

Zadanie 1.

a)  $\binom{52}{4}$

b)  $52 \cdot 51 \cdot 50 \cdot 49$

c)  $\binom{52+4-1}{4}$

d)  $52^4$

e)  $52 \cdot 51^3$

$$\frac{36}{2}^6$$

$$\frac{6^n}{n}$$

Zadanie 2.

a)  $8!$

$\binom{n+k-1}{k} = \binom{n+k-1}{n-1}$

b)  $10$

c)  $2! \cdot 2! \cdot 3!$

Zadanie 3.

a)  $6^n$

b)  $\binom{6+n-1}{n-1}$

$$\textcircled{5} \quad \binom{n}{k} = \binom{n-1}{k} + \binom{n-1}{k-1}$$

$$\frac{n!}{k!(n-k)!} = \frac{(n-1)!}{k!(n-1-k)!} + \frac{(n-1)!}{(k-1)!(n-k-2)!} \cdot k!$$

$$\frac{n!}{(n-k)!} = \frac{(n-1)!}{(n-k-1)!} + \frac{(n-1)^k}{(n-k-2)!} \cdot (n-k)!$$

$$n! = (n-1)! (n-k) + (n-1)! (n-k)^2 k$$

$$\textcircled{7} \quad \binom{6}{3} \cdot \frac{6!}{2^3}$$

$$3 \cdot \binom{4}{3} \cdot \binom{7}{3}$$

(9)

$$\binom{n+8}{8} \binom{k+n-1}{n-1}$$

(11)

$$\binom{k+n-1}{n-1} -$$

(12)

$$2^2 3^3 4^5 5^5$$

5

$11 \cdot 4 \cdot 6$

$2 \cdot 3 \cdot 7 \cdot 11 \cdot 17$

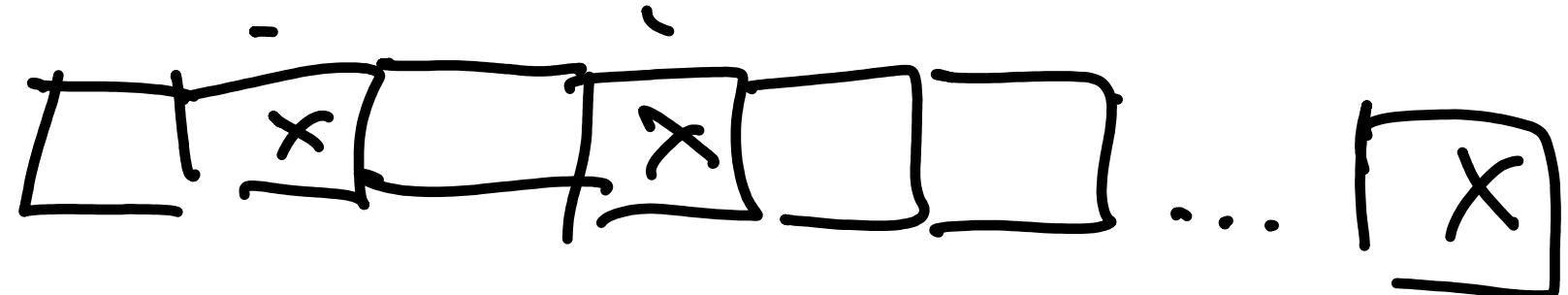
(13)

$$\binom{13}{3} \binom{4}{1} \left(\binom{13}{1}\right)^3$$

$$\left(\binom{13}{2}\right)^2 \binom{4}{2} \left(\binom{13}{2}\right)^2$$

~~(14)~~

15



$$\begin{pmatrix} n - k \cdot 2 - 1 \\ k \cdot 2 - 1 \end{pmatrix}$$

17

$$5^4 \quad 6 \cdot 7^3$$

22

wróćeli?

$$A = \{20, 21, \dots, 73\} \quad \textcircled{1} \quad \text{wykazuje } 3?$$

$$D = 3x$$

$$E = x3$$

$$B = D \cup E$$

$$|D| = 10$$

$$|E| = 6$$

$$|D \cup E| = 1$$

$$|B| = |D| + |E| - |D \cup E| = 10 + 6 - 1 - \begin{matrix} \text{Prawo} \\ \text{Sume} \end{matrix}$$

$$\textcircled{2} \quad \text{Nie ma } 3 \quad A = \{20, 21, \dots, 73\}$$

$$\{2, 4, 5, 6, 7\} \times \{0, 1, 2, 4, 5, 6, 7, 8, 9\}$$

$$|P| = 5 \cdot 9 = 45 - \text{Prawo iloczynu}$$

$$\textcircled{3} \quad \text{Prawo różnicę } |F| = |A / B| = |A| - |A - B|$$

$$\text{ponieważ } |A \cap B| = |B|, |F| = |A| - |B| = 60 - 45 = 15$$

⑤

$$A = 10$$

$$|A \vee B \vee C| = 20$$

$$B = A \wedge C$$

$$C = A \wedge B$$

$$B \wedge C = F$$

Zasada rozłączna:

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

Pniesztatczanie:

$$|A \cap B \cap C| = |A \cap B| + |A \cap C| + |B \cap C| - |A \cap B \cap C|$$

$$|A \vee B \vee C| =$$

$$20 - 10 + 7 = 17$$

⑥  $\{5000, 5002, 5004, \dots, 7998\}$

wszystkie:  $|\{5, 6, 7\}| \cdot |\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}|^2 \cdot |\{0, 2, 4, 6, 8\}|$   
 $= 1500$

Bez 5, 6, 7  $|\{7\}| \cdot |\{0, 1, 2, 4, 7, 8, 9\}|^2 \cdot |\{0, 2, 4, 8\}| = 136$

$$1500 - 136 = 1304$$

⑦ 20 cożków, 5 drogami  
każdy do 3 dróg

$$\frac{\sum_{i=1}^5 |W_i|}{5} = \frac{3 \cdot |V|}{5} = \frac{3 \cdot 20}{5} = 12$$

⑧ 25, 2 zespoły  
3 zespoły

$$\frac{\sum_{i=1}^3 |C_{kl}|}{3} \geq \frac{2 \cdot |K2C|}{3}, \quad 2 \cdot |K2C| = 50$$

$$⑨ \left| \bigcap_{v \in V_0} V_0 \right| = \prod |V_0|$$

1+

451 studentów

225 zdążyło MO = A

331 fgs = B

100% zdążyć 005

$$225 + 331 = 556 = 105$$

2+

5 zasobów

4 przydzielone zasoby

$$5^4 - 4^4$$

II

①

12 osók

7 int

5 grat

6 respol

2 int

1 grat

$$\binom{5}{4} \cdot \binom{7}{2} + \binom{5}{3} \cdot \binom{7}{3} + \binom{5}{2} \cdot \binom{7}{4} + \binom{5}{1} \cdot \binom{7}{5}$$

$$= 910$$

~~2~~

$$③ x^2 \cdot a^6 \quad (x^\alpha + a^\beta)^n = \sum_{k=0}^n \binom{n}{k} \cdot x^k \cdot a^{\beta(n-k)}$$

$$\alpha = \beta = 1$$

$$(x+a)^n = \sum_{k=0}^n \binom{n}{k} \cdot x^k \cdot a^{n-k}$$

$$\cancel{1} \quad \cancel{6} \quad \binom{k+j-2}{u-1} \cdot \frac{2}{u} \cdot \binom{(m-k)+(n-j)}{m-k}$$

③

7 -

3 -

2 -

$3^7 - 3^6$

$$\textcircled{2} \quad 06 \text{ Wcz } \begin{Bmatrix} 7 \\ 3 \end{Bmatrix} \quad \begin{Bmatrix} n-1 \\ u-1 \end{Bmatrix} + (n-1) \begin{Bmatrix} n-1 \\ u \end{Bmatrix}$$

$$\begin{Bmatrix} 6 \\ 2 \end{Bmatrix} + 6 \begin{Bmatrix} 6 \\ 3 \end{Bmatrix} = \overset{\textcircled{1}}{\left( \begin{Bmatrix} 5 \\ 1 \end{Bmatrix} + 5 \begin{Bmatrix} 5 \\ 2 \end{Bmatrix} \right)} + 6 \left( \begin{Bmatrix} 5 \\ 2 \end{Bmatrix} + 5 \begin{Bmatrix} 5 \\ 3 \end{Bmatrix} \right) =$$

$$24 + 5 \cdot \left( \begin{Bmatrix} 4 \\ 1 \end{Bmatrix} + 4 \begin{Bmatrix} 4 \\ 2 \end{Bmatrix} \right) = 24 + 30 + 20 \left( \begin{Bmatrix} 3 \\ 1 \end{Bmatrix} + 3 \begin{Bmatrix} 3 \\ 2 \end{Bmatrix} \right) =$$

$$= 54 + 80 + 60 \left( \begin{Bmatrix} 2 \\ 1 \end{Bmatrix} + 2 \begin{Bmatrix} 2 \\ 2 \end{Bmatrix} \right) = 54 + 80 + 60 + 120$$

$$\textcircled{3} \quad \begin{Bmatrix} 7 \\ 3 \end{Bmatrix} \quad \begin{Bmatrix} n \\ u \end{Bmatrix} = \begin{Bmatrix} n-1 \\ u-1 \end{Bmatrix} \cdot 6 \begin{Bmatrix} n-1 \\ u \end{Bmatrix}$$

$$\begin{Bmatrix} 7 \\ 3 \end{Bmatrix} = \begin{Bmatrix} 6 \\ 2 \end{Bmatrix} + 3 \begin{Bmatrix} 6 \\ 3 \end{Bmatrix} = \left( \begin{Bmatrix} 5 \\ 1 \end{Bmatrix} + 2 \begin{Bmatrix} 5 \\ 2 \end{Bmatrix} \right) + 3 \left( \begin{Bmatrix} 5 \\ 2 \end{Bmatrix} + 3 \begin{Bmatrix} 5 \\ 3 \end{Bmatrix} \right) =$$

$$= 1 + 2 \left( \begin{Bmatrix} 5 \\ 1 \end{Bmatrix} + 1 \cdot \begin{Bmatrix} 4 \\ 2 \end{Bmatrix} \right) \dots$$

# Liczby specjalne

Jle wynosi liczba podziałów zbioru  $n$ -elementowego na  $k$  podziałów.

Liczba Stirlinga II rodzaju -

$$\left\{ \begin{matrix} n \\ k \end{matrix} \right\} = \begin{cases} 0 & \text{dla } k=0 \vee k>n \\ \left\{ \begin{matrix} n-1 \\ k-1 \end{matrix} \right\} + k \cdot \left\{ \begin{matrix} n-1 \\ k \end{matrix} \right\} & \text{dla } 0 < k < n \end{cases}$$

$$\{abc\} = \{ab\} \cup \{c\} = \{a,c\}, \{c\} = \{a,c=a\} = \{b,c\} \cup \{c\}$$

$$\left\{ \begin{matrix} n \\ n-1 \end{matrix} \right\} = \binom{n}{2}; \left\{ \begin{matrix} n \\ n \end{matrix} \right\} = 1, \left\{ \begin{matrix} n \\ 1 \end{matrix} \right\} = 1$$

$$\left\{ \begin{matrix} 5 \\ 3 \end{matrix} \right\} = \left\{ \begin{matrix} 4 \\ 2 \end{matrix} \right\} + 3 \cdot \left\{ \begin{matrix} 4 \\ 3 \end{matrix} \right\} = \left\{ \begin{matrix} 3 \\ 1 \end{matrix} \right\} + 2 \left\{ \begin{matrix} 3 \\ 2 \end{matrix} \right\} + 3 \left( \binom{4}{2} \right) = 1 + 2 \cdot 3 + 3 \cdot 6 = 25$$

$$\left\{ \begin{matrix} n \\ 2 \end{matrix} \right\} = \frac{2^n - 2}{2} = 2^{n-1} - 1$$

Liczba Stirlinga I rodzaju - dzielącym na części

$$\left[ \begin{matrix} n \\ k \end{matrix} \right] = \begin{cases} 0 & \text{dla } k=0 \vee k>n \\ \left[ \begin{matrix} n-1 \\ k-1 \end{matrix} \right] + (n-1) \left[ \begin{matrix} n-1 \\ k \end{matrix} \right] & \text{dla } 0 < k < n \end{cases}$$

$\nearrow \lambda_1, \lambda_2, \lambda_3$  ilość możliwych cykli  $\frac{n!}{\lambda_1! \lambda_2! \lambda_3!} = (n-1)!$

ilość cykli po dodaniu 1 elementu w konkretnym miejscu:

$$\begin{array}{l} \xrightarrow{\text{Ca, G)(C)}} \left[ \begin{matrix} n \\ 1 \end{matrix} \right] = (n-1)! \\ (n-1) \quad \left[ \begin{matrix} n \\ n \end{matrix} \right] = 1 \end{array}$$

Praca domowa: Zrobić zestawienie liczb specjalnych + euler rekurencja, parzysta interpretacja

5. F na x grup

$$\{ \frac{7}{1} \} + \{ \frac{7}{2} \} + \{ \frac{7}{3} \} + \{ \frac{7}{4} \} + \{ \frac{7}{5} \} + \{ \frac{7}{6} \} + \{ \frac{7}{7} \}$$