%, Interest, Profit & Loss

10 August 2025

Revision: 1355

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Table of contents

1. Basics

1.1 Revision: Fractions and Decimals

A. Converting between fractions and decimals

These conversions are both useful and important when doing percentages.

Example 1.1: Fractions less than One

Convert the fractions below to decimals.

Denominators of 4

A. $\frac{3}{4}$
B. $\frac{1}{4}$
C. $\frac{2}{4}$

Denominators of 5

D. $\frac{2}{5}$
E. $\frac{1}{5}$
F. $\frac{4}{5}$

Denominator of 50

G. $\frac{4}{50}$
H. $\frac{9}{50}$
I. $\frac{11}{50}$
J. $\frac{27}{50}$

Denominator of 25

K. $\frac{4}{25}$
L. $\frac{7}{25}$
M. $\frac{11}{25}$
N. $\frac{23}{25}$

Denominator of 20

O. $\frac{7}{20}$
P. $\frac{13}{20}$
Q. $\frac{19}{20}$
R. $\frac{1}{20}$
S. $\frac{3}{20}$

$$\frac{3}{4} = \frac{75}{100} = 0.75$$
$$\frac{1}{4} = \frac{25}{100} = 0.25$$
$$\frac{2}{5} = \frac{40}{100} = 0.4$$
$$\frac{4}{25} = \frac{16}{100} = 0.16$$
$$\frac{7}{20} = \frac{35}{100} = 0.35$$

Example 1.2: Fractions more than One

Convert the fractions below to decimals.

Denominator of 10

A. $\frac{18}{10}$
B. $\frac{45}{10}$
C. $\frac{34}{10}$
D. $\frac{71}{10}$

Denominator of 5

E. $\frac{27}{5}$
F. $\frac{34}{5}$
G. $\frac{21}{5}$
H. $\frac{99}{5}$
I. $\frac{63}{5}$

Denominator of 20

J. $\frac{33}{20}$
K. $\frac{52}{20}$
L. $\frac{61}{20}$
M. $\frac{99}{20}$

Denominator of 25

N. $\frac{76}{25}$
O. $\frac{53}{25}$
P. $\frac{43}{25}$
Q. $\frac{56}{25}$

Denominator of 50

R. $\frac{67}{50}$
S. $\frac{110}{50}$

We do the first one of each type:

Denominator of 10:

$$\frac{18}{10} = 1.8$$

Denominator of 5

$$\frac{27}{5} = \underbrace{5\frac{2}{5}}_{Mixed\ Number} = \underbrace{5\frac{4}{10}}_{Make\ Denominator\ 10} = 5 + 0.4 = 5.4$$

Denominator of 20

$$\frac{33}{20} = 1\frac{13}{20} = 1.65$$

Denominator of 25

$$\frac{76}{25} = 3\frac{1}{25} = 3\frac{4}{100} = 3.04$$

Denominator of 50

$$\frac{67}{50} = 1\frac{17}{50} = 1\frac{34}{100} = 1.34$$

Example 1.3: Decimals to Fractions

Convert the decimals in the sets below to fractions in reduced form:

Single Decimal Digit

A. 0.6
B. 0.2
C. 0.7
D. 0.5
E. 0.4
F. 0.1
G. 0.8
H. 0.3

Two Decimal Digits

I. 0.25
J. 0.68
K. 0.45
L. 0.85
M. 0.65
N. 0.42
O. 0.58
P. 0.95

Q. 0.90
R. 0.73
S. 0.72

Decimal >1, 1 Decimal Digit

T. 1.2
U. 3.4
V. 2.7
W. 1.8

X. 2.9
Y. 1.5

Decimal >1, 2 Decimal Digit

Z. 2.45
AA. 1.35
BB. 4.12
CC. 3.35

$$0.6 = \frac{6}{10} = \frac{3}{5}, \quad 0.2 = \frac{2}{10} = \frac{1}{5}, \quad 0.25 = \frac{25}{100} = \frac{1}{4}$$
$$0.68 = \frac{68}{100} = \frac{34}{50}, \quad 0.45 = \frac{45}{100} = \frac{9}{20}, \quad 0.85 = \frac{85}{100} = \frac{17}{20}$$
$$1.2 = \frac{12}{10} = \frac{6}{5}, \quad 2.45 = 2\frac{45}{100} = 2\frac{9}{20}$$

1.2 Fractions/Decimals to Percentage

A. Basics

1.4: Percent

A fraction can have many denominators. But comparing fractions with different denominators is not easy. So, a common standard to compare fractions is to make the denominators 100.

$$\underbrace{\frac{3}{5}}_{Reduced\ Fraction} = \underbrace{\frac{6}{10}}_{Decimal\ Fraction\ out\ of\ 10} = \underbrace{\frac{60}{100}}_{Decimal\ Fraction\ out\ of\ 100} = \underbrace{60 \times \frac{1}{100}}_{Rewriting\ the\ fraction\ as\ a\ product} = 60\%$$

By definition, $\frac{1}{100}$ can be replaced with the percentage symbol, and vice versa:

$$\% = \frac{1}{100}$$

To convert x to a percentage:

- If x has a denominator which is a factor of 100, find an equivalent fraction that has denominator 100.
- If the fraction is improper, the percentage will be more than 100.

Example 1.5

Convert the following numbers into percentage. Shows Steps.

A. $\frac{3}{10}$
B. $\frac{3}{4}$
C. $\frac{7}{10}$

Part A

Denominator is 10. And that $10 \times 10 = 100$.

$$\therefore \frac{3}{10} = \frac{3 \times 10}{10 \times 10} = \underbrace{\frac{30}{100} = 30\%}$$

You can replace a denominator of 100 with the % sign

Part B

The denominator in $\frac{3}{4}$ is 4. We want the denominator to be 100.

If we want the denominator to be 100, then we need to multiply 4 by 25.

To keep the value of the fraction the same, we need to multiply the numerator also by 25.

$$\frac{3}{4} = \frac{3}{4} \times \frac{25}{25} = \frac{75}{100} = 75\%$$

Part C

$$\frac{7}{10} = \frac{7}{10} \times \frac{10}{10} = \frac{70}{100} = 70\%$$

Example 1.6

Convert each of the numbers in the following sets into percentage:

Denominator of 2

A. $\frac{1}{2}$

Denominator of 5

B. $\frac{2}{5}$
C. $\frac{4}{5}$
D. $\frac{1}{5}$
E. $\frac{6}{5}$
F. $\frac{3}{5}$

Denominator of 4

G. $\frac{1}{4}$
H. $\frac{3}{4}$
I. $\frac{2}{4}$
J. $\frac{5}{4}$

Denominator of 10

K. $\frac{6}{10}$
L. $\frac{3}{10}$
M. $\frac{7}{10}$
N. $\frac{9}{10}$
O. $\frac{12}{10}$

Denominator of 25

P. $\frac{4}{25}$
Q. $\frac{7}{25}$
R. $\frac{16}{25}$
S. $\frac{12}{25}$
T. $\frac{1}{25}$
U. $\frac{30}{25}$
V. $\frac{23}{25}$
W. $\frac{40}{25}$

Denominator of 20

X. $\frac{12}{20}$
Y. $\frac{8}{20}$
Z. $\frac{16}{20}$
AA. $\frac{1}{20}$
BB. $\frac{15}{20}$

Denominator of 50

CC. $\frac{16}{50}$
DD. $\frac{24}{50}$
EE. $\frac{10}{50}$
FF. $\frac{36}{50}$
GG. $\frac{60}{50}$
HH. $\frac{12}{50}$

$$\frac{1}{2} = \frac{50}{100} = 50\%$$

$$\frac{2}{5} = \frac{40}{100} = 40\%$$
$$\frac{1}{4} = \frac{25}{100} = 25\%$$
$$\frac{6}{10} = \frac{60}{100} = 60\%$$
$$\frac{4}{25} = \frac{16}{100} = 16\%$$
$$\frac{12}{20} = \frac{60}{100} = 60\%$$
$$\frac{16}{50} = \frac{32}{100} = 32\%$$

In the same way, we can also calculate percentages for whole numbers.

Example 1.7

A. Seven out of ten children in Maya's class play soccer. Find the percentage of children who play soccer.
B. I have twenty toy cars, out of which exactly eight are red. Find the percentage of cars which are red. Also, find the percentage which are not red.
C. At least 30% of the class must sign up for a picnic in order for the picnic to happen. Out of a class of 20 students, 5 students signed up. Are there enough students? If there aren't, how many more students are needed?
D. Twelve out of fifty patients found that a medicine had the side effect of making them drowsy. What percentage of the patients felt the side effect?

Part A

$$\frac{7}{10} = 70\%$$

Part B

$$Red\ Cars = \frac{8}{20} = \frac{40}{100} = 40\%$$
$$Not\ Red = 100\% - 40\% = 60\%$$

Part C

$$\frac{5}{20} = 25\%$$
$$30\% = \frac{30}{100} = \frac{6}{20} Students$$
$$1\ More\ Student$$

Part D

$$\frac{12}{50} = \frac{24}{100} = 24\%$$

1.8: Whole Numbers as a Percentage

Whole numbers can be converted into a percentage, by writing them first as a fraction, and then converting that fraction into a percentage.

$$4 = \frac{4}{1} = \frac{400}{100} = 400\%$$

Example 1.9

A. 1
B. 3
C. 7
D. 2
E. 5

$$1 = \frac{1}{1} = \frac{1}{1} \times \frac{100}{100} = \frac{100}{100} = 100\%$$
$$3 = \frac{3}{1} = \frac{300}{100} = 300\%$$
$$7 = \frac{7}{1} = \frac{700}{100} = 700\%$$
$$2 = 200\%$$
$$5 = 500\%$$

Example 1.10

A. A health aficionado needs to eat an avocado a day to meet his dietary requirements. He eats two avocados a day. What percent of his dietary requirements is he meeting?
B. I needed to get two cakes for my birthday party. The bakery wrote down the wrong order and sent me six cakes instead. What percent of the cakes that I needed to get did I actually get?

Part A

$$\frac{2}{1} = \frac{200}{100} = 200\%$$

Part B

$$\frac{Actually\ Got}{Needed\ to\ Get} = \frac{6}{2} = \frac{3}{1} = \frac{300}{100} = 300\%$$

1.11: Simplifying Numbers

If numbers are not simplified, then it is often easier to simplify before converting into a percentage

Example 1.12

A. $\frac{32}{40}$
B. $\frac{36}{60}$

$$\frac{32}{40} = \frac{16}{20} = \frac{80}{100} = 80\%$$
$$\frac{36}{60} = \frac{6}{10} = 60\%$$

Example 1.13

Out of thirty doctors in a survey, exactly 24 had a general practice. What percentage of the doctors did not have a general practice?

$$\frac{6}{30} = \frac{2}{10} = 20\%$$

Example 1.14

Convert the numbers below into a percentage. If necessary, simplify the fractions before converting into a percentage.

Proper Fractions

A. $\frac{7}{28}$
B. $\frac{63}{70}$
C. $\frac{19}{95}$
D. $\frac{51}{68}$
E. $\frac{36}{60}$
F. $\frac{68}{85}$
G. $\frac{26}{65}$
H. $\frac{23}{46}$

Improper Fractions

I. $\frac{7}{35}$
J. $\frac{11}{44}$
K. $\frac{33}{30}$
L. $\frac{25}{20}$
M. $\frac{122}{40}$
N. $\frac{168}{40}$
O. $\frac{255}{50}$

Proper Fractions

$$\frac{7}{28} = \frac{1}{4} = \frac{25}{100} = 25\%$$
$$\frac{63}{70} = \frac{9}{10} = \frac{90}{100} = 90\%$$
$$\frac{51}{68} = \frac{3}{4} = \frac{75}{100} = 75\%$$
$$\frac{36}{60} = \frac{6}{10} = \frac{60}{100} = 60\%$$
$$\frac{68}{85} = \frac{4}{5} = \frac{80}{100} = 80\%$$

Improper Fractions

We can simplify, convert into a mixed number, and then into a percentage:

$$\frac{33}{30} = \frac{11}{10} = 1\frac{1}{10} = 100\% + 10\% = 110\%$$

Sometimes, it is easier to convert to a mixed number first, and then simplify and convert into a percentage:

$$\frac{122}{40} = 3\frac{2}{40} = 3\frac{1}{20} = 300\% + 5\% = 305\%$$

Example 1.15

A. Li was supposed to complete 8 problems for homework. He completed 12 problems instead. What percentage of his target did he meet?
B. Dr. Chase is a medical resident, and she needs to work twelve hours shifts. However, if her colleague takes an off, she ends up working for twenty-four hours at a stretch. What percentage of her shift is she working?

Part A

$$\frac{12}{8} = \frac{3}{2} = \frac{150}{100} = 150\%$$

Part 2

$$\frac{24}{12} = \frac{2}{1} = 200\%$$

Example 1.16

Capacity utilization is defined as $\frac{Capacity\ Used}{Capacity}$. A classroom has a capacity of twenty students. Because a teacher is not well, students from two classes are combined, and it now sits thirty-five students. What is the capacity utilization of the classroom?

$$\frac{35}{20} = \frac{7}{4} = \frac{175}{100} = 175\%$$

1.17: Multiplying by 100 to get a percentage

When the denominator is not a factor of 100, we multiply by 100, and add a percentage sign

$$Some\ Number \times \frac{100}{100}$$

But, we can replace the denominator with a percentage sign:

$$(Some\ Number \times 100)\%$$

Example 1.18

Convert to a percentage:

A. $\frac{1}{3}$
B. $\frac{1}{6}$
C. $\frac{2}{3}$
D. $\frac{5}{6}$
E. $\frac{33}{99}$

Part A

$$\frac{1}{3} = \frac{\frac{1}{3} \times 100}{100} = \frac{\frac{100}{3}}{100} = \frac{100}{3}\% = 33\frac{1}{3}\% = 33.333\ldots\% = 33.\bar{3}\%$$

Part B

$$\frac{1}{6} = \frac{\frac{1}{6} \times 100}{100} = \frac{100}{6}\% = 16\frac{4}{6}\% = 16\frac{2}{3}\% = 16.\bar{6}\%$$

Part C

Direct Method

$$\frac{2}{3} = \frac{\frac{2}{3} \times 100}{100} = \frac{200}{3}\% = 66\frac{2}{3}\%$$

Shortcut Method

If we remember that $\frac{1}{3} = 33\frac{1}{3}\%$, then $\frac{2}{3}$ is just double of that:

$$\frac{2}{3} = 2 \times \frac{1}{3} = 2 \times 33\frac{1}{3}\% = 66\frac{2}{3}\%$$

Part D

$$\frac{5}{6} = \frac{500}{6}\% = 83\frac{1}{3}\%$$

Part E

$$\frac{33}{99} = \frac{3}{9} = \frac{1}{3} = 33\frac{1}{3}\%$$

Example 1.19

A. Out of the seven days in a week, I go jogging two days in a week, and do weight-lifting on three days. I rest the other days. What percentage of the days in a week do I rest?
B. I work twenty-two days in a month. If I consider a month to have thirty days, what percentage of the days in a month do I work?
C. I exercise 3 three days in a week. What percent of the days do I exercise?
D. Out of an eight hour shift, 90 minutes is the lunch break. What percentage of the shift is lunch?

Part A

$$\frac{2}{7}\times\frac{100}{100}=\frac{2\times100}{7}\times\frac{1}{100}=\frac{200}{7}\%$$

Part B

$$\frac{22}{30}\times\frac{100}{100}=\frac{2200}{30}\times\frac{1}{100}=\frac{220}{3}\%$$

Part C

$$\frac{3}{7}\times\frac{100}{100}=\frac{300}{7}\times\frac{1}{100}=\frac{300}{7}\%$$

Part D

$$\frac{90}{480}=\frac{9}{48}=\frac{3}{16}\times\frac{100}{100}=\frac{300}{16}\times\frac{1}{100}=\frac{300}{16}\%=\frac{75}{4}\%$$

B. Decimals to Percentage

1.20: Decimals to Percentage

When converting decimals to percentage, we multiply the number by 100.

Example 1.21

Convert the following decimals to percentage:

A. 3.56
B. 0.64
C. 0.32
D. 0.53
E. 5.02
F. 0.4
G. 0.321
H. 0.275

$$3.56=\frac{356}{100}=356\%$$
$$0.64=64\%$$
$$0.32=32\%$$
$$0.53=53\%$$
$$5.02=502\%$$
$$0.4=\frac{4}{10}=\frac{40}{100}=40\%$$
$$0.321=\frac{32.1}{100}=32.1\%$$
$$0.275=27.5\%$$

1.22: Percentage to Decimals

To convert a percentage to a decimal, divide the number by 100.

Example 1.23

Convert the following percentages to decimals:

A. 42%
B. 75%
C. 55%

D. 115%
E. 215%
F. 5%
G. 12.5%

$$42 = \frac{42}{100} = 0.42$$
$$75\% = \frac{75}{100} = 0.75$$
$$55\% = \frac{55}{100} = 0.55$$
$$115\% = \frac{115}{100} = 1.15$$
$$215\% = \frac{215}{100} = 2.15$$
$$12.5\% = \frac{12.5}{100} = 0.125$$

C. Revisiting the Definition

1.24: Percentage Definition

The percentage symbol (%) represents $\frac{1}{100}$.

In any calculation, you can replace a percent symbol with $\frac{1}{100}$.

Example 1.25: Converting between fractions, decimals and percentages

In each group below, fill the remaining columns with equivalent values.

Group 1			Group 2			Group 3		
Fraction	Decimal	%tage	Fraction	Decimal	%tage	Fraction	Decimal	%tage
$\frac{1}{2}$				0.1				25
$\frac{1}{3}$				0.25				$66\frac{2}{3}$
$\frac{1}{4}$				0.75				75
$\frac{1}{5}$				0.05				0.01
$\frac{1}{6}$				0.01				3000
$\frac{1}{7}$				0.02				50
$\frac{1}{8}$				0.0025				$33\frac{1}{3}$
$\frac{1}{9}$				0.45				$16\frac{2}{3}$
$\frac{1}{10}$				0.452				0.875
$\frac{1}{11}$				0.85				0.375

Group 1			Group 2			Group 3		
Fraction	Decimal	%tage	Fraction	Decimal	%tage	Fraction	Decimal	%tage
$\frac{1}{2}$	0.5	50%		0.1				25
$\frac{1}{3}$	$0.\overline{3}$	$33\frac{1}{3}\%$		0.25				66 2/3
$\frac{1}{4}$	0.25	25%		0.75				75
$\frac{1}{5}$	0.2	20%		0.05				0.01
$\frac{1}{6}$	$0.1\overline{6}$	$16\frac{1}{6}$		0.01				3000
$\frac{1}{7}$	$0.\overline{142857}$	14.285		0.02				50
$\frac{1}{8}$	0.125			0.0025				33 1/3
$\frac{1}{9}$	$0.\overline{1}$			0.45				0.8
$\frac{1}{10}$	0.1			0.452				0.875
$\frac{1}{11}$	$0.\overline{09}$			0.85				0.375

$$66\frac{2}{3}\% = \frac{200}{3}\% = \frac{200}{300} = \frac{2}{3}$$
$$16\frac{2}{3}\% = \frac{50}{3}\% = \frac{50}{300} = \frac{1}{6}$$
$$0.875\% = 0.00875 = \frac{875}{10{,}000} = \frac{7}{800}$$

1.3 Finding Percents

A. Calculating a Percentage of a Number

We often want to find a percentage of a number.

1.26: of in Maths

of means multiplication.

Hence, when we find 30% of a number, we are finding:

$$30\%\ of\ x = \frac{30}{100} \times x$$

Example 1.27

Find the following:

A. 50% of 100
B. 50% of 50

C. 50% of 200
D. 50% of 300
E. 50% of 500

$$50\%\ of\ 100 = \frac{50}{100} \times 100 = 50$$
$$50\%\ of\ 50 = \frac{50}{100} \times 50 = 25$$
$$50\%\ of\ 200 = \frac{50}{100} \times 200 = 100$$
$$50\%\ of\ 300 = \frac{50}{100} \times 300 = 150$$
$$50\%\ of\ 500 = \frac{50}{100} \times 500 = 250$$

Example 1.28

Find the following:

A. 25% of 40
B. 25% of 16
C. 25% of 64
D. 25% of 24

$$25\%\ of\ 40 = \frac{25}{100} \times 40 = \frac{1}{4} \times 40 = \frac{40}{4} = 10$$
$$25\%\ of\ 16 = \frac{25}{100} \times 16 = \frac{1}{4} \times 16 = 4$$
$$25\%\ of\ 64 = \frac{25}{100} \times 64 = \frac{1}{4} \times 64 = 64$$
$$25\%\ of\ 24 = \frac{25}{100} \times 24 = \frac{1}{4} \times 24 = 6$$

Example 1.29

Find the following:

A. 20% of 100
B. 20% of 50
C. 20% of 20
D. 20% of 200
E. 20% of 10

$$20\%\ \text{of}\ 100 = \frac{20}{100} \times 100 = 20$$
$$20\%\ \text{of}\ 50 = \frac{20}{100} \times 50 = 10$$
$$20\%\ \text{of}\ 20 = \frac{20}{100} \times 20 = 4$$
$$20\%\ \text{of}\ 200 = \frac{20}{100} \times 200 = 40$$
$$20\%\ \text{of}\ 200 = \frac{20}{100} \times 200 = \frac{1}{5} \times 200 = \frac{200}{5} = 40$$
$$20\%\ \text{of}\ 10 = \frac{20}{100} \times 10 = 2$$

Example 1.30

Find the following:

A. 30% of 40
B. 20% of 70
C. 25% of 300
D. 40% of 30

$$30\% \text{ of } 40 = \frac{30}{100} \times 40 = 3 \times 4 = 12$$
$$20\% \text{ of } 70 = \frac{20}{100} \times 70 = 2 \times 7 = 14$$
$$25\% \text{ of } 300 = \frac{25}{100} \times 300 = 75$$
$$40\% \text{ of } 30 = \frac{40}{100} \times 30 = 4 \times 3 = 12$$

1.31: Common Percentages and their Fractions

$$50\% = \frac{50}{100} = \frac{1}{2}$$
$$25\% = \frac{25}{100} = \frac{1}{4}$$
$$20\% = \frac{20}{100} = \frac{1}{5}$$
$$10\% = \frac{10}{100} = \frac{1}{10}$$
$$5\% = \frac{5}{100} = \frac{1}{20}$$

Example 1.32

A. 20% of 25
B. 25% of 16
C. 50% of 27
D. 50% of 33

$$\frac{1}{5} \times 25 = 5$$
$$\frac{1}{4} \times 16 = 4$$
$$\frac{1}{2} \times 27 = 13.5$$
$$\frac{1}{2} \times 33 = 16.5$$

1.33: Large Percentages

$$1000\% = \frac{1000}{100} = 10$$

Example 1.34

A collector offers to buy state quarters for 2000% of their face value. At that rate how much will Bryden get for his four state quarters? (AMC 8 2001/10)

$$Face\ Value = 4 \times 25\ cents = 100\ cents = 1\ Dollar$$

The purchase price is

$$1 \times 2000\% = 1 \times \frac{2000}{100} = 20\ Dollars$$

1.35: Some Less Common Percentages

$$33\frac{1}{3}\% = \frac{100}{3}\% = \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$$
$$66\frac{2}{3}\% = \frac{200}{3}\% = \frac{200}{3} \times \frac{1}{100} = \frac{2}{3}$$
$$14.\overline{285714}\% = \frac{1}{7}$$

Challenge 1.36

A. $33\frac{1}{3}\%$ of 81
B. $66\frac{2}{3}\%$ of 33
C. $14.\overline{285714}\%$ of 77

$$\frac{1}{3} \times 81 = 27$$
$$\frac{2}{3} \times 33 = 22$$
$$\frac{1}{7} \times 77 = 11$$

Example 1.37

A. Shankar likes to keep dogs. He raises dogs for a living, and sells them. He currently has 30 dogs. Out of the 30 dogs, 20% of the dogs are expensive. How many of his dogs are expensive?
B. At Elsie's Bakery, Jane can decorate 25% of a batch of five cakes in an hour. How many cakes can she decorate in a hour?

Part A

$$20\%\ of\ 30 = \frac{20}{100} \times 30 = 2 \times 3 = 6$$

Part B

$$25\%\ of\ 5 = \frac{1}{4} \times 5 = \frac{5}{4}\ Cakes$$

B. Calculating a Percentage of Zero

1.38: Finding Percent of Zero

Any percentage that you find of zero is always zero.
Zero is the only number that has this property.

Example 1.39

What is:

A. 10% of zero?

B. 50% of zero?
C. 0% of zero?

$$0$$
$$0$$
$$0$$

Challenge 1.40: Percentages of whole numbers

25% of the cakes that I had for breakfast are the same as 75% of the cakes that I had for breakfast. What is 65% of the cakes that I had for breakfast?

Suppose I had 4 cakes for breakfast. Then:

$$25\%\ of\ 4 = 1, \qquad 75\%\ of\ 4 = 3 \Rightarrow Answer\ is\ different.$$

The only number for which any two distinct percentages will be same is zero

$$0 \times \frac{25}{100} = 0 \times \frac{75}{100} = 0 \times \frac{65}{100} = 0$$

Algebra

$$0.25x = 0.75x$$
$$Case\ I: x \neq 0 \Rightarrow 0.25 = 0.75 \Rightarrow \boldsymbol{Contradiction}$$
$$Case\ II: x = 0 \Rightarrow LHS = 0.25 \times 0 = 0.75 \times 0 = RHS \Rightarrow \boldsymbol{Valid\ Solution}$$

Application: Rates

Example 1.41

A. Claire can dig 20% of a ditch in an hour. How long does she take to dig a ditch?
B. Peter can run 50% of a km in an hour. How long does he take to run a km?
C. At Elsie's Bakery, Jane can decorate 25% of a batch of five cakes in an hour. How many cakes can she decorate in a working day of eight hours?

Part A

$$20\%\ in\ 1\ hour = \frac{1}{5}\ in\ 1\ Hour = \underbrace{1\ Ditch\ in\ 5\ Hour}_{Multiply\ by\ 5}$$

Part B

$$50\%\ in\ 1\ hour = \frac{1}{2}\ in\ 1\ Hour = \underbrace{1\ Km\ in\ 2\ Hours}_{Multiply\ by\ 2}$$

Part C

$$In\ 1\ Hour: 25\%\ of\ 5 = \frac{1}{4} \times 5 = \frac{5}{4}\ Cakes$$
$$In\ 8\ Hours: \frac{5}{4} \times 8 = 10\ Cakes$$

Challenge 1.42

A. Claire can dig 20% of a ditch in 30% of an hour. How long does she take to dig a ditch?
B. Peter can run 40% of a km in 60% of an hour. How long does he take to run a km?

Part A

$$20\%\ of\ a\ ditch \rightarrow 30\%\ of\ an\ hour$$

Convert the percentage into fractions, and the hour into minutes:

$$\frac{1}{5}\ of\ a\ ditch \rightarrow \frac{3}{10} of\ 60\ minutes$$

Find the number of minutes:

$$\frac{1}{5}\ of\ a\ ditch \rightarrow 18\ minutes$$

Multiply both sides by 5:

$$1\ ditch \rightarrow 90\ minutes$$

Part B

$$40\%\ of\ 1\ km \leftrightarrow 60\%\ of\ an\ hour$$

Convert the percentage into fractions, and the hour into minutes:

$$\frac{4}{10}\ of\ 1\ km \leftrightarrow \frac{6}{10}\ of\ 60\ minutes$$

Find the number of minutes:

$$\frac{4}{10}\ of\ 1\ km \leftrightarrow 36\ minutes$$

Divide both sides 4:

$$\frac{1}{10}\ of\ 1\ km \leftrightarrow 9\ minutes$$

Multiply by 10:

$$1\ km \leftrightarrow 90\ minutes$$

Challenge 1.43

At Elsie's Bakery, Jane can decorate 40% of a batch of five cakes in 40% of an hour. How many cakes can she decorate in an hour?

Challenge 1.44

0.02% of those receive a vaccine get mild side-effects.

A. If 30,000 people are vaccinated, how many will get affected?
B. How many people must be vaccinated in order for 10 people to be affected?

$$30000 \times \frac{0.02}{100} = 300 \times 0.02 = 6$$

10,000 people = 2 people affected ⇒ 50,000 people = 10 people affected

C. Multi-Step Percentage Calculations

Sometimes we want to find a percentage of a percentage of a number.

Example 1.45

A. $50\%\ of\ 50\%\ of\ 200$
B. $60\%\ of\ 70\%\ of\ 500$
C. 20% of 40% of 100
D. 50% of 25% of 16
E. 30% of 80% of 400
F. 15% of 65% of 400
G. 55% of 45% of 500

$$\frac{1}{2} \times \frac{1}{2} \times 200 = \frac{200}{4} = 50$$

$$\frac{6}{10}\times\frac{7}{10}\times 500 = 6\times 7\times 5 = 210$$

$$\frac{1}{5}\times\frac{4}{10}\times 100 = 8$$
$$\frac{1}{2}\times\frac{1}{4}\times 16 = \frac{16}{8} = 2$$
$$\frac{3}{10}\times\frac{8}{10}\times 400 = 96$$
$$\frac{15}{100}\times\frac{65}{100}\times 400 = \frac{3}{20}\times\frac{13}{20}\times 400 = \frac{3\times 13}{400}\times 400 = 39$$
$$\frac{55}{100}\times\frac{45}{100}\times 500 = \frac{11}{20}\times 45\times 5 = \frac{495}{4}$$

Example 1.46

A. $16\frac{2}{3}\%$ of $9.\overline{09}\%$ of 7

B. $16\frac{2}{3}\%$ of $66\frac{2}{3}\%$ of $9.\overline{09}\%$ of $14.\overline{285714}\%$ of 21

$$\frac{1}{6}\times\frac{1}{11}\times 7 = 8$$
$$\frac{1}{6}\times\frac{2}{3}\times\frac{1}{11}\times\frac{1}{7}\text{ of }21 = \frac{2}{66}$$

D. Volume

Example 1.47

Find the following:

A. In ml: 25% of 16 Litre
B. In ml: 20% of 2.5 Litre
C. In Litres: 10% of 40,000 ml
D. In Litres: 40% of 25,000 ml

$$\frac{1}{4}\times 16 = 4\ Litres = 4000\ ml$$
$$\frac{1}{5}\times 2500\ ml = 500\ ml$$
$$\frac{1}{10}\times 40{,}000 = 4000\ ml = 4\ L$$
$$\frac{2}{5}\times 25{,}000 = 10{,}000\ ml = 10\ L$$

Example 1.48

Find the following:

A. In ml: 30% of 20% of 1.5 Litre
B. In ml: 20% of 25% of 2 Litre
C. In Liters: 10% of 20% of 30,000 ml
D. In Liters: 25% of 50% of 25,000 ml

Part A

Method I

Convert into ml at the beginning:

$$\frac{3}{10} \times \frac{1}{5} \times 1500\ ml = \frac{3 \times 150}{5} = 3 \times 30 = 90 \text{ ml}$$

Method II

Convert into ml at the end

$$\frac{3}{10} \times \frac{2}{10} \times \frac{15}{10} = \frac{90}{1000} = \frac{9}{100} = 0.09\ Liters = 90\ ml$$

Part B

$$\frac{2}{10} \times \frac{1}{4} \times 2000\ ml = 100\ ml$$

$$\frac{2}{10} \times \frac{1}{4} \times 2 = \frac{1}{10} Litres = 100\ ml$$

Part C

$$\frac{1}{10} \times \frac{1}{5} \times 30{,}000 = 6{,}00\ ml = 0.6\ Liters$$

Part D

$$\frac{1}{4} \times \frac{1}{2} \times 25{,}000 = 3{,}125\ ml = 3.125\ Liters$$

Example 1.49

Timothy is a milkman. He delivered 30% of a 9 Litre can of milk to customer A, and 40% of a 8 Litre can of milk to customer B. To which customer did he deliver more milk and by how much? (Answer in ml).

$$30\%\ of\ 9\ Litres = \frac{3}{10} \times 9000 = 2700\ ml$$
$$40\%\ of\ 8\ Litres = \frac{4}{10} \times 8000 = 3200\ ml$$
$$3200 - 2700 = 500\ ml$$

Example 1.50

In a measurement system used in the US, a gallon is a unit of volume equal to 4 quarts.

A. What percentage is a quart of a gallon?
B. What percentage is a gallon of a quart.

$$1\ gallon = 4\ quarts \Rightarrow \frac{1}{4}\ gallon = 1\ quart$$

$$\left(\frac{1}{4} \div 1\right) \times 100 = \frac{1}{4} \times 100 = 25\%$$
$$\frac{4}{1} \times 1004 = 400\%$$

E. Time

Example 1.51

Find the following in minutes:

A. 50% of 1 hour
B. 25% of 1 hour

C. 20% of 1 hour
D. 10% of 1 hour
E. 5% of 1 hour

$$\frac{1}{2} of\ 60\ min = 30\ min$$
$$\frac{1}{4}\ of\ 60\ min = 15\ min$$
$$\frac{1}{5}\ of\ 60\ \text{min} = 12\ min$$
$$\frac{1}{10} of\ 60\ min = 6\ min$$
$$\frac{1}{20} of\ 60\ min = 3\ min$$

Example 1.52

Find the following in hours:

A. 50% of a 24-hour day
B. 25% of a 12-hour day
C. 20% of a 24-hour day
D. 10% of a 24-hour day

$$\frac{1}{2} of\ 24 = 12$$
$$\frac{1}{4}\ of\ 12 = 3$$
$$\frac{1}{5} of\ 24 = \frac{24}{5} = 4\frac{4}{5} = 4.8\ hours$$
$$\frac{1}{10} \times 24 = 2.4\ hours$$

Example 1.53

Find the following in seconds:

A. 50% of 1 minute
B. 25% of 1 minute
C. 20% of 1 minute
D. 10% of 1 minute
E. 5% of 1 minute

$$\frac{1}{2} of\ 60\ seconds = 30\ seconds$$
$$\frac{1}{4}\ of\ 60\ seconds = 15\ seconds$$
$$\frac{1}{5}\ of\ 60\ seconds = 12\ seconds$$
$$\frac{1}{10} of\ 60\ seconds = 6\ seconds$$
$$\frac{1}{20} of\ 60\ seconds = 3 seconds$$

Challenge 1.54

A. In Seconds: $16\frac{2}{3}\%$ of $33\frac{1}{3}\%$ of one − tenth of an hour

$$\frac{1}{6}\times\frac{1}{3}\times\frac{1}{10}\times 3600 = 20 \text{ seconds}$$

F. Word Problems: Multi-Step Percentages

Example 1.55

A tanker of water with a capacity of 2000 Liters visited a village, and the water was distributed equally among ten drums. Each drum was used to fill jugs with a capacity of 4 L each. Find the number of jugs that can be filled from two drums.

$$\frac{2000}{10} = 200\ Liters$$
$$\frac{200}{4} = 50\ jugs\ per\ drum$$
$$Two\ drums = 2\times 50 = 100\ Jugs$$

Example 1.56

A milkman had two cans of milk, one with a capacity of nine liters, and the other with a capacity of six liters. When he reached the second last household that he had to give milk to, he was left with very little milk. The larger can had only 5% of its capacity left, and the smaller can had only 10% of its capacity left. The milkman divided the milk he had among the second-last and the last household. How much did each household get?

$$\frac{1}{20}\times 9000 = 450\ ml$$
$$\frac{1}{10}\times 6000 = 600\ ml$$
$$Total\ Milk = 450 + 600 = 1050\ ml$$
$$\frac{1050}{2} = 525\ ml$$

Example 1.57

When Roshan goes for swimming, he has a 90-minute routine, and when he goes for basketball, he has a two-hour routine. Today Roshan is going to go for both swimming and basketball, but he does not have time, so he spent only 30% of his usual swimming time, and 20% of his usual basketball time. How much time did he spend on basketball and swimming in all?

$$\frac{3}{10}\times 90 = 27\ \ minutes$$
$$\frac{1}{5}\times 120 = 24\ minutes$$
$$Total\ Time = 27 + 24 = 51\ Minutes$$

G. Integer Solutions

Example 1.58

One liter of medicinal alcohol was divided equally among ten doctors in a hospital. One doctor spilled 50% of

the alcohol that he got, so it was wasted. He needed 3 ml of alcohol for each patient to apply before an injection. How many patients could he inject?

$$\frac{1000\ ml}{10} = 100\ ml$$
$$\frac{1}{2} \times 100\ ml = 50\ ml$$
$$\frac{50}{3} = 16\frac{2}{3} \Rightarrow 16\ Patients$$

Example 1.59

A lift company rates its lifts on the basis that the average weight of a person is 70 kg. For example, a lift that can carry a load of 700 kg is rated to carry 10 Adults. A small lift has a capacity of 400 kg. A medium-sized lift has a capacity 50% more than the small lift. How many adults will it be rated to carry?

Capacity of Medium Lift:

$$= 150\%\ of\ Small\ Lift\ = \frac{150}{100} \times 400 = \frac{3}{2} \times 400 = 600\ kg$$

The number people it can carry is:

$$\frac{600}{70} = \frac{60}{7} = 8\frac{4}{7} \rightarrow 8\ People$$

Example 1.60

A small tanker has a capacity of 1000 liters. A large tanker has a capacity 50% more than the small tanker. An extra-large tanker has a capacity 50% more than the large tanker. A drum has a capacity of 100 liters. How many drums can be completely filled with the water from an extra larger tanker?

Capacity of extra-large tanker

$$= 150\%\ of\ large\ tanker$$
$$= 150\%\ of\ (150\%\ of\ small\ tanker)$$
$$= \frac{150}{100} \times \frac{150}{100} \times 1000 = \frac{3}{2} \times \frac{3}{2} \times 1000 = \frac{9}{4} \times 1000 = 2250$$

Number of Drums

$$= \frac{2250}{100} = 22\frac{1}{2} \Rightarrow 22\ Drums$$

H. Multi-Stage Calculations

Example 1.61

Vinayak inherited 20% of his father's land holding. And Vinayak's father inherited 25% of Vinayak's grandfather's land holdings.

A. What percent of his grandfather's land holdings does Vinayak have?
B. If Vinyak's grandfather had 10 acres of land, how much land does Vinayak have?

Part A

Let say

$$Vinayak = V$$

V's father got

$$25\%\ of\ V's\ Grandfather\ holdings = \frac{25}{100} = \frac{1}{4}$$

V got:

$$20\%\ of\ \frac{1}{4} = \frac{20}{100} \times \frac{1}{4} = \frac{1}{5} \times \frac{1}{4} = \frac{1}{20} = \frac{5}{100} = 5\%$$

Part B

$$\frac{1}{20} \times 10 = \frac{1}{2}\ an\ acre$$

Example 1.62

Suchi went to the market with Rs. 50. She spent 50% of the money on fruit. She spent 50% of the rest on vegetables. She spent 50% of the rest on coming back home. How much money did she have left with her?

At each stage, money spent is 50%. Hence, money remaining is

$$100\% - 50\% = 50\%$$

Money remaining after spending on fruit:

$$50\%\ of\ 50 = \frac{50}{100} \times 50 = \frac{1}{2} \times 50 = 25$$

Money remaining after spending on vegetables:

$$50\%\ of\ 25 = \frac{1}{2} \times 25 = \frac{25}{2}$$

Money remaining after spending on coming back home:

$$50\%\ of\ \frac{25}{2} = \frac{1}{2} \times \frac{25}{2} = \frac{25}{4} = 6\frac{1}{4}$$

Money remaining after spending on coming back home:

$$50\%\ of\ \frac{25}{2} = \frac{1}{2} \times \frac{25}{2} = \frac{25}{4} = 6\frac{1}{4}$$

Example 1.63

Suchi went to the market with Rs. 200. She spent 25% of the money on fruit. She spent 25% of the rest on vegetables. She spent 25% of the rest on coming back home. How much money did she have left with her?

At each stage, money spent is 25%. Hence, money remaining is

$$100\% - 25\% = 75\%$$

Money remaining after spending on fruit:

$$75\%\ of\ 200 = \frac{75}{100} \times 200 = \frac{3}{4} \times 200 = 150$$

Money remaining after spending on vegetables:

$$75\%\ of\ 150 = \frac{3}{4} \times 150 = \frac{3}{2} \times 75 = \frac{225}{2}$$

Money remaining after spending on coming back home:

$$75\%\ of\ \frac{225}{2} = \frac{3}{4} \times \frac{225}{2} = \frac{675}{8}$$

I. Equivalent Forms to find a percentage

Previously, we saw how to find some percent of a number. Now, we look at the different ways in which we can

find this.
To get the percentage of a number, we can multiply it using a percentage, a fraction, or a decimal. All three are useful depending on the situation.

$$x\%\ of\ y = x\% \times y = \frac{x}{100} \times y = \underbrace{z}_{z=\frac{x}{100}} \times y$$

Example 1.64
We want to find 7% of 37. Write an expression showing what you will multiply with 37 in order to find the required percentage. Show your multiplying factor as:

A. A percentage
B. A fraction
C. A decimal

$$\underbrace{7\%\ of\ 37}_{\text{"of" form}} = \underbrace{7\% \times 37}_{\text{Percent Form}} = \underbrace{\frac{7}{100} \times 37}_{\text{Fractional Form}} = \underbrace{0.07 \times 37}_{\text{Decimal Form}}$$

Convert the following expressions into percent form, fractional form and decimal form. Do not simplify the expressions:

A. $6\%\ of\ 42$
B. $12\%\ of\ 51$
C. $4\%\ of\ 24$

$$6\%\ of\ 42 = 6\% \times 42 = \frac{6}{100} \times 42 = 0.06 \times 42$$

$$12\%\ of\ 51 = 12\% \times 51 = \frac{12}{100} \times 51 = 0.12 \times 51$$

$$4\%\ of\ 24 = 4\% \times 24 = \frac{4}{100} \times 24 = 0.04 \times 24$$

Example 1.65
Convert the following expressions into percent form, fractional form and decimal form. Do not simplify the expressions:

A. $9\%\ of\ x$
B. $15\%\ of\ y$
C. $23\%\ of\ z$

1.4 Working with Percents

A. Finding x as a percentage of y

1.66: Finding x as a percentage of y

$$\frac{x}{y} \times 100\%$$

$$\frac{x}{y} = \frac{x \times 100}{100y} = \frac{\frac{x \times 100}{y}}{100} = \frac{x}{y} \times 100\%$$

Example 1.67

Percent of 50

A. 7 is what percent of 50?
B. 12 is what percent of 50?
C. 25 is what percent of 50?
D. 45 is what percent of 50?
E. 35 is what percent of 50?
F. 32 is what percent of 50?

Percent of 25

G. 5 is what percent of 25?
H. 12 is what percent of 25?
I. 20 is what percent of 25?
J. 15 is what percent of 25?

Percent of 40

K. 20 is what percent of 40?
L. 30 is what percent of 40?

Mixed Review

M. 20 is what percent of 25?
N. 10 is what percent of 50?
O. 40 is what percent of 80?
P. 33 is what percent of 55?
Q. 33 is what percent of 66?
R. 15 is what percent of 75?
S. 126 is what percent of 200?
T. 200 is what percent of 250?
U. 80 is what percent of 320?
V. 7 is what percent of 20?
W. 13 is what percent of 25?
X. 45 is what percent of 50?
Y. 120 is what percent of 200?

Percentages Out of 100

Z. 34 is what percent of 100?

Percentages Greater than 100

AA. 60 is what percent of 50?

Part A

7 out of 50

$$= \frac{7}{50} = \frac{14}{100} = 14\%$$

$$20 \; out \; of \; 25 \Rightarrow \frac{20}{25} = \frac{4}{5} = \frac{80}{100} = 80\%$$

Calculating a Number is what percent of another Number

4 is what percent of 40 has the same meaning as finding 4 is what share of 40. Hence, we can write it as a fraction.

$$\frac{4}{40}$$

Once we have a fraction, we can convert the fraction into a percentage in two ways:

Method I: Make the Denominator 100

$$\frac{4}{40} = \underbrace{\frac{1}{10}}_{Simplified} = \underbrace{\frac{10}{100}}_{\substack{Made\ the \\ Denominator\ 100}} = \underbrace{10\%}_{Convert\ to\ Percentage}$$

Method II: Multiply the Fraction by 100, and add a percentage sign

$$\frac{4}{40} \times 100\% = \frac{400}{40}\% = 10\%$$

Example 1.68

Find the following:

A. 5 is what percent of 20
B. 12 is what percent of 50
C. 25 is what percent of 200
D. 30 is what percent of 60

$$\frac{5}{20} \times 100 = 25\%$$
$$\frac{12}{50} \times 100 = 24\%$$
$$\frac{25}{200} \times 100 = \frac{25}{2} = 12.5\%$$
$$\frac{30}{60} \times 100 = 50\%$$

Example 1.69

A. 2 is what percent of 25?
B. 5 is what percent of 40?
C. What percent of 3 is 4.5
D. What percent of 9 is 72?

$$\frac{2}{25} \Rightarrow \frac{8}{100} \Rightarrow 8\%$$
$$\frac{5}{40} = \frac{1}{8} = 12.5\%$$
$$\frac{4.5}{3} = \frac{45}{30} = \frac{9}{6} = \frac{3}{2} = \frac{150}{100} = 150\%$$
$$\frac{72}{9} = 8 = 800\%$$

Example 1.70

A. Rishi has a greengrocer's shop where he sells apples. He currently has 25 apples with him to sell. Out of these apples, 5 apples are Washington apples. What percent of his apples are Washington Apples?
B. Out of 2.5 million liters of water in a swimming pool, only 2 million liters of water is left. What percent of the water has evaporated?

Part A

We need to find 5 is what percent of 25. Hence, we need to find:

$$\frac{5}{25} = \frac{1}{5} = \frac{20}{100} = 20\%$$

Part B

$$Water\ Evaporated = 2.5 - 2 = 0.5\ Million$$

The percentage of water evaporated

$$= \frac{0.5}{2.5} = \frac{5}{25} = \frac{1}{5} = \frac{20}{100} = 20\%$$

Example 1.71

Rohan likes toy cars. Rohan has five blue sports cars and ten red sports cars. He has 10 yellow vintage cars from the US and 15 brown vintage cars from the UK. What percent of his:

A. sports cars are blue?
B. sports cars are red?
C. vintage cars are from the US?
D. vintage cars are from the UK?
E. cars are sports cars?
F. cars are vintage cars?
G. cars are blue?

H. cars are red?
I. cars are brown?

Parts A and B[1]

Rohan has a total of

$$5 + 10 = 15\ sports\ cars$$

$$\%\ \text{which is blue} = \frac{5}{15} = \frac{1}{3} = 33\frac{1}{3}\%$$

$$\%\ \text{which is red} = \frac{10}{15} \times 100 = \frac{2}{3} \times 100 = \frac{200}{3} = 66\frac{2}{3}\%$$

Parts C and D

$$Total\ of\ 10 + 15 = 25\ vintage\ cars$$

$$\%\ \text{from the US} = \frac{10}{25} = \frac{2}{5} = \frac{40}{100} = 40\%$$

$$\%\ \text{from the US} = \frac{15}{25} = \frac{3}{5} = 60\%$$

Parts E and F

$$\%\ which\ are\ sports\ cars = \frac{15}{40} = \frac{3}{8} \times 100 = \frac{300}{8} = \frac{150}{4} = \frac{75}{2} = 37.5\%$$

$$\%\ which\ are\ sports\ cars = \frac{25}{40} = \frac{5}{8} \times 100 = \frac{500}{8} = \frac{250}{4} = \frac{125}{2} = 62.5\%$$

Parts G, H and I

$$\frac{5}{40} = 12.5\%$$

$$\frac{10}{40} = \frac{1}{4} = 25\%$$

$$\frac{15}{40} \times 100 = \frac{150}{4} = 37.5\%$$

Example 1.72

A 25 liter solution contains 20% alcohol. If 50 litres of water are added to the solution, what percent of the new solution is alcohol?

Quantity of Alcohol:

$$= 20\%\ of\ 25 = \frac{1}{5} \times 25 = 5\ Litres$$

Quantity of New Solution

$$= \underbrace{25\ Liters}_{\textit{Old Solution}} + \underbrace{50\ Liters}_{\textit{New Addition}} = 75\ Liters$$

Percentage of Alcohol in the new solution:

$$= \frac{5}{75} \times 100 = \frac{1}{15} \times 100 = \frac{100}{15} = \frac{20}{3} = 6\frac{2}{3}\%$$

B. Finding the number, given a percentage

Example 1.73

Numbers which are 50%

A. 70 is 50% of which number?

[1] The vintage car information is not related to this question and *must* be ignored to get the right answer.

B. 35 is 50% of which number?
C. 12 is 50% of which number?
D. 91 is 50% of which number?

Numbers which are 25%

E. 30 is 25% of which number?
F. 40 is 25% of which number?
G. 14 is 25% of which number?
H. 19 is 25% of which number?

Numbers which are 20%

I. 10 is 20% of which number?
J. 12 is 20% of which number?
K. 17 is 20% of which number?

Numbers which are 10%

L. 17 is 10% of which number?
M. 8 is 10% of which number?
N. 29 is 10% of which number?

Numbers which are 5%

O. 14 is 5% of which number?
P. 9 is 5% of which number?
Q. 27 is 5% of which number?

Numbers which are 4%

R. 4 is 4% of which number?
S. 1 is 4% of which number?
T. 22 is 4% of which number?

Numbers which are 2%

U. 7 is 2% of which number?
V. 17 is 2% of which number?
W. 4 is 2% of which number?

$$50\%\ of\ a\ Number = 70 \Rightarrow \frac{50}{100} of\ a\ Number = 70 \Rightarrow \frac{1}{2} of\ a\ Number = 70 \Rightarrow 140$$
$$\frac{25}{100} of\ a\ Number = 30 \Rightarrow \frac{1}{4} of\ a\ Number = 30 \Rightarrow Number = 30 \times 3 = 120$$
$$\frac{20}{100} of\ a\ Number = 10 \Rightarrow \frac{1}{5} of\ a\ Number = 10 \Rightarrow Number = 10 \times 5 = 50$$
$$\frac{10}{100} of\ a\ Number = 17 \Rightarrow \frac{1}{10} of\ a\ Number = 17 \Rightarrow Number = 10 \times 17 = 170$$
$$\frac{5}{100} of\ a\ Number = 14 \Rightarrow \frac{1}{20} of\ a\ Number = 14 \Rightarrow Number = 14 \times 20 = 280$$
$$\frac{4}{100} of\ a\ Number = 4 \Rightarrow \frac{1}{25} of\ a\ Number = 4 \Rightarrow Number = 4 \times 25 = 100$$
$$\frac{2}{100} of\ a\ Number = 7 \Rightarrow \frac{1}{50} of\ a\ Number = 7 \Rightarrow Number = 7 \times 50 = 350$$

C. Adding a Percentage to a Number

We often need to add a percentage to a number.

1.74: Adding Percents

We often need to add a percentage to a number.

For example:

- If a person works for a salary, then every year the increase in their salary is given as a percentage.
- If you look at prices (food items, gold prices, etc), the change in prices is expressed as a percentage.
- If you look at marks in a school exam, they are expressed as a percentage. If you want to compare last month's performance with this month's, you can do it by checking for increase or decrease in percentage.
- Tax (such as GST) is added on as an increase in the price of an item.
- If you go for dinner, the hotel can charge "Serving Charges" as an added percentage to the bill. You have the option to pay an extra tip over and above the bill.

Example 1.75

Last month I sold 500 books from my bookshop. This month I sold 5% more.

A. How many more books did I sell this month?
B. What is the total number of books sold this month?

$$5\%\ of\ 500 = \frac{5}{100} \times 500 = 25$$
$$500 + 25 = 525$$

Example 1.76

Shalu had a salary of Rs. 12,000 per month. Her salary was increased by 10%. Find:

A. The *increase* in the salary
B. The *increased* salary

Note that Part A asks for *increase* whereas Part B asks for *increased* salary

$$Increase\ in\ Salary = 10\%\ of\ 12000 = \frac{10}{100} \times 12000 = 1200$$
$$Increased\ Salary = \underbrace{12000}_{Old\ Salary} + \underbrace{1200}_{Increase} = \underbrace{13200}_{Increased\ Salary}$$

Example 1.77

I read twelve books in Dec 2020. In every month of 2021, I am planning to read 50% more books than the number of books I read in Dec 2020. Find the number of books I will read during the year.

Books read every month in 2021:

$$= \underbrace{12}_{\substack{Dec\\2020}} + \underbrace{50\%\ of\ 12}_{Increase} = 12 + 6 = 18$$

Books read during the year

$$= \underbrace{18}_{\substack{Every\\Month}} \times \underbrace{12}_{\substack{No.of\\Months}} = 18 \times 10 + 18 \times 2 = 180 + 36 = 216$$

Example 1.78

The number of students who were members of a Math club last year was 40. This year the number of students increased by 40%. Vidya calculated that the number of students this year was

$$40 + 40 = 80$$

Was she correct?
If she was not correct, find out by how many students, her answer was wrong.

Vidya chose the correct number of students for last year. So, we need to check the increase.
The increase in the number of students

$$= 40\%\ of\ 40 = \frac{40}{100}\ of\ 40 = \frac{2}{5} \times 40 = 16$$

Hence, Vidya was not correct.
Her answer was wrong by:

$$\underbrace{40}_{\substack{Vidya's\\Increase}} - \underbrace{16}_{\substack{Correct\\Increase}} = 24\ \text{Students}$$

Challenge 1.79

I have three holiday homes, one in Switzerland, one in Hawaii, and the third in Italy. If the number of holiday homes I have increases by $33\frac{1}{3}\%$, then find the total number of holiday homes I have.

$$33\frac{1}{3}\% = \frac{100}{3}\% = \frac{100}{3} \times \frac{1}{100} = \frac{1}{3}$$

Increase in holiday homes:

$$= \frac{1}{3} \times 3 = 1 \text{ Home}$$

Total homes

$$= 3 + 1 \text{ home}$$

Challenge 1.80

The average number of daily customers visiting a restaurant in June was 100. In July, this number increased by 10%. In August, this number increased by a further 10%.

A. Jennifer calculated the increase in August as 10% of the number in June. Find the total number of customers in August as per Jennifer.
B. Mary calculated the increase in August as 10% of the number in July. Find the total number of customers in August as per Mary.
C. What is the difference between Jennifer's answer and Mary's answer?
D. Since the question does not specify whether the August increase was on June or July, which answer is correct?

Part A

$$Jennifer: \underbrace{100}_{June} + 10 = \underbrace{110}_{July} + 10 = \underbrace{120}_{July}$$

Part B

$$Mary: \underbrace{100}_{June} + 10 = \underbrace{110}_{July} + 11 = \underbrace{121}_{July}$$

Part C

$$121 - 120 = 1$$

Part D

Mary's answer was correct.

1.81: Adding a percentage

To add $x\%$ of y to y, we find:

$$y + y \times x\% = y + y \times \frac{x}{100} = y\left(1 + \frac{x}{100}\right)$$

D. Subtracting a percentage from a number

Example 1.82

Kiran has 160 toy cars in his collection. His sister has 10% fewer toy cars in her collection. Find the number of cars in his sisters' collection.

Subtraction Method

$$\underbrace{160}_{\substack{Kiran's\\ Cars}} - \underbrace{10\% \text{ of } 160}_{Less} = 160 - \frac{10}{100} \times 160 = 160 - 16 = 144$$

Multiplication Method

$$\underbrace{100\%}_{\substack{Kiran's\\ Cars}} - \underbrace{10\%}_{Less} = 90\%$$

Hence, the number of cars that Kiran has is:

$$90\% \text{ of } 160 = \frac{90}{100} \times 160 = 9 \times 16 = 144$$

Example 1.83

The cost of regular green kiwis is Rs. 10 per piece. The cost of yellow kiwis is 20% less than the cost of green kiwis. The cost of export quality kiwis is 20% more than the cost of regular kiwis. What is the difference between the cost of regular yellow kiwis, and the cost of export quality green kiwis?

$$Regular\ green\ kiwis = Rs.10\ per\ piece$$

Cost of regular yellow kiwis is 20% les than cost of regular green kiwis:

$$= 10 - 20\%\ of\ 10 = 10 - 2 = 8\ Rs.$$

Cost of export quality green kiwis is more 20% more than cost of regular green kiwis:

$$= 10 + 20\%\ of\ 10 = 10 + 2 = 12\ Rs.$$

Difference

$$= \underbrace{12}_{\substack{Export\\ Green\ Kiwis}} - \underbrace{8}_{\substack{Regular\\ Yellow\ Kiwis}} = 4$$

1.84: Subtracting a percentage

To add $x\%$ of y to y, we find:

$$y - y \times x\% = y - y \times \frac{x}{100} = y\left(1 - \frac{x}{100}\right)$$

E. Finding Costs

Example 1.85

Sara bought a dozen oranges at a cost of Rs. 2 per orange every week in June. In July, the number of oranges she bought increased by 50%. And the cost of oranges also increased by 50%. Assuming four weeks in a month, find the difference in the cost of oranges in the two months.

Cost of oranges in June

$$Every\ Week\ of\ June = 12 \times 2 = 24$$

Cost of oranges in July

Number of Oranges every week in July

$$= 12 + 50\% \text{ of } 12 = 12 + 6 = 18$$

Cost of oranges in July

$$= 2 + 50\% \text{ of } 2 = 2 + 1 = 3$$

Cost of Oranges in July

$$Every\ Week\ of\ July = 18 \times 3 = 54$$

Difference between the two in one week:

$$= 54 - 24 = 30$$

Difference between the two in four weeks:

$$= 30 \times 4 = 120$$

Example 1.86

Last year Adah was in school and she needed 30 notebooks during the school year at a cost of Rs. 50 per notebook. This year, she has moved to college, and the number of notebooks she will need has increased by 50%. The cost of notebooks has increased by 900%. Find the amount she will spend on notebooks this year.

Notebooks needed this year

$$= 30 + 50\%\ of\ 30 = 30 + 15 = 45$$

Cost of Notebooks

$$= 50 + 900\%\ of\ 50 = 50 + 9 \times 50 = 50 + 450 = 500$$

Total Cost of Notebooks

$$= 45 \times 500 = 22{,}500$$

Example 1.87

Last year Adah was in school and she needed 30 notebooks during the school year at a cost of Rs. 50 per notebook. This year, she has moved to college, and the number of notebooks she will need has become 50% of what she needed last year. The cost of notebooks has become 900% of what the cost was last year. Find the amount she will spend on notebooks this year.

Notebooks needed this year

$$= 50\%\ of\ 30 = 15$$

Cost of Notebooks

$$= 900\%\ of\ 50 = 9 \times 50 = 450$$

Total Cost of Notebooks

$$= 15 \times 450 = 6{,}750$$

F. Integer Solutions

Challenge 1.88

The number of walkers at a park is approximately 20% more than the number of joggers at a park. If the number of joggers is 72, find the number of walkers.

$$20\%\ of\ 72 = \frac{20}{100} \times 72 = \frac{1}{5} \times 72 = \frac{72}{5} = 14.4$$
$$72 + 14.4 = 86.4 \approx 86$$

Verification

$$\frac{86}{72} = 19.44\%\ more$$
$$\frac{87}{72} = 20.83\%\ more$$

86 gives the answer which is closest to 20%, and hence we should take that.

G. Finding Percentage Increase

1.89: Percentage Change

When finding a percentage change, we use the old value to calculate the change:

$$Percentage\ Change = \frac{New\ Value - Old\ Value}{Old\ Value} \times 100$$

- "Using the old value" means that the old value goes in the denominator, not the new value.
- If the new value is
  - ✓ greater, than there is an increase, which will result in percentage increase.
  - ✓ smaller, than there is a decrease, which will result in percentage decrease.

Example 1.90

A value decreases from 1.8 to 1.2.

A. Is it a percentage increase or a percentage decrease?
B. Find the percentage change.

Part A

$$Percentage\ Decreases$$

Part B

$$\%\ change = \frac{1.8 - 1.2}{1.2} \times 100 = \frac{0.6}{1.2} \times 100 = \frac{1}{2} \times 100 = 50\%\ decrease$$

Example 1.91

I have 50 toy cars in my collection. I get a gift of 20 cars. By what percent has my collection of cars increased?

Increase in cars was:

$$20$$

Percentage Increase

$$= \frac{20}{50} \times 100 = \frac{2}{5} \times 100 = 40\%$$

Example 1.92

I have 10 red cars. The number of red cars I have increases by 30%. I have 15 blue cars. The number of additional blue cars is the same as the number of additional red cars. What is the percentage increase in the number of blue cars?

The number of red cars increased

$$30\%\ of\ 10 = 3\ Cars$$

The number of additional blue cars

$$= No.of\ Additional\ Red\ Cars = 3$$

Percentage Increase in number of blue cars:

$$\frac{Increase}{Original} = \frac{3}{15} \times 100 = 20\%$$

Example 1.93

Answer each part independently

My trees produced 50 green apples and 80 red apples last year. By what percent must my production of

A. green apples increase to make it the same as my production of red apples?
B. red apples decrease to make it the same as my production of green apples?

Part A

Percentage Increase

$$= \frac{30}{50} \times 100 = \frac{3}{5} \times 100 = 60\%$$

Part B

Percentage Increase

$$= \frac{30}{80} \times 100 = \frac{3}{8} \times 100 = \frac{300}{8} = \frac{75}{2} = 37.5\%$$

Example 1.94

I got 15 marks out of 20 in my Physics test last month. This month I got 17 marks out of 20. By what percent did my marks increase:

A. as a percentage of the maximum marks
B. compared to last time?

Part A

$$\frac{Increase}{Maximum} = \frac{2}{20} \times 100 = 10\%$$

Part B

$$= \frac{2}{15} \times 100 = \frac{40}{3}\% = 13\frac{1}{3}\%$$

1.5 Multiplying Factors

A. Addition and Subtraction

Example 1.95

Fill in the remaining values in each row below.			
Value	+50%	+25%	−50%
100			
1			
50			

25			

Value	+50%	+25%	-50%
100	150	125	50
1	1.5	1.25	0.5
50	75	62.5	25
25	37.5	31.25	12.5

Example 1.96
Last year I travelled 250 days in the year. This year, I did not travel so much. In fact, this year I travelled 50% less than last year. How many days did I travel across both the years?

This Year

$$100\% - 50\% = 50\% = \frac{1}{2}$$

Total

$$250 + 250 \cdot \frac{1}{2} = 250\left(1 + \frac{1}{2}\right) = 375$$

B. Challenging Questions

Example 1.97
A has a salary of Rs. 10,000. B has a salary which is 20% more than A. C has a salary which is 110% of A's salary. Find the sum of the salaries of B and C.

C. Equivalent Representations

Example 1.98
A. What is x increased by 10% of itself?
B. When 20% of x is added to x, what is the answer?
C. A quantity x is increased by 25%. Find the increased quantity.
D. I have y bananas. My brother has 30% of the bananas I have. We take them to the market together. Find, in terms of y, the number of bananas that we take to the market.

$$x + \underbrace{10\%\ of\ x}_{\textit{Percentage Form}} = x + \underbrace{\frac{10}{100}x}_{\textit{Fractional Form}} = x + \frac{x}{10} = x + \underbrace{0.1x}_{\textit{Decimal Form}} = 1.1x$$

$$x + \underbrace{20\%\ of\ x}_{\textit{Percentage Form}} = x + \underbrace{\frac{20}{100}x}_{\textit{Fractional Form}} = x + \underbrace{0.2x}_{\textit{Decimal Form}} = 1.2x$$

Example 1.99

A. Subtract 10% of x from itself.
B. I have t toys. I donate 20% of the toys to charity. Find the number of toys I have now.

$$x - \underbrace{10\%\ of\ x}_{\substack{Percentage\\Form}} = x - \underbrace{\frac{10}{100}x}_{\substack{Fractional\\Form}} = x - \frac{x}{10} = x - \underbrace{0.1x}_{\substack{Decimal\\Form}} = 0.9x$$

$$t - \underbrace{20\%\ of\ t}_{\substack{Percentage\\Form}} = x - \underbrace{\frac{20}{100}t}_{\substack{Fractional\\Form}} = t - \underbrace{0.2t}_{\substack{Decimal\\Form}} = 0.8t$$

D. Equivalent Representations: Applications

Example 1.100
In a village near the sea, the number of sailboats is 20% less than the number of fishing boats. The number of fishing boats is b. Find, in terms of b, the total number of boats. Your answer should not have a percentage sign.

$$\underbrace{b}_{\substack{Fishing\\Boats}} + \underbrace{0.8b}_{Sailboats} = 1.8b$$

Example 1.101
Charlie hikes h km every month. His sister hikes 20% more km than him, while his brother hikes 30% fewer km than him. Find, in terms of h, the total number of km that the three siblings hike.

The number of km that Charlie's sister hikes:

$$h + 20\%\ of\ h = h + \frac{20}{100}h = h + 0.2h = 1.2h$$

The number of km that Charlie's brother hikes:

$$h - 30\%\ of\ h = h - \frac{30}{100}h = h - 0.3h = 0.7h$$

$$h + 1.2h + 0.7h = 2.9h$$

E. Multiplying Factors for Addition
It is very convenient to convert addition into multiplication. This is done by means of multiplying factors, which are equivalent to addition.

Example 1.102
The price of a share which was Rs. 250 appreciated by 20%. Find the increased share price.

Method I: Find the increase, and then add

$$Increase = \frac{20}{100} \times 250 = 50 \Rightarrow Increased\ Price = 250 + 50 = 300$$

Method II: Find the single multiplying factor and then multiply
Consider adding 20% to a quantity.
We must start with 100% of the quantity, and we will add 20% to it, which means that our final answer will be:

$$\underbrace{100\%}_{\textit{Original Number}} + \underbrace{20\%}_{\textit{Increase}} = \underbrace{120\%}_{\textit{Final Value}}$$

And multiplying factors are often given as a decimal, so we convert it into a decimal:

$$1.2$$

And we find 1.2 times of the original share price:

$$250 \times 1.2 = 300$$

F. Calculating Multiplying Factors

We can find a single multiplying factor for a number that is the same as adding a percentage

Example 1.103

Find the multiplying factor for each of the following:

A. Adding 10%
B. Adding 20%
C. Adding 25%
D. Adding 175%

$$10\% + 100\% = 110\% = 1.1$$
$$20\% + 100\% = 120\% = 1.2$$
$$25\% + 100\% = 125\% = 1.25$$
$$175\% + 100\% = 275\% = 2.75$$

G. Multiplying Factors for Subtraction

Find the multiplying factor for each of the following:

Example 1.104

Find what is the value of 300 when it is reduced by 20% of itself using multiplying factors.

$$300 - 20\%\ of\ 300 = 300 - 0.2 \times 300 = 300(1 - 0.2) = 300(0.8) = 240$$

Example 1.105

Find the multiplying factor for subtracting each of the following:

A. Subtract 10%
B. Subtract 20%
C. Subtract 25%

$$100\% - 10\% = 90\% = 0.9$$
$$100\% - 20\% = 80\% = 0.8$$
$$100\% - 25\% = 75\% = 0.75$$

H. Using Multiplying Factors

Example 1.106

The price of a commodity was Rs. 100. It increased by 10%, and then again by 10%. Find the increased price.

Method I: Find the increase, and then add

We need to do this in two steps.

First Step: Add 10% to 100:

$$\underbrace{100+\frac{10}{100}\times 100=100+10=\mathbf{110}}_{\textit{First Increase}}$$

Then, add 10% to the value that we find:

$$\underbrace{110+\frac{10}{100}\times 110=110+11=\mathbf{121}}_{\textit{Second Increase}}$$

Method II: Find the single multiplying factor and then multiply

If we use multiplying factors, we can find the answer in a single step.
Recall that

$$\textit{Adding } 10\% \Rightarrow \textit{Multiplying by } 1.1$$

Hence, we multiply by 1.1 twice:

$$100\times 1.1\times 1.1=1\mathbf{00}\times\frac{11}{1\mathbf{0}}\times\frac{11}{1\mathbf{0}}=121$$

Example 1.107

The number 90 is reduced by 10% and the new number obtained is then further reduced by 90%. Find the final number.

Convert into multiplying factors:

$$\textit{Reducing by } 10\% \Rightarrow \textit{Multiplying by } 0.9$$
$$\textit{Reducing by } 90\% \Rightarrow \textit{Multiplying by } 0.1$$

And now that we have the two multiplying factors, we can find the answer in a single calculation:

$$90\times \underbrace{0.9}_{\substack{\textit{Reducing}\\ \textit{by } 10\%}}\times \underbrace{0.1}_{\substack{\textit{Reducing}\\ \textit{by } 90\%}}=8.1$$

Example 1.108

X has a salary of Rs. 20,000. His salary increases by 10% every year for four years.

A. After the fourth increase, what will be his new salary?
B. Calculate the percentage increase after the fourth increase compared to his original salary of 20,000.

$$\text{Multiplying factor}=1.1\times 1.1\times 1.1\times 1.1=\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}\times\frac{11}{10}=\frac{14641}{10{,}000}=1.4641$$

$$\text{New Salary}=20{,}000\times 1.4641=29{,}282$$

$$\text{B. Percentage Increase}=\frac{\text{New Value}-\text{Old Value}}{\text{Old Value}}=\frac{1.4641-1}{1}=0.4641=46.41\%$$

1.6 Unitary Method

A. Proportionality Concept

Consider

$$x\%\ of\ y$$

The proportionality concept, which applies to percentage, means that

- As y
  - ✓ increases, the final answer increases
  - ✓ decreases, the final answer increases
- As x
  - ✓ increases, the final answer increases

- ✓ decreases, the final answer increases

B. Finding the Original Number

Example 1.109

A. 50% of the students in a class are girls. There are 20 girls in the class. What is the total number of students in the class?
B. 25% of the students in a class play basketball. If the number of students who play basketball is 4, what is the total number of students in the class.
C. 25% of the people in a class are wearing red. The class has students and a teacher. If the number of people wearing red is 12, find the number of students in the class?

Part A

$$50\%\ of\ students = 20$$

Multiply both sides by 2:

$$100\%\ of\ students = 40$$
$$Total\ Number\ of\ Students = 40$$

Part B

$$25\%\ of\ students = 4$$
$$100\%\ of\ students = 16$$

Part C

$$25\%\ of\ people = 12$$
$$100\%\ of\ people = 48$$
$$No.of\ students = 48 - 1 = 47$$

Example 1.110

75% of the hiking trails on a mountain are closed in the winter. Irene arrives at the mountain, and finds that there are 12 trails closed.

A. What is the number of open trails?
B. What is the total number of trails?

Part A

$$Closed\ Trails = 75\% \Rightarrow Open\ Trails = 100 - 75 = 25\%$$

$$75\%\ of\ trails = 12$$

Divide both sides by 3:

$$Open\ Trails = 25\%\ of\ trails = 4$$

Part B

Total Number of Trails

$$= Closed\ Trails + Open\ Trails = 12 + 4 = 16$$

1.111: Using Variables

We can variables to make the presentation more systematic.

Example 1.112

A. 50% of the people in a neighborhood would like a new park to be built. If the number of people who would like a new park to be built is 1000, what is the total number of people in the neighborhood?
B. 25% of the students (that is, 10 students) in a class learn Mandarin Chinese. What is the total number of

students in the class?

C. A recessive gene is present in 10% of the population. If 100 people in an apartment complex have the gene, how many people in all are there in the complex?

Part A

Let the number of people be p.

$$50\%\ of\ p = 1000 \Rightarrow \frac{1}{2} \times p = 1000 \Rightarrow p = 2000$$

Part A

Let the number of students be s.

$$25\%\ of\ s = 10 \Rightarrow \frac{1}{4}s = 10 \Rightarrow s = 40$$

Part A

Let the number of people be p.

$$10\%\ of\ p = 100$$

Multiply both sides by 10:

$$100\%\ of\ p = 1000$$
$$p = 1000$$

C. Unitary Method

In the previous set of examples, we were able to multiply to get the value that we wanted. However, that may not always be possible.

In certain scenarios, we divide to "bring down" the value that we have been given, and then multiply.

Example 1.113

A. 40% of a number is 60. Find 60% of the number.

B. Ralph got 40% marks in an exam, and his score was 60. His sister got 60% marks in the same exam. Find her score.

Part A

$$40\%\ of\ the\ number = 60$$

Divide both sides by 4 to "bring it down" to 10%:

$$10\%\ of\ the\ number = 15$$

Multiply both sides by 6 to take it to 60%:

$$60\%\ of\ the\ number = 90$$

Part B

As above,

$$60\%\ of\ the\ number = 90$$

Example 1.114

Example 1.115

Reducing x by 10% makes it 100. Find the value of x when 10% is added to it.

Strategy

Originally, x must be 100%. Reducing it by 10% makes it:

$$100\% - 10\% = 90\%$$

On the other hand, we actually want to find:

$$100\% + 10\% = 110\%$$

Calculation

$$90\% \; of \; x = 100$$

Divide both sides by 90 to get 1%:

$$1\% \; of \; x = \frac{100}{90} = \frac{10}{9}$$

Multiply both sides by 110 to get 100%:

$$110\% \; of x = \frac{10}{9} \times 110 = \frac{1100}{9} = 122\frac{2}{9}$$

Example 1.116

Reducing a number by 20% makes it 40. What is the original number.

$$Original \; Number = 100\%$$
$$New \; Number = 100\% - 20\% = 80\%$$

$$80\% \; of \; the \; Number = 40$$
$$20\% \; of \; the \; number = 10$$
$$100\% \; of \; the \; number = 50$$

Example 1.117

Reducing a number by 40% makes it 60. What is the original number.

$$Original \; Number = 100\%$$
$$New \; Number = 100\% - 40\% = 60\%$$

$$60\% \; of \; the \; number \; = 60$$
$$10\% \; of \; the \; number = 10$$
$$100\% \; of \; the \; number = 100$$

Example 1.118

Increasing a number by 60% makes it 80. What is the original number?

Originally the number was:

$$100\%$$

Increasing it by 60% makes it:

$$100\% + 60\% = 160\%$$

And, we know that:

$$160\% \; of \; the \; number = 80$$

Divide by 80 both sides:

$$2\% \; of \; the \; number = 1$$

Multiply by 50 both sides:

$$100\% \; of \; the \; number = 50$$

Example 1.119

A. 43% of the cost of a bag is Rs. 129. Find the cost of the bag.
B. 40% of the cost of a jackfruit is Rs. 20. Find the cost of a single jackfruit.
C. 30% of the cost of a pair of shoes is Rs. 600. Find the cost of the pair of shoes.

Part A

$$43\% \; is \; 129$$

Divide both sides by 43:

$$1\% \; is \frac{129}{43} = 3$$

Multiply by 100 both sides:

$$100\% = Rs. 300$$

Part B

$$40\% \; is \; Rs. 20$$
$$1\% \; is \; Rs \frac{20}{40} = \frac{1}{2}$$
$$100\% = 100 \times \frac{1}{2} = 50$$

Part C

$$30\% \; is \; Rs. 600$$
$$1\% \; is \; Rs \frac{600}{30} = 20$$
$$100\% = 100 \times 20 = Rs. 2000$$

Example 1.120

A petrol drum with capacity 20 liters is 25% empty. If 5 Liters of petrol is emptied from the drum, and petrol is available at Rs. 100 per liter, what is the cost of the petrol inside the drum?

$$25\% \; empty = 75\% \; Full$$
$$75\% \; of \; 20 = \frac{3}{4} \times 20 = 15 \; Litres$$
$$15 \; Liters - 5 \; Litres = 10 \; Litres$$
$$10 \times 100 = Rs. 1000$$

Example 1.121

When a barrel is 40% empty, it contains 80 litres more than when it is 20% full. The full capacity of the barrel (in litres) is: **(NMTC Sub-Junior/Screening 2011/Part B/1)**

$$40\% \; Empty = 60\% \; Full$$

$$60\% \; Full - 20\% \; Full = 80 \; Litres$$
$$40\% \; Full = 80 \; Litres$$

Divide both sides by 40:

$$1\% \; Full = 2 \; Litres$$

Multiply both sides by 100:

$$100\% \; Full = 200 \; Litres$$

(Alternate Solution using Algebra) Example 1.122

When a barrel is 40% empty, it contains 80 litres more than when it is 20% full. The full capacity of the barrel (in litres) is: **(NMTC Sub-Junior/Screening 2011/Part B/1)**

Let the capacity of the barrel be x litres.
40% empty = 60% full = 0.6x
20% full = 0.2x
And this difference is equal to 80 litres

$$0.6x - 0.2x = 80 \Rightarrow 4x = 800 \Rightarrow x = 200$$

Example 1.123

When a barrel is 30% empty, it contains 50 litres more than when it is half full. The milk in the barrel costs $Rs.$ 10 per liter. What is the cost of five barrels of milk?

$$30\%\ Empty = 70\%\ Full$$

$$70\%\ Full - 50\%\ Full = 50\ Litres$$
$$20\%\ Full = 50\ Litres$$
$$100\%\ Full = 250\ Litres$$
$$Cost\ of\ a\ barrel = 10 \times 250 = 2500$$
$$Cost\ of\ 5\ barrels = 5 \times 2500 = 12{,}500\ Rs.$$

2. Applications

2.1 Discount

A. Discount

Stores will show the selling price of an item using item tag. This selling price is important. There are a number of changes that are applicable to selling price.

2.1: Selling Price

The price at which at item is sold is called the selling price.

$$Selling\ Price = SP$$

2.2: Discount

Discount is a reduction in the selling price of an item.

$$Discounted\ Price = DP = SP - Discount$$
$$Discount = SP - DP$$

Example 2.3

A. A toy car is sold in a shop for $Rs.$ 100 per car. On a sale for Christmas, the price of the car after discount is $Rs.$ 80. Find the discount.
B. Bananas are available at the greengrocers for $Rs.$ 15 per dozen. On a sale, the price is discounted to $Rs.$ 12 per dozen. Find the discount per banana.
C. Apples are available at Walmart in crates of 10 apples per crate. The cost of a crate is $15. On a weekend sale, the price of a crate is reduced to $10. Find the discount per apple.

Part A

$$Discount = 100 - 80 = 20$$

Part B

The discount

$$= SP - DP = \underbrace{15}_{\substack{Per\\Dozen}} - \underbrace{12}_{\substack{Per\\Dozen}} = 3\ per\ dozen$$

The discount per banana

$$= \frac{3}{12} = \frac{1}{4}\ Rs = 25\ paise\ per\ banana$$

Part C

The discount

$$= SP - DP = \underbrace{15}_{\substack{Per\\Dozen}} - \underbrace{10}_{\substack{Per\\Dozen}} = 5\ per\ crate$$

The discount per apple

$$= \frac{5}{10} = 0.5\ \$ = 50\ cents\ per\ apple$$

Example 2.4

Oranges cost $Rs.$ 18 per dozen. If you buy two dozen, there is a discount of $Rs.$ 2 on the entire transaction.

A. How much did you save per dozen?
B. How much did you save per orange?
C. What is the price per dozen?

Part A

The discount

$$= Rs.2 \ per \ 2 \ dozen$$
$$= Rs.1 \ per \ 1 \ dozen$$
$$= Rs.\frac{1}{12} \ per \ orange$$

Part B

The discounted price

$$= \underbrace{18}_{\substack{Per \\ Dozen}} - \underbrace{1}_{\substack{Per \\ Dozen}} = 17$$

Example 2.5

A coat had a price tag of $100. It was put on discount for an off-season sale at $\frac{9}{10}^{th}{}'s$ of its original price. Find:

A. The discounted price
B. The discount

$$Discounted \ Price = \frac{9}{10}^{th} \ of \ 100 = \frac{9}{10} \times 100 = 90$$
$$Discount = 10$$

Example 2.6

The selling price of a mango is Rs. 20. Due to discount, the price is brought down to Rs. 18. Find:

A. Discount per mango
B. Discount on a crate of 20 mangoes
C. Selling Price of 12 mangoes after discount

The discount per mango

$$= 20 - 18 = Rs.2$$

The discount on 20 mangoes

$$= 2 \times 20 = Rs.40$$

The selling price of 12 mangoes are discount

$$= \underbrace{12}_{\substack{No.of \\ Mangoes}} \times \underbrace{18}_{\substack{Price \\ per \ Mango}} = 216$$

Example 2.7

The selling price of a crate of dozen mangoes was Rs. 600. A customer bargained and brought the price down to Rs. 480. Find:

A. The total discount offered to the customer
B. The selling price per mango (before discount)
C. The discounted selling price per mango
D. The discount per mango

$$Total \ Discount = SP - DP = 600 - 480 = 120 \ Rs.$$
$$SP \ per \ Mango = \frac{Total \ Price}{No. of \ Mangoes} = \frac{600}{12} = 50$$
$$DP \ per \ Mango = \frac{Total \ DP}{No. of \ Mangoes} = \frac{480}{12} = 40$$

$$Discount\ per\ Mango = \frac{Total\ Discount}{No.\ of\ Mangoes} = \frac{120}{12} = 10$$

Example 2.8
Mangoes were being sold for Rs. 840 per dozen. Since a customer bought two dozen mangoes, the price was reduced to Rs. 1668. What is the discount offered per mango?

$$Discount\ per\ Mango = \frac{840 \times 2 - 1668}{2 \times 12} = \frac{1680 - 1668}{24} = \frac{12}{24} = 0.5\ Rs. = 50\ paise$$

Example 2.9
A coat with a price of Rs. 330 has a discount of 10%. Find the discount, and the discounted price.

$$\text{Discount } = \frac{10}{100} \times 330 = 33 \Rightarrow \text{DP} = \text{SP } - \text{ Discount} = 330 - 33 = 297$$

Example 2.10
A shirt with original price Rs. 800 was offered at 20% off. Find the discounted price.

$$20\%\ off = 20\%\ less = 80\%\ of\ original = 0.8 \times 800 = 640$$

Example 2.11
A car had a price tag of $90,000, when new. It is sold a year after being used, at a price 50% of its original price. Find the discounted price, and the discount.

$$Discounted\ Price = \$45{,}000$$
$$Discount = \$45{,}000$$

B. Discount Percent

2.12: Discount Percent
$$Discount\ Percent = \frac{SP - DP}{SP} = \frac{Discount}{Selling\ Price}$$
$$D = Discount$$
$$SP = Selling\ Price$$
$$DP = Discounted\ Price$$

Example 2.13
A blazer which had a price sticker of Rs. 900 was sold at a price of Rs. 720. Find the discount percent.

$$Discount\ \% = \frac{SP - DP}{SP} = \frac{900 - 720}{900} = \frac{180}{900} = \frac{20}{100} = 20\%$$

Example 2.14
A hotel room is offered for rent at Rs. 1000 per night. A customer asks for it to be reduced to Rs. 5600 for seven nights. After negotiation, the hotel agrees to offer half the discount the customer has asked for. Find the discount percent.

We can calculate on a per night basis:

$$5600\ for\ 7\ Nights = \frac{5600}{7} = 800\ per\ Night$$

The discount percent requested by the customer is:

$$= \frac{1000 - 800}{1000} = \frac{200}{1000} = 20\%$$

And the discount finally arrived at is half of the above discount:

$$= Half\ of\ 20\% = \ 10\%$$

Example 2.15

Violet buys seven dozen oranges from a wholesaler. Oranges are sold at Rs. 50 per dozen. However, 12 oranges are smaller than the usual, so the wholesaler agrees to give them at half price. Find the discount percent on the entire transaction.

The original price of the transaction

$$= 7 \times 50 = Rs.350$$

For the discount, six dozen oranges are sold at regular price. However, the seventh dozen is sold at half price. Hence, the discount is

$$\frac{50}{2} = 25$$

$$Discount\% = \frac{25}{350} = \frac{1}{14} = \frac{1}{14} \cdot \frac{100}{100} = \frac{100}{14}\% = \frac{50}{7}\%$$

Example 2.16

Magical mushrooms are sold at a price of Rs. 10 per mushroom. However, if you buy 10 mushrooms, you get a discount of Rs. 1 per mushroom. If you buy 20 mushrooms, you get a discount of Rs. 2 per mushroom. If you buy 30 mushrooms, you get a discount of Rs. 3 per mushroom, and so on. How many mushrooms must I buy so that they are all free?

Quantity	10	20	30	.	.	.	100
Discount per Mushroom	1	2	3	.	.	.	10
Price per Mushroom	9	8	7	.	.	.	0

Example 2.17

Five men can dig five holes in five days, and each man charges $100 per day. Tanisha asked for ten holes to be dug, and got a 10% discount on the overall cost. What is the money that she paid?

Cost of digging 5 holes

$$= \underbrace{5}_{Men} \times \underbrace{5}_{Day} \times 100 = \$2500$$

Cost of digging ten holes:

$$= 2 \times 2500 = \$5000$$

Discount that she got is:

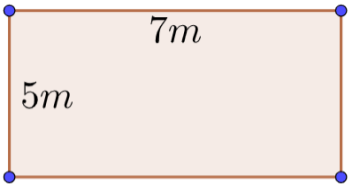
$$10\%\ of\ 5000 = 500\$$$

Amount that she paid is:

$$5000 - 500 = 4500$$

Example 2.18

Barbed wire is to be run around the fence on a rectangular garden three times. The length of the garden is $7m$, and the width of the garden is $5m$. The cost of the wire is 50 cent per meter. If the vendor offers a discount of 20% on the final price, what is the cost of the wire, in dollars?



The length of the fence

$$= Perimeter\ of\ the\ Garden = 2(7+5) = 2(12) = 24m$$

Length of wire that I need is:

$$= 3 \times Perimeter = 3 \times 24 = 72$$

Original Cost of Wire

$$= \frac{1}{2} \times 72 = \$36$$

Discount on the wire

$$= \frac{20}{100} \times 36 = \frac{1}{5} \times 36 = \frac{36}{5} = 7\frac{1}{5} = 7.2$$

Discounted price of the wire

$$= 36 - 7.2 = 28.8$$

Example 2.19

Regular members of a Maths Society get a discount of 10% on the prices of its book. Lifetime members of the society get a discount of 15% of on the prices of the books. You buy books worth 700 dollars. What is the additional discount that you get for being a lifetime member, as compared to a regular member?

Method I

$$15\%\ of\ 700 = \frac{15}{100} \times 700 = 105$$

$$10\%\ of\ 700 = \frac{10}{100} \times 700 = 70$$

$$Difference = 105 - 70 = 35$$

Method II

Additional discount percent

$$= 15 - 10 = 5\%$$

Additional discount

$$= \frac{5}{100} \times 700 = 35$$

C. Successive Discounts

Example 2.20

A mobile was being sold at \$200. A discount of 20% was offered on it. After that a discount of a further 10% was offered on the discounted price. Find the final price, and the final discount percent.

Original price of the mobile is 200. First discount is

$$= \frac{20}{100} \times 200 = 40$$

Second discount was

$$= \frac{10}{100} \times (200 - 40) = \frac{10}{100} \times 160 = 16$$

Final Price

$$= 160 - 16 = 144$$

Final Discount Percent

$$= \frac{Final\ Discount}{Original\ Price} = \frac{200 - 144}{200} = \frac{56}{200} = \frac{28}{100} = 28\%$$

Shortcut Solution

$$First\ Discount\ is\ 20\% \Rightarrow Discounted\ Price\ is\ 100 - 20 = 80\%$$
$$Second\ \ Discount\ is\ 10\% \Rightarrow Discounted\ Price\ is = 90\%$$

Hence, we find:

$$90\%\ of\ 80\%\ of\ 200 = \frac{90}{100} \times \frac{80}{100} \times 200 = 2 \times 8 \times 9 = 144$$

Example 2.21

A painting was offered for a million dollars. Since it was not sold, the painter offered a discount of 10% on it. A customer offered to buy it at a further 20% discount (on the discounted price). Find the price that the customer offered, and the discount on the original price that the customer asked for?

Price of the painting is 1,000,000. After discount, the price will be:

$$= \frac{90}{100} \times 1{,}000{,}000 = 900{,}000$$

The customer is willing to give $100\% - 20\% = 80\%$:

$$\frac{80}{100} \times 900{,}000 = 8 \times 90{,}000 = 720{,}000$$

Discount on original price

$$= \frac{1{,}000{,}000 - 720{,}000}{1{,}000{,}000} = \frac{280{,}000}{1{,}000{,}000} = \frac{28}{100} = 28\%$$

Example 2.22

Tablecloths are sold on the basis of their area. A blue tablecloth costs 70$. A red table cloth has area 50% of the blue table cloth. A green table cloth has area 50% of the red table cloth. A yellow table cloth has area 50% of the green table cloth. You buy a table cloth of each type. Find the total amount you need to pay.

Cost of blue tablecloth

$$= 70$$

Cost of red tablecloth

$$= \frac{50}{100} \times 70 = 35$$

Cost of green tablecloth

$$= \frac{50}{100} \times 35 = \frac{35}{2} = 17\frac{1}{2} = 17.5$$

Cost of yellow tablecloth

$$= \frac{50}{100} \times \frac{35}{2} = \frac{35}{4} = 8\frac{3}{4} = 8.75$$

Total Cost

$$= 70 + 35 + 17.5 + 8.75 = 131.25$$

Example 2.23

A blue tablecloth costs 70$. A red table cloth is offered at 50% discount to the blue tablecloth. A green table cloth is offered at a discount of 50% to the red table cloth. A yellow table cloth is offered at discount of 50% to the green table cloth. You buy a table cloth of each type. Find the total amount you need to pay.

Cost of blue tablecloth

$$= 70$$

Cost of red tablecloth

$$= 35$$

Cost of green tablecloth

$$= 17.5$$

Cost of yellow tablecloth

$$= 8.75$$

Total Cost

$$= 70 + 35 + 17.5 + 8.75 = 131.25$$

Example 2.24

Successive discounts of 10% and 20% are equivalent to a single discount of: **(AHSME 1950/22)**

Suppose the original price is 100.
The discount percent is 10. Hence the price will be $100 - 10 = 90\%$

$$= \frac{90}{100} \times 100 = 90$$

Second discount percent is 20. Hence, the price is $100 - 20 = 80\%$

$$\frac{80}{100} \times 90 = 8 \times 9 = 72$$

Hence, the final discount percent

$$= \frac{100 - 72}{100} = \frac{28}{100} = 28\%$$

Shortcut Method

$$100 \times \frac{90}{100} \times \frac{80}{100} = 1 \times 9 \times 8 = 72 \Rightarrow 28\%\ Discount$$

Example 2.25

A. Successive discounts of 50% and 50% are equivalent to a single discount of:
B. Successive discounts of 20% and 30% are equivalent to a single discount of:

Suppose the original price is 100:

$$\frac{50}{100} \times \frac{50}{100} \times 100 = 25 \Rightarrow Discount\ is\ 75\%.$$

$$\frac{80}{100} \times \frac{70}{100} \times 100 = 8 \times 7 = 56 \Rightarrow Discount\ is\ 44\%$$

D. Original Price

2.26: Original Price

$$\underbrace{SP}_{\text{Selling Price}} = \underbrace{DP}_{\text{Discounted Price}} + D$$

Example 2.27
A blazer is sold for Rs. 950 after offering a discount of 5% on the selling price. Find the original price of the blazer.

Method I: By Observation
950 is very close to 1000. Try 1000.

$$5\%\ of\ 1000 = \frac{5}{100} \times 1000 = 50$$
$$Discounted\ Price = 1000 - 50 = 950 \Rightarrow Works$$

Method I: By Calculation
Original Price is 100%. From the original price, 5% discount is given. So,

$$New\ Price = Original\ Price - Discount = 100\% - 5\% = 95\%$$

$$95\%\ of\ Original\ SP = 950$$

Divide both sides by 95:

$$1\%\ of\ Original\ SP = \frac{950}{95} = 10$$

Multiply both sides by 100:

$$100\%\ of\ Original\ SP = SP = 10 \times 100 = 1000$$

2.2 Markup

A. Basics

2.28: Mark-Up
Mark-up is the increase that a shop adds to its cost price in order to earn a profit.

$$SP = CP + Markup$$

2.29: Mark-Up Percent
The markup can be calculated as a percentage of the cost price:

$$Markup\ Percent = \frac{Markup}{Cost\ Price}$$

Example 2.30: Basics
A shopkeeper buys grain at Rs. 50 per kilo, and sells it at Rs. 72.

A. What is the markup?
B. What is the markup percent?

$$Markup = 72 - 50 = Rs.22$$
$$Markup\ Percent = \frac{Markup}{Cost\ Price} = \frac{22}{50} = \frac{44}{100} = 44\%$$

Example 2.31
A trader buys a fruit at Rs. 5 per piece, and applies a markup of 10% to the fruit. Find the selling price of a dozen fruits.

Method I

Price of a dozen fruit

$$= 12 \times 5 = 60$$

Mark up

$$= \frac{10}{100} \times 60 = 6$$

Final Selling Price

$$= 60 + 6 = 66$$

Method II

You can also add the markup to a single fruit, but the calculations are longer.

Mark up is 10%

$$= \frac{10}{100} \times 5 = \frac{1}{2}$$

Selling price per fruit

$$= 5 + \frac{1}{2} = 5\frac{1}{2} = \frac{11}{2}$$

Selling price of a dozen fruits

$$= 12 \times \frac{11}{2} = 6 \times 11 = 66$$

Example 2.32

A shopkeeper who trades in pets buys fish for $12 each, and has a markup of 25%. What is the price of twelve fish?

$$SP\ of\ one\ fish = 12 + 25\%\ of\ 12 = 12 + 3 = 15$$
$$SP\ of\ 12\ Fish = 15 \times 12 = \$180$$

2.33: Profit

$$Profit = Markup$$

The markup is above your cost price, and hence the markup should be the same as your profit for simple questions.

Example 2.34: Comparing Markup

Bananas are marked up 10% to their cost price, while mangoes are marked up 15% on their cost price. A shop buys Rs. 2000 worth of bananas, and Rs. 1000 worth of mangoes. From which fruit will the shop earn more profit and by how much?

Profit on bananas

$$= Markup\ on\ Bananas = 10\%\ of\ 2000 = 200$$

Profit on mangoes

$$= 15\%\ of\ 1000 = \frac{15}{100} \times 1000 = 150$$

The shop will earn more profit from the bananas and the profit is more by:

$$200 - 150 = Rs.\ 50$$

B. Successive Mark-up

Example 2.35

A farmer sells corn to a wholesaler. The wholesaler sells it to a grocery store at a markup of 10%. The grocery store sells it to retail customers at a markup 10% on its cost price. The farmer sold grain to the wholesaler for $500. Find the price at which the retail customer will buy it.

	Farmer	Wholesaler	Grocer
Cost Price		500	550
Selling Price	500	550	605

$$500 + 10\%\ of\ 500 = 500 + 50 = 550$$
$$550 + 10\%\ of\ 550 = 550 + 55 = 605$$

Example 2.36

Is a single markup of 20%, the same as a markup of 10%, followed by another markup of 10%.

No. Consider the same number as in the previous example, which is $500

Single markup of 20% will mean

$$500 + 20\%\ of\ 500 = 500 + 100 = 600$$

Two successive markups of 10% each will be

$$500 + 10\%\ of\ 500 = 500 + 50 = 550$$
$$550 + 10\%\ of\ 550 = 550 + 55 = 605$$

Example 2.37

Nita buys ingredients for chocolate at $20 per kilo. She makes homemade chocolate, and sells it to cafes at a markup of 25%. The cafes sell it to customers at a markup of 20% on their cost price. 1 kilo of chocolate contains six chocolate bars. Find the cost of a chocolate bar for the customer.

Cost price for the café:

$$20 + 25\%\ of\ 20 = 20 + 5 = 25$$

Cost price for the customer per kilo

$$25 + 20\%\ of\ 25 = 25 + 5 = 30$$

Cost price for the customer per chocolate bar:

$$\frac{30}{6} = \$5$$

Example 2.38

A greengrocer buys onions from a farmer's market at $200 per quintal. He sells to customers at a markup of 20%. A bistro buys the onions from the greengrocer to make onion rings. The onion rings have a markup of 350%. Find the cost to the bistro's customers.

Selling price for the greengrocer

$$= 200 + 20\%\ of\ 200 = 200 + 40 = 240$$

Selling price for the bistro

$$= 240 + 350\%\ of\ 240$$

$$= 240 + \frac{350}{100} \times 240$$
$$= 240 + 3.5 \times 240$$
$$= 240 + 35 \times 24$$
$$= 240 + 840$$
$$= 1080$$

2.3 Tips and Taxes

A. Taxes

2.39: Taxes

$$Final\ Price = SP + Tax$$

Where

$$SP = Selling\ price\ before\ tax$$

Taxes result in an increase in the price of an item.
They are charged on the selling price of an item.

One form of tax, which is applicable currently is:

$$Goods\ and\ Services\ Tax (GST)$$

2.40: Taxes with Discount

If there is a discount, taxes are always paid on the discounted price (unless the question mentions otherwise).

Example 2.41: Finding Tax and Final Price

Find the tax and the final price of the item in each case below:

A. A watch has a price tag of Rs. 2000, and 7% tax on selling price.
B. A phone with a base price of $600 has 3% tax.
C. A dress with a base price of Rs. 8000 has a 6% tax.
D. A computer has a selling price of $800. Tax at 9% is payable on the selling price.
E. A mountaineering company charges $250 for a nature trail per person. Tax at 4% of selling price is applicable. A family of three goes for the nature trail.
F. A hotel room in a hill station is available on rent for $100 per night during peak season. During off season, there is a 60% discount since very few customers are available. Also, there is a 10% tax on the price after discount. A customer stayed for a week in off season.
G. A pencil has a price tag of Rs. 2. Tax of 4.5% is payable on the selling price. What is the cost of a dozen pencils?

Part A: Watch

$$Tax = 7\%\ of\ SP = \frac{7}{100} \times 2000 = 140$$
$$Final\ Price = \underbrace{2000}_{\substack{Selling \\ Price}} + \underbrace{140}_{Tax} = 2{,}140\ Rs.$$

Part B: Phone

$$Tax = 3\%\ of\ 600 = \frac{3}{100} \times 600 = 18$$
$$Final\ Price\ = 600 + 18 = \$618$$

Part C: Dress

$$Tax = 6\%\ of\ 8000 = \frac{6}{100} \times 8000 = 480$$
$$Final\ Price = 8000 + 480 = 8480$$

Part D: Computer

$$Tax = 9\%\ of\ 800 = \frac{9}{100} \times 800 = 72$$
$$Final\ Price = 800 + 72 = 872$$

Part E: Computer

We calculate per person:

$$Tax = 4\%\ of\ 250 = \frac{4}{100} \times 250 = 10$$
$$Final\ Price = 250 + 10 = 260$$

For a family of three, the final price will be:

$$260 \times 3 = 780$$

Part F: Hotel

First, find the price for one night:

$$Discount = 60\%\ of\ 100 = 60$$
$$Price\ After\ Discount = 100 - 60 = 40$$
$$Tax = 10\%\ of\ 40 = \frac{10}{100} \times 40 = 4$$
$$Final\ Price = 40 + 4 = 44$$

The customer stayed for

$$1\ Week = 7\ Nights$$

Hence, the cost for seven nights

$$= 7 \times 44 = 308$$

Part G: Pencil

Calculate for 1 Pencil:

$$1\ Pencil = Rs.\ 2 = 200\ Paise$$
$$Tax = \frac{4.5}{100} \times 200 = \frac{900}{100} = 9\ Paise$$

$$Final\ Price = 200 + 9 = 209\ Paise$$

For a dozen pencils:

$$= 12 \times 209 = 2508\ Paise = 25.08\ Rs.$$

Example 2.42

Find the price before tax in the following situations:

A. If the tax rate is 10%, and a customer pays $50 in tax.
B. If the tax rate is 5%, and a customer pays 20 Rs. in tax.
C. If the tax rate is 7%, and a customer pays 14 Rs. in tax.
D. If the tax rate is 4%, and a customer pays 44 Rs. in tax.
E. A customer bought a doll for Rs. 210, inclusive of tax at 5%. What was the price before tax?
F.
G. A necklace was put on sale for a certain amount in 2010, but there were no buyers. It was offered again on sale in 2011, at a price 10% more than the price in 2010. It was sold in 2011, and the buyer paid the price after tax of 121 dollars. If the tax rate was 10%, what was the price offered in 2010?

Part A

$$Tax = 10\% = 50$$

$$Bill = 100\% = 50 \times 10 = \$500$$

Part B

$$5\% \; of \; Price = 20$$

Multiply both sides by 20:

$$(5\%)20 \; = 20(20)$$
$$100\% \; = 400$$

Part C

$$7\% \; is \; Rs. 14$$

Divide by 7:

$$1\% \; is \; Rs. 2$$

Multiply by 100:

$$100\% \; is \; Rs. 200$$

Part D

$$4\% \; is \; Rs. 44$$
$$1\% \; is \; Rs. 11$$
$$100\% \; is \; Rs. 1{,}100$$

Part E

$$210 \; is \; 105\%$$

Divide both sides by 21:

$$10 \; is \; 5\%$$

Multiply both sides by 20:

$$200 \; is \; 100\%$$

Part G

Part H

121 is price after tax.

$$Original \; Price = 100\%$$
$$Tax = 10\%$$
$$Price \; after \; tax = 110\%$$

$$121 \; is \; 110\% \; of \; the \; 2011 \; price$$

Divide both sides by 11:

$$11 \; is \; 10\% \; of \; the \; 2011 \; price$$
$$110 \; is \; 100\% \; of \; the \; 2011 \; price$$

Find the price in 2010:

$$110 \; is \; 110\% \; of \; 2010 \; price$$
$$100 \; is \; 100\% \; of \; 2010 \; price$$

2.43: Two Kinds of Taxes

Example 2.44

A hotel charges 10% service tax on a stay in the hotel. It charges 5% goods tax on items sold in the restaurant. A customer had a stay for one day in a room that cost $200 per day (before tax), and ordered items worth $50 (before tax) from the restaurant. Find the total cost after tax to the customer.

$$Room\ stay = 200 + 10\%\ of\ 200 = 200 + 20 = 220$$
$$Food = 50 + 5\%\ of\ 50 = 50 + 2.5 = 52.5$$

$$Total = 220 + 52.5 = 272.50$$

2.45: Two Step Questions

- Discount can be applied on the selling price
- Tax in this case will be applied on the selling price after discount.

If you are working backwards, you need to careful.

Example 2.46

A phone is available at a price of 100$. A discount of 10% is offered on the selling price. A tax of 10% is applied on the discounted price. A customer offers a $100 note for the phone. What is the change that he will receive?

$$Selling\ price = \$100$$
$$Discount = 10\%\ of\ 100 = \$10$$

$$Discounted\ price = 100 - 10 = 90\$$$
$$Tax = 10\%\ of\ 90 = 9\$$$

$$Price\ after\ Tax = 99\$$$

$$Change = 100 - 99 = 1\$$$

Example 2.47

A car is available at a selling price of 3000$. A discount of 20% is offered on the selling price. A tax of 10% is applied on the discounted price. What is the final price of the car?

$$Discount = 20\%\ of\ 3000 = 600$$
$$Discounted\ price = 3000 - 600 = 2400$$

$$Tax = 10\%\ of\ 2400 = 240$$
$$Final\ price = 2400 + 240 = 2640$$

Example 2.48

A phone was sold to a customer and he paid Rs. 270 in tax. The tax rate is 5%. The phone was sold at a discount of 10% to the maximum selling price. Find the maximum selling price?

$$270\ is\ 5\%\ of\ discounted\ price$$

Multiply by 20 both sides:

$$5400\ is\ 100\%\ of\ discounted\ price$$

Now, we need to find the maximum of selling price:

$$5400\ is\ 90\%\ of\ MSP$$

Divide both sides by 9:

$$600 \; is \; 10\% \; of \; MSP$$

Multiply both sides by 10:

$$6000 \; is \; 100\% \; of \; MSP$$

Example 2.49

A customer needs to pay 10% tax on a computer. He buys the computer for $1100 after tax. The computer was offered at a 20% discount to the price tag. What was the maximum selling price?

$$1100 \; is \; 110\% \; of \; Discounted \; Price$$
$$1000 \; is \; 100\% \; of \; Discounted \; Price$$

Now, we find the maximum selling price:

$$1000 \; is \; 80\% \; of \; MSP$$

Divide both sides by 8:

$$125 \; is \; 10\% \; of \; MSP$$
$$1250 \; is \; 100\% \; of \; MSP$$

2.50: Finding Tax Percent

$$Tax \; Percent = \frac{Tax \; Amount}{Price \; before \; Tax}$$

Example 2.51

Find the tax percent:

A. A customer settles a hotel bill, and pays Rs. 1060 after tax. The bill was Rs. 1,000 before tax.
B. A haircut which costs five dollars before tax costs five dollars and thirty-five cents after tax.

Part A

$$Tax = 1060 - 1000 = 60$$
$$Tax \; Percent = \frac{60}{1000} = \frac{6}{100} = 6\%$$

Part B

$$Tax = 5.35 - 5 = 0.35$$
$$Tax \; Percent = \frac{0.35}{5} = \frac{35}{500} = \frac{7}{100} = 7\%$$

Example 2.52: Two Step Questions

Find the tax percent:

A. A model ship is on display. It has a price tag of $70 before tax. A ten percent discount is offered on the price before tax. The final price paid by the customer is sixty-nine dollars and thirty cents.
B. A dress has a price tag of $Rs.3000$. A discount of 20% is offered on the price before tax. A customer who buys the dress pays 2460.

Part A

$$Discount = 10\% \; of \; 70 = 7$$
$$Price \; after \; discount = 70 - 7 = 63$$
$$Tax = 69.3 - 63 = 6.3$$

$$Tax\% = \frac{6.3}{63} \times 100 = \frac{630}{63} = 10\%$$

Part B

The discounted price

$$= 80\% \; of \; 3000 = 2400$$

Tax

$$= 2460 - 2400 = 60$$

$$Tax\ Percent = \frac{60}{2400} = \frac{6}{240} = \frac{1}{40} = \frac{0.5}{20} = \frac{2.5}{100} = 2.5\%$$

Example 2.53: Comparing Different Tax Rates

A. Maryland charges 7% tax, while Knoxville charges 8% tax. Nirav bought an item with a price before tax of $530 from Maryland. How much did he save compared to buying from Knoxville?
B. The difference between a 6.5% sales tax and a 6% sales tax on an item priced at $20 before tax is: **(AMC 8 1985/14)**
C. In my hometown, I need to pay 3.5% sales tax, and 2.75% surcharge on purchases. In the neighboring town, which is in the next state, I need to pay 3% sales tax only, but 4% surcharge on purchases. If I make purchases worth $700, where should I purchase from, and how much will I save by doing so?

Part A

Difference in tax percent

$$= 8\% - 7\% = 1\%$$

Saving = Difference in tax

$$= 1\% \; of \; 530 = 5.3$$

Part B

Difference in tax percent

$$= 6.5\% - 6\% = 0.5\%$$

Difference in tax

$$= 20 \times \frac{0.5}{100} = \frac{10}{100} = 0.1 \; Dollars$$

Part C

Tax in my hometown

$$= \underbrace{3.5\%}_{\substack{Sales \\ Tax}} + \underbrace{2.75\%}_{Surcharge} = 6.25\%$$

Tax in my neighboring town

$$= \underbrace{3\%}_{\substack{Sales \\ Tax}} + \underbrace{4\%}_{Surcharge} = 7\%$$

Difference in tax

$$= 7\% - 6.25\% = 0.75\%$$

Savings

$$\frac{0.75}{100} \times 700 = \frac{\frac{3}{4}}{100} \times 700 = \frac{21}{4} = 5.25$$

B. Tips

2.54: Tips

- Tips are added to a hotel bill and paid to the restaurant staff. Tips result in an increased outflow for the customer.
- Tips are calculated as a percentage of the bill.
- Tips are not paid on taxes.

Example 2.55: Finding Tip and Total Bill

Find the total cost to the customer in each case below:

A. A customer leaves a tip of 15% on a bill of Rs. 300.
B. A customer runs up a hotel bill of $770 and pays a tip of 10%.
C. A tip of 6% is paid on a bill of $220.
D. A tip of 10% is paid on a bill of Rs. 523.

Part A

$$300 + 15\% \; of \; 300 = 300 + 45 = 345$$

Part B

$$770 + 10\%\ of\ 770 = 770 + 77 = 847$$

Part C

$$6\%\ of 220 = \frac{6}{100} \times 220 = 13.2$$

$$Final\ Cost = 220 + 13.2 = 233.2$$

Part D

$$10\%\ of\ 523 = 52.3$$
$$523 + 52.3 = 575.3$$

Example 2.56: Finding the original bill

A. If a customer leaves a tip of 5%, amounting to 50 cents, what was the bill?
B. A customer tipped $2. If he usually tips 5%, but tipped only half of what he usually tips, what was the bill amount before tip?
C. A customer pays a tip in cents, which is as much as the bill, in dollars. If he tips a quarter of a dollar, what is his bill amount before tip? Also, how many percent is the customer tipping?

Part A

$$50\ cents = 5\%$$
$$10\ cents = 1\%$$
$$10\ Dollars = 100\%$$

Part B

$$Usual\ Tip = 5\%$$
$$Current\ Tip = \frac{5\%}{2} = 2.5\%$$
$$\$2 = 2.5\%$$

$$\$4 = 5\%$$
$$\$80 = 100\%$$

Part C

$$1\ Cent \Rightarrow 1\ Dollar$$
$$25\ Cents = 25\ Dollars$$

$$\% = \frac{1}{100} = 1\%$$

C. Taxes and Tips

Example 2.57

A customer pays tax at 10% on a bill of 700 dollars, and also tips 10%. Find the total cost if:

A. The tax and the tip are both on the base price of the bill.
B. The tip is paid on the bill after tax.

Part A

$$Tax = 10\%\ of\ 700 = 70$$
$$Tip = 10\%\ of\ 700 = 70$$
$$Total\ Cost = 700 + 70 + 70 = 840$$

Part B

Regular Method

$$Tax = 10\%\ of\ 700 = 70$$
$$Price\ after\ Tax = 700 + 70 = 770$$
$$Tip = 10\%\ of\ 770 = 77$$
$$Price\ after\ tax\ and\ tip = 770 + 77 = 847$$

Shortcut Method

$$700 \times \frac{11}{10} \times \frac{11}{10} = 7 \times 121 = 847$$

Example 2.58

Amy visits the Taj, and runs up a bill of Rs. 5000. She needs to pay GST (Tax) at 20% on it. She also wants to tip 10%. The cashier gives her two options. Find the tax, the tip and the total cost in each option below:

A. Pay the tax on the bill, then give a tip on the bill after tax.
B. Pay the tip, and then pay tax on the bill amount after tip.

Option A

$$Tax = 20\%\ of\ 5000 = 1000$$
$$Bill\ after\ Tax = 5000 + 1000 = 6000$$
$$Tip = 10\%\ of\ 6000 = 600$$
$$Total\ Cost = 6000 + 600 = 6600$$

Option B

$$Tip = 10\%\ of\ 5000 = 500$$
$$Bill\ after\ tip = 5000 + 500 = 5500$$
$$Tax = 10\%\ of\ 5500 = 1100$$
$$Total\ Cost = 5500 + 1100 = 6600$$

$$Total\ Cost\ is\ same\ in\ both\ options$$
$$Tax\ is\ more\ in\ Option\ B.$$

Example 2.59

A. A buffet dinner has a price tag of $40. A service tax of 5% is payable on the price tag, as are serving charges of 2.5% of the price tag. A customer has the buffet dinner and hands over a fifty dollar note. He asks the waiter to keep the change. What percent of the original price tag did he tip?
B. A customer paid a tax of 3.7% on a bill. He paid a tip of 6.3% on the bill. Both taxes and tips were paid on the original amount of the bill. If the combined value of the tax and the tip is eight dollars and fifty-two cents, then find the value of the bill after tax and tip.

Part A

$$Service\ Tax = 5\%\ of\ 40 = \frac{5}{100} \times 40 = 2$$
$$Serving\ Charges = \frac{1}{2} of\ Service\ Tax = \frac{1}{2} of\ 2 = 1$$
$$Bill + Tax + Serving\ Charge = 40 + 2 + 1 = 43$$
$$Tip = 50 - 43 = 7$$
$$\%\ Tip = \frac{7}{50} \times 100 = 14\%$$

Part B

$$\underbrace{3.7\%}_{Tax} + \underbrace{6.3\%}_{Tip} = 10\%$$
$$\$8.52 \rightarrow 10\%$$
$$\$85.2 \rightarrow 100\%$$
$$85.2 + 8.52 = 93.72$$

2.4 Commission and Brokerage

A. Brokerage

Brokerage is charged by a broker on the *sale* or *purchase* of a good, or commodity, or even a house.
The buyer of the goods has to pay the brokerage in addition to the purchase price.
Seller of the goods will have to pay the brokerage from the sale proceeds.

- Since the value of a house can be quite large in absolute terms, the numbers that we get in this section can also be quite large.

2.60: Brokerage

$$Cost\ for\ Buyer = SP + Brokerage$$
$$Receipts\ for\ Seller = SP - Brokerage$$

Example 2.61

A house was sold, for which the buyer paid Rs. 5,00,000. The broker charged brokerage of 3% on the sale price from the seller and 2% on the sale price from the buyer. Find the:

A. The money the seller received
B. The money the buyer paid
C. The brokerage earned by the broker

Part A

For the seller

$$Brokerage = 3\%\ of\ 5{,}00{,}000 = \frac{3}{100} \times 5{,}00{,}000 = Rs.\ 15{,}000$$
$$Seller = Selling\ Price - Brokerage = 5{,}00{,}000 - 15{,}000 = Rs.\ 4{,}85{,}000$$

Part B

$$Brokerage = 2\%\ of\ 5{,}00{,}000 = \frac{2}{100} \times 5{,}00{,}000 = Rs.\ 10{,}000$$
$$\text{Buyer Cost} = \text{ Selling Price} + \text{Brokerage} = 5{,}00{,}000 + 10{,}000 = \text{Rs. } 5{,}10{,}000$$

$$Brokerage = 15{,}000 + 10{,}000 = 25{,}000$$

Example 2.62

A house in Queens, New York was sold for a million dollars. The broker charged a commission of 1% oof the selling price from the seller. Calculate the brokerage received from the seller.

$$Selling\ Price = One\ Million\ Dollars = \$1{,}000{,}000$$

$$Brokerage = 1\%\ of\ 1{,}000{,}000 = \frac{1}{100} \times 1{,}000{,}000 = \$10{,}000$$

Example 2.63

A broker sold a house for $Rs.\ 25\ lakhs$. He charged a brokerage of 2% of the selling price. He had to pay $\frac{1}{4}^{th}$ of the brokerage that he earned to the contact who helped him in the sale. How much money did the broker earn from the transaction.

$$Brokerage = 2\%\ of\ 25{,}00{,}000 = \frac{2}{100} \times 25{,}00{,}000 = 50{,}000$$

$$Payment\ to\ Contact = \frac{1}{4} \times 50{,}000 = 12{,}500$$

$$Broker's\ Earning = 50{,}000 - 12{,}500 = 37{,}500$$

Example 2.64

A broker worked for an entire month to sell a house, and was very happy to finally see it sold for half a crore. During the month, he spent $Rs.\ 7500$ on transportation for himself and prospective buyers for the house. If the broker had no other incomes and no other expenses, how much money did he make in the month. The brokerage on the house sold was 1% of the sale price.

$$Brokerage = 1\% \times 50{,}00{,}000 = 50{,}000$$
$$Money\ made\ in\ the\ month = 50{,}000 - 7500 = 42{,}500$$

Example 2.65

Calculate the net profit for a broker who had the following incomes and expenses for the month of Jan 2023:

- Brokerage of 2% on a house sold for a quarter of a million dollars
- 2000 dollars spent on rent, power and water bills

$$2\%\ of\ 250{,}000 = \frac{2}{100} \times 250{,}000 = 5000$$
$$5000 - 2000 = \$3000$$

B. Commission

Commission is similar to brokerage. Except that commission is often charged on sale of goods.

Example 2.66

A. A shopkeeper purchased dry fruits from another country worth Rs. 2,00,000 through an agent. He paid commission of half a percent of the cost of the dry fruits. He had to pay service tax at 10% on the commission. He had to pay import duty of 5% of the dry fruit cost (before commission). What is the total cost to the shopkeeper?

B. A commission agent imports goods with a retail price of $1 Million, and sells the goods. There is spoilage of 10% of the goods, and he charges a commission of 2% of the total retail price of all the goods. What is the amount sent back to the company selling the goods?

C. A commission agent charges a commission of 2% for sourcing fresh fruits, and 1% for sourcing dry fruits. In the year 2021, he made Rs. 50,000 as commission from fresh fruits, and Rs. 75,000 from dry fruits. Find the value of each commodity that he helped source.

Part A

$$Commission = \frac{1}{2}\%\ of\ 200{,}000 = 1000$$
$$Service\ Tax = 10\%\ of\ 1000 = 100$$
$$Import\ Duty = 5\%\ of\ 200{,}000 = 10000$$

Total Cost
$$= 200{,}000 + 1{,}000 + 100 + 10{,}000 = 211{,}100$$

Part B

$$Spoilage = 10\%\ of\ 1\ Million = 100{,}000$$
$$Commission = 2\%\ of\ 1\ Million = 20{,}000$$

Money sent back
$$= 1{,}000{,}000 - 100{,}000 - 20{,}000 = 880{,}000$$

Part C

$$50000\ is\ 2\%$$
$$25{,}00{,}000\ is\ 100\%$$

$$75{,}000\ is\ 1\%$$
$$75{,}00{,}000\ is\ 100\%$$

2.5 Some Further Conversions

Denominator is not a factor of 100

If we want to convert a number which has a denominator which is a factor we 100, we find an equivalent fraction that has denominator 100.

For example, to convert $\frac{3}{10}$ into a percentage, we see that the denominator is 10. And that $10 \times 10 = 100$.

$$\therefore \frac{3}{10} = \frac{3 \times 10}{10 \times 10} = \underbrace{\frac{30}{100} = 30\%}$$

You can replace a denominator of 100 with the % sign

Example 2.67

Convert each of the numbers into percentage:

A. $\frac{1}{3}$
B. $\frac{4}{7}$
C. $\frac{2}{11}$
D. $\frac{5}{17}$

For each of the numbers above, we were able to easily convert it to have a denominator of 100. Here, it is not easily possible.
Hence, we use a different method

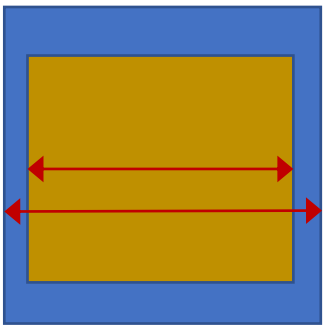
$$\frac{1}{3} = \frac{1}{3} \times 100 \times \frac{1}{100} = \frac{100}{3} \times \frac{1}{100} = \underbrace{\frac{100}{3}}_{\frac{1}{100}=\%}\% = 33.33333 \ldots \ldots = 33.\overline{3}$$

2.6 More Applications

A. Coordinate Systems and Positions

Example 2.68

Maria uses a square tablecloth of six feet on a rectangular table, with length 5 feet and width4 feet. (The shorter arrow shows the table, and the other arrow shows the tablecloth.)



A. Find the length of the tablecloth that will overhang each side of the table.
B. Find the percentage of the tablecloth as a percent of its side length that will overhang each side of the table.
C. Put a number line parallel to the arrows, such that the zero of the number line is at the top left corner of the table. If the tablecloth is held at the same level as the table, then at what position of the number line will the left most corner of the tablecloth be?

Part A

Overhang each side is

$$Lengthwise = \frac{6-5}{2} = \frac{1}{2} = 0.5$$
$$Widthwise = \frac{6-4}{2} = \frac{2}{2} = 1$$

Part B

$$\%\ length - wise\ overhang = \frac{1}{6} \times 100 = \frac{100}{6}\% = 16\frac{2}{3}\%$$
$$\%\ width - wise\ overhang = \frac{2}{6} \times 100 = \frac{200}{6}\% = 33\frac{}{3}\%$$

Part C

$$Top\ left\ corner = 0$$
$$Leftmost\ point\ of\ table = 0 - 0.5 = -0.5$$

B. Daily Balance in Bank Accounts

Example 2.69

The interest given on a checking account is 1% per month of the $\underbrace{average\ monthly\ balance}_{AMB}$ in the checking account. Calculate the interest for the following accounts for the period Jan to Mar. (Assume that one month has 30 days)

Year	Balance
Jan	100
Feb	150
Mar	80

$$AMB = \frac{100 + 150 + 80}{3} = \frac{330}{3} = 110$$
$$I = \frac{PNR}{100} = \frac{110 \times 3 \times 1}{100} = \frac{330}{100} = 3.3$$

Example 2.70

The interest given on a checking account is 20% per year of the $\underbrace{average\ monthly\ balance}_{AMB}$ in the checking account. Calculate the interest for the following accounts for six months from Jan to June. (Assume that one month has 30 days)

Year	Transaction Type	Amount
1st Jan	Starting Balance	100
1st April	Deposit	50
1st May	Withdrawal	70

Calculate the monthly Balance

Year	Transaction Type	Amount	Balance
1st Jan	Starting Balance	100	100
1st Feb		100	100
1st Mar		100	100
1st April	Deposit	50	150
1st May	Withdrawal	70	80
1st June			80

$$AMB = \frac{100 \times 3 + 150 + 80 \times 2}{6} = \frac{610}{6}$$
$$I = \frac{PNR}{100} = \frac{\frac{610}{6} \times 1 \times 10}{100} = \frac{61}{6}$$

3. Simple Interest

3.1 Calculating Interest

A. Interest

Depositing Money in the Bank

If you deposit money in your savings account with the bank, the bank can use your money. For this, the bank pays you interest.
In this case, Interest is an *income* for you. But interest is a *cost* for the bank.

Borrowing Money

If you borrow money from a bank, you will need to pay the bank the cost of using their money. For the cost, you will pay the bank interest.
In this case, interest is a *cost* for you. But interest is an *income* for the bank.

3.1: Time Period ($N = Number\ of\ Years$)

As the time period for which the money is borrowed increases, the interest also increases. So, if you are

- Depositing money for a longer period: The interest you receive is more.
- Borrowing money for a longer period: The interest you pay is more.

3.2: Principal ($= P$)

The money which has been borrowed is called the principal[2].
The borrowing is also called a loan.

As the money which is borrowed increases, the interest also increases. So, if you are:

- Depositing more money: The interest you receive is more
- Borrowing more money: The interest you pay is more

3.3: Rate of Interest ($= R$)

Rate of interest is the cost of borrowing money. Rate of interest is usually expressed in percentage.

$$Rate\ of\ Interest = 5\%\ \text{per year} \Rightarrow Interest\ on\ 100\ Rupees\ for\ one\ year = Rs.5$$

3.4: Rent on House

If a house is rented out, then the person living in the house pays rent to the owner.

- The person who pays the rent is the one who lives in the house.
- The person who receives the rent is the person who owns the house.

3.5: Interest

If money is borrowed, the "rent" that is paid on the money is called interest. Interest can be calculated using the following formula:

$$I = \frac{PNR}{100}$$
$$P = Principal$$
$$N = No.of\ Years$$
$$R = Rate\ of\ Interest$$

[2] Which is different from the word "principles". For example, my principles do not let me tell a lie.

Interest is usually expressed as percent $\underbrace{\text{per annum}}_{\textit{per year}}$. The above points can be mathematically converted into a formula for interest, given by:

$$\textbf{Interest} = I = \frac{\textbf{Principal}(P) \times \textbf{No. of Years}(N) \times \textbf{Rate of Interest}(R)}{100} = \frac{PNR}{100}$$

Example 3.6

Anand put $\underbrace{Rs.7000}_{=Principal=P}$ in his savings account for $\underbrace{two\ years}_{=No.of\ Years=N}$ at a rate of interest of $\underbrace{7\%\ per\ annum}_{=Rate\ of\ Interest=R}$. Find the interest received by him.

$$P = 7000, N = 2, R = 7\%$$

$$Interest = I = \frac{PNR}{100} = \frac{7000 \times 2 \times 7}{100} = 70 \times 2 \times 7 = 980\ Rs.$$

Example 3.7

Ram lends Rs. 300 to Shyam for 4 years at a rate of 5 $\underbrace{\%\ per\ annum}_{\textit{per cent per annum}}$. Find the interest he earned.

$$I = \frac{PNR}{100} = \frac{\underbrace{300}_{Principal} \times \underbrace{4}_{\substack{No.of\\Years}} \times \underbrace{5}_{\substack{Rate\ of\\Interest}}}{100} = 3 \times 4 \times 5 = 60\ Rs.$$

Example 3.8

Trishna puts Rs. 7000 in his savings bank account for four years at 3 $\underbrace{p.c.p.a.}_{\textit{per cent per annum}}$. Find the interest he earned.

$$I = \frac{7000 \times 4 \times 3}{100} = 840$$

Example 3.9

Dawn gives a friendly loan of $34,534 to her sister for a rate of 0% per annum for three months and twelve days. What is the interest?

$$Rate\ of\ interest\ is\ 0\% \Rightarrow I = \frac{PNR}{100} = \frac{PN \times 0}{100} = 0$$

Example 3.10

The bank offers Shristi a loan of Rs. 1,00,000 at an interest rate of 3% per annum. She takes half of the money offered by the bank for five years. Find the interest that Shristi must pay the bank after five years.

The principal

$$= P = \frac{1{,}00{,}000}{2} = 50{,}000$$

The interest is:

$$I = \frac{50000 \times 5 \times 3}{100} = 7500\ Rs.$$

Example 3.11
In each part, state who is borrowing money, who is lending the money and who will pay the interest to whom.

A. Anand put $\underbrace{Rs.\,7000}_{=Principal=P}$ in his savings account for $\underbrace{two\ years}_{=No.of\ Years=N}$ at a rate of interest of $\underbrace{7\%\ per\ annum}_{=Rate\ of\ Interest=R}$. Find the interest received by him.
B. Ram lends Rs. 300 to Shyam for 4 years at a rate of 5 $\underbrace{\%\ per\ annum}_{per\ cent\ per\ annum}$.
C. Trishna puts Rs. 7000 in his savings bank account for four years at 3 $\underbrace{p.c.p.a.}_{per\ cent\ per\ annum}$.
D. Dawn gives a friendly loan of $34,534 to her sister for a rate of 0% per annum for three months and twelve days. What is the interest?
E. The bank offers Shristi a loan of Rs. 1,00,000 at an interest rate of 3% per annum. She takes half of the money offered by the bank for five years. Find the interest that Shristi must pay the bank.

A. The bank is borrowing the money. Anand is lending the money. Bank will pay interest to Anand.
B. Shyam is borrowing money. Ram is lending the money. Shyam has to pay interest to Ram. He will pay the interest to Ram.
C. Bank is borrowing money. Trishna is lending the money. The bank will pay interest to Trishna.
D. Dawn's sister is borrowing money. Dawn is lending money. There is no interest being paid.
E. Shristi is borrowing money. She will pay interest.

B. Amount
If you borrow money from a bank (also called taking a loan), you must pay the bank interest. You must also pay the principal back.

3.12: Amount
The total money returned due to borrowing including both the principal and the interest is called the amount.

$$\underbrace{A}_{Amount} = \underbrace{P}_{Principal} + \underbrace{I}_{Interest}$$

Example 3.13
Anand deposited $\underbrace{Rs.\,7000}_{=P}$ in his savings account at a rate of interest of $\underbrace{7\%\ per\ annum}_{=R}$. Find the amount received by him after two years.

The interest is:

$$I = \frac{PNR}{100} = \frac{7000 \times 2 \times 7}{100} = 70 \times 2 \times 7 = 980\ Rs.$$

And then the amount is the sum of the principal and the interest:

$$Amount = A = P + I = 7000 + 980 = 7980\ Rs.$$

Example 3.14
Ram lends Rs. 300 to Shyam for 4 years at a rate of 5 $\underbrace{\%\ per\ annum}_{per\ cent\ per\ annum}$. Find the amount returned by Shyam.

$$I = \frac{300 \times 4 \times 5}{100} = 3 \times 4 \times 5 = 60$$
$$A = \underbrace{300}_{Principal} + \underbrace{60}_{Interest} = 360$$

Example 3.15
Trishna puts Rs. 7000 in his savings bank account at 3 $\underbrace{p.c.p.a.}_{per\ cent\ per\ annum}$. Find the balance in the account after four years.

$$I = \frac{7000 \times 3 \times 4}{100} = 840\ Rs.$$
$$A = 7000 + 840 = 7840\ Rs.$$

Example 3.16
Dawn gives a friendly loan of $34,534 to her sister for a rate of 0% per annum for three months and twelve days.

$$Rate\ of\ interest\ is\ 0\% \Rightarrow I = \frac{PNR}{100} = \frac{PN \times 0}{100} = 0$$
$$A = 34534 + 0 = 34534$$

Example 3.17
The bank offers Shristi a loan of Rs. 1,00,000 at an interest rate of 3% per annum. She takes half of the money offered by the bank for five years.

$$P = \frac{1{,}00{,}000}{2} = 50{,}000\ Rs.$$
$$I = \frac{50000 \times 5 \times 3}{100} = 7500\ Rs.$$
$$A = 50{,}000 + 7{,}500 = 57{,}500\ Rs.$$

Example 3.18
Sheela gave a loan of four hundred and forty-five rupees for five years at a rate of interest of two percent per annum. Find the interest.

$$I = \frac{PNR}{100} = \frac{445 \times 5 \times 2}{100} = \frac{445 \times 10}{100} = \frac{4450}{100} = 44.5\ Rs.$$

C. Fractions of Years
The number of years can be given in months. One strategy is to convert the number of months into years, and then proceed as usual.

3.19: Converting from Months to Years
$$Years = \frac{Months}{12}$$

Example 3.20

Rishabh gave $\underbrace{three\ thousand\ rupees}_{Principal}$ as loan to Rushabh for a period of $\underbrace{three\ and\ a\ half\ years}_{Number\ of\ Years=3.5=\frac{7}{2}}$ at a rate of interest of $\underbrace{6\ percent\ per\ annum}_{Rate\ of\ Interest}$. Find the amount at the end of the loan period.

Substitute $P = 3000, N = \frac{7}{2}, I = 6\%$

$$I = \frac{PNR}{100} = \frac{3000 \times \frac{7}{2} \times 6}{100} = 30 \times \frac{7}{2} \times 6 = 630\ Rs.$$

Example 3.21

Shital gave a loan of four thousand rupees to Sheetal for a time period of fifteen months at a rate of interest of three percent per annum. Find out who paid how much money to whom when the loan was repaid, including both principal and interest?

Convert the time period from months to years:

$$15\ months = \frac{15}{12} years = \frac{5}{4} years$$

Calculate the interest:

$$I = \frac{PNR}{100} = \frac{4000 \times \frac{5}{4} \times 3}{100} = 40 \times \frac{5}{4} \times 3 = 150$$

Calculate the amount:

$$A = P + I = 4000 + 150 = 4150 \Rightarrow Paid\ by\ Sheetal\ to\ Shital$$

Example 3.22

Find the amount on a loan of

A. 12,000 Rs. at a rate of interest of 4 per cent per annum for nine months
B. 6,000 Rs. at a rate of interest of 6 per cent per annum for fifteen months
C. 15,000 Rs. at a rate of interest of 9 per cent per annum for eighteen months
D. 18,000 Rs. at a rate of interest of 5 per cent per annum for thirty months

Part A

$$I = \frac{PNR}{100} = \frac{12000 \times 4 \times \frac{9}{12}}{100} = 360$$
$$A = P + I = 12000 + 360 = 12360$$

Part B

$$\frac{6000 \times 6 \times \frac{15}{12}}{100} = 450$$
$$A = P + I = 6450$$

Part C

$$\frac{15000 \times 9 \times \frac{18}{12}}{100} = 150 \times 9 \times \frac{3}{2} = \frac{1350 \times 3}{2} = \frac{4050}{2} = 2025$$
$$A = 17025$$

Part D

$$\frac{18000 \times 5 \times \frac{30}{12}}{100} = 15 \times 5 \times 30 = 2250$$

$$A = P + I = 18000 + 2250 = 20250$$

3.23: Monthly Rate of Interest

The monthly rate of interest is obtained by dividing the yearly rate of interest by 12.

$$Monthly\ ROI = Yearly \frac{ROI}{12}$$

Example 3.24

Find the monthly rate of interest for a yearly rate of interest of

A. 12 percent
B. 6 percent
C. 9 percent
D. 18 percent
E. 15 percent

$$\frac{12}{12} = 1\%$$
$$\frac{6}{12} = \frac{1}{2}\%$$
$$\frac{9}{12} = \frac{3}{4}\%$$
$$\frac{18}{12} = \frac{3}{2}\%$$
$$\frac{15}{12} = \frac{5}{4}\%$$

3.25: Yearly Rate of Interest

The yearly rate of interest is obtained by multiplying the monthly rate of interest by 12.

$$Yearly\ ROI = 12 \times Monthly\ ROI$$

Example 3.26

Find the yearly rate of interest for a monthly rate of interest of

A. 1 Percent
B. $\frac{1}{2}$ Percent
C. 2 Percent
D. $\frac{3}{4}$ Percent

$$1 \times 12 = 12\%$$
$$\frac{1}{2} \times 12 = 6\%$$
$$2 \times 12 = 24\%$$
$$\frac{3}{4} \times 12 = 9\ \%$$

D. Comparisons

3.27: Comparisons on Different Loans

We can compare the interest paid on two different loans. The formula for interest remains the same, but the calculations will increase.

Example 3.28

A. Hector and Priam were friends. Hector borrowed twenty rupees at a rate of interest of 5 percent per annum for five years from the bank. Priam borrowed thirty rupees at a rate of interest of 10 percent per annum for three years from the bank. Who paid more interest to the bank, and by how much?
B. Sohum and Madhav are friends. Sohum borrowed Rs. 700 from the bank at a rate of interest of 3% per annum for 4 years. Madhav borrowed Rs. 400 from the bank at a rate of interest of 6% for 5 years. Who paid more interest and by how much?

Part A

Calculate the interest paid by each friend:

$$H_{Interest} = \frac{PNR}{100} = \frac{20 \times 5 \times 5}{100} = Rs.5$$
$$P_{Interest} = \frac{PNR}{100} = \frac{30 \times 3 \times 10}{100} = Rs.9$$

The difference between the two values of interest is:

$$9 - 5 = Rs.4\ paid\ more\ by\ Priam$$

Part B

Calculate the interest paid by each friend:

$$S_{Interest} = \frac{PNR}{100} = \frac{700 \times 4 \times 3}{100} = Rs.84$$
$$M_{Interest} = \frac{PNR}{100} = \frac{400 \times 5 \times 6}{100} = Rs.120$$

The difference between the two values of interest is:

$$120 - 84 = Rs.36\ paid\ more\ by\ Madhav$$

3.29: Borrowing and Lending the Same amount (Arbitrage)

- If you borrow an amount at a lower rate of interest, and lend the same amount at a higher rate of interest, then you will make money on the entire transaction.
- This money (or profit) is called arbitrage.

Example 3.30

David borrowed 50,000 *pesos* from the bank at a rate of interest of 3% per annum for one year. He then lent the amount for the same time at a rate of interest of 10% per annum. What is the difference in the:

A. rate of interest paid by David, and received by David?
B. interest paid by David, and received by David?

Part A

$$\text{Difference} = ROI_{Money\ Lent} - ROI_{Borrowed} = 10\% - 3\% = 7\%$$

Part B

$$\text{Difference} = I_{Lent} - I_{Borrowed} = \frac{50000 \times 10 \times 1}{100} - \frac{50000 \times 3 \times 1}{100} = 5000 - 1500 = 3500$$

Shortcut Method

Instead of calculating the interest for the bank and the moneylender separately, you can calculate the difference directly by finding the interest of the difference in the rate of interest:

$$\frac{50000 \times 7 \times 1}{100} = 3500$$

3.31: Part Repayment

A loan can be paid in parts. These parts are called installments.

Example 3.32

The bank offers a loan of Rs. 10,000 to Prisha for ten years at a rate of interest of 5% per year. Interest is calculated on the principal outstanding at the beginning of the year. Prisha accepted the loan. After five years, she repaid half the principal. Calculate the total interest paid by Prisha.

Calculation of Principal

Prisha initially borrows Rs. 10,000. So, the principal is initially 10,000. After five years, she repays 5,000. So, the principal becomes 5000 only. Hence, we need to calculate the interest in two parts:

- First five years
- Next five years

Year	Principal	Repayment
1	10,000	
2	10,000	
3	10,000	
4	10,000	
5	10,000	
6	5,000	5000
7	5,000	
8	5,000	
9	5,000	
10	5,000	

Calculation of Interest

$$I = \underbrace{\frac{10{,}000 \times 5 \times 5}{100}}_{\textit{First Five Years}} + \underbrace{\frac{5{,}000 \times 5 \times 5}{100}}_{\textit{Next Five Years}} = 2500 + 1250 = 3750$$

Example 3.33

Jack borrowed a thousand dollars from Jill at a rate of interest of 4% per annum. Interest was payable on the principal outstanding at the beginning of the year. At the end of the first year, Jack paid the interest for the year, and also half of the principal. Determine the total interest paid by Jack.

Year	Principal	Repayment	Rate of Interest	Interest
1	1,000		4%	$\frac{1000 \times 1 \times 4}{100} = 40$
2	500	500	4%	$\frac{500 \times 1 \times 4}{100} = 20$

Total Interest

$$= 40 + 20 = 60$$

Example 3.34

Jack borrowed a thousand dollars from Jill at a rate of interest of 5% per annum. Interest was payable at the end of each year on the amount outstanding at the beginning of the year. If the interest is not paid, it gets added to the principal. Jack paid the entire amount at the end of two years. How much was it?

Year	Principal	Repayment	Rate of Interest	Interest
1	1,000		5%	$\frac{1000 \times 1 \times 5}{100} = 50$
2	1,050		5%	$\frac{1050 \times 1 \times 5}{100} = 52.5$

Total Interest

$$= 1050 + 52.5 = 1102.5$$

3.2 Calculating Rate of Interest

A. Calculating Rate of Interest

To find out the time period, we will substitute the information given into the formula for interest.

3.35: Rate of Interest

$$\frac{100I}{PN} = R$$

$$I = \frac{PNR}{100}$$

Multiply both sides by 100:

$$100I = \frac{PNR}{100} \times 100 \Rightarrow 100I = PNR$$

Divide both sides by PN:

$$\frac{100I}{PN} = \frac{PNR}{PN} \Rightarrow \frac{100I}{PN} = R$$

Example 3.36

A. Bhairavi put $\underbrace{Rs.5000}_{=Principal}$ in her savings account for $\underbrace{three\ years}_{=No.\ of\ Years}$ and received $\underbrace{Rs.150}_{=Interest}$ as interest. Find the rate of interest.

B. *Courtney* put $\underbrace{Rs.3000}_{=Principal}$ in her savings account for $\underbrace{two\ years}_{=No.\ of\ Years}$ and received $\underbrace{Rs.180}_{=Interest}$. Find the rate of interest?

C. We are borrowing ten thousand rupees for two years for interest of five hundred rupees. Find the rate of interest.

D. We are borrowing six thousand rupees for seven years for interest of four hundred and twenty rupees. Find the rate of interest.

Part A

$$I = \frac{PNR}{100}$$

Substitute $P = 5000, N = 3, I = 150$:

$$150 = \frac{5000 \times 3 \times R}{100}$$

Simplify the RHS:

$$150 = 150R$$

Therefore:

$$R = 1\%$$

Part B

$$I = \frac{PNR}{100} \Rightarrow 180 = \frac{3000 \times 2 \times R}{100} \Rightarrow 180 = 60R \Rightarrow R = 3\%$$
$$I = \frac{PNR}{100} \Rightarrow 500 = \frac{10{,}000 \times 2 \times R}{100} \Rightarrow 500 = 200R \Rightarrow R = 2.5\%$$
$$I = \frac{PNR}{100} \Rightarrow 420 = \frac{6{,}000 \times 7 \times R}{100} \Rightarrow 420 = 420R \Rightarrow R = 1\%$$

Example 3.37

A. Sudama borrowed 2 hundred thousand dollars from his rich friend. Calculate the rate of interest if, at the end of two years, he paid one thousand dollars as interest.
B. Find the rate of interest if a bank pays Rs. 700 as interest on a savings bank account for a period of three and a half years on a principal of ten thousand rupees.
C. Micheal borrowed twelve thousand dollars from his friend Robert for a period of three years, and paid interest of seven hundred and twenty dollars for one year. What is the rate of interest?

$$I = \frac{PNR}{100} \Rightarrow 1000 = \frac{200{,}000 \times 2 \times R}{100} \Rightarrow 1000 = 4000R \Rightarrow R = \frac{1}{4} = 0.25\%$$
$$I = \frac{PNR}{100} \Rightarrow 700 = \frac{10{,}000 \times \frac{7}{2} \times R}{100} \Rightarrow 700 = 350R \Rightarrow R = 2\%$$
$$I = \frac{PNR}{100} \Rightarrow 720 = \frac{12000 \times 1 \times R}{100} = 720 = 12000R \Rightarrow R = 6\%$$

Example 3.38

At what rate of interest will a sum of money double itself in 8 years?

$$A = P + I$$

If the sum of money doubles itself, then the amount will be double of the principal:

$$2P = P + I$$
$$P = I$$

Substitute $P = I$ in $I = \frac{PNR}{100}$

$$P = \frac{PNR}{100}$$
$$1 = \frac{NR}{100}$$

Substitute $N = 8$:

$$1 = \frac{8R}{100}$$
$$R = \frac{100}{8} = 12.5\%$$

Comparing Interest Rates

Challenge 3.39

Conan's friend offered him fourteen thousand *yen* for an interest of 280 *yen* over two years. A second friend offered him fifteen thousand *pesos* for interest of 900 *pesos* over three years. Compare the two rates of interest.

$$Yen: I = \frac{PNR}{100} \Rightarrow 280 = \frac{14{,}000 \times 2 \times R}{100} \Rightarrow 280 = 280R \Rightarrow R = 1\%$$
$$Pesos: I = \frac{PNR}{100} \Rightarrow 900 = \frac{15{,}000 \times 3 \times R}{100} \Rightarrow 900 = 450R \Rightarrow R = 2\%$$

Non-annual Interest Rates

Rate is usually expressed an annual rate. However, we can get question on time periods other than one year. One strategy to answer such questions is to convert everything to the same unit.

Example 3.40

Anshuman paid interest of Rs. 100 every month on a loan taken by him of principal six thousand rupees. What is the annual rate of interest as a percentage?

First find the annual interest

$$= Interest\ in\ 12\ Months = 12 \times 100 = 1200$$

Calculate the rate of interest using $I = \frac{PNR}{100}, I = 1200, P = 6000, N = 1$:

$$1{,}200 = \frac{6{,}000 \times 1 \times R}{100}$$
$$1200 = 60R$$
$$R = \frac{1200}{60} = 20\%$$

Challenge 3.41

Keerti paid interest of Rs. 120 every month on a loan taken by him of principal five thousand rupees. What is the annual rate of interest as a percentage?

First find the annual interest

$$= Interest\ in\ 12\ Months = 12 \times 120 = 1440$$

Calculate the rate of interest using $I = \frac{PNR}{100}, I = 1440, P = 5000, N = 1$:

$$1440 = \frac{5{,}000 \times 1 \times R}{100}$$
$$1440 = 50R$$
$$R = \frac{1440}{50} = 28.8\%$$

3.3 Calculating Time Period

Calculating Time Period

3.42: Calculating time period

To find the time period, we substitute the known information. The process is similar as for rate of interest:

$$I = \frac{PNR}{100}$$

Example 3.43

Dinesh put $\underbrace{Rs.5000}_{=Principal}$ in his savings account at a rate of interest of 3% per year and received $\underbrace{Rs.600}_{=Interest}$ as interest.

Find the time period for which the money was in the account.

Substitute $P = 5000, R = 3, I = 600$ in $I = \frac{PNR}{100}$:

$$600 = \frac{5{,}000 \times N \times 3}{100}$$
$$600 = 150N$$
$$N = 4\ years$$

***Example 3.44**

At 12.5%, an amount will triple itself in how many years?

Suppose the principal is $Rs.$ 100. For Rs. 100 to become Rs. 300

$$Interest = 300 - 100 = 200$$

Note that $Rs.$ 200 is double of Rs. 100. Hence, the interest must be double of the principal.

Substitute $I = 2P, R = 12.5$ in $I = \frac{PNR}{100}$

$$2P = \frac{PN \times 12.5}{100}$$

Divide by P both sides:

$$2 = \frac{N \times 12.5}{100}$$

Multiply by 100 in both sides:

$$200 = 12.5N$$
$$N = \frac{200}{12.5} = \frac{400}{25} = \frac{1600}{100} = 16$$

3.4 Challenging Questions

A. Back Calculations

Example 3.45

A sum of money amounts to Rs. 1680 in 3 years at simple interest, and Rs. 1920 in 7 years. What is the rate of interest, given that it is an integer percent?

$$P + \frac{7PR}{100} = 1920 \Rightarrow \underbrace{P\left(1 + \frac{7R}{100}\right) = 1920}_{\textit{Equation I}}$$
$$P + \frac{3PR}{100} = 1680 \Rightarrow \underbrace{P\left(1 + \frac{3R}{100}\right) = 1680}_{\textit{Equation II}}$$

Divide Equation I by Equation II:

$$\frac{1 + \frac{7R}{100}}{1 + \frac{3R}{100}} = \frac{1920}{1680} \Rightarrow \frac{\frac{100 + 7R}{100}}{\frac{100 + 3R}{100}} = \frac{8}{7} \Rightarrow \frac{100 + 7R}{100 + 3R} = \frac{8}{7}$$

$100 + 3R$ must be an integer multiple of 7.

$$R = 1 \Rightarrow 100 + 3R = 103$$
$$R = 2 \Rightarrow 100 + 3R = 106$$

$$R = 3 \Rightarrow 100 + 3R = 109$$
$$R = 4 \Rightarrow 100 + 3R = 112 = 7 \times 16 \Rightarrow 100 + 7R = 128 = 8 \times 16$$

B. Effective Interest Rate

Example 3.46

Annual Percentage Rate is 12%. Frequency of compounding is semi-annual. Calculate effective interest rate.

Assume your principal is 1 dollar. Amount you will get at the end of the year

$$= \left(1 + \frac{0.12}{2}\right)^2 = 1.1236$$

Interest earned during the year

$$= Amount - Principal = 1.1236 - 1 = 0.1236 = 12.36\%$$

In other words, an interest of 12.36%, compounded annually, is equal to an interest rate of 12%, compounded semi-annually.

4. Compound Interest

4.1 Calculating Interest

Interest as an Expense and an Income

4.1: Compound Interest

$$A = P\left(1 + \frac{R}{100}\right)^N$$
$$A = Amount$$
$$P = Principal$$
$$R = Rate\ of\ Interest, in\ percentage$$
$$N = Number\ of\ Time\ Periods$$

Example 4.2

X borrowed 10,000 from the bank at a rate of interest of 10% from the bank for two years. Find the:

A. Amount
B. Interest

$$A = P\left(1 + \frac{R}{100}\right)^N = 10{,}000\left(1 + \frac{10}{100}\right)^2 = 10{,}000(1.1)^2 = 10{,}000(1.21) = 12{,}100$$
$$A = P + I \Rightarrow I = A - P = 12{,}100 - 10{,}000 = 2100$$

Example 4.3

Find the compound interest on Rs. 4,000 at the rate of 5% for two years.

$$A = P\left(1 + \frac{R}{100}\right)^N = 4{,}000\left(1 + \frac{5}{100}\right)^2 = 4{,}000\left(\frac{105}{100}\right)^2 = 4{,}000\left(\frac{21}{20}\right)^2 = 4{,}410$$
$$I = A - P = 4{,}410 - 4000 = 410$$

Example 4.4

G borrowed 500 from the bank at a rate of 10% per year. The money was compounded six monthly.

A. Find the amount at the end of the year.
B. Find the difference if the amount had been borrowed at simple interest instead of compound interest
C. Find how many percent more did G earn because he got compound interest as compared to simple interest

$$I_{6\ Months}\ \frac{500 \times 5 \times 1}{100} = 25$$
$$I_{End\ of\ the\ year} = \frac{525 \times 5 \times 1}{100} = 5.25 \times 5 = 26.25$$
$$Amount = 500 + 25 + 26.25 = 551.25$$

$$\frac{500 \times 10 \times 1}{100} = 50 \Rightarrow Difference = 51.25 - 50 = 1.25$$

$$\frac{1.25}{50} \times 100 = 2.5\%$$

4.2 Finding Rate

Example 4.5
The compound interest on a principal of Rs. 3000 for two years is 1,320. Find the rate of interest.

$$A = P\left(1+\frac{R}{100}\right)^N \Rightarrow 4320 = 3000\left(1+\frac{r}{100}\right)^2$$
$$\left(1+\frac{r}{100}\right)^2 = \frac{4320}{3000} = \frac{432}{300} = \frac{216}{150} = \frac{72}{50} = \frac{144}{100}$$
$$\left(\frac{100+r}{100}\right)^2 = \frac{144}{100}$$
$$\frac{100+r}{100} = \frac{12}{10}$$
$$100+r = 120$$
$$r = 20$$

4.3 Finding Number of Years

Example 4.6
The compound interest on a principal of Rs. 4000 at a rate of 6% per annum is 494.40. Find the number of years.

Compare with equivalent simple interest

$$SI\ for\ 1\ Year = \frac{PNR}{100} = \frac{4000\times 1\times 6}{100} = 240$$
$$SI\ for\ 2\ Years = 240\times 2 = 480$$
$$SI\ for\ 3\ Years = 240\times 3 = 720$$

CI is between the SI for two years and three years. So, CI is for two years.

$$A = P\left(1+\frac{R}{100}\right)^N = 4{,}000\left(1+\frac{6}{100}\right)^2 = 4{,}000\left(\frac{106}{100}\right)^2 = 4494.4$$

4.4 Mixing

Example 4.7
The CI on a certain sum for 2 years at 10% per annum is 525. The SI on the same sum for double the time at half the rate percent per annum is:

$$SI = \frac{PNR}{100} = \frac{P(2N)\left(\frac{R}{2}\right)}{100}$$

Consider the CI on Rs. 1 for two years at 10% per annum

$$CI = 1.1^2 - 1 = 1.21 - 1 = 0.21$$
$$SI = 0.2$$

$$\frac{CI}{SI} = \frac{0.21}{0.2} = \frac{21}{20} \Rightarrow \frac{525}{SI} = \frac{21}{20}$$

4.5 Frequency of Compounding

(Calculator) Example 4.8

Find the compound interest on a sum of Rs. 1 at 5% per annum, if the interest is compounded:

A. Annually
B. Every six months
C. Once in 3 months
D. Monthly
E. Daily

Part A

$$A = P\left(1 + \frac{R}{100}\right)^N = 1\left(1 + \frac{5}{100}\right)^1 = 1 \times 1.05 = 1.05$$

Part B

If you compound it every six months, the rate of interest, which is:

$$5\%\ per\ 12\ months = \frac{5\%}{2}\ per\ six\ months = 2.5\%\ per\ 6\ months \Rightarrow R = 2.5$$

Also, since you compound it every six months, in 12 months, you will compound it

$$\frac{12}{6} = 2\ times \Rightarrow N = 2$$

$$A = P\left(1 + \frac{R}{100}\right)^N = 1\left(1 + \frac{2.5}{100}\right)^{1\times2} = 1.050625$$

$$A = P\left(1 + \frac{R}{100}\right)^N = 1\left(1 + \frac{5}{100 \times 4}\right)^{1\times4} = 1.05094$$

$$A = P\left(1 + \frac{R}{100}\right)^N = 1\left(1 + \frac{5}{100 \times 12}\right)^{1\times12} = 1.05116$$

A. Number *e*

(Calculator) Example 4.9

Find the compound interest on a sum of Rs. 1 at 100% per annum, if the interest is compounded:

A. Annually
B. Once in six months
C. Once in 3 months
D. Monthly
E. Daily

$$A = P\left(1 + \frac{R}{100}\right)^N = 1\left(1 + \frac{1}{1}\right)^1 = 1 \times 2 = 2$$

$$A = P\left(1 + \frac{R}{100}\right)^N = 1\left(1 + \frac{1}{2}\right)^{1\times2} = 2.25$$

$$A = P\left(1 + \frac{R}{100}\right)^N = 1\left(1 + \frac{1}{4}\right)^{1\times4} = 2.44$$

$$A = P\left(1 + \frac{R}{100}\right)^N = 1\left(1 + \frac{1}{12}\right)^{1\times12} = 2.61$$

$$A = P\left(1 + \frac{R}{100}\right)^N = 1\left(1 + \frac{1}{365}\right)^{1\times365} = 2.71$$

5. Profit and Loss

5.1 Profit and Loss

A. Profit

People engage in business in order to make some money. The money they make is called profit.

5.1: Profit

$$\underbrace{\textbf{\textit{Profit}}}_{=P} = \underbrace{\textbf{\textit{Selling Price}}}_{=SP} - \underbrace{\textbf{\textit{Cost Price}}}_{=CP}$$

$SP = Selling\ Price$ *is the price at which you sell goods*

$CP = Cost\ Price$ *is the price at which you buy the goods*

Example 5.2

Kavish bought a horse for Rs. 55,000 and sold it for Rs. 63,000. What is the profit that he made on the transaction?

$$P = SP - CP = 63{,}000 - 55{,}000 = 8{,}000$$

Example 5.3

Mark the correct option

You buy a horse for a cost price, and sell it for a selling price. You will make a profit if the cost price is

A. more than the selling price
B. equal to the selling price
C. less than the selling price
D. None of the above

Option C

5.4: Profit

$One\ Lakh = 1{,}00{,}000$ has five zeroes.
$One\ Million = 1{,}000{,}000$ has six zeroes.

Example 5.5

A. Kirti spent one-fourth of a lakh of rupees to buy a house, and sold it for one-half of a lakh of rupees. What is the profit that he made, in rupees?
B. Nirupam spent one-eighth of a million rupees to buy a house, and sold it for one-fifth of a million rupees. What is the profit that he made, in rupees?

Part A

$$\underbrace{\frac{1}{2} \times 1{,}00{,}000}_{\textit{Selling Price}} - \underbrace{\frac{1}{4} \times 1{,}00{,}000}_{\textit{Cost Price}} = 50{,}000 - 25{,}000 = 25{,}000\ Rs.$$

Part B

$$P = \underbrace{\frac{1}{5} \times 1{,}000{,}000}_{\textit{Selling Price}} - \underbrace{\frac{1}{8} \times 1{,}000{,}000}_{\textit{Cost Price}} = 200{,}000 - 125{,}000 = 75{,}000\ Rs.$$

B. Loss

Sometimes businesses do not make a profit, since the cost price is more than the selling price. In such

situations, you are said to have incurred a loss.

5.6: Loss

$$\underbrace{\boldsymbol{Loss}}_{=L} = \underbrace{\boldsymbol{Cost\ Price}}_{=CP} - \underbrace{\boldsymbol{Selling\ Price}}_{=SP}$$

Example 5.7

In the following transactions, classify whether the individuals involved made a profit or a loss.

A. Buying five watches for 34 pounds, and selling them for 36 pounds each.
B. Buying some oranges for Rupees 200, and selling them for Rs. 180.

$$34 < 36 \Rightarrow Profit$$
$$200 > 180 \Rightarrow Loss$$

C. Converting to Totals

If the costs are given in terms of each item, then you will need to convert to totals.

Example 5.8

Sharda runs a grocery shop. She buys five kg of potatoes at Rs. 20 per kilogram, and sells them at Rs. 18 per kg. Find the loss on the transaction.

Method I: Calculate on a $per - kg$ basis first

Calculate the profit for each kilogram, and multiply by the number of kilograms:

$$Loss\ per\ kg = SP - CP = 20 - 18 = 2$$
$$Total\ Loss = No.of\ Kg \times Loss\ per\ Kg = 5 \times 2 = 10$$

Method II: Calculate the totals first

Calculate the total costs, and the total selling price to get the profit directly:

$$Total\ Loss = Total\ CP - Total\ SP = 5 \times 20 - 5 \times 18 = 5(20 - 18) = 5 \times 2 = 10$$

Which method to use depends on your preference, and the question.

Example 5.9

Answer each part below independently

A. I buy a dozen apples for Rs. 100. I sell them for Rs. 10 each. Find the profit or loss.
B. I buy two dozen bananas for Rs. 24. If I want to make a profit of Rs. 2 on the entire transaction, at what rate what should I sell each banana?
C. My truck can accommodate 5 bushels of corn. I fill my truck with corn at the farmer's market (at $20 per bushel), and sell the entire lot for $110. How much profit or loss did I make?

Part A

$$Profit = \underbrace{12 \times 10}_{Selling\ Price} - 100 = 120 - 100 = 20$$

Part B

$$\underbrace{2}_{Profit} = SP - \underbrace{24}_{Cost\ Price} \Rightarrow SP = 26$$

$$SP\ per\ banana = \frac{Total\ Selling\ Price}{No.of\ Bananas} = \frac{26}{24} = \frac{13}{12}$$

Part C

$$110 - 5 \times 20 = 110 - 100 = 10\$\ profit$$

D. Breakage

5.10: Breakage

If some of the items are spoilt, or broken, or in some way not suitable for selling then you lose out on the selling price for those items. This must be considered when calculating profit.

Example 5.11

I buy a crate with three dozen mangoes in it. Three of the mangoes are spoilt. I sell the remaining mangoes for Rs. 5 each. If I made a profit of Rs. 20 on the transaction, find the cost price of each mango.

The number of unspoilt mangoes:

$$= \underbrace{12 \times 3}_{\substack{Total \\ Mangoes}} - \underbrace{3}_{\substack{Spoilt \\ Mangoes}} = 36 - 3 = 33$$

I sell the mangoes for:

$$Selling\ Price = 5 \times 33 = Rs.\ 165$$

Since I made a profit of Rs. 20, the cost price of the crate of mangoes must be less than selling price by Rs. 20

$$= 165 - 20 = 145$$

The cost of each mango

$$= \frac{145}{36}\ Rs.$$

Example 5.12

A shopkeeper bought 500 kiwis at Rs. 4 for 10 kiwis. Of these, 50 kiwis were found to be spoilt. If he sold the remaining kiwis at 60 paise each, what is the profit or loss that he made?

Because of the spoilt kiwis, we will need to calculate the total CP, and the total SP.

Cost Price

$$Rs.\ 4\ for\ 10\ Kiwis$$

Multiply both sides by 50:

$$Rs.\ 200\ for\ 500\ Kiwis$$

Selling Price

$$Rs.\ 0.6\ per\ kiwi$$

Multiply both sides by $500 - 50 = 450$:

$$60\ paise \times 450\ for\ 450\ Kiwis$$
$$27000\ paise\ for\ 450\ Kiwis$$
$$270\ Rs.\ for\ 450\ Kiwis$$

Since the SP is more than the cost price, we have a profit:

$$Profit = SP - CP = 270 - 200 = 70$$

E. Additional Costs

We need to subtract any additional costs from the profit which is being calculated.

5.13: Profit

$$\underbrace{\boldsymbol{Profit}}_{=P} = \underbrace{\boldsymbol{Selling\ Price}}_{=SP} - \underbrace{\boldsymbol{Cost\ Price}}_{=CP} - \underbrace{\boldsymbol{Additional\ Costs}}_{=AC}$$

Example 5.14

Abhay bought some muffins from a bakery shop, and sold them door to door. He bought each muffin at $3, and sold it for $4. If he bought twenty muffins in all, and spent $3 in travelling to and from the bakery, find his total profit.

Method I

$$P = SP - CP\ - Travelling\ Cost = 20 \times 4 - 20 \times 3 - 3 = 80 - 60 - 3 = 17$$

Method II

$$Profit_{1\ Muffin} = 4 - 3 = 1$$
$$Profit_{20\ Muffins} = 20$$
$$Net\ Profit = 20 - 3 = 17$$

Example 5.15

Sundar bought a horse for Rs. 10,000, and stabled it for ten days while he found a buyer. The cost of stabling and feed was Rs. 100 per day. The horse was sold for Rs. 11,500. Find the profit or the loss on the transaction.

The cost price of the horse is

$$Rs.\,10{,}000$$

The cost of stabling and feeding the horse is

$$10 \times 100 = 1000$$

The profit on the transaction:

$$Profit = \underbrace{11500}_{Selling\ Price} - \underbrace{10{,}000}_{Cost\ Price} - \underbrace{10 \times 100}_{\substack{Additional \\ Cost}} = 500$$

Example 5.16

A grocery shop purchased $10,000 worth of fruits. Storing the fruits in a warehouse cost $300. Transportation to and from the warehouse cost $100. The fruits were sold for $12,000. What is the net profit or loss on the transaction?

$$\underbrace{12{,}000}_{Selling\ Price} - \underbrace{10{,}000}_{Cost\ Price} - \underbrace{300}_{\substack{Additional \\ Cost\ 1}} - \underbrace{100}_{\substack{Additional \\ Cost\ 2}} = 1{,}600$$

F. Additional Income

5.17: Profit

$$\underbrace{\boldsymbol{Profit}}_{=P} = \underbrace{\boldsymbol{Selling\ Price}}_{=SP} + \underbrace{\boldsymbol{Additional\ Income}}_{=AI} - \underbrace{\boldsymbol{Cost\ Price}}_{=CP}$$

Example 5.18

Hari bought a house for 50,000 dollars. He rented it out for 1000 dollars per month while he looked for a buyer.

After a year, he was able to sell it for 5000 dollars more than the purchase price. What was the
A. selling price?
B. Overall profit or loss on the transaction

Rent Income

$$= 12 \times 1{,}000 = 12{,}000$$

The final profit is:

$$Profit = \underbrace{55{,}000}_{Selling\ Price} + \underbrace{12000}_{\substack{Additional \\ Income}} - \underbrace{50{,}000}_{Cost\ Price} = 17{,}000$$

Example 5.19

Pinaki bought twenty hens from the market at Rs. 200 each. He kept them with him for ten days. He spent Rs. 200 on feeding them, and made Rs. 250 from the eggs they laid. He sold the twenty hens for Rs. 5000. Find the profit or loss that he made.

The cost of the hens is

$$20 \times 200 = 4000$$

The additional income from the hens

$$= \underbrace{250}_{\substack{Income \\ from\ eggs}} - \underbrace{200}_{\substack{Cost \\ of\ feed}} = 50$$

The final profit is:

$$\underbrace{5000}_{Selling\ Price} + \underbrace{50}_{\substack{Additional \\ Income}} - \underbrace{4000}_{Cost\ Price} = 1050$$

G. Breakeven

5.20: Breakeven

A situation where a transaction does not result in either a profit or a loss is called breakeven:

$$Breakeven = Zero\ Profit = Zero\ Loss$$

In a breakeven scenario, the formula for profit, and also the formula for loss, will give a result of zero.

Example 5.21

Meenakshi bought three dozen eggs from the market at Rs. 20 per dozen. She sold them at Rs. 2 per egg. However, six eggs were spoilt and had to be thrown away. Find the total profit or loss on the transaction.

The cost is

$$3 \times 20 = 60$$

The selling price is

$$2(36 - 6) = 2(30) = 60$$

The profit is:

$$\underbrace{60}_{Selling\ Price} - \underbrace{60}_{Cost\ Price} = 0$$

$$Breakeven$$

H. Back-Calculations

5.22: Selling Price

The selling price will be above the cost price if there is a profit:

$$Selling\ Price = Cost\ Price + Profit$$

The selling price will be below the cost price if there is a loss:

$$Selling\ Price = Cost\ Price - Loss$$

Example 5.23

Kruti bought figs from the market at Rs. 300 per bunch, and made a profit of Rs. 20 per bunch. What is the selling price per bunch?

$$Selling\ Price = 300 + 20 = 320$$

Example 5.24

A grocery shops buys bananas at $Rs.$ 30 per dozen. They make a profit of Rs. 13 per box of bananas. Every box contains ten bananas. On Sunday, the grocery shop had no bananas in its inventory. On Monday, the grocery shop bought a whole number of dozens of bananas, and sold a whole number of boxes. What is the minimum value of the sales they could have made from the bananas?

The grocery store sold a whole number of boxes. The number of bananas they could have sold:

$$1\ Box = 10$$
$$2\ Boxes = 20$$
$$3\ Boxes = 30$$
$$4\ Boxes = 40$$
$$5\ Boxes = 50$$
$$6\ Boxes = 60$$

The grocery store bought bananas per dozen. The number of bananas they could have bought:

$$1\ Dozen = 12$$
$$2\ Dozen = 24$$
$$3\ Dozen = 36$$
$$4\ Dozen = 48$$
$$5\ Dozen = 60$$

(Shortcut: The minimum of bananas $= LCM(10,12) = 60$.

The cost price is

$$= 5 \times 30 = 150$$

The profit is

$$= 6 \times 13 = 78$$

The selling price is

$$= 150 + 78 = 228$$

5.25: Cost Price

$$Cost\ Price = Selling\ Price - Profit$$
$$Cost\ Price = Selling\ Price + Loss$$

Example 5.26
Vaishali bought some notebooks and sold them for Rs. 300. If she made a profit of Rs. 20 in the entire transaction, what is the money that she paid for the notebooks.

$$CP = 300 - 20 = 280$$

Example 5.27
Krutika bought oranges from the market at Rs. 200 per dozen, sold them for Rs. 1600, and made a profit of Rs. 200. What is the number of oranges she bought?

The total cost

$$= \underbrace{1600}_{Selling\ Price} - \underbrace{200}_{Price} = 1400$$

The number of dozens

$$= \frac{Total\ Cost}{Cost\ per\ Dozen} = \frac{1400}{200} = 7$$

The number of oranges

$$= 7\ dozen = 84$$

Example 5.28
A man buys apples at a rate of 3 kg for Rs. 21 and sells them at 5 kg for Rs. 50. How many kg of apples must he sell to earn Rs. 102 as his profit?

Calculate the cost price per kg:

$$3\ kg\ for\ Rs.21 \Rightarrow 1\ kg\ for\ Rs.7$$

Calculate the selling price per kg:

$$5\ kg\ for\ Rs.50 \Rightarrow 1\ kg\ for\ Rs.10$$

The profit per kg:

$$= 10 - 7 = 3$$

The number of kg required

$$= \frac{Total\ Profit}{Profit\ Per\ Kg} = \frac{102}{3} = 34$$

5.2 Profit and Loss Percentage

A. Profit Percentage
We have already discussed the calculation of profit and loss. Profit and loss can also be calculated as a percentage of the cost price. This allows for comparison of profit across different transactions.

5.29: Profit Percentage

$$Profit\ \% = \frac{Profit}{Cost\ Price} \times 100 = \frac{Selling\ Price - Cost\ Price}{Cost\ Price} \times 100$$

Example 5.30

Find the profit percent in each case:

A. Shruti bought bananas for Rs. 400 and sold them for Rs. 500.
B. Sheela bought mangoes for Rs. 100, and sold them for Rs. 123.
C. A shopkeeper bought apples at Rs. 12 per dozen, and sold them for Rs. 1.5 per apple.
D. A wholesaler bought 10 crates of bananas for Rs. 500, and sold the crates for Rs. 60 each.

Part A

$$Profit\ \% = \frac{Profit}{Cost\ Price} = \frac{500-400}{400} = \frac{100}{400} = \frac{1}{4} = 25\%$$

Part B

$$\frac{P}{CP} = \frac{23}{100} = 23\%$$

Part C

$$Rs.12\ per\ dozen\ apples = Rs.1\ per\ apple$$
$$\frac{SP-CP}{CP} = \frac{1.5-1}{1} = \frac{0.5}{1} = 0.5 = \frac{1}{2} = 50\%$$

Part D

$$Rs.500\ for\ 10\ Crates \Rightarrow Rs.50\ for\ 1\ Crate$$
$$Profit\ \% = \frac{60-50}{50} = \frac{10}{50} = \frac{1}{5} = \frac{20}{100} = 20\%$$

Example 5.31

Tanvi makes home-made chocolates. She bought chocolate mix for Rs. 2000, added dry fruits worth Rs. 500, and had miscellaneous expenses of Rs. 100. She divided the chocolates that she made into packets, and put a price of Rs. 290 on each packet. However, on selling, she gave a discount of Rs. 4 per packet. She made 11 packets in all (of which one was spoilt). Find the profit percent.

$$Total\ CP = 2000 + 500 + 100 = 2600$$

$$SP\ per\ Packet = 290 - 4 = 286$$
$$No.of\ Packets\ Sold = 11 - 1 = 10$$
$$Total\ SP = 286 \times 10 = 2860$$

$$Profit\% = \frac{2860-2600}{2600} = \frac{260}{2600} = \frac{1}{10} = 10\%$$

B. Loss Percentage

If there is a loss, instead of a profit, you can calculate the loss percentage. Loss percentage is also calculated using the cost price as a base.

5.32: Loss Percentage

$$Loss\ \% = \frac{Loss}{CP} \times 100 = \frac{CP-SP}{CP} \times 100$$

Example 5.33

Find the loss percentage in each case:

A. Monica bought widgets for Rs. 450 and sold them for Rs. 405.

B. Ajay bought 10 kilos of dragon fruit at Rs. 200 per kilo. He sold them at Rs. 220 per kilo. However, two kilos were spoilt in transportation.
C. Jay bought a horse for Rs. 10,000. He stabled it at a cost of Rs. 100 per day. He sold it after ten days for Rs. 10,500.

Part A

$$Loss\ \% = \frac{CP - SP}{CP} = \frac{450 - 405}{450} = \frac{45}{450} = \frac{1}{10} = 10\%$$

Part B

$$\frac{2000 - 220 \times 8}{2000} = \frac{2000 - 1760}{2000} = \frac{240}{2000} = \frac{12}{100} = 12\%$$

Part C

$$\frac{10{,}000 + 1000 - 10{,}500}{11{,}000} = \frac{500}{11000} = \frac{5}{110} = \frac{500}{110} = \frac{50}{11}\%$$

C. Deciding between Profit and Loss Percentage

It is not necessary that a question tells whether there is a profit or a loss. In this case, it is worked out first.

Example 5.34

Cho makes chairs. He bought wood worth Rs. 7,000 and other materials worth Rs. 500. He made seven chairs, and sold each for Rs. 900. He was very happy since he made a lot of money. What do you think? Calculate the profit or loss percentage.

$$CP = 7000 + 500 = 7500$$
$$SP = 7 \times 900 = 6300$$

We have a loss, not a profit.

$$Loss = \frac{7500 - 6300}{7500} = \frac{1200}{7500} = \frac{12}{75} = \frac{4}{25} = \frac{16}{100} = 16\%$$

So, he should not be so happy, because he made a loss.

Example 5.35

Tina bought tomatoes for Rs. 320 and sold them for Rs. 300. Did she make a profit or a loss. Find in percentage terms.

$$Loss = \frac{Loss}{Cost\ Price} = \frac{320 - 300}{320} = \frac{20}{320} = \frac{1}{16} = \frac{100}{16}\% = \frac{25}{4}\%$$

Example 5.36

A man buys 1000 candy bars at a price of 5 for Rs. 2. He sells all the candy bars at a price of 2 for Rs. 1. What was his profit/loss percent?

$$Cost\ Price: 5\ for\ Rs.\ 2 \Leftrightarrow 1\ for\ 40\ paise$$
$$Selling\ Price: 2\ for\ Rs.\ 1 \Leftrightarrow 1\ for\ 50\ paise$$

$$Profit = \frac{50 - 40}{40} = \frac{10}{40} = \frac{1}{4} = 25\%$$

Example 5.37
Adarsh bought seven dozen chickoos for Rs. 90 per dozen, and sold them for Rs. 110 per dozen. One dozen chickoos got spoilt, and could not be sold. Adarsh also spent Rs. 10 for travelling to purchase the chickoos. Find his profit or loss percentage.

$$Profit = \underbrace{110 \times 6}_{Selling\ Price} - \underbrace{90 \times 7}_{Cost\ Price} - \underbrace{10}_{Travel\ Cost} = 660 - 640 = 20$$

$$Profit\% = \frac{20}{640} = \frac{2}{64} = \frac{1}{32} = \frac{100}{32}\% = \frac{25}{8}\%$$

D. Finding Profit and Loss from Percentage
Multiplying the cost price by the profit percentage, or the loss percentage, will give us the profit or the loss.

5.38: Finding Profit or Loss
$$Profit = Cost\ Price \times Profit\%$$
$$Loss = Cost\ Price\%$$

Example 5.39
Tom had a profit percentage of 10% in a transaction where his cost price was Rs. 500. Find the profit and the selling price.

$$Profit = 10\%\ of\ CP = 10\%\ of\ 500 = 50\ Rs.$$
$$SP = CP + P = 500 + 50 = 550\ Rs.$$

Example 5.40
Srishti suffered a loss of 15% while selling her house in New York. If she bought the house for $1 Million, how much did she sell the house for?

$$Loss = 15\%\ of\ 1{,}000{,}000 = \frac{15}{100} \times 1{,}000{,}000 = 150{,}000$$
$$SP = CP - L = 1{,}000{,}000 - 150{,}000 = 850{,}000$$

E. If-Then Scenarios

Example 5.41
Shruti bought and sold vegetables from the market. She incurred a cost of Rs. 800, and had a profit percentage of 10%. If she had sold her vegetables for Rs. 40 more, what would have been her profit percentage?

Standard Method
$$Current\ Profit = 10\%\ of\ 800 = \frac{10}{100} \times 800 = 80$$
$$New\ Profit = Old\ Profit + 40 = 80 + 40 = 120$$
$$New\ Profit\ \% = \frac{120}{800} \times 100 = 15\%$$

Shortcut
$$Additional\ Profit\ \% = \frac{40}{800} \times 100 = \frac{40}{8} = 5\%$$
$$Total\ Profit\ \% = Old\ Profit\ \% + New\ Profit\ \% = 10\% + 5\% = 15\%$$

F. Comparison

Example 5.42

Divyansh and Siddharth are traders in the stock market. Divyansh bought 80 shares of $ABC\ LTD$ for Rs. 50 each. Siddharth bought 70 shares of $XYZ\ LTD$ for Rs. 60 each. In a stock market rally, they sold their shares for Rs. 60 and Rs. 70, respectively. Who made more profit and how much? Who had a higher profit percentage?

Profits

Divyansh:

$$Profit\ per\ share = 60 - 50 = 10$$
$$Total\ Profit = Profit\ per\ share \times No.of\ Shares = 10 \times 80 = 800$$

Siddharth:

$$No.of\ Shares \times Profit\ per\ Share = 70(70 - 60) = 70(10) = 700$$

Compare

Both Siddharth and Divyansh made a profit, but Divyansh's profit is more by Rs. 100

Profit Percentage

$$Divyansh: \frac{Profit}{CP} = \frac{10}{50} = 20\%$$
$$Siddharth: \frac{Profit}{CP} = \frac{10}{60} = \frac{1}{6} = \frac{100}{600} = \frac{100}{6}\% = 16\frac{2}{3}\% < 20\%$$

Compare:

Divyansh had the higher profit percentage.

G. Finding the Cost Price from Profit/Loss Percentage

5.43: Calculating Cost Price

If profit percentage is given along with selling price, the profit cannot be directly calculated since

$$Profit\% = \frac{Profit}{CP} \Rightarrow Profit = CP \cdot Profit\%$$

The formula above does not have selling price in it, so we cannot use the above formula to calculate profit.

Example 5.44

A pair of shoes is sold at a profit percentage of 20% and a selling price of Rs. 600. Find the cost price. If the same shoes are sold at a profit percentage of 10%, what will be the new selling price?

Method I: Assume values

In this method, we assume suitable values for the cost price that make our calculations easier. We will still the same profit percentage:

$$If\ CP = 100, then\ P = 20,\ \ and\ SP = 120$$

However, we want a selling price of Rs. 600, so multiply all values above by 5:

$$If\ CP = 500, then\ P = 100,\ \ and\ SP = 600$$

Method II: Find SP as a percentage of the CP

$$CP = 100\%, P = 20\% \Rightarrow SP = C + P = 120\%$$

$$SP = 120\% = 600 \Rightarrow 100\% = \frac{600}{120} \times 100 = 500$$

H. Discount

Example 5.45
Anubhav is in the business of buying and selling cars. He saw a car which was offered to him for Rs. 1,00,000. He negotiated a discount of 5%, and bought the car. He then offered to sell the car for Rs. 1,00,000 and had to offer a discount of 10%. What is the profit or loss that Anubhav made?

The cost price for Anubhav is:

$$1{,}00{,}000 - 5\% \; of \; 1{,}00{,}000 = 1{,}00{,}000 - 5{,}000 = 95{,}000$$

The selling price for Anubhav is:

$$1{,}00{,}000 - 10\% \; of \; 1{,}00{,}000 = 1{,}00{,}000 - 10{,}000 = 90{,}000$$

The loss is:

$$95{,}000 - 90{,}000 = 5000 \; Rs.$$

5.3 Businesses

A. Rent

Example 5.46
A man buys a house for 10,000 dollars and rents it. He puts $12\frac{1}{2}\%$ of each month's rent aside for repairs and upkeep; pays 325 dollars a year taxes and realizes $5\frac{1}{2}\%$ on his investment. The monthly rent is: **(AHSME 1950/23)**

$$5.5\% \; of \; 10{,}000 = \frac{5.5}{100} \times 10{,}000 = 550$$

$$550 + 325 = 875 \; is \; 87.5\% \; of \; the \; Yearly \; Rent$$

Type equation here.

$$Yearly \; Rent = 1000$$

$$Monthly \; Rent = \frac{1000}{12} = 83.33$$

6. Mixing Concepts

6.1 Ratios

A. Finding Ratios

Example 6.1

A. Lee got 80% marks in an exam, while Cho got 90% marks. What is the ratio of Lee's marks to Cho's marks?
B. Mary likes her coffee to be 10% cream, while her sister Macy prefers her coffee to be 15% cream. What is the ratio of the percentage of cream each sister wants in her coffee?

$$80\%:90\% = 80:90 = 8:9$$
$$\underbrace{10\%}_{Mary}:\underbrace{15\%}_{Macy} = 10:15 = 2:3$$

Example 6.2

Find the following ratios:

A. 30% to 45%
B. 75% to 99%
C. 0.2 to 2%
D. $\frac{1}{3}\%$ to 0.2%

$$30\%:45\% = 30:45 = 2:3$$
$$75\%:99\% = 75:99 = 25:33$$
$$0.2:2\% = 20\%:2\% = 20:2 = 10:1$$
$$\frac{1}{3}\%:0.2\% = \frac{1}{3}:\frac{1}{5} = \frac{1}{3}\times 15:\frac{1}{5}\times 15 = 5:3$$

B. Comparing

Example 6.3

10% of the acreage of $\underbrace{a\ farm}_{x}$ is the same as 25% of the acreage of $\underbrace{another\ farm}_{y}$. Find the ratio of the acreage of the two farms.

Conceptual Method

$$10\%\ of\ x = 25\%\ of\ y \Rightarrow \frac{1}{10}x = \frac{1}{4}y \Rightarrow \frac{x}{y} = \frac{10}{4} = \frac{5}{2} \Rightarrow x:y = 5:2$$

Conceptual Method

The second farm must be smaller than the first farm.
The ratio will be in the inverse of the required percentages:

$$25\%:10\% = 5:2$$

C. Unitary Method

Example 6.4

Miki has a dozen oranges of the same size and a dozen pears of the same size. Miki uses her juicer to extract 8 ounces of pear juice from 3 pears and 8 ounces of orange juice from 2 oranges. She makes a pear-orange juice

blend from an equal number of pears and oranges. What percent of the blend is pear juice? **(AMC 8 2002/24)**

To compare easily, find the juice when we have $LCM(2,3) = 6$ pears and 6 oranges.

	Quantity	Ounces of Juice
Pears	6	16
Oranges	6	24
Total		40

The percent of the blend that is pear juice is then:

$$\frac{16}{40} = \frac{8}{20} = \frac{40}{100} = 40\%$$

6.2 Change of Base

A. Changing the standard of comparison

Example 6.5

Bhola's age is 25% more than his brothers' age. If Bhola's age is 40, then his brother's age is what percent of his age?

Let the brother's age be 100%.
Bhola's age will be 125%

Brother's age as a percentage of Bhola's age

$$= \frac{100}{125} \times 100 = \frac{4}{5} \times 100 = 80\%$$

Example 6.6

There are two bookshops. The Blue bookshop sells 50% less books every day than the Red Bookshop. What percentage of the Blue Bookshop's sales does the Red Bookshop sell everyday?

$$\underbrace{100\%}_{Red} \rightarrow \underbrace{50\%}_{Blue}$$

$$\frac{100\%}{50\%} = 2 = 200\%$$

Example 6.7

In April, I jogged 50% more kilometers than I did in March. In May, I jogged 50% less kilometers than I did in April. By what percentage did the number of kilometers that I jogged in May change as compared to March?

$$\underbrace{100\%}_{March} \rightarrow \underbrace{150\%}_{April} \rightarrow \underbrace{75\%}_{May}$$

Percentage change in May as compared to March is

$$100\% - 75\% = 25\% \; Less$$

Example 6.8

The number of *Red Delicious* apples harvested this year is 10% more than the number of *Golden Delicious* apples harvested. And the number of *Honeycrisp* apples harvested is 10% less than the number of *Red Delicious* apples. Find the number of *Honeycrisp* apples as a percentage of the number of *Golden Delicious*?

$$\underbrace{100\%}_{\substack{Golden\\Delicious}} \rightarrow \underbrace{110\%}_{\substack{Red\\Delicious}} \rightarrow \underbrace{99\%}_{Honeycrisp}$$

Example 6.9
A's salary is 25% less than B's salary. How much is B's salary more than A's salary?

$$\underbrace{100}_{B's\ Salary} \Rightarrow \underbrace{75}_{A's\ Salary} \Rightarrow \frac{B's\ Salary}{A's\ Salary} = \frac{100}{75} = \frac{4}{3} = 1\frac{1}{3} \Rightarrow \frac{1}{3}\ extra = 33\%\ percent$$

B. Change of Base

Example 6.10
If A's salary is 25% more than B's salary, how much percent less is B's salary compared to A's salary.

Let $B = 100 \Rightarrow A = 125 \Rightarrow$ B's salary $= \frac{125 - 100}{125} = \frac{25}{125} = \frac{1}{5} = 20\%$ less

Example 6.11
A's salary is 50% more than B's salary. How much percent is B's salary below A's salary? **(NMTC-Primary, Screening, 2007/6)**

This is best done by assuming some numbers.
Let

$$B's\ salary = 100$$

Then A's salary is 50% more than $B's$ salary:

$$A = 100 + 50\%\ of\ 100 = 100 + 50 = 150$$

B's Salary is

$$150 - 100 = 50\ Less\ than\ A$$

And the percentage that B's salary is below A's salary:

$$\frac{50}{150} = \frac{1}{3} = 33\frac{1}{3}\%$$

Example 6.12
Two numbers are respectively 20% and 50% more than a third number.

A. What percentage is the first of the second? **(NMTC-Primary-III, Final)**
B. What percentage is the second of the first?

Let the third number be 100. Then:

$$1st\ No = 120, 2nd\ No. = 150$$

Part A

$$\frac{120}{150} \times 100 = 80\%$$

Part B

$$\frac{150}{120} \times 100 = 125\%$$

6.3 Distributing Items

A. Tiered Calculations

Example 6.13

Radha spends 40% of her salary of $2000 on expenses, and saves the rest.

A. What is the percentage that she saves?
B. What is the amount that she saves?
C. If Radha deposits 60% of her savings in her bank account, and invests the rest in mutual funds, how much does she invest in mutual funds?

Part A

Radha saves

$$100\% - 40\% = 60\% \; of \; her \; salary$$

Part B

$$60\% \; of \; 2000 = \$1200$$

Part C

$$40\% \; of \; 1200 = \$480$$

Example 6.14

Of the 3000 boys in a school, 30% play basketball, and 10% of those come for extra practice on weekends, and the basketball is to be selected from among these. 40 students who come for extra practice are not available to join the basketball team, since they need to attend a school event. From how many boys will the coach make his selection?

$$3000 \times 30\% \times 10\% - 40 = 90 - 40 = 50$$

Example 6.15

Large Numbers

Example 6.16

100,000 people enter a country. Out of these, 90% are vaccinated against disease X. 2% of the people who are not vaccinated get the disease.

A. What is the percentage that is not vaccinated?
B. What is the number of people who are not vaccinated?
C. How many people get the disease?

Part A

If 90% are vaccinated, then the percentage that are not vaccinated:

$$100\% - 90\% = 10\%$$

Part B

The number of people who are not vaccinated is

$$10\% \times 100{,}000 = 10{,}000$$

Part C

$$\frac{2}{100} \times 10{,}000 = 200$$

Example 6.17
The exports of dragon fruit from a country are worth $500,000. Out of these exports, 10% are of red dragon fruit. 3% of the red dragon fruit exports are spoilt on arrival.

A. Find the value of the red dragon fruit exports
B. Find the value of the red dragon fruits that are spoilt on arrival.

Part A

$$50{,}000$$

Part B

$$1500$$

Example 6.18
Light travels at approximately 300,000 km in 1 second. A human takes 10% of one second to blink.

A. Find the distance that light will travel in the time that one can blink an eye.
B. Find the distance that light will travel in 5% of the time that one can blink an eye.

B. Three Step Calculations

Example 6.19
10% of the 20,000 students in a school district sat for AMC. 3% of those who sat made it to the honor list. And 40% of those made it to the honor list, made it to the distinguished honor list. How many students made it to the distinguished honor list?

Example 6.20
1% of the people in a district describe themselves as readers. Of the readers, 30% read fiction, and out of those who read fiction, 20% read science fiction. 20% of those who read science fiction like Isaac Asimov. If there are 1,000,000 people in the district, how many like Isaac Asimov?

$$1{,}000{,}000 \times \frac{1}{100} \times$$

C. Back Calculations

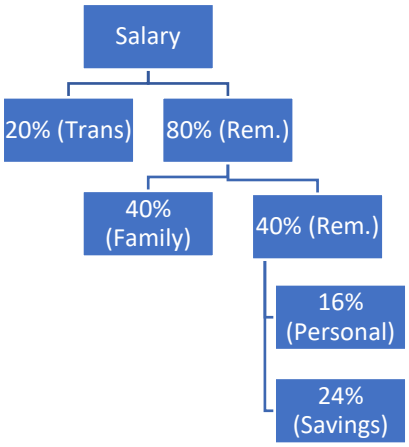
Example 6.21
Amul visited the market and spent 25% of the money that he had on buying fruits, 40% on buying vegetables, and Rs. 10 on transportation. He was left with Rs. 95? What is the money that he started with?

$$100\% - 40\% - 25\% = 95 \ + 10 \Rightarrow 35\% \ = \ 105 \Rightarrow 100\% \ = \ \frac{105}{35} \times 100 \ = \ 300$$

D. Distributing items

Example 6.22
Sheela spends 20% of her salary on transportation. She gives 50% of the remaining amount to her family for expenses. From the remaining, she spends 40% on her personal expenses, and saves the rest. If the amount she saves is a natural number, her salary must be divisible by:

$\text{Savings} = \frac{24}{100} \times \text{Salary} = \frac{6}{25} \times \text{Salary} \Rightarrow 25 \mid \text{Salary}$



6.4 Combining Percentages

A. Basics

Example 6.23

25% of the cost of 25% of a dozen bananas is Rs. 1. Find the cost of two dozen bananas.

$$25\%\ of\ 25\%\ of\ 1\ dozen = Rs.1$$
$$\frac{1}{4} of \frac{1}{4}\ of\ 1\ dozen = 1$$
$$\frac{1}{16}\ of\ 1\ dozen = 1$$

Multiply both sides by 32:

$$32 \times \frac{1}{16}\ of\ 1\ dozen = 32 \times 1$$
$$2\ dozen = 32$$

(Alternate Solution) Example 6.24

25% of the cost of 25% of a dozen bananas is Rs. 1. Find the cost of two dozen bananas.

$$1\ dozen = 12$$
$$25\%\ of\ a\ dozen = \frac{1}{4} \times 12 = 3\ Bananas$$
$$25\%\ of\ 3\ Bananas = \frac{1}{4} \times 3 = \frac{3}{4}$$
$$Cost\ of \frac{3}{4} bananas = Rs.1$$

Multiply both sides by 4:

$$Cost\ of\ 3\ bananas = Rs.4$$

Multiply both sides by 8:

$$Cost\ of\ 24\ bananas = Rs.32$$

Example 6.25

A water tank with capacity 1000 liters is 90% full. 90% of the water in the tank is used for watering. What percentage of the tank is now empty?

$$90\%\ Full = 90\%\ of\ 1000 = 900$$
$$90\%\ of\ 900 = 810$$
$$Water\ Remaining = 900 - 810 = 90\ Litres$$
$$Empty\ Capacity\ of\ Tank = 1000 - 90 = 910\ Litres$$
$$\%\ of\ Tank\ which\ is\ empty = \frac{910}{1000} = 91\%$$

(Alternate Solution) Example 6.26

A water tank with capacity 1000 liters is 90% full. 90% of the water in the tank is used for watering. What percentage of the tank is now empty?

Water used for watering

$$= 90\%\ of\ 90\% = \frac{9}{10} \times \frac{9}{10} = \frac{81}{100} = 81\%$$

Water remaining in the tank

$$= 90 - 81 = 9\%$$

Percentage of tank which is empty

$$= 100 - 9 = 91\%$$

B. Successive Increases

Example 6.27
Jacob joined a firm at a salary of 10,000 per month on 1st Jan 2000. On 1st Jan 2001, his salary increased by 10%. If every year, he gets the same percentage increase in salary on the same date, what will be his monthly salary in the year of 2004.

$$10{,}000 \times \frac{110}{100} \times \frac{110}{100} \times \frac{110}{100} \times \frac{110}{100}$$

Simplify:

$$10{,}000 \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10} \times \frac{11}{10}$$

$$10{,}000 \times \frac{11^4}{10{,}000} = 14641$$

Example 6.28
Mr. Green receives a 10% raise every year. His salary after four such raises has gone up by what percent? (AMC 8 1985/21)

$$\frac{S \times (1.1)^4 - S}{S} = \frac{0.4641S}{S} = 0.4641 = 46.41\%$$

C. Increases with Decreases

Example 6.29
Jane got a 10% hike for the year in Jan, but then due to the coronavirus her company had to reduce salaries, and she got a 10% pay cut. What is the overall change in her salary?

Assume a salary of 100 to make calculations easy:

$$100 \to 100 \times 1.1 = 110 \to 110 \times 0.9 = 99 \to Change\ is - \frac{1}{100} = -1\%$$

6.5 Challenging Questions

A. Challenging Applications

Example 6.30

Answer the questions below *independently.*
A husband's income is 150% of his wife's income in a family consisting of two members.

A. What is the wife's income as a percentage of the total income of the family?
B. The husband's income increased by 10%, while the wife's income increased by 50%. What is the husband's share of the total income of the family in the new situation?
C. The husband saves 30% of his income, while the wife saves 20% of her income. What is the saving of the family as a proportion of their income?

W = 100, H = 150
W as a % = 100 / (100 + 150) = 100/250 = 40%
Saving Percentage = (45 + 20) / 250 = 65/250 =260 / 1000 = 26%

Example 6.31

Convert the following into percentages

A. $\frac{1}{7}$

We can multiply $\frac{1}{7}$ by 1 without changing its value:

$$\frac{1}{7} \times 1$$

We can write 1 as $\frac{100}{100}$ without changing its value:

$$\frac{1}{7} \times \frac{100}{100}$$

We can interchange the position of the 100 and the 1 without changing its value:

$$= \frac{100}{7} \times \frac{1}{100}$$

Substitute $\frac{1}{100} = \%$:

$$= \frac{100}{7}\%$$

Example 6.32

Convert the following into percentages

A. $\frac{1}{6}$
B. $\frac{1}{3}$
C. $\frac{1}{9}$
D. $\frac{1}{11}$
E. $\frac{1}{14}$
F. $\frac{1}{12}$
G. $\frac{1}{35}$

$$\frac{1}{6} \times \frac{100}{100} = \frac{100}{6} \times \frac{1}{100} = \frac{100}{6}\% = 16\frac{2}{3}\%$$
$$\frac{1}{3} \times \frac{100}{100} = \frac{100}{3} \times \frac{1}{100} = \frac{100}{3}\% = 33\frac{1}{3}\%$$
$$\frac{1}{9} \times \frac{100}{100} = \frac{100}{9} \times \frac{1}{100} = \frac{100}{9}\% = 11\frac{1}{9}\%$$

$$\frac{1}{11}\times\frac{100}{100}=\frac{100}{11}\times\frac{1}{100}=\frac{100}{11}\%=9\frac{1}{11}\%$$
$$\frac{1}{14}\times\frac{100}{100}=\frac{100}{14}\times\frac{1}{100}=\frac{100}{14}\%=\frac{50}{7}\%$$
$$\frac{1}{12}\times\frac{100}{100}=\frac{100}{12}\times\frac{1}{100}=\frac{100}{12}\%=\frac{25}{3}\%$$
$$\frac{1}{35}\times\frac{100}{100}=\frac{100}{35}\times\frac{1}{100}=\frac{100}{35}\%=\frac{20}{7}\%$$

33 Examples