FRACTIONS

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1. Fractions

1.1 Basics

Definition

1.1: Definition
Fractions are when we talk about equal parts of a whole.

$$\underbrace{\frac{3}{4}}_{\substack{Read: 3\ over\ 4 \\ or \\ Read: Three-Fourths}} \Rightarrow 3 \to \underbrace{\text{Numerator}}_{\substack{Number\ of\ Parts \\ we\ are\ counting}}, \quad 4 \to \underbrace{\text{Denominator}}_{\substack{How\ many\ parts \\ in\ one\ whole}}$$

Example 1.2
Identify the numerator and the denominator in $\frac{5}{7}$

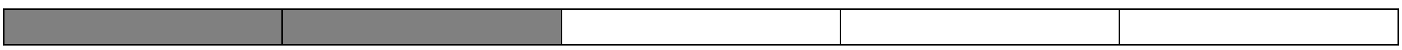
$$Numerator = 5$$
$$Denominator = 7$$

1.3: Alternate Definition
We can also define fractions as division
$$\frac{Numerator}{Denominator} = Numerator \div Denominator$$

$$\frac{12}{2} = 12 \div 2 = 6$$
$$\frac{4}{5} = 4 \div 5$$

1.4: Equal Parts
It is important for the parts to be equal. If the parts are not equal, you have to make equal.

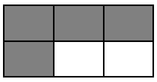
Example 1.5
In each shape below, what is the:
A. Shaded Fraction
B. Unshaded Fraction



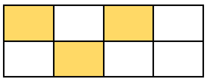
The shape is divided into five parts. The five parts are equal. Hence, we can say that
$$Shaded\ Fraction = \frac{2}{5}$$

$$Unshaded\ Fraction = \frac{3}{5}$$

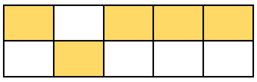
$$Shaded\ Fraction = \frac{3}{4}$$
$$Unshaded\ Fraction = \frac{1}{4}$$



$$Shaded\ Fraction = \frac{4}{6}$$
$$Unshaded\ Fraction = \frac{2}{6}$$



$$Shaded\ Fraction = \frac{3}{8}$$
$$Unshaded\ Fraction = \frac{5}{8}$$



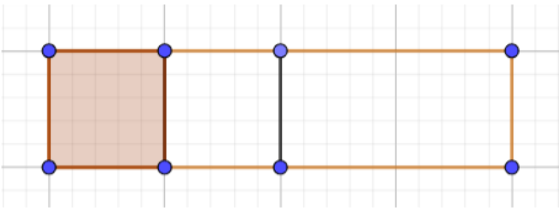
$$Shaded\ Fraction = \frac{5}{10}$$
$$Unshaded\ Fraction = \frac{5}{10}$$

Parts Not Equal

If the parts are not equal, then we first need to make them equal. This is very important since exam questions check your knowledge of this concept.

Example 1.6

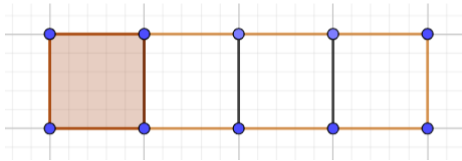
A. How many parts is the shape on the right divided into?
B. What fraction of the shape is shaded?



We need to make equal parts:

$$No.\,of\ Parts = 3$$

$$Shaded\ Fraction = \frac{1}{4}$$



A. Writing Fractions

Example 1.7

Alistair writes a fraction with the $\underbrace{predecessor\ of\ 2}_{\text{Number which comes just before 2}}$ for the numerator and the $\underbrace{successor\ of\ 3}_{\text{Number which comes just after 3}}$ for the denominator. What is the value of his fraction?

$$\frac{Numerator}{Denominator} = \frac{Predecessor\ of\ 2}{Successor\ of\ 3} = \frac{1}{4}$$

Example 1.8: Identifying Fractions

Shubham took a cake and divided into five equal parts. He gave two out of the five parts to his brother and ate three himself. Write the share of the cake that each person got as a fraction.

$$Shubham = \frac{3}{5}, \qquad Brother = \frac{2}{5}$$

Example 1.9: Understanding Numerator and Denominator

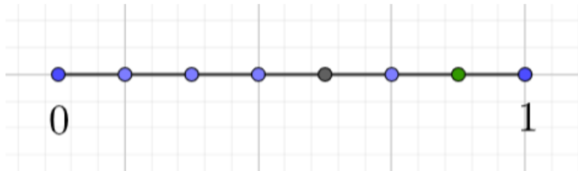
Shaurya had $\frac{3}{5}th$ of a pizza. What do 3 and 5 represent in this context?

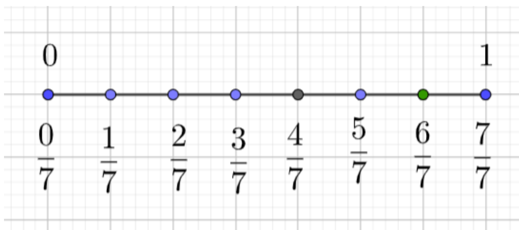
5 represents the number of equal parts that the pizza was divided into.
3 represents the number of equal parts that were eaten by Shaurya from the pizza.

B. Number Line

Example 1.10

A. What fraction does the green dot represent?
B. What fraction does the black dot represent?





$$Black\ Dot = \frac{4}{7}$$
$$Green\ Dot = \frac{6}{7}$$

Example 1.11

Consider the number $\frac{2}{7}$ on the number line. What do 2 and 7 represent in this context?

7 is the number of parts that each number on the number line is divided into.
2 is the number of parts at which the number $\frac{2}{7}$ lies.

C. Numerator and Denominator: Mixed Review

Example 1.12

Kaushalya had $\frac{4}{7}th$ of a cake. Kripa had $\frac{2}{7}th$ of a cake of the same cake as the one Kaushalya had.
Match the column with respect to the meaning of each number. Answer for Kaushalya, Kripa and both separately.

No.	Cake
2	Parts of the Cake
4	Parts eaten from the cake
6	Equal parts of the Cake
7	Equal parts eaten of the cake

Kaushalya

$$Equal\ Parts\ of\ the\ Cake = Parts\ of\ Cake = 7$$
$$Parts\ eaten\ from\ the\ cake = Equal\ Parts\ eather\ from\ the\ cake = 4$$

Kripa

$$Equal\ Parts\ of\ the\ Cake = Parts\ of\ Cake = 7$$
$$Parts\ eaten\ from\ the\ cake = Equal\ Parts\ eather\ from\ the\ cake = 2$$

D. Invalid Comparisons

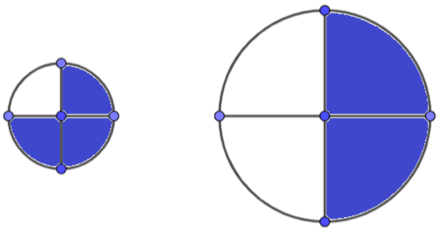
It is important when considering a fraction that the parts are equal. If the parts are not equal, then they need to be made equal, before considering them as a fraction.

Example 1.13

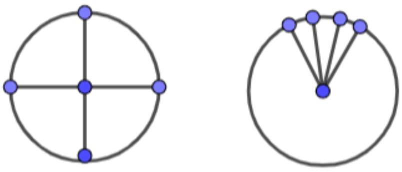
Ronald ate two slices from a pizza with four slices, while his brother, Richard had three slices from a pizza divided into four slices. Richard complained that Ronald had eaten more pizza than he did. Ronald replied that Richard had eaten more slices than he did. If *both Ronald and Richard were correct*, explain how?

Answer 1

If the pizzas were not of equal size, then both can be simultaneously correct.



Answer 2



If the pizzas slices were not equal, then also both can be correct.

Example 1.14

Mark the correct option

Rohan had $\frac{3}{5}th$ of a cake. Sohan had $\frac{1}{5}th$ of a cake. Who had more cake?

A. Rohan
B. Sohan
C. Both had equal cake
D. Not possible to determine with the given information

We have not been told that the size of cake that they ate is the same. Therefore, without more information, we cannot compare how much cake they ate.

E. Valid Comparisons

Example 1.15

Mark the correct option

Mary had $\frac{2}{7}th$ of a cake. Byron had had $\frac{3}{7}th$ of a cake of the same size that Mary had. Who had more cake?

A. Mary
B. Byron
C. Both had the same amount of cake
D. Not possible to determine with the given information

We can compare the fractions here, because the cake sizes are the same.

$$\frac{2}{7} < \frac{3}{7} \Rightarrow Byron\ had\ more\ cake.$$

F. Converting Fractions to Words

Example 1.16

Convert the given fractions into words

A. $\frac{1}{2}$
B. $\frac{1}{4}$
C. $\frac{1}{10}$
D. $\frac{1}{5}$
E. $\frac{1}{3}$
F. $\frac{1}{8}$
G. $\frac{1}{6}$
H. $\frac{1}{12}$
I. $\frac{1}{25}$
J. $\frac{1}{97}$

$$\frac{1}{2} \to One\ Half$$
$$\frac{1}{4} \to One\ Quarter\ Or\ One\ Fourth$$
$$\frac{1}{10} \to One\ Tenth$$
$$\frac{1}{5} \to One\ Fifth$$
$$\frac{1}{3} \to One\ Third$$
$$\frac{1}{8} \to One\ Eighth$$
$$\frac{1}{6} \to One\ Sixth$$
$$\frac{1}{12} \to One\ Twelfth$$
$$\frac{1}{25} = One\ Twenty\ Fifth$$

Example 1.17

A. $\frac{3}{4}$
B. $\frac{2}{5}$
C. $\frac{4}{7}$
D. $\frac{2}{25}$
E. $\frac{9}{34}$

$$\frac{3}{4} \to Three\ Quarters$$
$$\frac{2}{5} \to Two\ Fifths$$
$$\frac{4}{7} \to Four\ Sevenths$$
$$\frac{2}{25} = Two\ Twenty\ Fifths$$
$$\frac{9}{34} = Nine\ Thirty\ Fourths$$

Example 1.18

A. *Five Sevenths*
B. *One Quarter*

$$\text{Five Sevenths} = \frac{5}{7}$$
$$\text{One Quarter} = \frac{1}{4}$$

G. Word Problems on Shading

Example 1.19

Three out of seven equal parts of a circle are shaded. We want the shading to be $\frac{5}{7}$.

A. How many more parts must be shaded?
B. What is the fraction that must be shaded?

$$\underbrace{\frac{3}{7}}_{\substack{\text{Current}\\\text{Shading}}} \rightarrow \underbrace{\frac{5}{7}}_{\substack{\text{New}\\\text{Shading}}} \Rightarrow \underbrace{2 \text{ parts to be shaded}}_{\text{Part A}} \Rightarrow \underbrace{\frac{2}{7} \text{ to be shaded}}_{\text{Part B}}$$

The answer to Part A is in terms of number of Parts A, while the answer to Part B is in terms of a fraction. These two are the same concept, but their presentation is different.

Example 1.20

Four equal parts out of nine parts are shaded. Find the

A. Number of unshaded parts
B. Fraction which is not shaded.

$$\underbrace{\frac{4}{9}}_{\substack{\text{Current}\\\text{Shading}}} \rightarrow \underbrace{5 \text{ Parts}}_{\substack{\text{Unshaded}\\\text{Parts}}} \Rightarrow \underbrace{\frac{5}{9}}_{\substack{\text{Unshaded}\\\text{Fraction}}}$$

H. Grouped Questions

Make a distinction between questions which asks for fractions, and questions which ask for numbers.

Example 1.21

Answer the following questions. You may use information from one part in further parts.

A bakery shop has twelve cakes.

A. If five of the cakes are chocolates cakes, what is the fraction of chocolate cakes?
B. If four of the cakes have frosting on them, what fraction of cakes have frosting on them?
C. If two of the cakes are sold, what is the fraction of cakes that are yet to be sold?
D. If half of the cakes are 1 kg cakes, how many 1 kg cakes are there?
E. If one quarter of the cakes are half-kg cakes, how many half-kg cakes are there?
F. If half the cakes with frosting are chocolate cakes, how many chocolate cakes with frosting are there?
G. If $\frac{9}{12}$ of the cakes have vanilla essence, how many cakes have vanilla essence?

Parts A-C: Fractions as Answers

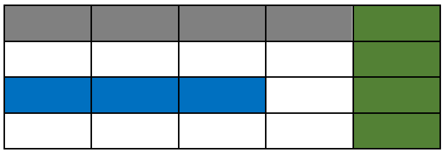
$$\frac{5}{12}$$
$$\frac{4}{12}$$

$$\frac{10}{12}$$

Parts D-G

$$Half\ of\ 12 = 6$$
$$One - quarter\ of\ 12 = 4$$
$$One - half\ of\ 4 = 2$$
$$Cakes\ with\ vanilla\ essence = 9$$

Example 1.22



A. What is the fraction of the boxes that are colored blue?
B. How many boxes are colored green?
C. Of the squares which are not colored white, what is the fraction that is colored gray?
D. How many more squares are colored green, than colored gray?
E. If one more box is colored blue out of the white boxes, what will be the fraction of boxes colored white?

$$Fraction\ coloured\ blue = \frac{3}{20}$$
$$No.of\ Boxes\ colored\ Green = 4$$
$$Fraction\ colored\ gray\ among\ coloured\ squares = \frac{4}{11}$$
$$No.of\ Green\ Squares - No.of\ Gray\ Squares = 4 - 4 = 0$$
$$Fraction\ of\ white\ boxes = \frac{8}{20}$$

1.2 Equivalent Fractions

Introduction

Fractions tell us the share in equal parts.

Example 1.23

An apple is shared among two siblings. The apple is cut into two parts. Each sibling gets one part. The fraction that the sibling has got is the shaded part or one-half of the apple.

$$\frac{1}{2}\text{ of the apple} = \text{one} - \text{half of the apple}$$

Now, suppose each of the parts is cut into further parts. Then, in all, there are four parts. Each sibling has two of those parts. Each sibling has

$$\frac{2}{4}\text{of the apple} = \text{two} - \text{fourths of the apple}$$

But these two fractions are equal. We can do this numerically by showing that we convert each part in the numerator and the denominator of the fraction into two equal parts.

$$\frac{1}{2}=\frac{1\times 2}{2\times 2}=\frac{2}{4}$$

Definition

Fractions which are equal are called equivalent fractions.
An equivalent fraction can be obtained by multiplying the numerator and denominator of a fraction by the same number.

Example 1.24

Find four equivalent fractions for $\frac{1}{2}$.

Multiply by 2	Multiply by 3	Multiply by 4	Multiply by 5
$\frac{1}{2}=\frac{1\times 2}{2\times 2}=\frac{2}{4}$	$\frac{1}{2}=\frac{1\times 3}{2\times 3}=\frac{3}{6}$	$\frac{1}{2}=\frac{1\times 4}{2\times 4}=\frac{4}{8}$	$\frac{1}{2}=\frac{1\times 5}{2\times 5}=\frac{5}{10}$

Example 1.25

Fill in the missing values in each part below:

↑ Numerator

A. $\frac{1}{2}=\frac{}{8}$
B. $\frac{2}{3}=\frac{}{6}$
C. $\frac{1}{3}=\frac{}{9}$
D. $\frac{1}{5}=\frac{}{20}$
E. $\frac{2}{5}=\frac{}{30}$
F. $\frac{4}{5}=\frac{}{15}$
G. $\frac{5}{7}=\frac{}{42}$
H. $\frac{2}{9}=\frac{}{18}$
I. $\frac{7}{11}=\frac{}{55}$
J. $\frac{4}{7}=\frac{}{28}$
K. $\frac{2}{10}=\frac{}{100}$
L. $\frac{4}{13}=\frac{}{26}$
M. $\frac{7}{19}=\frac{}{38}$

↓ Numerator

N. $\frac{}{4}=\frac{12}{16}$
O. $\frac{}{10}=\frac{18}{20}$
P. $\frac{}{3}=\frac{4}{6}$

↑Denominator

Q. $\frac{2}{3}=\frac{12}{}$
R. $\frac{4}{5}=\frac{12}{}$
S. $\frac{5}{7}=\frac{35}{}$

↓ Denominator

T. $\frac{7}{}=\frac{70}{100}$
U. $\frac{3}{}=\frac{30}{40}$
V. $\frac{1}{}=\frac{9}{18}$

$$\frac{1}{2}=\frac{4}{8}$$
$$\frac{2}{3}=\frac{4}{6}$$

Example 1.26

Surya had a cake divided into four equal parts. He gave one part to his brother. Then he divided each part into two equal parts. Then he ate a part.

A. What is the fraction of the cake that he has?
B. Find the sum of the numerator and the denominator of the reduced form of the fraction of cake that he has.

Surya divides a cake into four equal parts:

$$\frac{4}{4}\ Cake$$

He gives one part to his brother

$$\frac{3}{4}Cake$$

Then he divides each part into two equal parts

$$\frac{6}{8}Cake$$

Then he ate a part

$$\frac{5}{8}Cake$$

Sum of numerator and denominator

$$5+8=13$$

Example 1.27

Santosh was working with the fraction *five sevenths*. He found an equivalent fraction by multiplying the numerator and the denominator of the fraction by three. Find the sum of the numerator and denominator of the fraction that Santosh found.

$$\frac{5}{7}=\frac{15}{21}\Rightarrow 15+21=36$$

Example 1.28: Three Part Expressions

Fill in the missing values in each part below:

A. $\frac{1}{3}=\frac{10}{\ }=\frac{3}{\ }$
B. $\frac{2}{5}=\frac{6}{\ }=\frac{8}{\ }$
C. $\frac{5}{7}=\frac{25}{\ }=\frac{35}{\ }$
D. $\frac{5}{7}=\frac{\ }{14}=\frac{\ }{28}$
E. $\frac{3}{4}=\frac{\ }{12}=\frac{\ }{20}$
F. $\frac{7}{9}=\frac{63}{\ }=\frac{\ }{81}$
G. $\frac{\ }{7}=\frac{15}{21}=\frac{\ }{28}$

This question can be easier to solve if you break it into two different questions.

$$\frac{1}{3}=\frac{10}{\ }\Rightarrow\frac{1}{3}=\frac{1\times10}{3\times10}=\frac{10}{30}$$
$$\frac{1}{3}=\frac{3}{\ }\Rightarrow\frac{1}{3}=\frac{1\times3}{3\times3}=\frac{3}{9}$$

$$\frac{2}{5}=\frac{6}{\ }=\frac{8}{\ }$$

Simplifying Fractions

Is the reverse of finding equivalent fractions

Simplification		
Direct Multiple	One-Step	Multi-Step
$\frac{3}{6}=\frac{1\times\mathbf{3}}{2\times\mathbf{3}}=\frac{1}{2}\times\frac{\mathbf{3}}{\mathbf{3}}=\frac{1}{2}$ $\frac{4}{20}=\frac{1\times\mathbf{4}}{5\times\mathbf{4}}=\frac{1}{5}$ $\frac{6}{8}=\frac{3\times\mathbf{2}}{4\times\mathbf{2}}=\frac{3}{4}\times\frac{\mathbf{2}}{\mathbf{2}}=\frac{3}{4}$ $\frac{3}{12}=\frac{1\times3}{4\times3}=\frac{1}{4}$	$\frac{15}{25}=\frac{3\times\mathbf{5}}{5\times\mathbf{5}}=\frac{3}{5}$ $\frac{4}{14}=\frac{2}{7}$ $\frac{6}{16}=\frac{3}{8}$	$\frac{16}{24}=\frac{\mathbf{2}\times8}{\mathbf{2}\times12}=\frac{8}{12}=\frac{4}{6}=\frac{2}{3}$ $\frac{36}{72}=\frac{18}{36}=\frac{9}{18}=\frac{3}{6}=\frac{1}{2}$ $\frac{17}{51}=\frac{\mathbf{17}\times1}{\mathbf{17}\times3}=\frac{1}{3}$

Example 1.29
Simplify the following fractions

Whole Numbers

A. $\frac{6}{3}$
B. $\frac{9}{3}$
C. $\frac{42}{7}$
D. $\frac{27}{9}$
E. $\frac{64}{8}$
F. $\frac{30}{5}$
G. $\frac{35}{7}$

Unit Fractions

H. $\frac{4}{8}$
I. $\frac{3}{9}$
J. $\frac{5}{20}$
K. $\frac{7}{21}$

General

L. $\frac{6}{9}$
M. $\frac{9}{12}$
N. $\frac{25}{30}$
O. $\frac{32}{40}$
P. $\frac{100}{200}$
Q. $\frac{35}{55}$
R. $\frac{64}{72}$
S. $\frac{44}{77}$
T. $\frac{55}{121}$
U.

Whole Numbers

$$\frac{9}{3}=\frac{3}{1}=3$$
$$\frac{42}{7}=6$$

Unit Fractions

$$\frac{4}{8}=\frac{1}{2}$$

Multi-Step Simplification
When we are asked to reduce to lowest form, it is important to keep simplifying till there is no common factor between the numerator and the denominator.

Example 1.30
Simplify
$$\frac{34}{102}$$

We recognize that both numerator and denominator are divisible by 2. Hence, we get
$$\frac{34}{102}=\frac{17\times 2}{51\times 2}=\frac{17}{51}$$

But, we cannot stop here. Neither 17, nor 51 are divisible by 2. But when check for 3, sum of digits of $51=5+1=6$ is divisible by 51. Therefore, we find, by dividing that $51=17\times 3$.
Hence, we can simplify:
$$\frac{17}{51}=\frac{17\times 1}{17\times 3}=\frac{1}{3}$$

Example 1.31
$$\frac{38}{190}$$

$$\frac{38}{190}=\frac{2}{10}=\frac{1}{5}$$

Prime Numbers

It's important to know the prime numbers from 2 to 100, in order to recognize when a number cannot be simplified further.

Prime Numbers from 2 to 100									
2	11	23	31	41	53	61	71	83	97
3	13	29	37	43	59	67	73	89	
5	17			47			79		
7	19								

Example 1.32

Simplify, if possible, the following fractions $\left\{\frac{23}{67},\frac{31}{97},\frac{79}{41},\frac{61}{71},\frac{11}{43}\right\}$

The numerator and denominator of each of the fractions above is prime. Hence, they cannot be simplified further.

Even a single number (out of the numerator and the denominator) being prime is sufficient to tell us that the fraction cannot be simplified.

Example 1.33

Simplify, if possible, the following fractions $\left\{\frac{47}{22},\frac{69}{29},\frac{12}{89},\frac{15}{73}\right\}$

Either the numerator or the denominator of the fractions above is prime.

$$\left\{\frac{\mathbf{47}}{22},\frac{69}{\mathbf{29}},\frac{12}{\mathbf{89}},\frac{15}{\mathbf{73}}\right\}$$

Hence, the fractions cannot be simplified further.

Tests of Divisibility

You need to know the tests of divisibility to be able to simplify fractions. This is especially true if the numbers are a little complicated.

1.34: Test of Divisibility

- A number is divisible by 2 if it is even. A number is even if its last digit is any of $\{0,2,4,6,8\}$.
- A number is divisible by 5 if its last digit is zero or 5.
- A number is divisible by 3, if the sum of the digits of the number is divisible by 3.

Example 1.35

Simplify $\frac{111}{33}$

We know that $33=3\times 11$. Therefore, 33 is divisible by 11. We can check for divisibility of 111 by 3 by using the sum of the digits, which is $1+1+1=3$, which is divisible by 3.

$$\frac{111}{33}=\frac{37\times 3}{11\times 3}=\frac{37}{11}$$

We can't simplify this further, since both 37 and 11 are prime numbers.

Basics

Example 1.36

A. $\frac{1}{2} = \frac{2}{} = \frac{3}{}$
B. $\frac{1}{3} = \frac{10}{} = \frac{}{9}$
C. $\frac{2}{5} = \frac{}{30} = \frac{8}{}$

$$\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$$

Example 1.37

A. Find $a + b$ if $\frac{2}{3} = \frac{10}{a} = \frac{b}{18}$
B. Find $x + y$ if $\frac{5}{7} = \frac{50}{x} = \frac{y}{42}$
C. Find $p - q$ if $\frac{30}{50} = \frac{3}{p} = \frac{q}{5}$

$$\frac{2}{3} = \frac{10}{15} = \frac{12}{18} \Rightarrow a = 15, b = 18 \Rightarrow a + b = 15 + 18 = 33$$

$$\frac{5}{7} = \frac{50}{70} = \frac{30}{42} \Rightarrow x = 70, y = 30 \Rightarrow x + y = 70 + 30 = 100$$

$$\frac{30}{50} = \frac{3}{5} = \frac{3}{5} \Rightarrow p = 5, q = 3 \Rightarrow p - q = 5 - 3 = 2$$

Fractions as Division

Till now, we have learnt fractions as part of a whole. Another way of looking at fractions (which is the equivalent) is to look at fractions as division.

1.38: Fractions as Division

A fraction can be interpreted as its numerator divided by its denominator.

$$\frac{Numerator}{Denominator} = Numerator \div Denominator$$

Example 1.39

Consider the fraction $\frac{3}{5}$. Explain its meaning as

A. Parts of a whole
B. Division

$\frac{3}{5}$ when interpreted as parts of whole, means we have a whole divided into five parts, out of which we are looking at three parts under consideration.

When interpreted as division, we mean that we are going to divide the numerator by the denominator. $\frac{3}{5}$ means we are dividing three by five. For example, we could be dividing three apples among five people.

Example 1.40

A. $\frac{2}{7}$
B. $\frac{5}{6}$

C. $\frac{3}{11}$
D. $\frac{4}{9}$
E. $\frac{7}{13}$

$$2 \text{ parts out of } 7$$
$$5 \text{ parts out of } 6$$
$$3 \text{ parts out of } 11$$
$$4 \text{ parts out of } 9$$

Using Fractions as Division

Let's look at an example of simplifying fractions where the answers are whole numbers.

Example 1.41

Simplify each set in the fraction to below to arrive at a whole number

$$\left\{\frac{6}{3}, \frac{12}{4}, \frac{30}{6}, \frac{84}{12}, \frac{91}{7}, \frac{65}{13}\right\}$$

$$\left\{\frac{6}{3} = \frac{2}{1} = 2, \quad \frac{12}{4} = \frac{3}{1} = 3, \quad \frac{30}{6} = \frac{5}{1} = 5, \quad \frac{84}{12} = \frac{7}{1} = 7, \quad \frac{91}{7} = \frac{13}{1} = 13, \quad \frac{65}{13} = \frac{5}{1} = 5\right\}$$

The concept of fractions as division is directly useful in simplifying fractions which are actually whole numbers. Instead of doing the simplification, we can arrive at the same answer by doing division

$$\left\{\frac{6}{3} = 6 \div 3 = 2, \frac{12}{4} = 12 \div 4 = 3, \frac{30}{6} = 30 \div 6 = 5, \frac{84}{12} = 84 \div 12 = 7, \frac{91}{7} = 91 \div 7 = 13, \frac{65}{13} = 65 \div 13 = 5\right\}$$

Since we interpreted fractions as division, we now have a new way of looking at denominators. Denominators are numbers that you divide by.

For example, if we share 8 cakes among 2 hungry students, each one will get:

$$\frac{8}{2} = 8 \div 2 = 4$$

1.42: Writing Whole Numbers as Fractions

Whole Numbers can be written as a fraction. Since dividing by one does not change the value of a number, we can put 1 as the denominator of any number.

Example 1.43

Write the following as fractions:

A. 7
B. 4
C. 9
D. 2
E. 13

Write the following as fractions: {3,7,12,1}

$$\underbrace{3}_{\substack{Whole\\Number}} = \underbrace{\frac{3}{1}}_{Fraction}, \quad \underbrace{7}_{\substack{Whole\\Number}} = \underbrace{\frac{7}{1}}_{Fraction}, \quad 12 = \frac{12}{1}, \quad 1 = \frac{1}{1}$$

1.44: Writing Fractions with a Denominator of One as a Fraction

Since any number divided by one remains itself, we need not mention the denominator, and the number still remains the same.

Example 1.45

Write the following as whole numbers

A. $\frac{19}{1}$
B. $\frac{11}{1}$
C. $\frac{12}{1}$
D. $\frac{15}{1}$
E. $\frac{16}{1}$

Example 1.46

Write the following fractions as whole numbers: $\left\{\frac{5}{1}, \frac{8}{1}, \frac{13}{.1}, \frac{1}{1}\right\}$

$$\underbrace{\frac{5}{1}}_{Fraction} = \underbrace{5}_{\substack{Whole \\ Number}}, \quad \frac{8}{1} = 8, \quad \frac{13}{1} = \frac{13}{1}, \quad 1 = 1$$

Identifying Fractions versus Whole Numbers

Example 1.47

Some numbers are given below. Identify which numbers are equal. Also, state which is written as a whole number, and which is written as a fraction.

$$\left\{4, \frac{7}{1}, 8, \frac{4}{1}, 7, 5\right\}$$

The numbers which are equal are:

$$\underbrace{4}_{Whole\ Number} = \underbrace{\frac{4}{1}}_{Fraction}$$

1.48: Reciprocal of a Fraction

If I have a fraction, which is

$$\frac{Numerator}{Denominator}$$

Then its reciprocal is obtained by interchanging the numerator and the denominator, giving us:

$$\frac{Denominator}{Numerator}$$

Example 1.49

Find the reciprocal of the following fractions:

A. $\frac{7}{11}$
B. $\frac{4}{7}$

C. $\frac{9}{7}$
D. $\frac{3}{5}$
E. $\frac{4}{1}$

$$\frac{11}{7}$$
$$\frac{7}{4}$$
$$\frac{7}{9}$$
$$\frac{5}{3}$$
$$\frac{1}{4}$$

Example 1.50

Fill in the blank with the correct option

A fraction is _______ equal to its reciprocal.

A. Always
B. Never
C. Sometimes
D. None of the above

$$1 = \frac{1}{1} = \frac{1}{1} = 1$$
$$Reciprocal\ of\ \frac{2}{3} = \frac{3}{2} \neq \frac{2}{3}$$

Hence, a fraction is sometimes equal to its reciprocal.

$$Option\ C$$

Example 1.51

Are the fractions $\frac{1}{3}$ and $\frac{3}{1}$ the same?

$\frac{1}{3}$ means that one is being divided into three parts.

$\frac{3}{1}$ means that three is being divided into a single part.

These two fractions do not represent the same quantity, and hence they are not the same.

Example 1.52

Sunil gave three apples to A. His brother Anil gave a single apple to X, Y and Z who shared it equally among themselves. (The three of them together got a single apple). State and compare the shares that the people got.

A got three apples, which were not shared with anyone, so he got

$$\frac{3}{1} = 3\ Apples$$

X, Y and Z got one apple, which had to be shared among the three of them, and hence each of them got

$$\frac{1}{3}\ of\ an\ Apple$$

Shading

Example 1.53

A student drew a rectangle with five equal parts, and shaded three parts of the rectangle in answer to an exam question which required $\frac{6}{10}$ of the rectangle to be shaded.
Has he shaded the correct fraction?

$$\underbrace{\frac{6}{10}}_{\substack{Correct \\ Answer}} = \underbrace{\frac{3}{5}}_{\substack{Equivalent \\ Fraction}}$$

Example 1.54

Aatish has a rectangle with one part out of five shaded. He needs a rectangle that is shaded $\frac{8}{10}$. How many more parts must he shade in his current rectangle?

$$\frac{8}{10} = \frac{4}{5} \rightarrow Total\ 4\ parts\ to\ be\ shaded \rightarrow 3\ New\ parts\ to\ be\ shaded$$

Concept Questions

Example 1.55

Drishti took a chocolate bar and divided it into three equal pieces for her three friends, who turned up their noses since it was too less. So, Drishti divided up each piece of chocolate into two equal pieces. This time, happy at getting two pieces each, they accepted. Did the chocolate offered increase? Explain.

The fraction that Drishti offered her friends earlier was:

$$\frac{1}{3}$$

When she broke each piece into two pieces, she was offering them

$$\frac{2}{6}$$

But the two fractions are equivalent:

$$\frac{1}{3} = \frac{2}{6}$$

Hence, the chocolate offered did not increase.

Cumulative Review

Example 1.56

The following recipe shows the ingredients to make a single pudding.

Sugar	$\frac{1}{4}\ Cup$	*Salt*	To Taste
Flour	$\frac{9}{4}\ Cup$	Cornstarch	$\frac{5}{2}\ Tablespoons$
Spice Mix	$\frac{2}{3}\ Tablespoons$	*Vanilla Extract*	$\frac{3}{4}\ Teaspoon$

Making Puddings

A. Reema wants to make three puddings for her brother's birthday. What is the sugar required?

B. What is the flour required to make two puddings?
C. What is the spice mix needed to make six puddings?

Putting Ingredients Together

D. If the flour and the sugar for a single pudding are added together, how many cups will they be?

If-Then Scenarios

E. By mistake, the cornstarch has been written to be double of what it should actually be. How much cornstarch will you need for one pudding?

Multi-Step

F. Sean is making five puddings. How much spice mix and cornstarch will he need together?
G. Max is making three puddings. How much sugar and flour will he need together?

Sean

$$One\ Pudding = \frac{2}{3} + \frac{5}{2} = \frac{4}{6} + \frac{15}{6} = \frac{19}{6}$$
$$Five\ Puddings = \frac{19}{6} \times 5 = \frac{95}{6}$$

Max

$$One\ Pudding = \frac{1}{4} + \frac{9}{4} = \frac{10}{4} = \frac{5}{2}$$
$$Three\ Puddings = \frac{5}{2} \times 3 = \frac{15}{2} = 7\frac{1}{2}$$

1.3 Proper and Improper Fractions

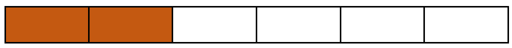
A. Proper Fractions

1.57: Proper Fractions

- A fraction where the numerator is less than the denominator is a proper fraction.
- A proper fraction is less than one.

$\frac{3}{5} =$

$\frac{2}{6} =$



Note that in each fraction above, the numerator is less than the denominator.
For example, in the first fraction

$$3 < 5$$

In the second fraction:

$$2 < 6$$

1.58: Improper Fractions: Type I

A fraction where the numerator is more than the denominator is an improper fraction.

An improper fraction is more than one whole.

$$\frac{8}{5} =$$

$$\frac{6}{4} =$$

1.59: Improper Fractions: Type II

Even if the numerator is equal to the denominator, then also it is an improper fraction.

$$Improper\ Fraction: \frac{5}{5} = 1$$

Example 1.60: Classification

Classify the following fractions as proper or improper:

A. $\frac{3}{5}$
B. $\frac{17}{20}$
C. $\frac{99}{98}$
D. $\frac{43}{97}$
E. $\frac{913}{913}$
F. $\frac{27}{23}$
G. $\frac{19}{20}$
H. $\frac{1}{875}$
I. 120

$$3 < 5 \Rightarrow \frac{3}{5} is\ a\ proper\ fraction$$
$$17 < 20 \Rightarrow \frac{17}{20} is\ a\ proper\ fraction$$
$$99 > 98 \Rightarrow \frac{99}{98} is\ an\ improper\ fraction$$
$$43 < 97 \Rightarrow \frac{43}{97}\ is\ a\ proper\ fraction$$
$$913 = 913 \Rightarrow \frac{913}{913} = 1\ is\ an\ improper\ fraction$$

Example 1.61

if a is a natural number. (Natural numbers are the counting numbers $\{1,2,3, \dots\}$)

A. Consider the proper fraction $\frac{a}{7}$. What is the largest possible value of a, if a is a natural number.
B. Consider the improper fraction $\frac{a}{11}$. What is the smallest possible value of a,
C. The fraction $\frac{a}{7}$ is improper, and the fraction $\frac{a}{11}$ is proper. Find the sum of all possible values of a.
D. The fraction $\frac{a}{7}$ is proper, and the fraction $\frac{a}{11}$ is improper. Find the possible values of a.

Part A
$a = 6$ is the largest value possible for a.
Part B
$b = 11$ is the largest value possible for a.
Part C

$$\frac{a}{7} \text{ is improper} \Rightarrow a \geq 7 \Rightarrow a \in \{7,8,9,10,11,\dots\}$$
$$\frac{a}{11} \text{ is proper} \Rightarrow a \leq 10 \Rightarrow \{\dots,6,7,8,9,10\}$$

Combine the two conditions:

$$a \in \{7,8,9,10\} \Rightarrow Sum = 7+8+9+10 = 34$$

Part D

$$\frac{a}{7} \text{ is proper} \Rightarrow a \leq 7 \Rightarrow a \in \{1,2,3,4,5,6,7\}$$
$$\frac{a}{11} \text{ is improper} \Rightarrow a \geq 11 \Rightarrow \{11,12,13,\dots\}$$

There are no common numbers.
Hence, there are no values of a that satisfy the given condition.

1.62: Comparing using one as a Benchmark

Given a fraction, you can check whether it is more than one, equal to one, or less than one using the following:

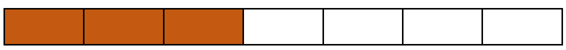
- If the numerator is less than the denominator (that is, it is a proper fraction), then the fraction is less than one.
- If the numerator is equal to the denominator, then the fraction is equal to one.
- If the numerator is more than the denominator, then the fraction is more than one.

Example 1.63: Classifying Fractions

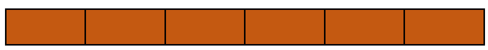
Classify the following fractions as less than one, equal to one, or greater than one:

A. $\frac{3}{7}$
B. $\frac{2}{9}$
C. $\frac{6}{6}$
D. $\frac{4}{3}$
E. $\frac{8}{9}$
F. $\frac{12}{10}$
G. $\frac{4}{5}$
H. $\frac{7}{3}$
I. $\frac{12}{4}$
J. $\frac{8}{8}$

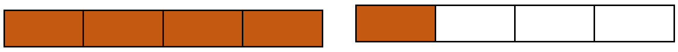
$$3 < 7 \Rightarrow \textit{Proper Fraction} \Rightarrow \textit{Less than One}$$



$$6 = 6 \Rightarrow \textit{Improper Fraction} \Rightarrow \textit{Equal to One}$$



$4 > 3 \Rightarrow Improper\ Fraction \Rightarrow Greater\ than\ One$



Example 1.64
Arrange the fractions $\left\{\frac{2}{5}, \frac{9}{7}, \frac{4}{4}\right\}$ in ascending order.

$$\left\{\underbrace{\frac{2}{5}<1}_{\substack{N<D\\ \text{Proper}\\ \text{Less than one}}},\ \underbrace{\frac{4}{4}=1}_{\substack{N=D\\ \text{Improper}\\ \text{Equal to One}}},\ \underbrace{\frac{9}{7}>1}_{\substack{N>D\\ \text{Improper}\\ \text{More than One}}}\right\} \Rightarrow \left\{\frac{2}{5}, \frac{4}{4}, \frac{9}{7}\right\}$$

Example 1.65
Arrange the fractions $\left\{\frac{11}{4}, \frac{2}{2}, \frac{3}{4}\right\}$ in descending order

B. Improper Fractions
Improper fractions are fractions which are more than one. In order to convert them into mixed numbers, we break them into wholes, and a proper fraction.

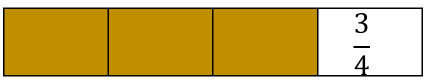
Example 1.66: Writing One as a Fraction
Write one whole with the following denominators:

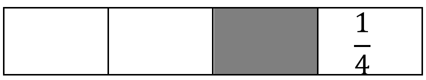
A. 3
B. 7
C. 2
D. 5
E. 9
F. 12
G. 4

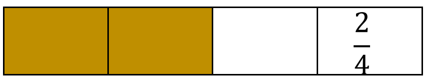
$$1 = \frac{3}{3} = \frac{7}{7} = \frac{2}{2} = \frac{5}{5} = \frac{9}{9} = \frac{12}{12} = \frac{4}{4}$$

1.4 Add and Subtract: Same Denominator

A. Basics







$$\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$$

B. Word Problems

Example 1.67
If an additional one-fourth of the circle is shaded, what fraction of the circle will be shaded.
(Add circle).

C. Number Lines

D. Addition

To add fractions, we need the denominator to be the same. Just add the numerators. You can add even if the numerator is more than the denominator.

$$\frac{4}{\underbrace{7}_{4\ parts\ of\ 7}} + \frac{2}{\underbrace{7}_{2\ parts\ of\ 7}} = \frac{4+2}{7} = \frac{6}{\underbrace{7}_{6\ parts\ of\ 7}}$$

1.68: Adding Fractions
To add two fractions with the same denominator:
- Add the numerators. This increases the number of "shaded" parts.
- The denominators do not change since the number of parts per whole do not change

We can only add fractions if the denominators are the same.

Example 1.69
For each part, add the two fractions:
A. $\frac{3}{7}+\frac{2}{7}$
B. $\frac{4}{7}+\frac{1}{7}$
C. $\frac{2}{13}+\frac{5}{13}$
D. $\frac{4}{9}+\frac{4}{9}$
E. $\frac{4}{17}+\frac{8}{17}$
F. $\frac{6}{11}+\frac{2}{11}$

$$\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$$
$$\frac{4}{7}+\frac{1}{7}=\frac{5}{7}$$
$$\frac{2}{13}+\frac{5}{13}=\frac{7}{13}$$
$$\frac{4}{9}+\frac{4}{9}=\frac{8}{9}$$
$$\frac{4}{17}+\frac{8}{17}=\frac{12}{17}$$
$$\frac{6}{11}+\frac{2}{11}=\frac{8}{11}$$
$$\frac{5}{7}+0=\frac{5}{7}$$

Example 1.70

A. What is the sum of one-fifth and two-fifth?
B. The total of three-seventh, and two-seventh is:
C. What should be added to $\frac{3}{7}$ to make it $\frac{5}{7}$?

$$\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$$
$$\frac{3}{7}+\frac{2}{7}=\frac{5}{7}$$
$$\frac{5}{7}-\frac{3}{7}=\frac{2}{7}$$

Example 1.71

For each part, add the fractions:

A. $\frac{3}{11}+\frac{1}{11}+\frac{2}{11}$
B. $\frac{4}{7}+\frac{1}{7}+\frac{1}{7}$

Example 1.72

Find the missing value

A. $\frac{3}{5}+\frac{a}{5}=\frac{4}{5}$
B. $\frac{2}{9}+\frac{b}{9}=\frac{6}{9}$
C. $\frac{c}{7}+\frac{2}{7}=\frac{2}{7}$

$$\frac{3}{5}+\frac{a}{5}=\frac{4}{5}$$
$$\frac{2}{9}+\frac{b}{9}=\frac{6}{9}\Rightarrow b=4$$

Example 1.73

Compare the two expressions below. Is their answer the same?

$$\frac{2}{5}+\frac{1}{5}$$
$$\frac{1}{5}+\frac{2}{5}$$

$$\frac{2}{5}+\frac{1}{5}=\frac{3}{5}$$
$$\frac{1}{5}+\frac{2}{5}=\frac{3}{5}$$

Yes, their answer is the same.

1.74: Commutative Property of Addition

When adding, the order in which we add does not matter.
This is called the commutative property of addition.

E. Subtraction

Subtracting works the same way as addition, but we are reducing parts rather than adding them. We still need the denominators to be the same.

1.75: Subtracting Fractions

To subtract one fraction from the another with the same denominator:

- Subtract the numerator of the fraction being subtracted from the other fraction. This decreases the number of "shaded" parts.
- The denominators do not change since the number of parts per whole do not change.

We can only subtract fractions if the denominators are the same.

Example 1.76

Simplify:

A. $\frac{3}{7}-\frac{1}{7}$
B. $\frac{4}{9}-\frac{2}{9}$
C. $\frac{9}{11}-\frac{3}{11}$
D. $\frac{2}{9}-\frac{1}{9}$
E. $\frac{12}{19}-\frac{5}{19}$
F. $\frac{17}{18}-\frac{4}{18}$

Example 1.77

A. What is the difference between five-ninths, and two-ninths?
B. How much more is three-eighths compared to one-eighths?

$$\frac{5}{9}-\frac{2}{9}=\frac{3}{9}$$
$$\frac{3}{8}-\frac{1}{8}=\frac{2}{8}$$

Example 1.78

A. $\frac{6}{7}-\frac{2}{7}-\frac{1}{7}$
B. $\frac{11}{12}-\frac{4}{12}-\frac{3}{12}$

Example 1.79

Find the missing value

A. $\frac{5}{9}-\frac{a}{9}=\frac{4}{9}$
B. $\frac{3}{9}-\frac{b}{9}=\frac{1}{9}$
C. $\frac{7}{11}-\frac{c}{11}=\frac{5}{11}$
D. $\frac{d}{12}-\frac{2}{12}=\frac{6}{12}$

E. $e - \frac{3}{5} = \frac{1}{5}$
F. $\frac{10}{13} - f = \frac{3}{13}$

Example 1.80

Compare the two expressions below. Is their answer the same?

$$\frac{2}{5} - \frac{1}{5}$$
$$\frac{1}{5} - \frac{2}{5}$$

$$\frac{2}{5} - \frac{1}{5} = \frac{1}{5}$$
$$\frac{1}{5} - \frac{2}{5} = ?? \Rightarrow We\ can't\ do\ this.$$ [1]

No, the answer is not the same.

1.81: Commutative Property of Subtraction.

Commutative property *does not hold* for subtraction.
Order does matter when subtracting.

F. Mixed Review

Example 1.82

Simplify:

A. $\frac{2}{3} - \frac{1}{3}$
B. $\frac{3}{6} + \frac{1}{6}$
C. $\frac{12}{15} + \frac{2}{15}$
D. $\frac{12}{15} - \frac{2}{15}$
E. $\frac{12}{17} + \frac{3}{17}$
F. $\frac{12}{17} - \frac{3}{17}$

Example 1.83

Find the missing operator: $+\ or\ -$ that will replace the question mark.

A. $\frac{3}{7} ? \frac{2}{7} = \frac{5}{7}$
B. $\frac{3}{7} ? \frac{2}{7} = \frac{1}{7}$
C. $\frac{5}{12} ? \frac{c}{12} = \frac{2}{12}$

G. Simplifying answers

Sometimes, after addition, the fraction that you get can be simplified. In such a case, it should be simplified.

[1] Till we learn negative numbers. Then we can.

Example 1.84

Add, and simplify:

A. $\frac{3}{12}+\frac{1}{12}$
B. $\frac{5}{12}+\frac{1}{12}$
C. $\frac{3}{8}+\frac{3}{8}$
D. $\frac{5}{9}+\frac{1}{9}$
E. $\frac{2}{10}+\frac{3}{10}$
F. $\frac{1}{9}+\frac{2}{9}$

$$\frac{3}{12}+\frac{1}{12}=\frac{4}{12}=\frac{1}{3}$$
$$\frac{5}{12}+\frac{1}{12}=\frac{6}{12}=\frac{1}{2}$$
$$\frac{3}{8}+\frac{3}{8}=\frac{6}{8}=\frac{3}{4}$$
$$\frac{5}{9}+\frac{1}{9}=\frac{6}{9}=\frac{2}{3}$$
$$\frac{2}{10}+\frac{3}{10}=\frac{5}{10}=\frac{1}{2}$$
$$\frac{1}{9}+\frac{5}{9}=\frac{6}{9}=\frac{2}{3}$$

Example 1.85

Subtract, and simplify:

A. $\frac{5}{8}-\frac{3}{8}$
B. $\frac{7}{8}-\frac{3}{8}$
C. $\frac{7}{9}-\frac{1}{9}$
D. $\frac{5}{6}-\frac{2}{6}$

$$\frac{5}{8}-\frac{3}{8}=\frac{2}{8}=\frac{1}{4}$$

H. Word Problems

Example 1.86

A recipe calls for one-seventh of a cup of vanilla essence, and two-sevenths of a cup of flavoring agent. These two are put together in a cup and mixed. How much of a cup is full now?

$$\frac{1}{7}+\frac{2}{7}=\frac{3}{7}$$

Example 1.87

Farmer A plowed seven-twelfth of his field on Monday, while Farmer B plowed five-twelfth of his field on the same day. The fields were the same size. How much more of his field did Farmer A plow as compared to Farmer

B.

$$\frac{7}{12} - \frac{5}{12} = \frac{2}{12} = \frac{1}{6}$$

Example 1.88

Charles waits two-seventh of an hour for the school bus, and then it takes the bus three-seventh of an hour to take him to school. What is the total time for Charles to reach school?

$$\frac{2}{7} + \frac{3}{7} = \frac{5}{7}\ of\ an\ hour$$

I. Zero

1.89: Adding and Subtracting Zero

- When we add zero to any number, the number does not change.
- When we subtract zero from any number, the number does not change.

Example 1.90

Simplify:

A. $\frac{5}{7} + 0$
B. $0 + \frac{3}{4}$
C. $\frac{2}{3} + 0$
D. $\frac{5}{11} - 0$

$$\frac{5}{7}$$
$$\frac{3}{4}$$
$$\frac{2}{3}$$

J. Getting Zero for an Answer

Remember that if the numerator and the denominator of a fraction are the same, the fraction is equal to one.

Example 1.91

Simplify

A. $\frac{3}{3}$
B. $\frac{5}{5}$
C. $\frac{9}{9}$
D. $\frac{10}{10}$

$$All\ 1$$

Example 1.92
Simplify:

A. $\frac{5}{5}-\frac{5}{5}$
B. $\frac{4}{4}-\frac{4}{4}$
C. $\frac{3}{3}-\frac{7}{7}$
D. $\frac{9}{9}-1$

$$All\ 0$$

K. Getting One as an Answer
If after the addition, the numerator is the same as the denominator, then the final answer is 1.

Example 1.93
Simplify:

A. $\frac{2}{3}+\frac{1}{3}$
B. $\frac{4}{6}+\frac{2}{6}$
C. $\frac{5}{7}+\frac{2}{7}$

$$\frac{2}{3}+\frac{1}{3}=\frac{3}{3}=1$$
$$\frac{4}{6}+\frac{2}{6}=\frac{6}{6}=1$$
$$\frac{5}{7}+\frac{2}{7}=\frac{7}{7}=1$$

Example 1.94
Find the missing value

A. $\frac{3}{5}+\frac{a}{5}=1$
B. $\frac{2}{9}+\frac{b}{9}=1$

$$\frac{3}{5}+\frac{a}{5}=1\Rightarrow\frac{3}{5}+\frac{2}{5}=\frac{5}{5}=1\Rightarrow a=1$$
$$\frac{2}{9}+\frac{b}{9}=1\Rightarrow b=7$$

L. Subtracting from One

Example 1.95
Two-fifth of the birds sitting on a branch flew away. What fraction of the birds were left?

$$1-\frac{2}{5}=\frac{3}{5}$$

Example 1.96
Sharon ate three-fifths of a cake, and April ate one-fifth of the same cake.

A. How much of the cake did Sharon and April eat in all?
B. How much of the cake was remaining after they finished eating?

$$\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$$
$$1-\frac{4}{5}=\frac{1}{5}$$

Example 1.97

Sarah has a bag with ten marbles, colored red, blue or green. If three of the marbles are red, and the number of red marbles is one more than the number of blue marbles, what fraction of the marbles are green?

$$1-\left(\frac{3}{7}+\frac{2}{7}\right)=1-\frac{5}{7}=\frac{2}{7}$$

Example 1.98

Will divided a cake into seven equal pieces. He ate one piece, and gave another to his brother to eat.

A. What fraction of the cake was eaten?
B. What fraction of the cake was left?

M. Odd One Out

Example 1.99

Pick the odd one out:

A. $\frac{2}{3}+\frac{1}{3}$
B. $\frac{5}{7}+\frac{2}{7}$
C. $\frac{7}{10}+\frac{2}{10}$
D. $\frac{5}{11}+\frac{6}{11}$

Option C.
All the others add up to 1.

Example 1.100

Find the value of Δ if:

A. $\frac{\Delta}{7}+\frac{\Delta}{7}=\frac{4}{7}$
B. $\Delta+\Delta=\frac{6}{9}$

$$2$$
$$\frac{3}{9}=\frac{1}{3}$$

Example 1.101

Find the value of the symbols if $\odot$ is an odd number, and $\otimes$ is an even number that is one more than $\odot$.

$$\frac{\odot}{9}+\frac{\otimes}{9}=\frac{7}{9}$$

N. Improper Fractions as Answers

If the two fractions that you add, add up to more than one, than the fraction will be an improper fraction. Ideally, the improper fraction should be converted into a mixed number after the addition.

Example 1.102

Add $\frac{5}{9}$ to $\frac{6}{9}$. Rewrite the answer as a mixed fraction.

$$\frac{5}{9}+\frac{6}{9}=\frac{11}{9}=1\frac{2}{9}$$

Example 1.103

Add, or subtract, as required. Convert improper fractions into mixed numbers.

A. $\frac{7}{6}+\frac{2}{6}$
B. $\frac{4}{5}+\frac{7}{5}$
C. $\frac{9}{11}+\frac{3}{11}$
D. $\frac{7}{12}-\frac{1}{12}$
E. $\frac{12}{15}-\frac{9}{15}$
F. $\frac{5}{6}+\frac{1}{6}$

$$\frac{7}{6}+\frac{2}{6}=\frac{9}{6}=\frac{6}{6}+\frac{3}{6}=1+\frac{3}{6}=1+\frac{1}{2}=1\frac{1}{2}$$
$$\frac{4}{5}+\frac{7}{5}=\frac{11}{5}=\frac{5}{5}+\frac{5}{5}+\frac{1}{5}=1+1+\frac{1}{5}=2+\frac{1}{5}=2\frac{1}{5}$$
$$\frac{9}{11}+\frac{3}{11}=\frac{12}{11}=\frac{11}{11}+\frac{1}{11}=1+\frac{1}{11}=1\frac{1}{11}$$

O. Getting One as an Answer

If after the subtraction, the numerator is the same as the denominator, then the final answer is 1.

Example 1.104

A. $\frac{8}{5}-\frac{3}{5}$
B. $\frac{12}{11}-\frac{1}{11}$
C. $\frac{5}{3}-\frac{2}{3}$

$$\frac{8}{5}-\frac{3}{5}=\frac{5}{5}=1$$

P. Applications: Lengths and Distances

Example 1.105

Gaurav is one half of a meter tall. Shalini is one half of a meter taller than Gaurav. What is the sum of the heights total height of Gaurav and Shalini?

$$\underbrace{\frac{1}{2}}_{Gaurav}+\underbrace{\frac{1}{2}+\frac{1}{2}}_{Shalini}=\frac{3}{2}$$

Example 1.106

Clara ran distances as per the following table:

Monday	Tuesday	Wednesday	Thursday
$\frac{1}{5}$ *units*	$\frac{2}{5}$	$\frac{3}{5}$	$\frac{4}{5}$

What is the total distance that she ran from Monday to Friday?

$$\frac{1}{5}+\frac{2}{5}+\frac{3}{5}+\frac{4}{5}=\frac{10}{5}=2$$

1.5 Mixed Numbers

A. Breaking Fractions

An improper fraction can be written one plus another fractions.

Example 1.107: Breaking Fractions

Write the following fractions as the sum of one whole and another fraction.

A. $\frac{8}{5}$
B. $\frac{7}{5}$
C. $\frac{9}{5}$
D. $\frac{6}{5}$
E. $\frac{10}{5}$
F. $\frac{10}{7}$
G. $\frac{8}{7}$
H. $\frac{12}{7}$
I. $\frac{9}{7}$
J. $\frac{13}{11}$
K. $\frac{5}{3}$
L. $\frac{11}{9}$
M. $\frac{12}{5}$
N. $\frac{17}{13}$
O. $\frac{13}{9}$
P. $\frac{19}{17}$
Q. $\frac{15}{11}$
R. $\frac{8}{6}$
S. $\frac{13}{8}$
T. $\frac{14}{9}$
U. $\frac{15}{9}$
V. $\frac{21}{13}$
W. $\frac{16}{12}$
X. $\frac{11}{7}$
Y. $\frac{12}{9}$
Z. $\frac{6}{4}$

Recall that $1=\frac{5}{5}$. Hence, write:

$$\frac{8}{5}=\frac{5+3}{5}=\frac{5}{5}+\frac{3}{5}$$

$$\frac{7}{5}=\frac{5+2}{5}=\frac{5}{5}+\frac{2}{5}$$

$$\frac{9}{5}=\frac{5}{5}+\frac{4}{5}$$

$$\frac{6}{5}=\frac{5+1}{5}=\frac{5}{5}+\frac{1}{5}=1+\frac{1}{5}=1\frac{1}{5}$$

$$\frac{10}{5}=\frac{5}{5}+\frac{5}{5}$$

$$\frac{10}{7}=\frac{7}{7}+\frac{3}{7},\quad \frac{8}{7}=\frac{7}{7}+\frac{1}{7},\quad \frac{12}{7}=\frac{7}{7}+\frac{5}{7},\quad \frac{9}{7}=\frac{7}{7}+\frac{2}{7}$$

$$\frac{8}{7}=\frac{7}{7}+\frac{1}{7}=1+\frac{1}{7}=1\frac{1}{7}$$

$$\frac{12}{7}=\frac{7}{7}+\frac{5}{7}=1+\frac{5}{7}=1\frac{5}{7}$$

B. Writing as a Mixed Number

One plus a proper fraction can be written in short form as a mixed number.

Example 1.108: Convert Broken Fractions to a whole plus a fraction

Write the following fractions as mixed numbers.

A. $\frac{4}{4}+\frac{1}{4}$
B. $\frac{4}{4}+\frac{3}{4}$
C. $\frac{4}{4}+\frac{2}{4}$
D. $\frac{6}{6}+\frac{2}{6}$
E. $\frac{6}{6}+\frac{5}{6}$

F. $\frac{6}{6}+\frac{3}{6}$
G. $\frac{5}{5}+\frac{2}{5}$
H. $\frac{5}{5}+\frac{3}{5}$
I. $\frac{5}{5}+\frac{1}{5}$
J. $\frac{7}{7}+\frac{2}{7}$

K. $\frac{7}{7}+\frac{5}{7}$
L. $\frac{11}{11}+\frac{3}{11}$
M. $\frac{11}{11}+\frac{7}{11}$
N. $\frac{9}{9}+\frac{4}{9}$
O. $\frac{11}{11}+\frac{4}{11}$

P. $\frac{13}{13}+\frac{5}{11}$
Q. $\frac{17}{17}+\frac{4}{17}$
R. $\frac{12}{12}+\frac{5}{12}$
S. $\frac{17}{17}+\frac{6}{17}$

$$\frac{4}{4}+\frac{1}{4}=1+\frac{1}{4}=1\frac{1}{4}$$
$$\frac{4}{4}+\frac{3}{4}=1+\frac{3}{4}=1\frac{3}{4}$$
$$\frac{4}{4}+\frac{2}{4}=1+\frac{2}{4}=1\frac{2}{4}$$
$$\frac{6}{6}+\frac{2}{6}=1\frac{2}{6}$$
$$\frac{6}{6}+\frac{5}{6}=1\frac{5}{6}$$
$$\frac{6}{6}+\frac{3}{6}=1\frac{3}{6}$$

C. Converting Improper Fractions into Mixed Numbers

Example 1.109

Convert the following improper fractions into mixed numbers:

A. $\frac{7}{5}$
B. $\frac{9}{6}$
C. $\frac{7}{4}$
D. $\frac{11}{6}$
E. $\frac{12}{7}$
F. $\frac{7}{6}$

G. $\frac{11}{9}$
H. $\frac{12}{11}$
I. $\frac{5}{3}$
J. $\frac{8}{5}$
K. $\frac{9}{5}$
L. $\frac{3}{2}$

M. $\frac{13}{7}$
N. $\frac{11}{7}$
O. $\frac{15}{9}$
P. $\frac{13}{11}$
Q. $\frac{17}{11}$
R. $\frac{9}{6}$

S. $\frac{12}{10}$
T. $\frac{15}{10}$
U. $\frac{15}{14}$
V. $\frac{19}{15}$
W. $\frac{21}{20}$

X. $\frac{12}{8}$
Y. $\frac{13}{9}$
Z. $\frac{11}{10}$

$$\underbrace{\frac{7}{5}=\frac{5}{5}+\frac{2}{5}=1+\frac{2}{5}=1\frac{2}{5}}_{Break\ up\ \frac{7}{5}\ as\ one\ whole\ plus\ a\ fraction}$$

$$\underbrace{\frac{9}{6}=\frac{6}{6}+\frac{3}{6}=1+\frac{3}{6}=1\frac{3}{6}}_{\text{Break up } \frac{9}{6} \text{ as one whole plus a fraction}}$$

$$\frac{7}{4}=\frac{4}{4}+\frac{3}{4}=1\frac{3}{4}$$

$$\frac{11}{6}=\frac{6}{6}+\frac{5}{6}=1\frac{5}{6}$$

$$\frac{12}{7}=\frac{7}{7}+\frac{5}{7}=1\frac{5}{7}$$

D. Fractions as Division

Fractions can also be thought of as division.

$$\frac{Numerator}{Denominator}=Numerator\div Denominator$$

Example 1.110

Convert the following fractions into whole numbers by dividing the numerator by the denominator.

A. $\frac{4}{2}$
B. $\frac{8}{2}$
C. $\frac{12}{3}$
D. $\frac{30}{5}$

E. $\frac{60}{6}$
F. $\frac{60}{5}$
G. $\frac{60}{10}$
H. $\frac{60}{3}$

I. $\frac{27}{3}$
J. $\frac{81}{9}$
K. $\frac{100}{2}$
L. $\frac{15}{3}$

M. $\frac{80}{2}$
N. $\frac{33}{3}$
O. $\frac{42}{3}$

$$\frac{4}{2}=4\div 2=2$$

$$\frac{8}{2}=8\div 2=4$$

$$\frac{12}{3}=12\div 3=4$$

$$\frac{30}{5}=30\div 5=6$$

Example 1.111: Fractions more than two

Convert the following numbers into mixed numbers:

A. $\frac{9}{4}$
B. $\frac{7}{3}$
C. $\frac{12}{5}$
D. $\frac{11}{4}$

E. $\frac{5}{2}$
F. $\frac{13}{2}$
G. $\frac{15}{4}$
H. $\frac{17}{6}$

I. $\frac{19}{5}$
J. $\frac{15}{2}$
K. $\frac{19}{11}$
L. $\frac{20}{13}$

M. $\frac{14}{5}$
N. $\frac{21}{17}$
O. $\frac{18}{7}$

$$\frac{9}{4}=\frac{8}{4}+\frac{1}{4}=2+\frac{1}{4}=2\frac{1}{4}$$

$$\frac{7}{3}=\frac{6}{3}+\frac{1}{3}=2+\frac{1}{3}=2\frac{1}{3}$$

$$\frac{12}{5}=\frac{10}{5}+\frac{2}{5}=2+\frac{2}{5}=2\frac{2}{5}$$

$$\frac{11}{4}=\frac{8}{4}+\frac{3}{4}=2+\frac{3}{4}=2\frac{3}{4}$$
$$\frac{5}{2}=\frac{4}{2}+\frac{1}{2}=2+\frac{1}{2}=2\frac{1}{2}$$
$$\frac{13}{2}=\frac{12}{2}+\frac{1}{2}=6+\frac{1}{2}=6\frac{1}{2}$$

E. Shortcut Method

We can divide the numerator by the denominator to find the number of wholes

Example 1.112: Fractions more than two

Convert $\frac{13}{4}$ into a mixed fraction

$$\underbrace{\frac{13}{4}=\frac{12}{4}+\frac{1}{4}=3+\frac{1}{4}=3\frac{1}{4}}_{\mathbf{13\div 4=3\ Remainder\ 1}}\Rightarrow With\ practice, you\ can\ directly\ write\ \frac{13}{4}=3\frac{1}{4}$$

Example 1.113

Convert the following fractions from improper to mixed

Denominator of Two

A. $\frac{15}{2}$
B. $\frac{19}{2}$
C. $\frac{11}{2}$
D. $\frac{7}{2}$
E. $\frac{9}{2}$
F. $\frac{5}{2}$

Denominator of Three

G. $\frac{11}{3}$
H. $\frac{8}{3}$
I. $\frac{16}{3}$
J. $\frac{17}{3}$
K. $\frac{28}{3}$
L. $\frac{5}{3}$
M. $\frac{31}{3}$

Denominator of Four

N. $\frac{13}{4}$
O. $\frac{39}{4}$
P. $\frac{23}{4}$
Q. $\frac{19}{4}$
R. $\frac{19}{4}$
S. $\frac{26}{4}$
T. $\frac{11}{4}$

Denominator of Five

U. $\frac{11}{5}$
V. $\frac{17}{5}$
W. $\frac{23}{5}$
X. $\frac{32}{5}$
Y. $\frac{6}{5}$
Z. $\frac{47}{5}$

Denominator of Six

AA. $\frac{19}{6}$

Mixed Review

BB. $\frac{12}{5}$
CC. $\frac{17}{4}$
DD. $\frac{20}{7}$
EE. $\frac{13}{2}$

$$15\div 2=7\ Remainder\ 1\Rightarrow\frac{15}{2}=7\frac{1}{2}$$
$$19\div 2=9\ Remainder\ 1\Rightarrow\frac{19}{2}=9\frac{1}{2}$$
$$11\div 2=5\ Remainder\ 1\Rightarrow\frac{11}{2}=5\frac{1}{2}$$
$$7\div 2=3\ Remainder\ 1\Rightarrow\frac{7}{2}=3\frac{1}{2}$$
$$9\div 2=4\ Remainder\ 1\Rightarrow\frac{9}{2}=4\frac{1}{2}$$

Example 1.114

Convert the following fractions from improper to mixed:

Denominator of Two

A. $\frac{21}{2}$

B. $\frac{35}{2}$

C. $\frac{47}{2}$

D. $\frac{57}{2}$

E. $\frac{94}{2}$

Denominator of Three

F. $\frac{47}{3}$

G. $\frac{52}{3}$

Denominator of Four

H. $\frac{78}{4}$

I. $\frac{62}{4}$

J. $\frac{91}{4}$

K. $\frac{49}{4}$

Denominator of Five

L. $\frac{64}{5}$

M. $\frac{72}{5}$

N. $\frac{98}{5}$

O. $\frac{71}{5}$

Denominator of Five

$$\frac{35}{2}=\frac{34}{2}+\frac{1}{2}=17+\frac{1}{2}=17\frac{1}{2}$$

$$\textit{Shortcut}: 35 \div 2 = 17\ \textit{Remainder}\ 1 \Rightarrow \frac{35}{2}=17\frac{1}{2}$$

F. Convert into using the same denominator

To convert a mixed fraction into an improper fraction, we need to write the whole number part of the fraction using the same denominator as the fractional part.

Example 1.115: Fractions more than one and less than two

Convert the following mixed numbers into improper fractions:

A. $1\frac{1}{5}$

B. $3\frac{2}{7}$

C. $3\frac{4}{11}$

$$1\frac{1}{5}=\underbrace{1+\frac{1}{5}}_{\substack{\textit{Write as}\\ \textit{addition}}}=\underbrace{\frac{1}{1}+\frac{1}{5}}_{\substack{\textit{1 can be}\\ \textit{denominator anywhere}}}=\underbrace{\frac{5}{5}+\frac{1}{5}}_{\substack{\textit{Make the}\\ \textit{denominators Same}}}=\underbrace{\frac{6}{5}}_{\substack{\textit{Adding the}\\ \textit{fractions}}}$$

$$3\frac{2}{7}=\underbrace{3+\frac{2}{7}}_{\substack{\textit{Write as}\\ \textit{addition}}}=\underbrace{\frac{3}{1}+\frac{2}{7}}_{\substack{\textit{1 can be the}\\ \textit{denominator anywhere}}}=\underbrace{\frac{21}{7}+\frac{2}{7}}_{\substack{\textit{Make the}\\ \textit{Denominators Same}}}=\underbrace{\frac{23}{7}}_{\substack{\textit{Add the}\\ \textit{fractions}}}$$

$$3\frac{4}{11}=\underbrace{3+\frac{4}{11}}_{\substack{\textit{Write as}\\ \textit{addition}}}=\underbrace{\frac{3}{1}+\frac{4}{11}}_{\substack{\textit{1 can be the}\\ \textit{denominator anywhere}}}=\underbrace{\frac{33}{11}+\frac{4}{11}}_{\substack{\textit{Make the}\\ \textit{Denominators Same}}}=\underbrace{\frac{37}{11}}_{\substack{\textit{Add the}\\ \textit{fractions}}}$$

G. Shortcut Method

We can multiply the whole number by the denominator to find the numerator of the whole number

Example 1.116: Fractions more than two

Convert $2\frac{4}{9}$ into an improper fraction using the regular method and the shortcut method

$$\underbrace{2\frac{4}{9}=2+\frac{4}{9}=\frac{2}{1}+\frac{4}{9}=\frac{18}{9}+\frac{4}{9}=\frac{22}{9}}_{\textit{Regular Method}},\quad \underbrace{2\frac{4}{9}=\frac{2\times 9+4}{9}=\frac{18+4}{9}=\frac{22}{9}}_{\textit{Shortcut Method}}$$

Example 1.117: Practice using the shortcut method

Convert the following mixed numbers to improper fractions using the shortcut method:

A. $1\frac{5}{9}$	E. $3\frac{2}{3}$	I. $10\frac{1}{2}$	M. $3\frac{4}{9}$
B. $2\frac{3}{4}$	F. $5\frac{2}{3}$	J. $11\frac{2}{3}$	N. $3\frac{2}{7}$
C. $7\frac{1}{8}$	G. $7\frac{1}{3}$	K. $1\frac{2}{9}$	O.
D. $3\frac{3}{4}$	H. $9\frac{2}{5}$	L. $2\frac{3}{7}$	

$$1\frac{5}{9}=\frac{1\times 9+5}{9}=\frac{9+5}{9}=\frac{14}{9}$$
$$2\frac{3}{4}=\frac{2\times 4+3}{4}=\frac{8+3}{4}=\frac{11}{4}$$

$$7\frac{1}{8}=\frac{7\times 8+1}{8}=\frac{56+1}{8}=\frac{57}{8}$$
$$3\frac{3}{4}=\frac{3\times 4+3}{4}=\frac{12+3}{4}=\frac{15}{4}$$
$$3\frac{2}{3}=\frac{3\times 3+2}{3}=\frac{9+2}{3}=\frac{11}{3}$$
$$5\frac{2}{3}=\frac{5\times 3+2}{3}=\frac{15+2}{3}=\frac{17}{3}$$

H. Concept Questions

Example 1.118

True or False

If the statement is false, give one example where it is false, and provide a corrected statement.

A. An improper fraction always has its numerator more than its denominator.
B. If you convert a mixed number into an improper fraction, the value increases.
C. If you add two positive improper fractions, you will always get an improper fraction.
D. If you add two proper fractions, you will always get a proper fraction.
E. The value of a mixed number is always more than one.
F. In the fractional part of a mixed number, the numerator can be more than the denominator.

Statement A: False
Counter-Example: $\frac{5}{5}$ is an improper fraction, but the numerator is not more than the denominator.
Corrected Version: An improper fraction always has its numerator *equal to or more* than its denominator.

Statement B: Incorrect
Counter-Example: $\underbrace{1\frac{3}{4}}_{\textit{Mixed Number}}=\underbrace{\frac{7}{4}}_{\textit{Improper Fraction}}\Rightarrow \textit{Both are the same value}$
Corrected Version: If you convert a mixed number into an improper fraction, the value remains the same.

Statement C: Correct

Statement D: Incorrect

Counter-Example: $\underbrace{\frac{5}{6}}_{\substack{Proper\\Fraction}} + \underbrace{\frac{4}{6}}_{\substack{Proper\\Fraction}} = \underbrace{\frac{9}{6}}_{\substack{Improper\\Fraction}}$

Statement E: Correct

Statement F: Incorrect
In a mixed number, you must ensure that the numerator is less than the denominator. Otherwise, it

$$2\frac{3}{4}+3\frac{2}{4}=2+3+\frac{3}{4}+\frac{2}{4}=5+\frac{5}{4}=5+1+\frac{1}{4}\quad\underbrace{=6\frac{1}{4}}_{\substack{Correct\\Presentation}}\quad\underbrace{\neq 5\frac{5}{4}}_{\substack{Incorrect\\Presentation}}$$

Example 1.119

Shalini had three cakes. She cut each cake into five pieces. She then ate two pieces and gave one piece to her brother to eat. Find the number of cakes left as:

A. a mixed number
B. an improper fraction

She started with three cakes, giving her:

$$3\ Cakes=\frac{3}{1}Cakes$$

She cut each cake into five pieces:

$$\frac{15}{5}Cakes$$

Then she ate two pieces:

$$\frac{15}{5}-\frac{2}{5}=\frac{13}{5}$$

Then she gave one piece to her brother:

$$\frac{13}{5}-\frac{1}{5}=\frac{12}{5}$$

And convert $\frac{12}{5}$ into a mixed number:

$$2\frac{2}{5}$$

Example 1.120

Rishabh wants to convert the fraction $\frac{11}{7}$ into a mixed number. For each of the following methods, decide whether it is appropriate or not appropriate?

A. Divide the numerator by the denominator. Write the quotient as a whole number, and the remainder as the numerator. Keep the denominator the same as before.
B. Divide the numerator by the denominator. Write the quotient as the numerator, and the remainder as a whole number. Keep the denominator the same as before.
C. Divide the denominator by the numerator. Write the quotient as a whole number, and the remainder as the numerator. Keep the denominator the same as before.

$$11\div 7=\underbrace{1}_{Quotient}\quad Remainder\ 4=1\frac{4}{7}$$

Option A: Appropriate
Option B: Inappropriate because we write the quotient as a whole number, and not as the numerator.

Option C: Is inappropriate since we divide the numerator by the denominator, and not *vice versa*.

Example 1.121

Puneet wants to convert the fraction $\frac{192}{91}$ into a mixed number. Use the table below to decide the correct combination of steps to be followed. In each of steps I, II and III decide which option is correct.

Step I	Step II	Step III
A. Divide the denominator by the numerator. B. Divide the numerator by the denominator. C. None of the above	A. Write the quotient as a whole number, and the remainder as the numerator. B. Write the quotient as the numerator, and the remainder as a whole number. C. None of the above	A. Make the numerator into the denominator. B. Keep the denominator the same as before. C. None of the above

$$Step\ I: B$$
$$Step\ II: A$$
$$Step\ III: B$$

I. Background

Comparing fractions is very important. There is one method, but there are many short based on special types of fractions. So, it is important to understand these shortcuts, because they are based on properties of fractions, and those properties are based on concepts.

Using One as a Benchmark ($N = Numerator,\ \ D = Denominator$)

We can use the following properties of fractions:

- If the numerator is less than the denominator, then the fraction is less than one.
$$Eg: \frac{3}{7} < 1, \frac{7}{9} < 1$$
- If the numerator is equal to the denominator, then the fraction is equal to one.
$$Eg: \frac{4}{4} = \frac{7}{7} = \frac{11}{11} = \frac{2}{2} = \frac{1}{1} = 1$$
- If the numerator is greater than the denominator, then the fraction is greater than one.
$$\frac{7}{5} > 1, \quad \frac{5}{3} > 1, \quad \frac{9}{7} > 1$$

$$\underbrace{\frac{1}{3} < \frac{1}{2}}_{\frac{2}{6} < \frac{3}{6}}, \underbrace{\frac{3}{4} < 1}_{\substack{N<D \\ Less\ than\ 1}}, \quad \underbrace{\frac{4}{4} = 1}_{\substack{N=D \\ Equal\ to\ One}}, \quad \underbrace{\frac{5}{4} > 1}_{\substack{N>D \\ Greater\ than\ One}}$$

This method is the fastest, but will not work in all situations.

Example 1.122

Compare $\left\{\frac{3}{7}, \frac{7}{5}, \frac{3}{3}\right\}$ using one as a benchmark, and arrange the fractions in ascending order

Compare each fraction with one

$$\left\{\underbrace{\frac{3}{7} < 1}_{N<D}, \quad \underbrace{\frac{7}{5} > 1}_{N>D}, \quad \underbrace{\frac{3}{3} = 1}_{N=D}\right\} \Rightarrow \left\{\frac{3}{7} < 1, \frac{3}{3} = 1, \frac{7}{5} > 1\right\}$$

Practice 1.123

Use 1 as a benchmark to arrange the following fractions in $\underbrace{ascending}_{smallest\ to\ largest}$ order:

A. $\frac{5}{6}, \frac{9}{7}, \frac{4}{4}$
B. $\frac{11}{9}, \frac{8}{8}, \frac{2}{3}$
C. $\frac{6}{6}, \frac{7}{11}, \frac{5}{4}$

$$\frac{5}{6} < 1, \qquad \frac{4}{4} = 1, \qquad \frac{9}{7} > 1$$

$$\underbrace{\frac{2}{3} < 1}_{N<D}, \qquad \underbrace{\frac{8}{8} = 1}_{N=D}, \qquad \underbrace{\frac{11}{9} > 1}_{N>D}$$

$$\frac{6}{6} = 1, \qquad \frac{7}{11} < 1, \qquad \frac{5}{4} > 1$$

Using Benchmark to Classify Fractions

Sometimes, if you have a few fractions, you cannot get the exact comparison using the benchmark method. But can check if the fractions are more than one, less than one, or equal to one.

Example 1.124

Compare $\left\{\frac{2}{7}, \frac{8}{3}, \frac{5}{5}, \frac{7}{9}, \frac{12}{7}\right\}$ using one as a benchmark, and group them based on whether they are less than one, equal to one, or more than one.

Compare each fraction with one

$$\left\{\underbrace{\frac{2}{7} < 1}_{N<D}, \underbrace{\frac{8}{3} > 1}_{N>D}, \underbrace{\frac{5}{5} = 1}_{N=D}, \underbrace{\frac{7}{9} < 1}_{N<D}, \underbrace{\frac{12}{7} > 1}_{N>D}\right\} \Rightarrow \underbrace{\left\{\frac{2}{7}, \frac{7}{9}\right\}}_{Less\ than\ One}, \underbrace{\left\{\frac{5}{5}\right\}}_{Equal\ to\ One}, \underbrace{\left\{\frac{8}{3}, \frac{12}{7}\right\}}_{More\ than\ One}$$

Using $\frac{1}{2}$ as a Benchmark $(N = Numerator, \; D = Denominator)$

We can use the following properties of fractions:

- If the denominator is more than twice the numerator, then the fraction is less than one-half.

$$Eg: \underbrace{\frac{3}{7} < \frac{1}{2}}_{3\times 2=6<7}, \underbrace{\frac{7}{16} < \frac{1}{2}}_{7\times 2=14}$$

- If the denominator is exactly twice the numerator, then the fraction is equal to one-half.

$$Eg: \underbrace{\frac{4}{8} = \frac{1}{2}}_{8\div 4=2}, \qquad \underbrace{\frac{5}{10} = \frac{1}{2}}_{10\div 5=2}$$

- If the denominator is less than twice the numerator, then the fraction is greater than one-half.

$$\underbrace{\frac{4}{5} > \frac{1}{2}}_{4\times 2=8>5}, \qquad \underbrace{\frac{3}{5} > \frac{1}{2}}_{3\times 2=6>5}$$

Example 1.125

Use 1 and $\frac{1}{2}$ as a benchmark to arrange the following fractions in $\underbrace{ascending}_{\text{smallest to largest}}$ order:

A. $\frac{3}{6}, \frac{7}{7}, \frac{8}{5}, \frac{1}{6}, \frac{9}{7}$
B. $\frac{2}{3}, \frac{6}{12}, \frac{21}{21}, \frac{23}{9}, \frac{3}{14}$
C. $\frac{3}{5}, \frac{9}{9}, \frac{12}{6}, \frac{2}{7}, \frac{12}{24}$

$$\frac{3}{6} = \frac{1}{2}, \quad \frac{7}{7} = 1, \quad \frac{8}{5} > 1, \quad \frac{1}{6} < \frac{1}{2}, \quad \frac{9}{7} > 1$$

$$1 > \frac{2}{3} > \frac{1}{2}, \quad 1 > \frac{6}{12} = \frac{1}{2}, \quad \frac{1}{2} < \frac{21}{21} = 1, \quad \frac{23}{9} > 1 > \frac{1}{2}, \quad \frac{3}{14} < \frac{1}{2} < 1$$

$$\frac{3}{5} < 1, \quad \frac{9}{9} = 1, \quad \frac{12}{6} > 1, \quad \frac{2}{7} < \frac{1}{2}, \quad \frac{12}{24} = \frac{1}{2}$$

Fractions with same Denominator

Fractions represent parts of a whole. So, if the numerator of a fraction is greater, than the fraction is also greater. However, we can only compare two fractions like this when the denominators are same. This is important because the denominator decides how many parts the whole is being divided into.

Fractions with same Denominator

If the denominator is the same, compare numerators

$$\textit{Greater Numerator} \Rightarrow \textit{Greater Fraction}$$

$$\underbrace{\frac{3}{4} > \frac{2}{4}}_{\substack{\text{Compare Numerators} \\ \text{Greater Numerator} \Rightarrow \textbf{Larger } \text{Fraction}}}$$

Three parts shaded out of Four $= \frac{3}{4}$						*Two parts shaded out of Four* $= \frac{2}{4}$		

Example 1.126

Compare each of the pairs of fractions below, and decide which one is greater:

$$\left\{\frac{3}{7}, \frac{5}{7}\right\}\left\{\frac{9}{6}, \frac{3}{6}\right\}\left\{\frac{2}{9}, \frac{8}{9}\right\}\left\{\frac{2}{3}, \frac{3}{3}\right\}$$

We can compare the numerator since the denominators are the same

$$\underbrace{\left\{\frac{3}{7} < \frac{5}{7}\right\}}_{3<5}\underbrace{\left\{\frac{9}{6} > \frac{3}{6}\right\}}_{9>3}\underbrace{\left\{\frac{2}{9} < \frac{8}{9}\right\}}_{2<8}\underbrace{\left\{\frac{2}{3} < \frac{3}{3}\right\}}_{2<3}$$

Fractions with same Numerator

If the numerator is the same, compare denominators

$$\textit{Greater } \textbf{Denominator} \Rightarrow \textbf{Smaller } \textit{Fraction}$$

$$\underbrace{\frac{3}{4} > \frac{3}{5}}_{\substack{\text{Compare Denominators} \\ \text{Lower Denominator} \Rightarrow \text{Larger Fraction}}}$$

$Three\ parts\ shaded\ out\ of\ Four = \frac{3}{4}$					$Three\ parts\ shaded\ out\ of\ Five = \frac{3}{5}$				

Example 1.127

Compare each of the pairs of fractions below, and decide which one is greater:

$$\left\{\frac{4}{5}, \frac{4}{7}\right\}\left\{\frac{6}{4}, \frac{6}{3}\right\}\left\{\frac{2}{3}, \frac{2}{9}\right\}\left\{\frac{3}{4}, \frac{3}{2}\right\}$$

We can compare the denominator since the numerators are the same

$$\left\{\frac{4}{5} > \frac{4}{7}\right\}\left\{\frac{6}{4} < \frac{6}{3}\right\}\left\{\frac{2}{3} > \frac{2}{9}\right\}\left\{\frac{3}{4} < \frac{3}{2}\right\}$$

1.128: Comparing Numerator and Denominator

For any fraction, as the

- Numerator increases, the fraction increases
- Numerator decreases, the fraction decreases
- Denominator increases, the fraction decreases
- Denominator decreases, the fraction increases

Example 1.129

Find the smallest fraction out of:

$$\frac{5}{14}, \frac{4}{7}, \frac{3}{8}, \frac{1}{16}, \frac{1}{4}$$

Largest denominator is 16. This makes the fraction smaller.
Smallest numerator is 1. This makes the fraction smaller.

In this case, largest denominator and smallest numerator both belong to same fraction, which is $\frac{1}{16}$.

Hence, smallest fraction is $\frac{1}{16}$.

Finding LCM

To make the denominators the same, we need to know how to find the LCM of two numbers.

1.130: LCM

The LCM of two numbers is the smallest number that is a multiple of both the numbers.

Example 1.131

Find the LCM of the pairs of numbers given below.
Notes:

1. The mixed review repeats earlier questions.
2. Needs comfort with multiplication and mental maths.
3. Practice, practice, practice.
4. Target 100% accuracy. Can take a little while.
5. Once accuracy is achieved target speed. Again, can take time.

Small Numbers
1) 6 and 8
2) 4 and 10
3) 6 and 9
4) 6 and 4
5) 10 and 8
6) 10 and 6

Co-Prime Numbers
7) 3 and 4
8) 6 and 11
9) 2 and 3
10) 8 and 9
11) 7 and 5
12) 9 and 4

Exact Multiples
13) 3 and 6
14) 4 and 8
15) 3 and 9
16) 7 and 21
17) 35 and 7
18) 5 and 10

Multiples of 10
19) 20 and 30
20) 30 and 60
21) 40 and 60
22) 20 and 100

Medium Numbers
23) 8 and 12
24) 9 and 15
25) 14 and 21
26) 10 and 15
27) 22 and 33
28) 8 and 12
29) 14 and 4
30) 13 and 2
31) 15 and 6
32) 16 and 4
33) 16 and 6
34) 10 and 15
35) 12 and 16
36) 15 and 20
37) 14 and 21
38) 18 and 9

Mixed Review
39) 8 and 12
40) 35 and 7
41) 10 and 8
42) 15 and 6
43) 2 and 3
44) 14 and 21
45) 16 and 4

Co-Prime Numbers-II
46) 12 and 7
47) 13 and 5
48) 17 and 2
49) 19 and 4
50) 7 and 11
51) 23 and 2

Larger Prime Factors
52) 26 and 39
53) 17 and 51
54) 11 and 13
55) 19 and 38
56) 13 and 52

Mixed Review-II

$$\{6,12,18,\mathbf{24},\}, \qquad \{8,16,\mathbf{24},\dots\}$$

The LCM is the smallest number that is a multiple of both 6 and 8, which is 24

6 is a multiple of 3. Hence, the LCM will be 6 itself.

$$\{3,6\} \Rightarrow LCM = 6$$

Note that 7 and 5 have no common factor. Hence, the LCM is the product of the two numbers.

$$\{7,5\} \Rightarrow LCM = 35$$

Example 1.132

The following sets have three numbers each. Find the lowest common multiples of the three numbers.

Small Numbers
A. 3, 6 and 9
B. 6, 8 and 12
C. 3, 5 and 10
D. 4, 14 and 6
E. 10, 15, and 20
F. 6, 12, and 16
G. 5, 6 and 10
H. 7, 14 and 10
I. 10, 20, and 30
J. 3, 5 and 7
K. 2, 6 and 12
L. 10, 12, 20

Making Denominators the Same

If the denominators are not the same, and any of the earlier methods is not working, then the method that will always work is to make the denominators the same, and then compare using the numerators.

Using LCM to Compare

We can use the LCM to compare two fractions.

- We convert the fractions to have the same denominator.
- Once the denominators are same, the fraction with the larger numerator is the larger fraction.

Example 1.133

Compare $\frac{2}{3}$ *with* $\frac{3}{6}$. First use LCM to make the denominators of the numbers same.

$$\underbrace{\left\{\frac{2}{3},\frac{3}{6}\right\}}_{LCM=6} = \underbrace{\left\{\frac{2}{3}\times\frac{2}{2},\frac{3}{6}\right\}}_{\substack{\text{No Need to change} \\ \text{the second fraction}}} = \left\{\frac{4}{6},\frac{3}{6}\right\} = \left\{\frac{4}{6} > \frac{3}{6}\right\}$$

Example 1.134

Compare $\frac{3}{4}$ *with* $\frac{2}{3}$. First use LCM to make the denominators of the numbers same.

$$\underbrace{\left\{\frac{3}{4},\frac{2}{3}\right\}}_{LCM=12} = \left\{\frac{9}{12},\frac{8}{12}\right\} = \underbrace{\left\{\frac{9}{12} > \frac{8}{12}\right\}}_{\substack{9>8 \\ \text{Denominators are same}}}$$

Example 1.135

Each question has two fractions given. Compare the fractions for each pair, and decide which one is greater, or if the two are equal.

Co-Prime Numbers

A. $\frac{3}{4}$ and $\frac{3}{5}$
B. $\frac{2}{7}$ and $\frac{3}{11}$
C. $\frac{4}{5}$ and $\frac{6}{7}$
D. $\frac{5}{9}$ and $\frac{7}{11}$
E. $\frac{3}{5}$ and $\frac{4}{7}$
F. $\frac{7}{9}$ and $\frac{4}{5}$

Direct Multiple

G. $\frac{7}{10}$ and $\frac{3}{5}$
H. $\frac{4}{5}$ and $\frac{6}{15}$
I. $\frac{3}{7}$ and $\frac{6}{14}$

Non-Co-Prime Numbers

J. $\frac{4}{6}$ and $\frac{3}{4}$
K. $\frac{5}{6}$ and $\frac{6}{8}$
L. $\frac{5}{6}$ and $\frac{4}{9}$
M. $\frac{5}{8}$ and $\frac{7}{9}$
N. $\frac{3}{8}$ and $\frac{5}{12}$
O. $\frac{2}{6}$ and $\frac{5}{8}$
P. $\frac{4}{9}$ and $\frac{5}{10}$

Part A

$$\frac{3}{4} \text{ and } \frac{3}{5}$$

In order to make the denominators common, we will first find the LCM of the denominators:

$$LCM(4,5) = 20$$

Then, we will rewrite both the fractions to have a denominator of 20:

$$\frac{3}{4} = \frac{15}{20} > \frac{12}{20} = \frac{3}{5}$$

Part B

In order to make the denominators common, we will first find the LCM of the denominators:

$$LCM(7,11) = 77$$

Then, we will rewrite both the fractions to have a denominator of 77:

$$\frac{2}{7} = \frac{22}{77} > \frac{21}{77} = \frac{3}{11}$$

Part C
In order to make the denominators common, we will first find the LCM of the denominators:

$$LCM(5,7) = 35$$

Then, we will rewrite both the fractions to have a denominator of 77:

$$\frac{4}{5} = \frac{28}{35} < \frac{30}{35} = \frac{6}{7}$$

Part D
In order to make the denominators common, we will first find the LCM of the denominators:

$$LCM(4,6) = 12$$

Then, we will rewrite both the fractions to have a denominator of 77:

$$\frac{4}{6} = \frac{8}{12} < \frac{9}{12} = \frac{3}{4}$$

Part E

$$\frac{5}{6} ? \frac{6}{8}$$

In order to make the denominators common, we will first find the LCM of the denominators:

$$LCM(6,8) = 24$$

Then, we will rewrite both the fractions to have a denominator of 77:

$$\frac{5}{6} = \frac{20}{24} > \frac{18}{24} = \frac{6}{8}$$

Part F

$$\frac{3}{5} \text{ and } \frac{4}{7}$$

In order to make the denominators common, we will first find the LCM of the denominators:

$$LCM(5,7) = 35$$

Then, we will rewrite both the fractions to have a denominator of 77:

$$\frac{3}{5} = \frac{21}{35} > \frac{20}{35} = \frac{4}{7}$$

Improper Fractions
To compare two fractions which are improper, it usually works best if you convert them to mixed numbers. This lets you compare the whole number part very quickly. You will then only compare the fractional part, if required.

Example 1.136
A. $\frac{7}{2}$ and $\frac{8}{3}$
B. $\frac{3}{2}$ and $\frac{4}{3}$

Part A

$$\frac{7}{2} = \mathbf{3}\frac{1}{2}, \qquad \frac{8}{3} = \mathbf{2}\frac{2}{3}$$

We first compare the whole number parts of the fractions. Since we see that:

$$3 > 2$$

We do not need to compare the fractional parts, and we can say:

$$3\frac{1}{2} > 2\frac{2}{3}$$

Part B

$$\frac{3}{2} = 1\frac{1}{2} \quad ??? \quad 1\frac{1}{3} = \frac{4}{3}$$

On comparing, we find that the whole number part is equal. Hence, we need to compare the fractions.

$$\frac{1}{2} > \frac{1}{3}$$

In the two fractions, above, the numerators are the same. Hence, the fraction with smaller denominator is the larger fraction.

Example 1.137

While working on his car, Sam tried the 1/2 inch wrench, and it was too large. He then tried the 3/8 wrench, and it was too small. Which wrench size should Sam try next?

A. 3/4 inch
B. 2/3 inch
C. 7/16 inch
D. 1/4 inch

So, we need a fraction that is between

$$\frac{3}{8} \, and \, \frac{1}{2} = \frac{4}{8}$$

$$\frac{3}{4} = \frac{6}{8} > \frac{4}{8}$$
$$\frac{2}{3} = \frac{16}{24} > \frac{4}{8} = \frac{12}{24}$$
$$\frac{3}{8} = \frac{6}{16} < \frac{7}{16} < \frac{4}{8} = \frac{8}{16}$$

Hence, option C is correct.

Mixed Review

Three Numbers

When we want to compare three fractions, we need to find the LCM of all three fractions. And we then need to compare the three fractions.

Example 1.138

$$\underbrace{\frac{3}{4}, \frac{5}{6}, \frac{6}{12}}_{LCM(4,6,12)=12} \Rightarrow \frac{3}{4} \times \frac{3}{3}, \frac{5}{6} \times \frac{2}{2}, \frac{6}{12} \times \frac{1}{1} \Rightarrow \underbrace{\frac{9}{12}}_{Middle}, \underbrace{\frac{10}{12}}_{Largest}, \underbrace{\frac{6}{12}}_{Smallest}$$

This method will always work, but is the lengthiest.

Example 1.139

$$\frac{5}{3}, \frac{10}{8}$$

The denominator of the first fraction is 3. The denominator of the second fraction is 8. Find the LCM of the two denominators.

3	6	9	12	15	18	**24**
8	16	**24**				

Convert both the fractions to have a denominator of 24.

$$\frac{5}{3}=\frac{40}{24}, \qquad \frac{10}{8}=\frac{30}{24}$$

Compare the two fractions

$$\frac{40}{24}>\frac{30}{24}\Rightarrow\frac{5}{3}>\frac{10}{8}$$

Many Numbers

When there are many numbers, it becomes difficult to find the LCM of all the fractions. Hence, you should group the fractions in the following categories:

- Fractions more than one
- Fractions less than one, but more than half
- Fractions less than half

Example 1.140

Order the following fractions from least to greatest:

$$\frac{3}{4},\frac{4}{5},\frac{2}{5},\frac{7}{6},\frac{11}{9},\frac{5}{11}$$

Group these fractions into categories.

Improper Fractions

The improper fractions are greater than one:

$$\frac{7}{6}>1,\frac{11}{9}>1\Rightarrow\frac{7}{6}=\frac{21}{18},\frac{22}{18}=\frac{11}{9}\Rightarrow\frac{7}{6}<\frac{11}{9}$$

Fractions less than half

Find the fractions which are less than half:

$$\frac{2}{5}=\frac{4}{10}<\frac{5}{10}=\frac{1}{2}$$

$$\frac{5}{11}=\frac{10}{22}<\frac{11}{22}=\frac{1}{2}$$

LCM of 5 and 11 is 55:

$$\frac{2}{5}=\frac{22}{55}<\frac{25}{55}=\frac{5}{11}\Rightarrow\frac{2}{5}<\frac{5}{11}$$

Fractions between half and one

The remaining fractions are between half and one:

$$\frac{3}{4}=\frac{15}{20}<\frac{16}{20}=\frac{4}{5}\Rightarrow\frac{3}{4}<\frac{4}{5}$$

$$\frac{2}{5} < \frac{5}{11} < \frac{1}{2} < \frac{3}{4} < \frac{4}{5} < 1 < \frac{7}{6} < \frac{11}{9}$$

1.6 Add and Subtract: Different Denominators

A. Making the denominator the same

Example 1.141

What is the answer to?

$$\frac{1}{3} + \frac{1}{3}$$

$$\frac{1}{3} + \frac{1}{3} = \frac{2}{3}$$

Are the denominators added when you add two fractions?

No.

1.142: Getting common denominators

- If want to add two fractions, with different denominators, we first have to make the denominators the same.
- We will make the denominators the same by finding the LCM of the two denominators.

Example 1.143

Add $\frac{3}{4} + \frac{1}{6}$

The denominators are not the same. We want to make them the same.
So, we find the LCM of the denominators of the two fractions:

$$LCM(4,6) = 12$$

Convert the first fraction to have a denominator of 12:

$$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$$

Convert the second fraction to have a denominator of 12:

$$\frac{1}{6} \times \frac{2}{2} = \frac{2}{12}$$

Now, we can complete the addition:

$$\frac{3}{4} + \frac{1}{6} = \frac{9}{12} + \frac{2}{12} = \frac{11}{12}$$

Practice 1.144

Perform the indicated operations

Direct Multiple LCM

A. $\frac{2}{5} + \frac{3}{10}$
B. $\frac{2}{3} + \frac{1}{6}$
C. $\frac{1}{2} + \frac{1}{4}$
D. $\frac{1}{3} + \frac{4}{9}$
E. $\frac{3}{4} + \frac{2}{8}$
F. $\frac{2}{7} + \frac{3}{14}$

Other LCM

G. $\frac{1}{2} + \frac{1}{3}$
H. $\frac{1}{3} + \frac{1}{4}$
I. $\frac{2}{3} + \frac{1}{2}$
J. $\frac{1}{2} + \frac{2}{5}$
K. $\frac{2}{5} + \frac{1}{3}$

L. $\frac{1}{4}+\frac{1}{5}$
M. $\frac{2}{7}+\frac{2}{3}$
N. $\frac{4}{5}+\frac{1}{7}$

O. $\frac{2}{3}+\frac{2}{7}$
P. $\frac{1}{9}+\frac{1}{4}$
Q. $\frac{2}{4}+\frac{2}{5}$

Improper Fractions for Answers

R. $\frac{3}{5}+\frac{1}{2}$
S. $\frac{3}{5}+\frac{4}{7}$

T. $\frac{2}{3}+\frac{3}{4}$
U. $\frac{7}{9}+\frac{2}{11}$
V. $\frac{1}{13}+\frac{1}{11}$

$$\frac{2}{5}+\frac{3}{10}=\frac{4}{10}+\frac{3}{10}=\frac{7}{10}$$
$$\frac{2}{3}+\frac{1}{6}=\frac{4}{6}+\frac{1}{6}=\frac{5}{6}$$
$$\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$$
$$\frac{1}{3}+\frac{4}{9}=\frac{3}{9}+\frac{4}{9}=\frac{7}{9}$$
$$\frac{3}{4}+\frac{2}{8}=\frac{6}{8}+\frac{2}{8}=\frac{8}{8}=1$$
$$\frac{2}{7}+\frac{3}{14}=\frac{4}{14}+\frac{3}{14}=\frac{7}{14}$$

B. Subtraction

Subtraction works very similar to addition. If the denominator is not the same, make it the same, by:

- Finding the lowest common multiple of the denominators
- Creating equivalent fractions that both have the same denominator

Example 1.145

Subtract $\frac{1}{3}$ from $\frac{7}{8}$

$$LCM(3,8)=24\Rightarrow\frac{7}{8}-\frac{1}{3}=\frac{21}{24}-\frac{8}{24}=\frac{13}{24}$$

Practice 1.146

Perform the indicated operations

A. $\frac{1}{2}-\frac{1}{4}$
B. $\frac{1}{3}-\frac{1}{9}$
C. $\frac{2}{5}-\frac{2}{10}$
D. $\frac{2}{7}-\frac{1}{21}$
E. $\frac{1}{11}-\frac{1}{22}$
F. $\frac{3}{7}-\frac{4}{14}$

$$\frac{1}{2}-\frac{1}{4}=\frac{2}{4}-\frac{1}{4}=\frac{1}{4}$$
$$\frac{1}{3}-\frac{1}{9}=\frac{3}{9}-\frac{1}{9}=\frac{2}{9}$$
$$\frac{2}{5}-\frac{2}{10}=\frac{2}{5}-\frac{1}{5}=\frac{1}{5}$$
$$\frac{2}{7}-\frac{1}{21}=\frac{6}{21}-\frac{1}{21}=\frac{5}{21}$$
$$\frac{1}{11}-\frac{1}{22}=\frac{2}{22}-\frac{1}{22}=\frac{1}{22}$$

$$\frac{3}{7}-\frac{4}{14}=\frac{6}{14}-\frac{4}{14}=\frac{2}{14}=\frac{1}{7}$$

C. Word Problems

Example 1.147

Travis ran $\frac{4}{3}$ laps in the morning, and $\frac{1}{5}$ of a lap in the evening. How many laps did he run in all during the day?

$$\frac{4}{3}+\frac{1}{5}=\frac{20}{15}+\frac{3}{15}=\frac{23}{15}=1\frac{8}{15}$$

D. Multi-Step Problems

Example 1.148

Charlie ate half of a cake. Charlotte ate one fourth of the same cake.

A. How much cake did the two of them eat together?
B. How much was left after the two of them finished eating?

$$\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$$

$$\underbrace{1}_{\substack{Whole\\Cake}}-\frac{3}{4}=\frac{4}{4}-\frac{3}{4}=\frac{1}{4}$$

Example 1.149

Three friends each ordered a large cheese pizza. Shauntee ate $\frac{2}{3}$ of her pizza, Carlos ate $\frac{8}{9}$ of his pizza and Rocco ate $\frac{26}{27}$ of his pizza. If the remaining portions from the three pizzas are put together, what fraction of a large pizza do they make? Express your answer as a common fraction. **(MathCounts 2005 Chapter Sprint)**

$$Pizza\ left\ over: \underbrace{1-\frac{2}{3}=\frac{1}{3}}_{Shauntee}, \quad \underbrace{1-\frac{8}{9}=\frac{1}{9}}_{Carlos}, \quad \underbrace{1-\frac{26}{27}=\frac{1}{27}}_{Rocco}$$

$$Putting\ them\ together: \frac{1}{3}+\frac{1}{9}+\frac{1}{27}=\frac{9}{27}+\frac{3}{27}+\frac{1}{27}=\frac{13}{27}$$

E. Challenge Problems

Example 1.150

The reciprocal of 8 is subtracted from the reciprocal of 4, and the result is added to the reciprocal of 10. What fraction do you get? **(MathCounts 2010 Chapter Countdown)**

$$\frac{1}{4}-\frac{1}{8}=\frac{1}{8}$$
$$\frac{1}{8}+\frac{1}{10}=\frac{9}{40}$$

1.7 Adding and Subtracting Mixed Numbers

Revision: Mixed Numbers

Recall that a mixed number is a whole number added to a fraction. If we want, we can write the mixed number as the addition of a whole number and a fraction. For example:

$$\underbrace{2\frac{3}{4}}_{Mixed\ Number}=\underbrace{2}_{Whole\ Number}+\underbrace{\frac{3}{4}}_{Fraction}$$

Mixed Numbers: Addition

When we want to add mixed numbers, we will

- Separate out the numbers from the fractions, and write the numbers together and the fractions together
- Add the numbers
- Add the fractions
- Combine the numbers the fractions

Example 1.151: Addition

Add $2\frac{3}{5}+1\frac{1}{5}$

Separate out the numbers from the fractions:

$$2\frac{3}{5}+1\frac{1}{5}=\underbrace{2+\frac{3}{5}}_{First\ Mixed\ Number}+\underbrace{1+\frac{1}{5}}_{Second\ Mixed\ Number}=\underbrace{2+1}_{Numbers\ Together}+\underbrace{\frac{3}{5}+\frac{1}{5}}_{Fractions\ Together}$$

Now, add the numbers(separately) and add the fractions (separately):

$$\underbrace{3+\frac{4}{5}}_{Writing\ as\ Addition}=\underbrace{3\frac{4}{5}}_{Writing\ as\ Mixed\ Number}$$

Practice 1.152

A. $2\frac{2}{7}+1\frac{3}{7}$
B. $1\frac{5}{9}+2\frac{1}{9}$
C. $4\frac{4}{11}+1\frac{3}{11}$
D. $1\frac{3}{5}+2\frac{1}{5}$

$$2\frac{2}{7}+1\frac{3}{7}=3\frac{5}{7}$$
$$1\frac{5}{9}+2\frac{1}{9}=3\frac{6}{9}=3\frac{2}{3}$$
$$4\frac{4}{11}+1\frac{3}{11}=5\frac{7}{11}$$
$$1\frac{3}{5}+2\frac{1}{5}=3\frac{4}{5}$$

Mixed Numbers: Subtraction

The process in subtraction is similar to addition. We separate out the whole numbers and the fractions, and do the subtraction of each separately. Then, we add to get the final answer.

Example 1.153: Subtraction

Compute $4\frac{5}{7}-2\frac{3}{7}$

Separate out the numbers from the fractions:

$$4\frac{5}{7}-2\frac{3}{7}=\underbrace{4+\frac{5}{7}}_{\text{First Mixed Number}}-\underbrace{\left(2+\frac{3}{7}\right)}_{\text{Second Mixed Number}}=\underbrace{4-2}_{\text{Numbers Together}}+\underbrace{\frac{5}{7}-\frac{3}{7}}_{\text{Fractions Together}}$$

Now, add the numbers(separately) and add the fractions (separately):

$$\underbrace{2+\frac{2}{7}}_{\text{Writing as Addition}}=\underbrace{2\frac{2}{7}}_{\text{Writing as Mixed Number}}$$

$$\textit{Method II}: 4\frac{5}{7}-2\frac{3}{7}=\frac{33}{7}-\frac{17}{7}=\frac{16}{7}=2\frac{2}{7}$$

Practice 1.154

A. $2\frac{3}{4}-1\frac{1}{4}$
B. $3\frac{2}{5}-2\frac{2}{5}$
C. $4\frac{3}{11}-2\frac{2}{11}$

$$2\frac{3}{4}-1\frac{1}{4}=2\frac{1}{4}$$
$$3\frac{2}{5}-2\frac{2}{5}=1$$
$$4\frac{3}{11}-2\frac{2}{11}=2\frac{1}{11}$$

Regrouping while Adding

Here, we need to rewrite the number after adding it because the fraction that we get is improper.

Example 1.155

A. $2\frac{5}{6}+1\frac{4}{6}$
B. $1\frac{4}{7}+2\frac{5}{7}$
C. $1\frac{4}{9}+1\frac{7}{9}$
D. $1\frac{3}{11}+1\frac{9}{11}$
E. $1\frac{7}{10}+1\frac{5}{10}$

Part A

$$2\frac{5}{6}+1\frac{4}{6}=\frac{17}{6}+\frac{10}{6}=\frac{27}{6}=\frac{9}{2}=4\frac{1}{2}$$
$$2\frac{5}{6}+1\frac{4}{6}=3\frac{9}{6}=3\frac{3}{2}=3+1\frac{1}{2}=4\frac{1}{2}$$

Part B

$$1\frac{4}{7}+2\frac{5}{7}=\frac{11}{7}+\frac{19}{7}=\frac{30}{7}=4\frac{2}{7}$$
$$1\frac{4}{7}+2\frac{5}{7}=3\frac{9}{7}=3+1\frac{2}{7}=4\frac{2}{7}$$

Part C

$$1\frac{4}{9}+1\frac{7}{9}=\frac{13}{9}+\frac{16}{9}=\frac{29}{9}=3\frac{2}{9}$$
$$1\frac{4}{9}+1\frac{7}{9}=2\frac{11}{9}=2+1\frac{2}{9}=3\frac{2}{9}$$

Part D

$$1\frac{3}{11}+1\frac{9}{11}=\frac{14}{11}+\frac{20}{11}=\frac{34}{11}=3\frac{1}{11}$$
$$1\frac{7}{10}+1\frac{5}{10}=\frac{17}{10}+\frac{15}{10}=\frac{32}{10}=3\frac{2}{10}=3\frac{1}{5}$$

Regrouping while Subtracting

Here, we need to borrow from one of the numbers.

Example 1.156

A. $2\frac{1}{3}-1\frac{2}{3}$
B. $5\frac{4}{7}-2\frac{6}{7}$
C. $7\frac{3}{11}-3\frac{7}{11}$

$$2\frac{1}{3}-1\frac{2}{3}=\underbrace{2-1}_{\substack{\textit{Numbers}\\\textit{Together}}}+\underbrace{\frac{1}{3}-\frac{2}{3}}_{\substack{\textit{Fractions}\\\textit{Together}}}=1+\frac{1}{3}-\frac{2}{3}=\frac{4}{3}-\frac{2}{3}=\frac{2}{3}$$

$$5\frac{4}{7}-2\frac{6}{7}=3\frac{4}{7}-\frac{6}{7}=2\frac{11}{7}-\frac{6}{7}=2\frac{5}{7}$$
$$7\frac{3}{11}-3\frac{7}{11}=4\frac{3}{11}-\frac{7}{11}=3\frac{14}{11}-\frac{7}{11}=3\frac{7}{11}$$

Example 1.157: Subtracting from a whole number

A. $2-\frac{2}{3}$
B. $5-\frac{3}{7}$
C. $4-\frac{5}{11}$
D. $7-3\frac{2}{7}$

$$2-\frac{2}{3}=\underbrace{1+\frac{3}{3}}_{\substack{\textit{Break up}\\\textit{the 2}}}-\frac{2}{3}=1\frac{1}{3}$$
$$5-\frac{3}{7}=4+\frac{7}{7}-\frac{3}{7}=4\frac{4}{7}$$

$$4-\frac{5}{11}=3+\frac{11}{11}-\frac{5}{11}=3\frac{6}{11}$$
$$7-3\frac{2}{7}=4-\frac{2}{7}=3+\frac{7}{7}-\frac{2}{7}=3\frac{5}{7}$$

Shortcut method for large numbers

Example 1.158

Add $3\frac{5}{8}+2\frac{7}{8}$

Separate out the numbers from the fractions:

$$\underbrace{3+2}_{\textit{Numbers Together}}+\underbrace{\frac{5}{8}+\frac{7}{8}}_{\textit{Fractions Together}}$$

Now, add the numbers(separately) and add the fractions (separately):

$$\underbrace{5+\frac{12}{8}}_{\substack{\textit{Writing as}\\ \textit{Addition}}}=\underbrace{5+\frac{3}{2}}_{\substack{\textit{Simplify the}\\ \textit{fraction}}}=\underbrace{5+1+\frac{1}{2}}_{\substack{\textit{Writing the improper}\\ \textit{fraction as Mixed Number}}}=\underbrace{6\frac{1}{2}}_{\textit{Add the Number}}$$

Example 1.159: Answer less than One

Subtract $1\frac{4}{5}$ from $2\frac{1}{5}$.

$$2\frac{1}{5}-1\frac{4}{5}=\underbrace{2+\frac{1}{5}-1-\frac{4}{5}}_{\textit{Broke up the fractions}}=\underbrace{2-1+\frac{1}{5}-\frac{4}{5}}_{\textit{Rearranged}}=\underbrace{1+\frac{1}{5}-\frac{4}{5}}_{\substack{\textit{Simplified the}\\ \textit{whole nunbers}}}=\underbrace{\frac{5}{5}+\frac{1}{5}-\frac{4}{5}}_{\substack{\textit{Rewrite one with}\\ \textit{denominator 5}}}=\underbrace{\frac{6}{5}-\frac{4}{5}}_{\substack{\textit{Wrote as a}\\ \textit{mixed fraction}}}=\frac{2}{5}$$

Example 1.160: Answer more than one

Subtract $2\frac{3}{7}$ from $5\frac{2}{7}$.

$$\textit{Method I}: 5\frac{2}{7}-2\frac{3}{7}=\underbrace{5+\frac{2}{7}-2-\frac{3}{7}}_{\textit{Broke up the fractions}}=\underbrace{5-2}_{\substack{\textit{Put the whole}\\ \textit{numbers together}}}+\underbrace{\frac{2}{7}-\frac{3}{7}}_{\substack{\textit{Put the fractions}\\ \textit{together}}}=\underbrace{3+\frac{2}{7}-\frac{3}{7}}_{\substack{\textit{Simplified the}\\ \textit{whole nunbers}}}$$

To subtract $\frac{3}{7}$ from $\frac{2}{7}$, we will borrow from the three:

$$\underbrace{2+1+\frac{2}{7}-\frac{3}{7}}_{\textit{Write } 3=2+1}=\underbrace{2+\frac{7}{7}+\frac{2}{7}-\frac{3}{7}}_{\textit{Write } 1=\frac{7}{7}}=\underbrace{2+\frac{9}{7}-\frac{3}{7}}_{\textit{Write } 1=\frac{7}{7}}=\underbrace{2+\frac{6}{7}}_{\substack{\textit{Do the}\\ \textit{subtraction}}}=\underbrace{2\frac{6}{7}}_{\substack{\textit{Write as a}\\ \textit{mixed numer}}}$$

$$\textit{Method II}: 5\frac{2}{7}-2\frac{3}{7}=\frac{37}{7}-\frac{17}{7}=\frac{20}{7}=2\frac{6}{7}$$

Practice 1.161: Answer more than one

$$5\frac{4}{8}-3\frac{7}{8}=5+\frac{4}{8}-3-\frac{7}{8}=5-3+\frac{4}{8}-\frac{7}{8}=2+\frac{4}{8}-\frac{7}{8}=1+1+\frac{4}{8}-\frac{7}{8}=1+\frac{8}{8}+\frac{4}{8}-\frac{7}{8}=1+\frac{5}{8}=1\frac{5}{8}$$

Addition

Example 1.162

A. $2\frac{3}{5} + 1\frac{2}{3}$
B. $1\frac{4}{5} + 1\frac{1}{4}$
C. $1\frac{3}{7} + 1\frac{4}{5}$

$$2\frac{3}{5} + 1\frac{2}{3} = 2 + 1 + \frac{3}{5} + \frac{2}{3} = 3 + \frac{9}{15} + \frac{10}{15} = 3\frac{19}{15} = 3 + 1\frac{4}{15} = 4\frac{4}{15}$$
$$1\frac{4}{5} + 1\frac{1}{4} = 2 + \frac{16}{20} + \frac{5}{20} = 2 + \frac{21}{20} = 2 + 1\frac{1}{20} = 3\frac{1}{20}$$
$$1\frac{3}{7} + 1\frac{4}{5} = 2 + \frac{15}{35} + \frac{28}{35} = 2 + \frac{43}{35} = 2 + 1\frac{8}{35} = 3\frac{8}{35}$$

Subtraction

Example 1.163

A. $5\frac{2}{7} - 2\frac{2}{3}$
B. $8\frac{1}{4} - 3\frac{2}{5}$
C. $3\frac{4}{9} - 1\frac{5}{6}$

$$5\frac{2}{7} - 2\frac{2}{3} = \underbrace{5-2}_{\substack{\text{Numbers}\\\text{Together}}} + \frac{2}{7} - \frac{2}{3} = 3 + \frac{6}{21} - \frac{14}{21} = \underbrace{2 + \frac{21}{21}}_{\substack{\text{Break the}\\3}} + \frac{6}{21} - \frac{14}{21} = 2\frac{13}{21}$$

$$8\frac{1}{4} - 3\frac{2}{5} = 8 - 3 + \frac{1}{4} - \frac{2}{5} = 5 + \frac{4}{20} - \frac{8}{20} = 4 + 1 + \frac{4}{20} - \frac{8}{20} = 4 + \frac{20}{20} + \frac{4}{20} - \frac{8}{20} = 4\frac{16}{20} = 4\frac{4}{5}$$

$$3\frac{4}{9} - 1\frac{5}{6} = 2\frac{4}{9} - \frac{5}{6} = 2\frac{8}{18} - \frac{15}{18} = \underbrace{1 + \frac{18}{18}}_{\text{Break the 2}} + \frac{8}{18} - \frac{15}{18} = 1\frac{11}{18}$$

Example 1.164

Find the value of Δ that makes the statement true

$$\frac{3}{5} + 14 = \frac{\Delta}{15} + 13$$
$$LHS = \frac{3}{5} + 14 = \frac{9}{15} + 1 + 13 = \underbrace{\frac{24}{15} + 13 = \frac{\Delta}{15} + 13}_{\text{Compare}} \Rightarrow \Delta = 24$$

Example 1.165

Last week, on Wednesday, I spent $\frac{3}{4}th$ of an hour swimming and $\frac{1}{3}^{\text{rd}}$ of an hour running. What is the total time, in hours, that I spent exercising on Wednesday?

$$\frac{3}{4} + \frac{1}{3} = \frac{9}{12} + \frac{4}{12} = \frac{13}{12} = 1\frac{1}{12}Hours$$

$$45 + 20 = 65\ minutes = 1\ hour\ 5\ minutes = 1\frac{1}{12}\ Hours$$

Example 1.166

Kirti takes a 5 cm long piece of paper, and makes an Origami frog that is $2\frac{1}{4}$ cm long. How much shorter is the frog compared to the paper used to make it?

$$5 - 2\frac{1}{4} = 5 - 2 - \frac{1}{4} = 3 - \frac{1}{4} = 2 + 1 - \frac{1}{4} = 2 + \frac{4}{4} - \frac{1}{4} = 2\frac{3}{4}$$

Example 1.167

I have two ropes, one of which is $3\frac{1}{4}$ meters long, and the other is $2\frac{4}{5}$ meters long. If I put the ropes end to end, what is the length from one end to another?

$$3\frac{1}{4} + 2\frac{4}{5} = 5 + \frac{5}{20} + \frac{16}{20} + 5\frac{21}{20} = 6\frac{1}{20}$$

Example 1.168

Reena has $4\frac{1}{5}$ jugs of milk. Sheela has $2\frac{3}{4}$ jugs of milk. How many more jugs of milk of does Reena have compared to Sheela?

$$4\frac{1}{5} - 2\frac{3}{4} = 2\frac{1}{5} - \frac{3}{4} = 1 + \frac{6}{5} - \frac{3}{4} = 1 + \frac{24}{20} - \frac{15}{20} = 1\frac{9}{20}$$

Example 1.169

A marble statute weighs $\frac{17}{3}$ pounds, while a bronze statute weighs $\frac{15}{2}$ pounds. Which one weighs more and by how much?

$$\underbrace{\frac{45}{6}}_{Bronze} - \underbrace{\frac{34}{6}}_{Marble} = \frac{11}{6} = 1\frac{5}{6}$$

Example 1.170

Ronak has $2\frac{3}{4}$ cups of juice every day for breakfast. Shalu has $1\frac{4}{5}$ cups less juice than Ronak does every day. What is the total juice that Ronak and Shalu will need for breakfast in one day?

$$Shalu = 2\frac{3}{4} - 1\frac{4}{5} = 1\frac{3}{4} - \frac{4}{5} = \frac{7}{4} - \frac{4}{5} = \frac{35}{20} - \frac{16}{20} = \frac{19}{20}$$
$$Total = 2\frac{3}{4} + \frac{19}{20} = \frac{11}{4} + \frac{19}{20} = \frac{55}{20} + \frac{19}{20} = \frac{74}{20} = 3\frac{14}{20} = 3\frac{7}{10}$$

Example 1.171

A shop has the following offers:

- A toy boat and two toy ships cost $3\frac{1}{3}$ Rs.
- A toy ship, a toy plane, and a toy helicopter cost $6\frac{1}{3}$ Rs.
- Two toy planes and a toy boat cost $8\frac{1}{3}$ Rs.

- A toy boat and two toy helicopters cost $5\frac{1}{3}$ Rs.

Find of cost of each individual toy.

Write out the information given in equation form

$$I: \textbf{Boat} + 2\ \textbf{Ships} = 3\frac{1}{3}$$

$$II: \textbf{Ship} + \textbf{Plane} + \textbf{Helicopter} = 6\frac{1}{3}$$

$$III: 2\ \textbf{Planes} + \textbf{Boat} = 8\frac{1}{3}$$

$$IV: \textbf{Boat} + 2\ \textbf{Helicopters} = 5\frac{1}{3}$$

Add all the equations

$$3\ \textbf{Boats} + 3\ \textbf{Ships} + 3\ \textbf{Planes} + 3\ \textbf{Helicopters} = 3\frac{1}{3} + 6\frac{1}{3} + 8\frac{1}{3} + 5\frac{1}{3} = 22 + \frac{4}{3} = 23\frac{1}{3} = \frac{70}{3}$$

Divide both sides of the equation by three:

$$V: 1\ Boat + 1\ Ship + 1\ Plane + 1\ Helicopter = \frac{70}{3} \div 3 = \frac{70}{3} \times \frac{1}{3} = \frac{70}{9}$$

$$II: \textbf{Ship} + \textbf{Plane} + \textbf{Helicopter} = 6\frac{1}{3}$$

$$1\ Boat = \frac{70}{9} - 6\frac{1}{3} = \frac{70}{9} - \frac{19}{3} = \frac{70}{3} - \frac{57}{9} = \frac{13}{9} = 1\frac{4}{9}$$

1.8 Multiplying Fractions

Revision: Multiplication

Multiplication can be considered repeated addition.

Example 1.172: Converting Addition to Multiplication

Write $7 + 7 + 7 + 7 + 7$ as multiplication

$$7 + 7 + 7 + 7 + 7 = 5 \times 7 = 7 \times 5$$

Example 1.173: Converting Multiplication to Addition

Write 3×4 as addition

$$3 \times 4 = \underbrace{4 + 4 + 4}_{Add\ 4\ three\ times} = \underbrace{3 + 3 + 3 + 3}_{Add\ 3\ four\ times}$$

Multiplying a Fraction with a whole Number

Multiplying a fraction with a whole number can be considered as repeated addition.
Shortcut: Multiply the numerator with the numerator, and ignore the denominator.

Example 1.174

Multiply $\frac{2}{5}$ with three.

$$\frac{2}{5} \times 3 = \underbrace{\frac{2}{5} + \frac{2}{5} + \frac{2}{5}}_{\substack{Converting\ multiplication \\ to\ addition}} = \underbrace{\frac{2+2+2}{5}}_{\substack{Denominator\ are\ same \\ Add\ the\ numerators}} = \underbrace{\frac{2 \times 3}{5}}_{\substack{Write\ addition \\ as\ multiplication}} = \frac{6}{5}$$

We can shorten this process by directly doing the multiplication:

$$\textit{Shortcut}: \frac{2}{5} \times 3 = \frac{2 \times 3}{5} = \frac{6}{5}$$

Example 1.175

Evaluate

A. $\frac{3}{4} \times 3$

B. $\frac{2}{5} \times 7$

C. $\frac{3}{5} \times 2$

D. $\frac{5}{8} \times 3$

$$\frac{3}{4} \times 3 = \frac{9}{4}$$
$$\frac{2}{5} \times 7 = \frac{14}{5}$$
$$\frac{3}{5} \times 2 = \frac{6}{5}$$
$$\frac{5}{8} \times 3 = \frac{15}{8}$$

Multiplying a Fraction with Unit Fraction

When we want to multiply a fraction with a unit fraction, we note that we are increasing the number of parts of each fraction. Hence, we will multiply the denominators.

Example 1.176

Find one-half of $\frac{1}{2}$.

$$\textit{One half of } \frac{1}{2}$$

Replace one half with $\frac{1}{2}$:

$$\frac{1}{2} \textit{ of } \frac{1}{2}$$

Replace "of" with multiplication:

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$$

Example 1.177

Find one-third of $\frac{2}{7}$.

Whenever we see of, we can replace it with the multiplication sign.

Each part in $\frac{2}{7}$ is getting divided into three further parts. Hence, we will multiply the denominators:

$$\frac{2}{\mathbf{7}} \times \frac{1}{\mathbf{3}} = \frac{2}{\mathbf{7 \times 3}} = \frac{2}{\mathbf{21}}$$

Example 1.178

A. Find one fourth of $\frac{1}{2}$

B. Find one fifth of $\frac{2}{5}$

C. Find one seventh of $\frac{2}{9}$
D. $\frac{1}{4}\times\frac{5}{7}$
E. $\frac{1}{5}\times\frac{2}{7}$

$$\frac{1}{4}of\frac{1}{2}=\frac{1}{4}\times\frac{1}{2}=\frac{1}{8}$$

Multiplying a Fraction with a Fraction

Multiplying a fraction with a fraction means we are multiplying by the numerator, and increasing the number of parts of the denominator:

$$\frac{N}{D}\times\frac{N}{D}=\frac{N\times N}{D\times D}$$

Example 1.179

A. $\frac{3}{7}\times\frac{2}{5}$
B. $\frac{4}{5}\times\frac{3}{7}$
C. $\frac{7}{4}\times\frac{2}{5}$

$$\frac{3}{7}\times\frac{2}{5}=\frac{3\times2}{7\times5}=\frac{6}{35}$$
$$\frac{4}{5}\times\frac{3}{7}=\frac{12}{35}$$
$$\frac{7}{4}\times\frac{2}{5}=\frac{14}{20}=\frac{7}{10}$$

Scaling

When we looked at fractions earlier, we used one as a benchmark and came to the conclusion that:

- Proper fractions are less than one
- Fractions where the numerator is equal to the denominator are equal to one
- Improper fractions are greater than one

We can use this in multiplication. Multiplying by:

- A fraction which is greater than one increases the fraction
- A fraction which is one keeps the fraction the same, and gives an equivalent fraction
- A fraction which is less than one makes the fraction smaller.

Example 1.180

Look at the following expressions, and arrange them in ascending order without actually carrying out the multiplication.

$$\frac{2}{3}\times\frac{5}{5},\quad\frac{2}{3}\times\frac{4}{5},\quad\frac{2}{3}\times\frac{7}{5}$$

We can arrange the fractions in ascending order as follows:

$$\underbrace{\frac{2}{3}\times\frac{4}{5}}_{\substack{Decreases\\Because\frac{4}{5}<1}},\quad\underbrace{\frac{2}{3}\times\frac{5}{5}}_{\substack{Remains\ same\\Because\frac{5}{5}=1}},\quad\underbrace{\frac{2}{3}\times\frac{7}{5}}_{\substack{Increases\\Because\frac{7}{5}>1}}$$

Cancellation in Multiplication

When the numerator and the denominator have a common factor, this can be **cancelled**. Cancelling means you divide by the common factor.

Example 1.181

A. $\frac{3}{5}\times\frac{2}{3}$
B. $\frac{6}{11}\times\frac{2}{9}$
C. $\frac{49}{4}\times\frac{8}{7}$

Part A

Three is common in both numerator and denominator. Therefore, we can cancel it:

$$\frac{3}{5}\times\frac{2}{3}=\frac{\mathbf{3}\times 2}{5\times\mathbf{3}}=\frac{\mathbf{1}\times 2}{5\times\mathbf{1}}=\frac{2}{5}$$

$$\frac{3}{5}\times\frac{2}{3}=\frac{6}{15}=\frac{2}{5}$$

Part B

$$\frac{6}{11}\times\frac{2}{9}=\frac{\mathbf{6}\times 2}{11\times\mathbf{9}}=\frac{2\times 2}{11\times 3}=\frac{4}{33}$$

Part C

$$\frac{\mathbf{49}}{\mathbf{4}}\times\frac{\mathbf{8}}{\mathbf{7}}=\frac{7}{1}\times\frac{2}{1}=14$$

Example 1.182

$\frac{3\times5}{9\times11}\times\frac{7\times9\times11}{3\times5\times7}=$ (AMC 8 1985/1)

$$\frac{\mathbf{3}\times\mathbf{5}}{\mathbf{9}\times\mathbf{11}}\times\frac{7\times\mathbf{9}\times\mathbf{11}}{\mathbf{3}\times\mathbf{5}\times 7}=7$$

Cancellation Trap

Example 1.183

Evaluate $\frac{3}{5}\times\frac{3}{5}$

Bogus Solution

$$\underbrace{\frac{3}{5}\times\frac{3}{5}=\frac{1}{1}\times\frac{1}{1}=1}_{\text{Bogus Solution}}$$

You cannot cancel, if the both the numbers are in the numerator, or both the numbers are in the denominator.

Valid Solution

$$\frac{3}{5}\times\frac{3}{5}=\frac{9}{25}$$

Improper Fractions

Improper fractions get multiplied the same way as proper fractions

Example 1.184

Mixed Numbers with Numbers

Mixed Numbers

To multiply numbers, which are mixed, we first convert them to improper fractions. Recall that improper fractions are multiplied, just like regular fractions.

Example 1.185

A. $4\frac{1}{3} \times 5$
B. $2\frac{1}{4} \times 3$
C. $3\frac{1}{3} \times 2$
D. $4\frac{1}{2} \times 3$

$$4\frac{1}{3} \times 5 = \frac{13}{3} \times 5 = \frac{65}{5}$$

$$2\frac{1}{4} \times 3 = \frac{9}{4} \times 3 = \frac{27}{4}$$

$$3\frac{1}{3} \times 2 = \frac{10}{3} \times 2 = \frac{20}{3}$$

$$4\frac{1}{2} \times 3 = \frac{9}{2} \times 3 = \frac{27}{2}$$

Example 1.186

A. $2\frac{1}{3} \times 3\frac{1}{4}$
B. $3\frac{1}{2} \times 1\frac{2}{5}$
C. $2\frac{2}{7} \times 3\frac{1}{5}$

$$2\frac{1}{3} \times 3\frac{1}{4} = \frac{7}{3} \times \frac{13}{4} = \frac{91}{12}$$

$$3\frac{1}{2} \times 1\frac{2}{5} = \frac{7}{2} \times \frac{7}{5} = \frac{49}{10}$$

$$2\frac{2}{7} \times 3\frac{1}{5} = \frac{16}{7} \times \frac{16}{5} = \frac{256}{35}$$

Cancellation

Example 1.187: Mixed Numbers with a Number

Evaluate $1\frac{7}{10} \times 5$

$$1\frac{7}{10} \times 5 = \frac{17}{10} \times 5 = \frac{85}{10} = \frac{17 \times 5}{2 \times 5} = \frac{17}{2}$$

Cancellation

$$1\frac{7}{10} \times 5 = \frac{17}{10} \times 5 = \frac{17 \times 5}{2 \times 5} = \frac{17}{2}$$

Example 1.188: Mixed Numbers with Mixed Numbers

$$2\frac{1}{3} \times 1\frac{3}{4} = \frac{7}{3} \times \frac{7}{4} = \frac{49}{12}$$

Example 1.189

What is $1/4^{th}$ $\underbrace{\text{of}}_{\textit{Multiplication}}$ 24?

$$\frac{1}{4}^{\text{th}} \text{ of } 24 = \frac{1}{4} \times 24 = 6$$

Basics

Example 1.190

Sridhar has a rope that is $\frac{2}{3}$ meters long. He puts three such ropes end to end. What is the total length from one end to another?

Example 1.191

The height of a tree is $\frac{4}{5}$ meters. What is the total height of four such trees?

Example 1.192

A cake is cut into seven equal pieces, and three of the pieces are eaten. Then half of the remaining cake is eaten. What fraction of the original cake is left?

Example 1.193

A farmer divided his land into three equal parts for his two sons and a daughter. The first son had two children, and he divided his share into two equal parts, one for each child. The second son had three children, and he divided his share into three equal parts, one for each child.

A. What fraction of the land did each of the farmer's children get?
B. What fraction of the land did each of the farmer's first son's children get?
C. What fraction of the land did each of the farmer's second son's children get?

Part A

Each one got

$$\frac{1}{3}$$

Part B
Each one got

$$\frac{1}{3}\times\frac{1}{2}=\frac{1}{6}$$

Part C
Each one got

$$\frac{1}{3}\times\frac{1}{3}=\frac{1}{9}$$

Mixed Numbers

Example 1.194
A day has twenty-four hours. A year has approximately three hundred and sixty-five, and a half days. Using this approximation, find the number of hours in a year.

$$Year=365\frac{1}{2}\ Days, Day=24\ Hours$$

You can convert $365\frac{1}{2}$ into an improper fraction, but that will increase the calculations. So, it is better to multiply separately:

Hence, the number of hours in a year

$$=\underbrace{365\frac{1}{2}}_{No.of\ Days}\times\underbrace{24}_{Hours\ per\ Day}=365\times 24+\frac{1}{2}\times 24=8760+12=8772$$

1.9 Division

Reciprocals

Reciprocals relate multiplication and division. They are very useful in converting division into multiplication and vice versa.

Reciprocals of Fractions

To find the reciprocal of a fraction, we *interchange* the numerator and the denominator. This is valid for both proper and improper fractions.

$$Reciprocal\ of\ \frac{3}{4}=\frac{4}{3}$$
$$Reciprocal\ of\ \frac{7}{5}=\frac{5}{7}$$

Example 1.195
Find the reciprocals of the following fractions:

A. $\frac{2}{3}$
B. $\frac{7}{12}$

C. $\frac{7}{4}$

$$Reciprocal\ of\ \frac{2}{3} = \frac{3}{2}$$
$$Reciprocal\ of\ \frac{7}{12} = \frac{12}{7}$$
$$Reciprocal\ of\ \frac{7}{4} = \frac{4}{7}$$

Reciprocals of Whole Numbers

To find the reciprocal of a whole number, recall that any number can be divided by one without changing its value, and hence, any number always has one as a denominator.

$$Reciprocal\ of\ 4 = \underbrace{Reciprocal\ of\ \frac{4}{1}}_{\substack{Make\ a\ fraction\ by \\ using\ denominator\ one}} = \frac{1}{4}$$

Example 1.196

Find the reciprocals of the following fractions:

A. 6
B. 7
C. 3

$$Reciprocal\ of\ 6 =$$
$$Reciprocal\ of\ 7 =$$
$$Reciprocal\ of\ 3 =$$

1.197: Product of a Number and Its Reciprocal

Product of a number and its reciprocal is always one.

$$\frac{3}{4} \times \frac{4}{3} = 1$$

Example 1.198

Verify that when the following fractions are multiplied by their reciprocals, the answer is 1.

A. $\frac{3}{7}$
B.

$$\frac{3}{7} \times \frac{7}{3} = 1$$

1.199: Identifying Reciprocals

When a number multiplied by another number gives one, the two numbers are reciprocals of each other.

Example 1.200

Find the value of x in the equation below

$$\frac{4}{9} \times x = 1$$

$$x = \frac{9}{4}$$

1.201: Reciprocals of improper and proper fractions

Reciprocal of a

- Proper fraction will be an improper fraction
- Improper fraction, which is not 1, will be a proper fraction
- Reciprocal of 1 will be 1

Example 1.202

Arrange the reciprocals of the following fractions in ascending order.

$$\frac{2}{9}, \frac{5}{3}, \frac{7}{7}$$

$$\frac{3}{5} < 1, \qquad \frac{7}{7} = 1, \qquad \frac{9}{2} > 1$$

Example 1.203

Tanisha finds the reciprocal of a fraction which is larger than one. Then the reciprocal will be:

A. Equal to one
B. Smaller than one
C. Larger than one
D. Cannot be determined

Example 1.204

Bhairav finds the reciprocal of a fraction which has a value of one. Then the reciprocal will be:

A. Equal to one
B. Smaller than one
C. Larger than one
D. Cannot be determined

Example 1.205

Bhairavi finds the reciprocal of a fraction which has a value less than one. Then the reciprocal will be:

A. Equal to one
B. Smaller than one
C. Larger than one
D. Cannot be determined

1.206: Comparing Reciprocals

When you take the reciprocal of two numbers, the smaller number will have the larger reciprocal, and the larger number will have the smaller reciprocal.

$$3 < 4 \Rightarrow \frac{1}{3} > \frac{1}{4}$$

Example 1.207

Without comparing the reciprocals, determine the larger among the reciprocals of the numbers below:

A. $4 \text{ and } 7$
B. $3 \text{ and } 5$

C. $\frac{2}{3}$ and $\frac{1}{3}$

$$4 < 7 \Rightarrow \frac{1}{4} > \frac{1}{7}$$

1.208: Comparing Reciprocals-II
When taking reciprocals, the smallest number will have the largest reciprocal, and the largest number will have the smallest reciprocal.

Example 1.209
Which of the following numbers has the largest reciprocal? (AMC 8 1986/2)

(A) $\frac{1}{3}$ (B) $\frac{2}{5}$ (C) 1 (D) 5 (E) 1986

Calculate the Reciprocals

$$R\left(\frac{1}{3}\right) = 3$$
$$R\left(\frac{2}{5}\right) = \frac{5}{2}$$
$$R(1) = 1$$
$$R(5) = \frac{1}{5}$$
$$R(1986) = \frac{1}{1986}$$

Shortcut
The reciprocal will be the largest when the number itself is the smallest. Hence, we are looking for the smallest number in the options, which is

$$\frac{1}{3}$$

1.210: Reciprocal of Zero does not exist
Zero does not have a reciprocal.

This is because we try to find the reciprocal, we will get:

$$\frac{1}{0}$$

And in this case, we will be dividing by zero.
Remember, that we can **never divide by zero**.

Reciprocals to convert between multiplication and division
We can convert division into multiplication by taking the reciprocal of the number that we are dividing by.

Example 1.211
Simplify:

A. $\frac{2}{5} \div \frac{11}{7}$
B. $\frac{3}{4} \div \frac{2}{11}$

C. $\frac{5}{7} \div \frac{8}{13}$

Part A
We can convert the division into multiplication by taking the reciprocal of the second fraction:

$$\frac{2}{5} \div \frac{11}{7} = \frac{2}{5} \times \frac{7}{11} = \frac{14}{55}$$

Part B

$$\frac{3}{4} \div \frac{2}{11} = \frac{3}{4} \times \frac{11}{2} = \frac{33}{8}$$

Part C

$$\frac{5}{7} \div \frac{8}{13} = \frac{5}{7} \times \frac{13}{8} = \frac{65}{56}$$

Reciprocals Trap: Do not reciprocate the first fraction
To convert division into multiplication, we need to find the reciprocal of the fraction that we are *dividing by*. We cannot take the reciprocal of the fraction that is *being divided*.

Example 1.212

Bogus Solution

$$\frac{3}{4} \div \frac{2}{5} = \underbrace{\frac{4}{3}}_{\substack{\textit{Take the}\\ \textit{Reciprocal}}} \times \frac{2}{5} = \frac{4 \times 2}{3 \times 5} = \frac{8}{15}$$

Valid Solution

$$\frac{3}{4} \div \frac{2}{5} = \frac{3}{4} \times \underbrace{\frac{5}{2}}_{\substack{\textit{Take the}\\ \textit{reciprocal}}} = \frac{3 \times 5}{4 \times 2} = \frac{15}{8}$$

Dividing fractions by a whole number
Dividing by a whole number has the same process. We convert the whole number into a fraction by using one as the denominator, and taking the reciprocal of that fraction.

Example 1.213
Simplify

A. $\frac{3}{4} \div 5$
B. $\frac{2}{5} \div 7$
C. $\frac{4}{9} \div 3$

Part A

$$\frac{3}{4} \div 5 = \frac{3}{4} \times \frac{1}{5} = \frac{3}{20}$$

Part B

$$\frac{2}{5} \div 7 = \frac{2}{5} \times \frac{1}{7} = \frac{2}{35}$$

Part C

$$\frac{4}{9} \div 3 = \frac{4}{27}$$

Dividing fractions by a Unit Fraction

When dividing by a unit fraction, the reciprocal will give us a whole number. We carry out the multiplication as usual.

Example 1.214

Simplify

A. $\frac{2}{5} \div \frac{1}{3}$
B. $\frac{3}{7} \div \frac{1}{4}$
C. $\frac{2}{11} \div \frac{1}{6}$

$$\frac{2}{5} \div \frac{1}{3} = \frac{2}{5} \times \frac{3}{1} = \frac{6}{5}$$
$$\frac{3}{7} \div \frac{1}{4} = \frac{12}{7}$$
$$\frac{2}{11} \div \frac{1}{6} = \frac{12}{11}$$

Example 1.215

$$8 \times 7 \times 15 \div 7 \div 3 \div 4$$

Convert all the division operations into multiplication operations by taking the reciprocal. This avoids any issue with respect to order of operations:

$$8 \times 7 \times 15 \times \frac{1}{7} \times \frac{1}{3} \times \frac{1}{4}$$
$$= 8 \times 7 \times 15 \times \frac{1}{7} \times \frac{1}{3} \times \frac{1}{4}$$
$$= 8 \times 15 \times \frac{1}{3} \times \frac{1}{4}$$
$$= 2 \times 15 \times \frac{1}{3}$$
$$= 2 \times 5$$
$$= 10$$

Mixed Numbers

Example 1.216

Simplify:

A. $1\frac{1}{2} \div 1\frac{1}{3}$
B. $1\frac{2}{5} \times 1\frac{3}{4}$
C. $2\frac{1}{3} \times 3\frac{2}{7}$

Part A

$$1\frac{1}{2} \div 1\frac{1}{3} = \frac{3}{2} \div \frac{4}{3} = \frac{3}{2} \times \frac{3}{4} = \frac{9}{8}$$

Part B

$$1\frac{2}{5} \div 1\frac{3}{4} = \frac{7}{5} \div \frac{7}{4} = \frac{7}{5} \times \frac{4}{7} = \frac{4}{5}$$

Part C

$$2\frac{1}{3} \div 3\frac{2}{7} = \frac{7}{3} \div \frac{23}{7} = \frac{7}{3} \times \frac{7}{23} = \frac{49}{69}$$

Fractions with fractions in the numerator and denominator

When we have fractions in both the numerator and denominator, we convert fractions into division, and then carry out the

Example 1.217

$$\frac{\frac{2}{3}}{\frac{5}{6}} = \frac{2}{3} \div \frac{5}{6} = \frac{2}{3} \times \frac{6}{5} = \frac{2}{1} \times \frac{2}{5} = \frac{4}{5}$$

Example 1.218

Shane had four-fifths of a pie. He shared it among his three brothers. How much did each one get?

$$\frac{4}{5} \div 3 = \frac{4}{5} \times \frac{1}{3} = \frac{4}{15}$$

Example 1.219

A road repair crew can repair 8 km of road in one week. How long will it take to repair:

A. 8 km
B. 16 km
C. 64 km
D. 4 km
E. 6 km
F. 22 km

$$\frac{No.of\ Km}{Kms\ per\ week}$$

$$\frac{8\ km}{8\ km} = 1\ Week$$

$$\frac{16\ km}{8\ km} = 2\ Weeks$$

$$\frac{64\ km}{8\ km} = 8\ Weeks$$

$$\frac{64\ km}{8\ km} = 8\ Weeks$$

$$\frac{4\ km}{8\ km} = \frac{1}{2}\ Week$$

$$\frac{6\ km}{8\ km} = \frac{3}{4}\ Week$$
$$\frac{22\ km}{8\ km} = \frac{11}{4}\ Week$$

Example 1.220

In an eating competition, you can $\frac{3}{4}^{th}$ of an apple in a minute. How many minutes will it take you to eat three apples?

$$\frac{No.of\ Apples}{Apples\ per\ Minute} = \frac{3}{\frac{3}{4}} = 3 \div \frac{3}{4} = 3 \times \frac{4}{3} = 4$$

Check

$$Apples\ eaten\ in\ 4\ minutes = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = 4 \times \frac{3}{4} = 3\ Apples$$

Example 1.221

A carpenter can make $\frac{2}{3}$rd of a chair in an hour. How long will it take him to make ten chairs?

$$Time = \frac{No.of\ Chairs}{Chairs\ per\ Hour} = \frac{10}{\frac{2}{3}} = 10 \div \frac{2}{3} = 10 \times \frac{3}{2} = 15\ hours$$

Example 1.222

Shakeel has the choicest, most delicious, gourmet apples that you can find across the length and breadth of the USA. He showcases his apples in tasting parties, and only invites connoisseurs to

A. If each guest is going eat three apples and he has 21 apples, how many guests can he call?
B. If each guest is going to eat two apples and he is going to invite eight guests, how many apples will he need?
C. If he has twelve apples, and he is going to invite seven guests, how many apples will each guest get?

(Give all answers in fractions.)

$$\frac{21}{3} = 7\ Guests$$
$$2 \times 8 = 16$$
$$12 \div 7 = \frac{12}{7}\ apples$$

Example 1.223

The Four Wheels Car manufacturing company makes cars at its factory. It has a working day of eight hours.

A. If $1\frac{1}{3}$ hours goes in lun/ch and other breaks, what is the actual working time?
B. If each worker can make $3\frac{2}{3}$ cars per hour of actual working time, how many cars can a worker make in a working day?
C.

(Give all answers in fractions.)

$$8 - 1\frac{1}{3} = 6\frac{2}{3}$$

Example 1.224

A hotel estimates that each guest in a buffet eats an average of $1\frac{3}{5}$ of a pizza. If he has:

A. seven guests in today's buffet, how many pizzas will he need?
B. has enough dough for $7\frac{3}{4}$ pizzas, how many guests can he serve?

(Calculate your answer as a mixed number, and then convert it to a whole number, as appropriate.)

$$1\frac{3}{5} \times 7 = \frac{8}{5} \times 7 = \frac{56}{5} = 11\frac{1}{5} \Rightarrow 12\ Pizzas$$

$$\frac{No. of\ Pizzas}{Pizzas\ per\ Guest} = \frac{7\frac{3}{4}}{1\frac{3}{5}} = 7\frac{3}{4} \div 1\frac{3}{5} = \frac{31}{4} \div \frac{8}{5} = \frac{31}{4} \times \frac{5}{8} = \frac{155}{32} = 4\frac{27}{32} \Rightarrow 4\ Guests$$

Example 1.225

A hotel estimates that each guest in a buffet eats an average of $1\frac{3}{5}$ of a pizza. If he has:

A. seven guests in today's buffet, how many pizzas will he need?
B. has enough dough for $7\frac{3}{4}$ pizzas, how many guests can he serve?

(Calculate your answer as a mixed number, and then convert it to a whole number, as appropriate.)

Example 1.226

One batch of oatmeals cookies needs 1/8 of a packet of vanilla powder. Shane has 1/2 of a packet of vanilla powder. How many batches can he make?

Method I
Make the denominators the same:
1/2 of a packet
= 4/8 of a packet

1 Batch takes 1/8 of a packet
2 Batches takes 2/8 of a packet
3 Batches takes 3/8 of a packet
4 Batches takes 4/8 of a packet

In all, Shane can make 4 batches

Method II
No. of Batches
= Total Quantity / (Quantity per Batch)
= 1/2 divided 1/8
= 1/2 * 8
= 4 Batches

2. Further Topics

2.1 Order of Operations

A. Same Denominator

Example 2.1

$$\frac{1}{2}+\frac{1}{2}\times 2$$

$$\frac{1}{2}+\underbrace{\frac{1}{2}\times 2}_{Multiplication}=\frac{1}{2}+\frac{2}{2}=\frac{3}{2}=1\frac{1}{2}$$

Example 2.2

$$\frac{1}{3}\times 3-\frac{1}{3}\times 2$$

$$\underbrace{\frac{1}{3}\times 3}_{Multiplication}-\underbrace{\frac{1}{3}\times 2}_{Multiplication}=\frac{3}{3}-\frac{2}{3}=\frac{1}{3}$$

Example 2.3

$$\frac{2}{5}\div 2+\frac{3}{5}\times\frac{1}{2}$$

$$\underbrace{\frac{2}{5}\div 2}_{Division}+\underbrace{\frac{3}{5}\times\frac{1}{2}}_{Multiplication}=\underbrace{\frac{2}{5}\times\frac{1}{2}}_{Multiplication}+\frac{3}{10}=\frac{2}{10}+\frac{3}{10}=\frac{5}{10}=\frac{1}{2}$$

B. Different Denominators

Example 2.4

$$\frac{3}{4}-\frac{1}{3}\times 2$$

$$\frac{3}{4}-\underbrace{\frac{1}{3}\times 2}_{Muliplication}=\frac{3}{4}-\frac{2}{3}=\frac{9}{12}-\frac{8}{12}=\frac{1}{12}$$

Example 2.5

$$\frac{1}{2}+\frac{1}{2}\times\frac{1}{3}-\frac{1}{4}\times 2-\frac{1}{3}\div 2$$

$$\frac{1}{2}+\underbrace{\frac{1}{2}\times\frac{1}{3}}_{Multiplication}-\underbrace{\frac{1}{4}\times 2}_{Multiplication}-\underbrace{\frac{1}{3}\div 2}_{Division}=\frac{1}{2}+\frac{1}{6}-\frac{1}{2}-\frac{1}{6}=0$$

2.6: Of

Of means multiplication

Example 2.7

$$\frac{3}{4}\ of\ \frac{2}{3}+\frac{1}{3}$$

$$\underbrace{\frac{3}{4}\times\frac{2}{3}}_{Multiplication}+\frac{1}{3}=\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$$

Example 2.8

$$\frac{1}{2}of\frac{2}{5}\div\frac{1}{2}$$

$$\frac{1}{2}\times\frac{2}{5}\div\frac{1}{2}=\frac{1}{5}\times 2=\frac{2}{5}$$

Example 2.9

$$\frac{2}{3}-\frac{1}{3}\ of\ \frac{1}{4}+\frac{1}{2}$$

$$\frac{2}{3}-\underbrace{\frac{1}{3}\times\frac{1}{4}}_{Multiplication}+\frac{1}{2}=\frac{2}{3}-\frac{1}{12}+\frac{1}{2}=\frac{8}{12}-\frac{1}{12}+\frac{6}{12}=\frac{13}{12}=1\frac{1}{12}$$

C. Brackets

With brackets, we need to simplify the innermost bracket first.

Example 2.10

$$\left(\frac{1}{2}+\frac{1}{3}\right)\times\frac{2}{5}$$

$$\left(\frac{5}{6}\right)\times\frac{2}{5}=\frac{2}{6}=\frac{1}{3}$$

Example 2.11

$$\frac{3}{2}-\left(\frac{2}{3}\ of\ \frac{3}{2}\right)\times\frac{1}{2}$$

$$\frac{3}{2}-\left(\frac{2}{3}\times\frac{3}{2}\right)\times\frac{1}{2}=\frac{3}{2}-\left(\frac{2}{3}\times\frac{3}{2}\right)\times\frac{1}{2}=\frac{3}{2}-\frac{1}{2}=\frac{2}{2}=1$$

Example 2.12

$$\frac{7}{5-\frac{8}{3}}\div\frac{3-\frac{2}{3-\frac{3}{2}}}{4-\frac{3}{2}}-\frac{5}{7}\ \text{of}\ \left\{\frac{1}{1\frac{3}{7}}+\frac{6}{5}\ \text{of}\ \frac{3\frac{1}{3}-2\frac{1}{2}}{\frac{47}{21}-2}\right\}(NMTC\ Primary-Final, Primary-III)$$

$$\frac{7}{\frac{15}{3}-\frac{8}{3}} \div \frac{3-\frac{2}{\frac{6}{2}-\frac{3}{2}}}{\frac{8}{2}-\frac{3}{2}} = \frac{7}{\frac{7}{3}} \div \frac{3-\frac{2}{\frac{3}{2}}}{\frac{5}{2}} = 3 \div \frac{\frac{9}{3}-\frac{4}{3}}{\frac{5}{2}} = 3 \div \frac{\frac{5}{3}}{\frac{5}{2}} = 3 \div \frac{2}{3} = 3 \times \frac{3}{2} = \frac{9}{2}$$

$$\frac{1}{\frac{10}{7}} + \frac{6}{5} \times \frac{\frac{10}{3}-\frac{5}{2}}{\frac{47}{21}-\frac{42}{21}} = \frac{7}{10} + \frac{6}{5} \times \frac{\frac{20}{6}-\frac{15}{6}}{\frac{5}{21}} = \frac{7}{10} + \frac{6}{5} \times \frac{\frac{5}{6}}{\frac{5}{21}} = \frac{7}{10} + \frac{6}{5} \times \frac{21}{6} = \frac{7}{10} + \frac{21}{5} = \frac{7}{10} + \frac{42}{10} = \frac{49}{10}$$

$$\frac{5}{7} \text{ of } \left\{ \frac{1}{1\frac{3}{7}} + \frac{6}{5} \text{ of } \frac{3\frac{1}{3}-2\frac{1}{2}}{\frac{47}{21}-2} \right\} = \frac{5}{7} \times \frac{49}{10} = \frac{7}{2}$$

$$\frac{9}{2} - \frac{7}{2} = \frac{2}{2} = 1$$

D. Nested Brackets

Example 2.13

$$\left[\left(\frac{2}{3} of\ 7\right) - \frac{1}{3}\right] + \frac{2}{7}$$

$$\left[\left(\frac{2}{3} \times 7\right) - \frac{1}{3}\right] + \frac{2}{7} = \left[\frac{14}{3} - \frac{1}{3}\right] + \frac{2}{7} = \frac{13}{3} + \frac{2}{7} = \frac{91}{21} + \frac{6}{21} = \frac{97}{21}$$

Example 2.14

$$\left\{\left[\left(\frac{1}{2} + \frac{2}{3}\right) \times \frac{1}{2}\right] - \frac{1}{3}\right\} \times 2$$

$$\left\{\left[\left(\frac{3}{6} + \frac{4}{6}\right) \times \frac{1}{2}\right] - \frac{1}{3}\right\} \times 2 = \left\{\left[\frac{7}{6} \times \frac{1}{2}\right] - \frac{1}{3}\right\} \times 2 = \left\{\frac{7}{12} - \frac{4}{12}\right\} \times 2 = \left\{\frac{3}{12}\right\} \times 2 = \frac{3}{6} = \frac{1}{2}$$

E. Division

2.15: Division as Fractions

$$a \div b = \frac{a}{b}$$

Division can always be converted into a fraction, and *vice versa*.
This property is very useful.

Example 2.16

A. $1 \div 2$
B. $\frac{2}{3} \div \frac{4}{5}$

$$1 \div 2 = 1 \times \frac{1}{2} = \frac{1}{2}$$
$$\frac{2}{3} \div \frac{4}{5} = \frac{2}{3} \times \frac{5}{4} = \frac{5}{6}$$

Example 2.17

$$5 \div 25 + 9 \div 27 + 4 \div 16 + 216 \div 6$$

Convert the division into fractions:

$$\frac{5}{25} + \frac{9}{27} + \frac{4}{16} + \frac{216}{6}$$

Simplify:

$$\frac{1}{5} + \frac{1}{3} + \frac{1}{4} + 36$$

Use $LCM\ (5,3,4) = 60$, and add:

$$\frac{12}{60} + \frac{20}{60} + \frac{15}{60} + 36 = \frac{47}{60} + 36$$

Rewrite as a mixed number:

$$36\frac{47}{60}$$

Example 2.18

$$3 \times 27 \times 9 \div 81 \times 2 + 1$$

$$3 \times 27 \times 9 \times \frac{1}{81} \times 2 + 1 = 18 + 1 = 19$$

Example 2.19

$$5 \times 25 \times 125 \div 625 + 2$$

Convert the division into multiplication by taking the reciprocal of 625:

$$5 \times 25 \times 125 \times \frac{1}{625} + 2$$

Combine the fractions:

$$\frac{5 \times 25 \times 125}{625} + 2$$

Multiply 5 and 125 to get 625:

$$625 \times 25 \times \frac{1}{625} + 2$$

Divide the 625 by 625 to get:

$$1 \times 25 \times \frac{1}{1} + 2$$

Multiplying anything by 1 gives the same answer:

$$25 + 2 = 27$$

Example 2.20

$$3 \times 27 \div 9 + 4 \times 64 \div 16 + 6 \times 216 \div 36$$

Convert the division into multiplication by taking the reciprocal:

$$\frac{3 \times 27}{9} + \frac{4 \times 64}{16} + \frac{6 \times 216}{36} = 3 \times 3 + 4 \times 4 + 6 \times 6 = 9 + 16 + 36 = 61$$

Example 2.21

$$\left(1\frac{5}{10} + 2\frac{5}{8}\right) \div \left(2\frac{9}{10} - \frac{3}{8}\right)$$

$$\left(3\frac{45}{45}\right) \div \left(2\frac{21}{40}\right) = \frac{165}{40} \times \frac{40}{101} = \frac{165}{101} = 1\frac{64}{101}$$

2.2 Shares

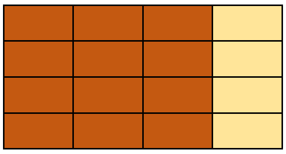
A. Word Application

Example 2.22

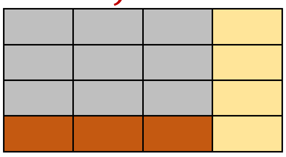
Stephen has a cake. He eats $\frac{1}{4}^{th}$ of it. Then he eats $\frac{3}{4}^{th}$ of what is left. How much cake is remaining?

Visual Method

Step I: Eat $1/4^{th}$. $3/4^{th}$ is left.



Step II: Eat $3/4^{th}$ of what is left. **(Not $3/4^{th}$ of the cake.)**



Algebraic Method

How much is left:

$$\frac{3}{4}th$$

He eats $3/4^{th}$ of what is left ($3/4^{th}$):

$$\frac{3}{4}th\ of\ \frac{3}{4}th = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

Cake left

$$\frac{3}{4} - \frac{9}{16} = \frac{12}{16} - \frac{9}{16} = \frac{3}{16}$$

Example 2.23

A boy has 24 apples on Monday. He eats half of them on Tuesday. On Wednesday, he eats half of the number of apples remaining on Tuesday. How many apples does he have on Thursday?

Example 2.24

Nishant left for South Africa for seventy days. He went to A for one-fifth of seventy days, and went to B for two seventh of his trip, and he went to C for four-ninth of the remaining days, and the rest of the days at D. Find the number of days spent at D.

$$Time\ at\ A{:}\ \frac{1}{5}th\ of\ 70 = \frac{1}{5} \times 70 = 14 \Rightarrow 70 - 14 = 56$$

$$Time\ at\ B{:}\ \frac{2}{7}th\ of\ 70 = \frac{2}{7} \times 70 = 20 \Rightarrow 56 - 20 = 36$$

$$Time\ at\ C: \frac{4}{9}th\ of\ 36 = \frac{4}{9} \times 36 = 16$$
$$Time\ at\ D: 36 - 16 = 20$$

Example 2.25

Shreya takes 64 biscuits while going camping. On the first day, she eats $\frac{3}{4}^{th}$ of the biscuits she has. On the second day, she eats $\frac{3}{4}^{th}$ of the remaining biscuits. How many biscuits did she have at the beginning of the third day?

In the beginning, she starts with one whole.

$$\text{At the end of the 1st day} = \frac{3}{4}th \text{ of the original}$$

At the end of the second day, she eats $\frac{3}{4}th$ of what is left, which is

$$\frac{3}{4}th\ of\ \frac{3}{4} = \frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$$

At the end of the second day what is left is:

$$\frac{3}{4} - \frac{9}{16} = \frac{12}{16} - \frac{9}{16} = \frac{3}{16}$$

This is the same as the number of biscuits she will have at the beginning of the third day.

Biscuits on third day

$$= 64 \times \frac{3}{16} = 12$$

Comparing Absolute and Proportions

Example 2.26

A. Nishant earns Rs. 12,000 as salary every month. He spends Rs. 2000 on travelling. From the rest, he saves one quarter. Find the money that Nishant saves every month.
B. Rishi earns Rs. 12,000 as salary every month. He spends Rs. 2000 on travelling. He saves one quarter of his salary. Find the money that Rishi saves every month.

Part A

Money remaining after travelling

$$= 12000 - 2000 = 10{,}000$$

Money saved

$$= 10{,}000 \times \frac{1}{4} = 2500$$

Part B

Rishi saves one quarter of his salary, which is

$$= 12{,}000 \times \frac{1}{4} = 3000$$

Going to the Market

Example 2.27

A. Shilpi went to the market with Rs. 100. She bought oranges with half of the money she had, and apples with half of the money left after purchasing the oranges. She spent Rs. 10 on a bus ticket to come back home. What is the money that she got back?

B. Siddesh went to the market with Rs. 100. He bought oranges with two fifths of the money she originally had, and apples with one fifth of the money she originally had. He spent Rs. 15 on a bus ticket to come back home. What is the money that he got back?

Part A

$$Oranges = 100 \times \frac{1}{2} = 50$$

Apples are bought using half of the money which is left $(50 - 50)$:

$$Apples = 50 \times \frac{1}{2} = 25$$

$$100 - 50 - 25 - 10 = 15$$

Part B

$$Oranges = 100 \times \frac{2}{5} = 40$$

Apples are bought using one fifth of the original amount:

$$Apples = 100 \times \frac{1}{5} = 20$$

$$100 - 40 - 20 - 15 = 25$$

Back Calculations

Example 2.28

Karuna wanted to read her aunt's entire collection of Science Fiction novels. In January she read half of the novels in the collection. In February, she read half of the remaining novels in the collection. In March, she read the remaining seven novels. How many novels does her aunt have in her collection?

Let the original number of novels in the collection be one whole.

In January, she reads $\frac{1}{2}$ of the novels, leaving her with

$$1 - \frac{1}{2} = \frac{1}{2}$$

In February, she reads $\frac{1}{2}$ of the remaining novels, which is

$$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}^{th}$$

Leaving her with

$$\frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

And this $\frac{1}{4}$, we are told is equal to seven books:

$$One\ fourth\ of\ the\ books = 7 \Rightarrow Total\ No.of\ Books = 28$$

Riddle-Type Back Calculations

Example 2.29

I have some cookies with me. On Monday, I ate half the cookies I had, and I gave one cookie to my pet hamster. On Tuesday, I ate half the cookies that were left, and also gave one cookie to my hamster. My cookies got over on Tuesday. What is the number of cookies I had originally?

We do this by working backwards.

I ended Tuesday with zero cookies. On Tuesday, the cookies eaten were

$$\underbrace{1}_{Hamster} + \underbrace{\frac{1}{2} of\ the\ Cookies}_{Me} \Rightarrow 2\ Cookies\ in\ all$$

2.3 Complex Expressions

Addition

Example 2.30

$$\frac{1}{7}+\frac{1}{8}+\frac{1}{9}+\frac{1}{10}+\frac{1}{11}+\frac{1}{12}+\frac{1}{14}+\frac{1}{15}+\frac{1}{18}+\frac{1}{22}+\frac{1}{24}+\frac{1}{28}+\frac{1}{33}\ (IPM)$$

This looks very intimdating. We certainly don't want to find

$$LCM\ (7,8,9,10,11,12,14,15,18,22,24,28,33) = Some\ very\ large\ number$$

So, look for a shortcut. What can we add with *minimum trouble*? Use the property that

$$LCM(a, xa) = xa$$

And select the fractions carefully.

Look at the first fraction. It has denominator 7. What are the multiples of 7, that are also denominators? Take all these fractions together:

$$\frac{1}{7}+\frac{1}{14}+\frac{1}{28}=\frac{4+2+1}{28}=\frac{7}{28}=\frac{1}{4}$$

We got a nice simple fraction above. Keep using the strategy. Next fraction has denominator 8:

$$\frac{1}{8}+\frac{1}{24}=\frac{3+1}{24}=\frac{4}{24}=\frac{1}{6}$$

And continue with the rest:

$$\frac{1}{9}+\frac{1}{18}=\frac{2+1}{18}=\frac{3}{18}=\frac{1}{6}$$
$$\frac{1}{11}+\frac{1}{22}+\frac{1}{33}=\frac{6+3+2}{66}=\frac{11}{66}=\frac{1}{6}$$
$$\frac{1}{10}+\frac{1}{12}+\frac{1}{15}=\frac{6+5+4}{60}=\frac{15}{60}=\frac{1}{4}$$

Add all the above fractions:

$$\frac{1}{4}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{4}=\frac{2}{4}+\frac{3}{6}=\frac{1}{2}+\frac{1}{2}=1$$

A. Nested Fractions

2.31: Nested Fractions

In a nested fraction, the numerator, or the denominator, or both, of the fraction are themselves fractions.
To simplify a nested fraction, we represent fractions as division. That is:

$$\frac{N}{D} = N \div D = N \times \frac{1}{D}$$

Example 2.32

Simplify the following

$$A=\frac{\frac{2}{3}}{\frac{3}{5}},\quad B=\frac{\frac{3}{4}}{\frac{2}{5}},\quad C=\frac{\frac{12}{27}}{\frac{16}{36}},\quad D=\frac{\frac{4}{5}}{\frac{8}{7}},\quad E=\frac{\frac{2}{9}}{\frac{4}{11}},\quad F=\frac{\frac{1}{7}}{\frac{3}{14}}$$

Part A

Convert the fraction into division:

$$\underbrace{\frac{2}{3}}_{Numerator}\div\underbrace{\frac{3}{5}}_{Denominator}=\frac{2}{3}\times\frac{5}{3}=\frac{10}{9}=1\frac{1}{9}$$

Part B

$$\frac{3}{4}\div\frac{2}{5}=\frac{3}{4}\times\frac{5}{2}=\frac{15}{8}$$

Part C

$$\frac{\frac{12}{27}}{\frac{16}{36}}=\frac{\frac{4}{9}}{\frac{4}{9}}=1$$

Part D

$$\frac{4}{5}\times\frac{7}{8}=\frac{7}{10}$$

Part E

$$\frac{2}{9}\times\frac{11}{4}=\frac{11}{18}$$

Part F

$$\frac{1}{7}\times\frac{14}{3}=\frac{2}{3}$$

Simplifying Complicated Fractions

Example 2.33

$$A=\frac{\frac{1}{2}+\frac{1}{3}}{\frac{2}{3}+\frac{3}{4}},\qquad B=\frac{\frac{1}{2}-\frac{2}{5}}{\frac{3}{4}-\frac{1}{6}}$$

$$A=\frac{\frac{1}{2}+\frac{1}{3}}{\frac{2}{3}+\frac{3}{4}}=\frac{\frac{5}{6}}{\frac{17}{12}}=\frac{5}{6}\div\frac{17}{12}=\frac{5}{6}\times\frac{12}{17}=\frac{10}{17}$$

$$B=\frac{\frac{1}{2}-\frac{2}{5}}{\frac{3}{4}-\frac{1}{6}}=\frac{\frac{5}{10}-\frac{4}{10}}{\frac{9}{12}-\frac{2}{12}}=\frac{\frac{1}{10}}{\frac{7}{12}}=\frac{1}{10}\times\frac{12}{7}=\frac{6}{35}$$

Nested Fraction Expressions

Example 2.34

$$\frac{2}{1-\frac{2}{3}}=$$ **(AMC 8 1986/6)**

$$\frac{2}{1-\frac{2}{3}}=\frac{2}{\frac{1}{3}}=2\div\frac{1}{3}=2\times 3=6$$

Example 2.35

$$\frac{1}{3}+\frac{\frac{1}{4}}{\frac{1}{5}+\frac{1}{2}}$$

$$\frac{1}{3}+\frac{\frac{1}{4}}{\frac{1}{5}+\frac{1}{2}}=\frac{1}{3}+\frac{\frac{1}{4}}{\frac{7}{10}}=\frac{1}{3}+\frac{1}{4}\div\frac{7}{10}=\frac{1}{3}+\frac{1}{4}\times\frac{10}{7}=\frac{1}{3}+\frac{10}{28}=\frac{28}{84}+\frac{30}{84}=\frac{58}{84}=\frac{29}{42}$$

B. Decimals

Example 2.36

$$\frac{0.1}{0.01}+\frac{0.01}{0.1}$$

Place Value Concept

If you move a digit to the left, it becomes 10 times the value that it had.

$$2\underbrace{3}_{=3}\to\underbrace{231}_{=30}$$

If you move a digit to the right, it becomes $\frac{1}{10}$ times the value that it had.

$$\underbrace{231}_{=30}\to\underbrace{3}_{=3}$$

$$0.1\ is\ 10\ times\ of\ 0.01\Rightarrow\frac{0.1}{0.01}=10$$
$$0.1\ is\ 10\ times\ of\ 0.01\Rightarrow\frac{0.1}{0.01}=10$$

Remove Decimals

$$\frac{0.1}{0.01}+\frac{0.01}{0.1}=\frac{0.1}{0.01}\times\frac{100}{100}+\frac{0.01}{0.1}\times\frac{100}{100}=\frac{10}{1}+\frac{1}{10}=10+0.1=10.1$$

Convert into Fractions

Write out the decimals as fractions:

$$\frac{\frac{1}{10}}{\frac{1}{100}}+\frac{\frac{1}{100}}{\frac{1}{10}}$$

Use the fact that fractions can be written as division:

$$\frac{1}{10}\div\frac{1}{100}+\frac{1}{100}\div\frac{1}{10}$$

Use the fact that dividing by a number is the same as multiplying by the reciprocal:

$$\frac{1}{10}\times 100+\frac{1}{100}\times 10=\frac{100}{10}+\frac{10}{100}=10+\frac{1}{10}=10+0.1=10.1$$

C. Continued Fractions

Example 2.37

$$1+\cfrac{1}{1+\cfrac{1}{3}}$$

$$1+\cfrac{1}{1+\cfrac{1}{3}}=1+\cfrac{1}{\cfrac{4}{3}}=1+1\times\frac{3}{4}=1+\frac{3}{4}=\frac{7}{4}$$

Example 2.38

$$1+\cfrac{1}{1-\cfrac{1}{3}}$$

$$1+\cfrac{1}{1-\cfrac{1}{3}}=1+\cfrac{1}{\cfrac{2}{3}}=1+1\times\frac{3}{2}=1+\frac{3}{2}=\frac{5}{2}$$

Example 2.39

Evaluate

$$\pi\approx 3+\cfrac{1}{7+\cfrac{1}{16}}$$

Add the fraction in the denominator by making their denominators same:

$$\pi\approx 3+\cfrac{1}{7+\cfrac{1}{16}}=3+\cfrac{1}{\cfrac{7}{1}+\cfrac{1}{16}}=3+\cfrac{1}{\cfrac{112}{16}+\cfrac{1}{16}}=3+\cfrac{1}{\cfrac{113}{16}}$$

We rewrite the fraction as a division, and then convert the division into multiplication by taking the reciprocal of the fraction on the right:

$$3+\left(1\div\frac{113}{16}\right)=3+\left(1\times\frac{16}{113}\right)=3+\frac{16}{113}=\frac{3}{1}+\frac{16}{113}=\frac{339}{113}+\frac{16}{113}=\frac{355}{113}$$

Example 2.40

$$1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3}}}$$

$$1+\cfrac{1}{2+\cfrac{1}{1+\cfrac{1}{3}}}=1+\cfrac{1}{2+\cfrac{1}{\cfrac{4}{3}}}=1+\cfrac{1}{2+\cfrac{3}{4}}=1+\cfrac{1}{\cfrac{11}{4}}=1+\frac{4}{11}=\frac{15}{11}$$

Example 2.41

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}}$$

$$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\frac{1}{2}}}}=1+\cfrac{1}{1+\cfrac{1}{1+\frac{2}{3}}}=1+\cfrac{1}{1+\frac{3}{5}}=1+\frac{5}{8}=\frac{13}{8}$$

Example 2.42

$$2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\frac{1}{3}}}}$$

$$2+\cfrac{1}{2+\cfrac{1}{2+\cfrac{1}{2+\frac{1}{3}}}}=2+\cfrac{1}{2+\cfrac{1}{2+\frac{3}{7}}}=2+\cfrac{1}{2+\frac{7}{17}}=2+\frac{17}{41}=\frac{99}{41}$$

2.4 Algebra

A. Fractions

Example 2.43

If $x=\frac{1}{3}$ and $y=\frac{1}{2}$, then find the value of:

A. $2x+3y$
B. $(2x)(3y)$
C. $xy+x-y$
D. $\frac{x}{y}+\frac{y}{x}$
E. $1+\{3-[2(x+y)]\}$
F. y^2-x^2

Parts A, B, C

$$2x+3y=2\left(\frac{1}{3}\right)+3\left(\frac{1}{2}\right)=\frac{2}{3}+\frac{3}{2}=\frac{4}{6}+\frac{9}{6}=\frac{13}{6}=2\frac{1}{6}$$

$$(2x)(3y)=(2)\left(\frac{1}{3}\right)(3)\left(\frac{1}{2}\right)=1$$

$$xy+x-y=\left(\frac{1}{3}\right)\left(\frac{1}{2}\right)+\frac{1}{3}-\frac{1}{2}=\frac{1}{6}+\frac{2}{6}-\frac{3}{6}=0$$

Parts D

Write the fractions as division:

$$x\div y+y\div x$$

Substitute $x=\frac{1}{3}$ and $y=\frac{1}{2}$:

$$\frac{1}{3} \div \frac{1}{2} + \frac{1}{2} \div \frac{1}{3}$$

Convert the division into multiplication by taking the reciprocal:

$$\frac{1}{3} \times 2 + \frac{1}{2} \times 3 = \frac{2}{3} + \frac{3}{2} = \frac{4}{6} + \frac{9}{6} = \frac{13}{6} = 2\frac{1}{6}$$

Part E

Substitute $x = \frac{1}{3}$ and $y = \frac{1}{2}$:

$$1 + \left\{3 - \left[2\left(\frac{1}{3} + \frac{1}{2}\right)\right]\right\} = 1 + \left\{3 - \left[2\left(\frac{5}{6}\right)\right]\right\} = 1 + \left\{3 - \left[\frac{5}{3}\right]\right\} = 1 + \left\{\frac{10}{3}\right\} = 1 + 3\frac{1}{3} = 4\frac{1}{3}$$

Part F

Substitute $x = \frac{1}{3}$ and $y = \frac{1}{2}$:

$$y^2 - x^2 = \left(\frac{1}{2}\right)^2 - \left(\frac{1}{3}\right)^2 = \frac{1}{2} \times \frac{1}{2} - \frac{1}{3} \times \frac{1}{3} = \frac{1}{4} - \frac{1}{9} = \frac{9}{36} - \frac{4}{36} = \frac{5}{36}$$

B. Mixed Numbers

Example 2.44

If $x = 1\frac{2}{3}$ and $y = 2\frac{1}{4}$, then find the value of:

A. $2x + 3y$
B. $(2x)(3y)$
C. $xy + x - y$
D. $\frac{x}{y} + \frac{y}{x}$

First, convert mixed numbers into improper fractions:

$$1\frac{2}{3} = \frac{5}{3}, 2\frac{1}{4} = \frac{9}{4}$$

Parts A, B, C

$$2x + 3y = 2\left(\frac{5}{3}\right) + 3\left(\frac{9}{4}\right) = \frac{10}{3} + \frac{27}{4} = 3\frac{1}{3} + 6\frac{3}{4} = 9 + \frac{4}{12} + \frac{9}{12} = 9\frac{13}{12} = 10\frac{1}{12}$$

$$(2x)(3y) = (2)\left(1\frac{1}{3}\right)(3)\left(2\frac{1}{4}\right) = (2)\left(\frac{5}{3}\right)(3)\left(\frac{9}{4}\right) = \frac{45}{2} = 22\frac{1}{2}$$

$$xy + x - y = \left(1\frac{1}{3}\right)\left(2\frac{1}{4}\right) + 1\frac{1}{3} - 2\frac{1}{4} = \left(\frac{5}{3}\right)\left(\frac{9}{4}\right) + \frac{5}{3} - \frac{9}{4} = \frac{15}{4} + \frac{5}{3} - \frac{9}{4} = \frac{6}{4} + \frac{5}{3} = \frac{18}{12} + \frac{20}{12} = \frac{38}{12} = 3\frac{1}{6}$$

Parts D

Write the fractions as division:

$$x \div y + y \div x$$

Substitute $x = 1\frac{1}{3}$ and $y = 2\frac{1}{4}$:

$$1\frac{1}{3} \div 2\frac{1}{4} + 2\frac{1}{4} \div 1\frac{1}{3}$$

Convert the division into multiplication by taking the reciprocal:

$$\frac{5}{3} \times \frac{4}{9} \quad + \frac{9}{4} \times \frac{3}{5} = \frac{20}{27} + \frac{27}{20} = \frac{400 + 729}{540} = \frac{1129}{540}$$

Example 2.45

Example 2.46
It is given that $5\frac{3}{a} \times b\frac{1}{2} = 19$ (where the two fractions are mixed fractions), then $a + b =$ **(NMTC Primary-Screening, 2007/1)**

$$5 < 5\frac{3}{a} < 6$$

$$5 \times 1 = 5$$
$$5 \times 2 = 10$$
$$5 \times 3 = 15$$
$$5 \times 4 = 20$$

From the above

$$b = 3$$

$$5\frac{3}{a} \times 3\frac{1}{2} = 19$$

$$\frac{5a+3}{a} = 19 \times \frac{2}{7}$$
$$\frac{5a+3}{a} = \frac{38}{7}$$
$$35a + 21 = 38a$$
$$21 = 3a$$
$$a = 7$$

Fractions
What is the correct ordering of the three numbers 5\19, $\frac{7}{21}$, and $\frac{9}{23}$, in increasing order?
(AMC 8 2012/20)
Ans = 5/19 < 7/21 < 9/23