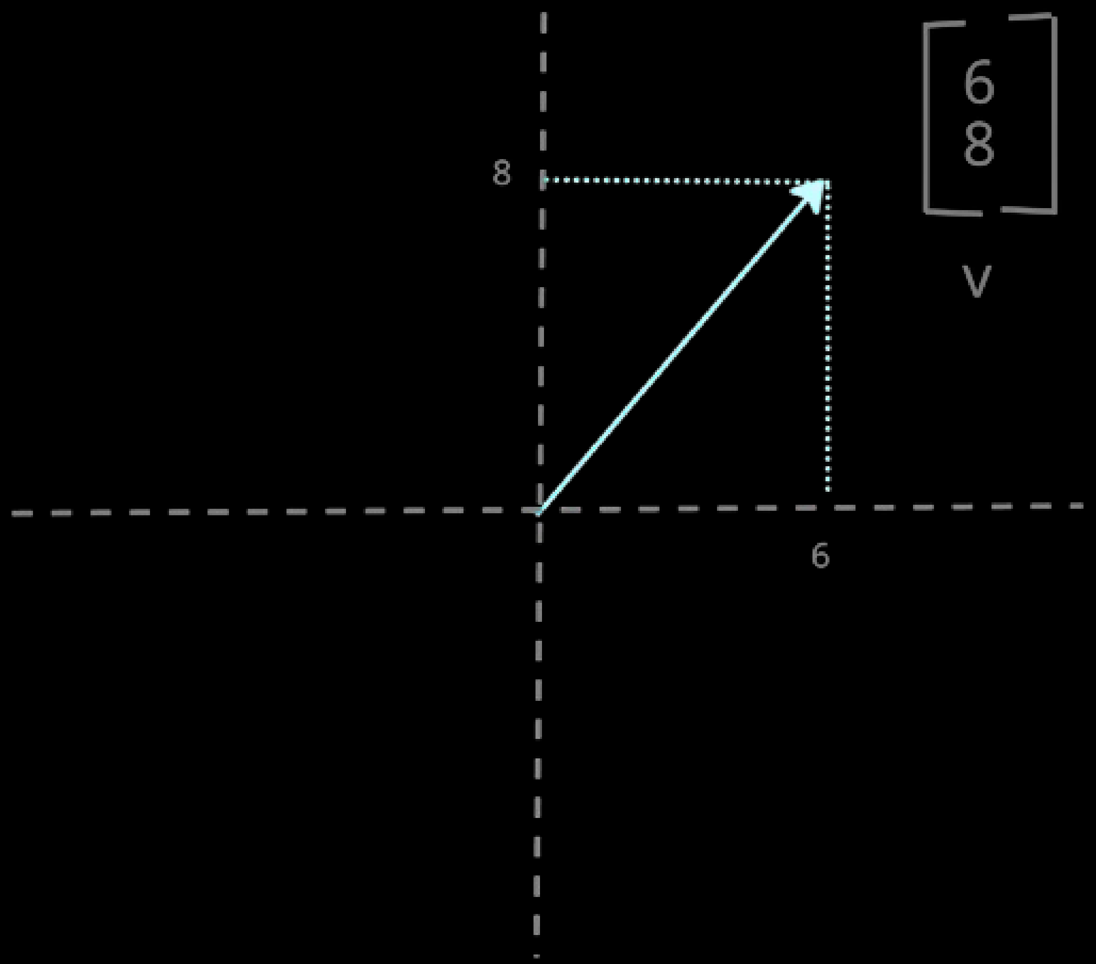
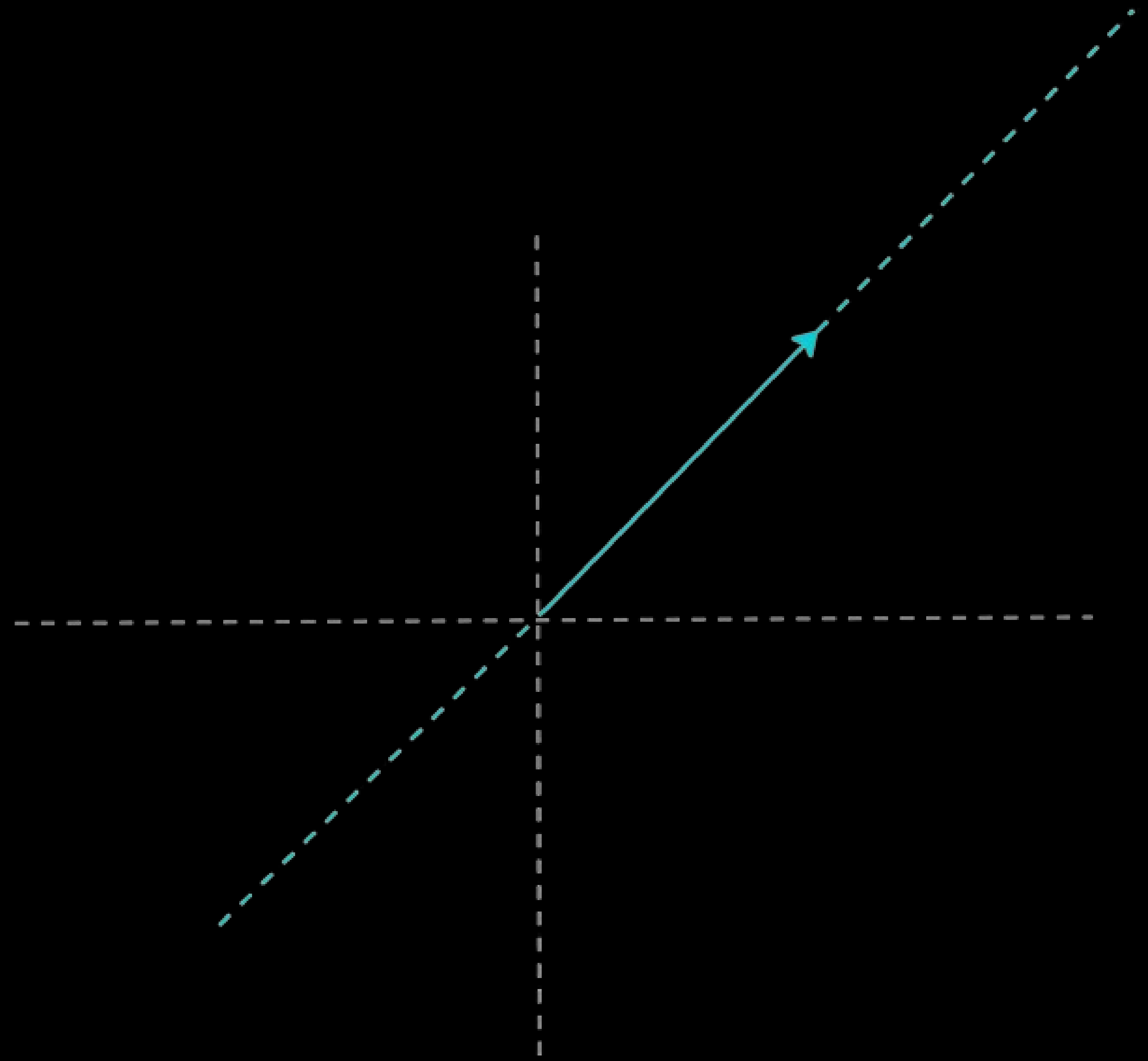
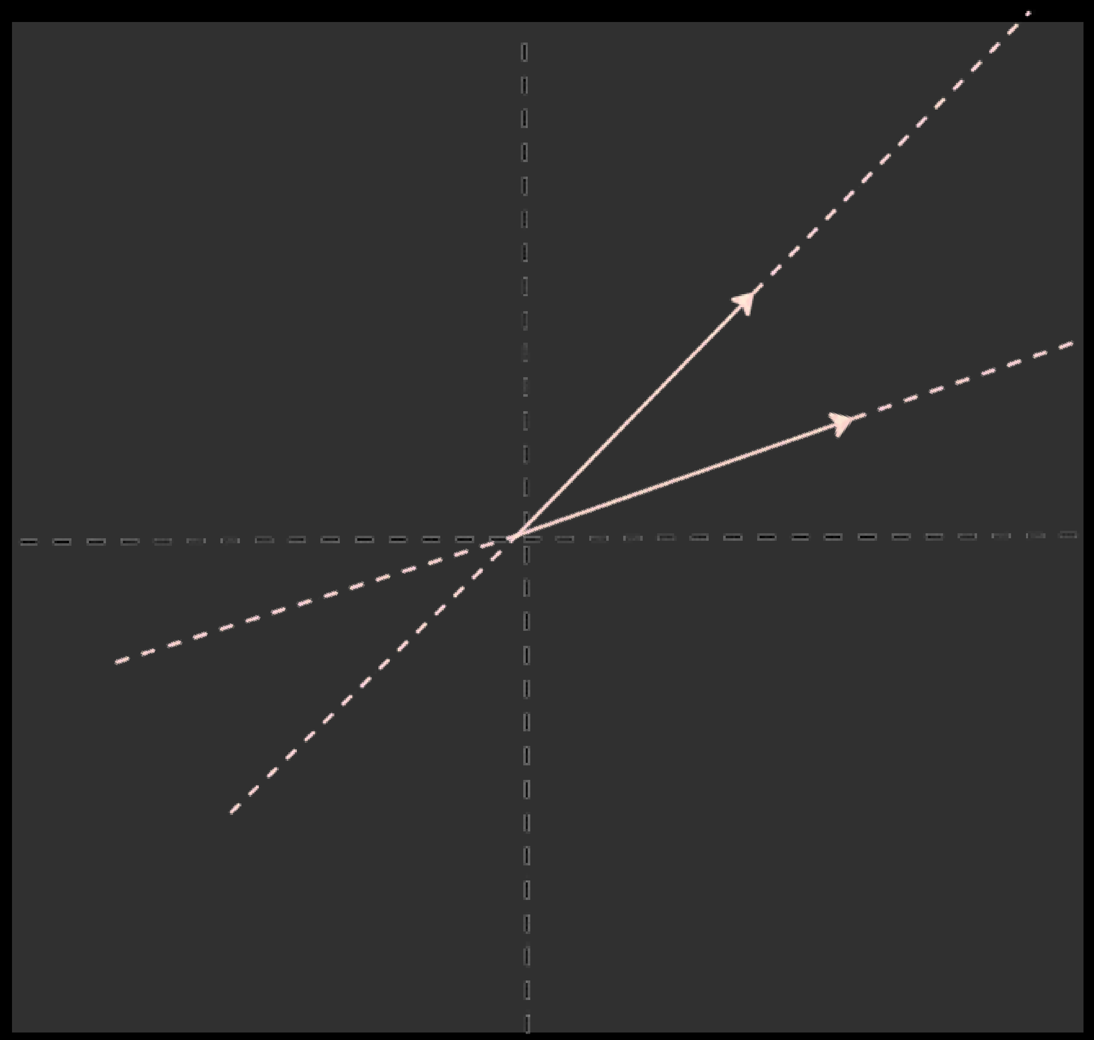
Applied Linear Algebra Series

Part 1 : Vectors







What are vectors ??

lets take an example : suppose a person can have many features such as its height , weight

Person / Features	Height cm	Weight kg
Person (1)	135 cm	75 Kg
Person (2)	98cm	58 Kg

Person(1)

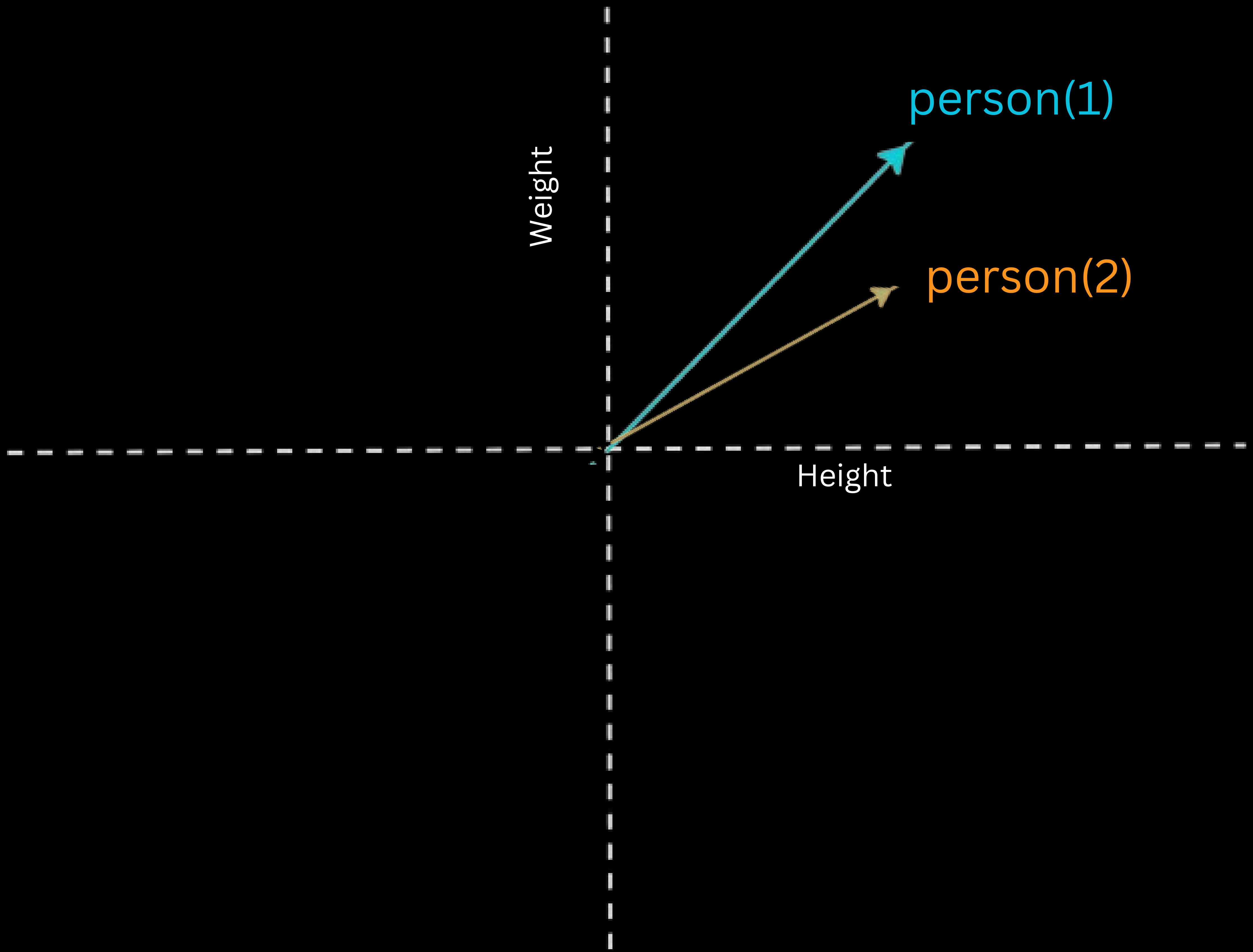
Person(2)

Lets model a person in vector space |R -2

$$\text{person\_vector} = \begin{bmatrix} \text{Height} \\ \text{Weight} \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix}$$

$$\text{Person(1)} = \begin{bmatrix} \text{135cm} \\ \text{75kg} \end{bmatrix} \quad \text{Person(2)} = \begin{bmatrix} \text{98cm} \\ \text{58kg} \end{bmatrix}$$

lets model x-axis as height and y-axis as weight



“ so a person can be represented as a vector based on its feature such as weight , height “

Lets Extend this idea of vector’s to a 3-dimensional vector space |R-3

we add another dimension by adding another feature of the person “Age”

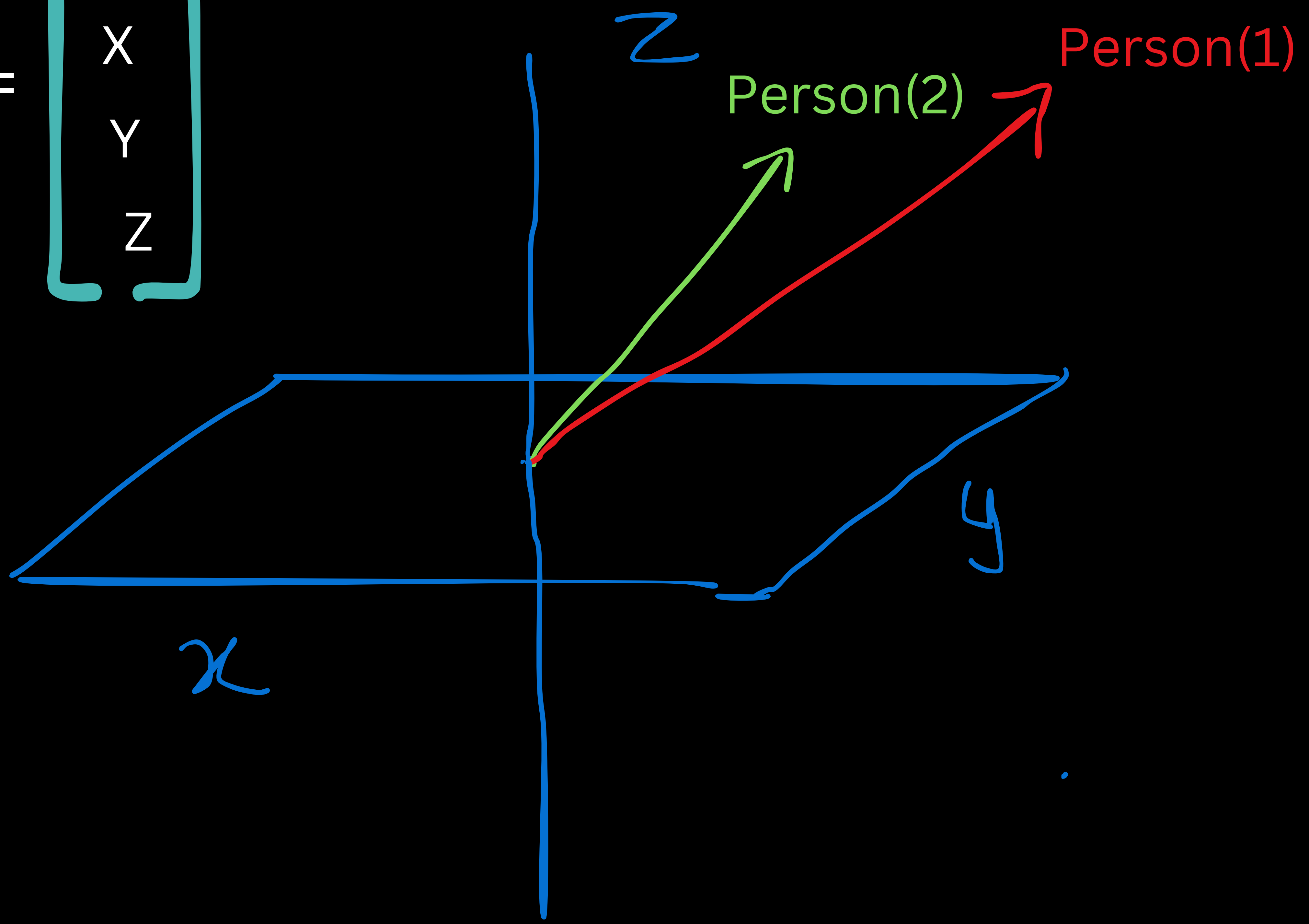
Person / Features	Height cm	Weight kg	Age (y)
Person (1)	135 cm	75 Kg	30 years
Person (2)	98cm	58 Kg	25 years

Person(1)

$$\text{Person} = \begin{bmatrix} \text{Height} \\ \text{Weight} \\ \text{Age} \end{bmatrix}$$

Person(2)

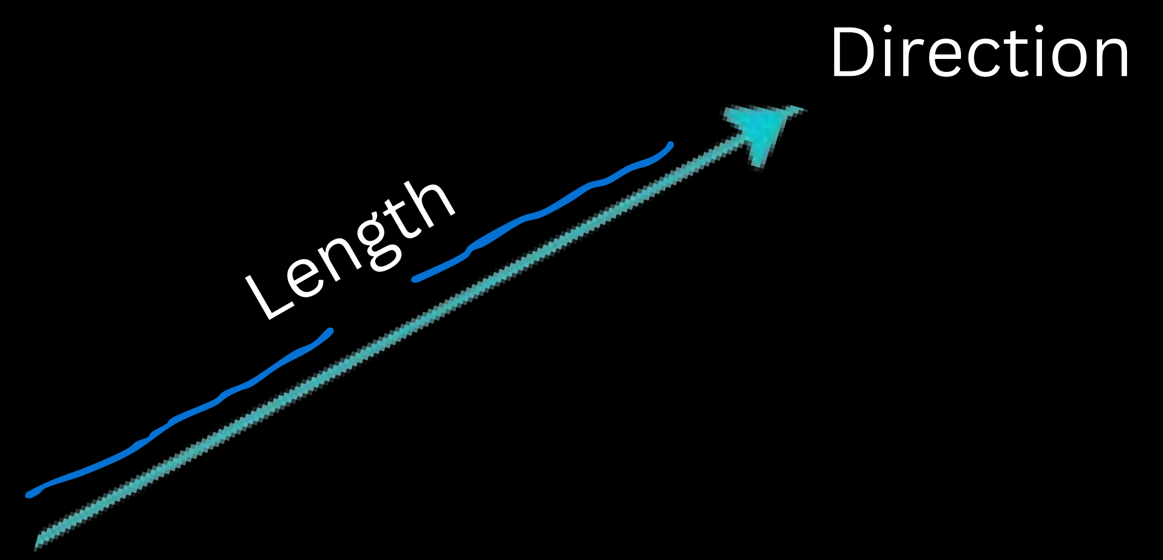
$$\text{Person} = \begin{bmatrix} \text{Height} \\ \text{Weight} \\ \text{Age} \end{bmatrix} = \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

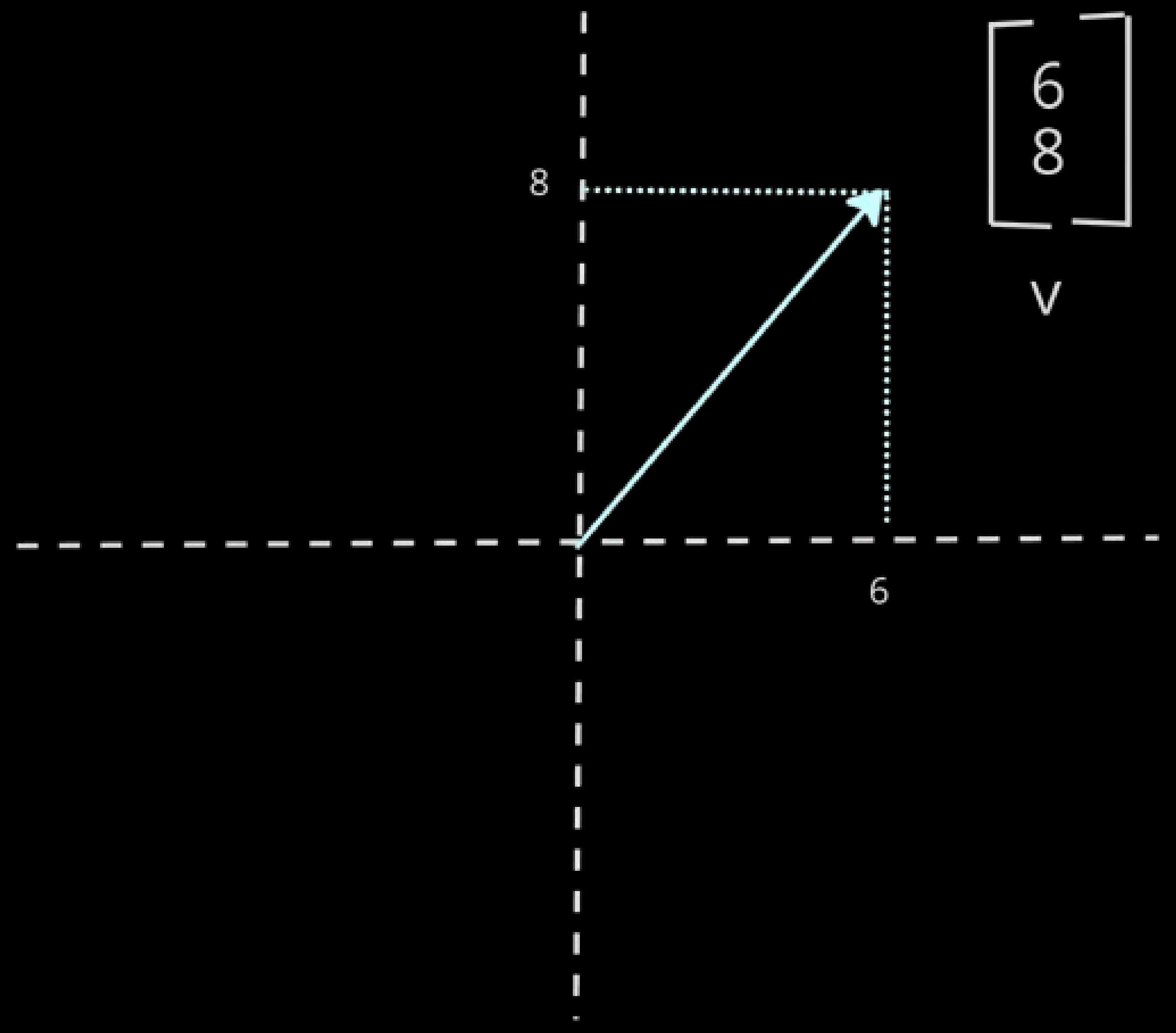


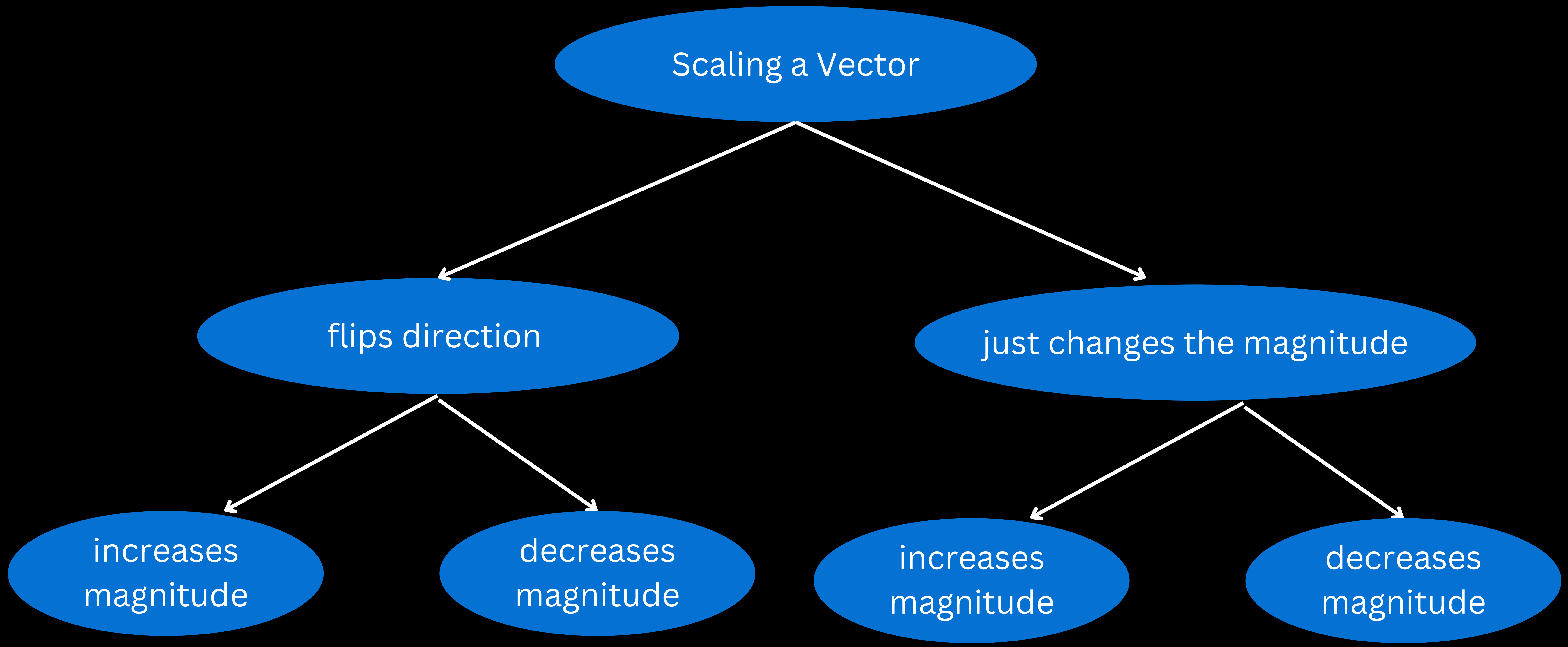
here the person is represented as a vector in R3 with the help of features such as height ,weight and age

Vector's: vectors can be represented as an arrow in a vector space where its tail sits at the origin and its head is pointing towards the direction of the vectors

- a vector has two properties one its magnitude the other one is its direction
- a vector can be used to represent an entity in n-dimension , and that entity has n features

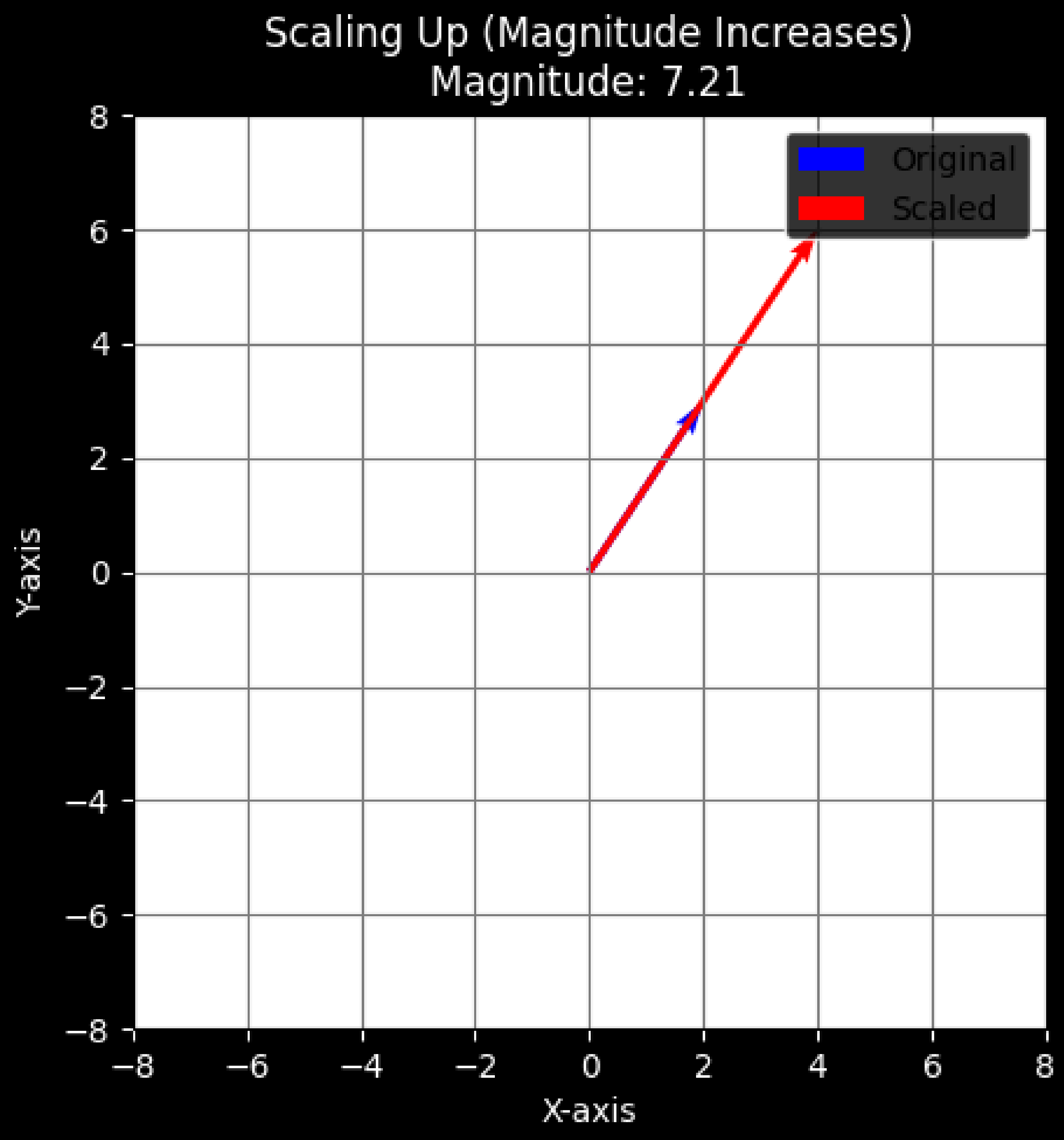





Scaling a Vector
flips direction
just changes the magnitude
increases magnitude
decreases magnitude
increases magnitude
decreases magnitude

lets see how we can scale vector

Increasing the magnitude / length



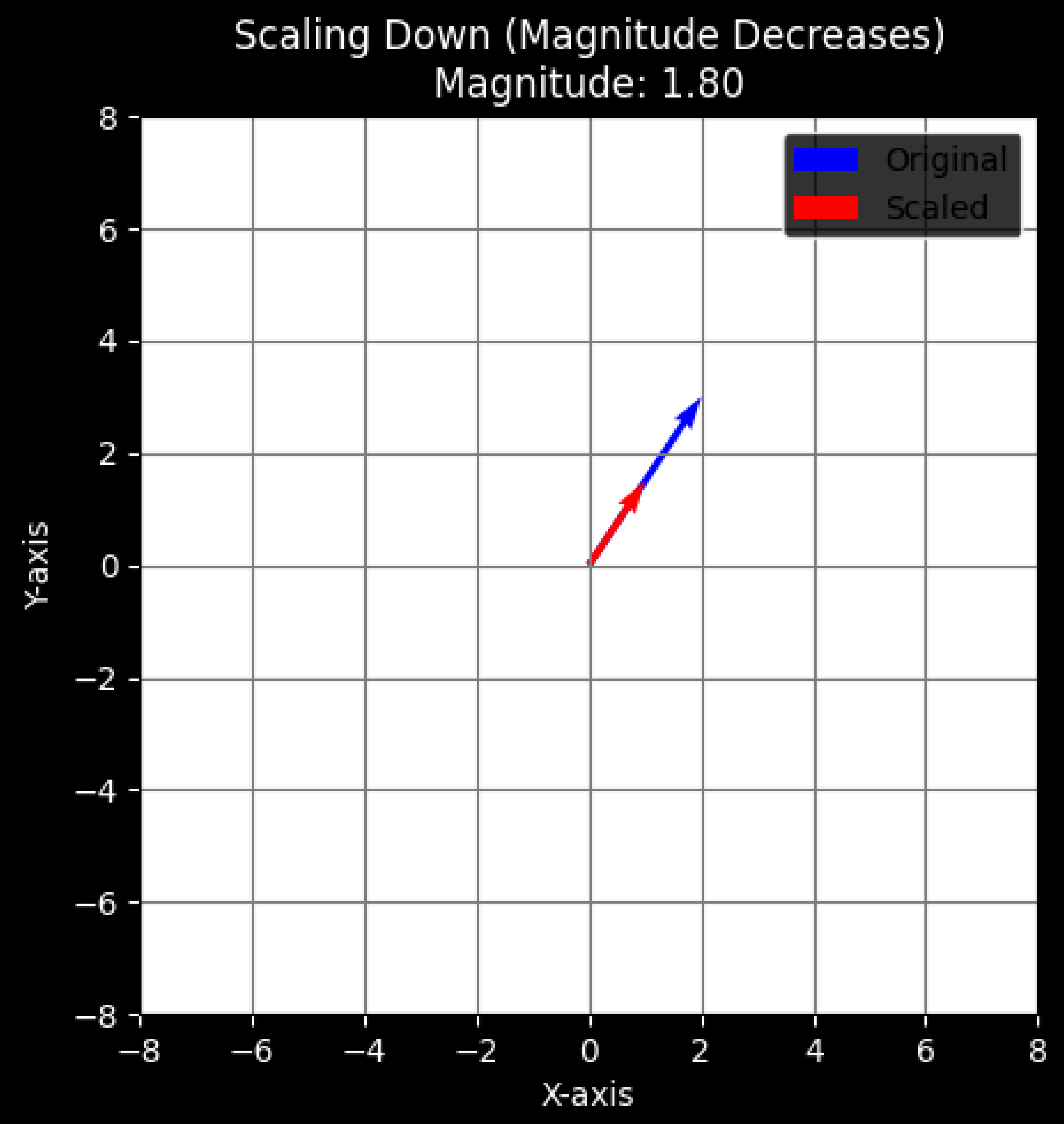
$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \qquad \vec{w} = \alpha \vec{v} \qquad \alpha = 2$$

$$\vec{w} = 2 \cdot \begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$$

- (w) is the scaled vector where we increase the length of vector (v) by a scaling factor of 2

lets see how we can scale vector

decreasing the magnitude / length

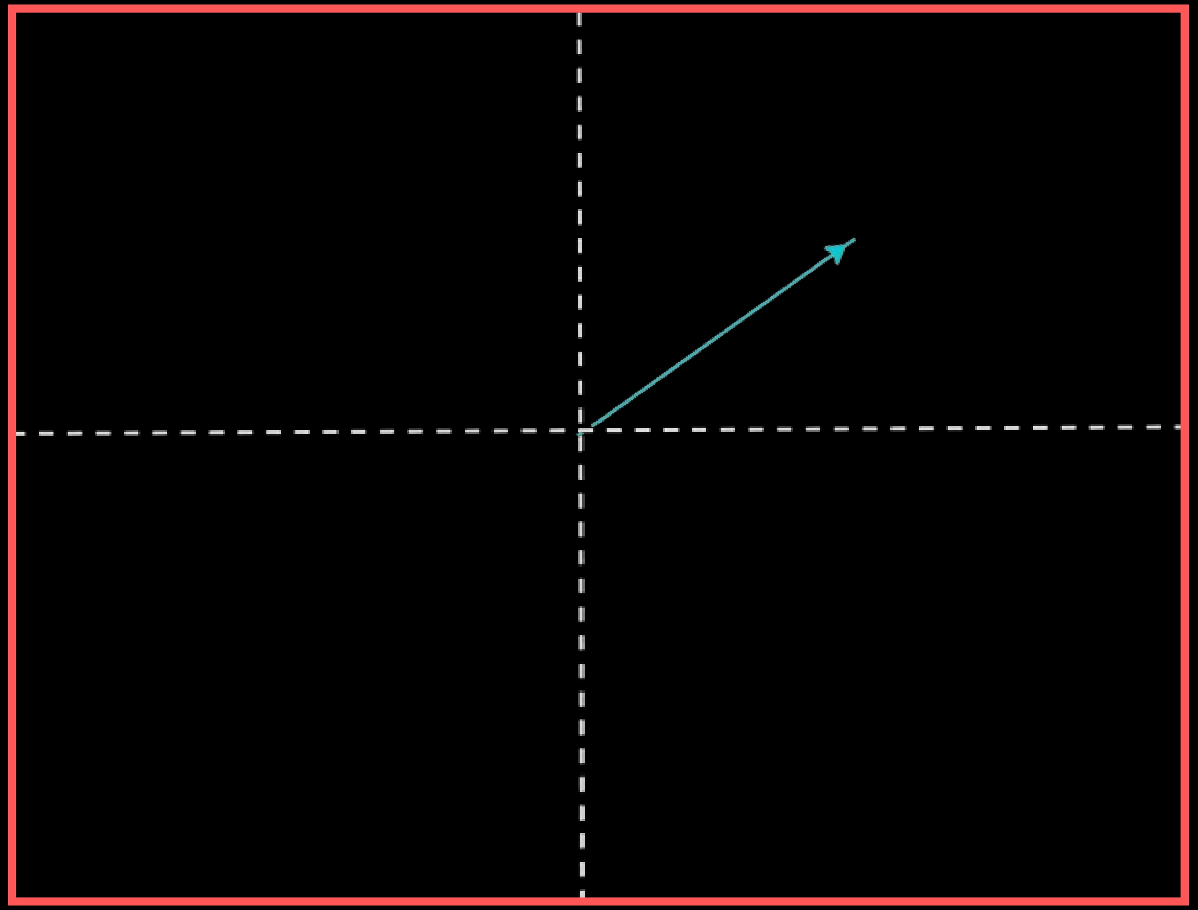


$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{w} = \alpha \vec{v} \quad \alpha = \frac{1}{2}$$

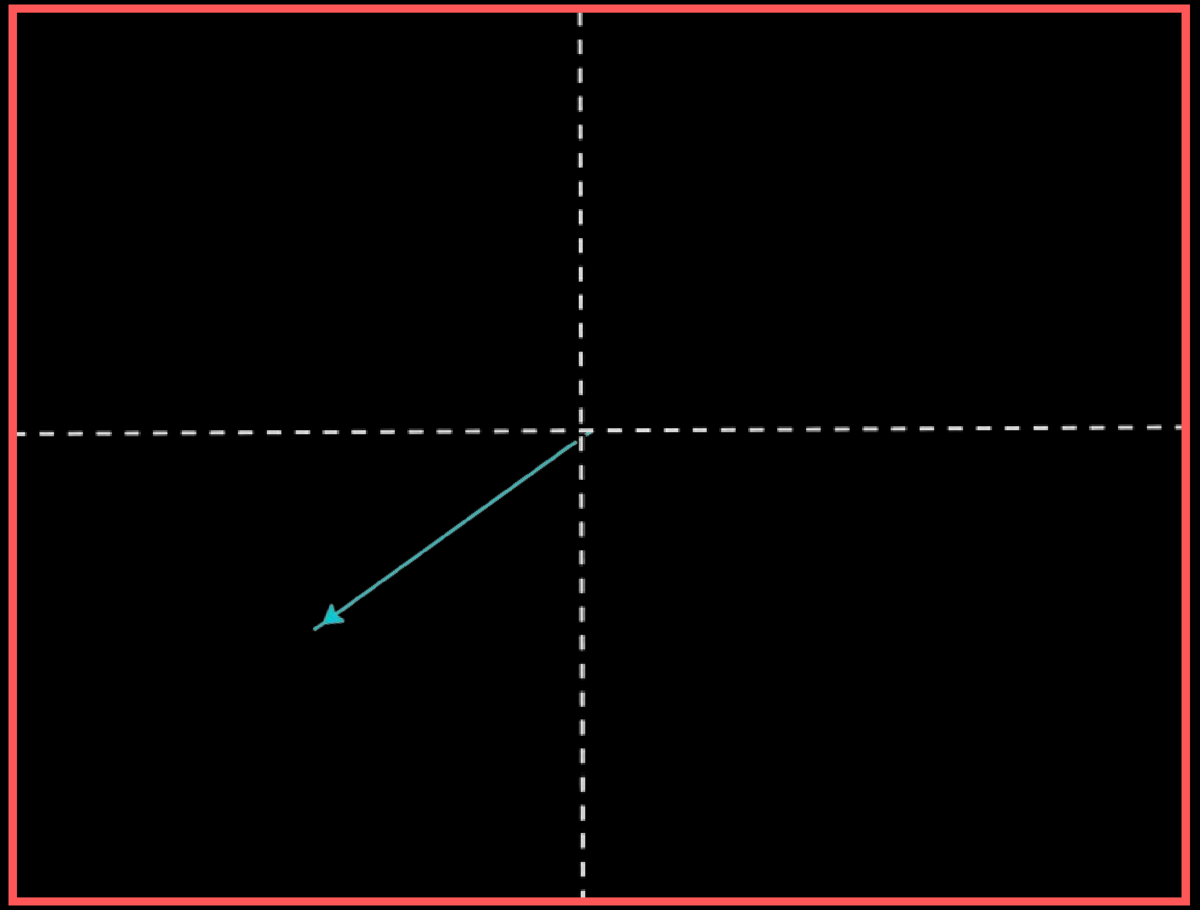
$$\vec{w} = \frac{1}{2}\begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} 1 \\ 3/2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1.5 \end{bmatrix}$$

- (w) is the scaled vector where we decrease the length of vector (v) by a scaling factor of (1/2)

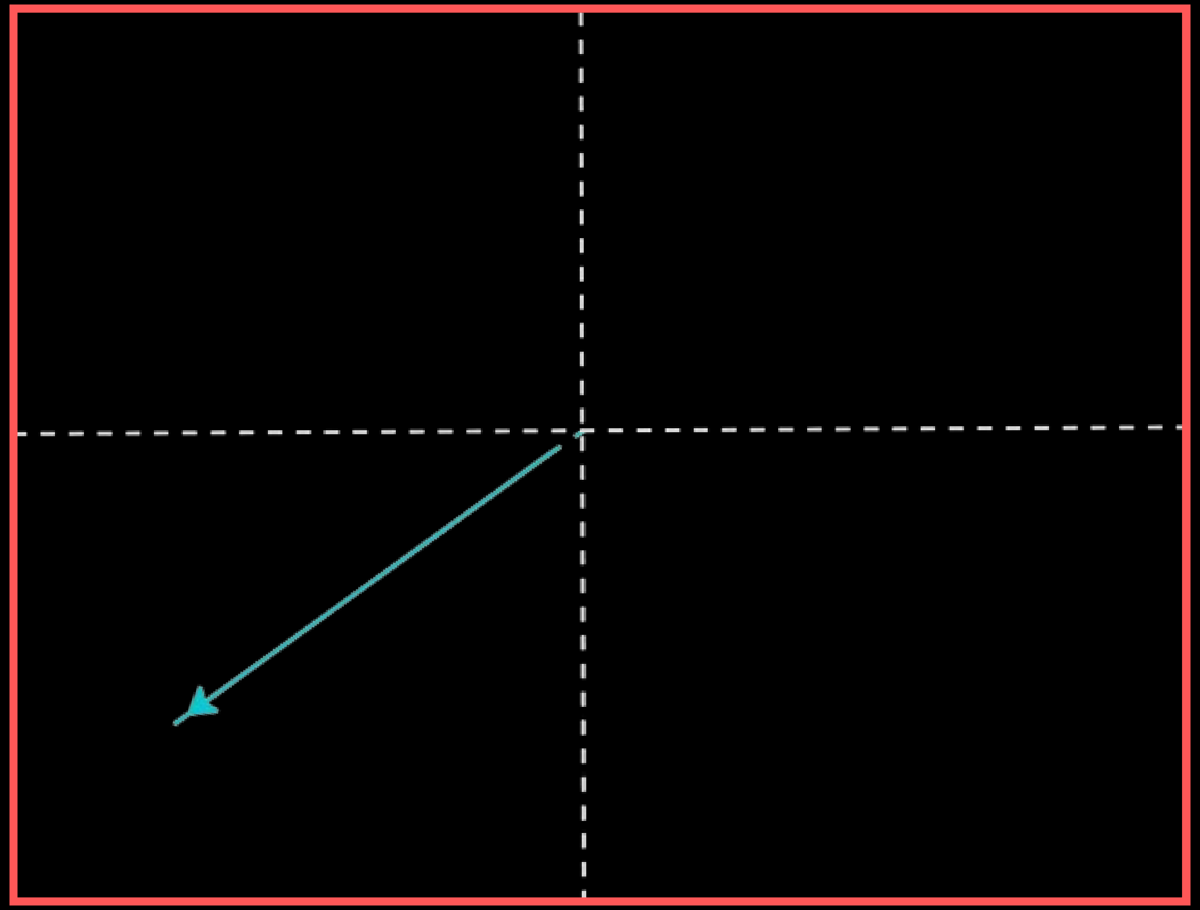
Lets see Flipping the direction and then scaling the vector



Original Vector (v)



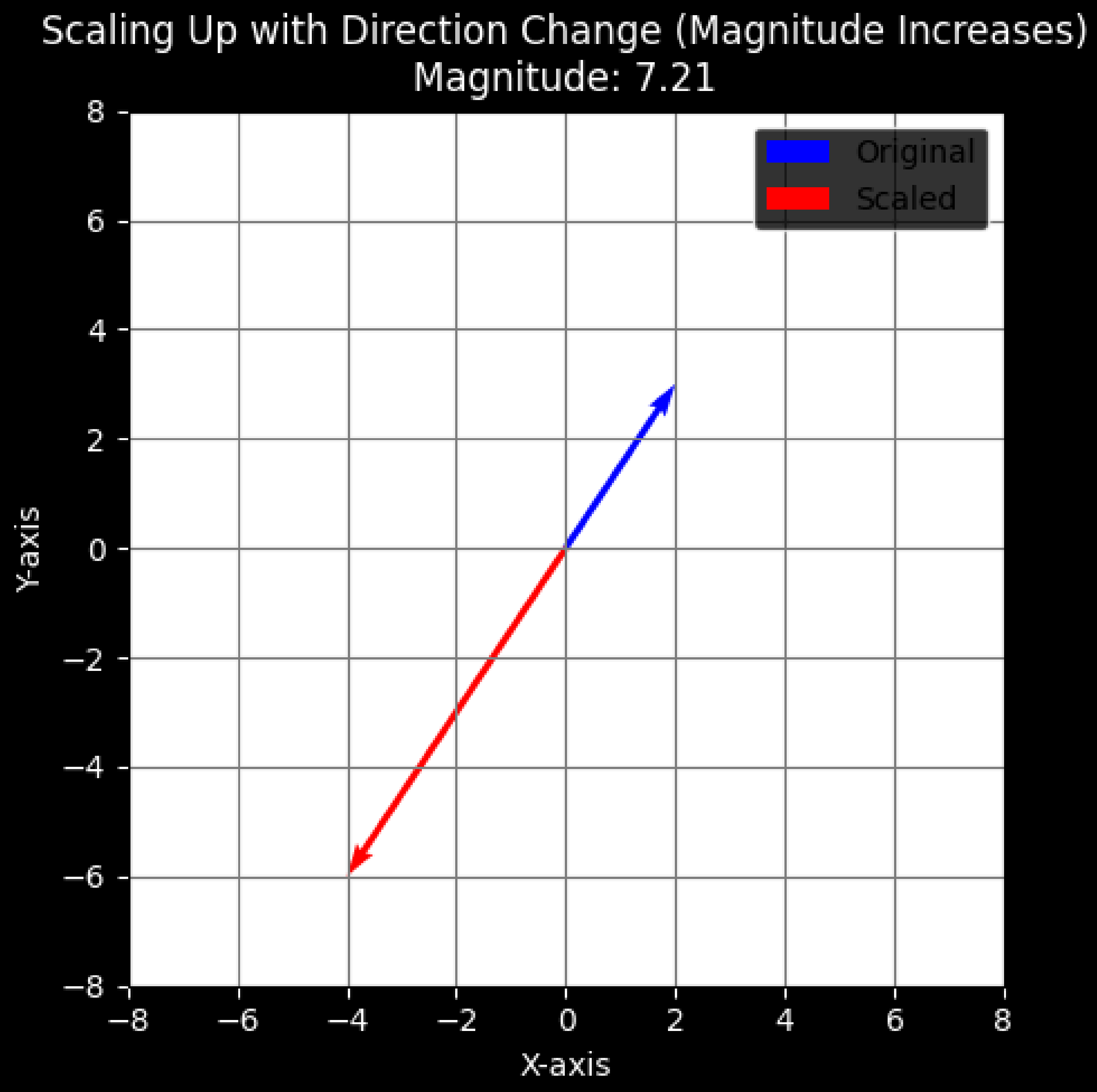
Fliiped vector (-v)



Scaled vector

lets see how we can scale vector

Fliping the direction (increasing magnitude)



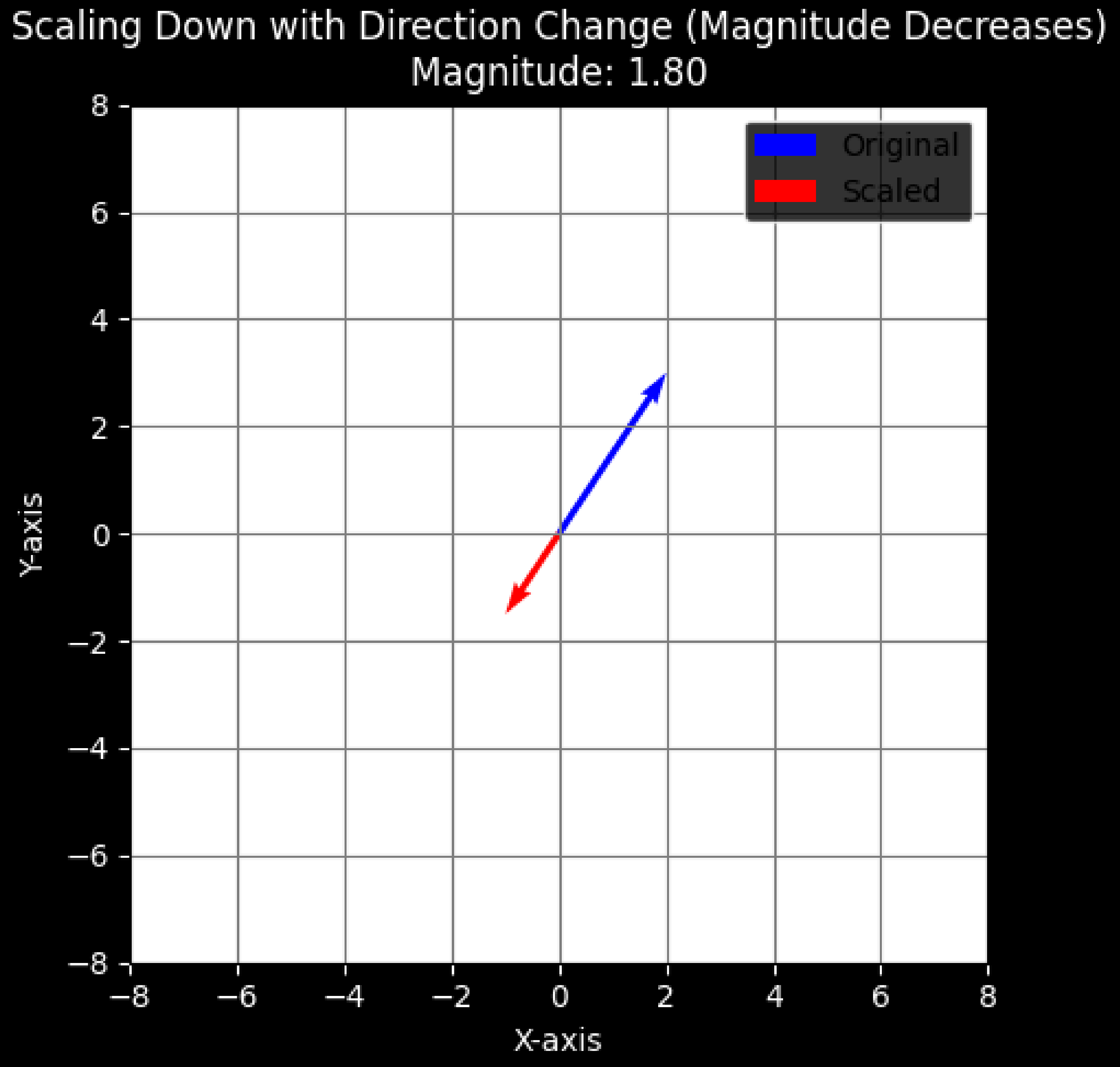
$$\vec{v} = \begin{bmatrix}2\\3\end{bmatrix} \quad \vec{w} = \alpha\vec{v} \quad \alpha = -1 \times 2$$

$$\vec{w} = -2\begin{bmatrix}2\\3\end{bmatrix} \quad \vec{w} = \begin{bmatrix}-4\\-6\end{bmatrix}$$

- (w) is the flipped scaled vector where we increase the length of vector (v) by a scaling factor of (-2)

lets see how we can scale vector

Fliping the direction (decreasing magnitude)

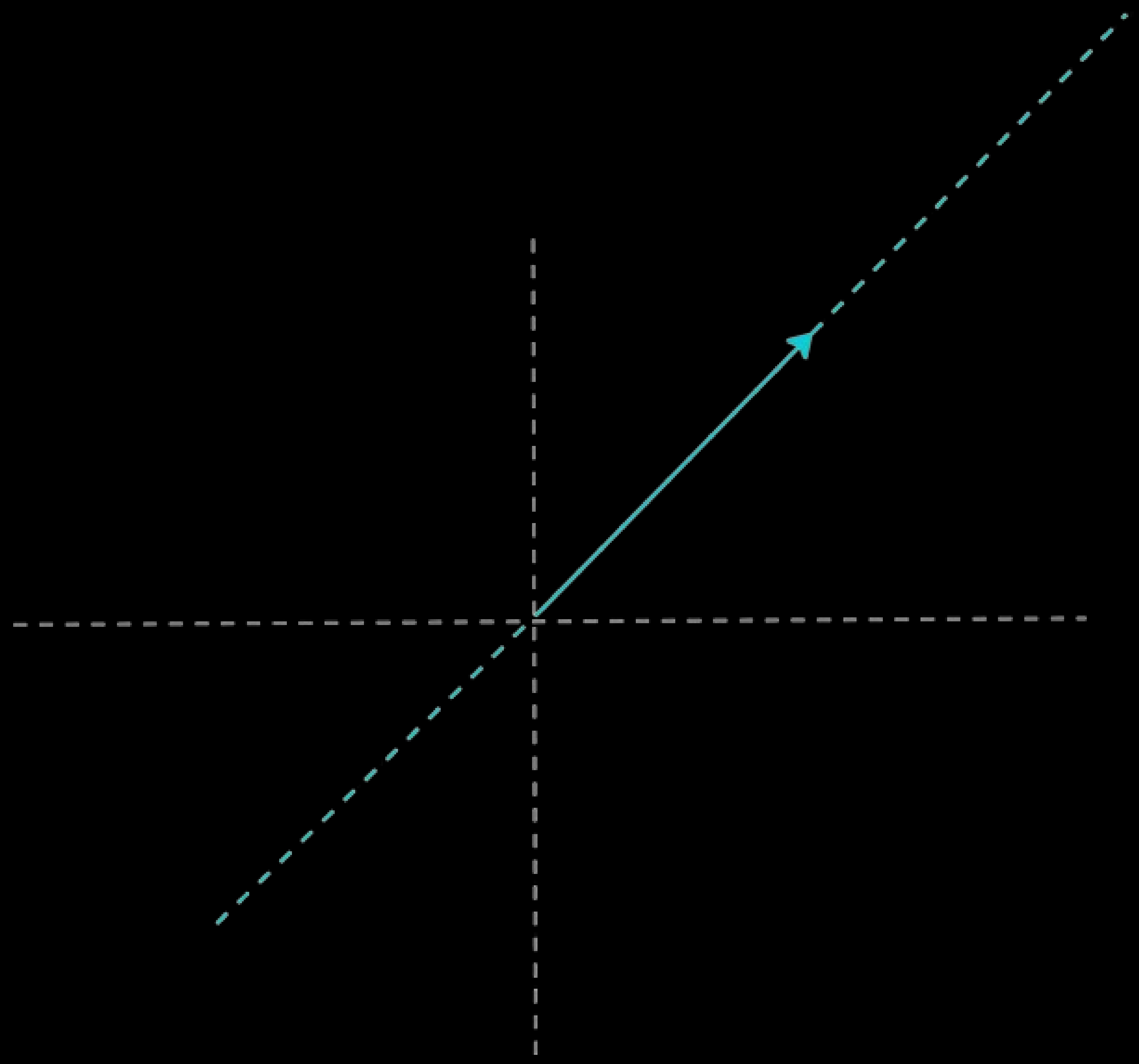


$$\vec{v} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{w} = \alpha \vec{v} \quad \alpha = -\frac{1}{2}$$

$$\vec{w} = -\frac{1}{2} \begin{bmatrix} 2 \\ 3 \end{bmatrix} \quad \vec{w} = \begin{bmatrix} -1 \\ -3/2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1.5 \end{bmatrix}$$

- (w) is the flipped scaled vector where we decrease the length of vector (v) by a scaling factor of (-1/2)

Lets Summarize Vector scaling



$$\vec{w} = \alpha \vec{v}$$

NOTE : for any values of α the vector $\vec{w}$ stays on this dashed line

Vector Addition

< what does it mean to add two vectors in linear algebra

$$\vec{s} = \vec{v} + \vec{w}$$

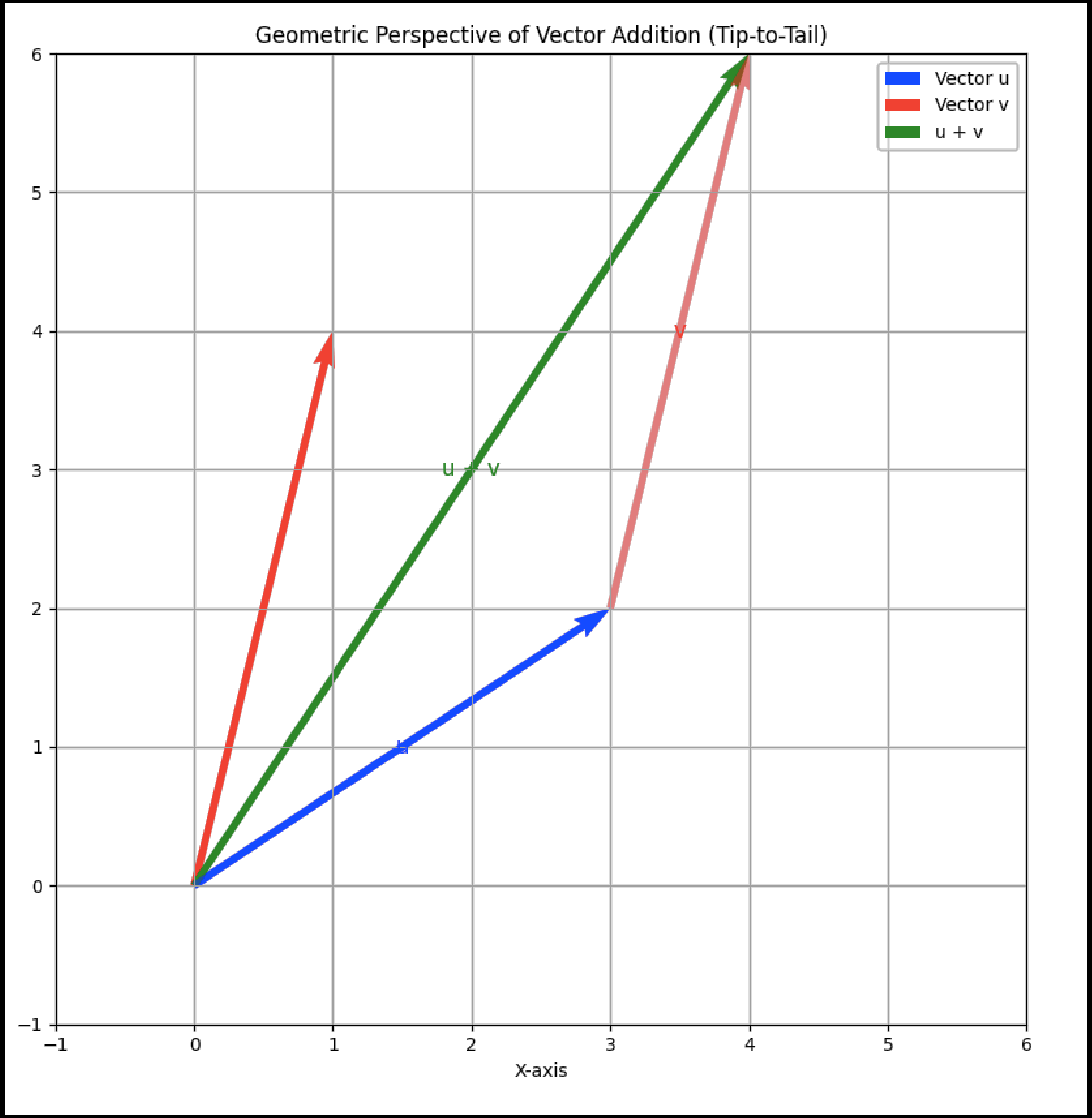
$$\vec{s} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\vec{s} = \begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \\ a_3 + b_3 \end{bmatrix}$$

Vector addition is similar to scalar addition it works element wise

Vector Addition

Geometric defination of vector addition



- Tip-to-Tail Addition: To add the vectors, place the tail of (v) at the tip of(u). The resultant vector, (u+v), is drawn from the tail of (u) to the tip of (v). This vector is shown in green.