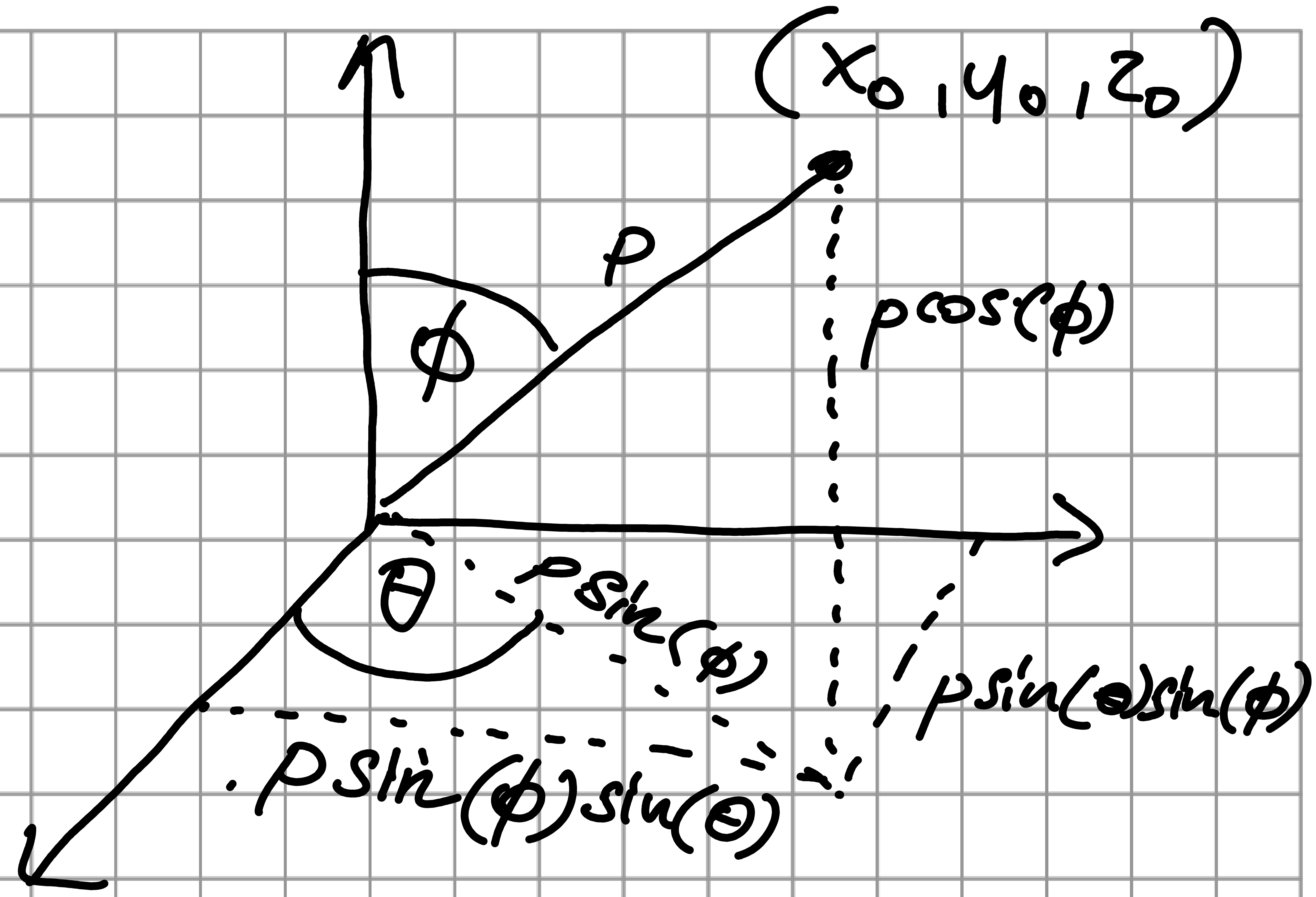
Czarni

Wielokrotne XD

Odkryte

Sferyczne



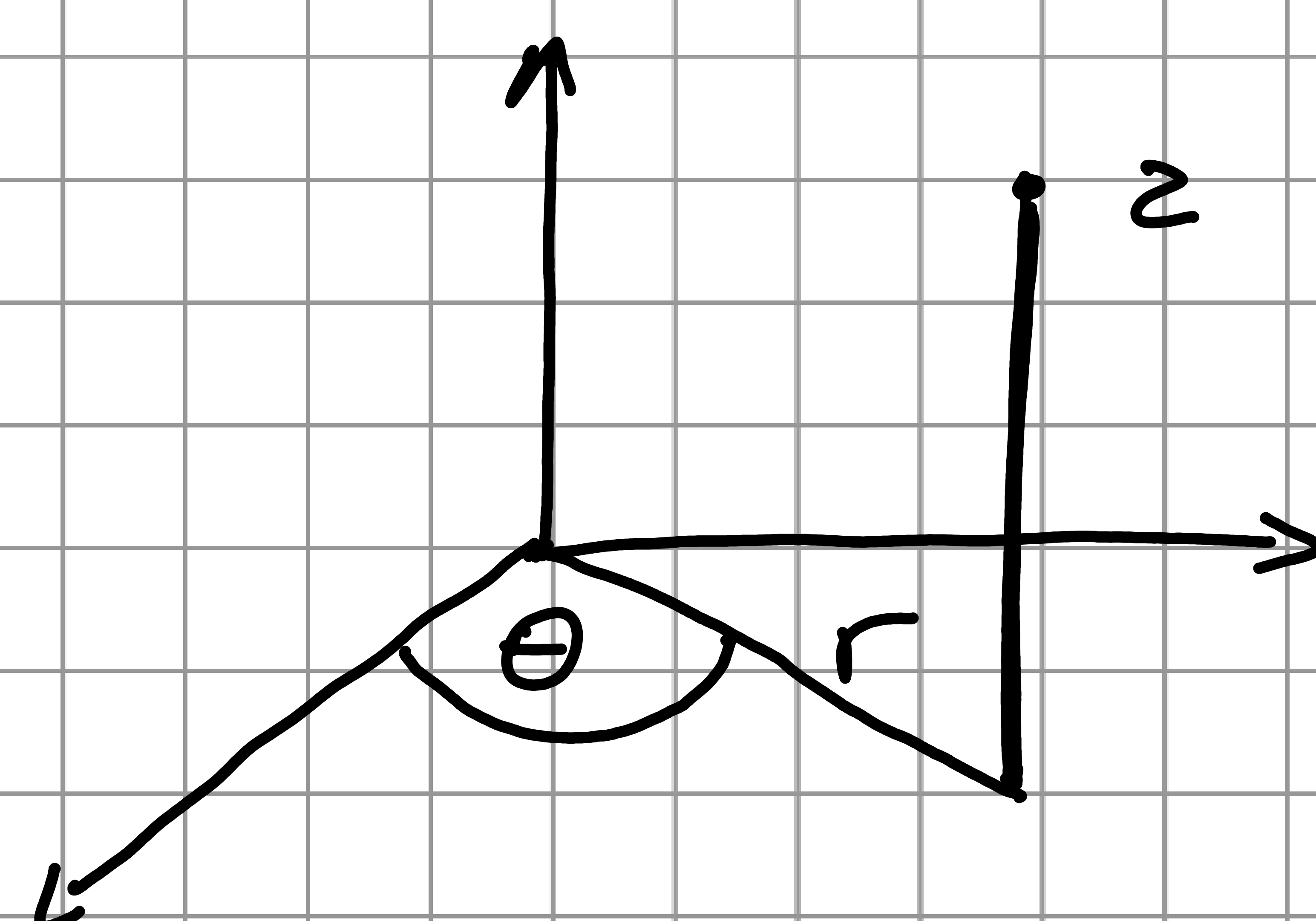
$x = p\sin(\theta)\sin(\phi)$

$y = p\sin(\theta)\sin(\phi)$

$z = p\cos(\phi)$

$j = p^2\sin(\phi)$

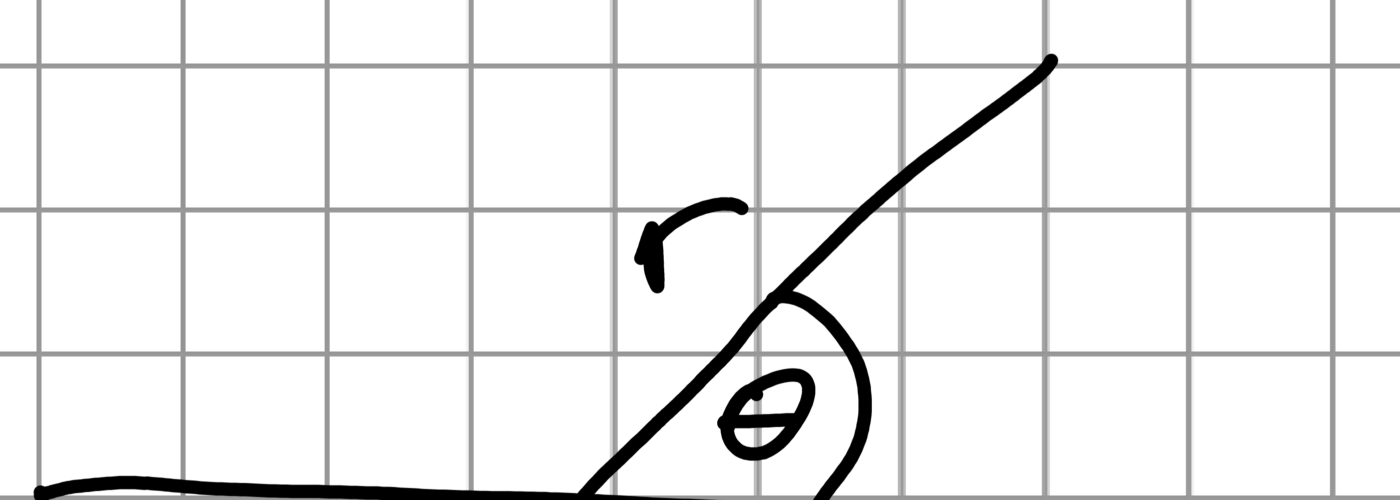
cylindryczne



$(x, y, z) \rightarrow (r, \theta, z)$

$j = r$

polar



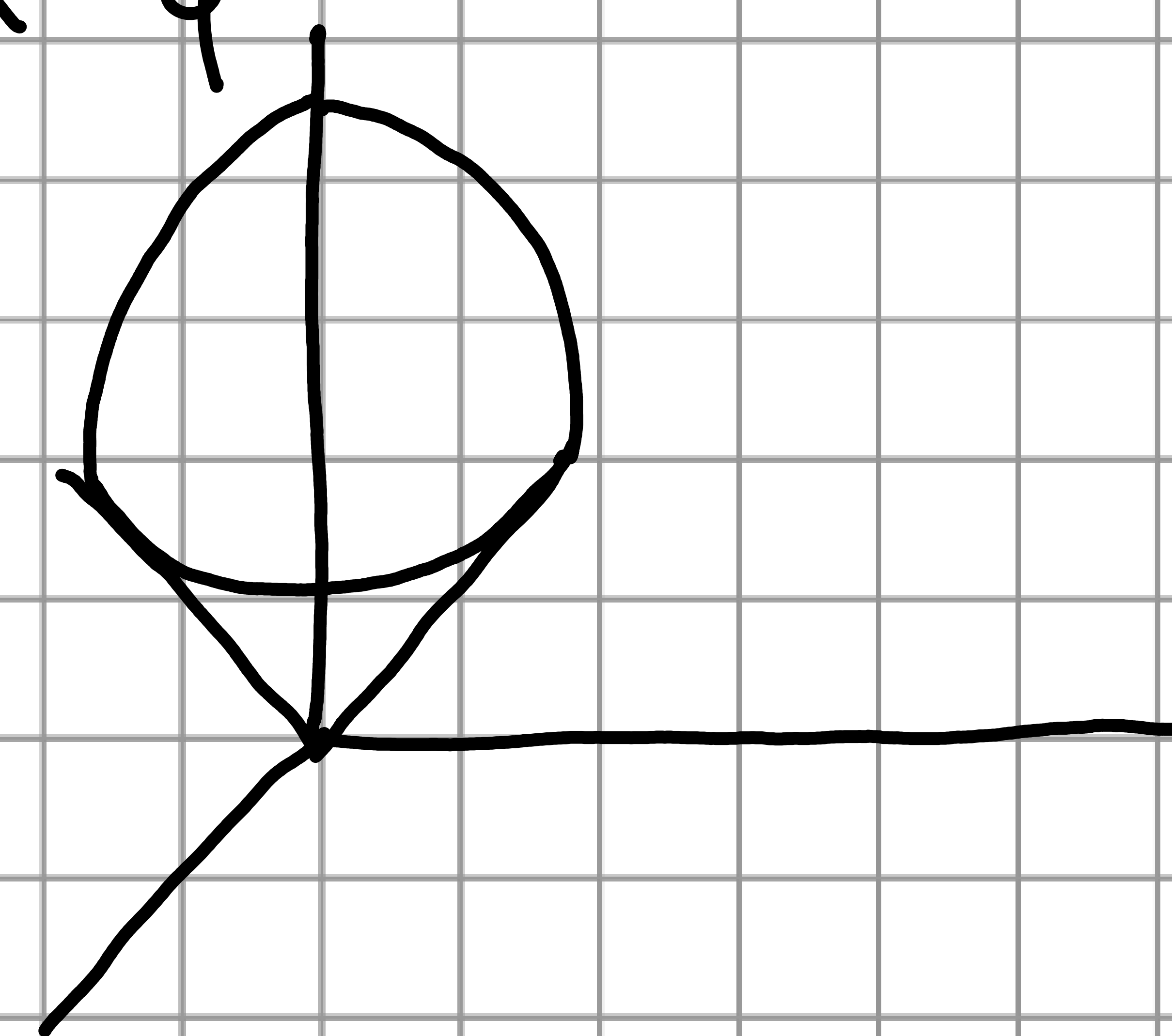
$x^2 + y^2 = r^2$

$x = r\cos(\theta)$

$y = r\sin(\theta)$

$j = r$

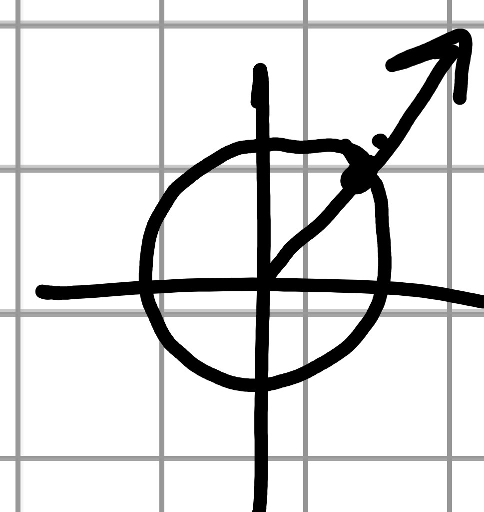
$z=\sqrt{x^2+y^2}$ oraz $z=6-x^2-y^2$



$z=\sqrt{x^2+y^2}=\sqrt{r^2}=r$

$z=6-(x^2+y^2)=6-r^2$

$$\int_0^{2\pi} dz \int_0^{2} r\,dr \int_r^{6-r^2} d\theta$$



$r=6-r^2 \Rightarrow r^2+r+6=0$

$(r+3)(r-\underline{2})=0$

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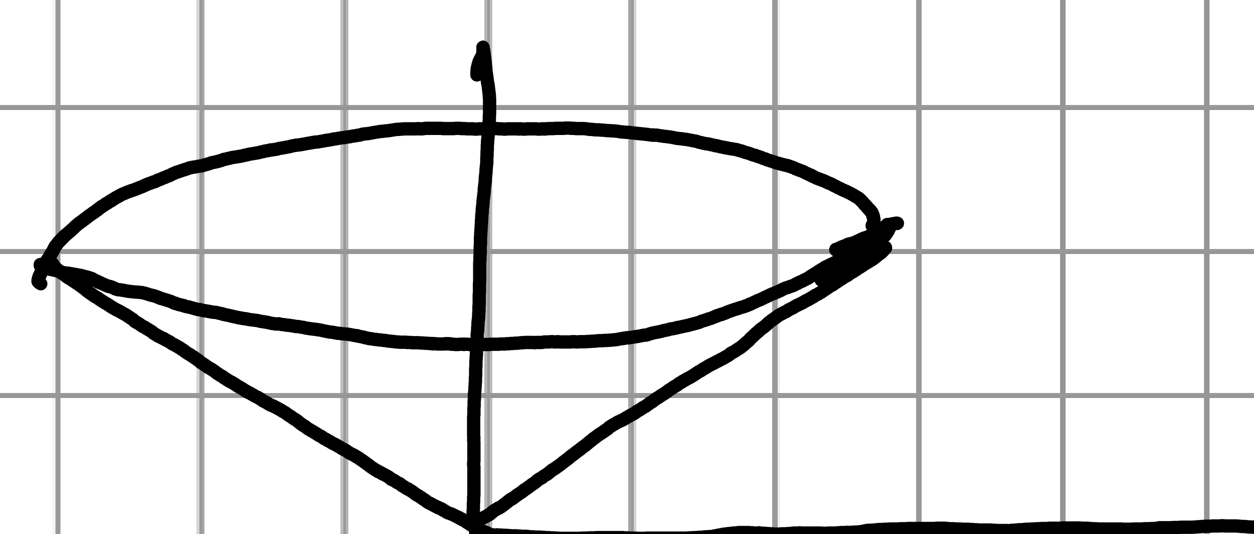
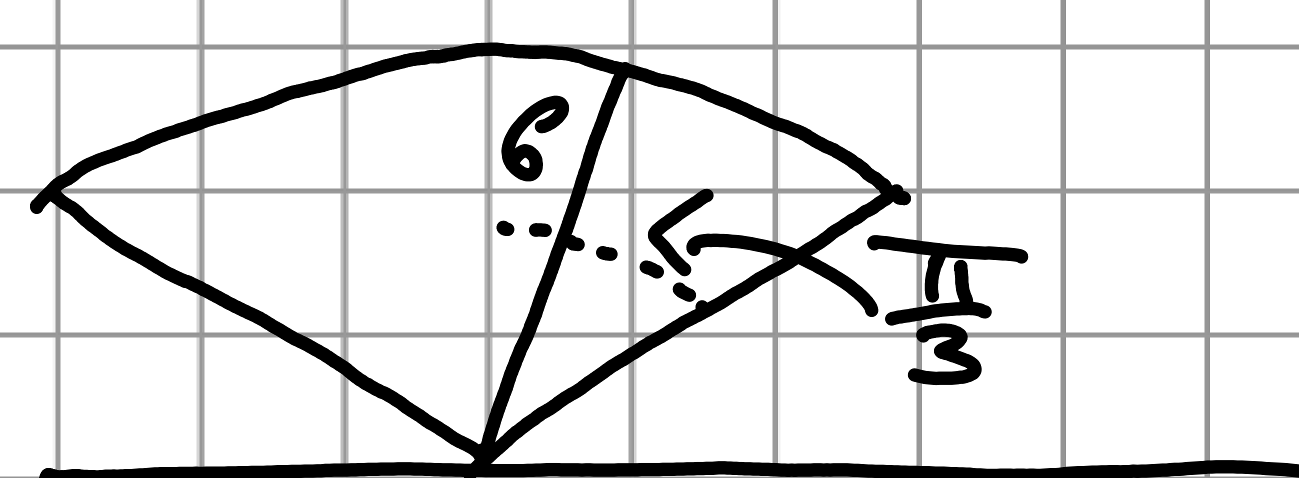
rozek $\phi=\frac{1}{3}$ i sfera $\rho=6$



$$\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{6} \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$$



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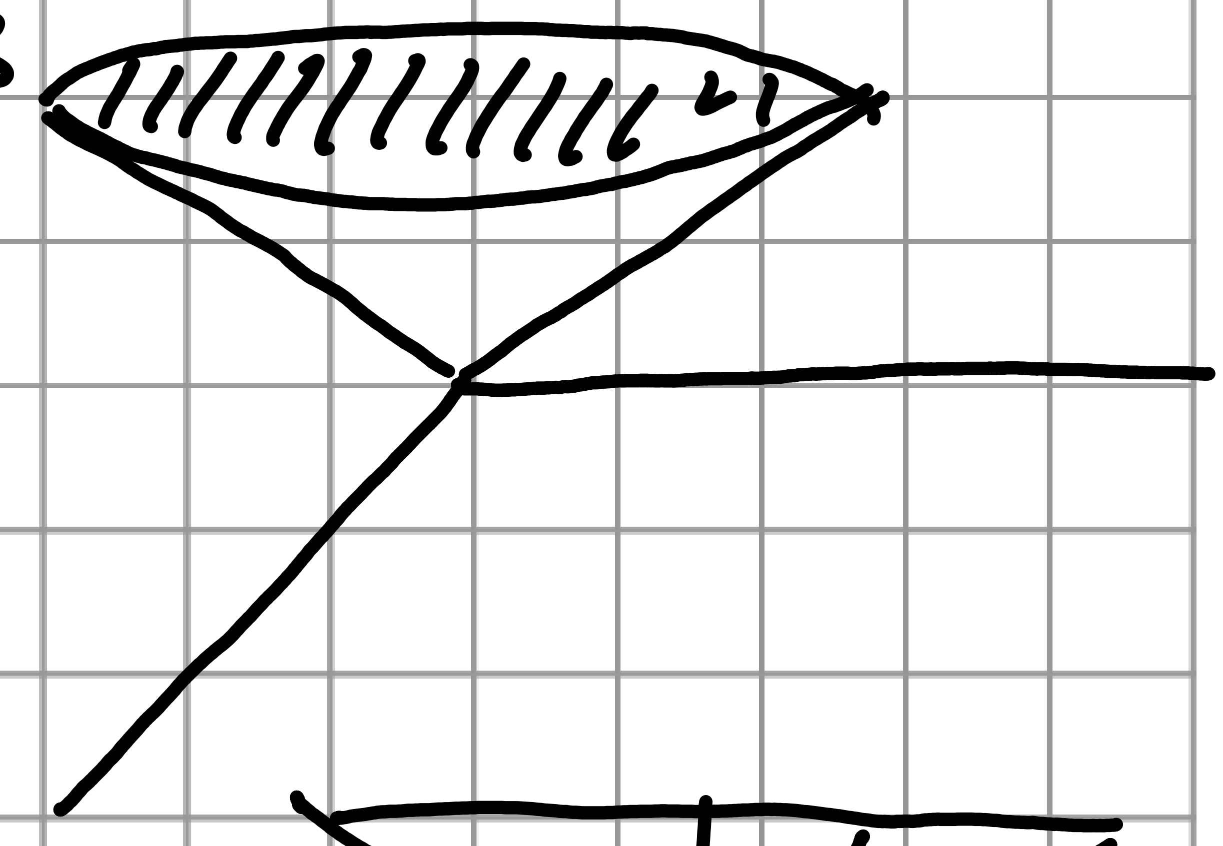
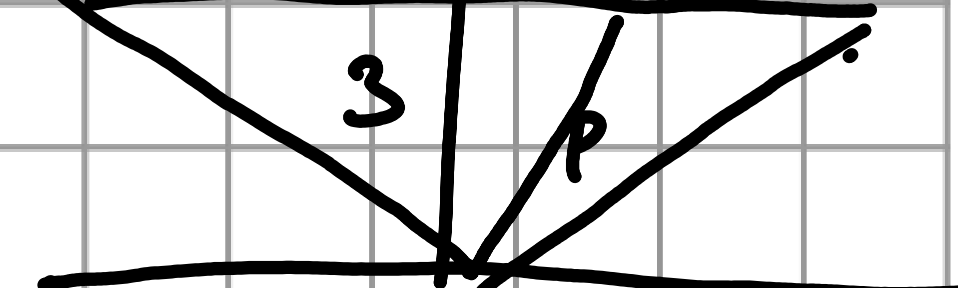
rozek $\phi=\frac{\pi}{3}$ oraz płaszczyzna $z=3$



$$\int_0^{2\pi}\int_0^{\frac{\pi}{3}}\int_0^{3\sec\phi} \rho^2\sin(\phi)\,d\rho\,d\phi\,d\theta$$

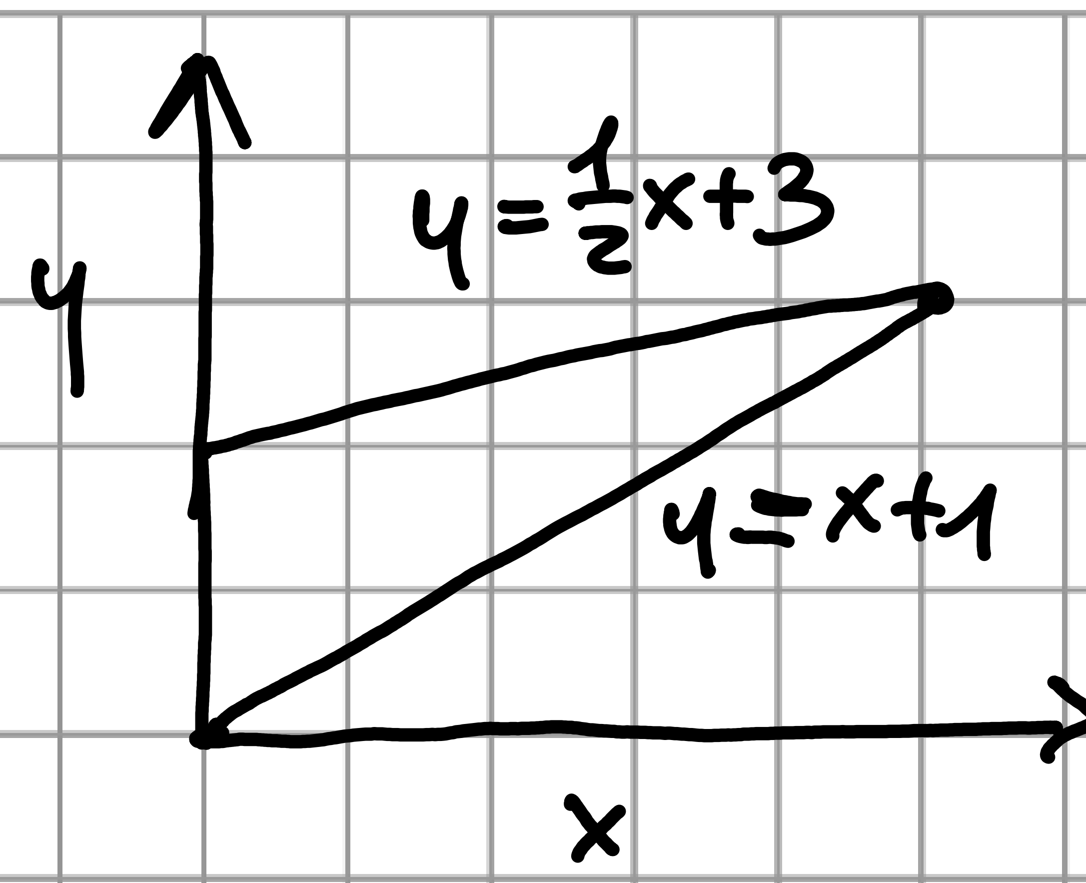


$\cos\phi=\frac{3}{\rho} \Rightarrow \rho=\frac{3}{\cos(\phi)}$

$$\frac{1}{2}x+3 = x+1$$

$$x = 4$$

$$\int_0^4 \int_{x+1}^{\frac{1}{2}x+3} dy\, dx$$



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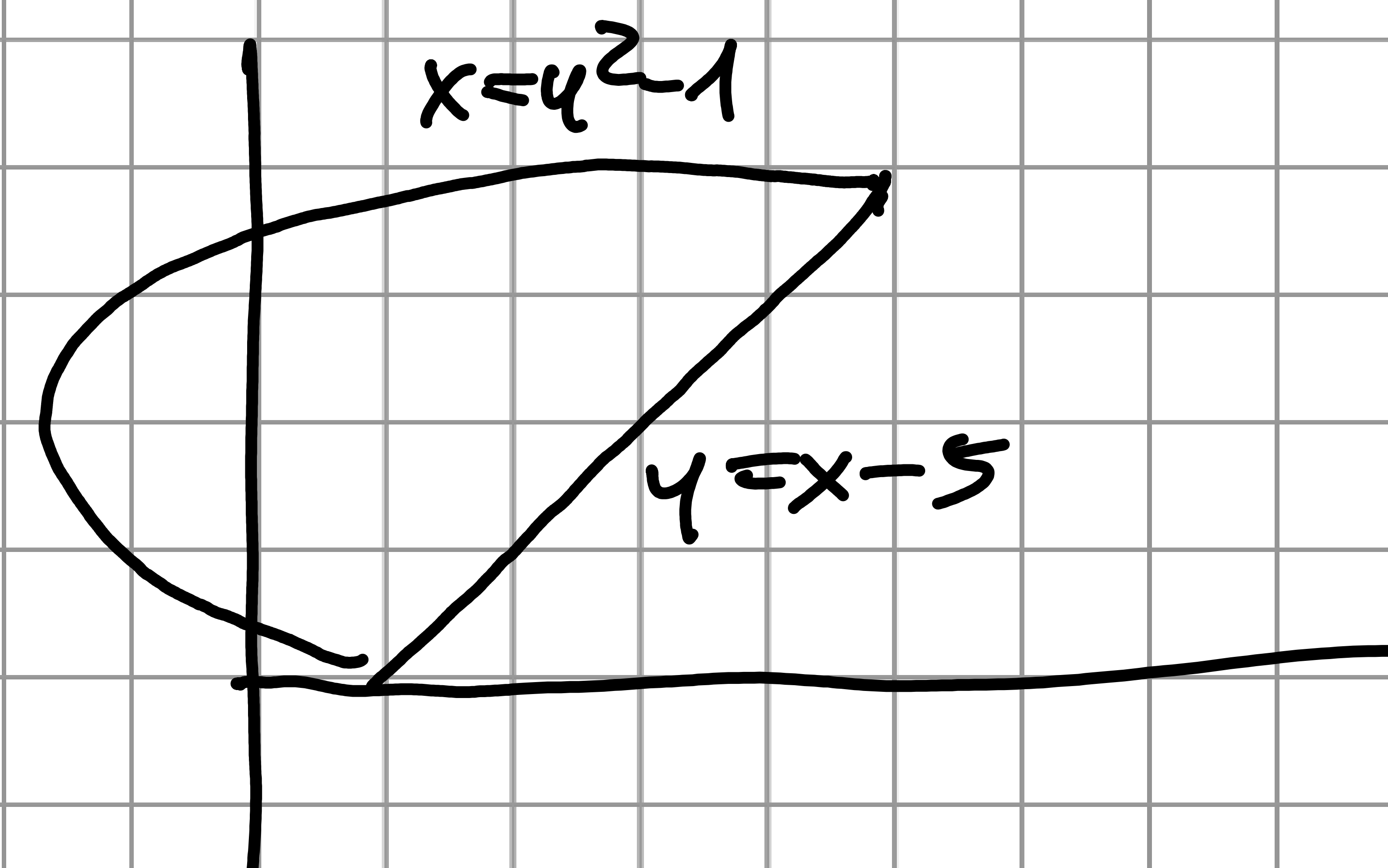
obszar ograniczony $x = y^2-1 \wedge y = x-5$

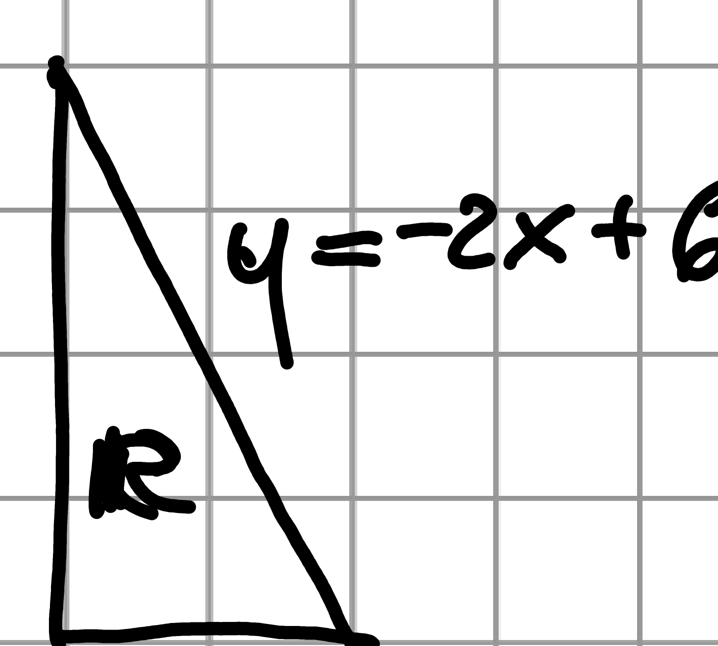
$$\int_{y=-2}^{y=3} \int_{x=y^2-1}^{x=y+5}$$



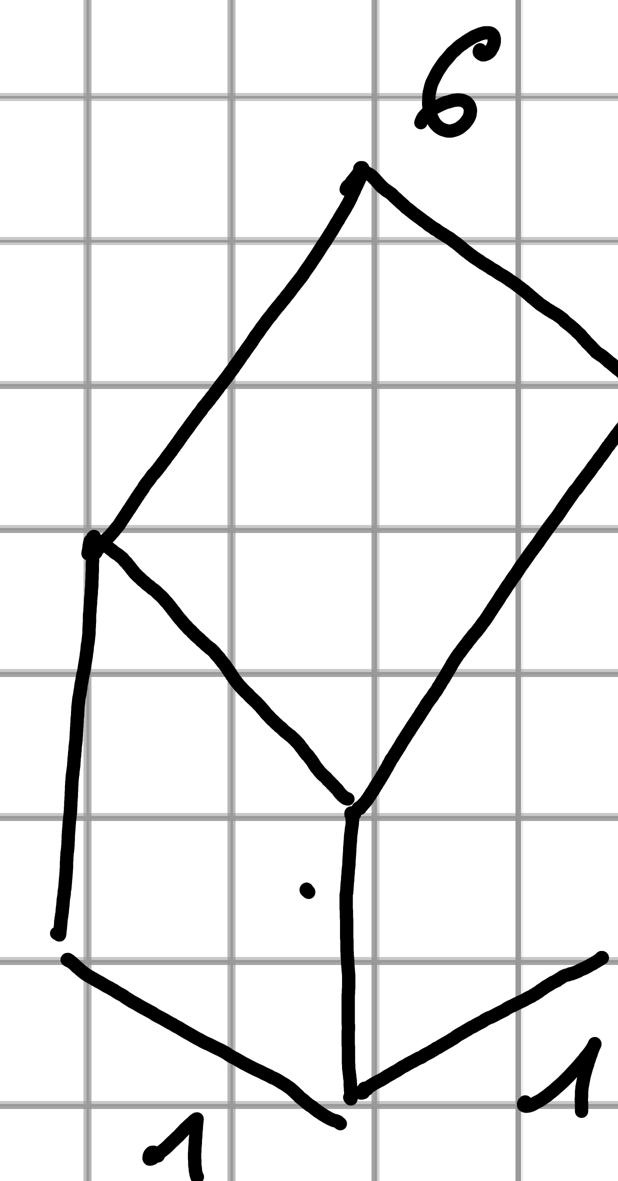
$$y^2-1 = y+5$$

$$y^2-y-6=0 \rightarrow (y-3)(y+2)=0$$

$$\int_0^3 \int_0^{-2x+6} dx\, dy$$



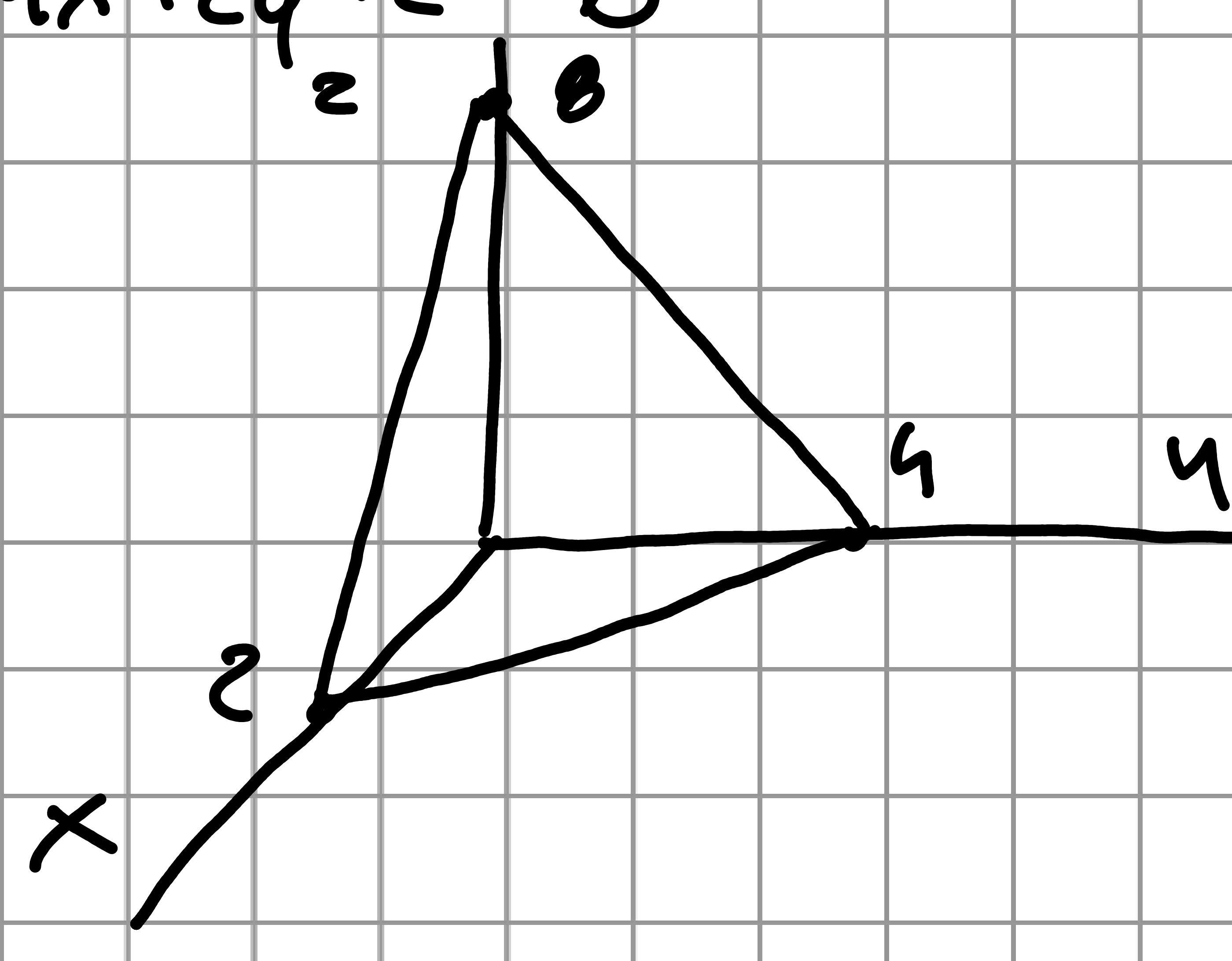
Obszar otoczony osiami oraz $z=6-3x-2y$ i powyżej kwadratu 1×1



$$\int_0^1\int_0^1 6-3x-2y\,dy\,dx =$$

$$= \int_0^1 6-3x-1\,dx = 6-\frac{3}{2}-1 = 3\frac{1}{2}$$

Ograniczony osiami oraz $4x+2y+z=8$

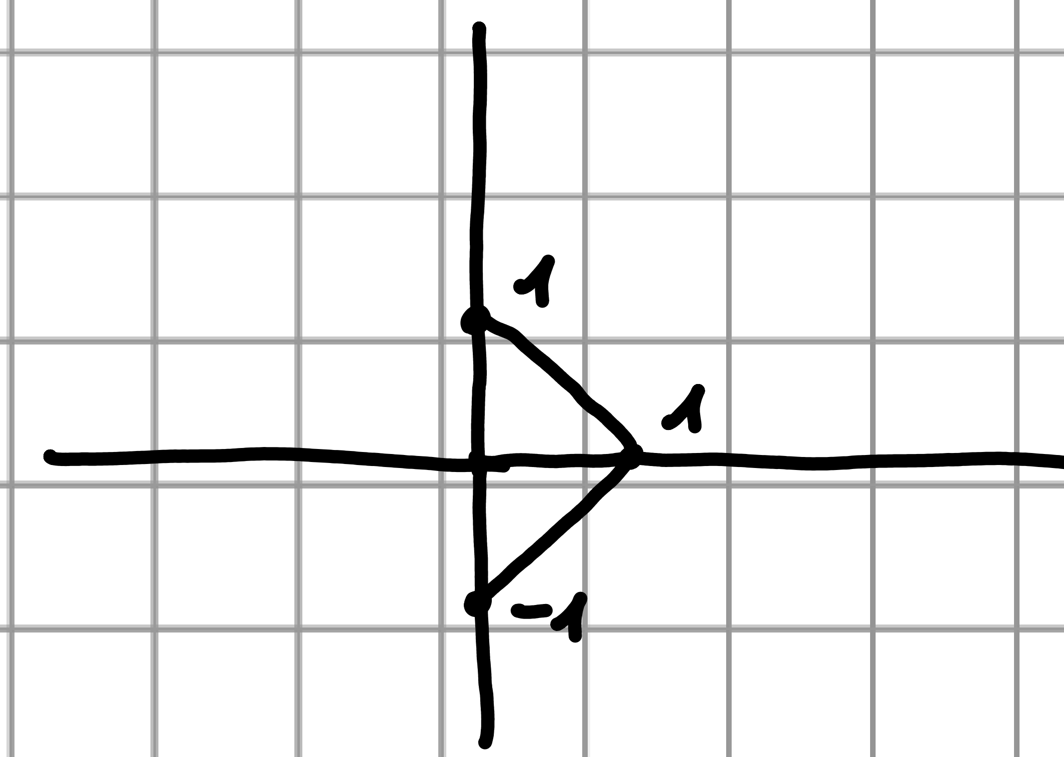


$z=8-4x-2y \Rightarrow y=-2x+4$ (bez z)

$$\int_0^{-2x+4}\int_0^{4} 8-4x-2y\,dx\,dy$$

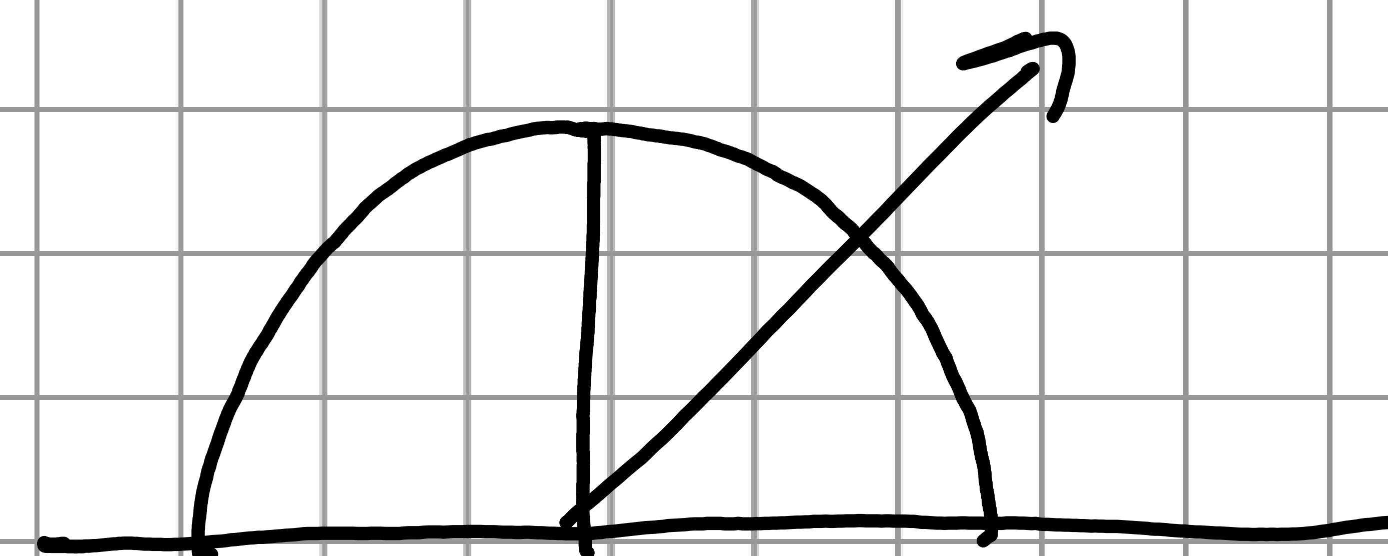
Pole pod $z=x^2$ i obszarem trójkąta $(1,0),(0,1),(0,-1)$

$$\int_0^1 \int_{x-1}^{-x+1} x^2 \, dy\, dx$$



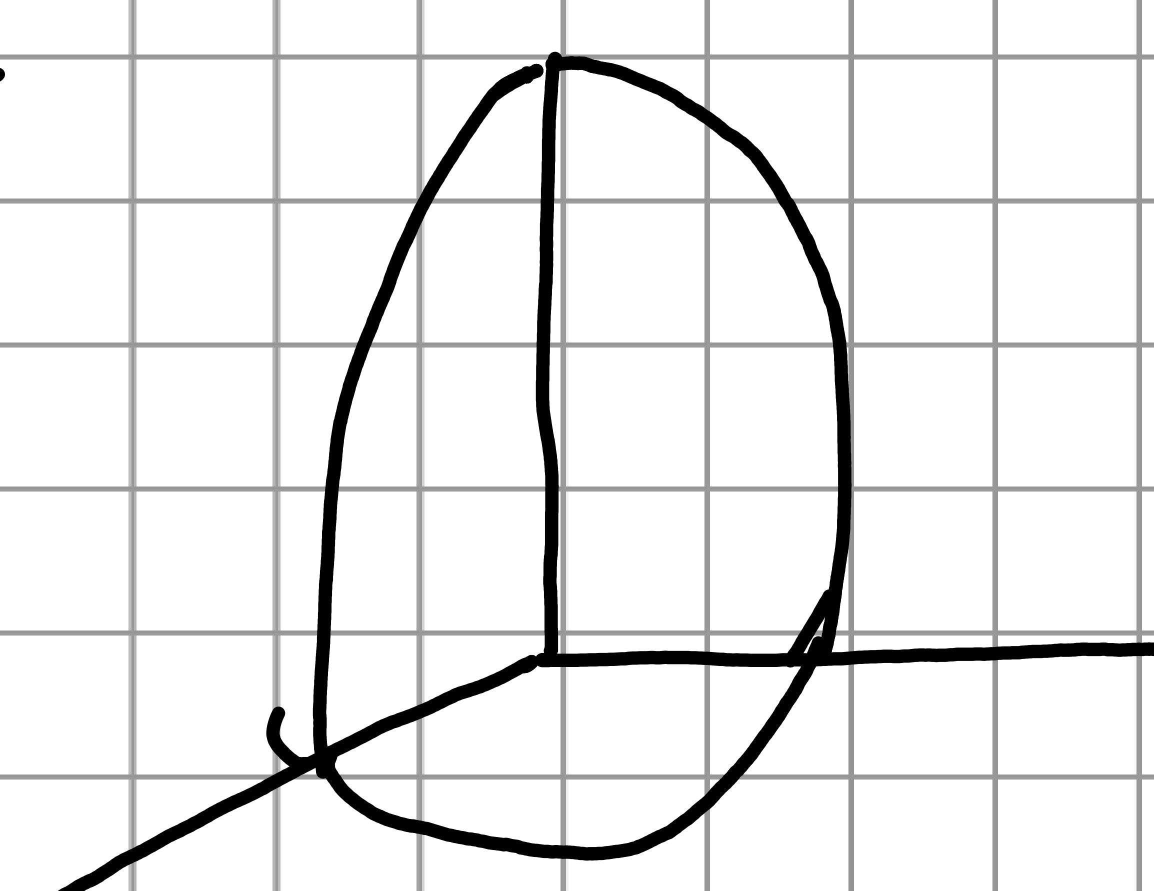
$$\iint_R (1-r^2)\, dA$$ – górna część koła

$$\int_0^{\pi} \int_0^1 (r-r^3)\, dr\, d\theta$$



Obszar między osiami, $z=4-x^2-y^2$

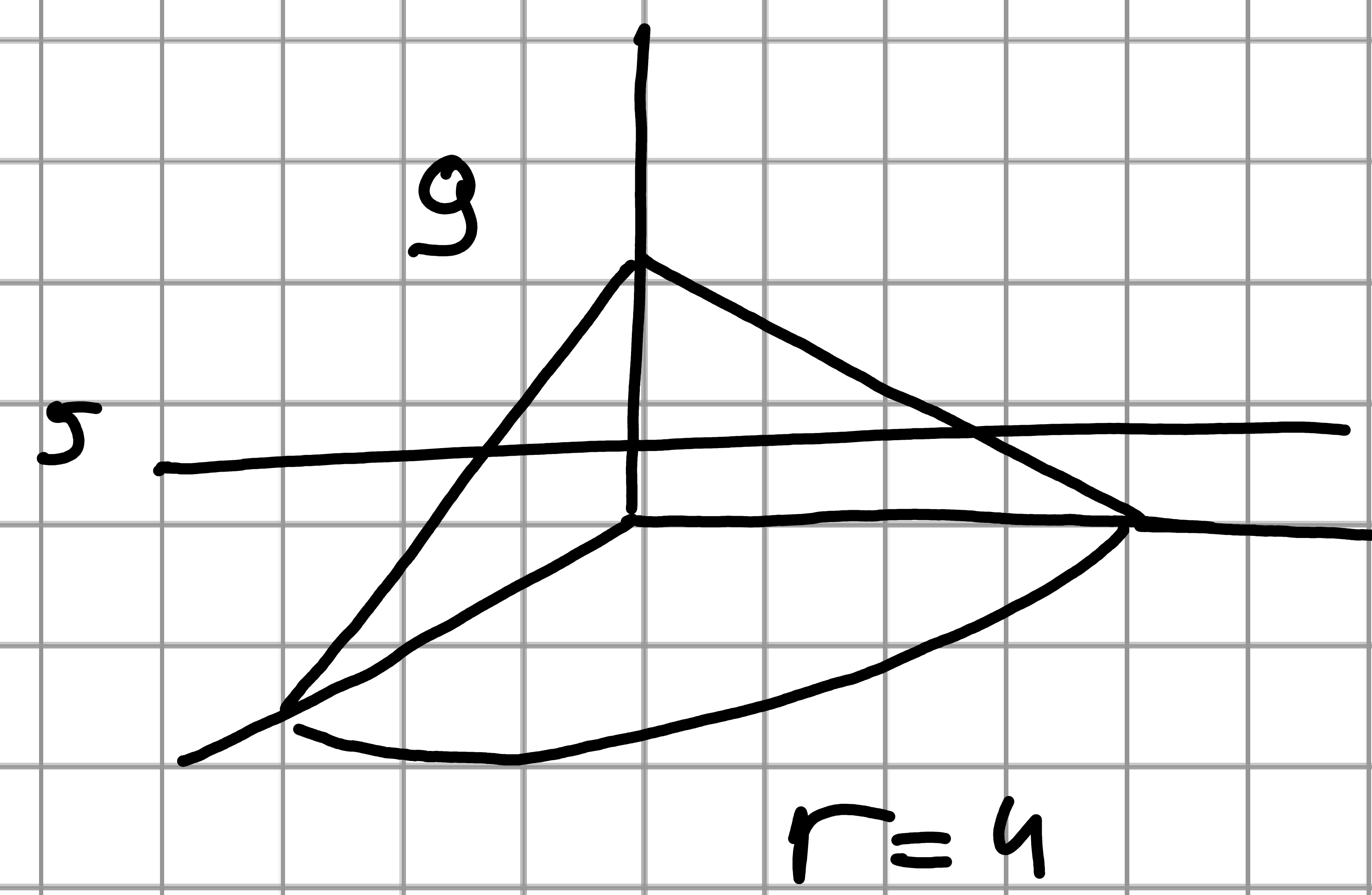
$$\int_0^{2\pi} \int_0^2 (4-r^2) r \, dr\, d\theta$$



$z = 9 - \sqrt{x^2+y^2}$ oraz $z = 5$

$$\int\int (\text{Top} - \text{Bottom})\, r\, dr\, d\theta$$

$$\int_0^{2\pi}\int_0^4 (9 - r - 5)\, r\, dr\, d\theta$$



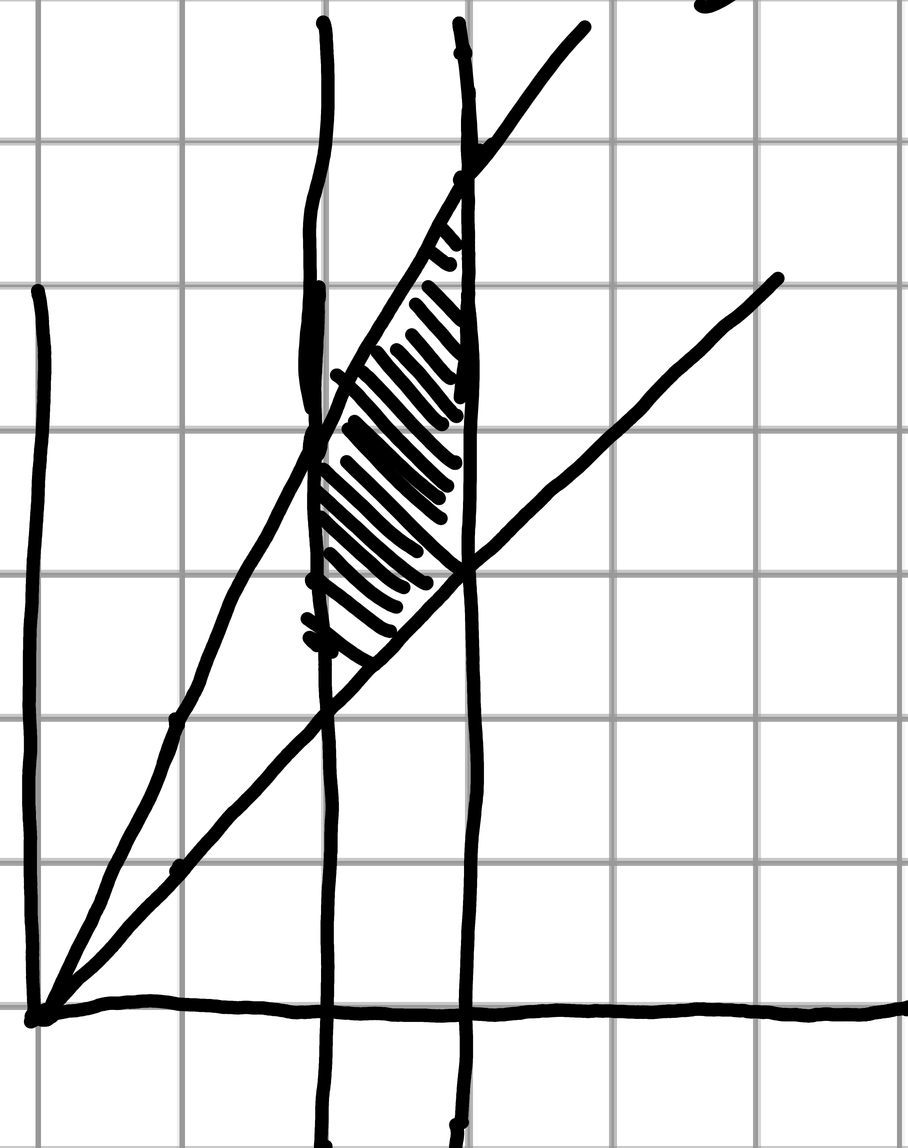
$$= \int_0^{2\pi}\int_0^4 (4r - r^2)\, dr\, d\theta =$$

$$= \int_0^{2\pi} \left[2r^2 - \frac{1}{3}r^2\right]_0^4 d\theta = \int_0^{2\pi} 32 - \frac{64}{3}\, d\theta = \int_0^{2\pi} \frac{32}{3}\, d\theta$$

$$= \frac{64}{3}\pi$$

$$\iint ax(x+2y)\,dD$$

$x=2,\ x=3,\ y=x,\ y=2x$



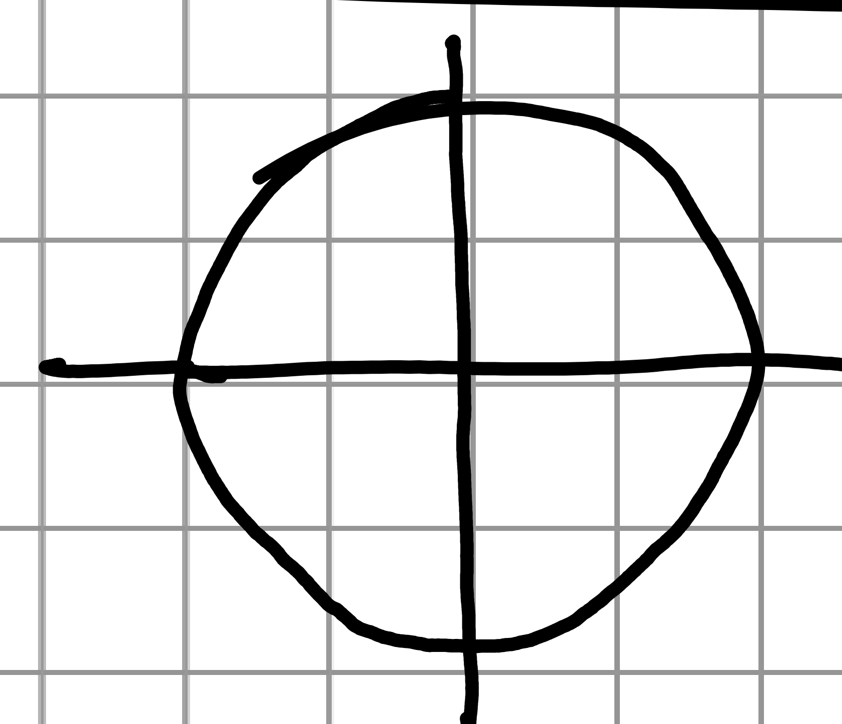
$$\int_2^3\int_x^{2x} a\cdot(x+2y)\,dy\,dx$$

$$= a\cdot\int_2^3\int_x^{2x} x+2y\,dy\,dx = \ldots$$

NIE MOGĘ ZOSTAĆ Z

X/Y NA KONIEC ?

$$\iint_D (h-x-y)\cdot dD \text{ dla } x^2+y^2=a^2$$



$$\int_0^{2\pi}\int_0^{a} (h-r\cos(\varphi)-r\sin(\varphi))\,r\,dr\,d\theta$$

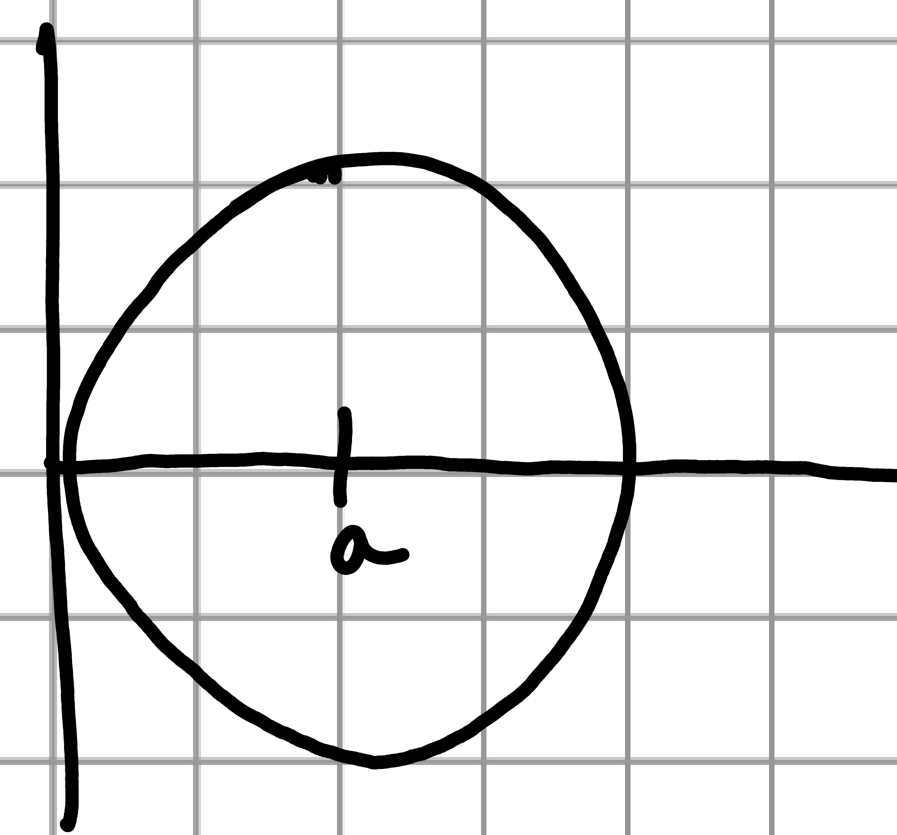
$r-a^2=0$

$x=r\cos(\varphi)$

$y=r\sin(\varphi)$

Biegunowe

$$\iint_D \left(a - x + y^3\right)\cdot dD, \quad x^2 + y^2 = 2a\cdot x, \quad a > 0$$

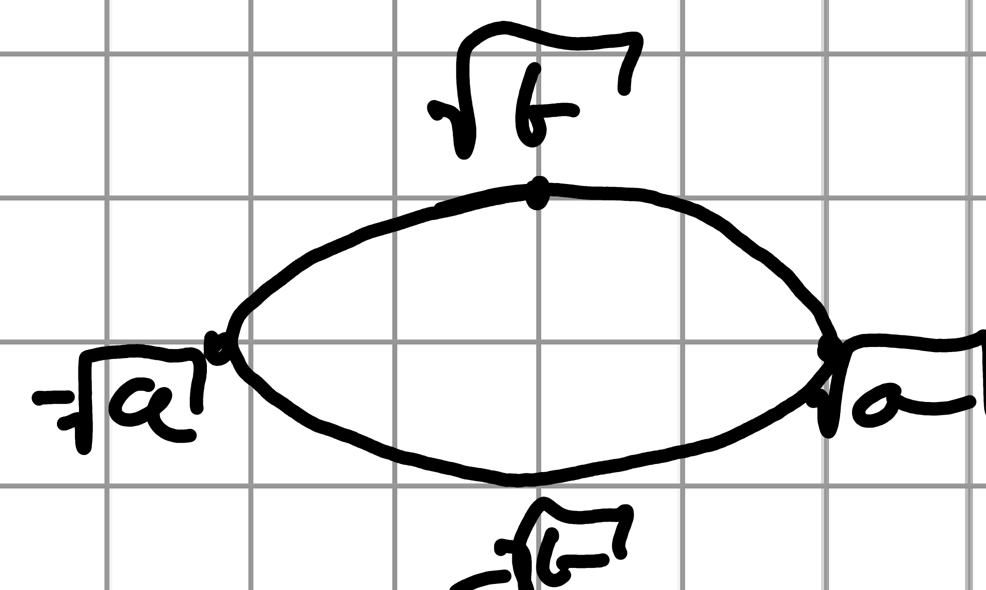


$$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{2a\cos(\varphi)} \left(a - r\cos(\alpha) + (r\cos(\alpha))^3 \quad r \, dr \, d\varphi\right.$$

$x = r\cos(\alpha)$

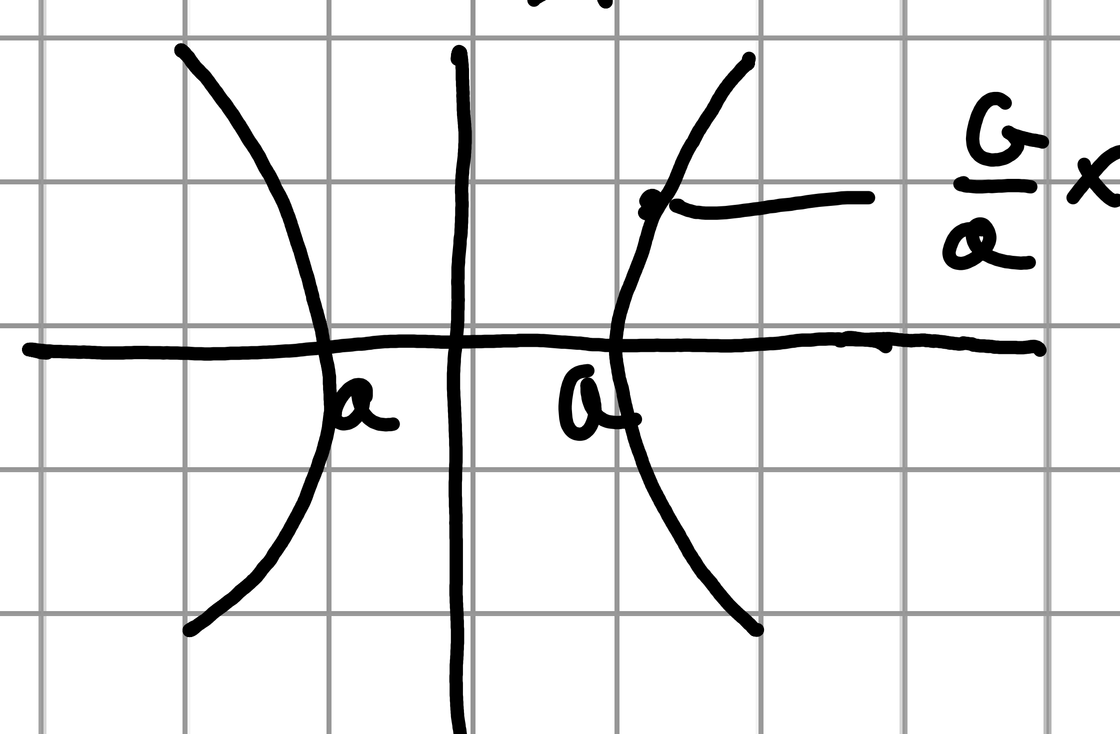
$y = r\sin(\alpha)$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$



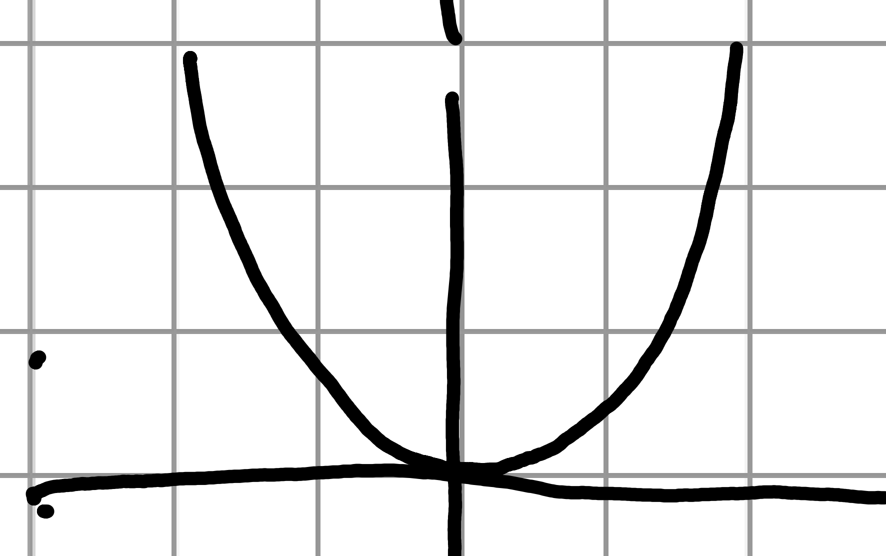
Elipsa

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



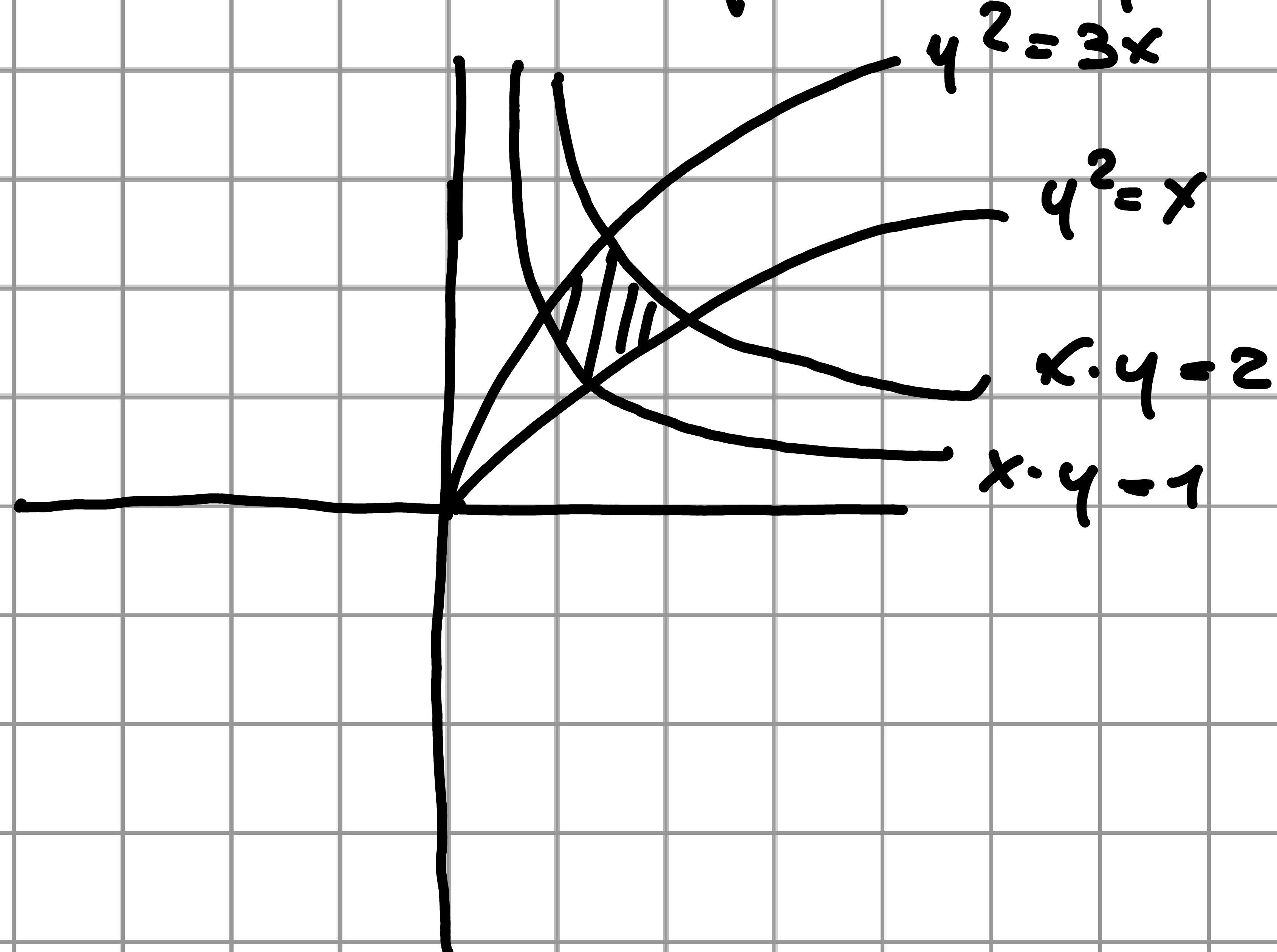
Hiperbola

$$x^2 = 2py$$



$$\iint_D \left(\frac{2y^4}{x^2} + \frac{3y^2}{x} \right) \cdot e^{x \cdot y} \, dD$$

po obszare $y^2 = 3x$, $y^2 = x$, $x \cdot y = 1$, $x \cdot y = 2$



Walec $x^2+y^2=a^2$ oraz $\frac{y}{a}+\frac{z}{c}=1$

$x = r\cos(\theta)$

$y = r\sin(\theta)$

$z = r$

$j = r$

$\int\int\int (r\cos(\theta), r\sin(\theta), z) r \, dz dr d\theta$

$\int$