Zadanie 1.

a) $\binom{52}{4}$

b) $52 \cdot 51 \cdot 50 \cdot 49$

c) $\binom{52+4-1}{4}$

d) 52^4

e) $52 \cdot 51^3$

6

$\frac{36}{2}$

$\frac{6^n}{n}$

Zadanie 2.

a) $8!$

b) $\frac{10}{2!\,2!\,3!}$

$$\binom{n+k-1}{k} = \binom{n+k-1}{n-1}$$

Zadanie 3.

a) 6^n

b) $\binom{6+n-1}{n-1}$

(5) $\binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$

$$\frac{n!}{k!\,(n-k)!} = \frac{(n-1)!}{k!\,(n-1-k)!} + \frac{(n-1)!}{(k-1)!\,(n-k-2)!} \quad \Big| \cdot k!$$

$$\frac{n!}{(n-k)!} = \frac{(n-1)!}{(n-k-1)!} + \frac{(n-1)k}{(n-k-2)!} \quad \Big| \cdot (n-k)!$$

$$n! = (n-1)!\,(n-k) + (n-1)!\,(n-k)^2 k$$

(7) $\binom{6}{n} \cdot \frac{6!}{2}$

$$3 \cdot \binom{4}{3} \cdot \binom{7}{3}$$

9

$$\binom{n+8}{8}\binom{k+n-1}{n-1}$$

11 $\binom{k+n-1}{n-1}$ -

12 $2^2 3^3 4^4 5^5$

$11 \cdot 4 \cdot 6$

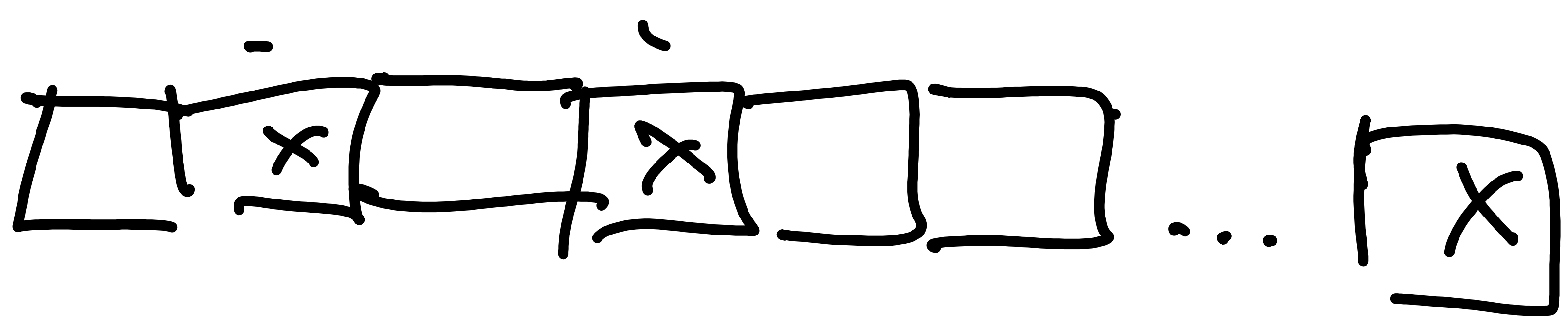
$2 \cdot 3 \cdot 7 \cdot 11 \cdot 17$

13 $\binom{13}{3}\binom{4}{1}\binom{13}{1}^3$

$\binom{13}{2}^2\binom{4}{2}\binom{13}{2}^2$

~~14~~

15



$$\binom{n-k\cdot 2-1}{k\cdot 2-1}$$

17

5^4 $\quad 6\cdot 7^3$

22

wróblczki ❗
$A = \{20, 21, \dots, 79\}$ ① występuje 3?

$D = 3X$ $B = D \cup E$ $|D| = 10$
$E = X3$ $|E| = 6$
$|D \cup E| = 1$

$|B| = |D| + |E| - |D \cup E| = 10 + 6 - 1$ - Prawo sumy

② Nie ma 3 $A = \{20, 21 \dots 79\}$

$\{2, 4, 5, 6, 7,\} \times \{0, 1, 2, 4, 5, 6, 7, 8, 9\}$

$|P| = 5 \cdot 9 = 45$ - Prawo iloczynu

③ Prawo różnicy $|F| = |A / B| = |A| - |A - B|$

ponieważ $|A \cap B| = |B|$, $|F| = |A| - |B| = 60 - 45 = 15$

5

$A = 10$ $\qquad$ $|A \vee B \vee C| = 20$

$B = A \wedge C$

$C = A \wedge B$

$B \wedge C = 7$

Zasada włączeń:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \wedge B| - |A \wedge C| - |B \wedge C| + |A \wedge B \wedge C|$$

Przekształcenie:

$$|A \wedge B \wedge C| = |A \wedge B| + |A \wedge C| + |B \wedge C| - |A| - |B| - |C| + |A \vee B \vee C| =$$

$$20 - 10 + 7 = 17$$

6 $\{5000, 5002, 5004, \ldots, 7998\}$

wszystkie: $|\{5,6,7\}| \cdot |\{0,1,2,3,4,5,6,7,8,9\}|^2 \cdot |\{0,2,4,6,8\}|$

$= 1500$

Bez 5,6,3 $\quad |\{7\}| \cdot |\{0,1,2,4,7,8,9\}|^2 \cdot |\{0,2,4,8\}| = 196$

$1500 - 196 = 1304$

(7) 20 węzłów, 5 drogami

węzły do 3 dróg

$$\frac{\sum_{i=1}^{5} |W_i|}{5} = \frac{3 \cdot |V|}{5} = \frac{3 \cdot 20}{5} = 12$$

(8) 25, 2 zespołach

3 zespoły

50

$$\frac{\sum_{k=1}^{3} |C_k|}{3} \geq \frac{2 \cdot |KZC|}{3} ? \quad 2 \cdot |KZC| = 50$$

(9) $$\left| \prod_{v \in U} V_v \right| = \prod |V_v|$$

(1+) 451 studentów

225 zdało MD = A

331 tgs = B

100% zdało coś

$225 + 331 = 556 = 105$

(2+) 5 zasobów

4 przydziałowie zasobów

$5^4 - 4^4$

II

① 12 osób — 7 inf, 5 grat
6 zespół — 2 inf, 1 grat

$$\binom{5}{4}\cdot\binom{7}{2}+\binom{5}{3}\cdot\binom{7}{3}+\binom{5}{2}\cdot\binom{7}{4}+\binom{5}{1}\cdot\binom{7}{5}$$

$$=910$$

~~②~~

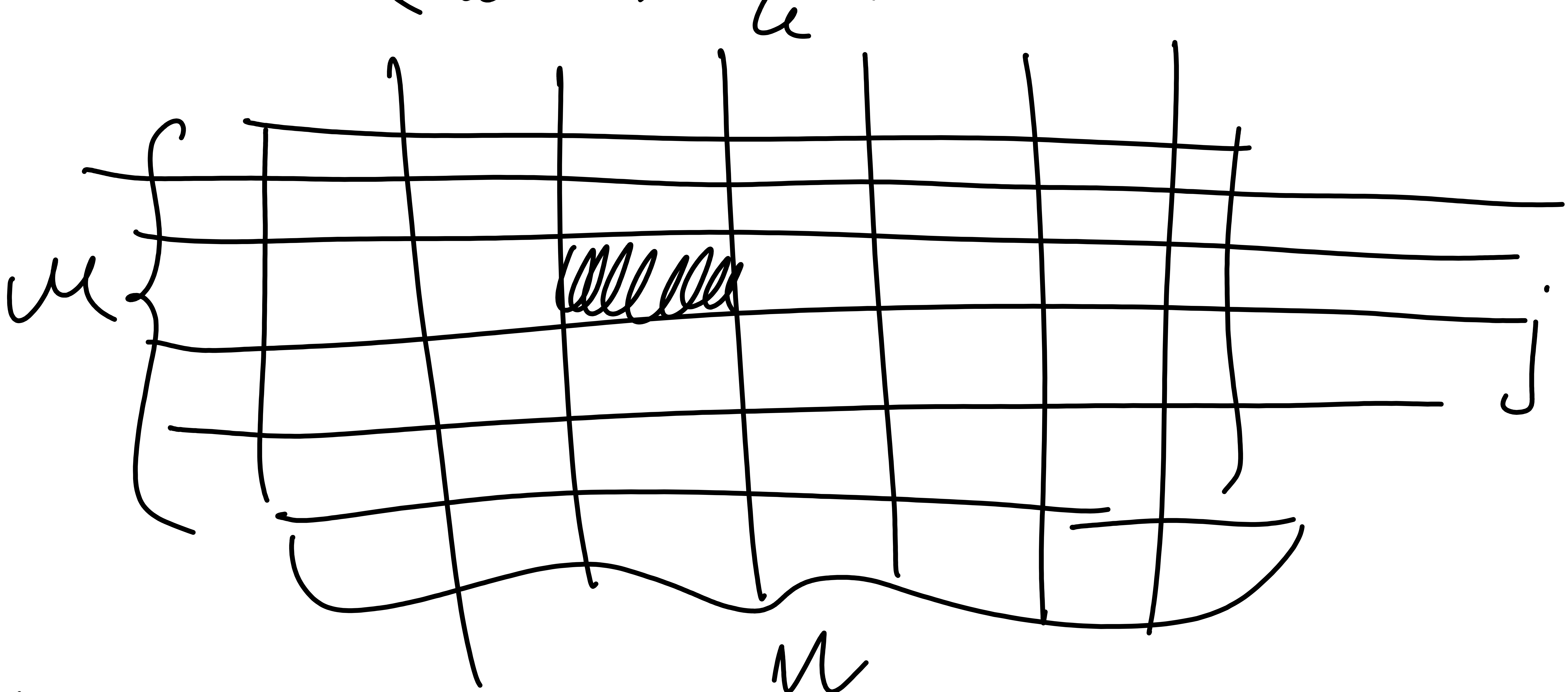
③ $x^2\cdot a^6$ $\quad (x^\alpha+a^\beta)^n=\sum_{k=0}^{n}\binom{n}{k}\cdot x^k\cdot a^{\beta(n-k)}$

$\alpha=\beta=1$

$$(x+a)^n=\sum_{k=0}^{n}\binom{n}{k}\cdot x^k\cdot a^{(n-k)}$$

~~4~~ (6

$$\binom{k+j-2}{k-1} \cdot 2 \cdot \binom{(m-k)+(n-j)}{m-k}$$



9

7 –

3 –

2 –

$3^7 - 3^6$

3 Oblicz $\left[{7 \atop 3}\right]$ $\qquad \left[{n-1 \atop k-1}\right] + (n-1)\left[{n-1 \atop k}\right]$

$$\left[{6 \atop 2}\right] + 6\left[{6 \atop 3}\right] = \left(\overset{1}{\left[{5 \atop 1}\right]} + 5\left[{5 \atop 2}\right]\right) + 6\left(\left[{5 \atop 2}\right] + 5\left[{5 \atop 3}\right]\right) =$$

$$24 + 5\cdot\left(\left[{4 \atop 1}\right] + 4\left[{4 \atop 2}\right]\right) = 24 + 30 + 20\left(\left[{3 \atop 1}\right] + 3\left[{3 \atop 2}\right]\right) =$$

$$= 54 + 80 + 60\left(\left[{2 \atop 1}\right] + 2\left[{2 \atop 2}\right]\right) = 54 + 80 + 60 + 120$$

4 $\left\{{7 \atop 3}\right\}$ $\qquad \left\{{n \atop k}\right\} = \left\{{n-1 \atop k-1}\right\} \cdot k\left\{{n-1 \atop k}\right\}$

$$\left\{{7 \atop 3}\right\} = \left\{{6 \atop 2}\right\} + 3\left\{{6 \atop 3}\right\} = \left(\left\{{5 \atop 1}\right\} + 2\left\{{5 \atop 2}\right\}\right) + 3\left(\left\{{5 \atop 2}\right\} + 3\left\{{5 \atop 3}\right\}\right) =$$

$$= 1 + 2\left(\left\{{5 \atop 1}\right\} + 1\cdot\left\{{4 \atop 2}\right\}\right) \ldots$$

Liczby specjalne

Ile wynosi liczba podziałów zbioru n-elementowego na k podziałów.

Liczba Stirlinga II rodzaju -

$$\left\{ n \atop k \right\} = \begin{cases} 0 & \text{dla } k=0 \lor k>n \\ \left\{ n-1 \atop k-1 \right\} + k \cdot \left\{ n-1 \atop k \right\} & \text{dla } 0<k<n \end{cases}$$

$$\{a,b,c\} = \{a,b\} \cup \{c\} = \{a,c\},\{b\} = \{b,c=a\} = \{b,c\} \cup \{a\}$$

$$\left\{ n \atop n-1 \right\} = \binom{n}{2} ; \left\{ n \atop n \right\} \ 1 , \left\{ n \atop 1 \right\} = 1$$

$$\left\{ 5 \atop 3 \right\} = \left\{ 4 \atop 2 \right\} + 3 \cdot \left\{ 4 \atop 3 \right\} = \left\{ 3 \atop 1 \right\} + 2\left\{ 3 \atop 2 \right\} + 3\binom{4}{2} = 1 + 2\cdot 3 + 3 \cdot 6 = 25$$

$$\left\{ n \atop 2 \right\} = \frac{2^n - 2}{2} = 2^{n-1} - 1$$

Liczba Stirlinga I rodzaju - dzielimy na cykle

$$\left[n \atop k \right] = \begin{cases} 0 & \text{dla } k=0 \lor k>n \\ \left[n-1 \atop k-1 \right] + (n-1) \left[n-1 \atop k \right] & \text{dla } 0<k<n \end{cases}$$

$1,2,3$ ilość możliwych cykli $\frac{n!}{n} = (n-1)!$

ilość cykli po dodaniu 1 elementu w konkretnym miejscu:

$(a,b)(c)$

$(n-1)$

$$\left[n \atop 1 \right] = (n-1)!$$

$$\left[n \atop n \right] = 1$$

Praca domowa: Zrobić zestawienie liczb specjalnych + Euler

rekurencja, przykłady, interpretacja

5 7 na x grup

$$\left\{{7 \atop 1}\right\} + \left\{{7 \atop 2}\right\} + \left\{{7 \atop 3}\right\} + \left\{{7 \atop 4}\right\} + \left\{{7 \atop 5}\right\} + \left\{{7 \atop 6}\right\} + \left\{{7 \atop 7}\right\}$$