

Transition

X_{t-1}	X_t	$P(X_t X_{t-1})$
h	h	0.7
h	f	0.3
f	h	0.4
f	f	0.6

Emission

X_t	E_t	$P(E_t X_t)$
H	Dzzy	0.1
	Cold	0.4
	Normal	0.5
F	Dzzy	0.6
	Cold	0.3
	Normal	0.1

$$X_0 = [0.6, 0.4]$$

$$P(X_t) = \sum_{X_{t-1}} P(X_t | X_{t-1}) P(X_{t-1})$$

$$P(X_1) = \sum P(X_1 | X_0) P(X_0)$$

$$\text{Outline: } P(X_t | e_{1:t}) = \alpha \prod_{i=1}^t P(e_i | X_i) P(X_i)$$

$$\text{By Bayes: } = \alpha \frac{P(e_t | X_t) \cdot P(X_t)}{P(e_{1:t})}$$

$$P(e_{1:t})$$



unknown

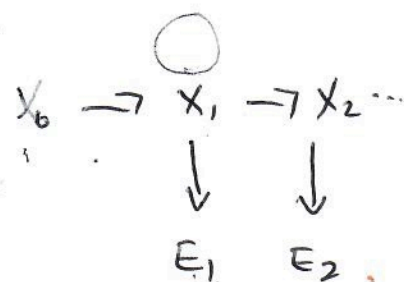
$$\text{Day 1: } X_0 \rightarrow [0.6, 0.4]$$

$$P(X_1) = \sum_{X_0} P(X_1 | X_0) \cdot P(X_0)$$

$$= [\text{h} \rightarrow \text{h}, \text{f} \rightarrow \text{h}, \text{h} \rightarrow \text{f} + \text{f} \rightarrow \text{f}]$$

$$= [0.7 * 0.6 + 0.4 * 0.4, 0.3 * 0.6 + 0.6 * 0.4]$$

$$= [0.58, 0.42]$$



Evidence $P(X_1 | \text{normal}) = \alpha P(\text{Normal} | X_1) \cdot P(X_1)$

(2)

$$= \alpha [0.5, 0.1] [0.58, 0.42]$$

$$= \alpha [0.5 \times 0.58, 0.1 \times 0.42]$$

$$= \alpha [0.29, 0.042] \quad \alpha = 0.29 + 0.042$$

$$= 0.332$$

Normalizing $\frac{[0.29, 0.42]}{\alpha}$

$$= [0.87, 0.13]$$

$$P(X_1 = \text{healthy} | \text{normal}) = \frac{P(\text{Normal} | \text{healthy}) \cdot P(\text{healthy})}{P(\text{Normal})}$$

$$= \frac{0.5 \cdot 0.58}{0.332} = 0.87$$

$P(X_2 | E_{1:t} = \text{normal}, \text{dizzy})$

$$P(X_2 | \text{normal}) = \sum_{X_1} P(X_2 | X_1) \cdot P(X_1 | \text{normal})$$

$$= [0.7 \times 0.87 + 0.4 \times 0.13, \text{from } 0.3 \times 0.87 + 0.6 \times 0.13]$$

$$= [0.661, 0.339]$$

Evidence: $P(X_2 | \text{normal}, \text{dizzy}) =$

$$= \alpha P(x_3 | x_2) P(x_2 | \text{normal})$$

$$= \alpha P[0.1 * 0.661, 0.6 * 0.339]$$

$$= \alpha [0.0661, 0.2034]$$

$$\alpha = 0.0661 + 0.2034 = 0.2695$$

$$P(x_2 = \text{healthy} | n, d) = \frac{0.1 * 0.661}{0.2695} = 0.245$$

$$P(x_2 = \text{Fever} | n, d) = \frac{0.6 * 0.339}{0.2695} = 0.755$$

$$P(x_2 | n, d) = [0.245, 0.755]$$

$$P(x_3 | n, d, n)$$

$$P(x_3 | n, d) = \sum P(x_3 | x_2) \cdot P(x_2 | n, d)$$

$$= [0.7 * 0.245 + 0.4 * 0.755, 0.3 * 0.755 + 0.6 * 0.245]$$

$$= [0.47, 0.53]$$

Evidence

$$P(x_3 | n, d, n) = \alpha P(n | x_3) \cdot P(x_3 | n, d)$$

$$= [0.5 * 0.47, 0.1 * 0.53]$$

(4)

$$\lambda = 0.24 + 0.053 = 0.293$$

$$= \left[\frac{0.24}{0.293}, \frac{0.053}{0.293} \right]$$

$$= [0.82, 0.18]$$

$$\begin{array}{cc} \uparrow & \nwarrow \\ P(X_3=H | n, d, n) & P(X_3=F | n, d, n) \end{array}$$

Deficit Equation to incorporate evidence in $P(X_4 | n, d, n, c)$

$$P(X_4 | n, d, n, c) = \alpha P(\text{cold} | X_4) \cdot P(X_4 | n, d, n)$$

$$= \alpha P(\text{cold} | X_4) \sum_{X_3} P(X_4 | X_3) P(X_3 | n, d, n)$$

What is your belief in X_4 without evidence?

$$P(X_4 | n, d, n) = \sum P(X_4 | X_3) \cdot P(X_3 | n, d, n)$$