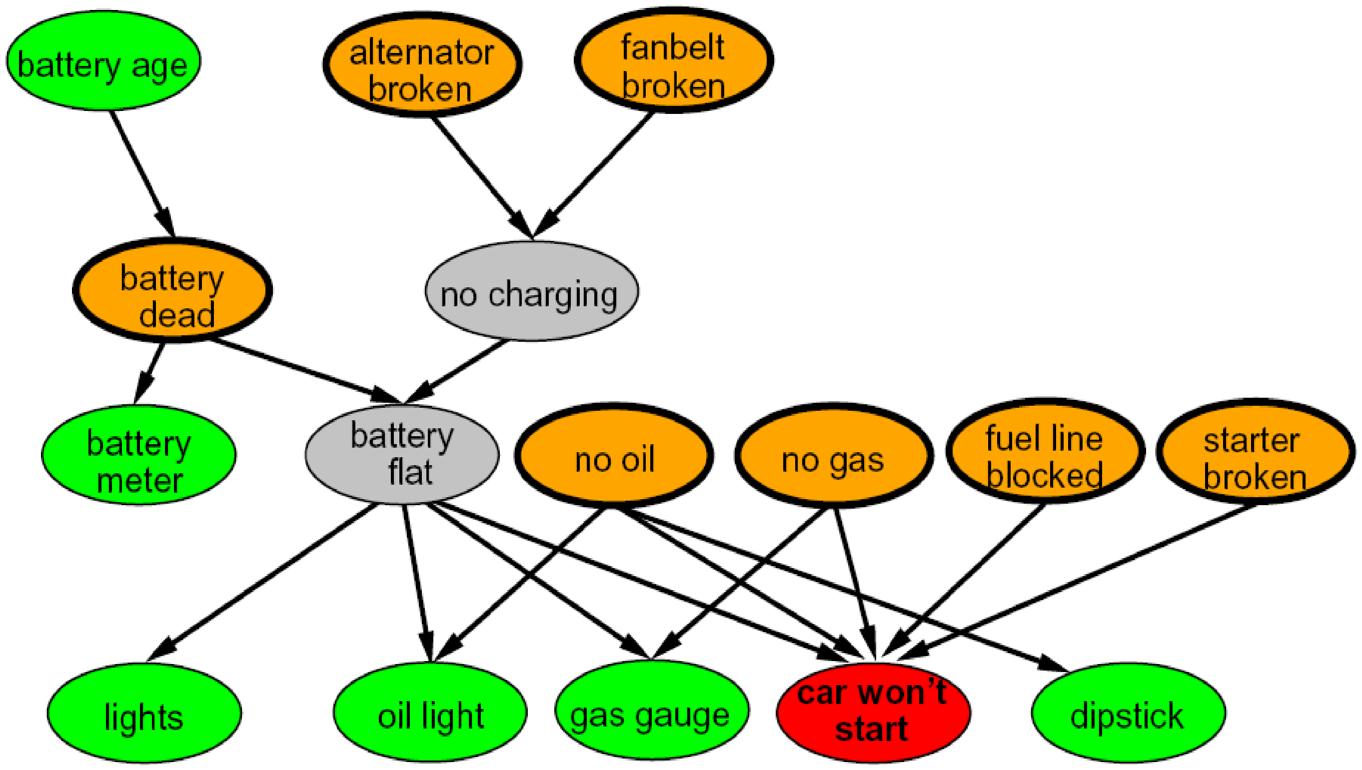
CSCI 3202: Intro to Artificial Intelligence

Bayesian Networks, Part II

Rhonda Hoenigman

Department of Computer Science

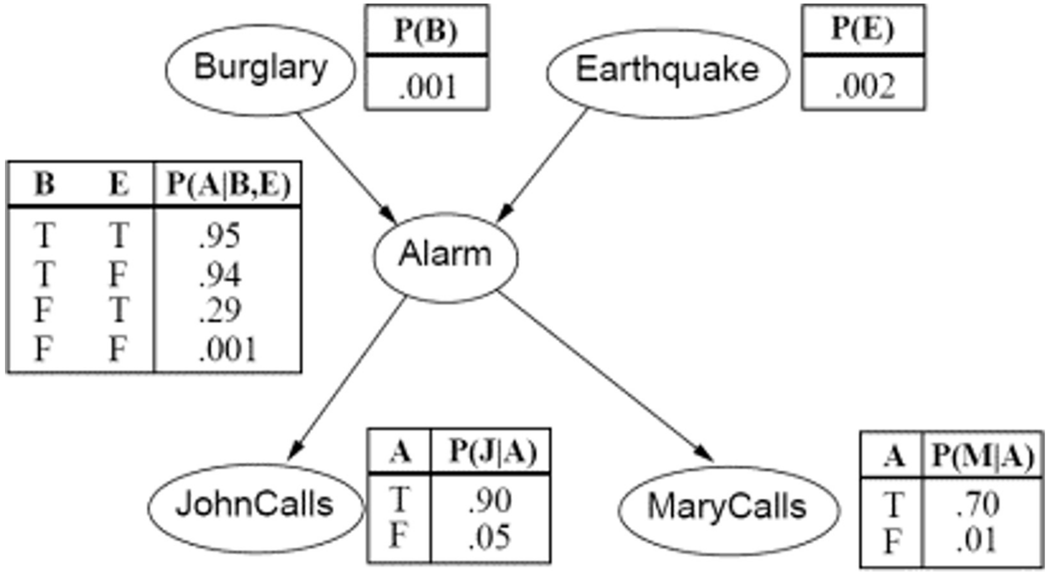


Bayesian Networks – example

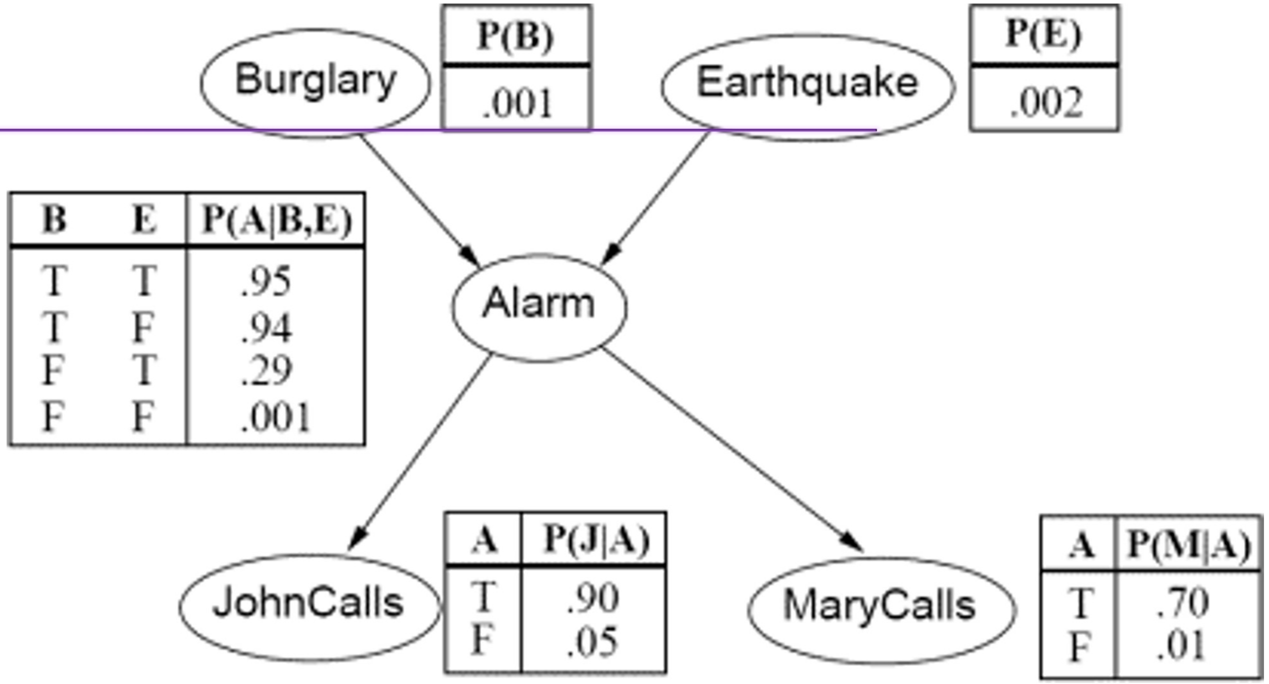
Bayes nets implicitly encode joint distributions as a product of the local conditional distributions:

$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$$

P(b | j, m) =



Bayesian Networks



Example: *S'pose* we know the alarm has gone off, and both John and Mary have called to warn us. What is the probability that we have been burgled?

→ $P(+b \mid +j, +m) = ?$

Query variables: what we want the **posterior** probability of X, **given** some **evidence**

$X = B$

Evidence variables: the variables we are given an assignment of (the **data**)

$\mathbf{E} = [+j, +m]$

Hidden variables: the non-evidence, non-query variables

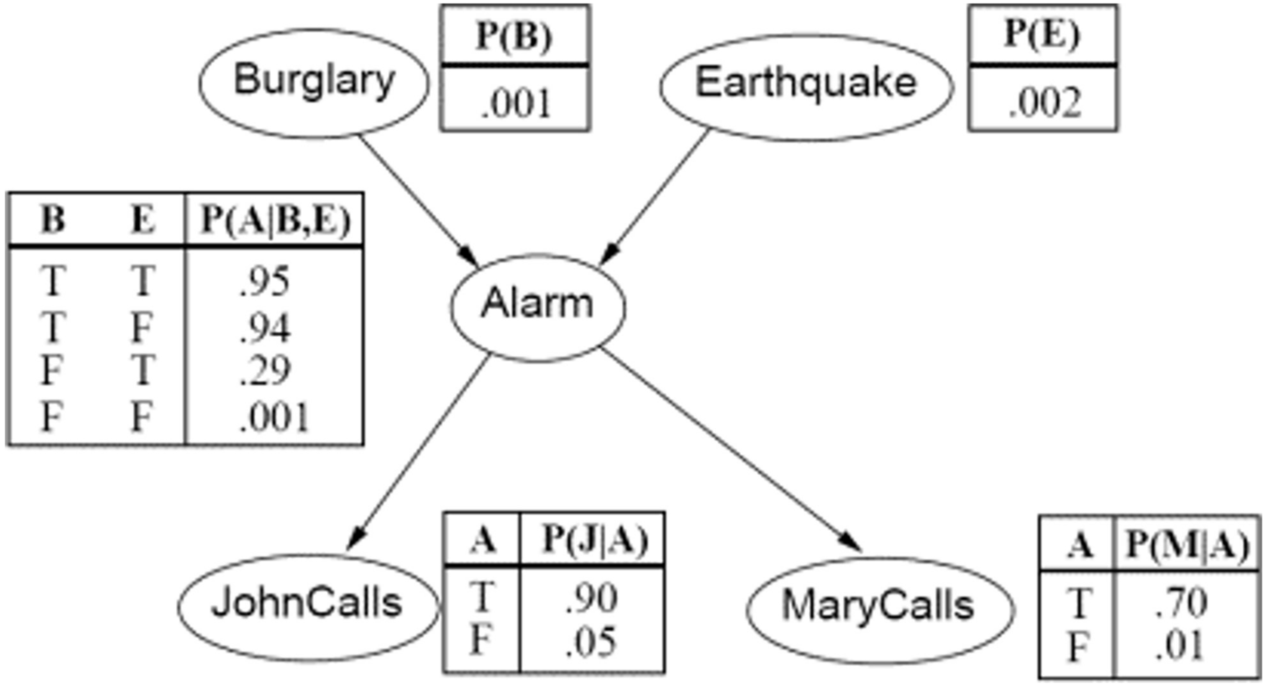
$\mathbf{y} = [E, A]$

Bayesian Networks

Example: *S'pose* we know the alarm has gone off, and both John and Mary have called to warn us. What is the probability that we have been burgled?

→ $P(+b \mid +j, +m) = ?$

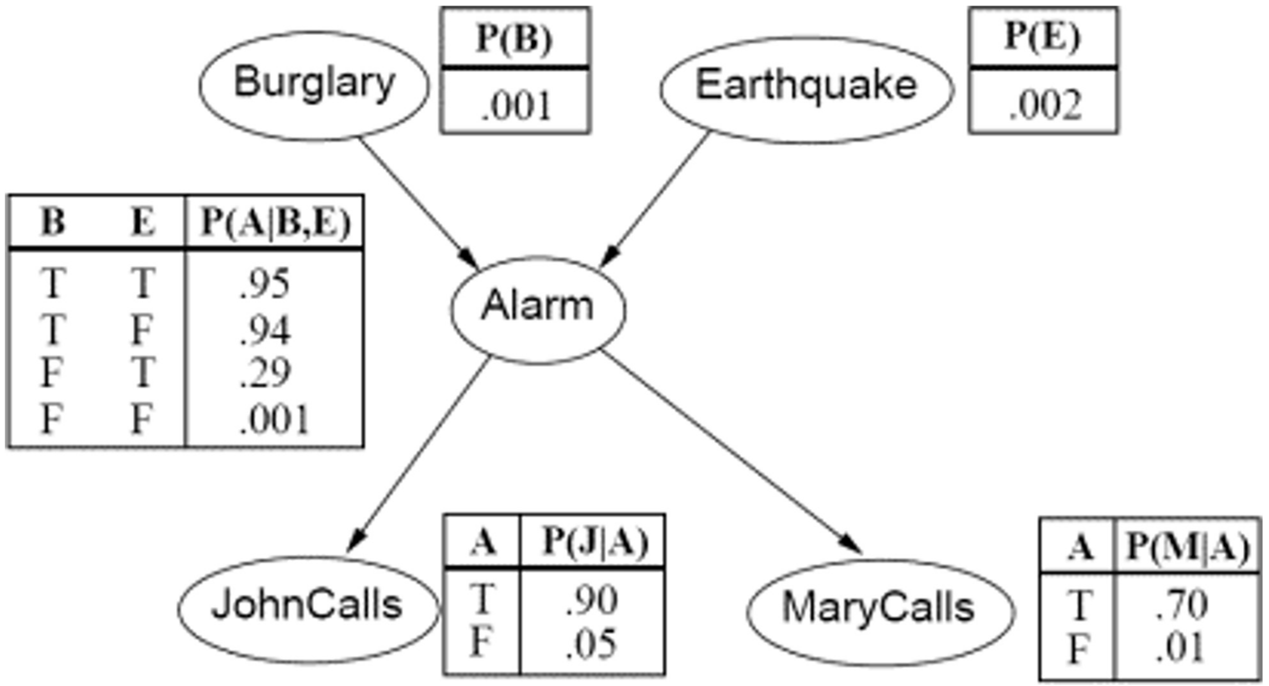
Calculation by **enumeration:**



Bayesian Networks

Example: S'pose we know the alarm has gone off, and both John and Mary have called to warn us. What is the probability that we have been burgled?

→ *P*(+b | +j, +m) = ?



Calculation by **enumeration:**

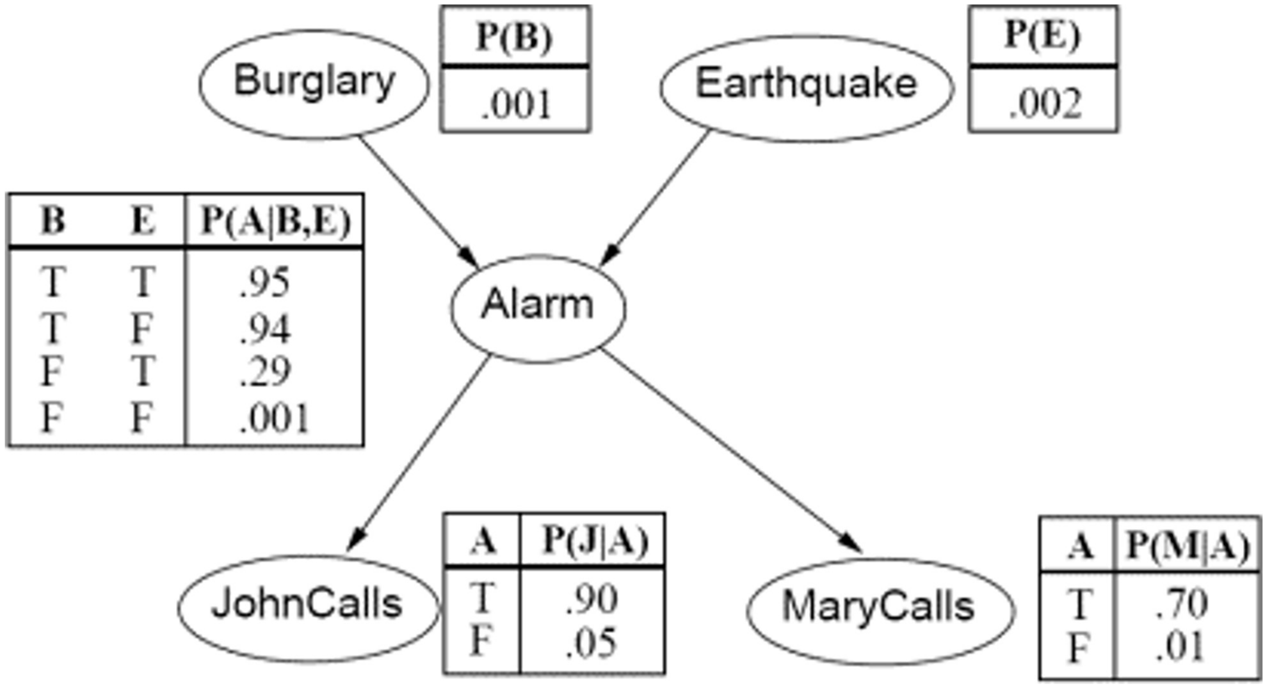
$$P(B \mid j, m) = \frac{P(B, j, m)}{P(j, m)} \quad \Rightarrow \quad \alpha = \frac{1}{P(j, m)} \quad \Rightarrow \quad P(B \mid j, m) = \alpha P(B, j, m)$$

We'll do our thing, then figure out the normalizing constant α later

Bayesian Networks

<u>Example</u>: Suppose we know the alarm has gone off, and both John and Mary have called to warn us. What is the probability that we have been burgled?

→ *P*(+b | +j, +m) = ?



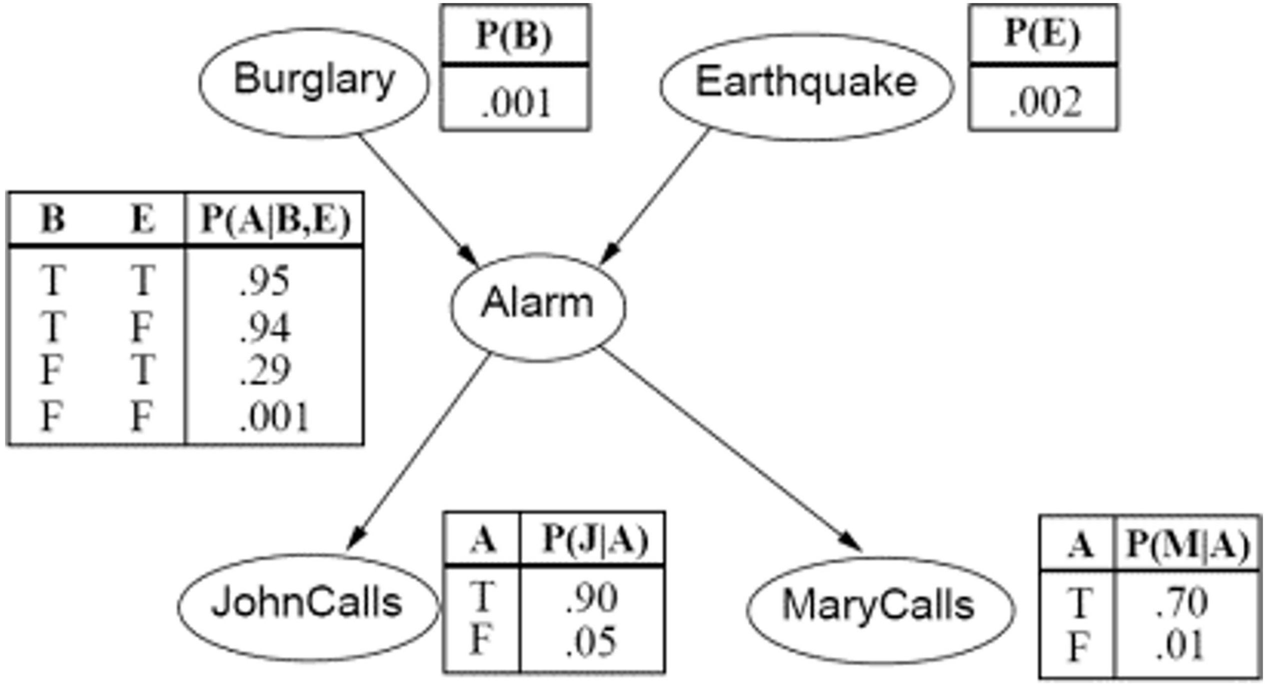
Calculation by **enumeration:**

$$\begin{aligned} P(B \mid j, m) &= \alpha P(B, j, m) \\ &= \alpha \sum_a P(B, j, m \mid a) P(a) \\ &= \alpha \sum_a P(B, j, m, a) \end{aligned}$$

Bayesian Networks

Example: Suppose we know the alarm has gone off, and both John and Mary have called to warn us. What is the probability that we have been burgled?

→ $P(+b \mid +j, +m) = ?$



Calculation by **enumeration:**

$$P(B \mid j, m) = \alpha \sum_a P(B, j, m, a)$$

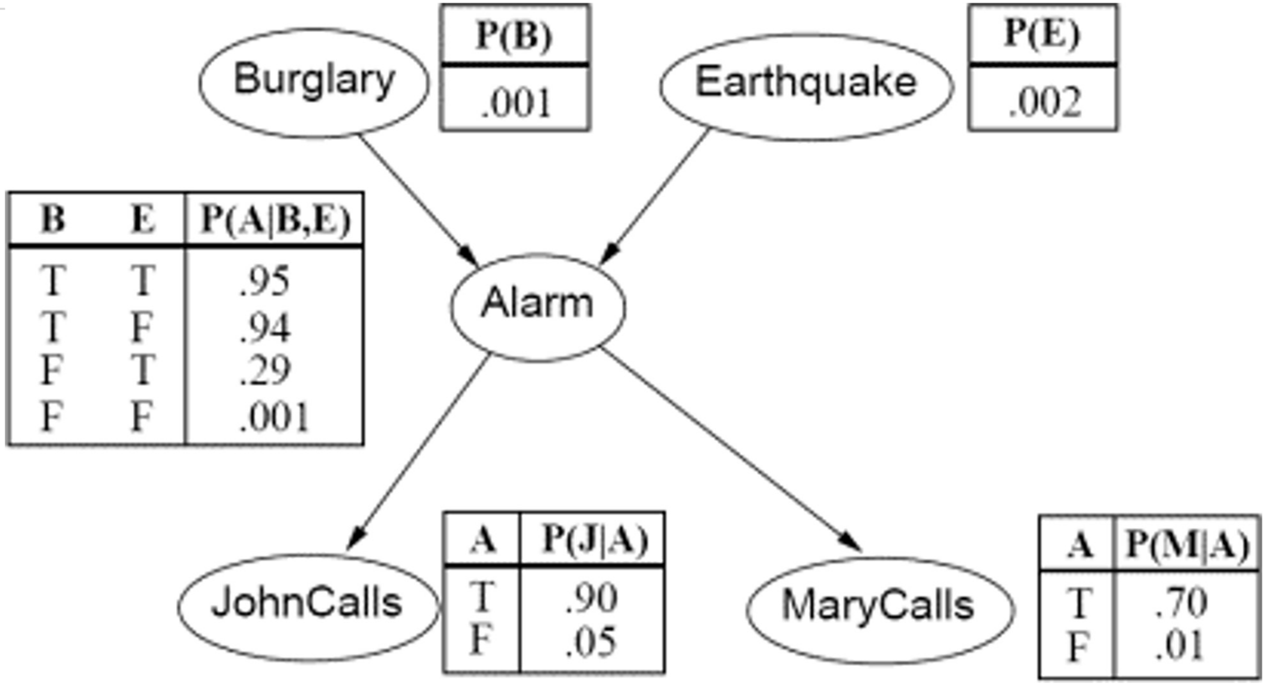
$$= \alpha \sum_e \sum_a P(B, j, m, a \mid e) P(e)$$

$$= \alpha \sum_e \sum_a P(B, j, m, a, e)$$

Bayesian Networks

Example: Suppose we know the alarm has gone off, and both John and Mary have called to warn us. What is the probability that we have been burgled?

→ *P*(+b | +j, +m) = ?



FINALLY we have:

$$P(B \mid j, m) = \alpha \sum_e \sum_a P(B, j, m, a, e)$$

From the conditional independence of the Bayes net:

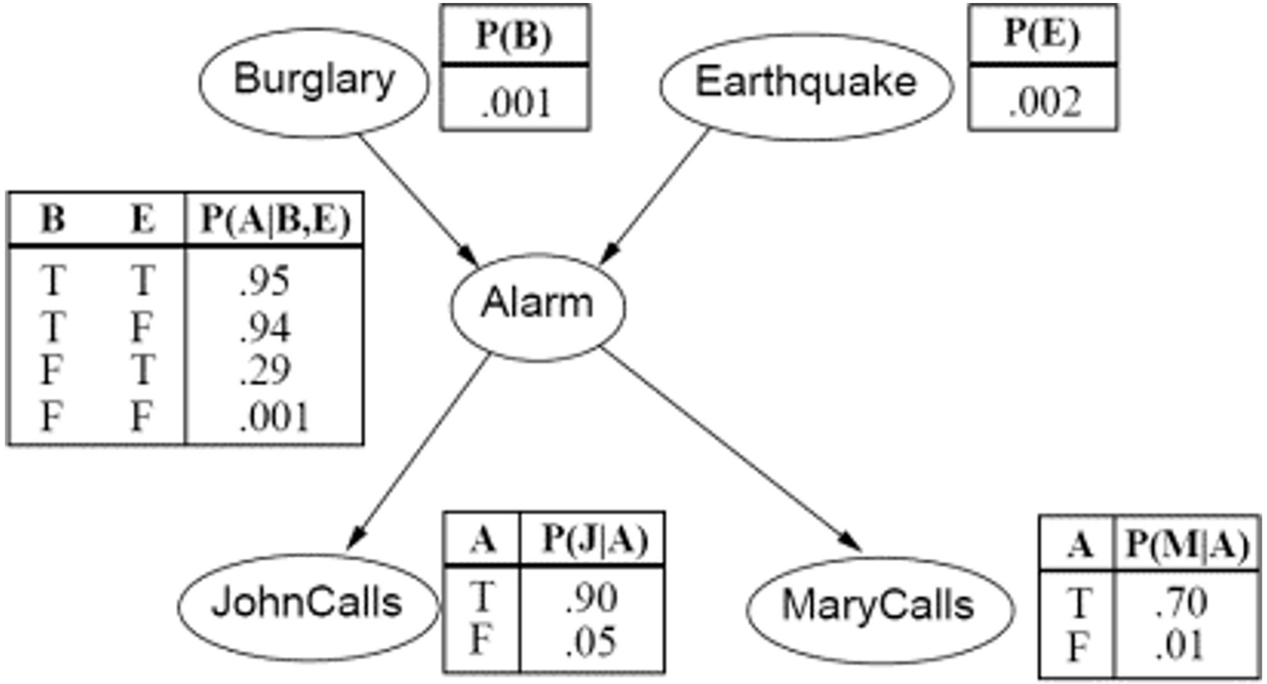
$$P(B \mid j, m) = \alpha \sum_e \sum_a \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$$

Bayesian Networks

Example: Suppose we know the alarm has gone off, and both John and Mary have called to warn us. What is the probability that we have been burgled?

→ *P*(+b | +j, +m) = ?

So for this problem…



$$P(B \mid j, m) = \alpha \sum_e \sum_a \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$$

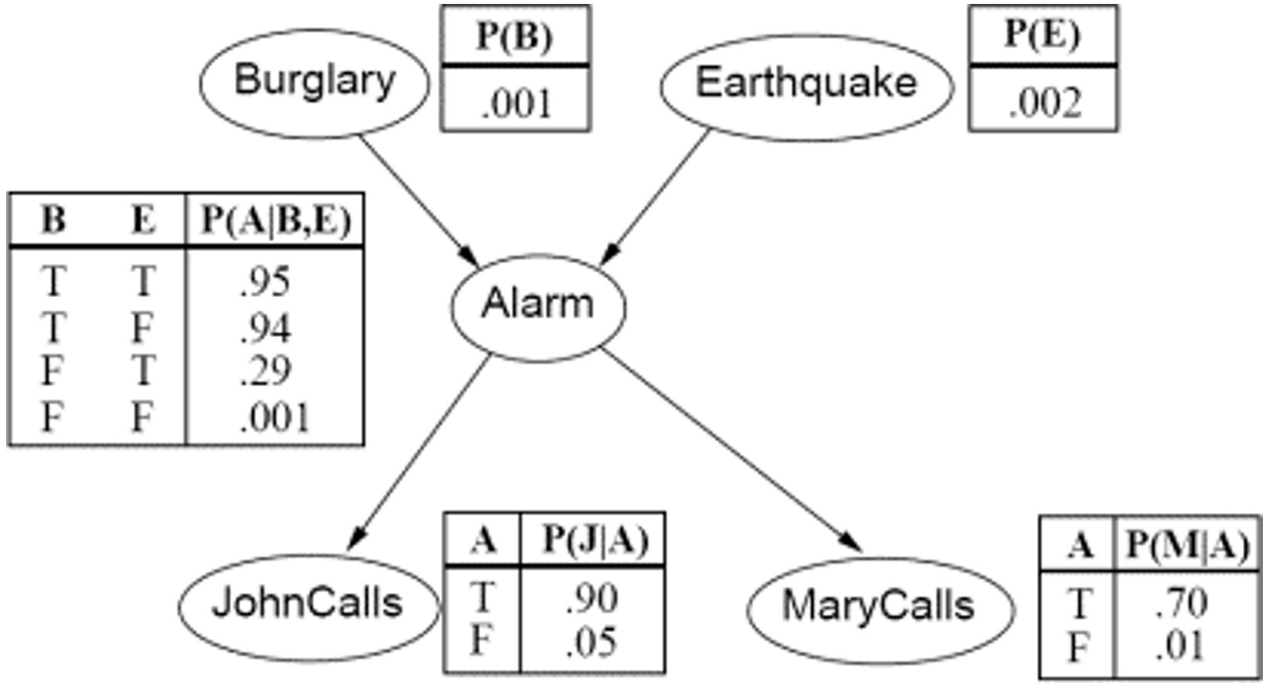
$$= \alpha \sum_e \sum_a P(B)P(e)P(a \mid B, e)P(j \mid a)P(m \mid a)$$

$$= \alpha P(B) \sum_e P(e) \sum_a P(a \mid B, e)P(j \mid a)P(m \mid a)$$

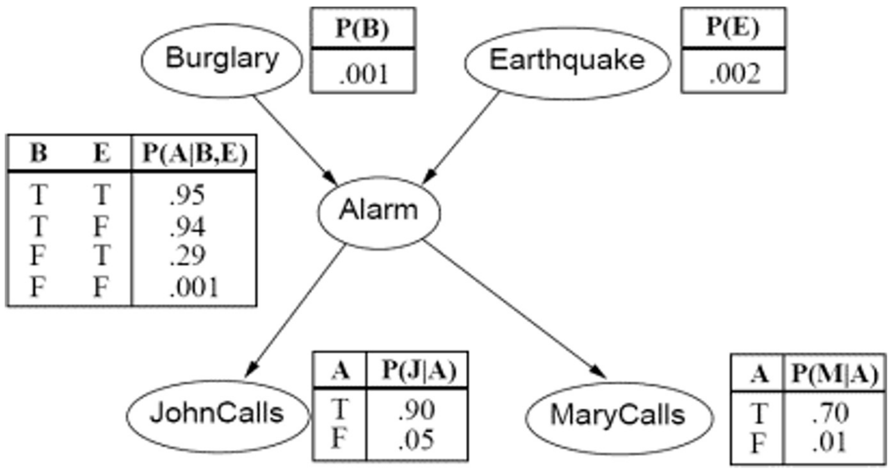
Bayesian Networks

Example:

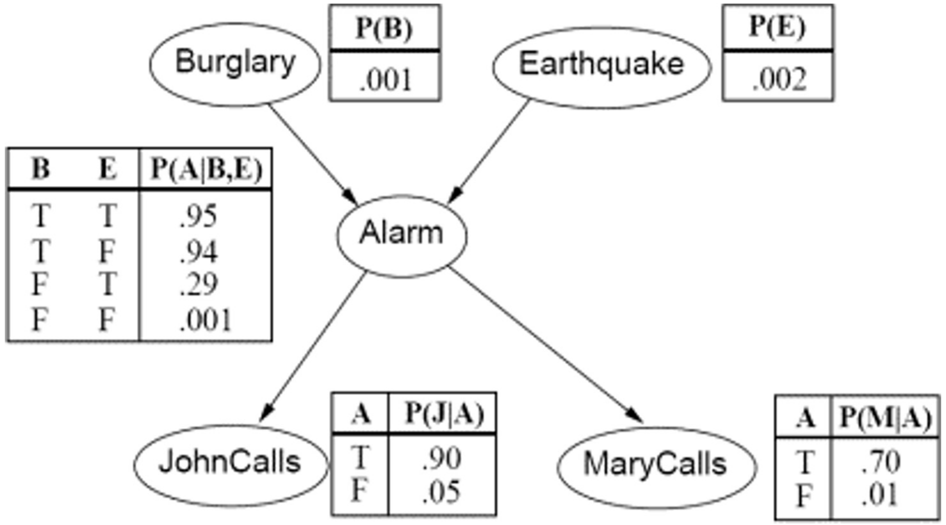
$$P(B \mid j, m) = \alpha P(B) \sum_e P(e) \sum_a P(a \mid B, e) P(j \mid a) P(m \mid a)$$



Bayesian Networks

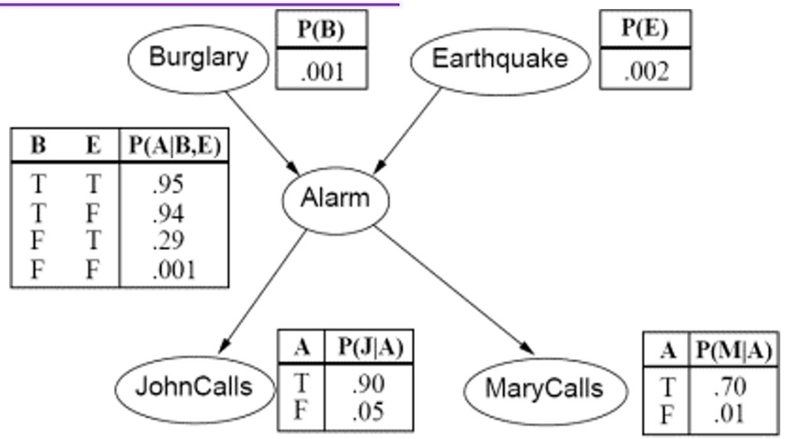


Bayesian Networks



Bayesian Networks

Example: Find $P(Burglary = T \mid JohnCalls = T, MaryCalls = T)$



Entire Joint Probability Distribution:

$$P(B, E, A, J, M) = P(B)P(E)\, P(A \mid B, E)\, P(J \mid A)\, P(M \mid A)$$

$$P(Burglary = T \mid JohnCalls = T, MaryCalls = T) = \frac{P(Burglary = T, JohnCalls = T, MaryCalls = T)}{P(JohnCalls = T, MaryCalls = T)}$$

$$= \frac{P(Burglary = T, \boldsymbol{Earthquake = T\ or\ F}, \boldsymbol{Alarm = T\ or\ F}, JohnCalls = T, MaryCalls = T)}{P(\boldsymbol{Burglary = T\ or\ F}, \boldsymbol{Earthquake = T\ or\ F}, \boldsymbol{Alarm = T\ or\ F}, JohnCalls = T, MaryCalls = T)}$$

$$= \frac{P(+B,+E,+A,+J,+M) + P(+B,+E,-A,+J,+M) + \boldsymbol{P(+B,-E,+A,+J,+M)} + P(+B,-E,-A,+J,+M)}{P(+B,+E,+A,+J,+M) + P(+B,+E,-A,+J,+M) + P(+B,-E,+A,+J,+M) + P(+B,-E,-A,+J,+M) + P(-B,+E,+A,+J,+M) + P(-B,+E,-A,+J,+M) + P(-B,-E,+A,+J,+M) + P(-B,-E,-A,+J,+M)}$$

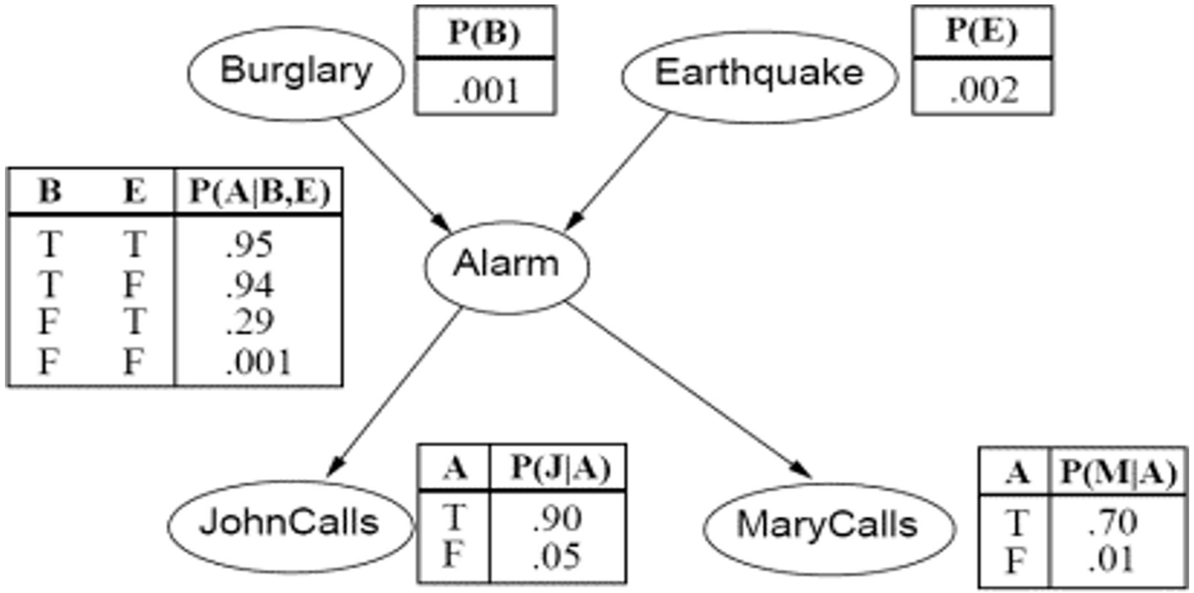
$$= \frac{\boldsymbol{p(+B)p(+E)p(+A|+B,+E)p(+J|+A)p(+M|+A) + p(+B)p(+E)p(-A|+B,+E)p(+J|-A)p(+M|-A) + p(+B)p(-E)p(+A|+B,-E)p(+J|+A)p(+M|+A) + p(+B)p(-E)p(-A|+B,-E)p(+J|-A)p(+M|-A)}}{\begin{array}{c}\boldsymbol{p(+B)p(+E)p(+A|+B,+E)p(+J|+A)p(+M|+A) + p(+B)p(+E)p(-A|+B,+E)p(+J|-A)p(+M|-A) + p(+B)p(-E)p(+A|+B,-E)p(+J|+A)p(+M|+A) + p(+B)p(-E)p(-A|+B,-E)p(+J|-A)p(+M|-A)} \\ + p(-B)p(+E)p(+A|-B,+E)p(+J|+A)p(+M|+A) + p(-B)p(+E)p(-A|-B,+E)p(+J|-A)p(+M|-A) + p(-B)p(-E)p(+A|-B,-E)p(+J|+A)p(+M|+A) + p(-B)p(-E)p(-A|-B,-E)p(+J|-A)p(+M|-A)\end{array}}$$

$$= \frac{\boldsymbol{(0.001)(0.002)(0.95)(0.90)(0.70) + (0.001)(0.002)(1-0.95)(0.05)(0.01) + (0.001)(1-0.002)(0.94)(0.90)(0.70) + (0.001)(1-0.002)(1-0.94)(0.05)(0.01)}}{\begin{array}{c}\boldsymbol{(0.001)(0.002)(0.95)(0.90)(0.70) + (0.001)(0.002)(1-0.95)(0.05)(0.01) + (0.001)(1-0.002)(0.94)(0.90)(0.70) + (0.001)(1-0.002)(1-0.94)(0.05)(0.01)} \\ + (1-0.001)(0.002)(0.29)(0.90)(0.70) + (1-0.001)(0.002)(1-0.29)(0.05)(0.01) + (1-0.001)(1-0.002)(0.001)(0.90)(0.70) + (1-0.001)(1-0.002)(1-0.001)(0.05)(0.01)\end{array}}$$

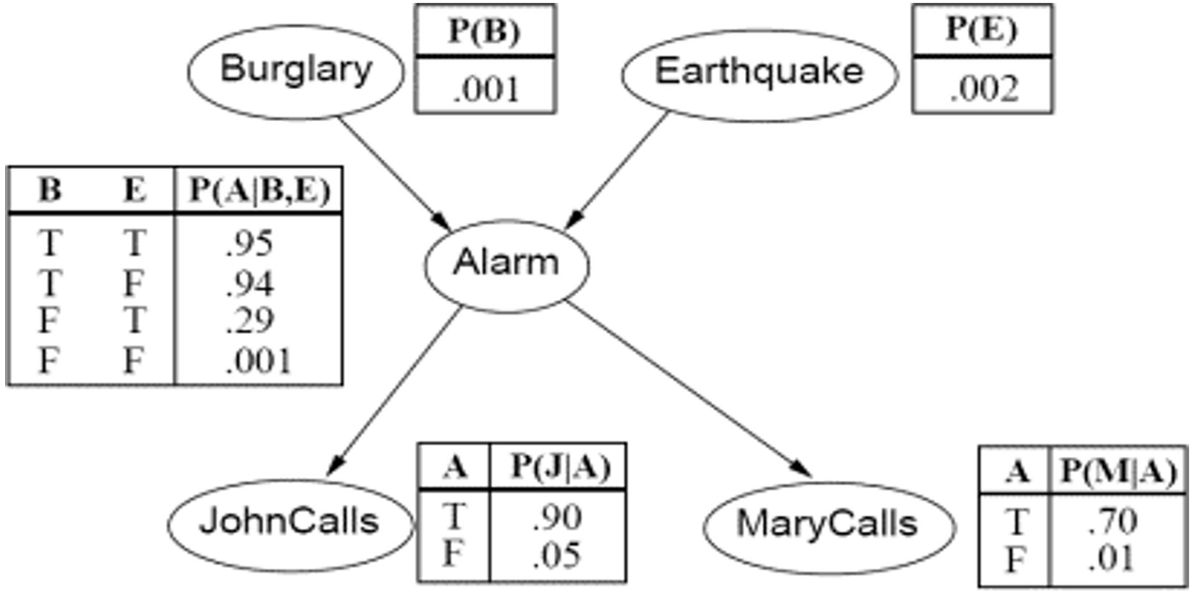
$$= \frac{\boldsymbol{0.0000012 + 0.000000001 + 0.0005910156 + 0.00000002994}}{\boldsymbol{0.0000012 + 0.000000001 + 0.0005910156 + 0.00000002994} + 0.0003650346 + 0.00000070929 + 0.00062811126 + 0.000498002499} = \frac{0.00059224654}{0.002084104189} = \boldsymbol{0.284173}$$

Bayesian Networks

Example: $P(+b|+j)$

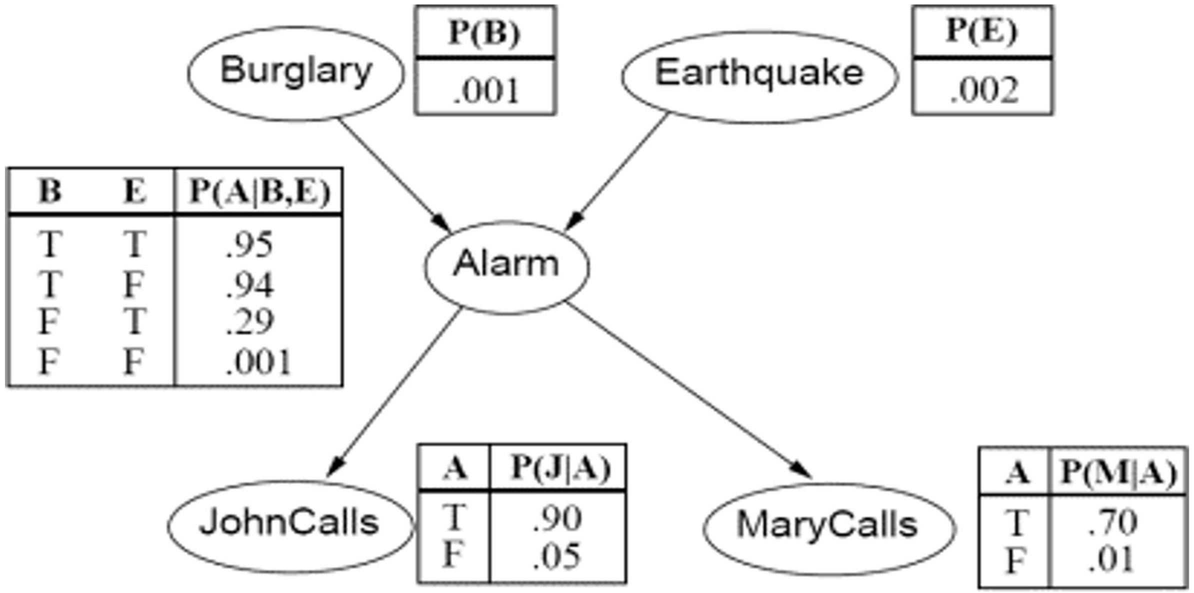


Bayesian Networks

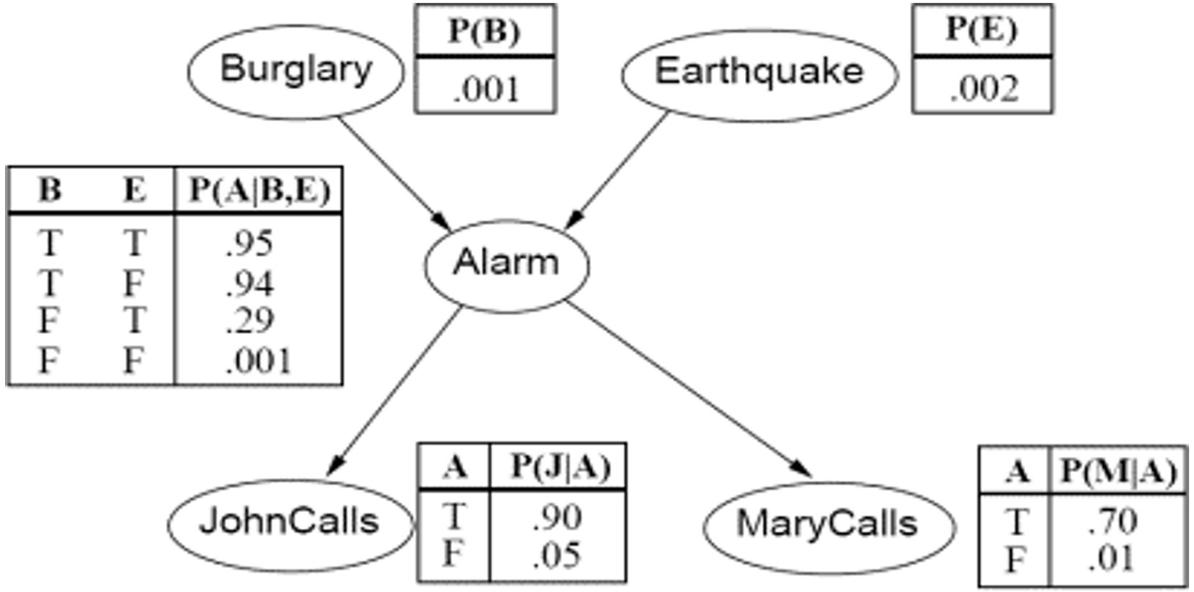


Bayesian Networks

Example: $P(+b|+a)$



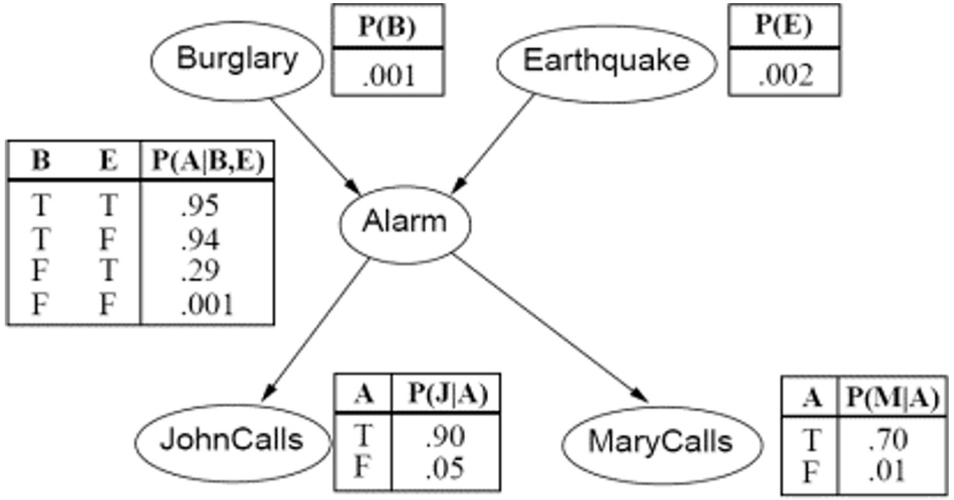
Bayesian Networks



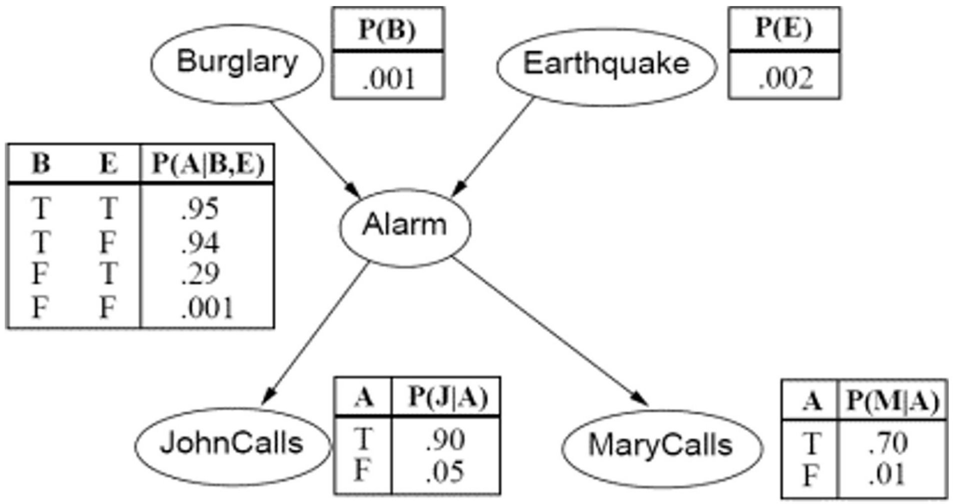
Bayesian Networks

Example: Find $P(Earthquake = T \mid Burglary = T, Alarm = T)$
And $P(Earthquake = T \mid Alarm = T)$

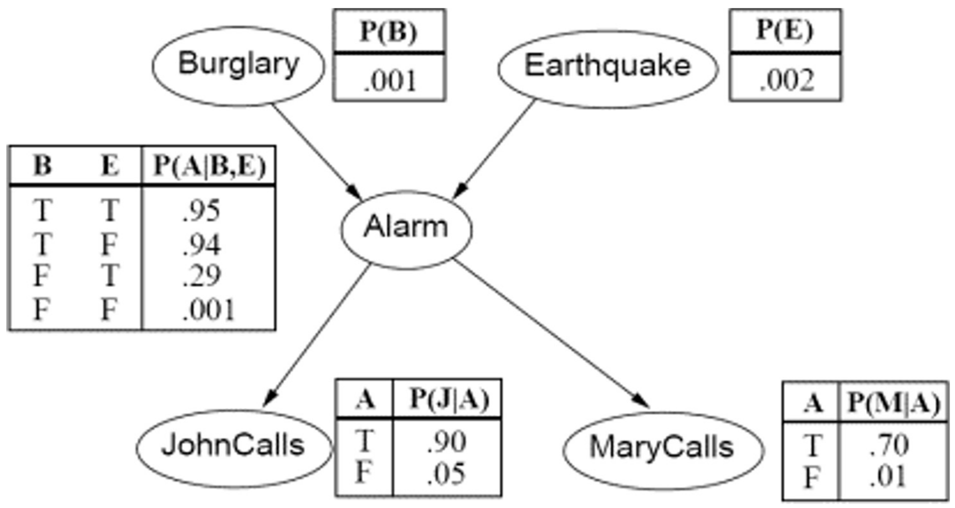
Is it true that Burglary = T explains away Earthquake = T?



Bayesian Networks



Bayesian Networks



Bayesian Networks – construction

Bayes nets implicitly encode joint distributions as a product of the local conditional distributions:

$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid x_{i-1}, x_{i-2}, \ldots, x_2, x_1) = \ldots?$$

Node ordering: write in such a way that

$$\text{parents}(X_i) \subseteq \{X_{i-1}, X_{i-2}, \ldots, X_2, X_1\}$$

$\rightarrow$

$$\prod_{i=1}^{n} P(x_i \mid x_{i-1}, x_{i-2}, \ldots, x_2, x_1) = \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$$

This statement:

$$P(X_i \mid X_{i-1}, X_{i-2}, \ldots, X_2, X_1) = P(X_i \mid \text{parents}(X_i))$$

is key: **each node is conditionally independent**

of its other predecessors, given its parents

Bayesian Networks: Construction

Show a "flow" from cause to effect: **Pearl's Network Construction Algorithm**

Nodes: What is the set of variables we need to model?

Order them: $\{X_1, X_2, X_3, \ldots, X_n\}$

Best if ordered such that **causes precede effects**

Links: For each node X_i , do:

- Choose a minimal set of parents $\text{parents}(X_i) \subseteq \{X_{i-1}, X_{i-2}, \ldots, X_2, X_1\}$
  such that $P(x_i \mid x_{i-1}, x_{i-2}, \ldots, x_1) = P(x_i \mid \text{parents}(X_i))$
- For each parent, insert arcs (links) from parent to X_i
- Write down CPT $P(X_i \mid \text{parents}(X_i))$

Bayesian Networks: Construction

Example: Suppose we have an old motorcycle that might either blow a head gasket (H) or have a broken thermometer (T). Either one would cause the bike to overheat (O). If the bike overheats, then it might blow smoke (S) and/or run weak (W).

Construct a Bayesian network for this situation.

1. Node ordering: {H, T, O, W, S}
2. Insert arcs

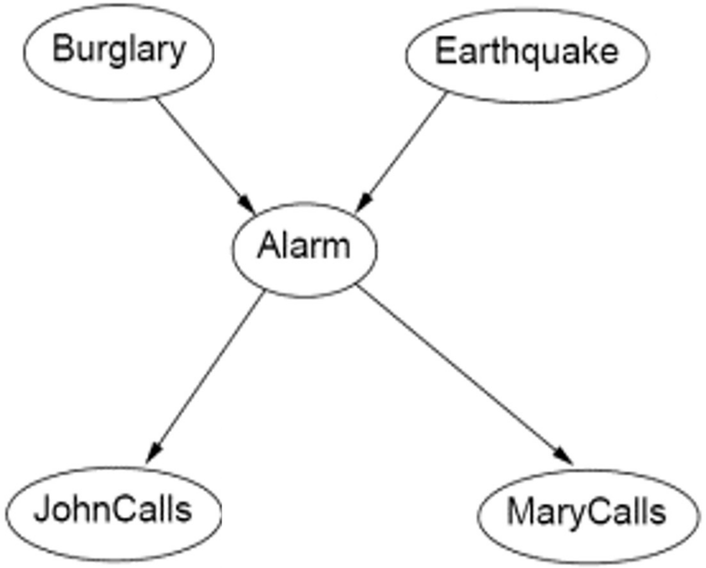
Bayesian Networks: Construction

Here, we chose to put the causes before effects:

{Burglary, Earthquake, Alarm, JohnCalls, MaryCalls}

What if instead we did the following?

{MaryCalls, JohnCalls, Alarm,
Burglary, Earthquake}



Bayesian Networks: Types of reasoning – diagnostic

Bayesian Networks: Types of reasoning – predictive

Bayesian Networks: Types of reasoning – intercausal

Bayesian Networks: Types of reasoning – combined