 = 120\ Rupees$$

B. Change

Example 10
Michelle goes to the store to buy sweets worth Rs. 36. She pays with a fifty rupee note. What is the change she will receive from the shopkeeper?

$$Change = Money\ Given - Cost\ of\ Item = 50 - 36 = 14$$

Example 11
A chocolate costs Rs. 5.

A. What is the cost of eight such chocolates?
B. If you pay for the eight chocolates using a hundred rupee note, what is the change you will receive?

$$5 \times 8 = 40$$

Change

$$= Money\ Given - Cost\ of\ Chocolates = \ 100 - 40 = 60$$

4.5 Word Problems: Concepts

A. Concepts

Example 12
A. Shirley multiplied the number seven by another number, and got seven as her answer. What number could she have multiplied by?
B. Ashok multiplied a hundred and twelve by another number and got zero as his answer. What number did he multiply by?

$$7 \times 1 = 7 \Rightarrow Number = 1$$
$$112 \times 0 = 0 \Rightarrow Number = 0$$

4.6 Word Problems: Grouped

A. Group Questions

In a group, we generally have some information given to us at the beginning. After the information, we get two or more questions, based on the information which is given.
Group questions are generally longer than usual questions, so you need to be more careful.

Example 13

Read the information below, and answer the questions that follow.

A row of houses along a street has a yellow house, a red house, a white house, and a green house. Each house has two pets. Each house also has three cars. The total number of people living in all the houses is twelve. There are seven adults in all of the houses put together, and the rest are children.

A. Pets
   I. What is the total number of pets?
   II. Two of the pets are parrots, and the rest of the pets are dogs. How many dogs are there?
   III. How many legs do the pets have together?

B. Cars
   I. How many cars are there?
   II. Each car has four wheels, and also an extra wheel is kept in each car as a spare tire for tire flats. How many wheels are there in total?

C. People
   I. How many children live in the row of houses?
   II. If the yellow house has a single adult living in it, and the rest of the houses have the same number of adults, then what is the number of adults in the green house and the white house combined?

Part A: Pets

$$No.of\ Pets = \underbrace{4}_{\substack{No.of\\Houses}} \times \underbrace{2}_{\substack{Pets\ per\\House}} = 8$$

$$No.of\ Dogs = Total\ Pets - No.Of\ Parrots = 8 - 2 = 6$$

$$No.of\ Legs = \underbrace{2 \times 2}_{Parrots} + \underbrace{6 \times 4}_{Dogs} = 4 + 24 = 28$$

Part B: Cars

$$No.of\ Cars = 3 \times 4 = 12$$

$$No.of\ Wheels = 12 \times 5 = 60$$

Part C: People

$$No.of\ Children = 12 - 7 = 5$$

$$Adults\ not\ in\ Yellow\ House = 7 - 1 = 6$$

$$Adults\ in\ each\ house\ non - Yellow - House = \frac{6}{3} = 2$$

$$Adults\ in\ Green\ House + White\ House = 2 + 2 = 4$$

4.7 Word Problems: Number Theory

A. Digits

The digits of a number are the digits that make up the number.

Example 14

Find the sum of 4 and 5.

$$4 + 5 = 9$$

Find the difference between 8 and 3

$$8 - 3 = 5$$

Example 15: Working with Digits

In each number below:

- Identify the digits
- State the number of digits
- Find the sum of the digits.
- Find the difference between the largest digit and the smallest digit
- Find the product of the digits

A. 27
B. 48
C. 81
D. 342
E. 103

Digits	No. of Digits	Sum of Digits	Difference	Product of Digits
2,7		$2+7=9$	$7-2=5$	$2\times 7=14$
4,8		$4+8=12$		$4\times 8=32$
8,1		$8+1=9$		$8\times 1=8$
3,4,2		$3+4+2=9$		
1,0,3				

$$,\qquad Product\ of\ Digits = 3\times 4\times 2 = 24$$

The digits in 103 are

$$1, 0\ and\ 3 \Rightarrow 3\ Digits, \qquad Sum\ of\ Digits = 1+0+3=4, \qquad Product\ of\ Digits = 1\times 0\times 3 = 0$$

B. Sum of Digits

Example 16

Find the two-digit number that has

A. Maximum sum of digits
B. Minimum sum of digits

Maximum value of a digit is 9. Hence, the number with maximum sum of digits will be

$$99 \rightarrow 9+9=18$$

Maximum value of a digit is 0. But the first digit cannot be zero, since the number is a two-digit number. Hence, the number with minimum sum of digits will be

$$10 \rightarrow 1+0=1$$

Example 17

The sum of digits of a two-digit number is three. Find the possible values of the number.

$$12 \rightarrow Digits\ are\ 1\ and\ 2 \rightarrow Sum = 3$$
$$21 \rightarrow Digits\ are\ 2\ and\ 1 \rightarrow Sum = 3$$
$$30 \rightarrow Digits\ are\ 3\ and\ 0 \rightarrow Sum = 3$$

Example 18

The sum of digits of a two-digit number is four. Find the possible values of the number.

$$13 \rightarrow Digits\ are\ 1\ and\ 3 \rightarrow Sum = 3$$
$$22 \rightarrow Digits\ are\ 2\ and\ 2 \rightarrow Sum = 3$$
$$31 \rightarrow Digits\ are\ 3\ and\ 1 \rightarrow Sum = 3$$
$$40 \rightarrow Digits\ are\ 4\ and\ 0 \rightarrow Sum = 3$$

Example 19

The sum of digits of a two-digit number is five. Find the possible values of the number.

$$14 \rightarrow Digits\ are\ 1\ and\ 2 \rightarrow Sum = 3$$
$$23 \rightarrow Digits\ are\ 1\ and\ 2 \rightarrow Sum = 3$$
$$32 \rightarrow Digits\ are\ 3\ and\ 0 \rightarrow Sum = 3$$
$$41 \rightarrow Digits\ are\ 3\ and\ 0 \rightarrow Sum = 3$$
$$50 \rightarrow Digits\ are\ 2\ and\ 1 \rightarrow Sum = 3$$

Example 20

The sum of digits of a two-digit number is six. Find how many such numbers are there?

$$60,\quad 51,\quad 42,\quad 33,\quad 24,\quad 15 \Rightarrow 6\ Numbers$$

Example 21

The sum of digits of a two-digit number is seven. Find how many such numbers are there?

$$70,\quad 61,\quad 52,\quad 43,\quad 34,\quad 25,\quad 16 \Rightarrow\ Numbers$$

C. Difference of Digits

D. Product of Digits

Example 22

Find the two-digit number with

A. Maximum product of digits
B. Minimum product of digits

$$99 \rightarrow 9 \times 9 = 81$$

There are quite a few numbers that have product of digits zero. These numbers are:

10,20,30,40,50,60,70,80,90

Example 23

The product of the digits of a number is 2. Find the possible values of that number.

$$12,\quad 21$$

Example 24

The product of the digits of a number is 3. Find the possible values of that number.

13, 31

Example 25

The product of the digits of a number is 4. Find the possible values of that number.

14, 22, 41

Example 26

The product of the digits of a number is 5. Find the possible values of that number.

51, 15

Example 27

The product of the digits of a number is 6. Find the possible values of that number.

61, 32, 23, 16

Example 28

The product of the digits of a number is 7. Find the possible values of that number.

71, 17

Example 29

The product of the digits of a number is 8. Find the possible values of that number.

81, 42, , 24, 18

5. Division

5.1 Direct Division

A. Division Calculations

Example 1

$$12 \div 2 = 6$$
$$21 \div 3 = 7$$
$$25 \div 5 = 5$$
$$28 \div 4 = 7$$

5.2 Long Division

A. Two Digits by One Digit

Example 2

Divide:

A. 48 by 4
B. 57 by 4
C. 78 by 6
D. 29 by 2
E. 91 by 7
F. 56 by 4
G. 73 by 4

	1	2
4	4	8
	−4	
	0	8
		−8
		0

	1	4
4	5	7
	−4	
	1	7
	−1	6
		1

B. Three Digits by One Digit

Example 3

Divide:

A. 136 by 3
B. 753 by 2
C. 719 by 8
D. 285 by 3
E. 554 by 4
F. 456 by 6
G. 918 by 4
H. 819 by 9

	0	4	5
3	1	3	6
	−1	2	
		1	6
		−1	5
			1

	3	7	6
2	7	5	3
	6		
	1	5	
	1	4	
		1	3
		1	2
			1

	0	8	9
8	7	1	9
	6	4	
		7	9
		7	2
			7

	0	9	5
3	2	8	5
	2	7	
		1	5
		1	5
			0

	1	3	8
4	5	5	4
	4		
	1	5	
	1	2	
		3	4
		3	2
			2

Example 4

Divide 4671 by 6.

	0	7	7	8
6	4	6	7	1
	4	2		
		4	7	
		4	2	
			5	1
			4	8
				3

5.3 Word Problems

A. Real Life Scenarios

Example 5

Farmer John has an orchard with six apple trees in it. He needs to share the trees among his two children, Shalu and Neelu. How many trees will each child inherit?

Tree 1	Tree 2	Tree 3	Tree 4	Tree 5	Tree 6

Shalu	Tree 1	Tree 2	Tree 3
Neelu	Tree 4	Tree 5	Tree 6

Shorter Method:

$$2 \times 3 = 6 \Rightarrow 6 \div 3 = 2$$

Example 6

Neerav is doing his HW. He has 40 questions to do. He is able to do five questions every minute. How many minutes will he take to complete his HW?

$$40 \div 5 = \frac{40}{5} = 8$$

Example 7

A classroom has 30 benches and five rows of benches. Find the number of benches in each row?

$$\frac{30}{5} = 6$$

Example 8

100 strawberries are packed in boxes of six. Find the number of complete boxes that can be packed.

$$\frac{100}{6} = 16\ Rem\ 4$$

Example 9

A picnic has a group of people travelling by boat as follows:

$$\underbrace{3}_{Boat\ 1}\ ,\ \underbrace{5}_{Boat\ 2}\ ,\ \underbrace{4}_{Boat\ 3}\ ,\ \underbrace{8}_{Boat\ 4}$$

If the people change their seating so that there are an equal number of people in each boat, then how many will there be?

Total Number of People is

$$3 + 5 + 4 + 8 = 20$$

Number of people in each boat

$$\frac{20}{4} = 5$$

5.4 Number Sentences

A. Introduction

Example 10

50 chocolates are distributed among five children. Which is the number sentence that can be used to find the number of chocolates that each child got?

$$\underbrace{50}_{Chocolates} \div \underbrace{5}_{Children} = \underbrace{10}_{\substack{Chocolates \\ Each}}$$

B. Remainder

Example 11

A Monster-Sized Pizza has 28 slices. Vivina is with her friends. Each friend wants to eat at least three slices of pizza. How many friends can share a Monster Pizza?

$$\frac{28}{3} = 9\ Remainder\ 1 \Rightarrow 10\ Friends$$

Example 12

A Monster-Pizza costs $3. Vivina has 28 dollars with her. What is the number of pizzas that she can buy?

$$\frac{28}{3} = 9\ Remainder\ 1 \Rightarrow 9\ Pizzas$$

Example 13

A Monster-Sized Pizza has 28 slices. Vivina is with her friends. Each friend wants to eat at most three slices of pizza. What is the number of friends needed to completely eat a Monster Pizza?

$$\frac{28}{3} = 9\ Remainder\ 1 \Rightarrow 10\ Friends$$

Example 14

Rushabh has 26 blocks, each of which has one of the letters of the English alphabet on it. He puts his blocks in groups of four.

A. How many groups of four blocks can he make?
B. After putting his blocks in groups of four, he has a few left over. What is the number of blocks left over?

$$\frac{26}{4} = 6\ Rem\ 2 \Rightarrow 6\ Groups$$
$$2\ Blocks\ left\ over$$

C. Remainder: Applications

Example 15

Riddhi has 25 chewing gums which she got from her trip to the USA. She eats three chewing gums every week, because she really wants them to last.

A. For how many weeks will she get to eat three chewing gums?
B. How many chewing gums will she eat in the last week?

Number of Weeks

$$No.of\ Weeks = \frac{25}{3} = 8\ Rem\ 1$$

We interested in the number of weeks she will get to eat three chewing gums, and hence that is eight.

Chewing Gums in the last week

In eight weeks, Riddhi will eat

$$24\ chewing\ gums$$

Leaving her with

$$25 - 24 = 1\ Chewing\ gum$$

For the last week

Example 16

Ayan has some pencils. His school works from Monday to Friday. He decides to use take the same number of pencils each day to school. He also decides never to take the same pencil to school twice. After the first week of school, Ayan has three pencils which he has not taken to school. What is the smallest number of pencils that Ayan can have?

To find the smallest number of pencils, assume that Ayan takes 1 pencil to school each day:

$$\underbrace{1}_{Mon} + \underbrace{1}_{Tue} + \underbrace{1}_{Wed} + \underbrace{1}_{Thu} + \underbrace{1}_{Fri} = 5$$

And he also has three pencils left over:

$$\underbrace{5}_{\substack{Taken\ to\\ School}} + \underbrace{3}_{At\ Home} = \underbrace{8}_{Total}$$

In the previous question, you found that the smallest number of pencils that Ayan can have is eight. Write a number sentence to confirm that the answer that you have found is correct.

$$\frac{8}{5} = 1\ Remainder\ 3$$

Example 17

Twenty-three children are going on a picnic. They get onto four buses. The children are divided among the buses equally as far as possible. Find the number of children in each bus.

We have twenty-three children, and hence we have:

$$\frac{23}{4} = 5\ Remainder\ 3$$

If there were twenty children, we could have five children on each bus.

$$\frac{20}{4} = 5\ Remainder\ 0 \Rightarrow \left\{ \underbrace{5}_{Bus\ 1}, \underbrace{5}_{Bus\ 2}, \underbrace{5}_{Bus\ 3}, \underbrace{5}_{Bus\ 4} \right\}$$

We need to distribute the remaining three children among the four buses, and hence the best we can do is:

$$\left\{ \underbrace{6}_{Bus\ 1}, \underbrace{6}_{Bus\ 2}, \underbrace{6}_{Bus\ 3}, \underbrace{5}_{Bus\ 4} \right\}$$

So, three buses will have six children each, and one bus will have five children.

5.5 Properties

A. Terminology

Properties of Division

$$\frac{Dividend}{Divisor} = Quotient + Remainder$$
$$Divisor \times Quotient + Remainder = Dividend$$

Example 1.18

I have thirty-six apples to be shared among nine children. How many apples will each child get? Write this out in the form of a number sentence. Explain what it means.

$$\frac{36}{9} = 4 + 0$$

When I share 36 apples among 9 children, each child will get 4 apples, and there are no apples left over.

$$\frac{Dividend}{Divisor} = Quotient + Remainder$$

$$\underbrace{36}_{Dividend} \div \underbrace{9}_{Divisor} = \underbrace{4}_{Quotient} + \underbrace{0}_{Remainder}$$

When I divide 36 by 9, I am able to make the division without any remainder.

In the case of apples, I am able to share the apples equally, without any apples left over.

Consider the statement given above, where:

$$\frac{36}{9} = 4 + 0$$

Write this statement in the form:

$$Divisor \times Quotient + Remainder = Dividend$$

$$\underbrace{4}_{Divisor} \times \underbrace{9}_{Quotient} + \underbrace{0}_{Remainder} = \underbrace{36}_{Dividend}$$

If I multiply the divisor with the quotient, and add my remainder, I will get back my original number.

In the case of apples, each child has

$$4\ Apples$$

And there are

$$9\ Children$$

Hence, the number of apples with the children is:

$$4 \times 9 = 36$$

And, there are no apples left over, so the total number of apples is:

$$36 + 0 = 36$$

B. Verifying Division

The property

$$Divisor \times Quotient + Remainder = Dividend$$

Gives a way of checking that a division has been carried out correctly, by performing a multiplication. Since the calculations are different, the chances of making the same mistake go down substantially.

Example 1.19

$\frac{28}{5} = Quotient\ 5\ Remainder\ 3$

Verify the above division using a suitable multiplication.

$$5 \times 5 + 3 = 25 + 3 = 28$$

Example 1.20

In each question below, verify the given division using a suitable multiplication.

A. $\frac{27}{6} = Quotient\ 4\ Remainder\ 3$

B.

Example 1.21

Note the division of $34 \div 4$ that has been carried out using the long division algorithm. Verify this division using a suitable multiplication.

	0	8
4	3	4
−	3	2
		2

$$5 \times 5 + 3 = 25 + 3 = 28$$

C. Remainder

Example 1.22

There are a large number of students from a school (more than 20), that have gone for a picnic. While the number of students is not known, it is known that it is odd. On reaching the hiking trail, the teacher asks them to walk in pairs for their safety. Then:

A. The students will not be able to form pairs.
B. The students will be able to form pairs without the teacher.
C. The students will be able to form pairs only if the teacher is in one of the pairs.
D. The students will not be able to form pairs even if the teacher is in one of the pairs.

$$Students \rightarrow Odd \rightarrow Cannot\ form\ pairs, One\ left\ over$$
$$Students + Teacher \rightarrow Even \rightarrow Can\ form\ pairs, None\ left\ over$$

Example 1.23

A group of school students are travelling to a theme park in SUV's, such that each SUV has a capacity of seven people, including the driver. None of the students are driving. There are a few SUV's that are all full, and the last SUV has four people in it. If there are a minimum of 50 students in the group, what is the smallest number of students that could be in the group?

Note that the number of students in a full SUV is

$$7 - 1 = 6$$

Hence, the number of students in the full SUV's can be:

$$6 \times 8 = 48, which\ is\ less\ than\ 50$$

And, the number of students in the last SUV is

$$4 - 1 = 3$$

And hence the smallest value that the total number of students can take is:

$$48 + 3 = 51$$

Example 1.24

I have some muffins to be shared among a group of students. Which of the statements below correctly represents what will happen when I share the muffins? (More than one answer may be correct).

A. Twenty-seven muffins shared among five students will result in six muffins each, with two left over.
B. Twenty-seven muffins shared among four students will result in six muffins each, with three left over.
C. Twenty muffins shared among seven students will result in six muffins each, with two left over.
D. Twenty muffins shared among seven students will result in two muffins each, with six left over.

Option A

$$27 \div 5 = Quotient\ 5\ Remainder\ 2$$

Option says six muffins, not 5.
Hence, it is incorrect.

Option B

$$27 \div 4 = Quotient\ 6\ Remainder\ 3$$

Option says six muffins, with 3 left over.
Hence, it is correct.

Option C and D

$$20 \div 7 = Quotient\ 2\ Remainder\ 6$$

Option C says six muffins, with 2 left over.
Option D says two muffins, with 6 left over.
Hence, option C is incorrect.
Option D is correct.

5.6 Mixed Problems: Multiplication and Division

Example 1.25

Wribhu has six wooden bookcases that have eight books each. He also has three metal bookcases that have four books each. He takes all the books and divides them among his five siblings. How many books will each sibling get?

First, let's calculate the total number of books:

$$\underbrace{6 \times 8}_{\substack{Wooden \\ Bookcases}} + \underbrace{3 \times 4}_{\substack{Metal \\ Bookcases}} = 48 + 12 = 60$$

Then, let's divide the books among the five siblings:

$$\frac{60}{5} = 12$$

Example 26

Sheela got fifteen crackers for lunch. She shared her lunch with her friends. First, she divided the crackers equally among them. But Nilesh was not hungry, and just nibbled at his crackers. Finally, he ended up having two less than his fair share. Rakesh was voraciously hungry. He ate two more than his fair share. And Sheela just ate what she had given herself. Find the number of biscuits that each person ate.

A fair share of biscuits would have been

$$\frac{15}{3} = 5\ Biscuits$$

However, we know that Nilesh ate only

$$5 - 2 = 3\ Biscuits$$

And Rakesh ate

$$5 + 2 = 7\ Biscuits$$

And, finally Sheela ate her fair share

$$= 5\ Biscuits$$

Example 27

Three friends go for a picnic. Each of them has three cakes. The first one cut his cakes into three pieces each, the second one cut his cakes into four pieces each, and the third one cuts his cakes into five pieces each. Each of the friends eats nine pieces. What is the total number of pieces left?

$$First\ Friend = 3 \times 3 = 9\ Pieces \rightarrow 9 - 9 = 0\ Pieces$$
$$Second\ Friend = 4 \times 3 = 12 \rightarrow 12 - = 3\ Pieces\ left$$
$$Third\ Friend = 5 \times 3 = 15 \rightarrow 15 - 9 = 6\ Pieces\ left$$

$$Total\ Pieces\ Left = 0 + 3 + 6 = 9\ Pieces$$

6. Mixed Operations

6.1 Order of Operations

A. Background

Consider the expression

$$3 + 2 \times 3$$

If we do the multiplication first, we get:

$$3 + \underbrace{2 \times 3}_{\substack{Multplication \\ First}} = 3 + 6 = 9 = Answer$$

On the other hand, if we do the addition first, we get:

$$\underbrace{3 + 2}_{\substack{Addition \\ First}} \times 3 = 5 \times 3 = 15 = Answer$$

Hence, depending upon which operation we do first, we get different answers.
A fundamental principle in Maths is that any calculations has to have precisely one answer. There cannot be multiple answers.

Hence, we decide an order of priority for arithmetic operations. This is covered next.

B. PEMDAS/BODMAS/BIDMAS

As seen above, we need to have an order of priority. This given by

- **B**rackets **(Highest order of priority)**
- **O**f
- **D**ivision/**M**ultiplication
- **A**ddition/**S**ubtraction **(Lowest Order of Priority)**

If you put the first few letters of the operations together, you get:

BODMAS
BEODMAS

Example 6.1
Simplify $1 + 1 - 1 + 1$

If you do:

$$1 + 1 - 1 + 1 = 2 - 1 + 1 = 1 + 1 = 2 \ (\boldsymbol{Correct})$$

On the other hand, if you add the last 1 to the 1 before it, that is incorrect, since this ignores the minus sign before that number.

$$1 + 1 - 1 + 1 = 2 - 2 = 0 \ (\boldsymbol{Incorrect})$$

Example 6.2
Simplify:

A. $3 + 2 \times 3$
B. $8 - 2 \times 3$
C. $3 \times 2 - 2 \div 2$

D. $12 \div 4 - 12 \div 6$
E. $4 \times 5 - 2 \div 1 + 3$
F. $2 + 3 + 2 \times 2 - 6 \div 3$
G. $6 \div 3 + 2 \div 1 - 8 \div 2 + 2 \times 4$

Part A

$$3 + \underbrace{2 \times 3}_{Multiplication} = 3 + 6 = 9$$

Part B

$$8 - \underbrace{2 \times 3}_{Multiplication} = 8 - 6 = 2$$

Part C

$$\underbrace{3 \times 2}_{Multiplication} - \underbrace{2 \div 2}_{Division} = 6 - 1 = 5$$

Part D

$$\underbrace{12 \div 4}_{Division} - \underbrace{12 \div 6}_{Division} = 3 - 2 = 1$$

Part E

$$\underbrace{4 \times 5}_{Multplication} - \underbrace{2 \div 1}_{Division} + 3 = 20 - 2 + 3 = 18 + 3 = 21$$

Part F

$$2 + 3 + \underbrace{2 \times 2}_{Multplication} - \underbrace{6 \div 3}_{Division} = 2 + 3 + 4 - 2 = 7$$

Part G

$$\underbrace{6 \div 3}_{Division} + \underbrace{2 \div 1}_{Divsion} - \underbrace{8 \div 2}_{Division} + \underbrace{2 \times 4}_{Multiplication} = 2 + 2 - 4 + 8 = 8$$

C. Series

Series is when you have numbers together. A series of numbers generally has a particular rule that it follows. Sometimes, applying ideas or tricks helps us find the value of a series faster.

Example 6.3

Simplify the following expressions.

A. $10 - 9 + 8 - 7 + 6 - 5 + 4 - 3 + 2 - 1$
B. $1 - 2 + 3 - 4 + 5 - 6 + 7$
C. $1 \times 1 + 2 \times 2 - 3 \times 3 + 4 \times 4$
D. $1 \div 1 + 2 \div 2 - 3 \div 3 + 4 \div 4$
E.

Part A

This becomes much easier, if we make pairs out of the numbers:

$$\underbrace{10 - 9}_{Pair\ 1} + \underbrace{8 - 7}_{Pair\ 2} + \underbrace{6 - 5}_{Pair\ 3} + \underbrace{4 - 3}_{Pair\ 4} + \underbrace{2 - 1}_{Pair\ 5} = 1 + 1 + 1 + 1 + 1 = 5$$

Part B

There are two ways of doing this.
First method is to do all the addition first:

$$\underbrace{1 + 3 + 5 + 7}_{Addition} - \underbrace{2 - 4 - 6}_{Subtraction} = 16 - 2 - 4 - 6 = 4$$

Another way is to change the order of the numbers so that we have the rightmost number first. Remember that

we do not change the sign of the numbers:

$$7-6+5-4+3-2+1=1+1+1+1=4$$

Part C

$$\underbrace{1\times 1}_{Multiplication}+\underbrace{2\times 2}_{Multiplication}-\underbrace{3\times 3}_{Multiplication}+\underbrace{4\times 4}_{Multiplication}=1+4-9+16=12$$

Part D

$$\underbrace{1\div 1}_{Division}+\underbrace{2\div 2}_{Division}-\underbrace{3\div 3}_{Division}+\underbrace{4\div 4}_{Division}=1+1-1+1=2$$

D. Brackets

Brackets are used to change the order of priority in the regular precedence given by the order of operations.

Example 6.4

A. $3+2\times 3$
B. $(3+2)\times 3$

$$3+\underbrace{2\times 3}_{This\ First}=3+6=9$$
$$\underbrace{(3+2)}_{This\ First}\times 3=5\times 3=15$$

Example 6.5

Simplify $(4+3)\times 2-(4+3\times 2)$ and write your answer as a single number.

$$(7)\times 2-(4+6)=14-10=4$$

Example 6.6

Let

$$a=3\times 3+2,\qquad b=3\times(3+2)$$

Which of a and b is greater and by how much?

$$a=9+2=11$$
$$b=3\times 5=15$$

b is greater than a by:

$$15-11=4$$

Example 6.7

In which of the expressions below will removing the brackets not result in a change in the final answer. (There may be more than one correct answer).

A. $13\times(4+3)$
B. $13+(4\times 3)$
C. $13-(4\times 3)$
D. $13-(4-3)$
E. $13\times(4-3)$

This question does not require us to calculate the answer. All we need to do is find out whether removing the brackets will change the final answer.

Part A

$$13 \times \underbrace{(4+3)}_{Addition\ First} \rightarrow \underbrace{13 \times 4}_{\substack{Multiplication \\ First}} + 3 \Rightarrow Answers\ Changes$$

Part B

$$13 + \underbrace{(4 \times 3)}_{\substack{Multiplication \\ First}} \rightarrow 13 + \underbrace{4 \times 3}_{\substack{Multiplication \\ First}}$$

Part C

$$13 - \underbrace{(4 \times 3)}_{\substack{Multiplication \\ First}} \rightarrow 13 - \underbrace{4 \times 3}_{\substack{Multiplication \\ First}}$$

Part D

$$13 - (4 - 3) = 13 - 1 = 12$$
$$13 - 4 - 3 = 6$$

The two answers are different.
Hence, removing the brackets changes the answer here.

E. Nested Brackets

When you have brackets inside brackets, you simplify the inner brackets before simplify the outer brackets.

Example 6.8

A. $5 + [(3 - 2) \times 2]$
B. $10 - \{[7 - (2 + 1) \times 2] - 1\}$

Part A

$$5 + [1 \times 2] = 5 + 2 = 7$$

Part B

$$10 - \{[7 - 3 \times 2] - 1\} = 10 - \{[7 - 6] - 1\} = 10 - \{1 - 1\} = 10 - 0 = 10$$

F. Vinculum

When you have more than 3 brackets, it is customary to use a vinculum to show priority.

Example 6.9

Simplify

A. $\{[(\overline{2+3} \times 4) - 2] \times 2\} - 4$

$$\left\{\left[\left(\underbrace{\overline{2+3}}_{This\ First} \times 4\right) - 2\right] \times 2\right\} - 4$$
$$= \{[(5 \times 4) - 2] \times 2\} - 4$$
$$= \{[20 - 2] \times 2\} - 4 = \{18 \times 2\} - 4 = 36 - 4 = 32$$

G. Multiplying by Zero

Example 10

A. $[3 - (2 + 1) \times 0] + 1$
B. $[97 - (98 + 99) \times 0] + 100$

C.

$$[3-(2+1)\times 0]+1=[3-0]+1=3+1=4$$

$$[97-(98+99)\times 0]+100=[97-0]+100=97+100=197$$

H. Factoring

Example 11

A. $99\times 7+99\times 3$

$$99\times 7+99\times 3=99\times 10=990$$

I. Comparison

Example 12

Compare the two expressions and insert the correct sign $(<,>,=)$ between them

A. $5-4\times 1 \quad 12-2\times 2$
B. $4-4\div 4 \quad 8-8\div 8$
C. $5\times 5-5 \quad 8+8\div 8$
D. $3+3\times 3-3\div 3 \quad 2\times 2\times 2+2-2\div 2$

Part B

$$4-4\div 4 \quad 8-8\div 8\Rightarrow 4-1 \quad 8-1\Rightarrow 3<7$$

J. Correct Expressions

Example 13

Identify whether the expression is correct.

A.

K. Fill in the Blanks

Example 14

L. Word Problems

Example 15

Consider the expression:

6.2 Mixed Operations

A. Mixed Word Problems

In mixed words, you can have a mix of:

- Addition
- Subtraction

- Multiplication
- Division

You need to read the problem and understand which arithmetic operation is to be applied. Further, the order in which you apply the operations is important.

Note that

$$Doubled\ means\ multiply\ by\ 2$$
$$Tripled\ means\ multiply\ by\ 3$$
$$Quadrupled\ means\ multiply\ by\ 4$$
$$Quintupled\ means\ multiply\ by\ 5$$

Example 6.16

Jason Grace had seven friends. He moved to a new school so the number of friends he had doubled. Then, he made three more friends. Find the number of friends he has now.

$$Original\ Friends = 7$$
$$Doubled\ Friends = 7 \times 2 = 14$$
$$Add\ Three\ Friends = 14 + 3 = 17$$

Example 6.17

Timothy takes care of stray dogs on the street. He saw that a road had twelve stray dogs on it. He got six of them adopted. And from the rest, he noticed that half were puppies. How many adult dogs were left on that road?

$$Original\ Dogs = 12$$
$$New\ No.of\ Dogs = 12 - 6 = 6$$
$$No.of\ Puppies = \frac{6}{2} = 3$$
$$Adult\ Dogs = 6 - 3 = 3$$

Example 6.18

Siddhartha bakes cakes for his bakery. He baked twelve cakes on Monday, and sold seven of them. He baked nine cakes on Tuesday, and ten cakes were sold from the bakery. If the bakery had three cakes when it closed on Sunday night, and one cake each got spoilt on Monday and Tuesday, how many cakes does the bakery have it closes Tuesday night?

$$Sunday\ Night: 3$$
$$Monday: 3 + 12 - 7 - 1 = 7$$
$$Tuesday: 7 + 9 - 10 - 1 = 5$$

Example 6.19

Shaun loves books. He has a hundred books. Half of the books that he has are fiction, and the other half are non-fiction. Of the fiction books that he has, one-fifth are fantasy. Three of his fantasy books are from the *Percy Jackson* series.

A. How many fiction books does Shaun have?
B. How many fantasy books does Shaun have?
C. How many fantasy books does Shaun have that are not Percy Jackson?

$$Original\ Books = 100$$
$$Fiction\ Books = Half\ of\ 100 = 50$$
$$Fantasy\ Books = One = Fifth\ of\ 50 = \frac{50}{5} = 10$$
$$Non - Percy\ Jackson\ Fantasy\ Books = 10 - 3 = 7$$

B. Back Calculations

Example 6.20

A number is doubled, then two is added to it. Then it is tripled, and three is added to it. Finally, the result is 27. Find the original number.

$$Result\ is\ 27$$
$$Before\ adding\ 3, it\ was\ 27 - 3 = 24$$
$$Before\ tripling, it\ was \frac{24}{3} = 8$$
$$Before\ adding\ two, it\ was\ 8 - 2 = 6$$
$$Before\ doubling, it\ was \frac{6}{2} = 3$$

Example 6.21

At 8.00 pm, there were a few patients waiting for Dr. Doolittle, but the beloved doctor had not reached his clinic. At 9.00 pm, the number of patients had doubled, all patiently waited for him. The doctor arrived at 9.00 pm. No patients arrived after 9.00 pm, but still ten patients were left even after the doctor dispensed medicine to six patients. How many patients did the clinic have at 8.00 pm?

We need to work backwards:

$$Ending\ Patients = 10$$
$$Patients\ before\ dispensing\ medicine = 10 + 6 = 16$$
$$Patients\ at\ 8.00\ pm = Half\ of\ patients\ at\ 9.00\ pm = Half\ of\ 16 = 8$$

C. Group Questions

Example 6.22

A hotel has three types of rooms: standard rooms, deluxe rooms, and suites. The hotel has four floors. The bottom two floors each have 3 standard rooms, and 2 deluxe rooms. The top two floors each have 2 deluxe rooms, and one suite. What is the number of rooms of each type?

Bottom Two Floors:

$$Standard\ Rooms = 3 \times 2 = 6$$
$$Deluxe\ Rooms = 2 \times 2 = 4$$

Top Two Floors:

$$Deluxe\ Rooms = 2 \times 2 = 4$$
$$Suites = 1 \times 2 = 2$$

Hence, the final answer is:

$$Standard\ Rooms = 6 + 0 = 6$$

$$Deluxe\ Rooms = 4 + 4 = 8$$
$$Suites = 2$$

A standard room costs $50, a deluxe room costs $100, and a suite costs $150 per night. On a particular night, one suite, two deluxe rooms, and three standard rooms are empty. What is the money made by the hotel?

Standard Rooms occupied are

$$6 - 3 = 3$$
$$Money\ from\ Standard\ Room: 3 \times 50 = 150$$

Deluxe Rooms occupied are:

$$8 - 2 = 6$$
$$Money\ from\ Deluxe\ Rooms: 6 \times 100 = 600$$

Suites occupied are:

$$2 - 1 = 1$$
$$From\ Suites: 1 \times 150 = 150$$

Total Money

$$= 150 + 600 + 150 = 900$$

Every day the hotel has to spend 600 dollars on its expenses, including staff, electricity and other expenses. On the night mentioned above, how much money did the hotel make?

Money hotel made

$$= Revenue - Expenses = 900 - 600 = 300$$

6.3 Basics of Ratios

Example 6.23

Sam has three blue cars for every five yellow cars he has. If he has a total of forty cars, find:

A. The number of blue cars
B. The number of yellow cars
C. How many more yellow cars he has than blue.

						Total	Multiplication
Blue	3	3	3	3	3	15	3×5
Yellow	5	5	5	5	5	25	5×5
Total	8	8	8	8	8	40	8×5

$$Blue\ Cars = 15, Yellow\ Cars = 25, Difference = 10$$

Example 24

Harmeet has four non-fiction books for every three fiction books he has. If he has thirty-five books in all, find:

A. The number of non-fiction books he has
B. The number of fiction books he has
C. How many fiction books he must buy so that the number of non-fiction and fiction books are the same? Once he makes the purchase, how many books will he have in all.

						Total

Non-Fiction Books	4	4	4	4	4	20
Fiction Books	3	3	3	3	3	15
	7	7	7	7	7	35

Example 25

In a garden, for every 7 roses, there are 4 sunflowers. If there are a total of 42 tulips:

A. What is the total of number of sunflowers?
B. What is the total number of flowers?
C. What is the difference between the number of roses and the number of sunflowers?

							Total
Roses	7	7	7	7	7	7	42
Sunflowers	4	4	4	4	4	4	24
Total	11	11	11	11	11	11	66

Part III: Properties of Numbers

7. Place Value and Comparisons

7.1 Number Sense

A. Predecessor and Successor

The number that comes just before another number is called its predecessor.
The number that comes just after another number is called it successor.

Example 7.1

Find the predecessor and successor of the following numbers

A. 7
B. 19
C. 100
D. 541
E. 1000
F. 6999
G. 10,000

B. Place Value Chart

Indian	Ten Crore	Crore	Ten Lakhs	Lakhs	Ten Thousands	Thousands	Hundreds	Tens	Ones
International	Hundred Million	Ten Millions	Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones

Example 7.2

Find the predecessor and successor of the following numbers:

A. 1,00,000
B. 1,000,000

Part A

$$Predecessor\ of\ 100{,}000 = 99{,}999$$
$$Sucessor\ of\ 100{,}000 = 1{,}00{,}001$$

Part B

$$Predecessor\ of\ 1{,}000{,}000 = 999{,}999$$
$$Sucessor\ of\ 1{,}000{,}000 = 1{,}000{,}001$$

Example 7.3

Convert the following numbers into words in both Indian notation, and international notation.

Six Figure Numbers

A. 5,34,612
B. 7,00,002

Seven Figure Numbers

C. 1,000,002
D. 8,456,142

Eight Figure Numbers

E.

$$534{,}612 = Five\ Lakhs, Thirty\ Four\ Thousand, Six\ Hundred\ and\ Twelve$$
$$534{,}612 = Five\ and\ Thirty\ Four\ Thousand, Six\ Hundred\ and\ Twelve$$

7.2 Understand Place Value

A. Expanded Notation

The number 43 can be expanded as

$$43 = 40 + 3$$

Example 7.4: Two Digit

Write each of the following numbers in expanded notation:

A. 23
B. 67
C. 81
D. 90
E. 39
F. 43
G. 7
H. 80

$$23 = 20 + 3$$
$$67 = 60 + 7$$
$$81 = 80 + 1$$
$$90 = 90 + 0$$
$$39 = 30 + 9$$
$$43 = 40 + 3$$
$$7 = 7$$
$$80 = 80 + 0$$

Example 7.5: Three Digit Numbers

Write each of the following numbers in expanded notation:

A. 458
B. 247
C. 910
D. 803
E. 600
F. 281
G. 752
H. 704

$$458 = 400 + 50 + 8$$
$$247 = 200 + 40 + 7$$
$$910 = 900 + 10 + 0$$
$$803 = 800 + 0 + 3$$
$$600 = 600 + 0 + 0$$

$$281 = 200 + 80 + 1$$
$$752 = 700 + 50 + 2$$
$$704 = 700 + 0 + 4$$

Example 7.6: Four Digit Numbers

Write each of the following numbers in expanded notation

A. 2987
B. 3092
C. 5087
D. 6004
E. 4000
F. 2300

$$2987 = 2000 + 900 + 80 + 7$$
$$3092 = 3000 + 0 + 90 + 2$$
$$5087 = 5000 + 0 + 80 + 7$$
$$6004 = 6000 + 0 + 0 + 4$$
$$4000 = 4000 + 0 + 0 + 0$$
$$2300 = 2000 + 300 + 0 + 0$$

B. Expanded Notation with Place Value

We can write the expanded notation of a number to show the place value and the face value of its digits.

Example 7.7

Write out the following numbers in expanded notation and show their place value and their face value

A. 47
B. 352

$$47 = \underbrace{4}_{Tens}\ \underbrace{7}_{Ones} = 40 + 7 = \underbrace{4}_{Face\ Value} \times \underbrace{10}_{Place\ Value} + \underbrace{7}_{Face\ Value} \times \underbrace{1}_{Place\ Value}$$

$$352 = 300 + 50 + 2 = \underbrace{3}_{Face\ Value} \times \underbrace{100}_{Place\ Value} + \underbrace{5}_{Face\ Value} \times \underbrace{10}_{Place\ Value} + \underbrace{2}_{Face\ Value} \times \underbrace{1}_{Place\ Value}$$

C. Digits versus Numbers

Consider the number

$$42$$

This number is made using the digits

$$4\ and\ 2$$

Example 7.8

Identify which digits are used in each of the numbers below:

A. 56
B. 72
C. 481
D. 309
E. 455

$$56 \rightarrow Digits\ are\ 5\ and\ 6$$
$$72 \rightarrow Digits\ are\ 7\ and\ 2$$
$$481 \rightarrow digits\ are\ 4, 8\ and\ 1$$
$$309 \rightarrow digits\ are\ 3, 0\ and\ 9$$

$$455 \rightarrow Digits\ are\ 4{,}5\ and\ 5\ (note\ the\ 5\ is\ repeated)$$

Example 7.9: Repeated Digits

If a digit occurs twice, it has to be counted twice. Identify which digits are used in each of the numbers below.

$$\{455\}$$

$$455 \rightarrow Digits\ are\ 4{,}5\ and\ 5\ (note\ the\ 5\ is\ repeated)$$

D. Face Value of a Digit

$$43 = 40 + 3$$

If consider only the value of the digit, and not the value because of its position, that is called its face value.

Example 7.10

Identify the digits and their face values. Note that the face value of the digit is the same as the digit itself.

A. 74
B. 62

$$74 \rightarrow Digits\ are\ 7\ and\ 4, \qquad Face\ value\ is\ also\ 7\ and\ 4$$
$$62 \rightarrow Digits\ are\ 6\ and\ 2, \qquad Face\ value\ is\ also\ 6\ and\ 2$$

E. Place Value of a Digit

Place value of a digit comes from its position in the number. For example, in the number below, the digit in the ten's place has place value 10 and the digit in the one's place has place value 1.

$$\underbrace{4}_{\substack{Ten's\\Place}}\ \underbrace{3}_{\substack{Ones\\Place}} = 40 + 3$$

We can rewrite what we have written in terms of expanded notation to show the combination of face value and place value:

$$\underbrace{4}_{\substack{Ten's\\Place}}\ \underbrace{3}_{\substack{Ones\\Place}} = 40 + 3 = \underbrace{4}_{\substack{Face\\Value}} \times \underbrace{10}_{\substack{Place\\Value}} + \underbrace{3}_{\substack{Face\\Value}} \times \underbrace{1}_{\substack{Place\\Value}}$$

Example 7.11

Write the following numbers in terms of their place value and their face value

A. 57
B. 63
C. 87
D. 26
E. 75

$$57 = 50 + 7 = \underbrace{5}_{\substack{Face\\Value}} \times \underbrace{10}_{\substack{Place\\Value}} + \underbrace{7}_{\substack{Face\\Value}} \times \underbrace{1}_{\substack{Place\\Value}}$$
$$63 = 60 + 3 = 6 \times 10 + 3 + 1$$

$$87 = 80 + 7 = 8 \times 10 + 7 \times 1$$
$$26 = 20 + 6 = 2 \times 10 + 6 \times 1$$

Example 7.12

Write the following numbers in terms of their place value and their face value

$$\{164, \quad 289, \quad 7831, \quad 9604\}$$

Example 7.13

A. What is the place value of 7 in the number 3716?
B. What is the place value of 4 in the number 2148?
C. What is the place value of 3 in the number 3410?
D. What is the place value of 4 in the number 3410?

Exam-Style 7.14

A. What is the difference of the place values of the number 3 in 4313?
B. What is the difference of the place values of the number 4 in 44?
C. What is the difference of the place values of the number 8 in 8187?
D. What is the difference of the place values of the number 3 in 3238?

There are two 3's in 4313.
The first 3 is at the hundred's place and it has value 300.
The second 3 is at one's place, and it has value 3.
Therefore, the difference in their place values is

$$300 - 3 = 297$$

$$40 - 4 = 36$$

$$8000 - 80 = 7920$$

Exam-Style 7.15

A. The difference of the place values of 4X3X is 495, where X represents a digit. Find X.
B. The difference of the place values of 23XX is 72, where X represents a digit. Find the number.
C. The difference of the place values of the digit X in a two-digit number, XX, is 81. Find the number.
D. X occurs twice in a three-digit number. The difference of the place values of the digit X is 693. Find X. Find the positions of the digit X.

Part A

495, if rounded up to the nearest hundred is equal to:

$$500 = 5 \times 100 = 495 + 5$$

Hence, the digit which will work is

$$5$$

Part B

The next ten which comes after 72 is 80. Try 8:

$$80 - 8 = 72$$
$$X = 8$$
$$Number = 2388$$

Part C

$$81 = 90 - 9 \Rightarrow Number\ is\ 99$$

700-7=693
X is at Hundred's place
And X is at One's Place

F. Writing a Number in Terms of Ones, Tens, Hundreds

Example 7.16

Write the following numbers as tens and ones:

A. 34
B. 59
C. 38

$$34 = 3\ Tens + 4\ Ones$$
$$59 = 5\ Tens + 9\ Ones$$
$$38 = 3\ Tens + 8\ Ones$$

Example 7.17

Write the following numbers as hundred, tens, and ones.

$$381 = 3\ Hundreds + 8\ Tens + 1\ Ones$$
$$234 = 2\ Hundreds\ 3\ Tens\ and\ 4\ Ones$$
$$567 = 5\ Hundreds\ 6\ tens\ and\ 7\ Ones$$
$$781 = 7\ Hundreds, 8$$
$$284$$

G. Converting Ones, Tens and Hundreds into a Number

Example 7.18

$$\{8\ Tens + 4\ Ones, \quad 3\ Tens + 2\ Ones, \quad 9\ Tens + 7\ Ones\}$$

$$8\ Tens + 4\ Ones = 84$$
$$3\ Tens + 2\ Ones = 32$$
$$9\ Tens + 7\ Ones = 97$$

Example 7.19

$$\{4\ Hundreds + 3\ Tens + 2\ Ones, \quad 2\ \ Hundreds + 2\ Ones\ \}$$

$$4\ Hundreds + 3\ Tens + 2\ Ones = 432$$
$$2\ \ Hundreds + 2\ Ones = 202$$

H. Compare Numbers using Ones, Tens and Hundreds

Example 7.20

Compare the two numbers, and identify which one is greater, or if they are both equal

$3\ Tens + 2\ Ones \qquad 2\ Ones\ + 3\ Tens$

$4\ Tens + 5\ Ones \qquad 6\ Ones\ + 4\ Tens$

Example 7.21

Arrange the numbers given below in ascending order:

$$a = 3\ Hundreds + 2\ Tens + 3\ Ones$$
$$b = 3\ Hundreds + 2\ Ones + 3\ Tens$$
$$c = 3\ Tens + 2\ Hundreds + 3\ Ones$$
$$d = 3\ Tens + 2\ Ones + 3\ Hundreds$$
$$e = 3\ Ones + 2\ Hundreds + 3\ Tens$$
$$f = 3\ Ones + 2\ Tens + 3\ Hundreds$$

$$a = 323, \qquad b = 332, \qquad c = 233, \qquad d = 332, \qquad e = 233, \qquad f = 323$$

7.3 Conversion

A. Converting Tens into Ones

Example 7.22

$$1\ Tens = 10$$
$$5\ Tens = 50$$
$$9\ Tens = 90$$
$$10\ Tens = 100$$

Example 7.23

Convert into ones

$$\{12\ Tens, 17\ Tens, 23\ Tens, 54\ Tens\}$$

$$12\ Tens = 10\ Tens + 2\ Tens = 100 + 20 = 120$$
$$17\ Tens = 170$$
$$23\ Tens = 230$$
$$54\ Tens = 540$$

B. Converting Hundreds into Tens

Example 7.24

$$\{1\ Hundred, \qquad 3\ Hundred, \qquad 8\ Hundred\}$$

$$1\ Hundred = 10\ Tens$$
$$3\ Hundred = 30\ Tens$$
$$8\ Hundred = 80\ Tens$$

Example 7.25

$$\{2\ Hundreds + 3\ Tens, \qquad 3\ Hundreds + 4\ Tens\}$$

$$2\ Hundreds + 3\ Tens = 23\ Tens$$
$$3\ Hundreds + 4\ Tens = 34\ Tens$$

Example 7.26

Convert the following into regular numbers:

$$2\ Hundreds, 31\ Tens, 55\ Ones$$
$$4\ Hundreds, 23\ Tens, 67\ Ones$$
$$3\ Hundreds, 76\ Tens, 12\ Ones$$
$$7\ Hundreds, 14\ Tens, 45\ Ones$$

$$2\ Hundreds, 31\ Tens, 55\ Ones = 200 + 310 + 55 = 565$$
$$4\ Hundreds, 23\ Tens, 67\ Ones = 400 + 230 + 67$$
$$3\ Hundreds, 76\ Tens, 12\ Ones = 300 + 760 + 12 = 1072$$
$$7\ Hundreds, 14\ Tens, 45\ Ones = 700 + 140 + 45 = 885$$

C. Compare

Example 7.27

$$45\ Tens \quad 34\ Tens$$

Example 7.28

Compare the two numbers, and put one of the three signs below

$$<, >, =$$

$3\ Hundreds\ 25\ Tens$	$4\ Hundreds\ 15\ Tens$
$2\ Hundreds\ 12\ Tens$	$1\ Hundred\ 14\ Tens$
$5\ Hundreds\ 17\ Tens$	$3\ Hundreds\ 29\ Tens$
$3\ Hundreds\ 14\ Tens$	$2\ Hundreds\ 24\ Tens$

7.4 Applying Place Value

A. Identifying Numbers

Example 7.29

Find the number in each of the following cases:

A. The units digit is five. The tens digit is three.
B. The tens digit is four. The units digit is two.
C. The tens digit is the largest single digit prime, and the units digit is the smallest single digit prime.
D. The tens digit is the smallest possible odd number and the units digit is the largest possible odd number.
E. The tens digit is the smallest possible even number and the units digit is the largest possible even number.
F. The units digit is seven. The hundreds digit is three. The tens digit is zero.

Example 7.30

Find the number in each of the following cases:

A. The units digit of a number is 7. The ten's digit of the number is 2 more than the Units Digit.

B. The units digit of a number is 5. The ten's digit of the number is 3 less than the Units Digit.
C. The ten's digit of a number is 3. The unit's digit is double of the ten's digit.

$$Units\ Digit = 7, \quad Ten's\ Digit = 9 \Rightarrow \underbrace{9}_{Ten's\ Digit}\ \underbrace{7}_{Unit's\ Digit}$$

$$Units\ Digit = 5, \quad Ten's\ Digit = 5 - 3 = 2 \Rightarrow \underbrace{2}_{Ten's\ Digit}\ \underbrace{5}_{Unit's\ Digit}$$

$$Ten's\ Digit = 3, Units\ Digit = 6 \Rightarrow \underbrace{3}_{Ten's\ Digit}\ \underbrace{6}_{Unit's\ Digit}$$

Example 7.31

A. In a two-digit number, the unit's digits of the number is double the tens digit. The tens digit is an even number. The number is greater than thirty.

If the units digit is double of the tens digit, the numbers which are possible are:

$$12, 24, 36, 48$$

If the tens digit is an even number, we are left with only:

$$24, 48$$

And the number that we want is greater than 30, hence it must be:

$$48$$

Check this:

$$\underbrace{4}_{Ten's\ Digit}\ \underbrace{8}_{Unit's\ Digit} \Rightarrow 8\ is\ double\ of\ 4.$$

B. Smallest and Largest Number

Example 7.32

Find the

A. Smallest Three Digit Number
B. Greatest Four Digit Number
C. Smallest Odd Five Digit Number
D. Greatest Odd Four Digit Number
E. Smallest Even Three Digit Number
F. Greatest Even Three Digit Number
G. Difference between the largest four-digit number and the largest three-digit number.
H. Sum of the largest four-digit number and the largest three-digit number.

9999+999=10,998

Example 7.33

Use the digits in each bracket to make the smallest possible number using the all the digits.

$$\underbrace{(2,3)}_{Two\ Digit\ Number}, \underbrace{(5,4,2), (7,9,3)}_{Three\ Digit\ Number}\ \underbrace{(7,92,0)}_{Four\ Digit\ Number}$$

$$23, \quad 245, \quad 379, \quad 2079$$

Example 7.34

Consider the four digits:

3, 0, 4, 5

Using each of these digits exactly once to form a four-digit number, what is the:

A. Greatest Number less than 5000
B. Smallest Number more than 4000

4530
4035

Example 7.35

A. What is the smallest odd number that can be made using each digit once from 6,3 *and* 8.
B. What is the largest odd number that can be made using each digit from 3,2, and 5

683
532

C. Using fewer digits

Example 7.36

Make the smallest possible two-digit number using only the digits

A. 3, 4, and 7. Digits may not be repeated.
B. 4, 6, and 9. Digits may be repeated.

D. Finding Numbers between Two Numbers

E. Identifying Numbers given clues

Concept 4: Counting Number of Numbers
Consider the numbers greater then zero. What is the number of:
A. One Digit Numbers
B. Two Digit Numbers
C. Three Digit Numbers
D. Four Digit Numbers

Consider the natural numbers. How many odd and even numbers are there which are:
A. One Digit
B. Two Digits
C. Three Digits
D. Four Digits

One Digit:
Five Odd Numbers: 1,3, 5, 7,9
Four Even Numbers: 2, 4, 6,8

Two Digit Numbers
You can make pairs:
10 and 11
12 and 13

.
.
.
98 and 99

Hence, the 90 two digits get divided equally between odd and even.
45 Odd Two digit numbers
45 Even Two Digits Number

8. Rounding

8.1 Rounding to Nearest Ten

A. Rounding to Nearest Ten

Concept 1

Round 34 to the nearest ten using the number below. Note that 35 is exactly between 30 and 40, and it is marked with a green dot.

If you go backwards from 34, you will reach 30 as the nearest ten.
You need to go 4 units to reach 30.

If you go forward from 34, you will reach 40 as the nearest ten.
You need to go forward 6 units to reach 40.

34 is nearer to 30 than it is to 40. So, we round 34 to the nearest ten as:

30

Concept 2

Round 35 to the nearest ten using the number line below.

35 is equally distant from 30 and 40. They are each five units in each direction.
However, if a number is exactly in the middle, then we round to the larger number.

Hence, 35 when rounded to nearest ten becomes:

40

Practice 3

Round the following numbers to the nearest ten:

Two Digit Numbers

A. 27
B. 75
C. 88
D. 7
E. 40
F. 63
G. 84
H. 55
I. 4
J. 39
K. 92
L. 15
M. 3

Three Digit Numbers

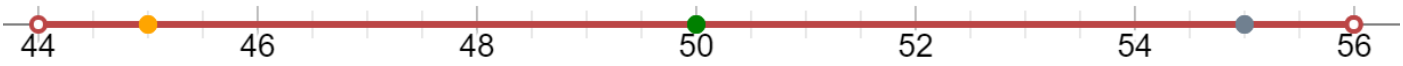
N. 112
O. 236
P. 865
Q. 451
R. 870
S. 905
T. 992

B. Finding Smallest and Largest Number

Concept 4

Consider the number line below.

A. Find the largest natural number that rounds to 50 when rounding to the nearest ten.
B. Find the smallest natural number that rounds to 50 when rounding to the nearest ten.



Consider rounding numbers which are near 50:

Number	44	45	46	47	48	49	50	51	52	53	54	55
Rounded Number	40	50	50	50	50	50	50	50	50	50	50	60

Practice 5

A. Find the largest number that rounds to 70 when rounded to the nearest ten.
B. Find the smallest number that rounds to 70 when rounded to the nearest ten.

Number	64	65	66	67	68	69	70	71	72	73	74	75
Rounded Number	60	70	70	70	70	70	70	70	70	70	70	80

Practice 6

A. Find the largest number that rounds to 20 when rounded to the nearest ten.
B. Find the smallest number that rounds to 20 when rounded to the nearest ten.

$Part\ A: 24$
$Part\ B: 15$

Practice 7

A. Find the largest number that rounds to 90 when rounded to the nearest ten.
B. Find the smallest number that rounds to 90 when rounded to the nearest ten.

$Part\ A: 94$
$Part\ B: 85$

Practice 8

A. Find the largest number that rounds to 60 when rounded to the nearest ten.
B. Find the smallest number that rounds to 60 when rounded to the nearest ten.

$Part\ A: 64$
$Part\ B: 55$

Practice 9

A. Find the largest number that rounds to 30 when rounded to the nearest ten.
B. Find the smallest number that rounds to 30 when rounded to the nearest ten.

$Part\ A: 34$
$Part\ B: 25$

Practice 10

A. Find the largest number that rounds to 40 when rounded to the nearest ten.
B. Find the smallest number that rounds to 40 when rounded to the nearest ten.

$Part\ A: 44$
$Part\ B: 35$

8.2 Rounding to Nearest Hundred

A. Rounding

Concept 11

A. Round 271 to the nearest ten
B. Round 271 to the nearest hundred

Practice 12

Round each of the following numbers to the nearest ten, and also the nearest hundred. Answer separately for each.

A. 537
B. 851
C. 388
D. 674
E. 449

We want to round 449 to the nearest hundred:

$$\underbrace{4}_{Hundred's\ Digit:Round\ to\ here}\quad\underbrace{4}_{Ten's\ Digit:\ Check\ this\ digit}\quad 9 \rightarrow 400$$

B. Smallest and Largest Numbers

Concept 13

When rounded to the nearest hundred, what is the smallest number that rounds to:

A. 300
B. 800
C. 500
D. 900
E. 700

Concept 14

When rounded to the nearest hundred, what is the largest number that rounds to:

A. 400
B. 600
C. 900
D. 200
E. 700

Round 449 to the nearest hundred:

$$\underbrace{4}_{Hundred's\ Digit:Round\ to\ here}\quad\underbrace{4}_{Ten's\ Digit:\ Check\ this\ digit}\quad 9 \rightarrow 400$$

Round 450 to the nearest hundred:

$$\underbrace{4}_{Hundred's\ Digit:Round\ to\ here}\quad\underbrace{5}_{Ten's\ Digit:\ Check\ this\ digit}\quad 0 \rightarrow 500$$

This means that 449 rounds to 400, but 450 rounds to 500.
Hence, the largest number that rounds to 400 is 449.

8.3 Rounding to Nearest Thousand

A. Rounding

We will always check the digit to the right of the one that we rounding to. For example:

- To round to Tens, we will check the one's digit
- To round to hundreds, we will check the ten's digit
- To round to thousands, we will check the hundreds digit

Whenever we round to a particular digit, we want only zeroes to the right of that digit. For example:

- If we round to nearest ten, the ones digit has to always be zero
  - ✓ 340
  - ✓ 230
  - ✓ 120
- If we round to nearest hundred, the ones digit and the tens digit have to always be zero
  - ✓ 300
  - ✓ 200
  - ✓ 100
- If we round to nearest thousand, the ones digit and the tens digit and the hundred's digit have to always be zero
  - ✓ 3000
  - ✓ 2000
  - ✓ 1000

Concept 15

A. Round 2738 to the nearest ten
B. Round 2738 to the nearest hundred
C. Round 2738 to the nearest thousand

Round to the Nearest Ten

$$27\ \underbrace{3}_{Round\ to\ Ten's\ Digit}\ \underbrace{8}_{Check\ this\ Digit}$$

Since 8 is more than 5, we increase ten's digit by 1, which gives us:

2740

Round to the Nearest Hundred

$$2\ \underbrace{7}_{Round\ to\ Hundred's\ Digit}\ \underbrace{3}_{Check\ this\ Digit}\ 0$$

Since 3 is less than 5, we keep the hundred's digit the same, which gives us:

2700

Round to the Nearest Hundred

$$\underbrace{2}_{\text{Round to Thousand's Digit}}\ \underbrace{7}_{\text{Check this Digit}}\ 38$$

Since 7 is more than 5, we increase the thousand's digit the same, which gives us:

3000

Practice 16

Round each of the following numbers to the nearest ten, the nearest hundred, and then nearest thousand. Answer separately for each.

A. 8172
B. 7241
C. 6762
D. 8456
E. 2381

B. Smallest and Largest Numbers

Concept 17

When rounded to the nearest thousand, what is the smallest number that rounds to:

A. 3000
B. 8000
C. 5000
D. 9000
E. 7000

Concept 18

When rounded to the nearest thousand, what is the largest number that rounds to:

A. 4000
B. 6000
C. 9000
D. 2000
E. 7000

8.4 Rounding to Larger Numbers

A. Rounding to Nearest Ten Thousand

Practice 19

Round 38159 to the nearest ten, the nearest hundred, the nearest thousand, and the nearest ten thousand. Answer separately for each.

$$\underbrace{\text{Rounding to this Digit}}\ \underbrace{\text{Digit to check}}$$

Rounding to Nearest Ten

$$38158 \rightarrow 38160$$

Because 8 is greater than five, we increase the ten's digit.

Rounding to Nearest Hundred

$$38158 \rightarrow 38200$$

Because 5 is greater than or equal five, we increase the hundred's digit.

Rounding to Nearest Thousand

$$38158 \rightarrow 38000$$

Because 1 is less than five, we keep the thousand's digit the same.

Rounding to Nearest Ten Thousand

$$38158 \rightarrow 40000$$

Because 8 is more than five, we keep the ten thousand's digit by one.

Practice 20

Round the following numbers to the nearest ten, the nearest hundred, the nearest thousand, and the nearest ten thousand. Answer separately for each.

A. 53,147
B. 38,163
C. 27,816
D. 71,836
E. 36,824

Part C

Rounding to Nearest Ten

$$27{,}816 \rightarrow 27{,}820$$

Rounding to Nearest Hundred

$$27{,}816 \rightarrow 27800$$

Rounding to Nearest Thousand

$$27{,}816 \rightarrow 28{,}000$$

Rounding to Nearest Ten Thousand

$$27{,}816 \rightarrow 30{,}000$$

9. Word Problems/Applications

9.1 Counting and Positions

A. Counting and Positions

Example 1

Consider the sentence

Mr. and Mrs. Dursley of number four, Privet Drive, were proud to say that they were perfectly normal, thank you very much.

A. How many words does the sentence have?
B. What is the fifth word in the sentence?
C. From the right, what is the seventh word in the sentence?
D. What is the second letter (from the left) of the third word in the sentence (from the right)?
E. How many capital letters does the sentence have, if a repeated letter is counted each time it occurs? Which is the second last capital letter in the sentence?

Example 2

Consider the sequence below, which consists of the numbers from 1 to 15 written out next to each other:

123456789101112131415

A. Which digit is third from the right?

9.2 Unitary Method

A. Cost of many items

Example 3

A. I am buying chairs. One chair costs Rs. 100 What is the cost of five chairs?

Here, we are given the price of one chair, and we need to find the price of many chairs.

B. Cost of one item

A. I am buying utensils. I need to pay Rs. 100 for 5 utensils. What is the cost of one utensil?
B. I am buying bananas from the market. Bananas are sold in dozens, where one dozen represents twelve bananas. Buying a dozen bananas costs Rs. 24. What is the cost of one banana?

Here we are given the price of many utensils, and we need to find cost of one utensil.

$$Cost\ of\ One\ Banana = \frac{Cost\ of\ Twelve\ Bananas}{12} = \frac{24}{12} = 2$$

C. Cost of X Items, given cost of Y Items.

Example 4

Ruchita sews dresses for which she gets paid Rs. 700 for seven dresses. How much will she get paid for sewing twelve dresses.

Cost of One Dress = 700/7 = 100
Cost of Twelve Dresses = 100 * 12 = 1200

D. Market Visits

Example 5

Palak went to the market and bought:

A. 7 Cakes at a cost of Rs. 12 per cake from the bakery.
B. 5 Kilos of Wheat at a cost of Rs. 14 per kilo from the grocery shop
C. Three dozen bananas at a cost of Rs. 8 per dozen from the fruit vendor

Question 1: Find the total cost of Palak's visit to the market.

$$\underbrace{7 \times 12}_{Cakes} + \underbrace{5 \times 14}_{Wheat} + \underbrace{3 \times 8}_{Bananas} = 84 + 70 + 24 = 178$$

Question 2: Palak gave a hundred rupee note at each shop she visited. Find the change that she got back from each shop.

$$\underbrace{100 - 84}_{Bakery} + \underbrace{100 - 70}_{Grocery} + \underbrace{100 - 24}_{Fruit\ Vendor} = 16 + 30 + 76 = 122$$

E. Hotels

Example 6

A hotel charges $120 for a night for a regular room, and $140 for a deluxe room. Find the cost of renting:

A. two regular rooms for a night.
B. a regular room for three nights.
C. a deluxe room for five nights.
D. three deluxe rooms for a night.
E. two regular rooms for three days, and three deluxe rooms for two days.

Example 7

David visited the Ecotel Motel and stayed for two nights. He paid a total of $250 for his stay.

A. How much did his stay cost him per day?
B. If David spent $50 on dining, and the dining cost was included in the $250, how much did he spend per day on room rent?

10. Squares and Square Roots

10.1 Squares and Square Roots

A. Squares

The square of a number is the number multiplied by itself. The table below gives the square of the first few numbers.

Number	0	1	2	3	4	5	6	7	8	9	10	11	12
Square	0	1	4	9	16	25	36	49	64	81	100	121	144

Example 1

Find the squares of each of the following numbers: {5,7,2,9,12,0,4,20,10,6,8,3,1,11,30,0}

{25,49,4,81,144,0,16,400,100,36,64,9,1,121,900,0}

Example 2

A. Priyesh ate five chocolates each day for five days. What is the number of chocolates that he ate in all?
B. Siddarth has seven candles to celebrate his seventh birthday. Each candle is seven inches long. If the candles are lined up end to end, how long will the entire object be?

B. Square Roots

A square root is the opposite of a square. When finding a square root, we find a number that will multiply to give the number that we have

For example:

$$Square\ Root\ of\ 4 = 2 \Rightarrow \underbrace{\sqrt{4}}_{\textbf{Square Root Symbol}} = 2$$

Example 3

Find the square roots of each of the following numbers: {64,36,81,4,16,9,25,100,0,1,121,49,400,900}

10.2 Cubes and Cube Roots

A. Cubes

The cube of a number is when you multiply the number by itself three times. For example, to find the cube of 2, we multiply it by itself three times

$$2\ cubed = 2 \times 2 \times 2 = 8$$

The cubes of the first few numbers are given below.

Number	0	1	2	3	4	5			
Cube	0	1	8	27	64				

Example 4

A. I have six bags. Each bag has six compartments. Each compartment has six apples. What is the total number of apples that I have?
B. A *fictional* Martian race lives in herds of eight animals each. Each animal has eight legs, and each leg

has eight toes. They want to apply nail polish to their claws. Each toe has exactly one claw. It takes the nail polish expert two minutes apply nail polish to one claw. What is the total number of minutes taken to apply nail polish to the entire herd?

11. Problem Solving

11.1 More and Less

12. Properties and Identities

12.1 Parity

A. Even and Odd Numbers

- Numbers which are divisible by two are called even numbers.
  - ✓ Even numbers end in the digits {0,2,4,6,8}
- Numbers which are not divisible by two are called odd numbers.
  - ✓ Odd numbers end in the digits {1,3,5,7,9}

Parity

Parity is the mathematical term for whether a number is odd or even.
It is not necessary to do the calculations in order to decide the parity. The information on whether the numbers being added/subtracted are odd, or even, is sufficient to decide the parity of the answer.

Example 1

Identify the numbers in the set below as odd or even

{35, 672, 870, 651, 904]

B. Parity: Addition and Subtraction

Two numbers which are added, or subtracted, will be:

- Even, if they have the same parity
  - ✓ $Odd \pm Odd = Even$
  - ✓ $Even \pm Even = Even$
- Odd, if they have different parity
  - ✓ $Odd \pm Even = Odd$
  - ✓ $Even \pm Odd = Odd$

Example 2

Identify whether the expressions given below are even or odd

A. $345 + 761$
B. $871 - 20$
C. $918 + 7$
D. $456 - 241$

Example 3

Identify the parity of the answer in each case below

A. Shivam added an even number to an odd number.
B. Rishi subtracted an odd number from an odd number.

C. Parity: Multiplication

The product of two numbers will be

- odd, only when both numbers multiplied are odd
  - ✓ $Odd \times Odd = Odd$
- even, in all cases except when both numbers are od
  - ✓ $Even \times Even = Even$
  - ✓ $Odd \times Even = Even$
  - ✓ $Even \times Odd = Even$

Example 4

Identify whether the expressions given below are even or odd

A. 7×8
B. 22×44
C. 441×663
D. 342×781
E. 1001×555

Example 5

Identify the parity of the answer in each case below

A. Avnish multiplied an even number with an odd number.
B. Poonam multiplied an odd number with an odd number.

D. Parity: Division

12.2 Terms

A. Comparing Numbers

We can compare two numbers to see which one is greater.

- The number which is greater
  - ✓ comes to the right on the number line of the number which is smaller
  - ✓ Is said to come before the number
- The number which is smaller
  - ✓ comes to the left on the number line of the number which is greater
  - ✓ Is said to come before the number which is greater

Example 6

In each question, compare the two numbers and state which one is further to the right on the number line. Also, state, which one is further to the left on the number line.

A. 34 and 77
B. 910 and 5
C. 34 and 92

B. Predecessor and Successor

The number that comes

- immediately before a number is called its predecessor
- immediately after a number is called its successor

Example 7

Find the predecessor of each number in {27; 100; 472; 5; 340; 1000}

12.3 Properties

A. Summary

Commutative property applies when order of operations for two numbers does not matter

- It holds for $\underbrace{addition}_{a+b=b+a}$ and $\underbrace{multiplication}_{a\times b=b\times a}$

- It does not hold for $\underbrace{subtraction}_{a-b\neq b-a}$ and $\underbrace{division}_{\frac{a}{b}\neq\frac{b}{a}}$

Associative Property applies when out of three numbers, which we do the operation first does not matter

- It holds for $\underbrace{addition}$ and $\underbrace{multiplication}$
- It does not hold for $\underbrace{subtraction}$ and $\underbrace{division}$

B. Commutative Property

Commutative Property applies to two numbers on which are we performing some operations such as:

- Addition
- Subtraction
- Multiplication
- Division

If the order of the numbers does not matter, then the commutative property holds. If changing the order of the numbers can change the final answer, then the commutative property does not hold.

Addition & Multiplication: Commutative Property holds

Order of addition does not matter

$Example: 2 + 3 = 5 = 3 + 2$

Order of multiplication does not matter

$Example: 2 \times 3 = 6 = 3 \times 2$

Subtraction and Division: Commutative Property does not hold

Order of subtraction does matter

$$2 = 5 - 3 \quad \underbrace{\neq}_{Symbol: Not\ Equal\ To} \quad 3 - 5 = -2$$

Order of division does matter

$$Example: 2 = 6 \div 3 \neq 3 \div 6 = \frac{3}{6} = \frac{1}{2}$$

Example 8

In each of the statements below, identify which property, or lack or property is being demonstrated.

A. $3 - 4 \neq 4 - 3$
B. $33 \times 43 = 43 \times 33$
C. $3 + 4 = 4 + 3$
D. $3 \div 5 \neq 5 \div 3$

A. Lack of Commutative Property for Subtraction
B. Commutative Property of Multiplication
C. Commutative Property of Addition
D. Lack of Commutative Property of Division

C. Associative Property

Associative Property applies to three numbers. If the result on applying an operation to the first two numbers first, is the same as the result of applying the operation to the last two numbers first, then the associative property holds.

Addition and Multiplication: Associative Property holds

Order of **addition**, as shown in the example below, does not matter for three or more numbers

$$\underbrace{2+3}_{\textit{Add this first}} + 5 = 5 + 5 = 10 \Leftrightarrow 2 + \underbrace{3+5}_{\textit{Add this first}} = 2 + 8 = 10$$

Order of **multiplication**, as shown in the example below, does not matter for three or more numbers

$$\underbrace{2\times 3}_{\textit{Multiply this first}} \times 5 = 6 \times 5 = 30 \Leftrightarrow 2 \times \underbrace{3\times 5}_{\textit{Multiply this first}} = 2 \times 15 = 30$$

Subtraction and Division: Associative Property does not hold

Order of **subtraction**, as shown in the example below, does matter for three or more numbers

$$\underbrace{2+3}_{\textit{Add this first}} + 5 = 5 + 5 = 10 \neq 2 + \underbrace{3+5}_{\textit{Add this first}} = 2 + 8 = 10$$

Order of **division**, as shown in the example below, does not matter for three or more numbers

$$\underbrace{20\div 2}_{\textit{Divide this first}} \div 2 = 10 \div 2 = 5 \neq 20 \div \underbrace{2\div 2}_{\textit{Multiply this first}} = 20 \div 1 = 20$$

Example 9

In each of the statements below, identify which property, or lack or property is being demonstrated.

A. $(4 + 6) + 10 = 4 + (6 + 10)$
B. $(3 \div 2) \div 7 \neq 3 \div (2 \div 7)$
C. $(5 - 3) - 12 \neq 5 - (3 - 12)$
D. $(3 \times 4) \times 5 = 3 \times (4 \times 5)$

A. Associative Property of Addition
B. Lack of Associative Property of Division
C. Lack of Associative Property of Subtraction
D. Associative Property of Multiplication

D. Application: Word Problems

Example 10

A. Raghu was going to school. He counted seven shops in the lane from his house to the main road, and twelve shops on the main road before school, making a total of nineteen shops. While coming back from school, he counted the same twelve shops on the main road, and seven shops in the lane. This also added up to nineteen.
B. Nishant was arranging chairs for a marriage. He made twelve rows of eleven chairs each, using one hundred and thirty-two chairs. Later, for the reception, he arranged chairs in eleven rows of twelve chairs each, and still needed the same number of chairs.
C. Sheela needed to share twelve apples equally among her three children. She took three apples and made twelve parts of each. But she was not successful in sharing the apples, and her children complained.
D. Parth was on the fifth floor of a building. He needed to go down three floors to visit his friend. The next day, Parth was on the third floor of the same building. So, he decided to go down five floors to visit the same friend. But, instead he ended up getting to the basement.

S1:

A. Commutative Property of Addition
B. $12 \times 11 = 132 = 11 \times 12 \Rightarrow$Commutative Property of Multplication
C. $4 = 12 \div 3 \neq 3 \div 12 = \frac{1}{4} \Rightarrow$ Lack of Commutative Property of Division
D. $5 - 3 = 2 \neq 3 - 5 = -2 \Rightarrow$ Lack of Commutative Property of Subtraction

E. Application: Variables

Example 11

In each of the statements below, identify which property, or lack or property is being demonstrated.

A. $a - b \neq b - a$
B. $x \times y = y \times x \Leftrightarrow xy = yx$
C. $p + q = q + p$
D. $m \div n \neq n \div m$

A. Lack of Commutative Property for Subtraction
B. Commutative Property of Multiplication
C. Commutative Property of Addition
D. Lack of Commutative Property of Division

Example 12

In each of the statements below, identify which property, or lack or property is being demonstrated.

A. $(a + b) + c = a + (b + c)$
B. $(p \div q) \div r \neq p \div (q \div r)$
C. $(f - g) - h \neq f - (g - h)$
D. $(x \times y) \times z = x \times (y \times z) \Leftrightarrow (xy)z = x(yz)$

A. Associative Property of Addition
B. Lack of Associative Property of Division
C. Lack of Associative Property of Subtraction
D. Associative Property of Multiplication

12.4 Distributive and Inverse Properties

A. Distributive Property

Distributive means that the order of operations inside a bracket does not matter. You can perform the operation inside the bracket first, or you can perform the operation on each item inside the bracket.

Addition and Subtraction: Distributive Property holds

$$\underbrace{2(3+4) = 2(7)}_{\textit{Do the addition first}} = 14, \qquad \underbrace{2(3+4) = 2\times 3 + 2\times 4 = 6 + 8 = 14}_{\textit{Do the multiplication first}}$$

$$\underbrace{3(7-2) = 3(5)}_{\textit{Do the subtraction first}} = 15, \qquad \underbrace{3(7-2) = 3\times 7 - 3\times 2 = 21 - 6 = 15}_{\textit{Do the multiplication first}}$$

Multiplication and Division: Distributive Property does not hold

$$\underbrace{2(3\times 4) = 2(12)}_{\textit{Simplify the bracket first}} = 24, \qquad \underbrace{2(3+4) = \mathbf{2\times 3\times 2\times 4 = 6\times 8 = 48}}_{\textit{Not Correct}}$$

$$\underbrace{4(8\div 4) = 4(2)}_{\textit{Simplify the bracket first}} = 8, \qquad \underbrace{4(8\div 4) = \mathbf{4\times 8\div 4\times 4 = 32\div 1 = 32}}_{\textit{Not Correct}}$$

Example 13

Apply the distributive property to create an equivalent expression

B. Distributive Property

12.5 Identities

A. Additive Identity

Additive Identity means if you add the identity to any number, the number remains unchanged.
Additive Identity is zero.

$$\underbrace{5 + 0 = 5}_{\textit{Number is unchanged}}$$

B. Multiplicative Identity

Multiplicative Identity means if you multiply the identity with any number, the number remains unchanged.
Multiplicative Identity is one.

$$\underbrace{5 \times 1 = 5}_{\textit{Number is unchanged}}$$

Example 14

In each of the statements below, identify which property is being demonstrated.

A. $7 \times 1 = 7$
B. $12 + 0 = 12$
C. $a + 0 = a$
D. $b \times 1 = b$

A. Multiplicative Identity
B. Additive Identity
C. Additive Identity
D. Multiplicative Identity

Example 15

Q1: In each of the statements below, identify which property is being demonstrated.

A. Saloni bought 12 boxes of sweets, each with the same number of sweets. She got 12 sweets in all.
B. Rishabh had seven apples. He got x more apples. The number of apples he had did not increase.
C. $12 + 0 = 12$

S2:

A. $12 \times 1 = 12 \Rightarrow$ Multiplicative Identity
B. $7 + x = 7 \Rightarrow$ Additive Identity

13. Average

13.1 Basics

A. Uni

Example 1

Find the number between 4 and 10.

4	5	6	7	8	9	10
0	1	2	3			
			3	2	1	0

$$\frac{4+10}{2}=\frac{14}{2}=7$$

Practice 2

In each case, find the number between the two numbers.

A. 5 and 7
B. 3 and 11

Example 3

A. 13 and 25
B. 12 and 20
C. 40 and 100

Example 4

A. 4 and 5

$$\frac{4+5}{2}=\frac{9}{2}=4\frac{1}{2}=4.5$$

Example 5

A. 0.5 and 1.5
B. 0.7 and 0.9

0.6	0.7	0.8	0.9	1.0

Example 6

A. $\frac{1}{2}$ and $\frac{1}{3}$

First, add the two fractions:

$$\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$$

Divide the sum by 2:

$$\frac{5}{6} \div 2 = \frac{5}{6} \times \frac{1}{2} = \frac{5}{12}$$

B. Average of Three Numbers

Example 7

A. 3,4,11
B. 2,5,14

$$\frac{3+4+11}{3} = \frac{18}{3} = 6$$

Example 8

A. $0.3, 0.4\ and\ 0.9$

$$\frac{0.3+0.4+0.9}{3} = \frac{1.5}{3} = \frac{15}{30} = \frac{1}{2} = 0.5$$

Example 9

A. $\frac{1}{2}, \frac{1}{3}, and\ \frac{1}{4}$

First, find the total of the three numbers:

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{13}{12}$$

Divide the total by 3:

$$\frac{13}{12} \div 3 = \frac{13}{12} \times \frac{1}{3} = \frac{13}{36}$$

C. Word Problems

Example 10

Three children in a school are working on

D. Finding Totals

Example 11

Ritu saw TV for an average of 40 minutes each day over the weekend (Saturday and Sunday).

A. How much time did she watch TV over the weekend?
B. If she saw TV for 22 minutes on Saturday, how many minutes did she watch it on Sunday?

$$\frac{Sat + Sun}{2} = 40 \Rightarrow Sat + Sun = 40 \times 2 = 80$$

$$Sat + Sun = 80 \Rightarrow 22 + Sun = 80 \Rightarrow Sun = 80 - 22 = 58$$

14. Roman Numerals

14.1 Summary

Symbol	Value	Symbol	Value
I	1	V	5
X	10	L	50
C	100	D	500
M	1000		

Decimal		Roman
10	10	X
20	10 + 10	XX
30	10 + 10 + 10	XXX
40	50 − 10	XL
50	50	L
60	50 + 10	LX
70	50 + 10 + 10	LXX
80	50 + 10 + 10 + 10	LXXX
90	100 − 10	XC
100	100	C

Decimal		Roman	Decimal	Roman
100	100	C	1000	M
200	100 + 100	CC	2000	MM
300	100 + 100 + 100	CCC	3000	MMM
400	500 − 100	CD		
500	500	D		
600	500 + 100	DC		
700	500 + 100 + 100	DCC		
800	500 + 100 + 100 + 100	DCCC		
900	1000 − 100	CM		
1000	1000	M		

14.2 Numerals from 1 to 11

A. Symbols

The first symbols that we need to learn are

$$I = 1 = One$$
$$V = 5 = Five$$
$$X = 10 = Ten$$

14.1: Addition Rule

When two symbols of the same value are repeated, we add their values.
Any symbol can be repeated consecutively a maximum of three times.

Example 1

Using the addition rule, write the numbers 1 to 3 in Roman.

The symbol for 1 is

$$I$$

If you want 2, you repeat the symbol for 1 twice:

$$II = I + I = 1 + 1 = 2$$

If you want 3, you repeat the symbol for 1 thrice:

$$III = I + I + I = 1 + 1 + 1 = 3$$

14.2: Dissimilar Values Rule

If a smaller number is written

- to the left of a larger number, it is subtracted from the larger number.
- to the right of a larger number, it is added to the larger number.

Example 2

Write the number 5 in Roman

$$V$$

Example 3

Write the number 6 in Roman

$$VI$$

Example 4

Write the number 4 in Roman

Bogus Solution

If you want 4, you might want to repeat the symbol for 1 four times:

$$IIII = I + I + I + I = 1 + 1 + 1 + 1 = 4$$

But this is incorrect.

Because any symbol can be repeated a maximum of three times consecutively.

Correct Solution

Instead, the correct way to write 4 is using the symbol for 5:

$$IV = V - I = 5 - 1 = 4$$

Example 5

Write the number 7 in Roman

$$VII$$

Example 6

Write the number 8 in Roman

$$VIII$$

Example 7

Write the number 10 in Roman

X

Example 8

Write the number 11 in Roman

XI

Example 9

Write the number 9 in Roman

IX

Decimal		Roman
1	1	I
2	1+1	II
3	1+1+1	III
4	5-1	IV
5	5	V
6	5+1	VI
7	6+1+1	VII
8	5+1+1+1	VIII
9	10-1	IX
10	10	X

14.3 Numerals from 11 to 30

Example 10

Write the numbers from 10 to 20 in Roman

X
XI
XII
XIII
XIV
XV
XVI
XVII
XVIII
XIX
XX

Example 11

Write the numbers from 21 to 30 in Roman

XXI

$XXII$
$XXIII$
$XXIV$
XXV
$XXVI$
$XXVII$
$XVIII$
$XXIX$
XXX

Example 12

Write the numbers from 31 to 39 in Roman

$XXXI$
$XXXII$
$XXXIII$
$XXXIV$
$XXXV$
$XXXVI$
$XXXVII$
$XXXVIII$
$XXXIX$

14.4 Numerals from 40 to 100

A. Symbols

$I = 1 = One$
$V = 5 = Five$
$X = 10 = Ten$
$\mathbf{L = 50 = Fifty}$

Example 13

Write the numbers {50,60,40,70,80} in Roman

$50 = L$
$60 = LX$
$40 = XL$
$70 = LXX$
$80 = LXXX$

14.3: Converting from Decimal into Roman

When converting from Decimal system to Roman system, use the following steps:

- Write the Decimal Number in Expanded Notation (using the Decimal System only)
- Write each part of the Decimal Number as a Roman Numeral
- Put the different Roman Numerals together

Example 14

Write 35 in Roman using the Conversion Method

Write the number using Expanded Notation:

$$35 = 30 + 5$$

Convert each part of the number into Roman separately:

$$30 = XXX$$
$$5 = V$$

Put the Roman Numerals together

$$XXX + V = XXXV$$

Example 15

Write 39 in Roman using the Conversion Method

Write the number using Expanded Notation:

$$39 = 30 + 9$$

Convert each part of the number into Roman separately:

$$30 = XXX$$
$$5 = IX$$

Put the Roman Numerals together

$$XXX + IX = XXXIX$$

Example 16

Write 42 in Roman using the Conversion Method

Write the number using Expanded Notation:

$$42 = 40 + 2$$

Convert each part of the number into Roman separately:

$$40 = XL$$
$$2 = II$$

Put the Roman Numerals together

$$XLII$$

Practice 17

Write the following numbers in Roman Numerals

A. 34
B. 52
C. 71
D. 49
E. 85
F. 63
G. 89

$$34 = 30 + 4 = XXX + IV = XXXIV$$

$$52 = 50 + 2 = L + II = LII$$
$$71 = 70 + 1 = LXX + I = LXXI$$
$$49 = 40 + 9 = XL + IX = XLIX$$
$$85 = 80 + 5 = LXXX + V = LXXXV$$
$$63 = 60 + 3 = LX + III = LXIII$$
$$89 = 80 + 9 = LXXX + IX = LXXXIX$$

Practice 18

1. IX
2. XVI
3. XXII
4. LI
5. XXXVIII
6. LXI
7. LXXIX

$$IX = 9$$
$$XVI = 14$$
$$XXII = 22$$
$$LI = 51$$
$$XXXVIII = 38$$
$$LXI = 59$$
$$LXXIX = 79$$

14.5 Numerals from 90 to 899

A. Symbols

$$I = 1 = One$$
$$V = 5 = Five$$
$$X = 10 = Ten$$
$$L = 50 = Fifty$$
$$\boldsymbol{C = 100}$$
$$\boldsymbol{D = 500}$$

Example 19

Convert the following numbers into Roman:

$$\{100,200,300, \ldots ,800\}$$

$$100 = C$$
$$200 = CC$$
$$300 = CCC$$
$$400 = CD$$
$$500 = D$$
$$600 = DC$$
$$700 = DCC$$
$$800 = DCCC$$

Example 20

Convert the following numbers into Roman:

$$\{90,94,99\}$$

$$90 = XC$$
$$94 = 90 + 4 = XC + IV = XCIV$$
$$99 = 90 + 9 = XC + IX = XCIX$$

Bogus Solution without using Place Value

$$IC = 100 - 1 = 99$$

This solution is incorrect because you have to first convert into expanded notation.

Example 21

Write the following in Roman Numerals

A. 745
B. 861
C. 345
D. 561
E. 781

$$745 = 700 + 40 + 5 = DCC + XL + V = DCCXLV$$
$$861 = 800 + 60 + 1 = DCCC + LX + I$$
$$345 = 300 + 40 + 5 = CCC + XL + V = CCCXLV$$
$$561 = 500 + 60 + 1 = D + LX + I = DLXI$$
$$781 = 700 + 80 + 1 = DCC + LXXX + I = DCCLXXXI$$

Example 22

Convert the following into the Decimal System

1. CCI
2. CCIX
3. CCCLXVI
4. $DCXLIX$
5. $DCCCLXXXVIII$
6. $CDXLIV$

$$CCI = 201$$
$$CCIX = 209$$
$$CCCLXVI = 364$$
$$DCXLIX = 649$$
$$DCCCLXXXVIII = 888$$
$$CDXLIV = 444$$

14.6 Numerals from 900 to 3000

A. Symbols

$$I = 1 = One$$
$$V = 5 = Five$$
$$X = 10 = Ten$$
$$L = 50 = Fifty$$

$$C = 100 = One\ Hundred$$
$$D = 500 = Five\ Hundred$$
$$M = 1000 = One\ Thousand$$

Example 23

Write the following numbers in Roman Numerals

A. 1000
B. 2000
C. 3000
D. 900
E. 990
F. 999

$$1000 = M$$
$$2000 = MM$$
$$3000 = MMM$$
$$900 = CM$$
$$990 = 900 + 90 = CM + XC = CMXC$$
$$999 = 900 + 90 + 9 = CM + XC + IX = CMXCIX$$

Example 24

Write the following numbers in Roman Numerals

A. 1001
B. 1300
C. 1405
D. 1920
E. 2456
F. 2894

$$1001 = MI$$
$$1300 = MCCC$$
$$1405 = 1000 + 400 + 5 = M + CD + V = MCDV$$
$$1920 = 1000 + 900 + 20 = MCMXX$$
$$2456 = 2000 + 400 + 50 + 6 = MMCDLVI$$
$$2894 = 2000 + 800 + 90 + 4 = MMDCCCXCIV$$

Example 25

Write the following numbers in the Decimal System

1. MI
2. CM
3. CMXCIX
4. MMDCCLXXVIII
5. MMMCDI
6. MMMXLI

$$MI = 1001$$
$$CM = 900$$
$$CMXCIX = 999$$
$$MMDCCLXXVIII = 2778$$

$$MMMCDI = 3401$$
$$MMMXLI = 3041$$

14.7 Numerals above 3000

14.8 Theoretical Questions

A. Repetition

- A symbol can be repeated to increase its value

$$II = 1 + 1 = 2$$
$$XX = 10 + 10 = 20$$
$$CCC = 100 + 100 + 100 = 300$$

- The maximum number of times a symbol can be used consecutively is three.

$$IIII = 1 + 1 + 1 + 1 = 4$$
$$XXXX = 10 + 10 + 10 + 10 = 50$$
$$CCCC = 100 + 100 + 100 + 100 = 400$$

Example 26

A. The maximum number of times that a symbol can be used consecutively is ___.
B. Riddhi used a symbol four times consecutively in Roman Numerals. Is her number valid?

Example 27

Identify which of the following numbers are valid Roman Numerals?

1. CCCLXXXVIII
2. MMCMCCCCVIIII
3. MMMMDCCI
4. MMMDCCCIX

B. Addition Rules

When symbols are written in decreasing order, then the values of the symbols are added.

$$VI = 5 + 1 = 6$$
$$XII = 10 + 1 + 1 = 12$$
$$CI = 100 + 1 = 101$$
$$MDCLXI = 1000 + 500 + 100 + 50 + 10 + 1 = 1661$$

Example 28

A. In Roman Numerals, if you write a smaller number to the immediate left of a bigger number, you should add the smaller number to the bigger number. State True or False.

C. Subtraction Rules

When you write a smaller number to the left of a larger number, subtract the smaller number.

- If you need a variant of 4, subtract it from a variant of 5
  - ✓ If you need 4, use 5 – 1 (IV)
  - ✓ If you need 40, use 50 – 10 (XL)
  - ✓ If you need 400, use 500 – 100 (CD)
- If you need 9, subtract it from a variant of 10
  - ✓ $9 = 10 - 1 (IX)$

- ✓ $90 = 100 - 10\ (XC)$
- ✓ $900 = 1000 - 100\ (CM)$

D. Converting to Roman Numerals from Decimal

- ➢ Write the decimal in expanded form
- ➢ Convert each part of the expanded form into a Roman Numeral

Put all the parts together

15. Arithmetic

15.1 Arithmetic with Roman Numerals

A. T

B. Arithmetic Operations

- Convert from Roman to Decimal
- Perform the indicated operations
- Convert back to Roman

Example 1

Simplify the following, and write the answer in Roman Numerals

$$LXXII + LXI$$

Example 2

Simplify the following, and write the answer in Roman Numerals

XLVII – XXIV

$$XLVII = 40 + 7 = 47$$
$$XXIV = 20 + 4 = 24$$
$$47 - 24 = 23$$
$$23 = 20 + 3 = XXIII$$

Example 3

$$XVIII + IV + V = 18 + 4 + 5 = 27 = XXVII$$