Decimals

12 June 2023

Aziz Manva

azizmanva@gmail.com

TABLE OF CONTENTS

1. DECIMALS

1.1 Tenths

A. Fractions and ways of representing them

We have earlier learnt about fractions and different ways of representing them.

Numeric Form

For example, consider the fraction below, which is written in numeric form

$$\frac{2}{3}$$

Word Form

The fraction $\frac{2}{3}$ can represented in word form as:

➢ Two-thirds
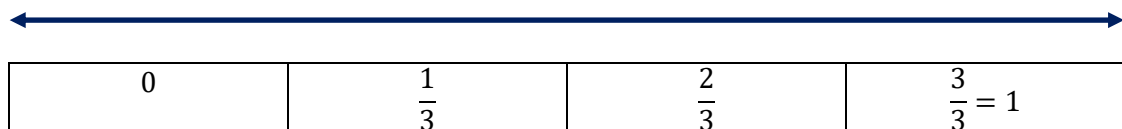➢ Two out of three
➢ Two parts out of three
➢ Two by three

Visual Models

It can also be represented using visual models:

Number Line

And finally, it can be represented on the number line:

| 0	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{3}{3} = 1$

The space on the number between 0 and 1 (and in fact, any two numbers) is divided into three parts. This is because the denominator for the fraction that we want is three, and hence, we want to divide it into three equal parts.

B. Denominator of 4
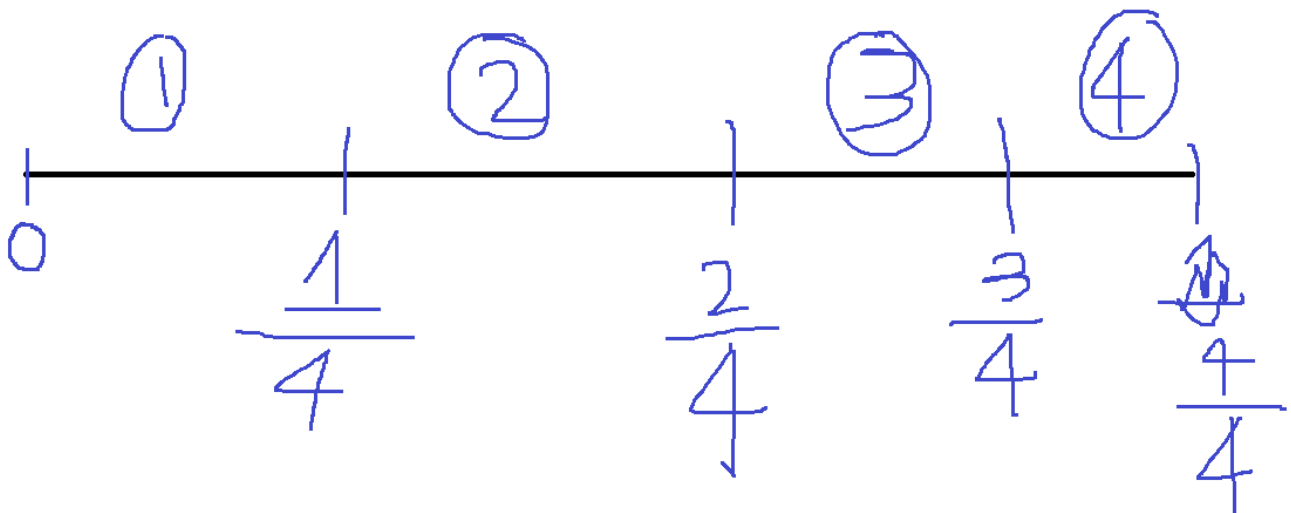
C. Denominator of 10

Fractions with a denominator of 10 are decimal fractions. We will learn how to convert fractions into decimals and vice versa.
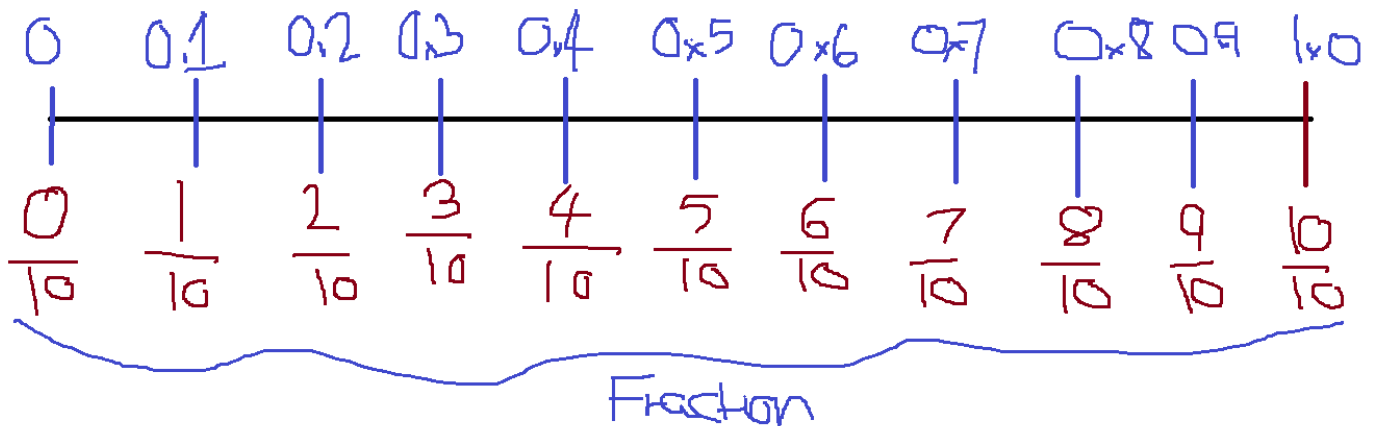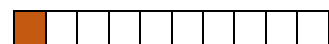

Example 1.1

Write the following fractions in words, and as a decimal:

A. $\frac{1}{10}$

B. $\frac{3}{10}$

C. $\frac{2}{10}$
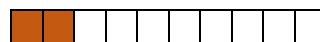
D. $\frac{7}{10}$

E. $\frac{4}{10}$

F. $\frac{9}{10}$

$$\frac{1}{10} = One\ Tenth = 0.1$$

$$\frac{3}{10} = Three\ Tenths = 0.3$$

$$\frac{2}{10} = Two\ Tenths = 0.2$$

$$\frac{7}{10} = Seven\ Tenths = 0.7$$

$$\frac{4}{10} = Four\ Tenths = 0.4$$

$$\frac{9}{10} = Nine\ Tenths = 0.9$$

Example 1.2

Represent the fractions below on a a number line

$$a = \frac{1}{10}, \quad b = \frac{3}{10}, \quad c = \frac{2}{10}, \quad d = \frac{7}{10}, \quad e = \frac{4}{10}, \quad f = \frac{9}{10}$$

0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$\frac{0}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$
	a	c	b	e			d		f	

Example 1.3

  A. $0.2 + 0.5$
  B. $0.8 - 0.3$

$$0.2 + 0.5 = \frac{2}{10} + \frac{5}{10} = \frac{7}{10} = 0.7$$
$$0.2 + 0.5 = 0.7$$

$$0.8 - 0.3 = \frac{8}{10} - \frac{3}{10} = \frac{5}{10} = 0.5$$
$$0.8 - 0.3 = 0.5$$

$$0.3 \times 3 = \frac{3}{10} \times 3 = \frac{9}{10} = 0.9$$
$$0.8 \div 2 = \frac{8}{10} \div 2 = \frac{8}{10} \times \frac{1}{2} = \frac{4}{10} = 0.4$$

Example 1.4

Simplify:

Addition
  A. $0.1 + 0.3$
  B. $0.5 + 0.1$
  C. $0.2 + 0.7$
  D. $0.4 + 0.5$
  E. $0.2 + 0.3$

Subtraction
  F. $0.8 - 0.3$
  G. $0.9 - 0.1$
  H. $0.7 - 0.5$
  I. $0.6 - 0.4$

Multiplication

  J. 0.2×3
  K. 0.3×2
  L. 0.3×3
  M. 0.1×6

Division
  N. $0.6 \div 2$

O. $0.9 \div 3$ | P. $0.8 \div 2$ | Q. $0.8 \div 4$

Addition

$$0.1 + 0.3 = \frac{1}{10} + \frac{3}{10} = \frac{4}{10} = 0.4$$

$$0.5 + 0.1 = \frac{5}{10} + \frac{1}{10} = \frac{6}{10} = 0.6$$

$$0.2 + 0.7 = \frac{2}{10} + \frac{7}{10} = \frac{9}{10} = 0.9$$

$$0.4 + 0.5 = \frac{4}{10} + \frac{5}{10} = \frac{9}{10} = 0.9$$

$$0.2 + 0.3 = \frac{2}{10} + \frac{3}{10} = \frac{5}{10} = 0.5$$

Subtraction

$$0.8 - 0.3 = \frac{8}{10} - \frac{3}{10} = \frac{5}{10} = 0.5$$

$$0.9 - 0.1 = \frac{9}{10} - \frac{1}{10} = \frac{8}{10} = 0.8$$

$$0.7 - 0.5 = \frac{7}{10} - \frac{5}{10} = \frac{2}{10} = 0.2$$

$$0.6 - 0.4 = \frac{6}{10} - \frac{4}{10} = \frac{2}{10} = 0.2$$

Multiplication

Write the multiplication as repeated addition:
$$0.2 \times 3 = 0.2 + 0.2 + 0.2$$
Write the decimals as fractions, add them, and then convert back to decimals:

$$\frac{2}{10} + \frac{2}{10} + \frac{2}{10} = \frac{6}{10} = 0.6$$

$$0.3 \times 2 = 0.3 + 0.3 = \frac{3}{10} + \frac{3}{10} = \frac{6}{10} = 0.6$$

$$0.3 \times 3 = \frac{9}{10} = 0.9$$

Division

$$0.6 \div 2 = Half \ of \ 0.6 = 0.3$$
$$0.9 \div 3 = 0.3$$
$$0.8 \div 2 = 0.4$$

D. Place Value in Decimals

1.5: Place Value System

The place value system is such that the place value increases as we go to the left, and decreases as we go to the right.

For example, consider the number
$$23 = 20 + 3 = \underbrace{2}_{Tens} + \underbrace{3}_{Ones}$$

1.6: Place Value in Decimals

If we want to represent fractions with a denominator of 10, we can extend the place system by introducing a decimal point, and adding one more place value to the right of the decimal point which has value $\frac{1}{10}$ for each number.

For example,
$$23.4 = 20 + 3 + 0.4 = \underbrace{2}_{Tens} \ \underbrace{3}_{Ones} . \ \underbrace{4}_{Tenths}$$

Example 1.7

In the number below, what is the difference between the place value name for the digit 2, and the place value name for the digit 4.

$$\underbrace{2}_{Tens} \ \underbrace{3}_{Ones} . \ \underbrace{4}_{Tenths}$$

$$Digit \ 2 = Tens$$
$$Digit \ 4 = Tenths$$

Example 1.8
Write the numbers below in expanded form to reflect their decimal place values.

7.2

$$7.2 = 7 + 0.2 = 7 + \frac{2}{10}$$

Example 1.9
Write the numbers below in expanded form to reflect their decimal place values.

Group-I
A. 3.7
B. 2.9

C. 1.8
D. 5.6
E. 9.2

F. 8.4
G. 0.3
Group-II

H. 15.8
I. 28.4
J. 10.7

Group I

$3.7 = 3 + 0.7 = \underset{Ones}{3} \ and \ \underset{Tenths}{7}$

$2.9 = 2 + 0.9 = \underset{Ones}{2} \ and \ \underset{Tenths}{9}$

$1.8 = 1 + 0.8 = \underset{Ones}{1} \ and \ \underset{Tenths}{8}$

$5.6 = 5 + 0.6 = \underset{Ones}{5} \ and \ \underset{Tenths}{6}$

$9.2 = 9 + 0.2 = \underset{Ones}{9} \ and \ \underset{Tenths}{2}$

$8.4 = 8 + 0.4 = \underset{Ones}{8} \ and \ \underset{Tenths}{4}$

$0.3 = 0 + 0.3 = \underset{Ones}{0} \ and \ \underset{Tenths}{3}$

Group-II

$15.8 = 10 + 5 + 0.8 = \underset{Tens}{1} + \underset{Ones}{5} + \underset{Tenths}{8}$

$28.4 = 20 + 8 + 0.4 = \underset{Tens}{2} + \underset{Ones}{8} + \underset{Tenths}{4}$

$10.7 = 10 + 0 + 0.7 = \underset{Tens}{1} + \underset{Ones}{0} + \underset{Tenths}{7}$

Example 1.10
Write the following as decimals:

Using Place Value Notation
A. Seven and four tenths
B. Twelve and two tenths
C. Ten and five tenths
D. Three and one tenths

E. Zero and nine tenths
F. Sixteen and zero tenths
Using Fraction Notation
G. Six and a half
H. Four and a quarter

I. Seven and three-fourths
J. A tenth of a dozen
K. Half a litre

Using Place Value Notation

$7\frac{4}{10} = 7.4$

$12\frac{2}{10} = 12.2$

$10\frac{5}{10} = 10.5$

$3\frac{1}{10} = 3.1$

0.9
16.0
Using Fraction Notation
6.5
4.25
7.75

$\frac{12}{10} = 1\frac{2}{10} = 1.2$

0.5

E. Decimals more than 1
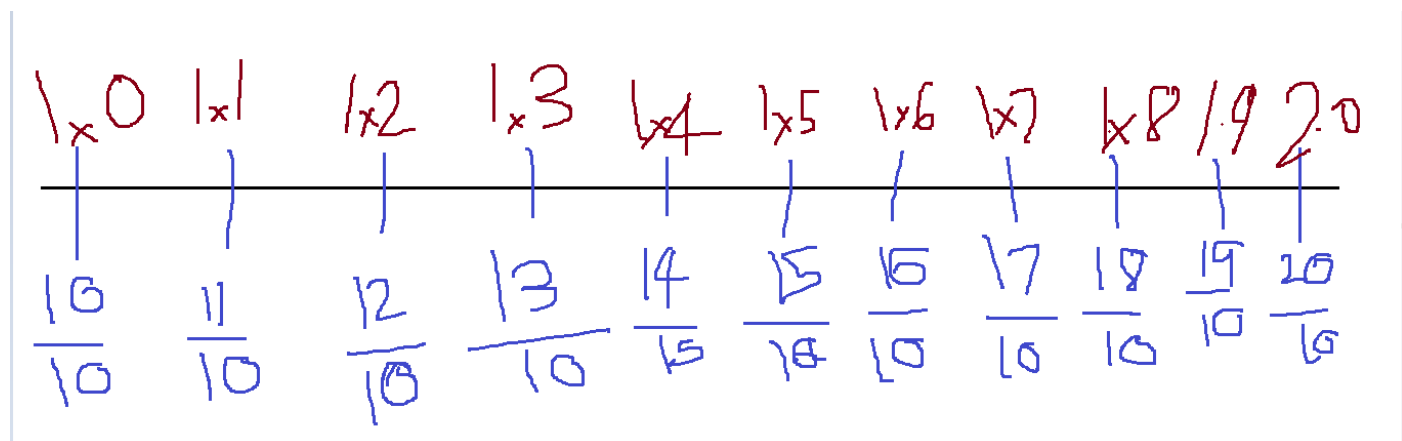
F. Regrouping

Example 1.11

A. $0.6 + 0.7$
B. $1.4 - 0.6$
C. 1.2×3
D. $1.2 \div 3$

$$0.6 + 0.7 = \frac{6}{10} + \frac{7}{10} = \frac{13}{10} = 1\frac{3}{10} = 1 + 0.3 = 1.3$$
$$0.6 + 0.7 = \mathbf{0.13} \Rightarrow \mathbf{\textit{Wrong}}$$

$$1.4 - 0.6 = \frac{14}{10} - \frac{6}{10} = \frac{8}{10} = 0.8$$

$$1.2 \times 3 = \frac{12}{10} \times 3 = \frac{36}{10} = 3.6$$

$$1.2 \div 3 = \frac{12}{10} \div 3 = \frac{12}{10} \times \frac{1}{3} = \frac{4}{10} = 0.4$$

Example 1.12

Addition
A. $0.8 + 0.4$
B. $0.9 + 0.6$
C. $0.5 + 0.8$
D. $0.8 + 0.9$
Subtraction

E. $1.8 - 0.9$
F. $1.5 - 0.8$
G. $1.2 - 0.5$
H. $1.6 - 0.7$
Multiplication
I. 1.2×3

J. 1.8×3
K. 1.6×4
L. 1.3×5
M. 1.7×4
Division
N. $1.2 \div 3$

O. $2.4 \div 4$
P. $3.8 \div 2$
Q. $2.6 \div 2$

Addition

$$0.8 + 0.4 = \frac{8}{10} + \frac{4}{10} = \frac{12}{10} = 1.2$$

$$0.9 + 0.6 = \frac{9}{10} + \frac{6}{10} = \frac{15}{10} = 1.5$$

$$1.8 \times 3 = \frac{18}{10} \times 3 = \frac{54}{10} = 5.4$$

$$0.5 + 0.8 = \frac{5}{10} + \frac{8}{10} = \frac{13}{10} = 1.3$$

$$1.6 \times 4 = \frac{16}{10} \times 4 = \frac{64}{10} = 6.4$$

$$0.8 + 0.9 = \frac{8}{10} + \frac{9}{10} = \frac{17}{10} = 1.7$$

$$1.3 \times 5 = \frac{13}{10} \times 5 = \frac{65}{10} = 6.5$$

Subtraction

$$1.7 \times 4 = \frac{17}{10} \times 4 = \frac{68}{10} = 6.8$$

$$1.8 - 0.9 = \frac{18}{10} - \frac{9}{10} = \frac{9}{10} = 0.9$$

Division

$$1.2 - 0.5 = \frac{12}{10} - \frac{5}{10} = \frac{7}{10} = 0.7$$

$$2.4 \div 4 = \frac{24}{10} \div 4 = \frac{6}{10} = 0.6$$

$$1.6 - 0.7 = \frac{16}{10} - \frac{7}{10} = \frac{9}{10} = 0.9$$

Multiplication

G. Comparing

Example 1.13:
In each pair below, identify which of the following is greater, or say equal if they are both equal.

Decimals
- A. 0.3 & 0.7
- B. 0.9 & 0.2
- C. 2.0 & 0.2
- D. 0.9 & 9

Words

E. *Two Tenths & Five Tenths*

Fractions

Decimals and Fractions

- F. $\frac{3}{10}$ & 0.7
- G. 0.4 & $\frac{5}{10}$

H. 2 & $\frac{21}{10}$

Decimals and Words

Words and Fractions

H. Word Problems

Example 1.14
- A. A housefly is 0.3 cm long. The distance from point A to point B is three times the body length of the housefly. The housefly flies from point A to point B, and back. What is the total distance travelled by the fly, in cm?
- B. What is the perimeter of a square with side length 0.4 meters?
- C. A snail crawls 0.7 meters every hour. It starts climbing a tree which is three and a half meters tall at noon. What is the time when it reaches the top of the tree?
- D. I have a blue rope that is 0.7 meters, and a red rope that is 0.4 meters. How much shorter is the red rope as compared to the blue rope?
- E. Vinod measures a gas sample to contain 0.3 liters of nitrogen, and 0.9 liters of oxygen. If Vinod has in-correctly measured the gas, so that the actual nitrogen is four times the measurement, and the actual oxygen is twice the measurement, what is the total volume of gas in the sample?

Part A

$$Total\ Volume = 0.3 \times 4 + 0.9 \times 2 = 1.2 + 1.8 = 3.0\ Litres$$

Example 1.15

Arnav's school is 3.7 km from his house. And Arnav's library is 2.4 km from his house. Arnav's school is 1.9 km from his library.
   A. Arnav goes in the morning to school, and comes back by afternoon. What is the distance he has travelled?
   B. Arnav visits his school, and his library, and then comes back home. What is the shortest distance that he can travel?
   C. How much further is his school from his house as compared to his library?
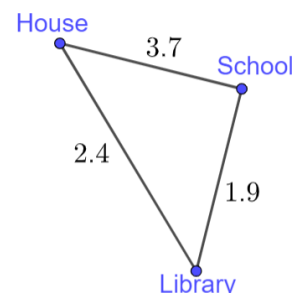
Part A
$$3.7 \times 2 = 7.4$$

Part B
$$\underbrace{3.7}_{\substack{House \\ to\ School}} + \underbrace{1.9}_{\substack{School \\ to\ Library}} + \underbrace{2.4}_{\substack{Library \\ to\ House}} = 8\ km$$

Part C
$$3.7 - 2.4 = 1.3\ km$$

Example 1.16

A monkey is jumping up a

1.2 Hundredths

A. Hundredths Place Value

We can extend the decimal place value further to right of tenths by introducing one place that has value $\frac{1}{100}$

$$23.47 = \underbrace{2}_{Tens}\ \underbrace{3}_{Ones}\ .\ \underbrace{4}_{Tenths}\ \underbrace{7}_{Hundredths}$$
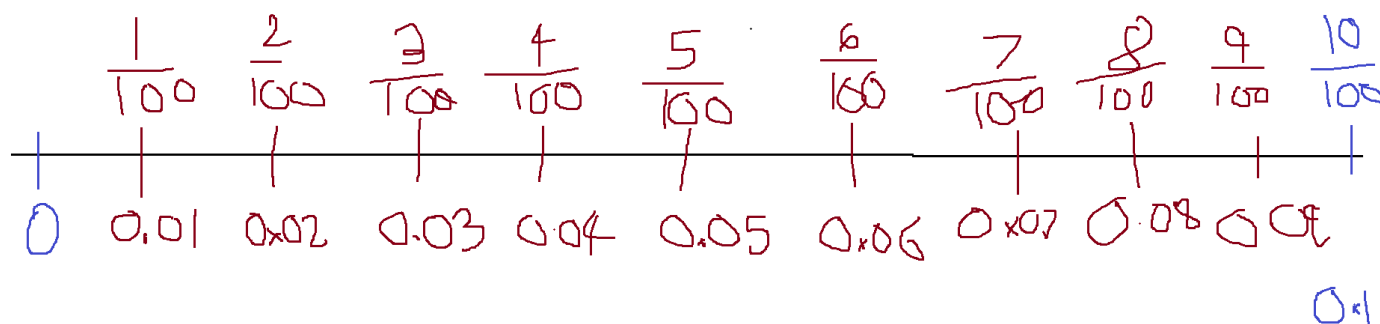
Example 1.17

Find the value of the blue dot in the number line along-side.
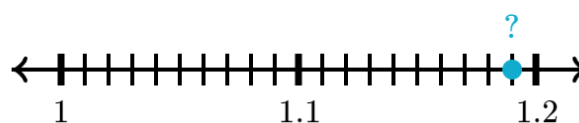
Rewrite the decimals as fractions:

$$1.1 = \frac{11}{10} = \frac{110}{100}$$
$$1.2 = \frac{12}{10} = \frac{120}{100}$$

And the blue dot is between 110 *and* 120, hence, we count and get:

$$\frac{119}{100} = 1.19$$
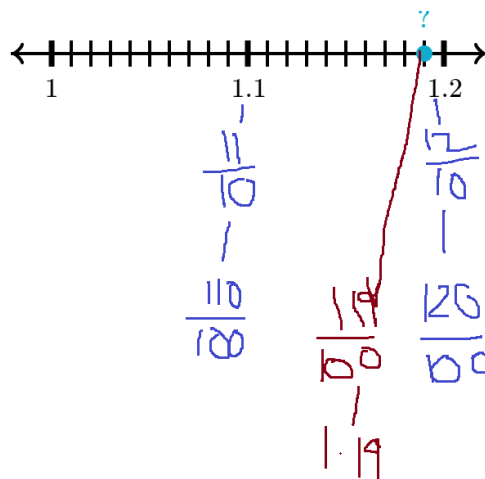
Example 1.18: Conversions
Write the following fractions as Decimals, and in Words

Smaller Numbers
A. $\frac{7}{100}$
B. $\frac{2}{100}$
C. $\frac{5}{100}$
D. $\frac{3}{100}$

E. $\frac{9}{100}$
F. $\frac{4}{100}$

Larger Numbers
G. $\frac{27}{100}$
H. $\frac{13}{100}$

I. $\frac{84}{100}$

Tenths and Hundredths
J. $\frac{9}{10}$
K. $\frac{9}{100}$

L. $\frac{7}{100}$
M. $\frac{7}{10}$
N. $\frac{3}{10}$
O. $\frac{3}{100}$

Smaller Numbers

$$\frac{7}{100} = Seven\ Hundredths = 0.07$$
$$\frac{2}{100} = Two\ Hundredths = 0.02$$
$$\frac{5}{100} = Five\ Hundredths = 0.05$$
$$\frac{3}{100} = Three\ Hundredths = 0.03$$
$$\frac{9}{100} = Nine\ Hundredths = 0.09$$
$$\frac{4}{100} = Four\ Hundredths = 0.04$$

Larger Numbers

$$\frac{27}{100} = 0.27 = Twenty\ Seven\ Hundredths = \underbrace{2}_{Tenths}\ and\ \underbrace{7}_{Hundredths}$$
$$\frac{13}{100} = 0.13 = Thirteeen\ Hundredths = \underbrace{1}_{Tenths}\ and\ \underbrace{3}_{Hundredths}$$
$$\frac{84}{100} = 0.84 = Eighty\ Four\ Hundredths$$

Comparing Tenths and Hundredths

$$\frac{9}{10} = 0.9$$

$$\frac{9}{100} = 0.09 = Nine\ Hundredths$$

$$\frac{7}{10} = 0.7$$

$$\frac{7}{100} = 0.07 = Seven\ Hundredths$$

$$\frac{3}{10} = 0.3$$

$$\frac{3}{100} = 0.03 = Three\ Hundredths$$

Example 1.19: Addition and Subtraction
Simplify

Addition (No Regrouping)
- A. $0.12 + 0.23$
- B. $0.17 + 0.31$
- C. $0.34 + 0.51$
- D. $0.60 + 0.32$
- E. $1.23 + 2.61$
- F. $1.4 + 1.2$

Subtraction (No Regrouping)
- G. $0.88 - 0.23$

- H. $0.75 - 0.24$
- I. $1.82 - 0.51$
- J. $0.96 - 0.24$
- K. $2.9 - 1.6$
- L. $1.4 - 0.2$

Addition (With Regrouping)
- M. $0.85 + 0.25$
- N. $0.25 + 0.45$
- O. $0.88 + 0.12$

- P. $0.23 + 0.39$
- Q. $0.92 + 0.3$
- R. $0.60 + 0.92$

Subtraction (With Regrouping)
- S. $0.53 - 0.27$
- T. $0.90 - 0.42$
- U. $1 - 0.3$
- V. $1 - 0.02$

$$0.12 + 0.23 = \frac{12}{100} + \frac{23}{100} = \frac{35}{100} = 0.35$$

$$0.17 + 0.31 = \frac{17}{100} + \frac{31}{100} = \frac{48}{100}$$

$$0.23 + 0.39 = \frac{23}{100} + \frac{39}{100} = \frac{62}{100} = 0.62$$

$$0.90 - 0.42 = \frac{90}{100} - \frac{42}{100} = \frac{48}{100} = 0.48$$

$$1 - 0.02 = \frac{100}{100} - \frac{2}{100} = \frac{98}{100} = 0.98$$

Example 1.20: Multiplication and Division
Simplify

Multiplication
- A. 1.2×3
- B. 2.3×2
- C. 1.02×3
- D. 4.03×2
- E. 3.12×3

Division

- F. $2.4 \div 4$
- G. $3.6 \div 2$
- H. $6.18 \div 3$
- I. $5.10 \div 5$
- J. $9.54 \div 3$
- K. $2.58 \div 2$
- L. $3.09 \div 3$

Multiplication by 10 & 100 & 1000
- M. Type equation here.

Example 1.21: Moving the Decimal Point

$Multiplying\ by\ 10 \rightarrow Moving\ the\ Decimal\ Point\ 1\ place\ to\ the\ right$

Multiplying by Ten

Multiply each number below by 10
- A. 2.31
- B. 4.3
- C. 0.7
- D. 2.9
- E. 0.04
- F. 9.2
- G. 0.07

- H. 0.25

Multiplying by Hundred

Multiply each number below by 100
- I. 2.341
- J.

Multiplying by Thousand

Multiply each number below by 1000

$$2.31 \times 10 = \frac{231}{100} \times 10 = \frac{23}{10} = 23.1$$
$$4.3 \times 10 = \frac{43}{10} \times 10 = 43$$

Example 1.22: Place Value

$$23.47 = \underset{Tens}{2}\ \underset{Ones}{3}\ .\ \underset{Tenths}{4}\ \underset{Hundredths}{7}$$

If you have tenths, and you write it as a fraction, the denominator is ten.
$$Four\ Tenths = \frac{4}{10} = 0.4$$
$$Twelve\ Tenths = \frac{12}{10} = 1.2$$
If you have hundredth, and you write it as a fraction, the denominator is hundred.
$$Five\ Hundredths = \frac{5}{100} = 0.05$$
$$Twelve\ Hundredths = \frac{12}{100} = 0.12$$
$$One\ hundred\ and\ five\ hundred\ Hundredths = \frac{105}{100} = 1.05$$

Tenths Revision
- A. Seven and five tenths
- B. Four and two tenths
- C. Nine and Fourteen Tenths
- D. Three and thirteen Tenths

Hundredths
- E. Five and three hundredths
- F. Eight and nine hundredths

- G. Six and eleven hundredths

Mixing Tenths and Hundredths
- H. Two and five tenths and four hundredths
- I. Seven and three tenths and nine hundredths
- J. Twelve and seven tenths and two hundredths
- K. Two and six tenths and seven hundredths
- L. Five and four tenths and three hundredths

Tenths Revision

$$7\frac{5}{10} = 7 + 0.5 = 7.5$$

$$4\frac{2}{10} = 4 + 0.2 = 4.2$$

$$9\frac{14}{10} = 9 + 1.4 = 10.4$$

$$3\frac{13}{10} = 3 + 1.3 = 4.3$$

Hundredths Revision

$$5\frac{3}{100} = 5 + 0.03 = 5.03$$

$$8\frac{9}{100} = 8 + 0.09 = 8.09$$

$$6\frac{11}{100} = 6 + 0.11 = 6.11$$

Mixing Tenths and Hundredths

$$2 + \frac{5}{10} + \frac{4}{100} = 2 + 0.5 + 0.04 = 2.54$$

$$
\begin{array}{r}
2.00 \\
0.50 \\
0.04 \\
2.54
\end{array}
$$

$$7 + \frac{3}{10} + \frac{9}{100} = 7 + 0.3 + 0.09 = 7.39$$

$$12 + \frac{7}{10} + \frac{2}{100} = 12 + 0.7 + 0.02 = 12.72$$

$$
\begin{array}{r}
12.00 \\
0.70 \\
0.02
\end{array}
$$

$$2 + \frac{6}{10} + \frac{7}{100} = 2 + 0.6 + 0.07 = 2.67$$

Example 1.23: Expanded Notation With Decimals

Write the following numbers in expanded notation

A. 37
B. 2.4
C. 3.7
D. 7.81
E. 4.53
F. 2.04

$$
\begin{array}{r}
37 = 30 + 7 \\
2.4 = 2 + 0.4 \\
3.7 = 3 + 0.7 \\
7.81 = 7 + 0.8 + 0.01 \\
4.53 = 4 + 0.5 + 0.03 \\
2.04 = 2 + 0 + 0.04
\end{array}
$$

Example 1.24: Expanded Notation in Multiplication Form

Write the following numbers in expanded notation:

A. 37
B. 64
C. 89
D. 472
E. 2.4
F. 3.7
G. 7.81
H. 4.53

I. 2.04

$$37 = 30 + 7 = \underbrace{3 \times 10}_{\substack{Tens \\ Place}} + \underbrace{7 \times 1}_{\substack{Units \\ Place}}$$
$$23 = 20 + 3 = 2 \times 20 + 3 \times 1$$
$$64 = 60 + 4 = 6 \times 10 + 4 \times 1$$
$$89 = 80 + 9 = 8 \times 10 + 9 \times 1$$
$$472 = 400 + 70 + 2 = 4 \times 100 + 7 \times 10 + 2 \times 1$$

Part A

$$2.4 = 2 + 0.4 = 2 + \frac{4}{10} = \left(\underbrace{2}_{\substack{Face \\ Value}} \times \underbrace{1}_{\substack{Place \\ Value}} \right) + \left(\underbrace{4}_{\substack{Face \\ Value}} \times \underbrace{\frac{1}{10}}_{\substack{Place \\ Value}} \right)$$

We write the same thing in table form like this:

	Ones	Tenths
2.4	$\underbrace{2}_{\substack{Face \\ Value}} \times \underbrace{1}_{\substack{Place \\ Value}}$	$\underbrace{4}_{\substack{Face \\ Value}} \times \underbrace{\frac{1}{10}}_{\substack{Place \\ Value}}$

Part B

$$3.7 = 3 + 0.7 = 3 + \frac{7}{10} = (3 \times 1) + \left(7 \times \frac{1}{10} \right)$$

Part C

$$7.81 = 7 + 0.8 + 0.01 = 7 + \frac{8}{10} + \frac{1}{100} = \left(\underbrace{7}_{\substack{Face \\ Value}} \times \underbrace{1}_{\substack{Place \\ Value}} \right) + \left(\underbrace{8}_{\substack{Face \\ Value}} \times \underbrace{\frac{1}{10}}_{\substack{Place \\ Value}} \right) + \left(\underbrace{1}_{\substack{Face \\ Value}} \times \underbrace{\frac{1}{100}}_{\substack{Place \\ Value}} \right)$$

Part D

$$4.53 = 4 + 0.5 + 0.03 = 4 + \frac{5}{10} + \frac{3}{100} = \left(\underbrace{4}_{\substack{Face \\ Value}} \times \underbrace{1}_{\substack{Place \\ Value}} \right) + \left(\underbrace{5}_{\substack{Face \\ Value}} \times \underbrace{\frac{1}{10}}_{\substack{Place \\ Value}} \right) + \left(\underbrace{3}_{\substack{Face \\ Value}} \times \underbrace{\frac{1}{100}}_{\substack{Place \\ Value}} \right)$$

Part E

$$2.04 = 2 + 0.04 = 2 \times 1 + \frac{4}{100}$$

$$3.724 = 3 + 0.7 + 0.02 + 0.004 = 3 + \frac{7}{10} + \frac{2}{100} + \frac{4}{1000}$$
$$= 3 + \left(7 \times \frac{1}{10} \right) + \left(2 \times \frac{1}{100} \right) + \left(4 \times \frac{1}{1000} \right)$$

B. Comparing Decimals

Example 1.25: Comparision

In part, decide which of the given numbers is greater

Same Place Value
- A. *Seven Tenths* and *Five Tenths*
- B. *Two Tenths* and *Six Tenths*
- C. *Five Hundredths* and *Twelve Hundredths*
- D. 0.3 and 0.7
- E. 0.5 and 0.9

Different Place Values
- F. *Three* and *Three Tenths*
- G. *Five Tenths* and *Five Hundredths*
- H. *Two Tenths* and *Seven Hundredths*
- I. 4 and 0.4
- J. 0.4 and 0.09

Example 1.26: Comparision
- A. By how much is 0.4 greater than 0.04?
- B. I have 0.2 of a bitcoin. My sister has 0.03 of a bitcoin. How much more of bitcoins do I have compared to my sister.
- C. How many times is 0.5 of 0.05?

Part A

$$
\begin{array}{r}
0.40 \\
-0.04 \\
\hline
0.36
\end{array}
$$

Part B

$$
\begin{array}{r}
0.20 \\
-0.03 \\
\hline
0.17
\end{array}
$$

Example 1.27: Comparision
Ascending and Descending Order

1.3 Adding Decimals

A. Adding Decimals

Example 1.28
Add the following decimals
$$0.3 + 0.5$$

Method I
We can convert the decimals back into fractions, add the fractions, and then convert the fraction back into a decimal:

$$0.3 + 0.5 = \frac{3}{10} + \frac{5}{10} = \frac{8}{10} = 0.8$$

Method II
You can also write the numbers one below the other in table format, and make sure that you put the place values correctly one below the other.

0	.	5
0	.	3
0	.	8

B. Carryover

Example 1.29

$$0.7 + 0.5$$

Method I

$$0.7 + 0.5 = \frac{7}{10} + \frac{5}{10} = \frac{12}{10} = 1\frac{2}{10} = 1 + 0.2 = 1.2$$

Method II

	Ones		Tenths
Carryover	1		
	0	.	7
	0	.	5
	1	.	2

Practice 1.30

A. $0.8 + 0.5$
B. $0.9 + 0.4$
C. $1.2 + 0.8$
D. $2.3 + 0.9$
E. $1.1 + 0.8$

	Ones		Tenths
Carryover	1		
	1	.	2
	0	.	8
	2	.	0

C. Hundredths

The important thing when adding decimals with more than one digit to the right of the decimal point is that you need to match place values.
This is done the easiest by matching the decimal point.

Example 1.31

A. $0.34 + 0.69$

	Ones		Tenths	Hundredths
Carryover	1		1	
	0	.	3	4
	0	.	6	9
	1	.	0	3

Example 1.32

Add 0.8 and 0.26

We need to be careful since one number has one digit to the right of the decimal point and the other has two.

Method I: Using Fractions

This method is more conceptual, but we won't need it everytime.

$$0.8 + 0.26 = \frac{8}{10} + \frac{26}{100} = \frac{80}{100} + \frac{26}{100} = \frac{106}{100} = 1\frac{6}{100} = 1.06$$

Bogus Solution using Decimals

The solution below is incorrect because the place values are not matched correctly.

	Ones		Tenths
Carryover	1		
	0		8
0	2		6
0	3		4

Method II: Using Decimals

	Ones		Tenths	Hundredths
Carryover	1			
	0	.	8	0
	0	.	2	6
	1	.	0	6

Example 1.33

A. $0.5 + 0.61$

$$0.5 + 0.61 = \frac{5}{10} + \frac{61}{100} = \frac{50}{100} + \frac{61}{100} = \frac{111}{100} = 1\frac{11}{100} = 1 + 0.11 = 1.11$$

	Ones		Tenths	Hundredths
Carryover	1			
	0	.	5	0
	0	.	6	1
	1	.	1	1

Example 1.34

A. $0.4 + 0.63$

	Ones		Tenths	Hundredths
Carryover	1			
	0	.	4	0
	0	.	6	3
	1	.	0	3

D. Thousandths

Example 1.35

Add 0.4 and 0.003

	Ones		Tenths	Hundredths	Thousandths
Carryover	0	.	4	0	0
	0	.	0	0	3
	0	.	4	0	3

Example 1.36
Add 0.6 and 0.159

$$6 + 159 = 165 \Rightarrow 0.6 + 0.159 = 0.165$$
We cannot add like this directly because it does not place value into account.
The correct place for the 6 is in the tenths place, and not the thousandths place.

Correct Solution

	Ones		Tenths	Hundredths	Thousandths
Carryover	0	.	6	0	0
	0	.	1	5	9
	0	.	7	5	9

Example 1.37
Add $0.42 + 0.009$

	Ones		Tenths	Hundredths	Thousandths
Carryover	0	.	4	2	0
	0	.	0	0	9
	0	.	4	2	9

1.4 Subtracting Decimals

A. Basics

Example 1.38
$$0.5 - 0.3$$

Method I: Using Fractions
$$0.5 - 0.3 = \frac{5}{10} - \frac{3}{10} = \frac{2}{10} = 0.2$$

Method II: Using the Table Method

	0	.	5
−	0	.	3

	0	.	2

B. Borrowing

Example 1.40

$$1.2 - 0.8$$

Method I: Using Fractions

$$1.2 - 0.8 = \frac{12}{10} - \frac{8}{10} = \frac{4}{10} = 0.4$$

Method II: Using the Table Method

			10
	10	.	2
−	0	.	8
	0	.	4

Method III
Both numbers have a single decimal point. Remove the decimal point to get:
$$12 - 8 = 4$$

Now, put back the decimal point:
$$0.4$$

C. Hundredths

Example 1.41
Subtract 0.04 from 0.8

Bogus Solution without using Place Value
$$8 - 4 = 4 \Rightarrow 0.8 - 0.04 = 0.4$$
We cannot subtract like this because it does not place value into account.

Using Decimals

	Ones		Tenths	Hundredths
Borrowing				10
	0	.	8 7	0
−	0	.	0	4
	0	.	7	6

Remove the decimal point

$$0.8 - 0.04$$

The first number has a single digit after the decimal point. The second number has two digits after the decimal point. We have to make the number of digits after the decimal point the same:

$$= 0.80 - 0.04$$

Now, we remove the decimal point:

$$80 - 4 = 76$$

Now, we add back two decimal points:

$$0.76$$

Practice 1.42

    A. $0.3 - 0.01$
    B. $0.5 - 0.08$
    C. $1 - 0.02$

	Ones		Tenths	Hundredths
Borrowing				**10**
	0	.	**3 2**	0
−	0	.	0	1
	0	.	2	9

	Ones		Tenths	Hundredths
Borrowing				**10**
	0	.	**5 4**	0
−	0	.	0	8
	0	.	4	2

	Ones		Tenths	Hundredths
Borrowing			**10 9**	**10**
	1 0	.	**0**	0
−	0	.	0	2
	0	.	9	8

D. Thousandths

Example 1.43

Subtract 0.003 from 0.7

$$7 - 3 = 4 \Rightarrow 0.7 - 0.003 = 0.4$$

We cannot subtract like this because it does not place value into account.

Using Decimals

This question requires us to double borrow. We need to subtract in the thousandths place, which has a zero, and

hence, we need to borrow.
But the hundredths place is also zero, and hence, we need to borrow from the tenths place.

	Ones		Tenths	Hundredths	Thou- sandths
Borrowing				**10** 9	**10**
	0	.	**7** 6	0	0
	0	.	0	0	3
	0	.	6	9	7

E. Matching the decimal point

If we have numbers greater than one, then match the decimal point of one number below the decimal point of the other number:
  ➤ Numbers to the left of the decimal point should remain left
  ➤ Numbers to the right of the decimal point should remain right

Example 1.44

$$34.6 - 7.03$$

Tens	Ones		Tenths	Hundredths
	10			**10**
3 2	4	.	**6** 5	0
	7	.	0	3
2	7		5	7

1.5 Place Value

A. Place Value Names

$$\underbrace{8}_{Hundreds} \quad \underbrace{2}_{Tens} \quad \underbrace{3}_{Ones} \quad . \quad \underbrace{7}_{Tenths} \quad \underbrace{1}_{Hundredths} \quad \underbrace{9}_{Thousandths}$$

B. Place Value without Decimals

A digit increases in value as it moves to the left.

Example 1.45

$$2 \underbrace{3}_{Ones} \rightarrow 3 \; has \; value \; 3$$
$$2 \underbrace{3}_{Tens} 1 \rightarrow 3 \; has \; value \; 30$$
$$2 \underbrace{3}_{Hundreds} 1 5 \rightarrow 3 \; has \; value \; 300$$

C. Place Value with Decimals

Example 1.46

$$2. \quad \underset{Tenths}{3} \quad \rightarrow 3 \ has \ value \ 0.3 = \frac{3}{10} = 3 \ tenths$$

$$2.1 \quad \underset{Hundredths}{3} \quad \rightarrow 3 \ has \ value \ 0.03 = \frac{3}{100} = 3 \ hundredths$$

$$2.17 \quad \underset{Thousandths}{3} \quad \rightarrow 3 \ has \ value \ 0.003 = \frac{3}{1000} = 3 \ Thousandths$$

D. Compare Place Value without Decimals

Example 1.47

Compare the place value of the 6 in the two numbers below:

$$216, \quad 763$$

The value of 6 in 216 is

$$6$$

The value of 6 in 763 is

$$60$$

Therefore, the 6 in 763 is 10 times the value of the 6 in 216.

Example 1.48

Compare the place of the 6 in the two numbers below:

$$2.16, \quad 7.63$$

The value of 6 in 2.1 $\underset{Hundredths}{6}$ is

$$0.06$$

The value of 6 in 7. $\underset{Tenths}{6}$ 3 is

$$0.6$$

Therefore, the 6 in 763 is 10 times the value of the 6 in 216.

Example 1.49

Write the following in decimals
- A. *Seven and four hundredths*
- B. *Two and five tenths*
- C. *Twenty and one hundredth*

$$Seven \ and \ four \ hundredths = 7\frac{4}{100} = 7.04$$

$$7.004$$
$$2.05$$
$$20.01$$
$$12.12$$

Example 1.50
A. *Twelve and twelve hundredths*

$$Twelve = 12$$

$$Twelve\ hundredths = \frac{12}{100} = 0.12$$
$$12 + 0.12 = 12.12$$

1.6 Number Line

A. Tenths

Example 1.51

Find 0.4 on the number line.

$$0.4 = \frac{4}{10}$$

To position $\frac{4}{10}$ on the number line, you will need to divide intervals between 0 and 1 into ten parts.

	0				0.4						1
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Tenths	$\frac{0}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$
Hundredths	0.0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90	1.00
	$\frac{0}{100}$	$\frac{10}{100}$	$\frac{20}{100}$	$\frac{30}{100}$	$\frac{40}{100}$	$\frac{05}{100}$	$\frac{60}{100}$	$\frac{70}{100}$	$\frac{80}{100}$	$\frac{90}{100}$	$\frac{100}{100}$
Thousands	0.00	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000
	$\frac{0}{1000}$	$\frac{100}{1000}$	$\frac{200}{1000}$	$\frac{300}{1000}$	$\frac{400}{1000}$	$\frac{500}{1000}$	$\frac{600}{1000}$	$\frac{700}{1000}$	$\frac{800}{1000}$	$\frac{900}{1000}$	$\frac{1000}{1000}$

B. Hundredths

Example 1.52

Find 0.43 on the number line.

$$0.43 = \frac{43}{100}$$

To position $\frac{43}{100}$ on the number line, you will need to divide intervals between 0.4 and 0.5 into ten parts.

Divide into Tenths

First, as above, we divide the number line into ten parts:

	0				0.4	0.5					1
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Tenths	$\frac{0}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$

Divide into Hundredths

We are interested in finding

$$0.43$$

Which has to be in between 0.4 and 0.5 on the number line
.

Hence, we focus on the part of the number line from 0.4 to 0.5 only.

	0.4										0.5
Tenths	$\frac{4}{10}$										$\frac{5}{10}$
	$\frac{40}{100}$	$\frac{41}{100}$	$\frac{42}{100}$	$\frac{43}{100}$	$\frac{44}{100}$	$\frac{45}{100}$	$\frac{46}{100}$	$\frac{47}{100}$	$\frac{48}{100}$	$\frac{49}{100}$	$\frac{50}{100}$
Hundredths	0.40	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.50

C. Thousandths

Example 1.53

Find 0.432 on the number line.

$$0.432 = \frac{432}{1000}$$

To position $\frac{432}{1000}$ on the number line, we need to divide intervals between 0.43 and 0.44 into ten parts.

Divide into Tenths

First, as above, we divide the number line into ten parts.

	0										1
	0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
Tenths	$\frac{0}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{3}{10}$	$\frac{4}{10}$	$\frac{5}{10}$	$\frac{6}{10}$	$\frac{7}{10}$	$\frac{8}{10}$	$\frac{9}{10}$	$\frac{10}{10}$

Divide into Hundredths

We are interested in finding

$$0.432$$

Which has to be in between 0.4 and 0.5 on the number line
.

Hence, we focus on the part of the number line from 0.4 to 0.5 only.

	0.4										0.5
Tenths	$\frac{4}{10}$										$\frac{5}{10}$

	$\frac{40}{100}$	$\frac{41}{100}$	$\frac{42}{100}$	$\frac{43}{100}$	$\frac{44}{100}$	$\frac{45}{100}$	$\frac{46}{100}$	$\frac{47}{100}$	$\frac{48}{100}$	$\frac{49}{100}$	$\frac{50}{100}$
	0.40	0.41	0.42	0.43	0.44	0.45	0.46	0.47	0.48	0.49	0.5

Divide into Thousandths
We are interested in finding

$$0.432$$

Which has to be in between 0.43 and 0.44 on the number line
.

Hence, we focus on the part of the number line from 0.43 to 0.44 only.

Hundredths	0.43 $\frac{43}{100}$										0.44 $\frac{44}{100}$
	$\frac{430}{1000}$	$\frac{431}{1000}$	$\frac{432}{1000}$	$\frac{433}{1000}$	$\frac{434}{1000}$	$\frac{435}{1000}$	$\frac{436}{1000}$	$\frac{437}{1000}$	$\frac{438}{1000}$	$\frac{439}{1000}$	$\frac{440}{1000}$
Thou-sandths	0.430	0.431	0.432	0.433	0.434	0.435	0.436	0.437	0.438	0.439	0.440

1.7 Rounding

A. Introduction
When we want to round, we are giving an approximate number instead of a precise number.

For example:

A school has 308 *students.*

Instead of saying 308, if someone asks us, "How many hundred student are there in the school" we might say that:

The school has around 300 *students*

B. Revision: Rounding of Whole Numbers
Rounding on a number line tells us a number is closer to which number on the number line.

Example 1.54
Round 43 to the nearest ten on the number line.

Method I
Since we are counting in terms of tens, we will count

    0,   10,   20,   30,   40,   50,   60, …

Perfect Ten										Perfect Ten
$40 = 4 \times 10$	41	42	43	44	45	46	47	48	49	$50 = 5 \times 10$
3	2	1	0							
			0	1	2	3	4	5	6	7

Method II
Since we are rounding to the nearest ten, we check the digit just to the right of the ten's digit. This digit is the

Unit's digit.

$$\underbrace{4}_{\substack{Tens\ Digits \\ Rounding\ Here}} \quad \underbrace{3}_{\substack{Units\ Digit \\ Digit\ to\ check}}$$

Since

$$3 < 5$$

We will round down.

Therefore, we will go back on the number line, till we find the ten that comes before 43. This is

$$43, \quad 42, \quad 41, \quad 40$$

Example 1.55
Round 45 to the nearest ten on the number line.

Perfect Ten	Going Left					Going Right				Perfect Ten
$40 = 4 \times 10$	41	42	43	44	45	46	47	48	49	$50 = 5 \times 10$
5	4	3	2	1	0	1	2	3	4	5
					0					

C. Rules
To round a number, we need to know the place value to which the number is being rounded.
We can round at:
- Tens
- Hundreds
- Thousands

etc

Steps in Rounding
Check the digit to the right of the place value that we are rounding to. If the digit is
- 4 or less, we will keep the digit on the left unchanged.
- 5 or more, we increase the value of the digit on the left by one.

All digits to the right of the place value that we are rounding to will become zero.

Example 1.56
Round 345 to the nearest
- Ten
- Hundred

Round to the nearest Ten
We are rounding to the **nearest ten**, which has digit 4 in its place.
Since we are rounding to the nearest ten, we will check the digit to the right of the tens place, which is the **one's place**.

$$345 = \underbrace{3}_{Hundreds} \underbrace{4}_{Tens} \underbrace{5}_{Ones}$$

The digit in the ones place is

$$5 \Rightarrow 5 \ or \ more$$

Therefore, we increase the digit in the tens place by 1:
$$345 \to 350$$

Round to the nearest Hundred

We are rounding to the nearest hundred, which has digit 3 in its place.
Since we are rounding to the nearest hundred, we will check the digit to the right of the hundred's place, which is the ten's place.

$$345 = \quad \underbrace{3}_{Hundreds} \quad \underbrace{4}_{Tens} \quad \underbrace{5}_{Ones}$$

The digit in the ten's place is
$$4 \Rightarrow less\ than\ 5$$
Therefore, we keep the digit in the hundred's place to have the same value:
$$345 \to 300$$

Example 1.57

Round each of the following numbers to the nearest
- ➢ Ten
- ➢ Hundred
- ➢ Thousand

A. 2174
B. 3761
C. 2073

Part A
$$2174 \to 2170$$
$$2174 \to 2200$$
$$2174 \to 2000$$

Part B
$$3761 \to 3760$$
$$3761 \to 3700$$
$$3761 \to 4000$$

Part C

$$2073$$

D. Rounding of Decimals

Example 1.58

Round the following numbers as indicated:
To the nearest one and the nearest ten:
A. 3.1
B. 3.07
C. 23.7
D. 34.81
To the nearest tenth and the nearest whole number

A. 2.45
B. 3.05

E. Largest and Smallest Number

1.8 Converting between Decimals and Fractions

A. Common Decimals to Fractions

$$0.2_{\text{Decimal}} = \frac{2}{10}_{\substack{\text{Decimal} \\ \text{Fraction}}} = \frac{1}{5}_{\substack{\text{Simplified} \\ \text{Fraction}}}$$

$$0.75 = \frac{75}{100} = \frac{25 \times 3}{25 \times 4} = \frac{3}{4}$$

B. Common Fractions to Decimals

$$\frac{1}{5}_{\text{Fraction}} = \frac{1}{5} \times \frac{2}{2} = \frac{2}{10}_{\substack{\text{Equivalent} \\ \text{Decimal Fraction}}} = 0.2_{\text{Decimal}}$$

	Multiples of $\frac{1}{4}$				Multiples of $\frac{1}{5}$				
Decimal	Decimal Fraction	Simplified Fraction		Decimal	Decimal Fraction	Simplified Fraction			
0.25	$\frac{25}{100}$	$\frac{1}{4}$		0.2	$\frac{2}{10}$	$\frac{1}{5}$			
0.50	$\frac{50}{100}$	$\frac{1}{2}$		0.4	$\frac{4}{10}$	$\frac{2}{5}$			
0.75	$\frac{75}{100}$	$\frac{3}{4}$		0.6	$\frac{6}{10}$	$\frac{3}{5}$			
				0.8	$\frac{8}{10}$	$\frac{4}{5}$			

C. General Decimals to Fractions
To convert a number from a decimal to a fraction:
➢ Remove the decimal point
➢ Divide by 1 followed as many zeroes as there are digits after the decimal point

Example 1.59
Convert the following decimals to fractions

Single Decimal Point
A. 0.4
B. 0.7
C. 1.3
D. 9.4
E. 7.8
F. 6.7

G. 12.7
Two Decimal Points
H. 0.43
I. 0.32
J. 0.85
K. 1.23
L. 2.45

M. 3.04
Three Decimal Points
N. 0.314
O. 0.201
P. 0.700
Q. 2.340
Mixed

R. 0.04
S. 3.5
T. 2.001
U. 5.6
V. 6.22

$$0.4 = \frac{4}{10}$$

$$0.7 = \frac{7}{10}$$

$$1.3 = \frac{13}{10}$$

$$9.4 = \frac{94}{10}$$

$$7.8 = \frac{78}{10}$$

$$6.7 = \frac{67}{10}$$

D. Fractions to Decimals

Example 1.60
Convert the following decimals to fractions

Single Decimal Point

A. $\dfrac{6}{10}$

B. $\dfrac{9}{10}$

C. $\dfrac{3}{10}$

D. $\dfrac{48}{10}$

E. $\dfrac{39}{10}$

F. $\dfrac{12}{10}$

G. $\dfrac{123}{100}$

H. $\dfrac{371}{100}$

I. $\dfrac{236}{10}$

Example 1.61
Convert the following fractions to decimals

Denominator of 10

A. $\dfrac{3}{10}$

B. $\dfrac{7}{10}$

C. $\dfrac{12}{10}$

Denominator of 100

D. $\dfrac{72}{100}$

E. $\dfrac{40}{100}$

F. $\dfrac{65}{100}$

Denominator of 2

G. $\dfrac{1}{2}$

H. $\dfrac{3}{2}$

I. $\dfrac{7}{2}$

J. $\dfrac{9}{2}$

K. $\dfrac{15}{2}$

Denominator of 5

L. $\dfrac{3}{5}$

M. $\dfrac{2}{5}$

N. $\dfrac{1}{5}$

O. $\dfrac{4}{5}$

P. $\dfrac{7}{5}$

Q. $\dfrac{14}{5}$

R. $\dfrac{21}{5}$

S. $\dfrac{31}{5}$

Denominator of 20

T. $\dfrac{3}{20}$

U. $\dfrac{7}{20}$

V. $\dfrac{11}{20}$

W. $\dfrac{13}{20}$

X. $\dfrac{23}{20}$

Y. $\dfrac{35}{20}$

Denominator of 25

Z. $\dfrac{4}{25}$

AA. $\dfrac{7}{25}$

BB. $\dfrac{12}{25}$

CC. $\dfrac{15}{25}$

DD. $\dfrac{9}{25}$

EE. $\dfrac{31}{25}$

FF. $\dfrac{56}{25}$

Denominator of 4

GG.

Denominator of 10

Denominator of 100

Denominator of 2

$$\frac{1}{2} = \frac{5}{10} = 0.5$$

$$\frac{3}{2} = 1\frac{1}{2} = 1 + 0.5 = 1.5$$

$$\frac{7}{2} = 3\frac{1}{2} = 3 + 0.5 = 3.5$$

Denominator of 5

$$\frac{3}{5} = \frac{6}{10} = 0.6$$

$$\frac{2}{5} = \frac{4}{10} = 0.4$$

$$\frac{1}{5} = 0.2$$
$$\frac{4}{5} = 0.8$$

$$\frac{7}{5} = 1\frac{2}{5} = 1.4$$
$$\frac{14}{5} = 2\frac{4}{5} = 2.8$$

1.9 Multiplication

A. Revision

1.62: Multiplying and Dividing by 10
When we multiply by 10, we add one zero to the number.
When we divide by 10, we take away one zero from the number.

For example
$$32 \times 10 = 320$$
And
$$670 \div 10 = 67$$

Example 1.63: Working with 10
Carry out the following multiplications and divisions
   A. 21×10
   B. 57×10
   C. 634×10
   D. $340 \div 10$
   E. $720 \div 10$
   F. $3500 \div 10$
   G. $1200 \div 10$

<div align="center">

210
570
6340
34
72
350
120

</div>

1.64: Multiplying and Dividing by 100
When we multiply by 100, we add two zeros to the number.
When we divide by 100, we take away two zeros from the number.

Example 1.65: Working with 100
   A. 22×100
   B. 78×100
   C. 20×100
   D. $3400 \div 100$
   E. $7200 \div 100$
   F. $30,000 \div 100$

<div align="center">

2200

</div>

7800
2000
34
72
300

1.66: Multiplying and Dividing by 1000

When we multiply by 1000, we add three zeros to the number.
When we divide by 1000, we take away three zeros from the number.

Example 1.67

A. 45×1000
B. 123×1000
C. 451×1000
D. $34000 \div 1000$
E. $45000 \div 1000$
F. $29000 \div 1000$

45000
123,000
451,000
34
45
29

Example 1.68: Mixed Review

Carry out the indicated operation
A. $2 \times 32 \times 10$
B. $4 \times 16 \times 10$
C. $25 \div 10 \times 4$
D. $125 \times 2 \div 10$
E. $3 \div 10 \times 10$

Part A
$$2 \times 32 \times 10 = 64 \times 10 = 640$$

Part B
$$4 \times 16 \times 10 = 64 \times 10 = 640$$

Part C
$$25 \div 10 \times 4 = 25 \times 4 \div 10 = 100 \div 10 = 10$$
$$25 \div 10 \times 4 = \frac{25}{10} \times 4 = \frac{100}{10} = 10$$

Part D
The easier way to do this to carry out the multiplication first:
$$125 \times 2 \div 10 = 250 \div 10 = 25$$

The long method to do this is to carry out the division first. Note that it gives the right answer but is substantially more difficult in terms of the calculations.
$$125 \times 2 \div 10 = 125 \times \frac{2}{10} = 125 \times \frac{1}{5} = \frac{125}{5} = 25$$

Part E
Method I

$$3 \div 10 \times 10 = \frac{3}{10} \times 10 = 3$$

Method II

$$3 \underbrace{\div 10}_{\substack{Dividing \\ by\ 10}} \underbrace{\times 10}_{\substack{Multiplying \\ by\ 10}} = 3$$

1.69: Multiplying by a Power

To multiply by 10 or 100, or 1000:
  ➢ Convert the decimal into a decimal fraction
  ➢ Cancel the zeros where possible
  ➢ Convert back into a decimal

$$3.75 \times 10 = \frac{375}{10\textbf{0}} \times 1\textbf{0} = \frac{375}{10} = 37.5$$

1.70: Shortcut

To multiply by 10, move the decimal point one place to the right.

$$3 \underset{\substack{Decimal \\ Point}}{.} 75 \times 10 = 37 \underset{\substack{Decimal \\ Point}}{.} 5$$

$$6 \underset{\substack{Decimal \\ Point}}{.} 3 \times 10 = 63 \underset{\substack{Decimal \\ Point}}{.} 0$$

$$0 \underset{\substack{Decimal \\ Point}}{.} 47 \times 10 = 4 \underset{\substack{Decimal \\ Point}}{.} 7$$

Example 1.71: Multiplying by a Power of Ten

Multiply the following numbers by 10:
  A. 89.4
  B. 0.3
  C. 0.34
  D. 79.21
  E. 8.4
  F. 0.9
  G. 0.51
  H. 5.4
  I. 0.6
  J. 0.41

$$6.3 \times 10 = \frac{63}{1\textbf{0}} \times 1\textbf{0} = \frac{63}{1} = 63$$

$$89.4 \times 10 = \frac{894}{10} \times 10 = 894$$

$$0.3 \times 10 = \frac{3}{10} \times 10 = 3$$

$$0.34 \times 10 = \frac{34}{100} \times 10 = \frac{34}{10} = 3.4$$
$$79.21 \times 10 = \frac{7921}{100} \times 10 = \frac{7921}{10} = 792.1$$
$$8.4 \times 10 = \frac{84}{10} \times 10 = 84$$
$$0.9 \times 10 = 9$$
$$0.51 \times 10 = \frac{51}{100} \times 10 = 5.1$$
$$5.4$$

1.72: Multiply by 100
To multiply by 100, move the decimal point two places to the right.

$$5 \quad . \quad 6 \times 100 = 56 \times 10 = 560 \quad . \quad 0$$
Decimal Point   *Decimal Point*

Example 1.73
Multiply the numbers below by 100:
- A. 5.6
- B. 34.2
- C. 10.2
- D. 0.4
- E. 0.41
- F. 0.417
- G. 0.8
- H. 0.82
- I. 0.829
- J. 0.7
- K. 0.72
- L. 0.714
- M. 0.5
- N. 0.53
- O. 0.531

$$5.6 \times 100 = 56 \times 10 = 560$$
$$34.2 \times 100 = 342 \times 10 = 3420$$
$$10.2 \times 100 = 1020$$
$$0.7 \times 100 = 7 \times 10 = 70$$
$$0.72 \times 100 = 7.2 \times 10 = 72$$
$$0.714 \times 100 = 71.4 \times 10 = 71.4$$
$$0.41 \times 100 = 4.1 \times 10 = 41$$

Example 1.74
Multiply the numbers below by 100:
- A. 0.04
- B. 0.041
- C. 0.0417

Example 1.75
Multiply the numbers below by 100:
- A. 23
- B. 0.23
- C. 0.5
- D. 0.71

1.76: Multiply by 100
To multiply by 1000, move the decimal point three places to the right.

$$5 \overset{.}{\underset{\substack{Decimal \\ Point}}{}} 6 \times 100 = 56 \times 10 = 560 \overset{.}{\underset{\substack{Decimal \\ Point}}{}} 0$$

Example 1.77
Multiply the numbers below by 1000.

Greater than 1
- A. 5.6
- B. 34.2
- C. 10.2

Smaller than 1
- D. 0.4
- E. 0.41

- F. 0.417
- G. 0.8
- H. 0.82
- I. 0.829
- J. 0.7
- K. 0.72
- L. 0.714

- M. 0.5
- N. 0.53
- O. 0.531

Smaller than 0.1
- P. 0.04
- Q. 0.041
- R. 0.0417

Mixed
- S. 23
- T. 0.23
- U. 0.5
- V. 0.71

$$5.600 \rightarrow 5600$$
$$34.200 \rightarrow 34,200$$
$$10.200 \rightarrow 10,200$$
$$5.6 \times 1000 = \frac{56}{1\textcolor{red}{0}} \times 100\textcolor{red}{0} = \frac{56}{1} \times 100 = 5600$$
$$34.2 \times 1000 = \frac{342}{1\textcolor{red}{0}} \times 100\textcolor{red}{0} = \frac{342}{1} \times 100 = 34200$$

B. Multiplying by $0.1, 0.01$ and 0.001

1.78: Multiply by 0.1, 0.01, and 0.001
To multiply by 0.1, we move the decimal point one place to left.
To multiply by 0.01, we move the decimal point two places to the left.
To multiply by 0.001, we move the decimal point three places to the left.

$$0.1 = \frac{1}{10}$$

Hence, multiplying by 0.1 is the same as dividing by 10.

$$0.01 = \frac{1}{100}$$

Multiplying by 0.01 is the same as dividing by 100.

$$0.001 = \frac{1}{1000}$$

Multiplying by 0.001 is the same as dividing by 1000.

Example 1.79
What is the difference between multiplying by 10, and multiplying by 0.1?

When we multiply by 10, we move the decimal point one place to the right. The number increases.
When we multiply by 0.1, we move the decimal point one place to the left. The number decreases.

$$3.4 \times 10 = 34$$
$$3.4 \times 0.1 = 0.34$$

Example 1.80
Multiply the decimal numbers in the following exercise as indicated:

Multiply by 0.1		**Multiply by 0.01**	
A. 3.75	G. 79.21	**Multiply by 0.01**	T. 0.714
B. 7	H. 8.4	N. 5.6	**Multiply by 0.001**
C. 6.3	I. 0.9	O. 8	U. 23.1
D. 89.4	J. 0.51	P. 34.2	V. 341
E. 0.3	K. 5.4	Q. 10.2	W. 23.4
F. 0.34	L. 0.6	R. 0.7	X. 6.75
	M. 0.47	S. 0.72	

$$3.75 \times 0.1 = \frac{375}{100} \times \frac{1}{10} = \frac{375}{1000} = 0.375$$
$$6.3 \times 01 = 6.3 \div 10 = 0.63$$
$$89.4 \div 10 = 8.94$$
$$0.3 \div 10 = 0.03$$
$$0.34 \div 0.034$$

$$23.1 \div 10 = 2.31$$
$$2.31 \div 10 = 0.231$$
$$0.231 \div 10 = 0.0231$$

$$341 \div 10 = 341.0 \div 10 = 34.10$$
$$34.1 \div 10 = 3.41$$
$$3.41 \div 10 = 0.341$$

Example 1.81
Find each expression below
   A. $37 \div 10$
   B. 37×0.1
   C. $37 \times \frac{1}{10}$

$$37 \div 10 = \frac{37}{10} = 3.7$$
$$37 \times 0.1 = 37 \times \frac{1}{10} = \frac{37}{10} = 3.7$$
$$37 \times \frac{1}{10} = \frac{37}{10} = 3.7$$

1.82: Comparison

$$a \div 10 = a \times 0.1 = a \times \frac{1}{10}$$

Example 1.83
Divide the following numbers by 10.

Multiples of Ten
- A. 30
- B. 60

Two Digit Numbers
- C. 41
- D. 89
- E. 78
- F. 53

One Decimal Point
- G. 2.3
- H. 7.4
- I. 8.9

Two Digit Numbers with One Decimal Point
- J. 45.6
- K. 89.1
- L. 90.3

$$41 \div 10 = 41.0 \div 10 = 4.10 = 4.1$$

$$2.3 \div 10 = 0.23$$

Example 1.84
Adivait took the number 23.04 and divided it by 10. Bholu took the same number that Adivait started with and multiplied it by the decimal 0.1. Cheentu took the same number that Bholu started with and multiplied it by one-tenth. And Dhyanchand took the same number that Bholu ended with and found one-tenth of the number. Arrange the numbers that Adivait, Bholu, Cheentu and Dhyandchand found from least to greatest.

$$Adivait = 23.04 \div 10 = 2.304$$
$$Bholu = 23.04 \times 0.1 = 2.304$$
$$Cheentu = 23.04 \times \frac{1}{10} = 2.304$$
$$Dhyanchand = One - tenth \ of \ 2.304 = \frac{1}{10} \times 2.304 = 0.2304$$

$$Adivait = Bholu = Cheentu > Dhyanchand$$

Therefore, their answers will be the same.

C. Single Decimal Digit

1.85: Decimal Multiplication
➢ Count the number of decimal digits
➢ Remove the decimal point
➢ Carry out the multiplication
➢ Add back the sum of the decimal digits

Decimal digits mean number of non-zero digits to the right of the decimal point.

Example 1.86
$$0.3 \times 2$$

Count the decimal digit:
$$0.3 \rightarrow 1 \; decimal \; digit$$
$$2 \rightarrow 0 \; decimal \; digits$$
$$Total = 1 \; decimal \; digit$$

$$3 \times 2 = 6$$
Add back the decimal digit:
$$0.6$$

Example 1.87
Carry out the following multiplication:
A. 0.4×3
B. 0.5×3
C. 0.6×4
D. 0.7×5
E. 0.4×8
F. 0.9×2

$$0.3 \times 2 = \frac{3}{10} \times \frac{2}{1} = \frac{6}{10} = 0.6$$
$$0.4 \times 3 = \frac{4}{10} \times \frac{3}{1} = \frac{12}{10} = 1.2$$

Example 1.88
$$0.12 \times 2$$

Remove the decimal point. There are 2 digits to the right of the decimal point:
$$12 \times 2 = 24$$
Put the decimal point back
$$= 0.24$$

Example 1.89
  A. 0.11×3
  B. 0.23×2
  C. 0.21×5
  D. 0.12×30
  E. 0.14×30

$$\frac{12}{100} \times 2 = \frac{24}{100} = 0.24$$

Example 1.90

$$0.2 \times 0.3$$

$$0.2 \to 1 \text{ DD}$$
$$0.3 \to 1 \ DD$$
$$Total = 1 + 1 = 2 \ DD$$

Remove the decimal point:
$$2 \times 3 = 6$$

Put the two decimal points back. We need two digits to the right of the decimal point:
$$0.06$$

Example 1.91
  A. 0.4×0.3
  B. 0.7×0.3
  C. 0.1×0.2
  D. 0.8×0.2
  E. 0.7×0.6

$$0.2 \times 0.3 = \frac{2}{10} \times \frac{3}{10} = \frac{6}{100} = 0.06$$

$$0.4 \times 0.3 = \frac{4}{10} \times \frac{3}{10} = \frac{12}{100} = 0.12$$

Example 1.92
More Complicated Decimals
  A. 0.2×4.2
  B. 0.3×1.2
  C. 0.06×0.3

D. Shortcuts in Multiplication

Example 1.93

$$12 \times 0.75 = 12 \times \frac{75}{100} = \mathbf{12} \times \frac{3}{\mathbf{4}} = 3 \times \frac{3}{1} = \frac{9}{1} = 9$$

$$18 \times 0.75 = 18 \times \frac{75}{100} = \mathbf{18} \times \frac{3}{\mathbf{4}} = 9 \times \frac{3}{2} = \frac{27}{2} = 13\frac{1}{2} = 13.5$$

E. Estimation

Example 1.94
The product $(1.8)(40.3 + .07)$ is closest to (AMC 8 1986/4)
- A. 7
- B. 42
- C. 74
- D. 84
- E. 737

$$2 \times 41 = 82$$

But note that 1.8 is 10% less than 2. Therefore, if we subtract 10% from 82, we get

$$82 - 8.2 = 73.8$$

Which is closest to 74.

F. Introducing Algebra in Decimals

Example 1.95
A number is equal to 2.15. Find:
- ➢ Ten times the number
- ➢ Hundred times the number
- ➢ Thousand times the number

$$2.15 \times 10 = 21.5$$
$$2.15 \times 100 = 215$$
$$2.15 \times 1000 = 2150$$

Example 1.96
Let $x = 2.15$. Find $10x$, $100x$, and $1000x$.

This is the same question as the previous one, stated in terms of variables. Hence, the answer remains the same.

1.10 Division

A. Long Division with Decimals

> **Example 1.97**
>
> Convert the following into decimals using long division:
>
> A. $\frac{1}{2}$
> B. $\frac{1}{4}$
> C. $\frac{1}{8}$
> D. $\frac{2}{5}$
> E. $\frac{1}{5}$
> F. $\frac{3}{4}$
> G. $\frac{3}{5}$
> H. $\frac{2}{4}$
> I. $\frac{7}{5}$
> J. $\frac{3}{2}$
> K. $\frac{8}{5}$
> L. $\frac{5}{4}$
> M. $\frac{25}{4}$
> N. $\frac{35}{8}$
> O. $\frac{6}{5}$

> **Example 1.98**

Convert $\frac{1}{16}$ into a decimal using long division

		0.	0	6	2	5
	16	1.	0	0	0	0
			9	6		
				4	0	
				3	2	
					8	0

					8	0
					0	0

Practice 1.99
Convert the following fractions to decimals using long division:

B. Decimals in Numerator and Denominator

Example 1.100
Simplify:
- A. $0.6 \div 2$
- B. $0.9 \div 3$
- C. $0.8 \div 2$
- D. $0.8 \div 4$
- E. $0.12 \div 2$
- F. $0.24 \div 4$
- G. $0.36 \div 4$
- H. $0.36 \div 9$
- I. $0.36 \div 2$
- J. $0.36 \div 6$
- K. $0.36 \div 12$
- L. $0.48 \div 2$
- M. $0.48 \div 4$
- N. $0.48 \div 6$
- O. $0.48 \div 8$
- P. $0.48 \div 16$
- Q. $0.48 \div 3$

$$6 \div 2 = 3 \Rightarrow 0.6 \div 2 = 0.3$$
$$12 \div 2 = 6 \Rightarrow 0.12 \div 2 = 0.06$$
$$\frac{0.12}{2} = \frac{0.12 \times 100}{2 \times 100} = \frac{12}{200} = \frac{6}{100} = 0.06$$

Example 1.101
Convert into decimal:
- A. $\dfrac{0.3}{0.5}$
- B. $\dfrac{0.4}{0.8}$
- C. $\dfrac{0.45}{0.25}$
- D. $\dfrac{0.3}{0.9}$
- E. $\dfrac{0.25}{0.75}$
- F.

$$\frac{0.3}{0.5} = \frac{0.3 \times 10}{0.5 \times 10} = \frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} = 0.6$$
$$\frac{0.4}{0.8} = \frac{4}{8} = \frac{1}{2} = \frac{5}{10} = 0.5$$

$$\frac{0.45}{0.25} = \frac{45}{25} = \frac{9}{5} = \frac{18}{10} = 1.8$$

$$\frac{0.45}{0.25} = \frac{45}{25} = \frac{180}{100} = 1.8$$

$$\frac{0.3}{0.9} = \frac{3}{9} = \frac{1}{3}$$

$$\frac{0.25}{0.75}$$

C. Dividing using Long Division

Example 1.102
Divide 93 by 0.16

$$\frac{93}{0.16}$$

We don't want decimals in our fraction. So, to eliminate the decimal, we multiply numerator and denominator by 100:

$$\frac{93}{0.16} \times \frac{100}{100} = \frac{9300}{16} = \frac{4650}{8} = \frac{2325}{4}$$

D. Recurring Decimals

Example 1.103
Convert $\frac{1}{3}$ into a recurring decimal using long division.

If you keep dividing, you get an infinite sequence of 3's in the quotient, giving us

$$\frac{1}{3} = 0.3333\ldots$$

Which is also written as

$$0.333\ldots = 0.\overline{3}$$

			0.	3	3
	3	1.	0	0
			9	
			1	0
				9

Example 1.104
Convert $\frac{1}{6}$ into a recurring decimal using long division.

If you keep dividing, you get an infinite sequence of 6's in the quotient, giving us

$$\frac{1}{6} = 0.1666\ldots$$

Which is also written as

$$0.1666\ldots = 0.1\overline{6}$$

		0.	1	6	6
	6	1.	0	0	0
			6		
			4	0	
			3	6	
				4	0
				3	6
					4

Example 1.105

Convert $\frac{8}{9}$ into a decimal.

$\frac{8}{9}$ is the same as $8 \div 9$. So, we will do this using Decimal Long Division.

$$0.88888888\ldots = 0.\bar{8}$$

		0.	8	8	8	8
	9	8.	0	0	0	0
		7	2			
			8	0		
			7	2		
				8	0	
				7	2	
					8	0
					7	2

Example 1.106
Convert $\frac{2}{3}$ into a decimal.

	0.	6	6
3	2.	0	0
	1	8	
		2	0
		1	8

Example 1.107
Convert $\frac{1}{7}$ into a decimal.

$$\frac{1}{7} = 0.142857142857\ldots = 0.\overline{142857}$$

	0.	1	4	2	8	5	7
7	1.	0	0	0	0	0	0
		7					
		3	0				
		2	8				
			2	0			
			1	4			
				6	0		
				5	6		
					4	0	
					3	5	
						5	0
						4	9
							1