

Critical Thinking Questions

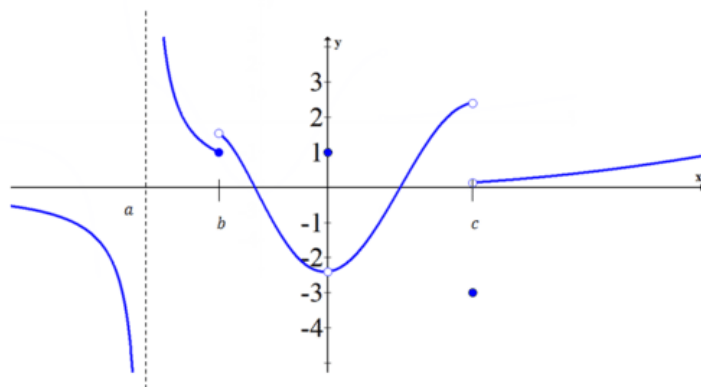


Figure 1: The Squeeze Theorem

- Let's examine the function $v = \frac{x}{3x^2+x+1}$
- This is the symbol for all real numbers: \mathbb{R}
- This is the symbol for the set of integers: \mathbb{Z}
- This is the symbol for the set of rational numbers: \mathbb{Q}
- Is it possible for a sequence to converge to two different numbers? If so, give an example. If not, explain why not.
- Explain how to use partial sums to determine if a series converges or diverges. Give an example
- Explain why $\int_1^{\infty} f(x) dx$ and $\sum_{n=1}^{\infty} a_n$ need not converge to the same value, even if they are both convergent.
- In your own words, explain the Alternating Series Remainder Theorem. How is this theorem useful?
- Explain the difference between absolute and conditional convergence. Give an example of each.
- The Ratio Test is inconclusive if $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$. Give an example of one convergent series and one divergent series for which $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 1$. Explain how you determined your examples.