

Flyea

PPF

Zoania

# ZADANIA ĆW 2

## zadanie 1

$v_u = 108 \text{ km/h}$  - predkość końcowa

$$t = 8s$$

z jakim przyspieszeniem jechał?

$$a \approx \frac{dv}{dt} \Rightarrow v = at + v_0$$

$$v = \frac{ds}{dt} \Rightarrow ds = v dt \Rightarrow ds = (at + v_0) dt \Rightarrow$$

$$s = \frac{1}{2}at^2 + v_0 t + s_0$$

$$v_0 = 0, s_0 = 0 \Rightarrow s = \frac{1}{2}at^2 \Rightarrow s = 32a$$

$$v_u = 30 \frac{\text{m}}{\text{s}}$$

$$v_u = at \Rightarrow \frac{30}{8} = 3,75 \frac{\text{m}}{\text{s}^2}$$

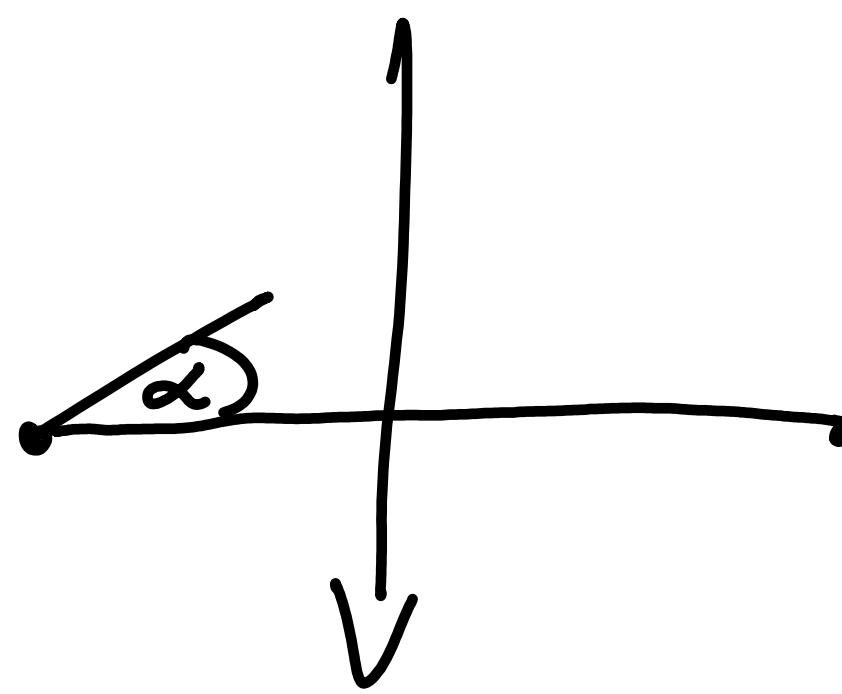
$$s = 120 \text{ m}$$

## Zadanie 2

$$d = 100 \text{ m.}$$

$$v_1 = 10 \text{ km/h}$$

$$v_2 = 6 \text{ km/h}$$



$$v_{\text{wyn}} = v_1 + v_2$$

$$|v_{\text{wyn}}| = |v_1| \cos(\alpha) = \sqrt{v_1^2 + v_2^2} = 8 \text{ km/h} = 2.22 \frac{\text{m}}{\text{s}}$$

$$\cos \alpha = \frac{\sqrt{v_1^2 + v_2^2}}{v_1} = 36^\circ$$

$$t = \frac{d}{v_{\text{wyn}}} = \frac{d}{\sqrt{v_1^2 + v_2^2}} \approx 15 \text{ s}$$

### Zadanie 3

$\omega_0$  - 300 obrótów na minutę

Po wyjściu - 75 obrótów.

$$\omega = \omega_0 - \epsilon t ; \alpha = \omega_0 t - \frac{\epsilon \epsilon^2}{2}$$

$$\omega_0 = \frac{300 \cdot 2\pi}{60} = 30\pi \frac{\text{rad}}{\text{s}}$$

$$\omega_k = 0$$

$$\alpha = 75 \text{ obr} = 150\pi \frac{\text{rad}}{\text{s}^2}$$

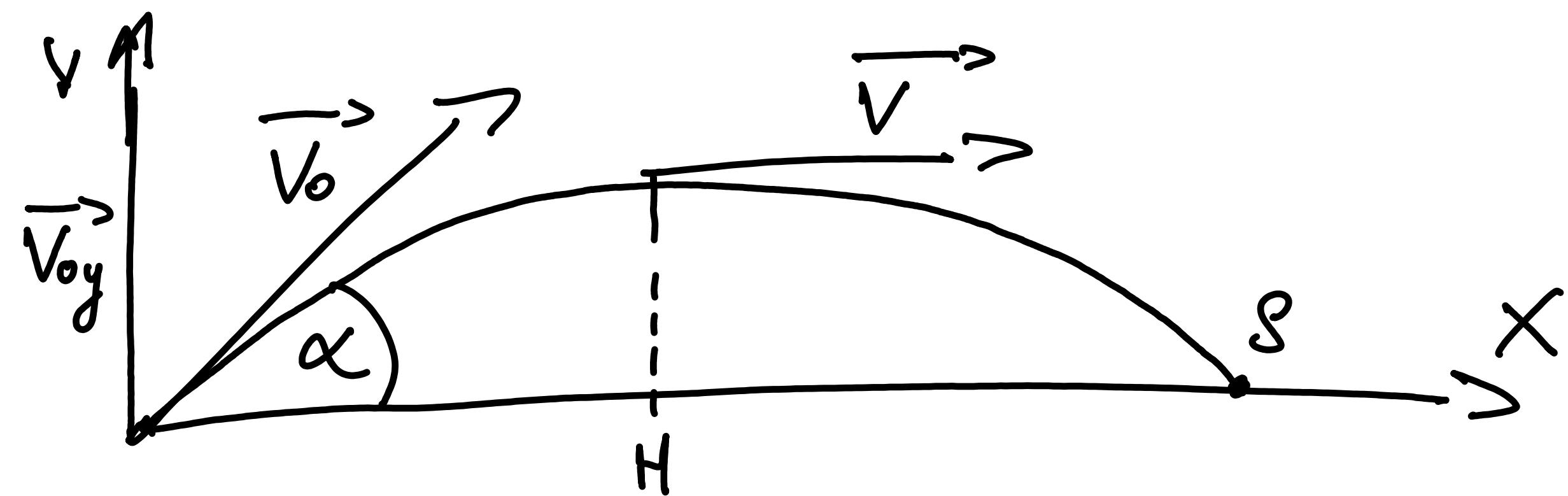
$$\epsilon = \frac{\omega_0}{t}$$

$$\alpha = \omega_0 t - \cancel{\frac{\omega_0}{t} \cdot \frac{t^2}{2}} = \frac{\omega_0 t}{2} \Rightarrow t = \frac{2\alpha}{\omega_0} = \\ = \frac{300}{30} = 10 \text{ s}$$

Zadanie 4.

Pociąk wystrzelono z  $v_0$  pod kątem  $\alpha$ .

Znaleźć czas  $S$ , na który przeszedł toru  $H$  oraz czas lotu  $t$ .



$$V_x = V_{0x} = V_0 \cos(\alpha)$$

$$x = V_{0x} \cdot t = V_0 \cos(\alpha) \cdot t$$

$$V_{0y} = V_0 \sin(\alpha)$$

wyliczamy czas

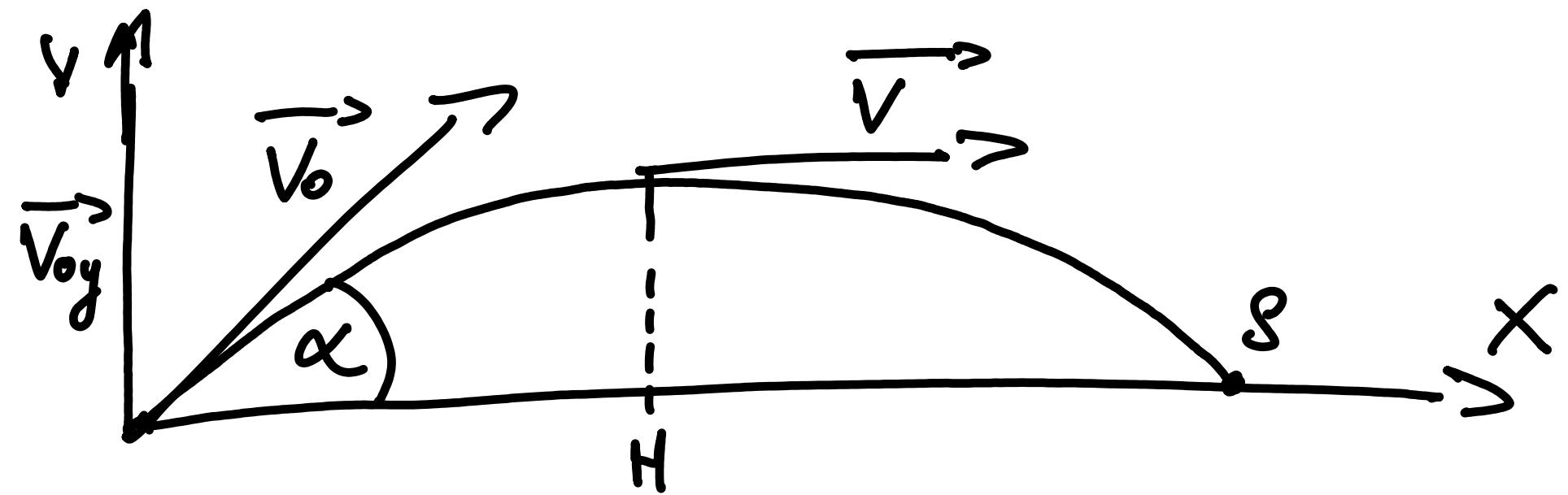
$$V_y = V_{0y} - gt$$

$$y = V_{0y}t - \frac{1}{2}gt^2$$

verte

Pociąk wystrzelono z  $v_0$  pod kątem  $\alpha$ .

Znaleźć radię  $S$ , maksymalne przeniesienie toru  $H$  oraz czas lotu  $t$ .



$$V_{0x} = V_0 \cos(\alpha)$$

$$V_{0y} = V_0 \sin(\alpha)$$

$$V_x = V_{0x}$$

$$V_y = V_{0y} - gt$$

$$\begin{cases} x = V_x t = V_0 \cos(\alpha) t \\ y = V_y t - \frac{1}{2}gt^2 \end{cases}$$

$$t = \frac{x}{\cos(\alpha) V_0} \quad \text{--- wyliczamy z tego } y$$

$$y = \frac{V_0 \sin(\alpha) x}{V_0 \cos(\alpha)} - \frac{g \left( \frac{x}{V_0 \cos \alpha} \right)^2}{2}$$

$$\begin{cases} x = V_0 \cos(\alpha) t \\ y = V_0 \sin(\alpha) t - \frac{gt^2}{2} \end{cases}$$

$$t = \frac{x}{V_0 \cos(\alpha)}$$

$$y = \frac{V_0 \sin(\alpha) x}{V_0 \cos(\alpha)} - \frac{g \left( \frac{x}{V_0 \cos \alpha} \right)^2}{2} = \dots$$

obliczamy  $\delta$ :

$$x + g(\alpha) = \frac{1}{2} \left( \frac{x}{V_0 \cos(\alpha)} \right)^2$$

$$x = \delta = 2 \frac{V_0^2}{g} \sin \alpha \cos \alpha = \frac{V_0^2}{g} \sin(2\alpha)$$

# Ćwiczenia A 2

Zadanie 1.

Prędkość  $v$ .

zawalnia  $- \frac{1}{3}t$

$$a = \frac{dv}{dt} \Rightarrow v = -\frac{1}{3} \int t dt = -\frac{1}{6}t^2 + c$$

$$v = \frac{ds}{dt} \Rightarrow s = -\frac{1}{6} \int (t^2 + c) dt = \\ = -\frac{1}{18}t^3 + ct$$

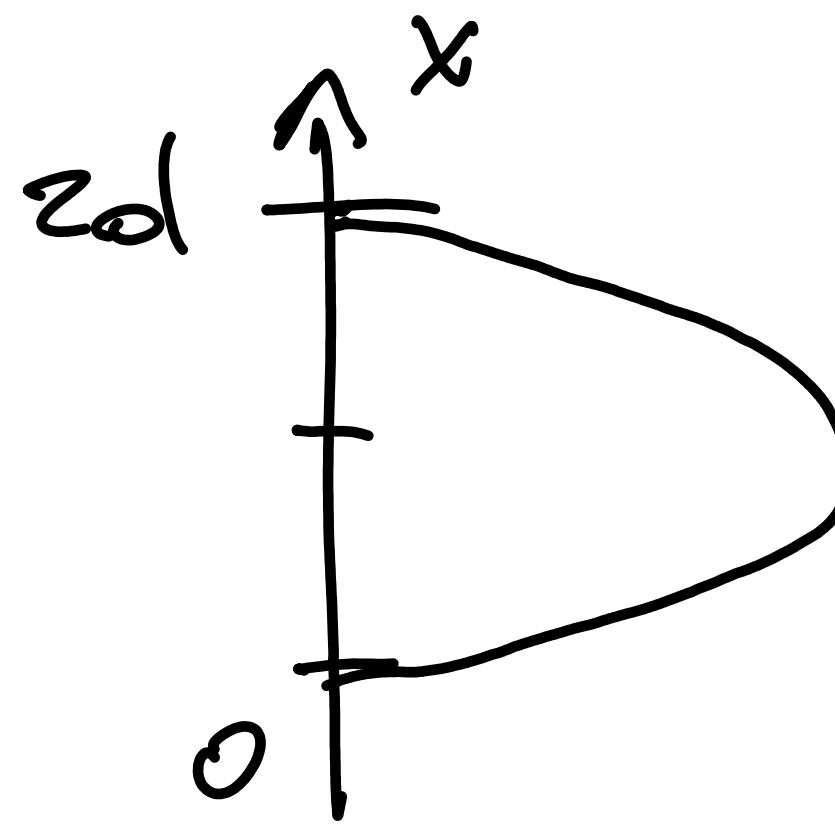
$$t_u = \sqrt[3]{60}$$

$$s = -\frac{1}{18}(\sqrt[3]{60})^3 + 0 \cdot \sqrt[3]{60}$$

$$s = -\frac{1}{18} \cdot 60 \sqrt[3]{60}$$

## ZADANIE 2

$$V(2d) = V(0) = 0$$



$$V(d) = V_1$$

$$V = ax^2 + bx + c$$

$$V(0) = a \cdot 0 + b \cdot 0 + c \Rightarrow c = 0$$

$$V(d) = ad^2 + bd$$

$$V(2d) = 4ad^2 + 2db \Rightarrow b = -2ad$$

$$V_1 = ad^2 - 2ad^2 \Rightarrow a = \frac{-V_1}{d^2}, \quad b = \frac{V_1}{d}$$

$$V_x = \frac{-V_1}{d^2} x^2 + \frac{V_1}{d} x - \text{prędkość wody}$$

$V_L$  - prędkość Códki

$$V(t) = \frac{-V_1}{d^2} (V_L t)^2 + \frac{V_1}{d} (V_L \cdot t)$$

$$y = \int V dt$$

$$y = \frac{1}{3} V_L^2 +$$

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## ZADANIE 4

$$F = ma$$

w tym przypadku:  $F = -kU$

$$-kU = ma$$

$$a = \frac{dU}{dt}$$

$$-\frac{k}{m} U = \frac{dU}{dt} \Rightarrow -\frac{k}{m} dt = \frac{dU}{U} \quad | \int$$

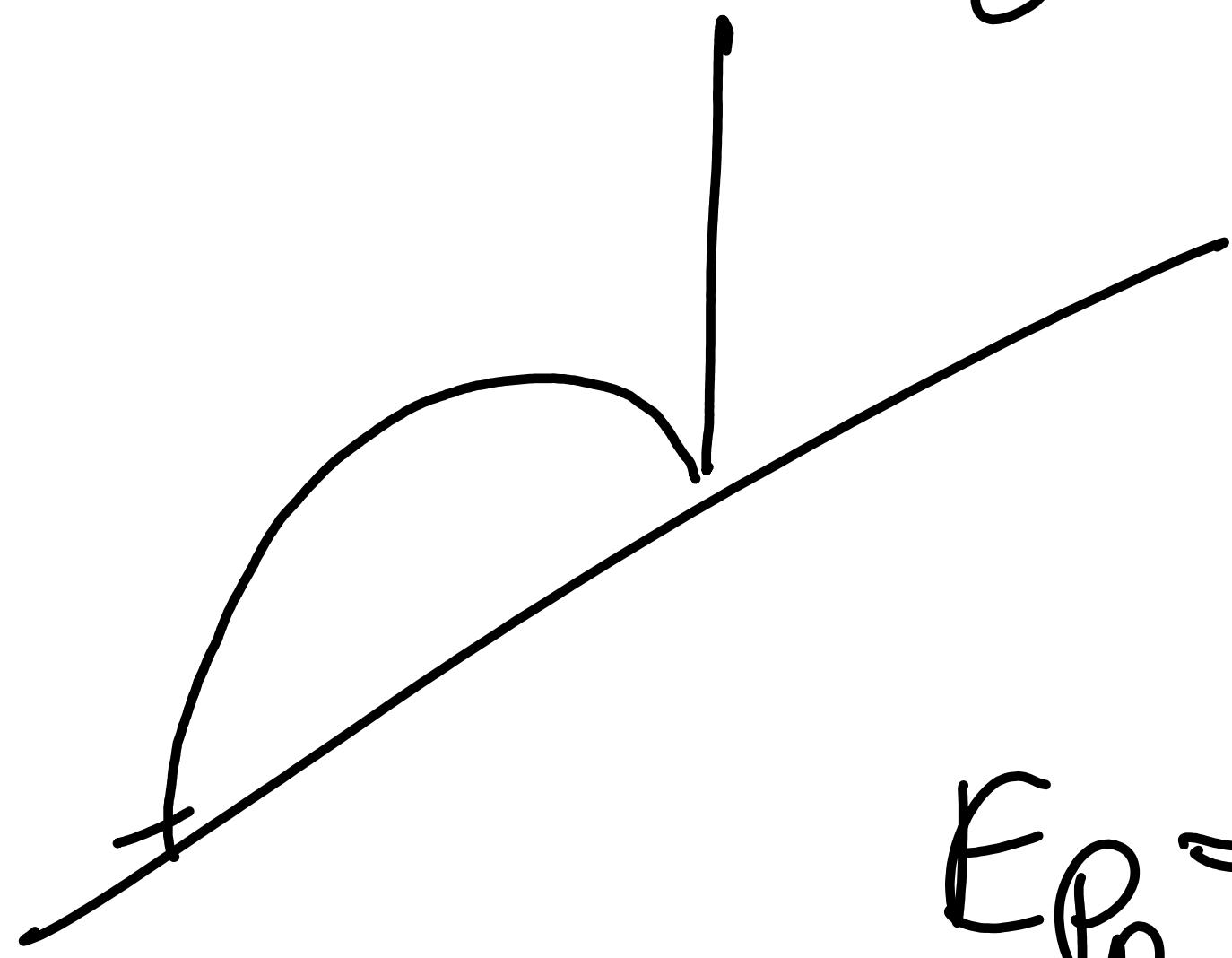
$$-\frac{kt}{m} = \ln U + C \Rightarrow U = Ce^{-\frac{kt}{m}}$$

$$U(t=0) = U_0 = C$$

$$S = \int_0^{\infty} U_0 e^{-\frac{kt}{m}} dt$$

## ZADANIE 5

Wielka spada z H, odbiła się o α  
obliczyć odległość wielki.



$$E_k = \frac{1}{2} m v^2$$

$$E_P = mgh$$

$$E_{P_A} = mgh \quad E_{k_A} = 0$$

$$E_{P_B} = 0 \quad E_{k_B} = \frac{1}{2} m V_B^2$$

$$mgh = \frac{1}{2} m V_B^2$$

$$gh = \frac{1}{2} m V_B^2$$

$$V_B^2 = 2gh$$

$$V_B = \sqrt{2gh}$$

## CWICZENIA 3

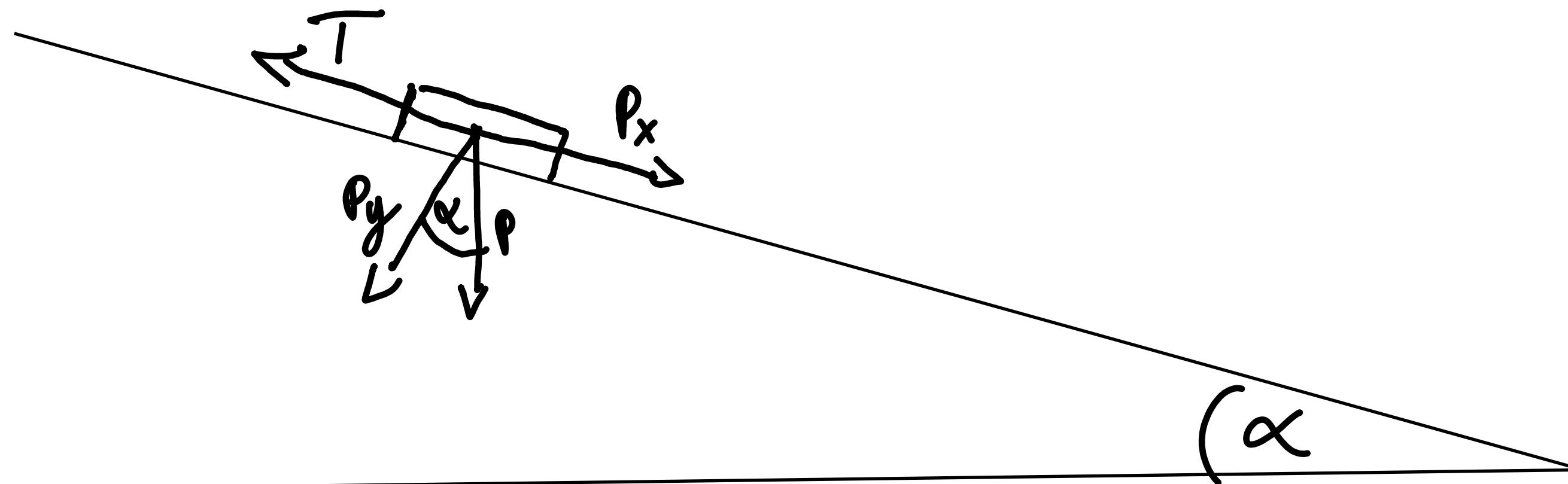
Zadanie 2

$$\begin{aligned} F &= am_c \\ m_c &= m + M \quad \left. \right\} \quad Mg = a(m + M) \\ F &= mg \end{aligned}$$

$$a = \frac{gM}{(m + M)}$$

Zadanie 3 Z jakim przyspieszeniem  
zsuwa się ciasto?

f - siła tarcia



$$F = \rho m \quad P = mg$$

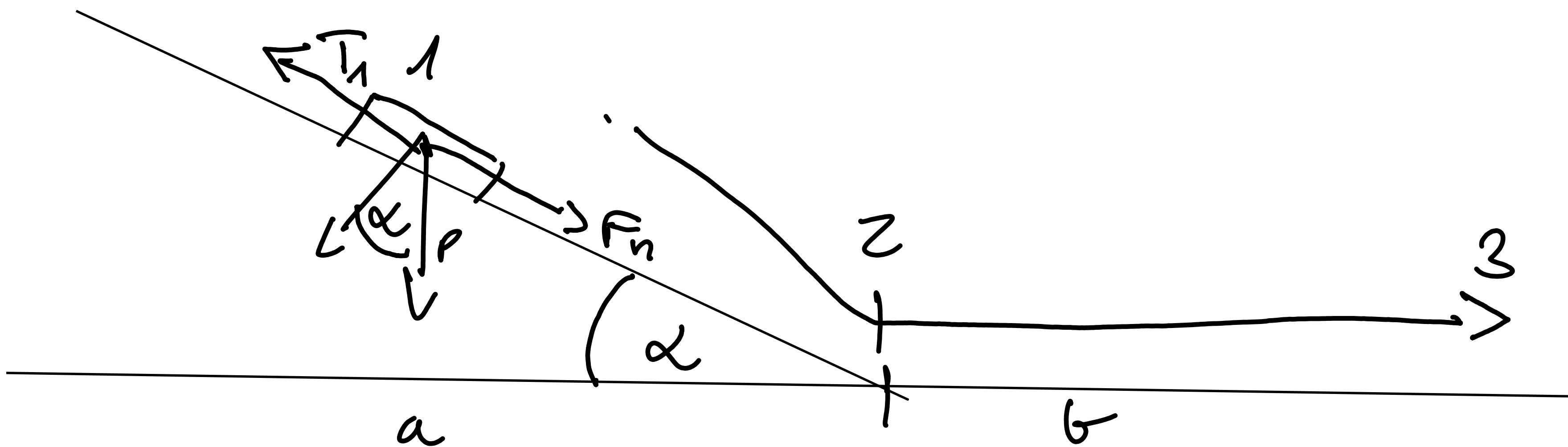
$$P_x = P \sin(\alpha) \quad P_y = P \cos(\alpha)$$

$$T = f \quad P_y = f mg \cos(\alpha)$$

$$F = P_x - T = mg \sin(\alpha) - f mg \cos(\alpha) = mg (\sin(\alpha) - f \cos(\alpha))$$

$$a = \frac{mg (\sin(\alpha) - f \cos(\alpha))}{m} = g (\sin(\alpha) - f \cos(\alpha))$$

Zadanie 4. Oblicz przyspieszenie  
szeru na odcinku planowym.



$$T_1 = \mu N_1 = \mu mg \cos(\alpha) \quad N = mg \cos(\alpha)$$

$$T_2 = \mu P = \mu mg$$

$$E_{P1} = mgh \quad E_{P2} = 0$$

$$E_{k2} = \frac{mv^2}{2} \quad E_{P1} = E_{P2} + E_{k2} + W_{1,2}$$

$$E_{k2} = E_{P1} - W_{1,2} \quad W = \vec{F} \cdot \vec{s}$$

$$E_{k2} = W_{1,3}$$

$$E_{P1} = W_{1,2} + W_{2,3}$$

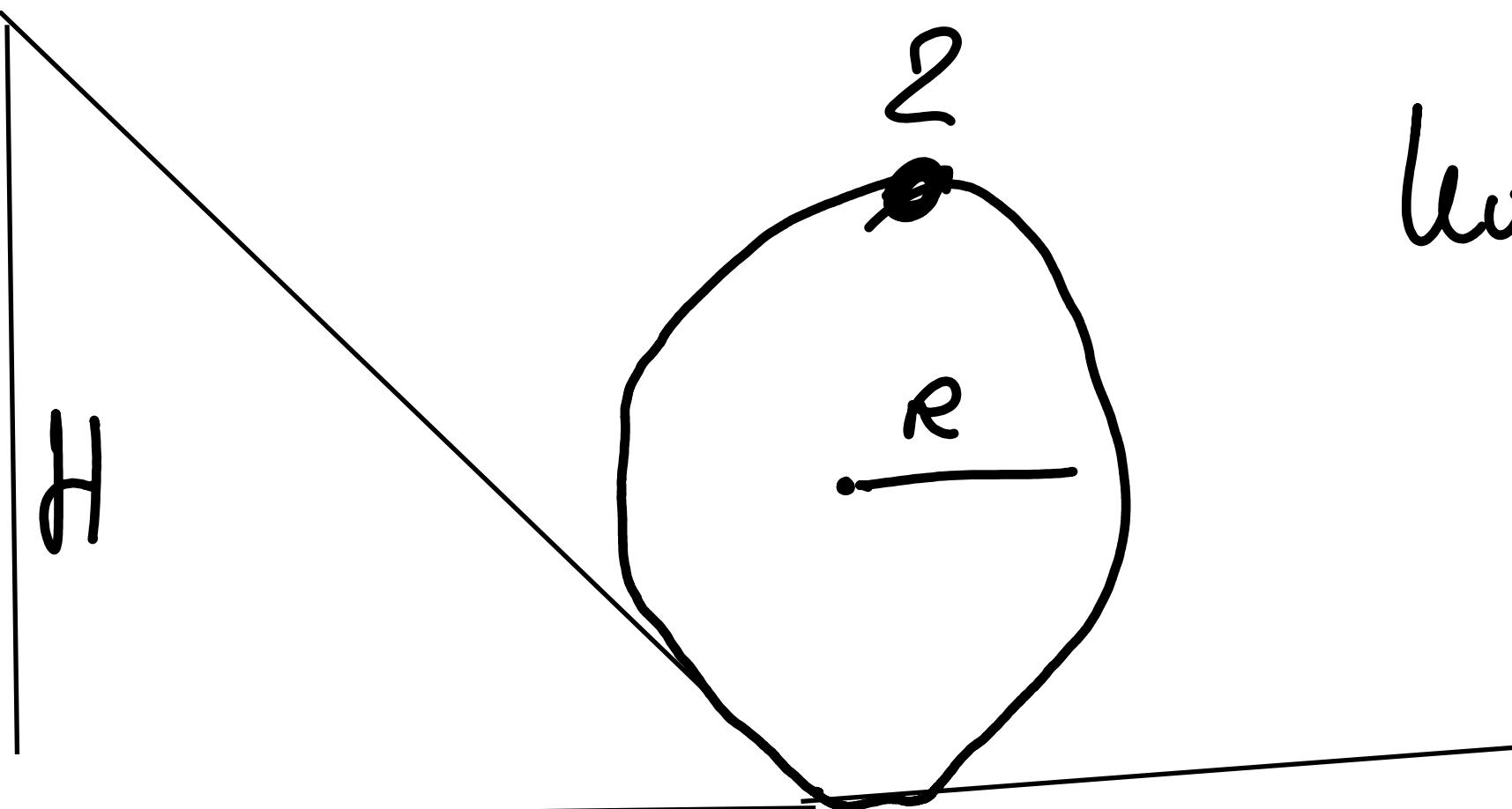
$$W_{1,2} = T_1 \cdot S_{1,2} \quad S_{1,2} = \frac{\mu}{\sin(\alpha)}$$

# Zadanie 5

1

Jakie powinno być  $H$ , aby kulka przebyła pętlę?

Kulka porusza się bez tarcia.



warunek minimum

$$mg = \frac{mv^2}{R} \Rightarrow v = \sqrt{gR} *$$

$$E_{K1} = 0 \quad E_{P1} = mgh$$

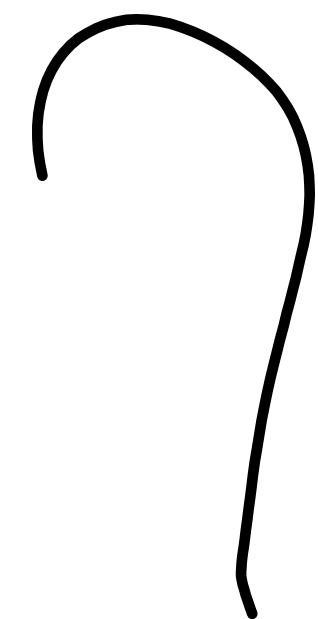
$$E_{K2} = \frac{mv^2}{2} \quad E_p = mg2R$$

$$E_{K1} + E_{P1} = E_{K2} + E_{P2} \Rightarrow mgh = \frac{mv^2}{2} + mg2R$$

$$v^2 = 2g(H - 2R) \Rightarrow gR = 2g(H - 2R)$$

$$R = 2H - 4R \Rightarrow H = 2,5R$$

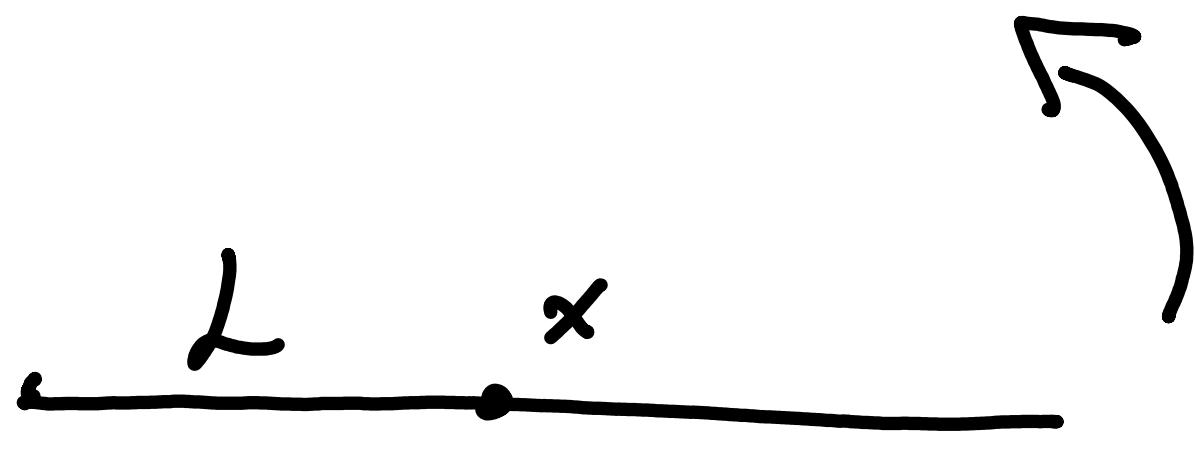
Zadanie 6



6

# ĆWICZENIA 4

Zadanie 1.



$\frac{m}{L}$  - gęstość cieplna

dzielimy na części -  $dm = \left(\frac{m}{L}\right) dx$

$$dI = x^2 dm$$

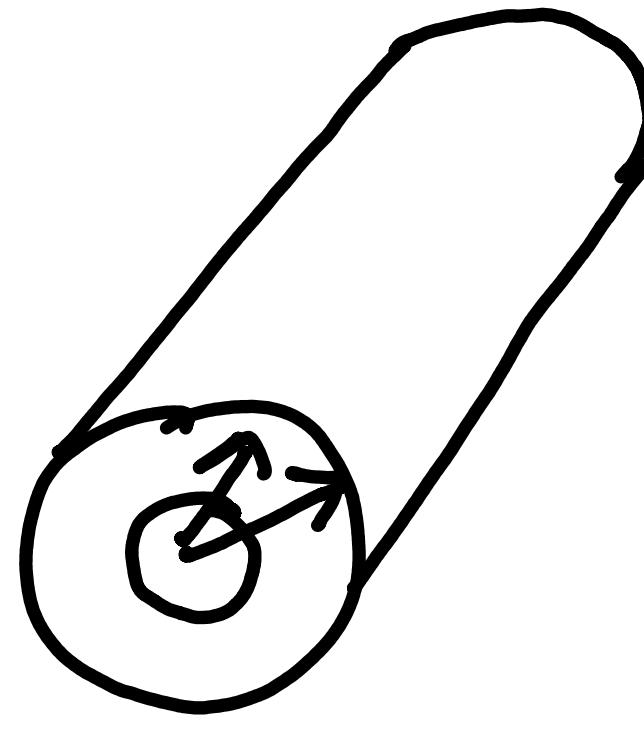
$$dI = x^2 dm = \left(\frac{m}{L}\right) x^2 dx$$

$$I = \int_{-\frac{1}{2}L}^{\frac{1}{2}L} \left(\frac{m}{L}\right) x^2 dx = \frac{ML^2}{12} - \text{dla odcięcia w środku}$$

$$I = \int_0^L \left(\frac{m}{L}\right) x^2 dx = \frac{1}{3}ML^2 - \text{dla odcięcia na końcu}$$

## Zadanie 2. P-gestośc'

$$dm = \rho dV = 2\pi \rho r l dr$$



$$2 \text{ definicj}i: dI = dm \cdot r^2 = 2\pi \rho r^3 l dr$$

$$I = \int_{R_w}^{R_z} 2\pi \rho r^3 l dr = \frac{1}{2} \pi \rho (R_z^4 - R_w^4)$$

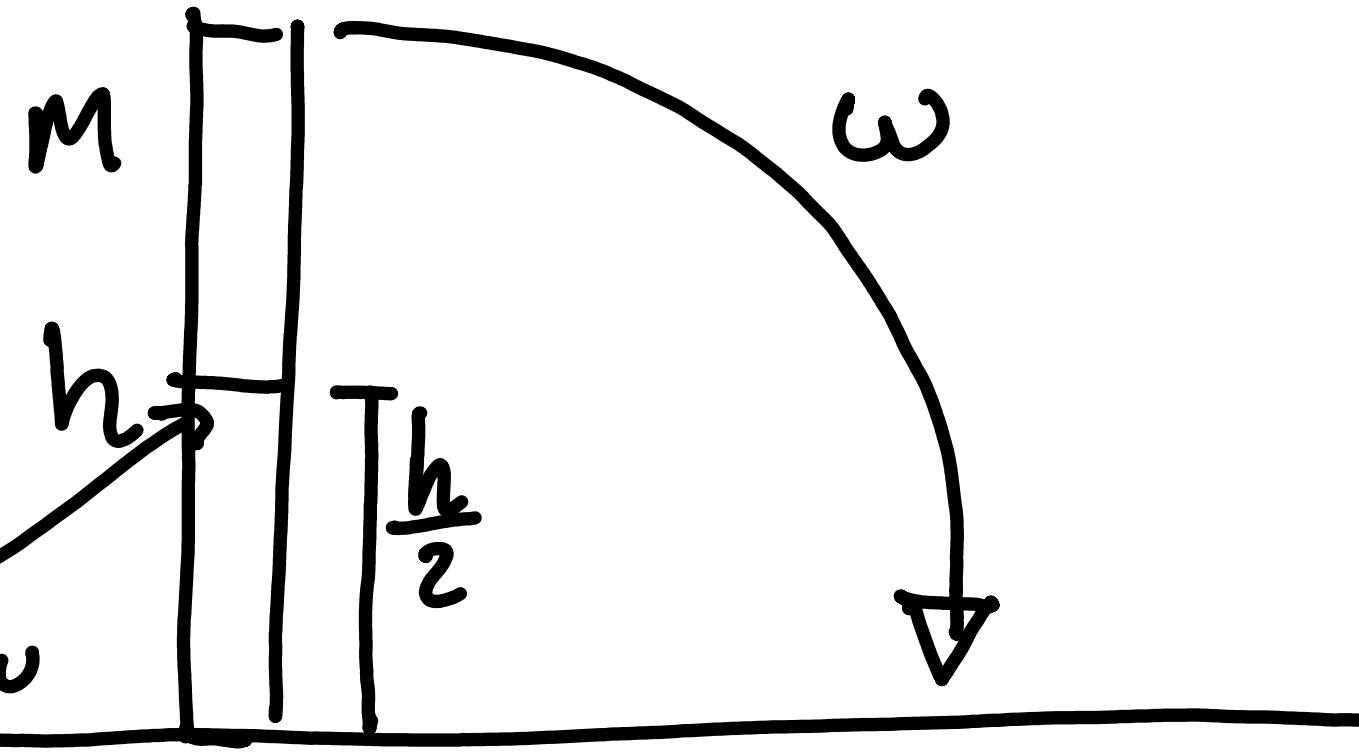
Odejmijemy puste pole:  $M = \pi \rho l (R_z^2 - R_w^2)$

$$I = \frac{1}{2} M (R_z^2 + R_w^2)$$

### Zadanie 3:

Z jaką prędkością

upadnie wierzchołek? osi obrotu



$E_{kin} = \frac{1}{2} \omega^2 I$  - energia kinetyczna w ruchu obrotowym.

$$\omega = \frac{v}{r} \quad \Delta E_{kin} + \Delta E_{pot} = 0$$

$$V = \omega h, \quad I_0 = \frac{m h^2}{12}$$

$$I = I_0 + \frac{m h^2}{4} = \frac{m h^2}{3}$$

$$E_{kin} = \frac{1}{2} \omega^2 I = \frac{m h^2 \omega^2}{6}$$

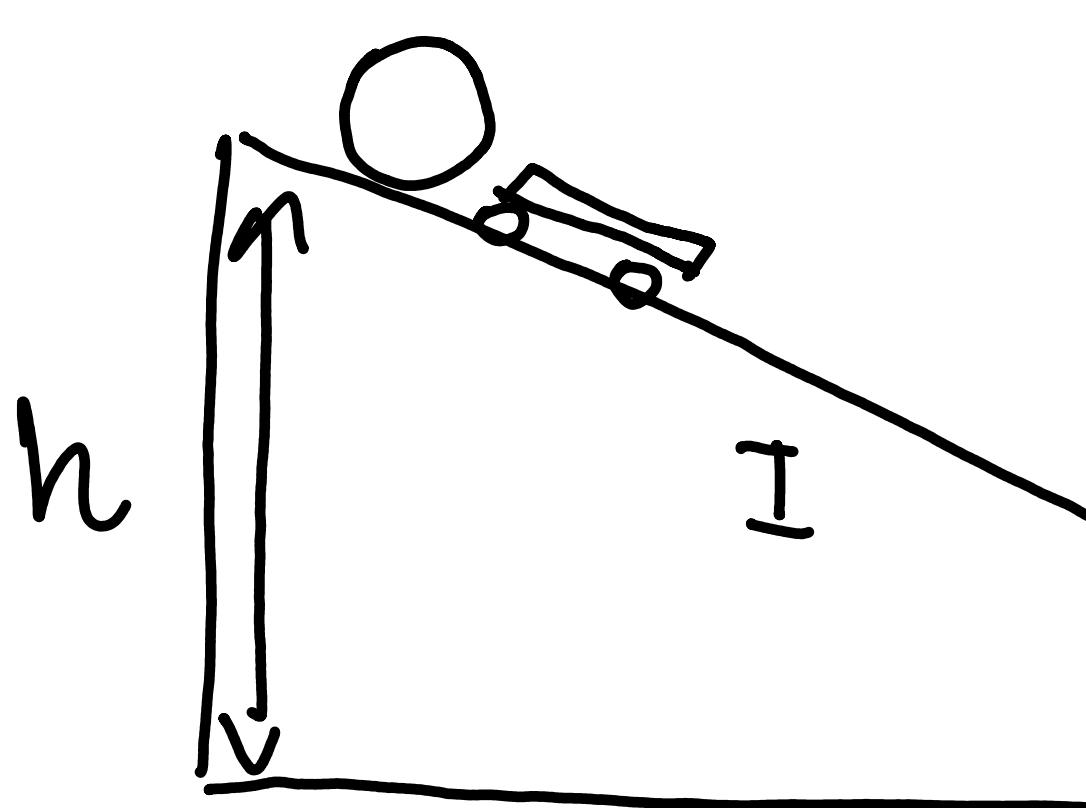
$$\text{Steiner-}I = m d^2 + I_0$$

???

.. .

Zadanie 4.

$$\omega = \frac{v_1}{r} \quad I = mr^2 b$$



$$\Delta E_{kin} = \frac{mv_1^2}{2} + \frac{\omega^2 I}{2}$$

Dla wózka:  $\Delta E_{kin} + \Delta E_{pot} = 0$

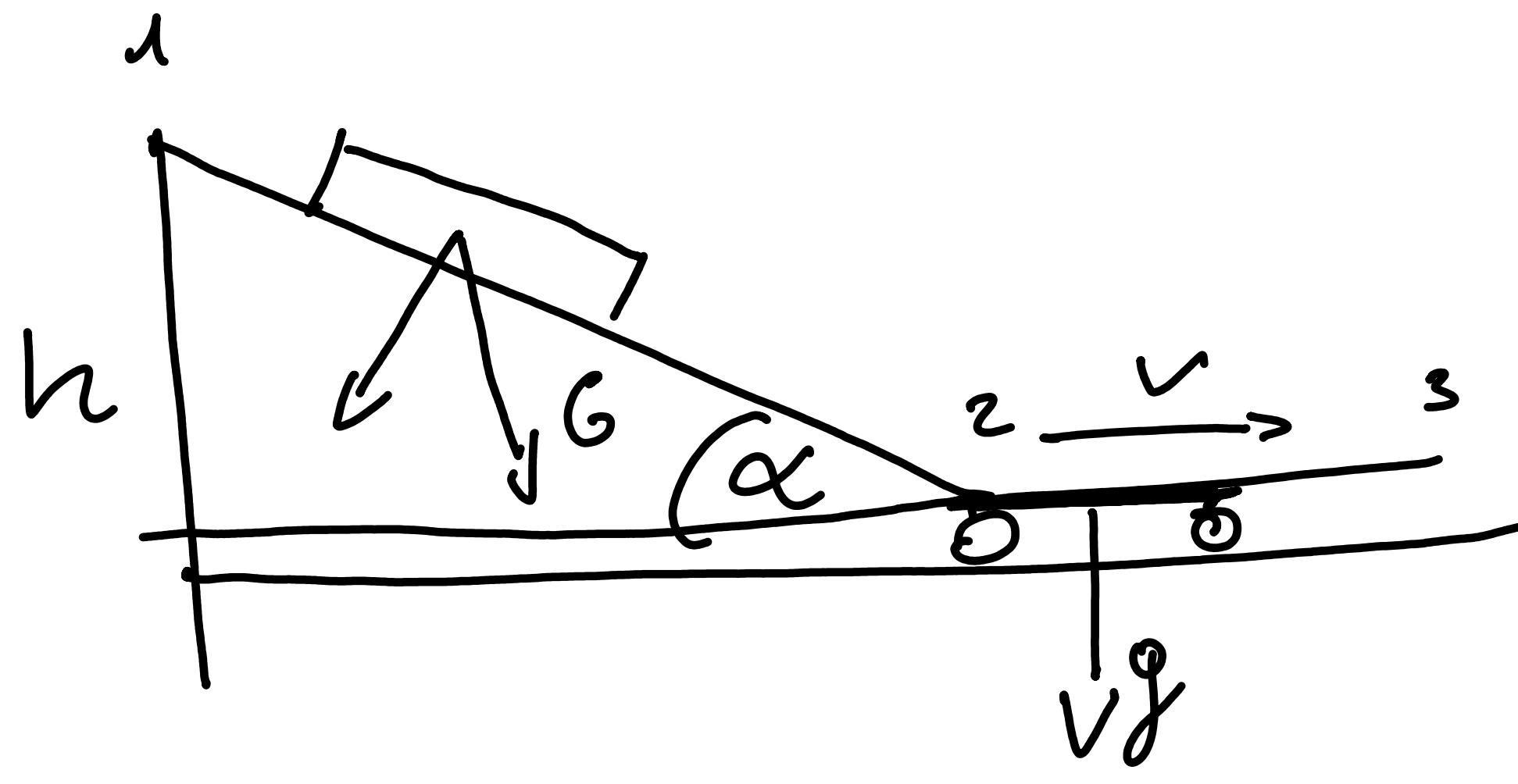
$$\Delta E_{kin} = -mgh$$

$$\Delta E_{pot} = \frac{mv^2}{2}$$

$$-mgh + \frac{mv_2^2}{2} = 0 \Rightarrow v_2 = \sqrt{2gh}$$

# ĆWICZENIA 5

zadanie 1.



$$M = \frac{G}{g}$$

$$mgh = \frac{mv^2}{2} \quad v_1 = \sqrt{2gh}$$

w chwili upadku cięzko ma pęd:  $\vec{p}_1 = m\vec{v}_1$

$$P_{1x} = mv_1 \cos(\alpha)$$

$$P_{1y} = mv_1 \sin(\alpha)$$

Cięzar po upadku na platformie:  $M = \frac{G+G_1}{g}$

$$\vec{p}_2 = M\vec{v}_2$$

$$V_{x1} = V_1 \cdot \cos(\alpha) = \sqrt{2gh} \cdot \cos(\alpha)$$

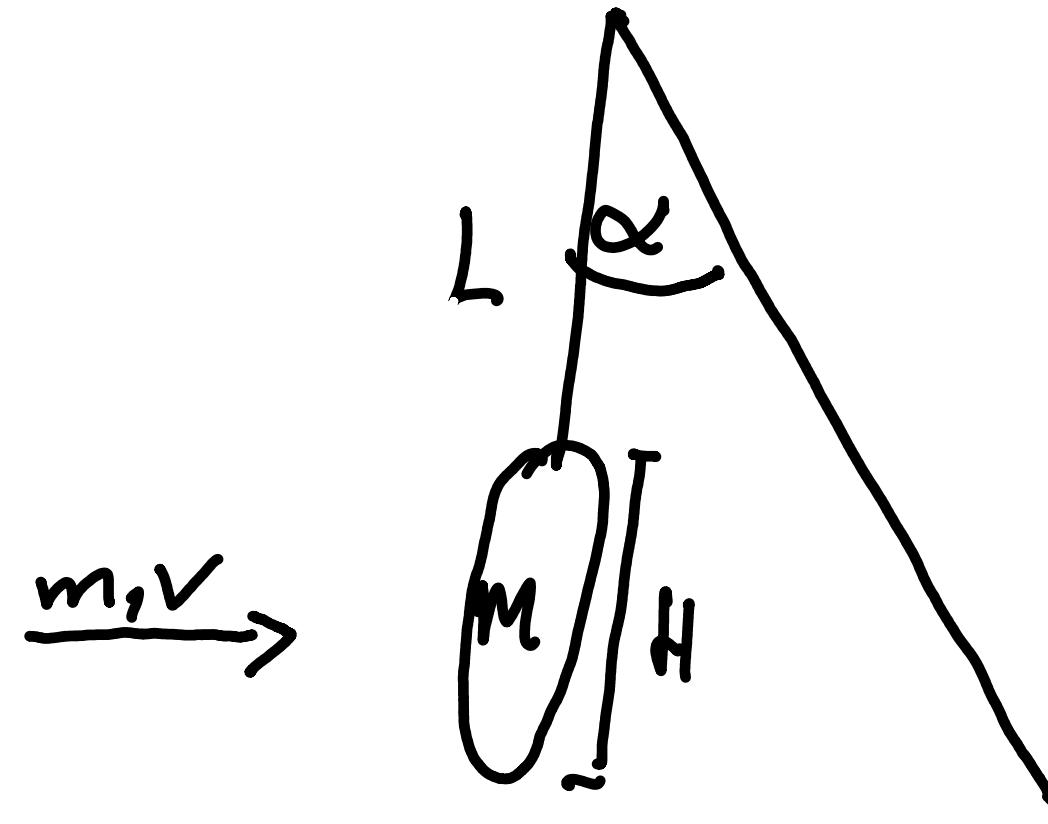
$$M \cdot V_x = m V_{1x}$$

$$\frac{G+G_1}{g} \cdot V = \frac{G}{g} V_{1x} \Rightarrow \frac{G+G_1}{g} \cdot V = \frac{G}{g} \sqrt{2gh} \cdot \cos(\alpha)$$

## Zadanie 2

nieprezyste, zasada zachowania

pełdu.



$$mV = (M+m) \cdot U \Rightarrow U = \frac{m}{M+m} V$$

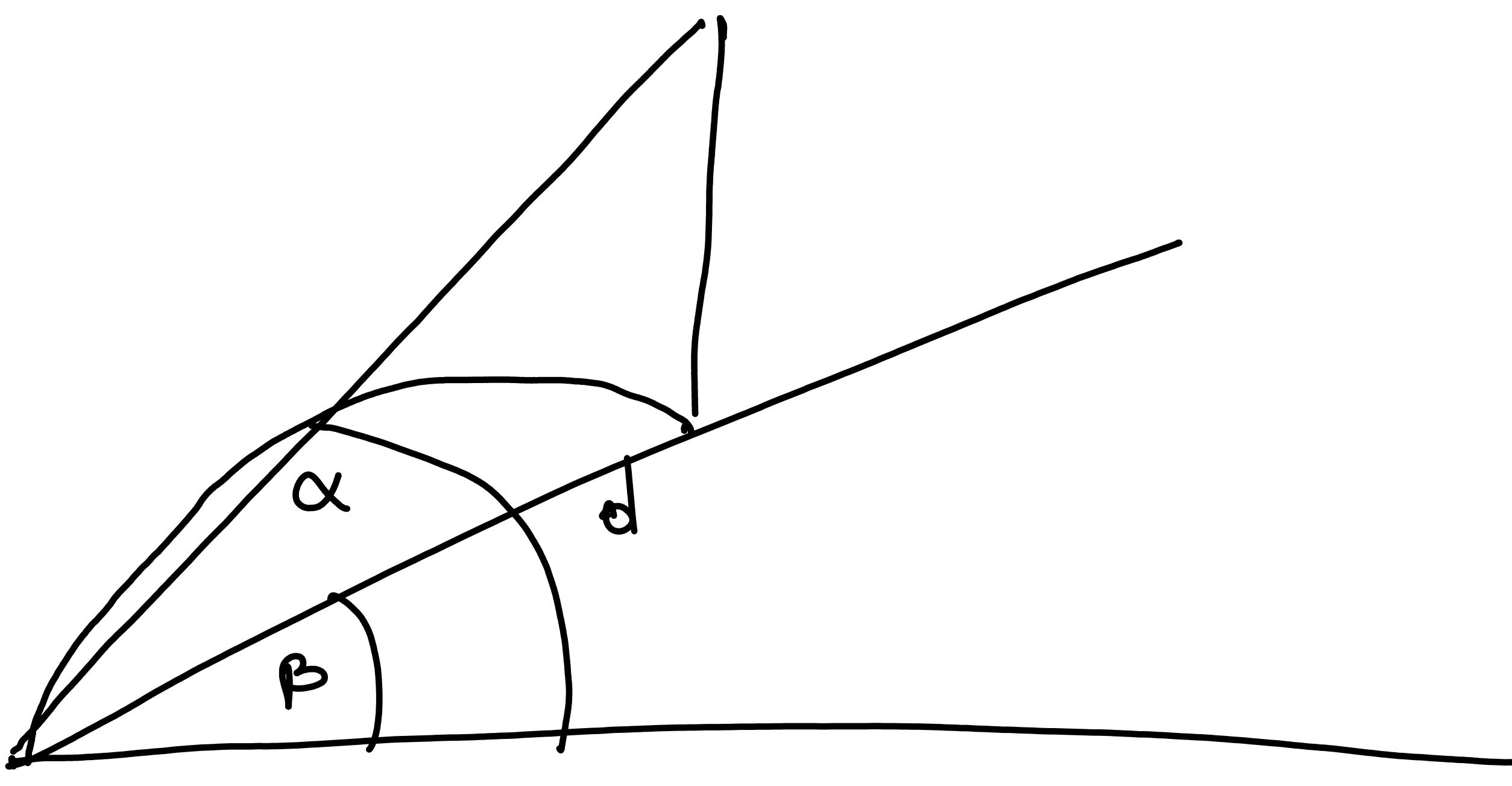
$$E_k = \int m \cdot V \, dt = \frac{(m+m) \left[ \frac{m}{(m+m)} V \right]^2}{2} = \frac{1}{2} \frac{m^2 V^2}{m+m}$$

$$E_p = (m+M)gh$$

$$\text{z zależnością tryg: } \cos(\alpha) = \frac{L-H}{L} = 1 - \frac{H}{L}$$

$$E_p = (m+M)gL(1 - \cos(\alpha))$$

przyrównać, obliczyć V.



$$V_x = V_{0x} = V_0 \cos(\alpha)$$

$$\rightarrow x = V_0 t \cos(\alpha)$$

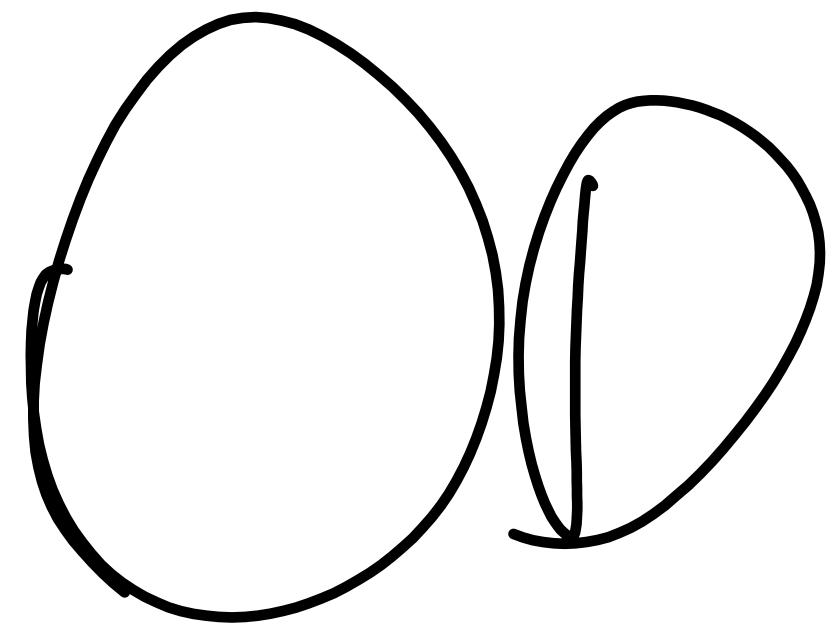
$$V_y = V_0 \sin(\alpha) - gt$$

$$\rightarrow y = V_0 t \sin(\alpha) - \frac{1}{2} g t^2$$

wyliczamy  $t$

$$y = x \operatorname{tg}(\beta)$$

$$y_A = x \operatorname{tg}(\beta) = x_n \operatorname{tg}(\alpha) - \frac{1}{2} \frac{g}{t^2}$$



Nowra

A handwritten signature in black ink, reading "Nowra". The letters are fluid and cursive, with varying line thicknesses.