558 A1

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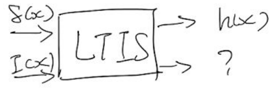
Q1

Q 1

a)



We can write $I(x) = \sum_i I(x_i)\,\delta(x-x_i)$

where $I(x_i) \neq 0$

We denote $h(x) = H(\delta(x))$, where $h(x)$ is the output of system when the input is $\delta(x)$

$H(I(x)) = H\left(\sum_i I(x_i)\,\delta(x-x_i)\right)$

Because the system is linear:

$H(I(x)) = \sum_i I(x_i)\,H(\delta(x-x_i))$

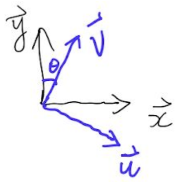
Because the system is time-invariant:

$H(I(x)) = \sum_i I(x_i)\,h(x-x_i)$

Then we can conclude that:

$h(x) \circledast I(x) = \sum h(x-u)\,I(u)$

b)



As shown in the figure above,

(x, y) is the original coordinates.

After rotate original coordinate clockwisely with degree of θ

we can get a new coordinate (u, v)

For a point (x, y), the new coordinate is (u, v). we have

$$x = u\cos\theta - v\sin\theta$$

$$y = u\sin\theta + v\cos\theta$$

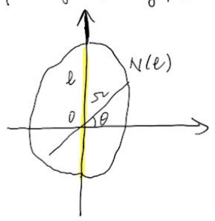
So the laplacian operator in (u, v) coordinate is:

$$
\begin{aligned}
\nabla^2 f &= \frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2} \\
&= \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{\partial f}{\partial v}\right) \\
&= \frac{\partial}{\partial u}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right) + \frac{\partial}{\partial v}\left(\frac{\partial f}{\partial x}(-\sin\theta) + \frac{\partial f}{\partial y}\cos\theta\right) \\
&= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)\frac{\partial x}{\partial u} + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)\frac{\partial y}{\partial x} \\
&\quad + \frac{\partial}{\partial x}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial y}\right)\frac{\partial x}{\partial v} + \frac{\partial}{\partial y}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial y}\right)\frac{\partial y}{\partial v} \\
&= \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)\cos\theta + \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\cos\theta + \frac{\partial f}{\partial y}\sin\theta\right)\sin\theta + \frac{\partial}{\partial x}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial y}\right)(-\sin\theta) + \frac{\partial}{\partial y}\left(-\sin\theta\frac{\partial f}{\partial x} + \cos\theta\frac{\partial f}{\partial y}\right)\cos\theta \\
&= \frac{\partial}{\partial x}\frac{\partial f}{\partial x} + \frac{\partial}{\partial y}\frac{\partial f}{\partial y} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}
\end{aligned}
$$

c) It is condition of linear variation:

the intensity variation near and parallel to the line of zero-crossings should locally be linear.

Proof: Without loss of generality, Let's consider following situation:



as shown above, l is a segment of the y-axis. (in yellow color)

Let $f(x,y)$ be a real-valued, twice continuously differentiable function.

we have $\nabla^2 f = 0$ on l. (l is zero-crossings)

the intensity variation near and parallel to l is linear. ($\frac{\partial f}{\partial y}$ is constant)

Then we can know

$$\nabla^2 f = \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial x^2} = 0$$

$$\frac{\partial^2 f}{\partial y^2} = 0$$

so $\frac{\partial^2 f}{\partial x^2} = 0$ on l

Consider the line segment $\Omega = (r\cos\theta, r\sin\theta)$ for fixed θ and values of r sufficiently small that Ω lies entirely in $N(l)$

we have:

$$(\partial^2 f / \partial \Omega^2) = (f_{xx}\cos^2\theta + f_{xy}\, 2\sin\theta\cos\theta + f_{yy}\sin^2\theta)_{r,\theta}$$
$$= (f_{xx}\cos^2\theta)_{r,\theta}$$

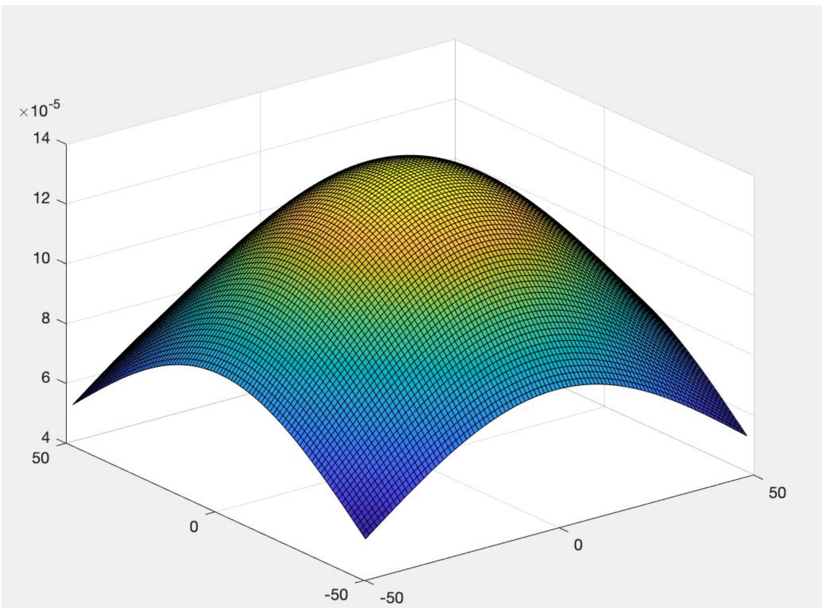
Since the condition that f_y is constant implies $f_{xy} = f_{yy} = 0$

As required, the above quantity is zero at $r=0$ and has maximum slop when $\theta = 0$, which implies the orientation is perpendicular to zero-crossings.

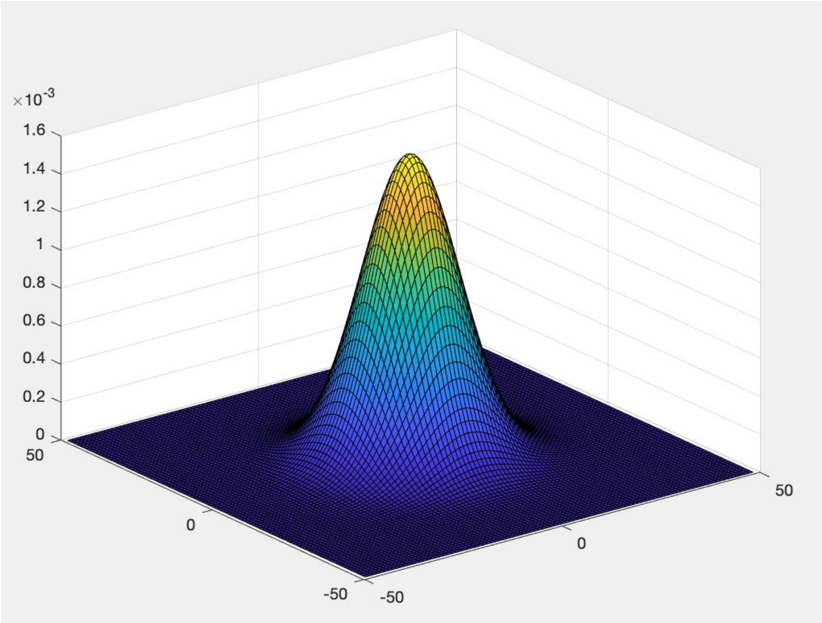
Q2

a) 2d Gaussian

sigma = 50 N = 99

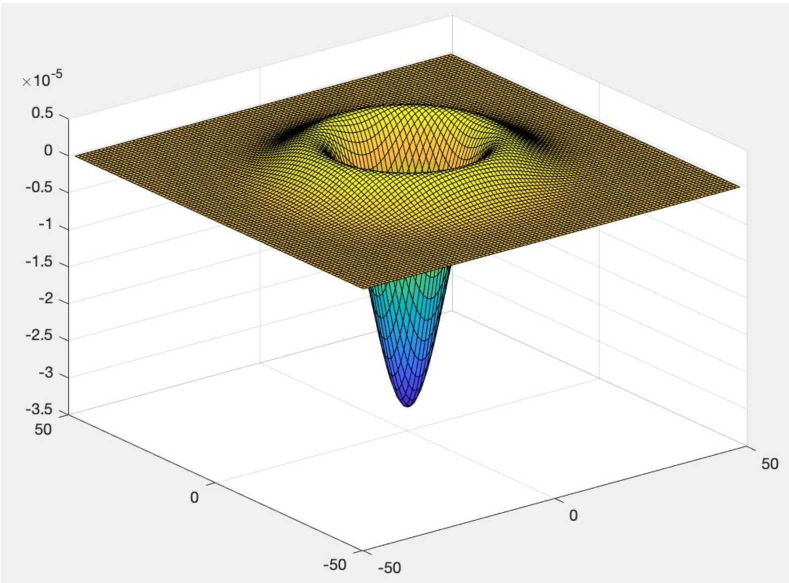


sigma = 10 N = 99

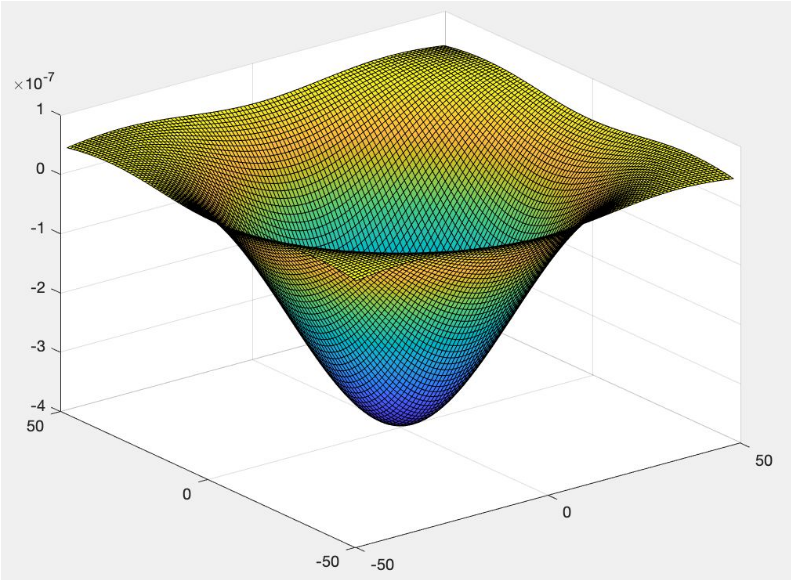


b) 2d Laplacian

sigma = 10, N = 99

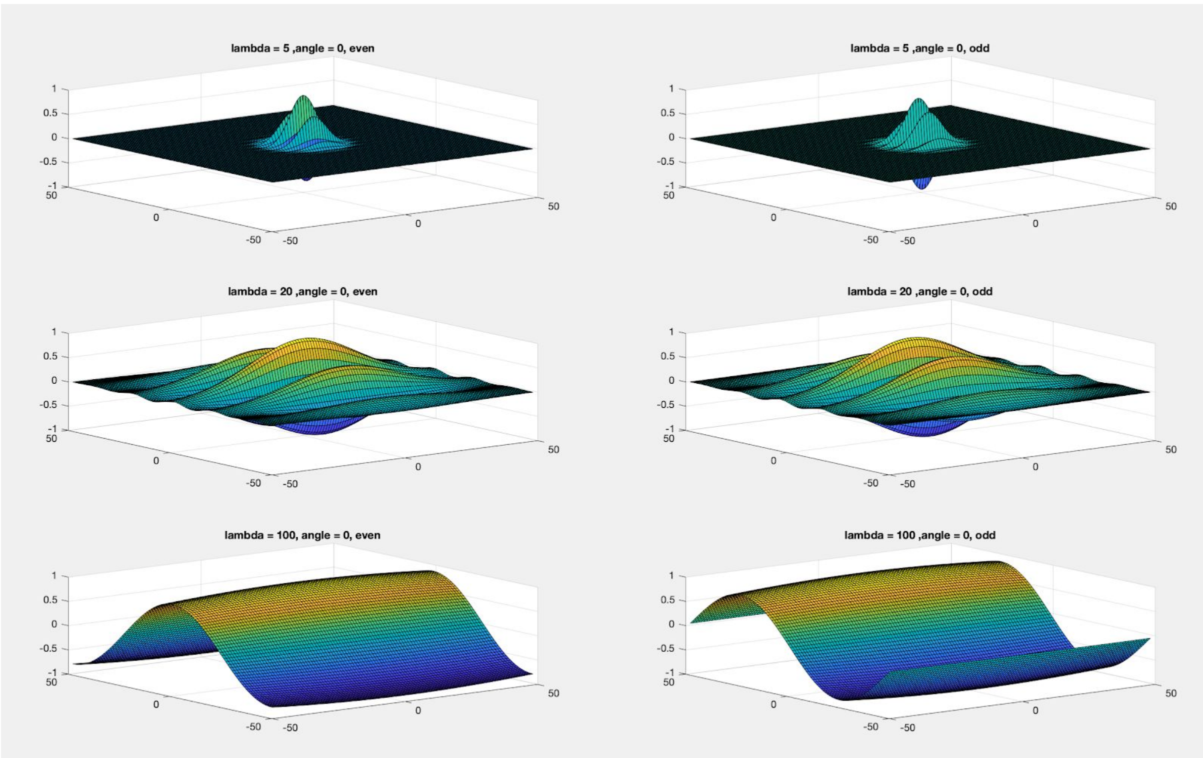


sigma = 30, N = 99

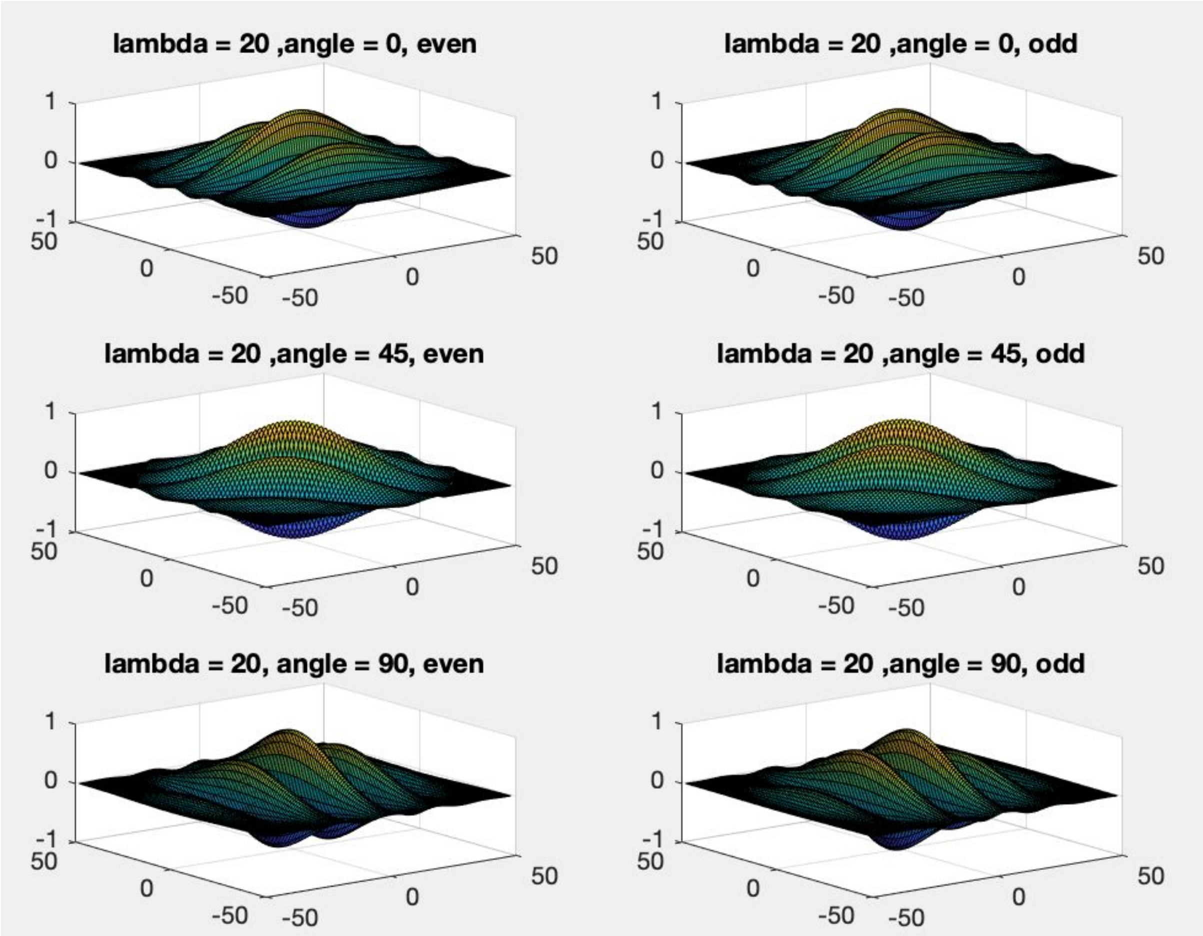


c) 2d Gabor

different wavelength

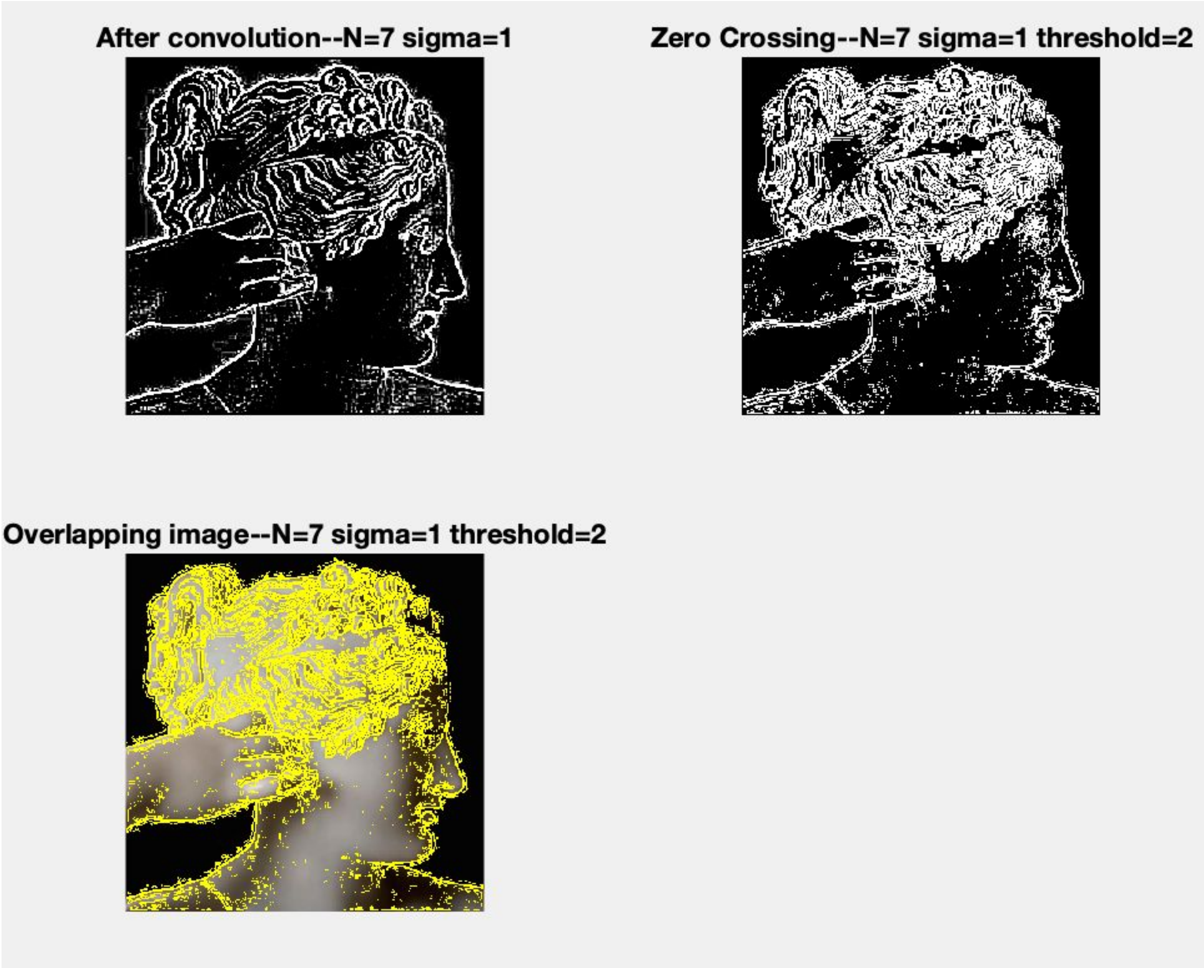


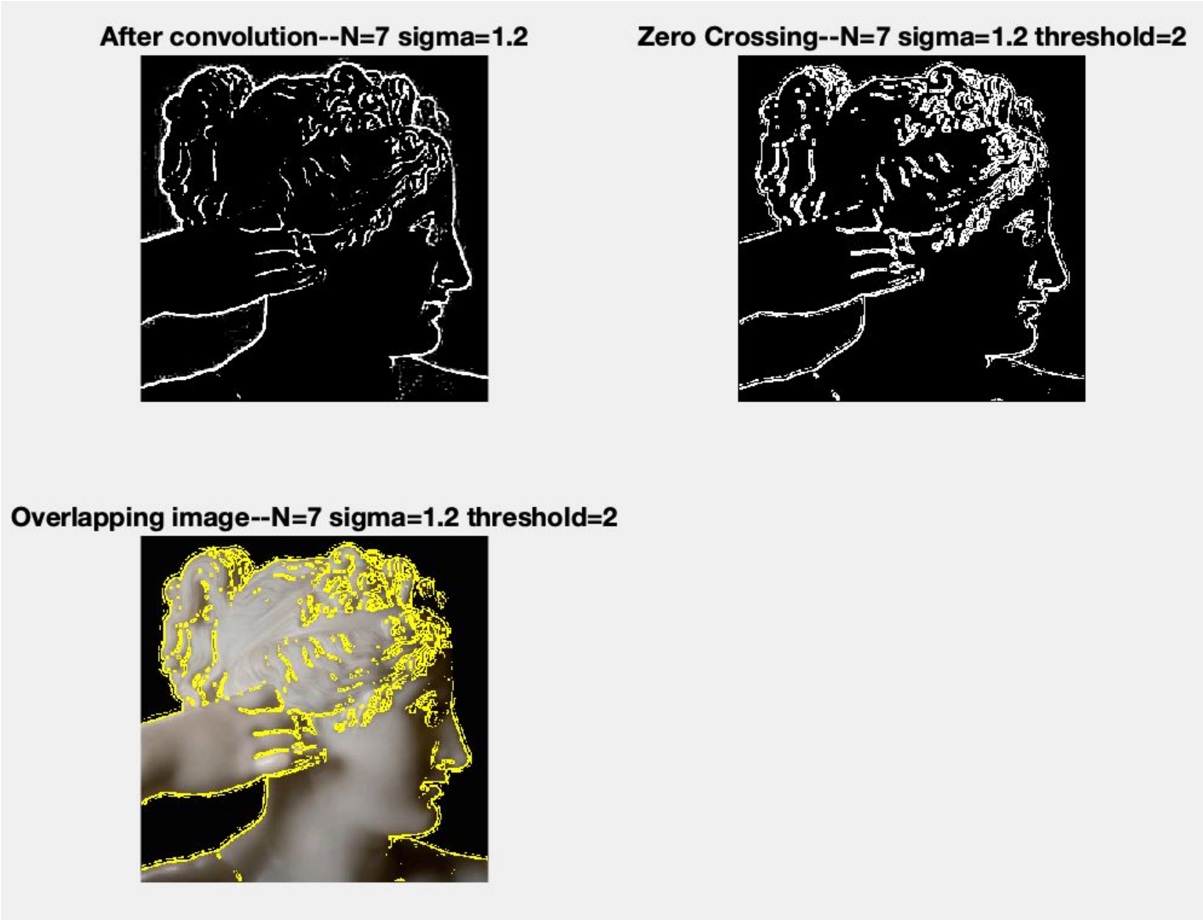
different angle:

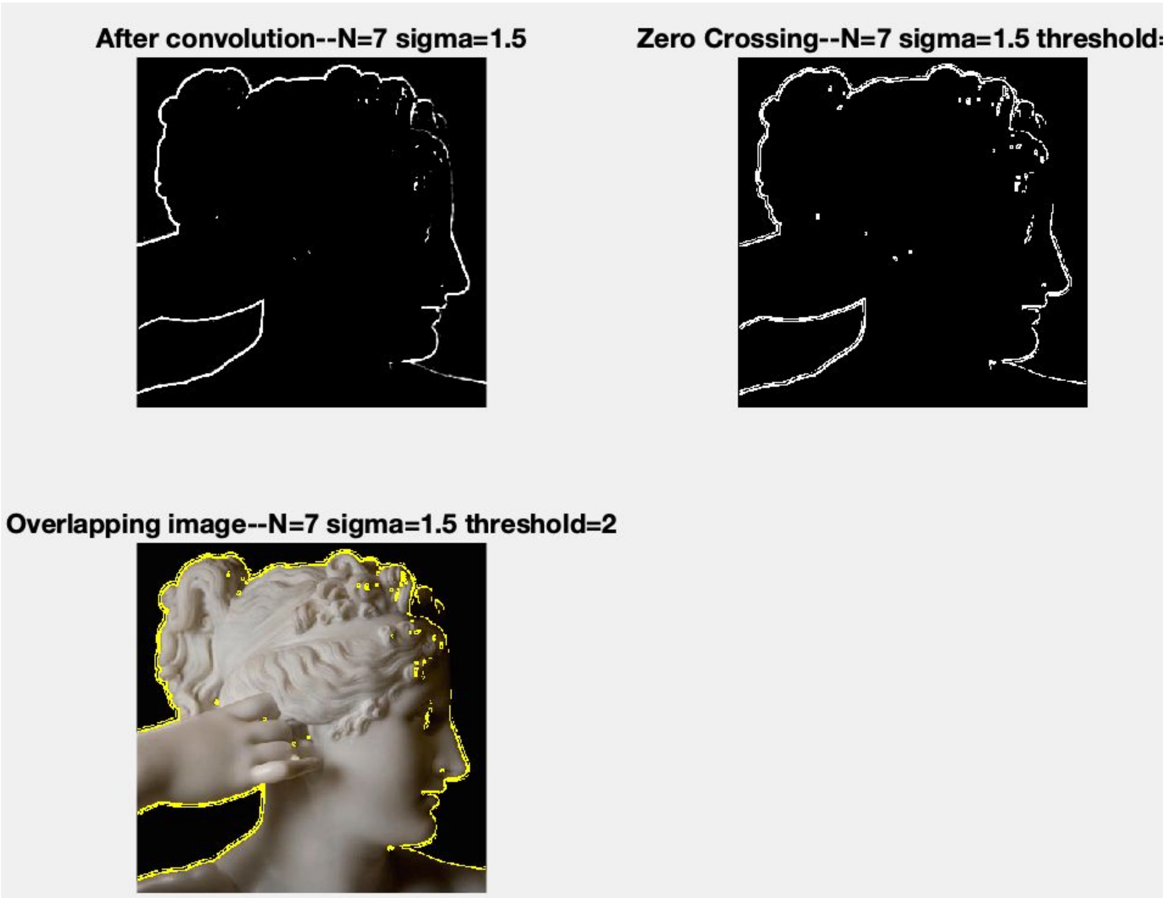


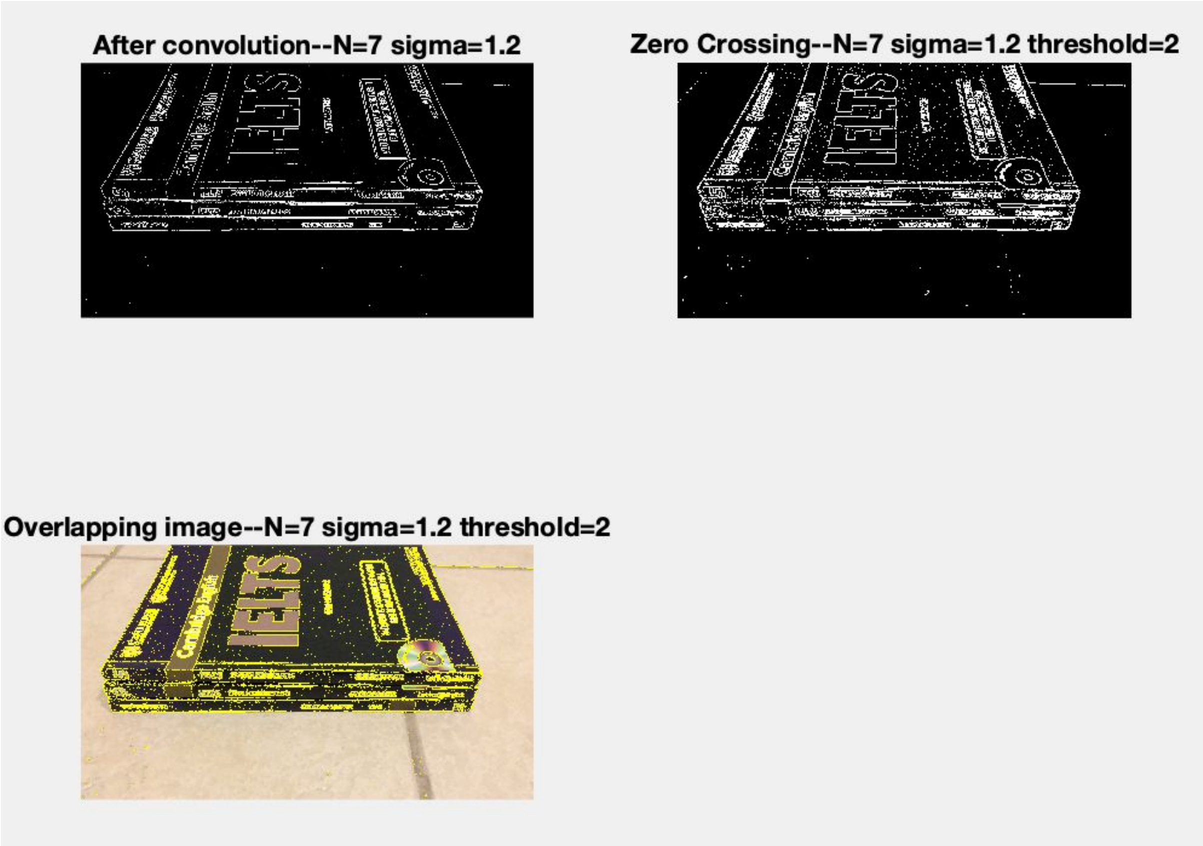
q3

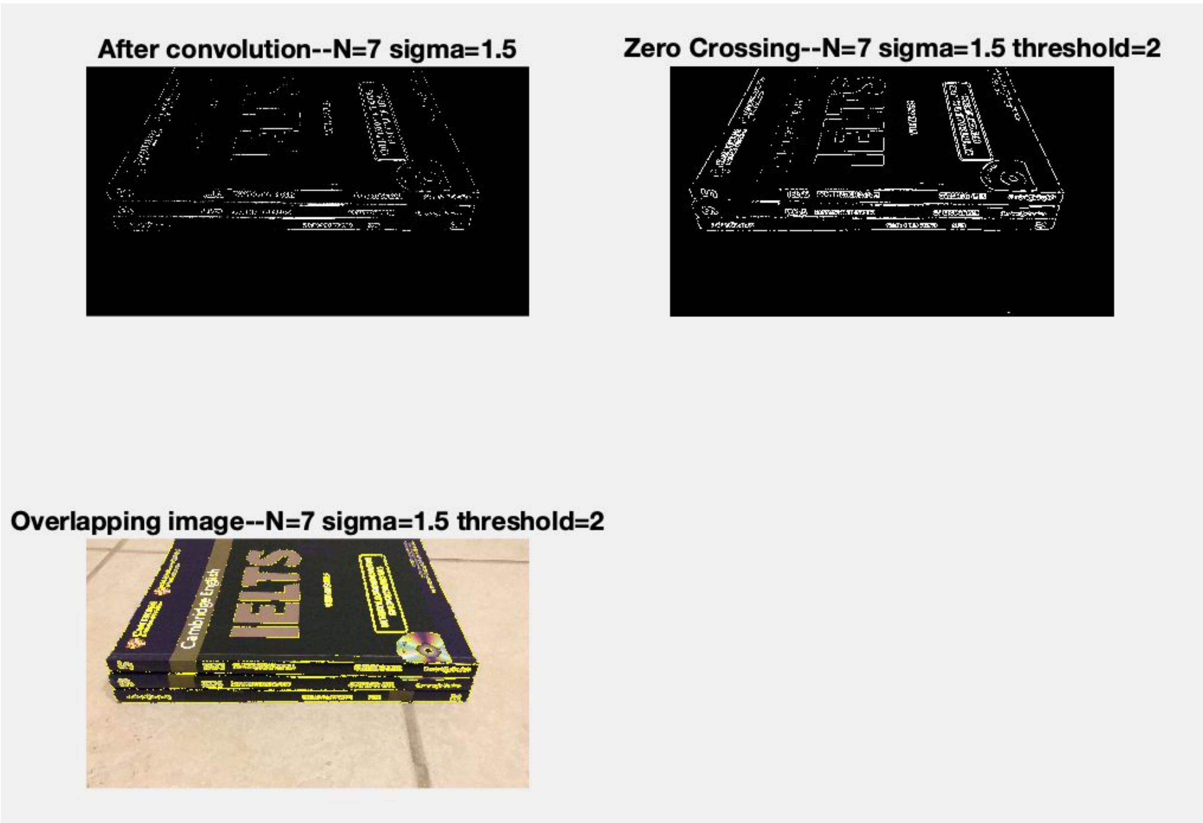
a) &b) Images after convolution with LOG and then finding zero-crossing.
Different sigma:


After convolution--N=7 sigma=1
Zero Crossing--N=7 sigma=1 threshold=2
Overlapping image--N=7 sigma=1 threshold=2


After convolution--N=7 sigma=1.2
Zero Crossing--N=7 sigma=1.2 threshold=2
Overlapping image--N=7 sigma=1.2 threshold=2


After convolution--N=7 sigma=1.5
Zero Crossing--N=7 sigma=1.5 threshold
Overlapping image--N=7 sigma=1.5 threshold=2

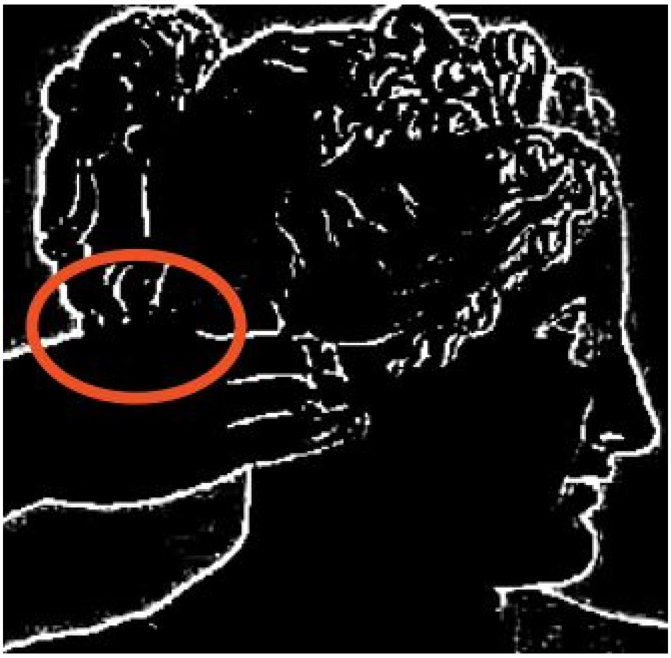




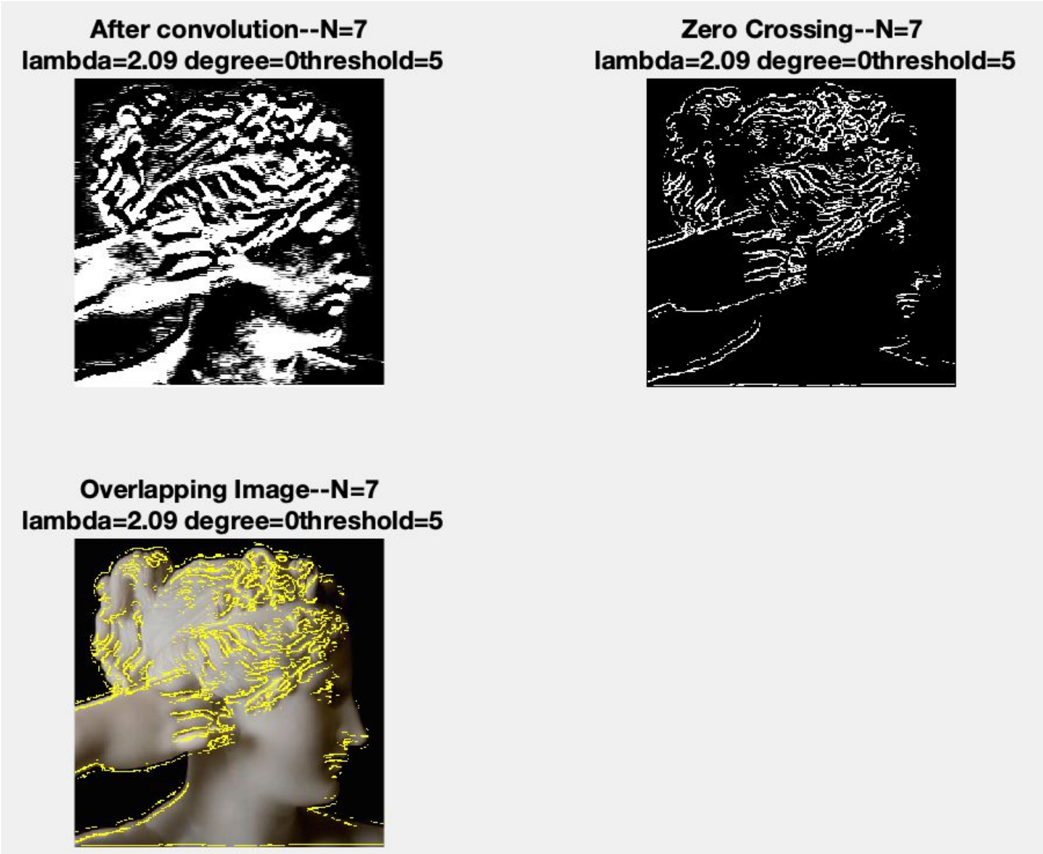
c)

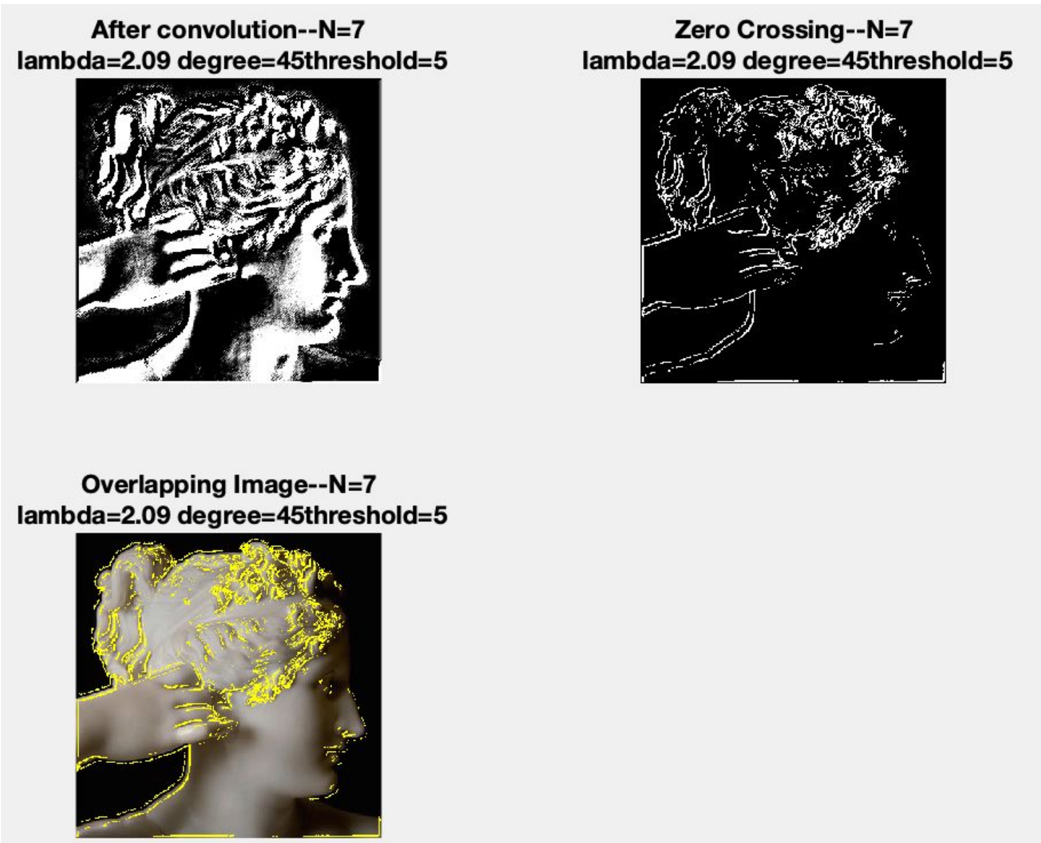
From figures in b), we can observe the following phenomenons:

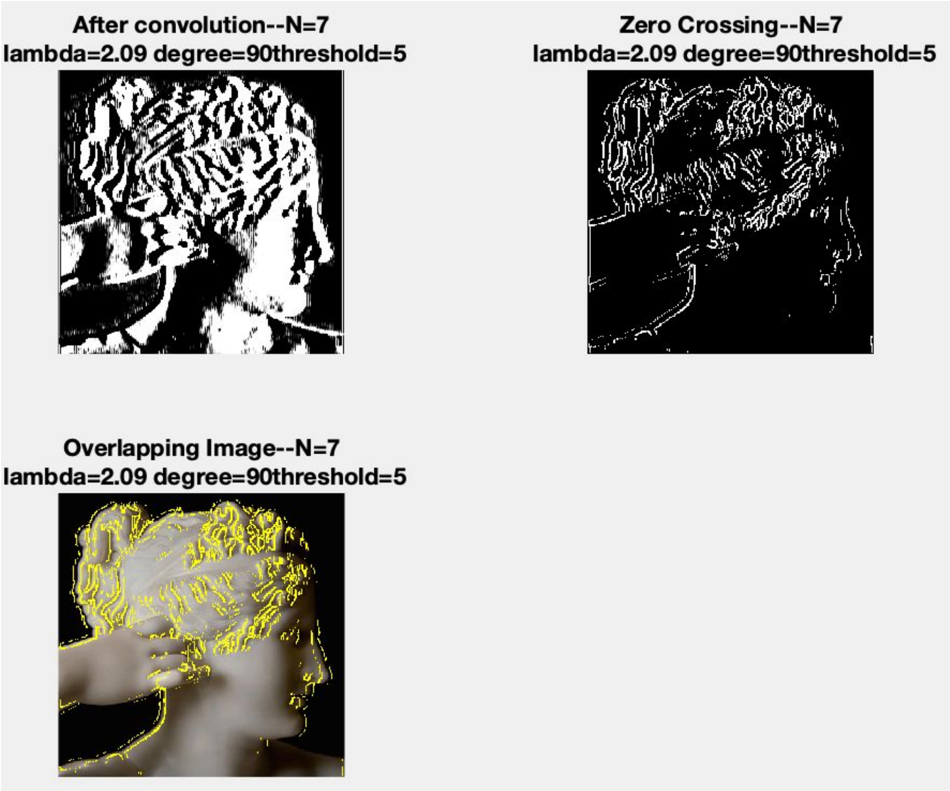
1. When the sigma is small(sigma = 1.0), the detected information can be very noisy but every edge information can be detected. When the sigma becomes larger (1.2, 1.5), there are no too many noisy points, but some details like hairs and eyes edge information are lost. This is because larger sigma will make pictures more smooth so that some detailed information is not preserved.
2. Although sigma = 1.2 seems to yield the best result, there are still failures in the picture, which I circled in red color shown below. Based on question 1 c), we know that the Laplacian operator can perform best when the change of intensity in the orientation parallel to the edge is linear, so when there are two edges(edge of hair and edge of hand) perpendicular to each other, the surrounding zero-crossing points can be hard to detect, which account for the failure.

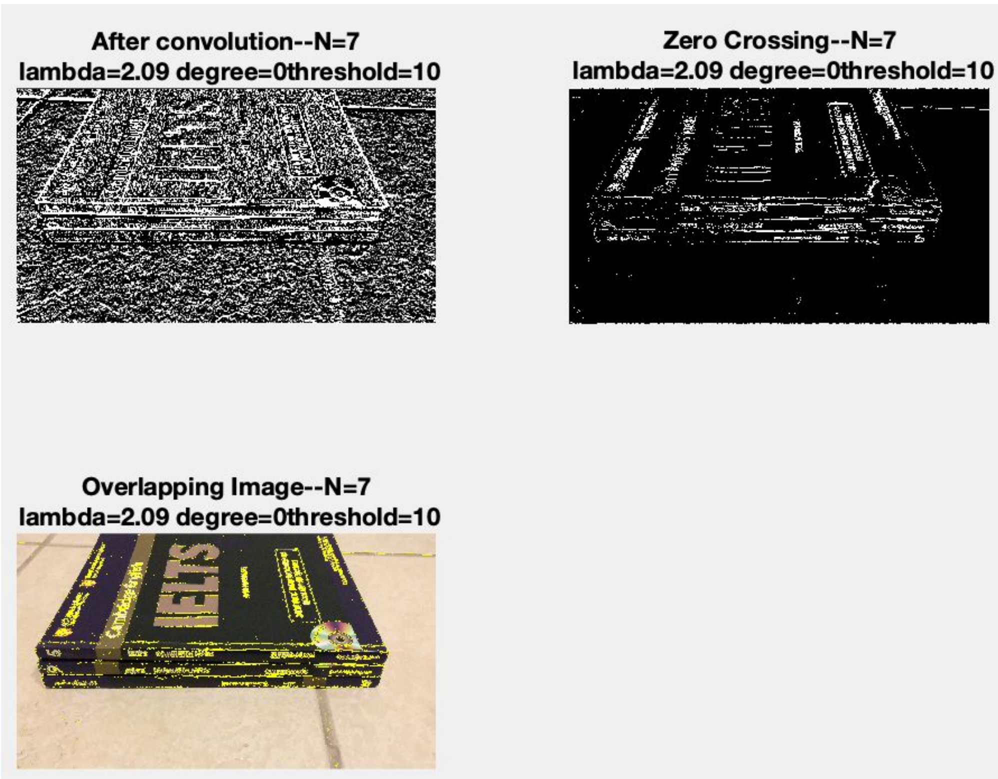


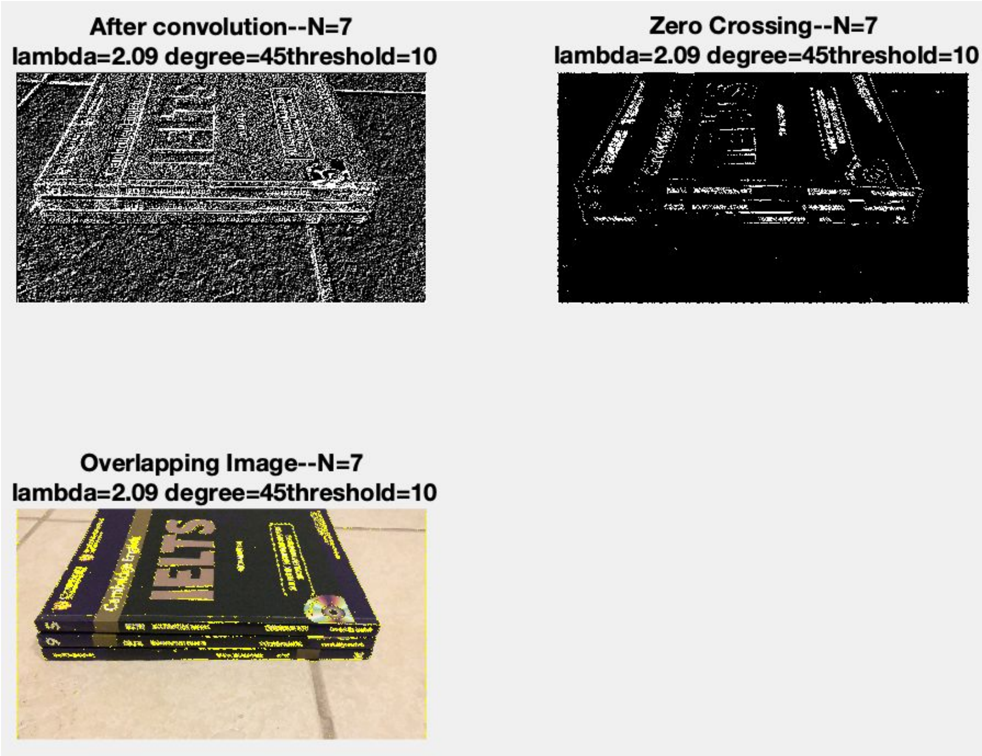
d) Display images after convolution with Gabor and then finding zero crossings.
Different degree:

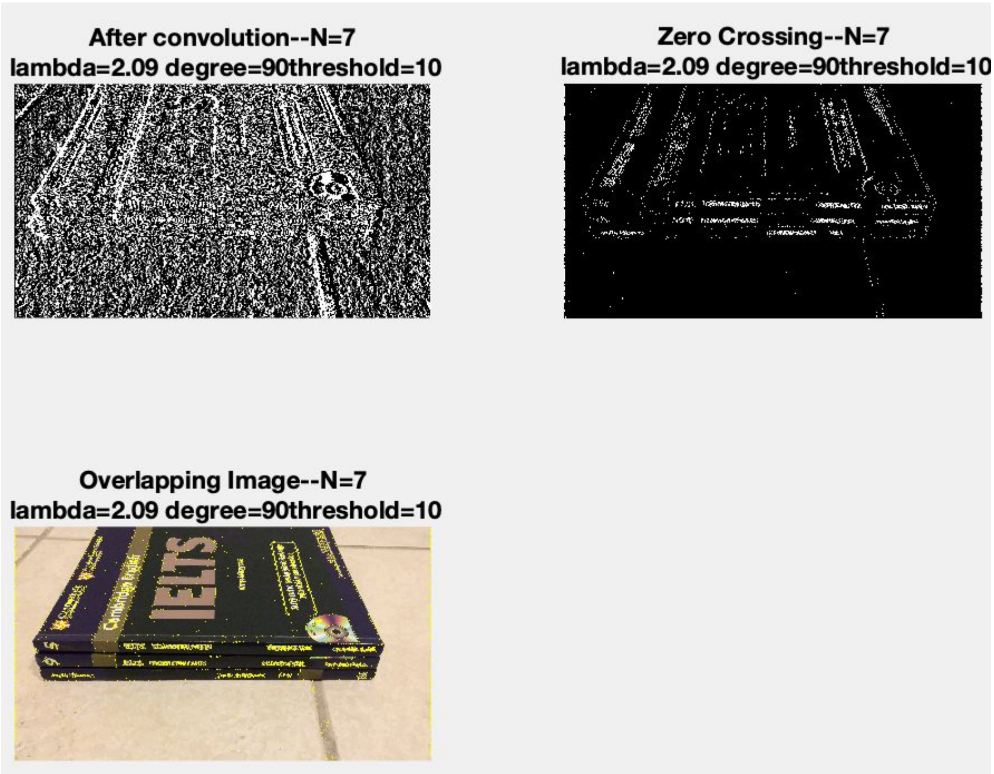

After convolution--N=7
lambda=2.09 degree=0threshold=5
Zero Crossing--N=7
lambda=2.09 degree=0threshold=5
Overlapping Image--N=7
lambda=2.09 degree=0threshold=5


After convolution--N=7
lambda=2.09 degree=45threshold=5
Zero Crossing--N=7
lambda=2.09 degree=45threshold=5
Overlapping Image--N=7
lambda=2.09 degree=45threshold=5

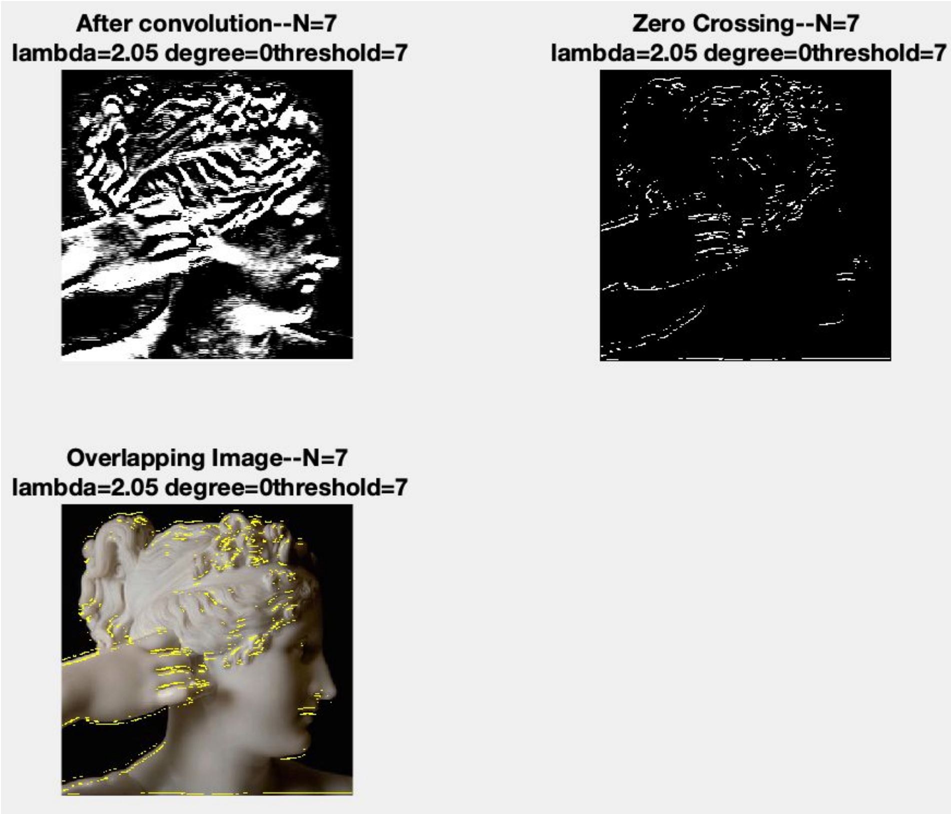

After convolution--N=7
lambda=2.09 degree=90threshold=5
Zero Crossing--N=7
lambda=2.09 degree=90threshold=5
Overlapping Image--N=7
lambda=2.09 degree=90threshold=5

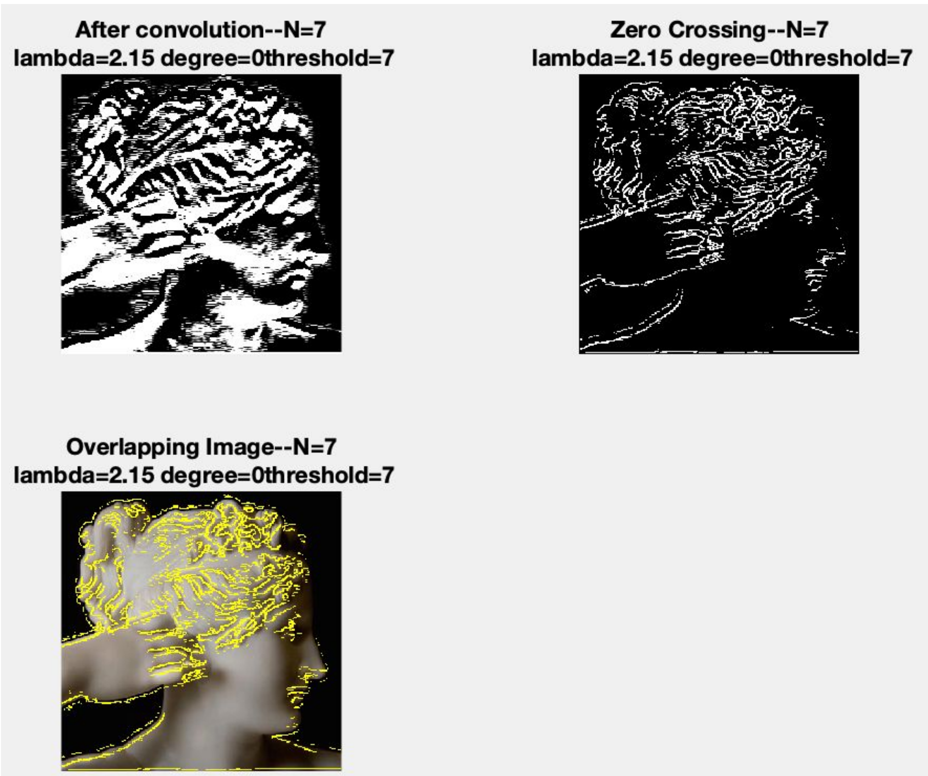

After convolution--N=7
lambda=2.09 degree=0threshold=10
Zero Crossing--N=7
lambda=2.09 degree=0threshold=10
Overlapping Image--N=7
lambda=2.09 degree=0threshold=10





Different wavelength:

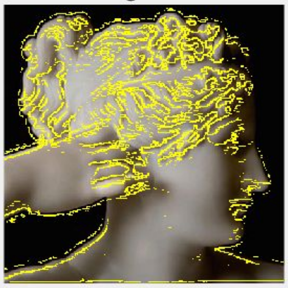

After convolution--N=7
lambda=2.05 degree=0threshold=7
Zero Crossing--N=7
lambda=2.05 degree=0threshold=7
Overlapping Image--N=7
lambda=2.05 degree=0threshold=7


After convolution--N=7
lambda=2.15 degree=0threshold=7
Zero Crossing--N=7
lambda=2.15 degree=0threshold=7
Overlapping Image--N=7
lambda=2.15 degree=0threshold=7

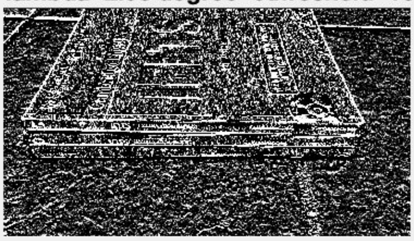
After convolution--N=7
lambda=2.2 degree=0threshold=7

Zero Crossing--N=7
lambda=2.2 degree=0threshold=7

Overlapping Image--N=7
lambda=2.2 degree=0threshold=7

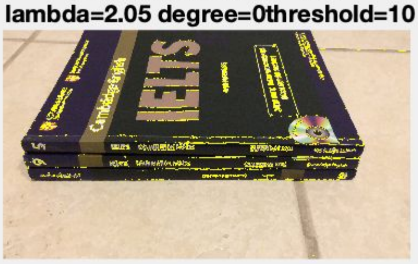


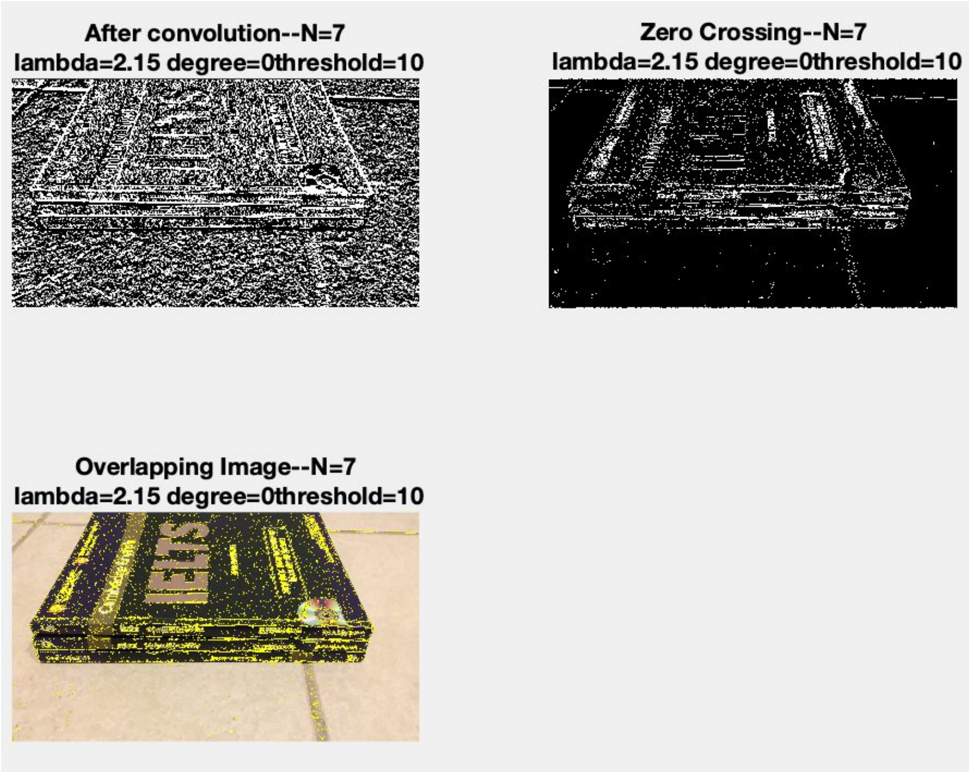
After convolution--N=7
lambda=2.05 degree=0threshold=10

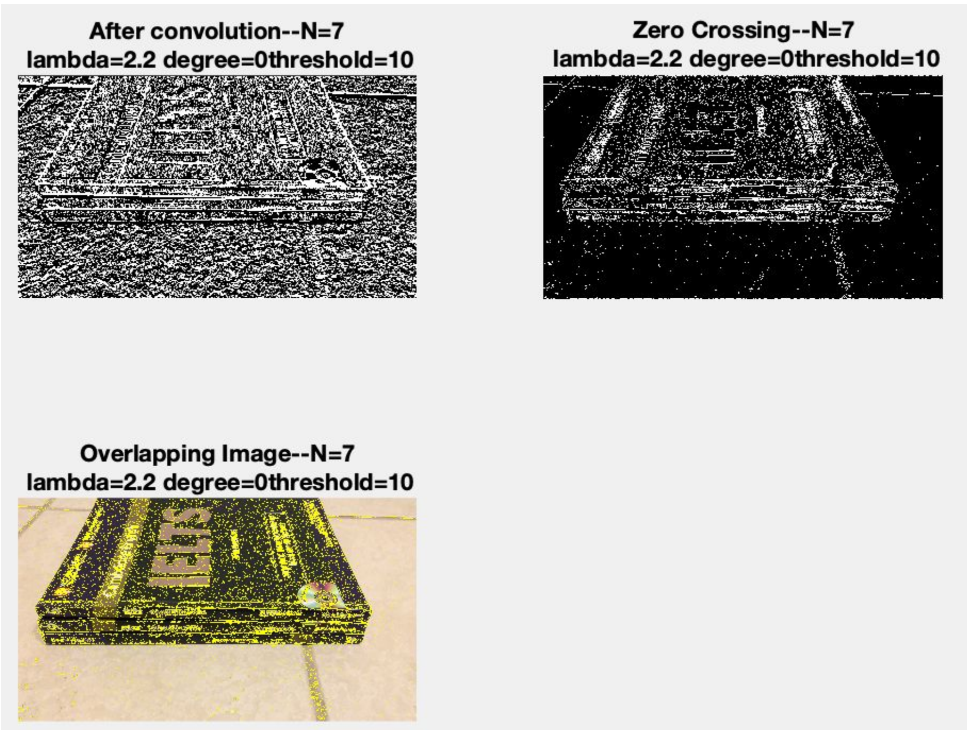


Zero Crossing--N=7
lambda=2.05 degree=0threshold=10

Overlapping Image--N=7
lambda=2.05 degree=0threshold=10







From the image above, we can see that the bigger the lambda, the more zero crossings detected. However, this will also introduce a lot of noisy point. We can also observe that a single Gabor filter tends to highlight edges with the same orientation of degree. For example, a 0 degree Gabor tend to highlight edges parallel to the x-axis. So maybe a group of Gabor operators with different angels will yield a better result.

Furthermore, because we set sigma of Gabor to be equal to lambda, so the lambda is the standard derivation of Gaussian kernel(sigma). It can control the size of the Gabor kernel(number of stripes).