CSCI 3202: Intro to Artificial Intelligence

Introduction to Bayesian Networks

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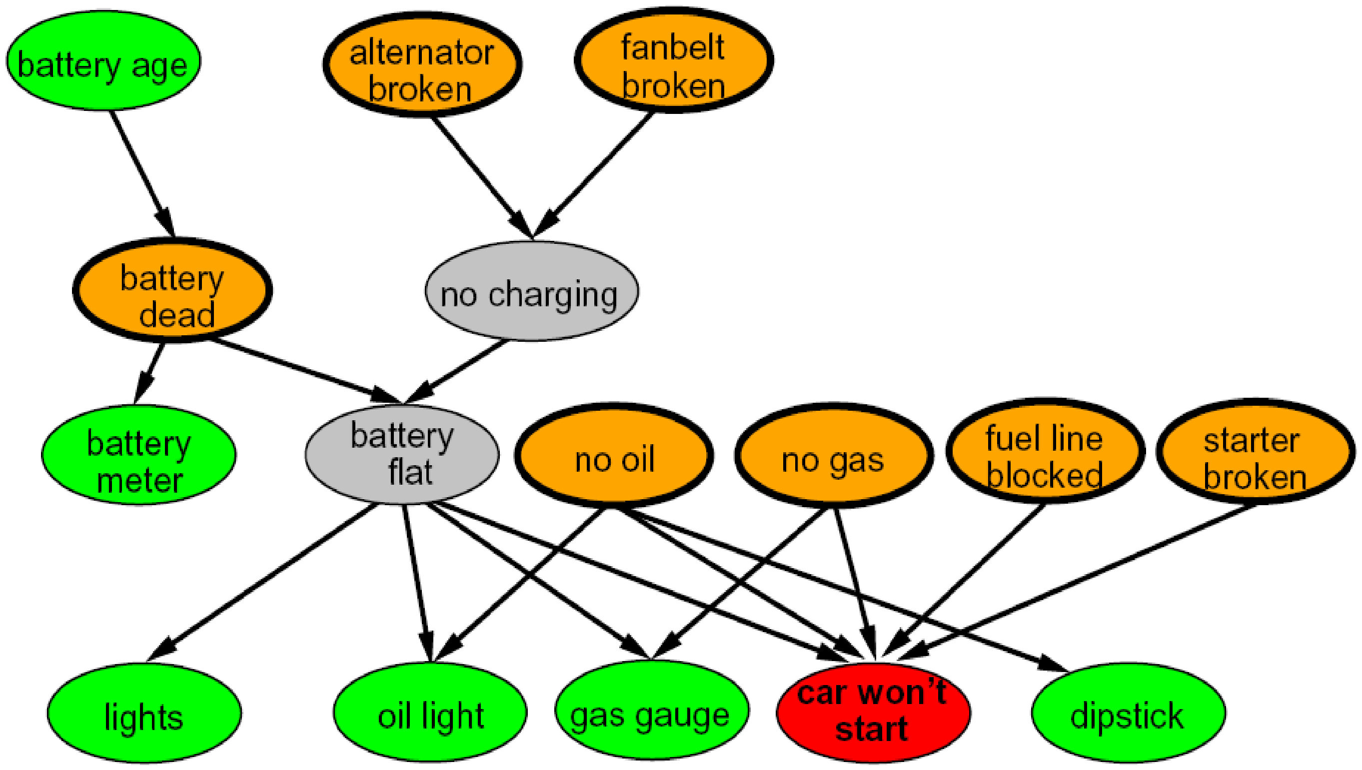


Bayes Nets

Bayes Nets

Bayes Nets – dealing with Uncertainty

Probabilistic reasoning framework for managing uncertain beliefs and knowledge.

a.k.a - belief network, decision network

In general:

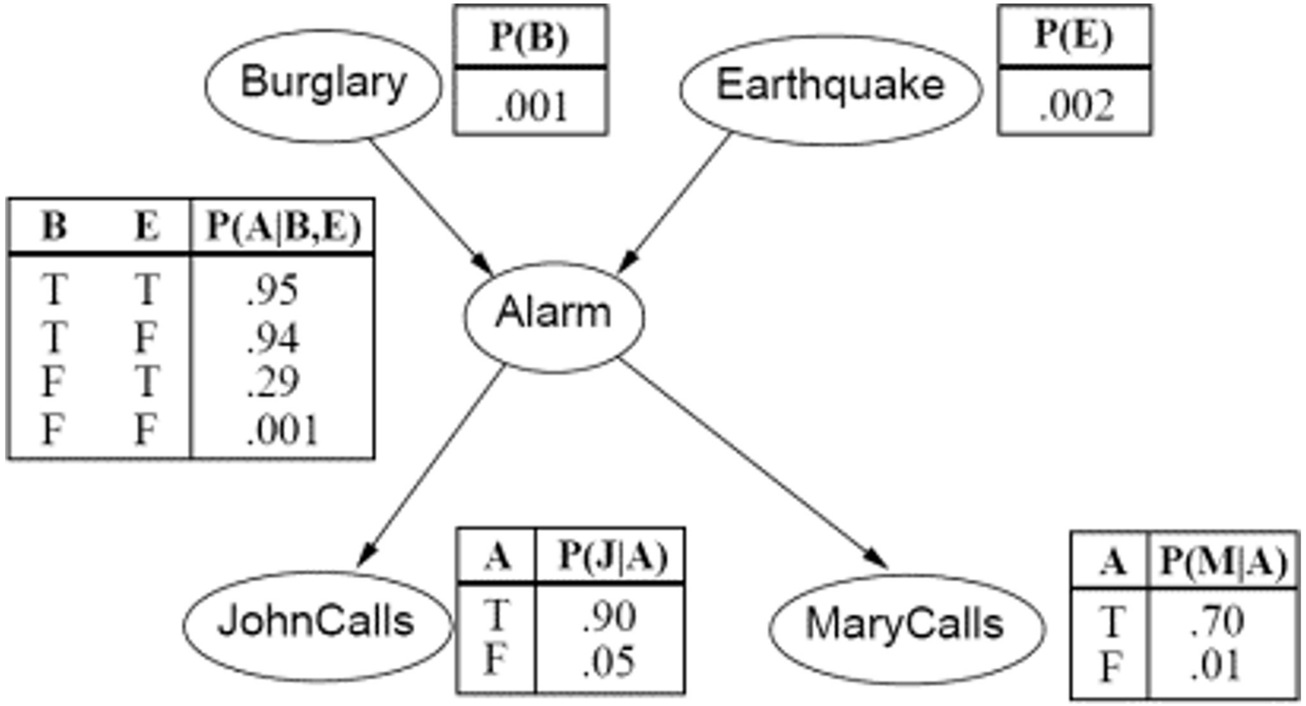
- **Observed variables (evidence)**: Agent knows certain things about the state of the world (e.g. sensor readings or symptoms)
- **Unobserved variables**: Agent needs to reason about other aspects (e.g. where an object is or what disease is present)
- **Model**: Agent knows something about how the known variables relate to the unknown variables

Bayesian Networks

The point of Bayes nets is to **represent full joint probability distributions**, *and*

to encode an interrelated set of **conditional independence/probability statements**

- Directed, acyclic graph
- Consists of **nodes** (events), and
- Each edge is conditional dependency
- **conditional probability tables (CPTs)**, relating those events
- Describe **local** variable interactions
- Chain together local interactions to estimate **global**, **indirect** interactions

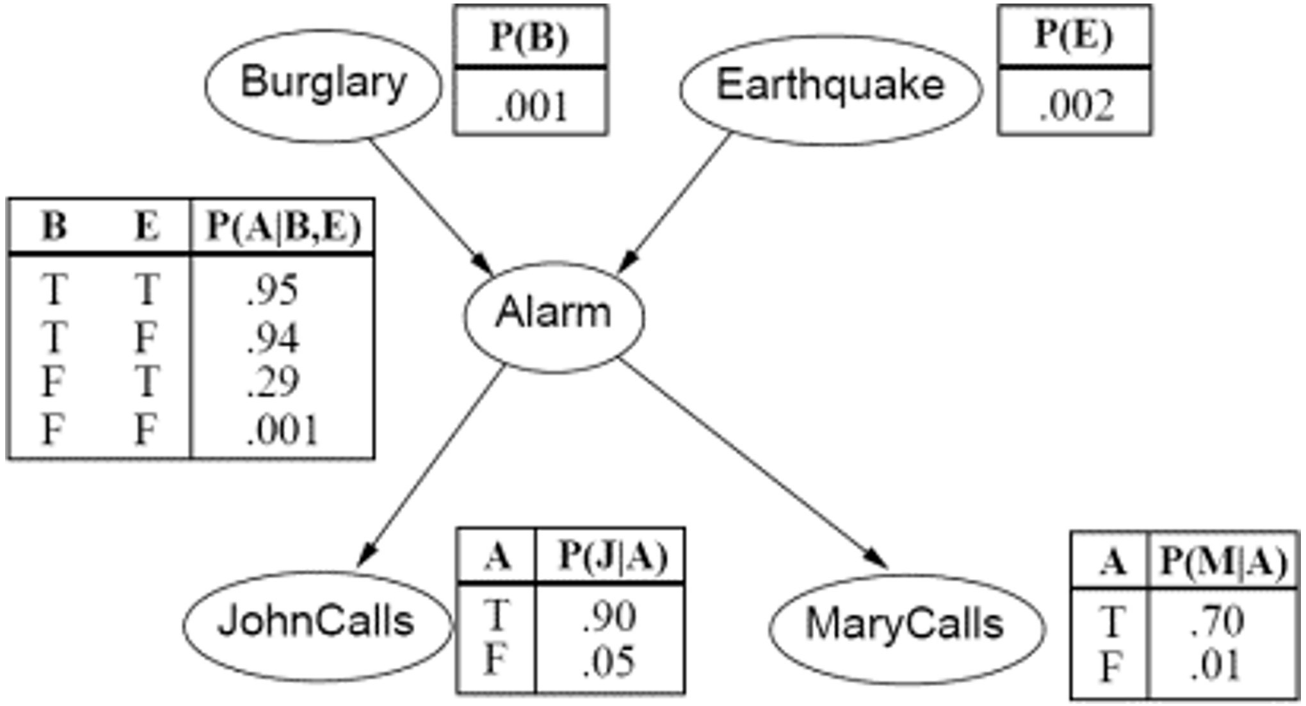


Bayesian Networks

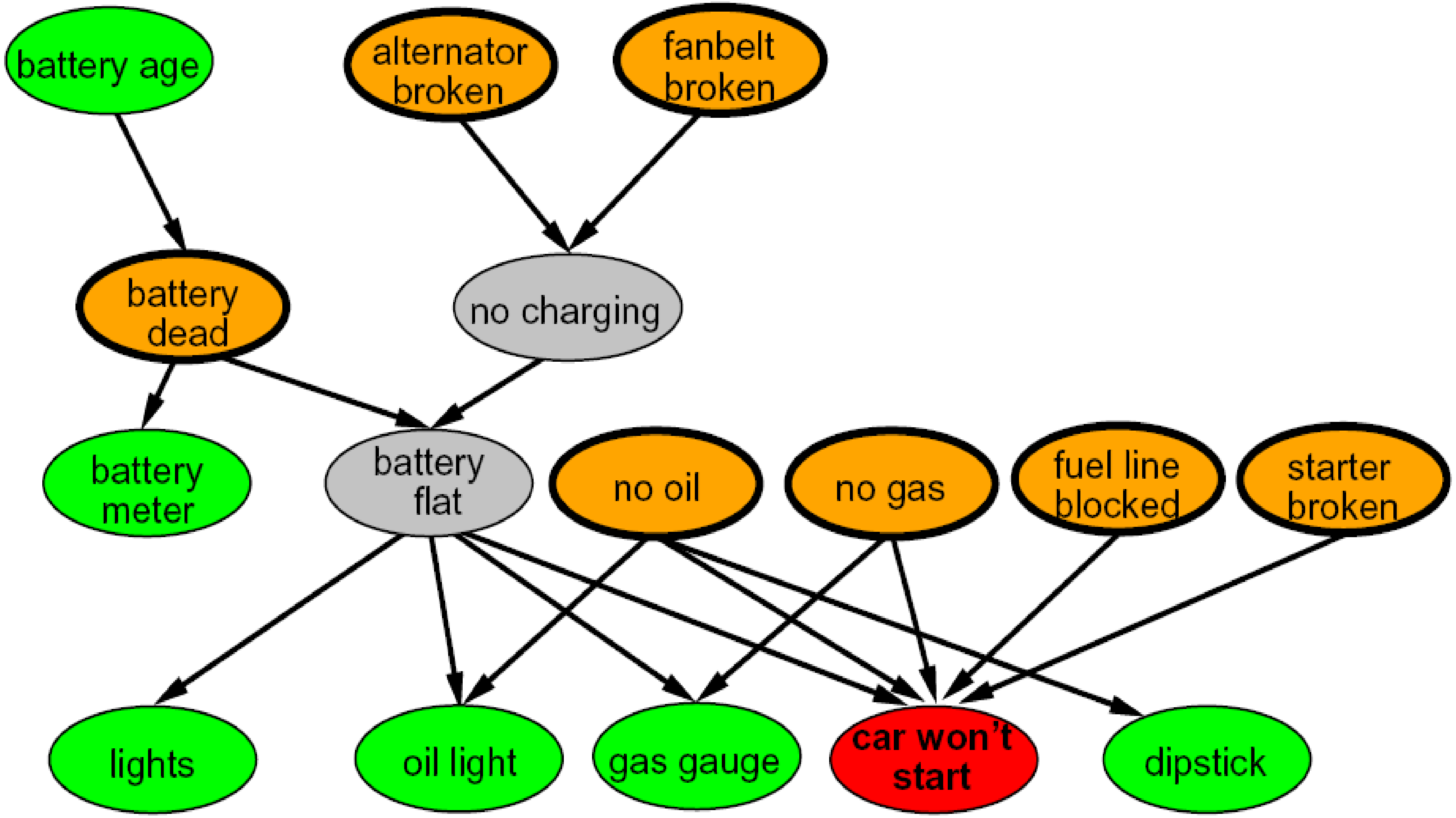
The point of Bayes nets is to **represent full joint probability distributions**, *and*

to encode an interrelated set of **conditional independence/probability statements**

- P(a | b, e)
- P(a)
- P(b | a)
- P(J | b)



Graphical Model Notation – more complicated example



Independence and conditional independence

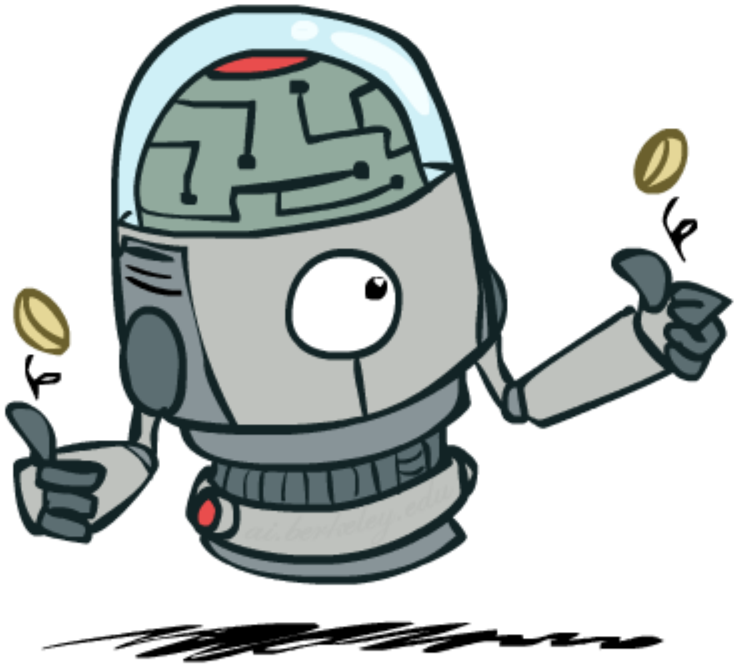
- Two variables are *independent* if:

$$\forall x, y : P(x, y) = P(x)P(y)$$

  - This says that their joint distribution *factors* into a product two simpler distributions
  - Another form:

$$\forall x, y : P(x|y) = P(x)$$

- Independence is a simplifying *modeling assumption*
  - *Empirical* joint distributions: at best "close" to independent
  - What could we assume for {Weather, Traffic, Cavity, Toothache}?



Conditional Independence

- Unconditional (absolute) independence very rare (why?)

- *Conditional independence* is our most basic and robust form of knowledge about uncertain environments.

- X is conditionally independent of Y given Z

  if and only if: $\forall x, y, z : P(x, y|z) = P(x|z)P(y|z)$

  or, equivalently, if and only if $\forall x, y, z : P(x|z, y) = P(x|z)$

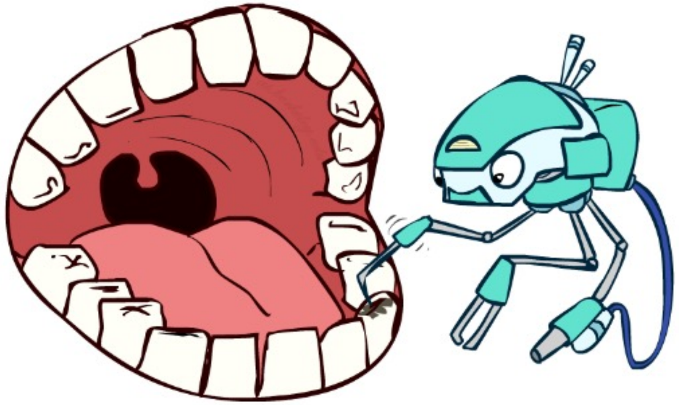
Conditional Independence

- P(Toothache, Cavity, Catch)

- If I have a cavity, the probability that the probe catches in it doesn't depend on whether I have a toothache:
  - P(+catch | +toothache, +cavity) = P(+catch | +cavity)

- The same independence holds if I don't have a cavity:
  - P(+catch | +toothache, -cavity) = P(+catch| -cavity)

- Catch is *conditionally independent* of Toothache given Cavity:
  - P(Catch | Toothache, Cavity) = P(Catch | Cavity)



- Equivalent statements:
  - P(Toothache | Catch , Cavity) = P(Toothache | Cavity)
  - P(Toothache, Catch | Cavity) = P(Toothache | Cavity) P(Catch | Cavity)
  - One can be derived from the other easily

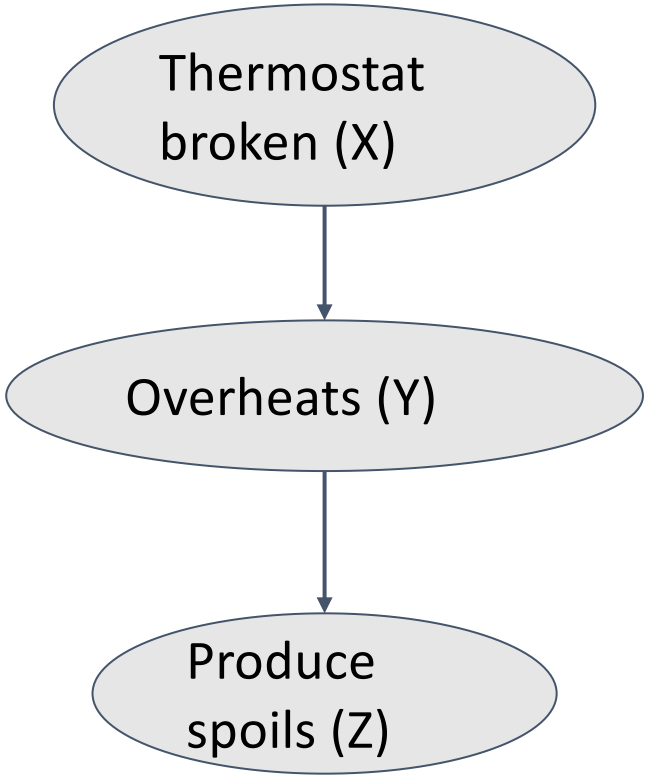
Conditional Independence examples

Bayesian Networks: Canonical Cases

Important Bayes net question: Are two nodes independent *given* certain evidence?

- If yes -- can prove using algebra
- If no -- can prove using a counterexample

Example: Are X and Z necessarily independent?



Bayesian Networks: Canonical Cases

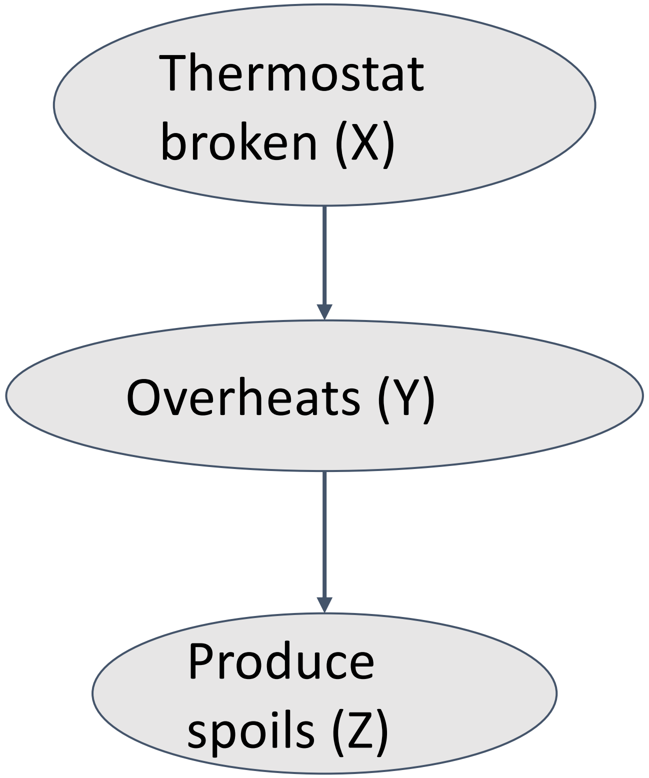
Important Bayes net question: Are two nodes independent *given* certain evidence?

- If yes -- can prove using algebra
- If no -- can prove using a counterexample

Example: Are X and Z necessarily independent?

No!

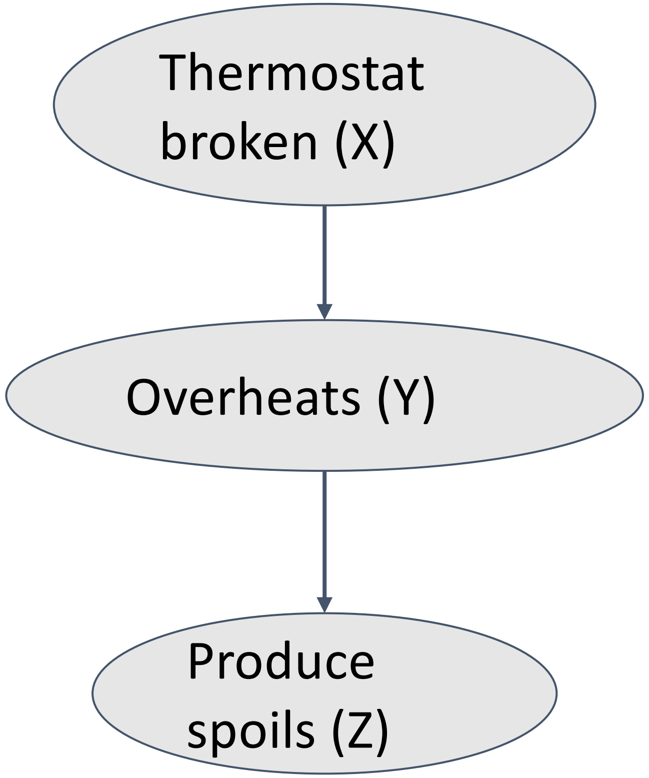
- X certainly influences Y, which influences Z
- Also, knowledge of Z influences beliefs about X (through Y)



Bayesian Networks: Canonical Cases

Example, rebooted: What about X and Z, ***given*** Y?

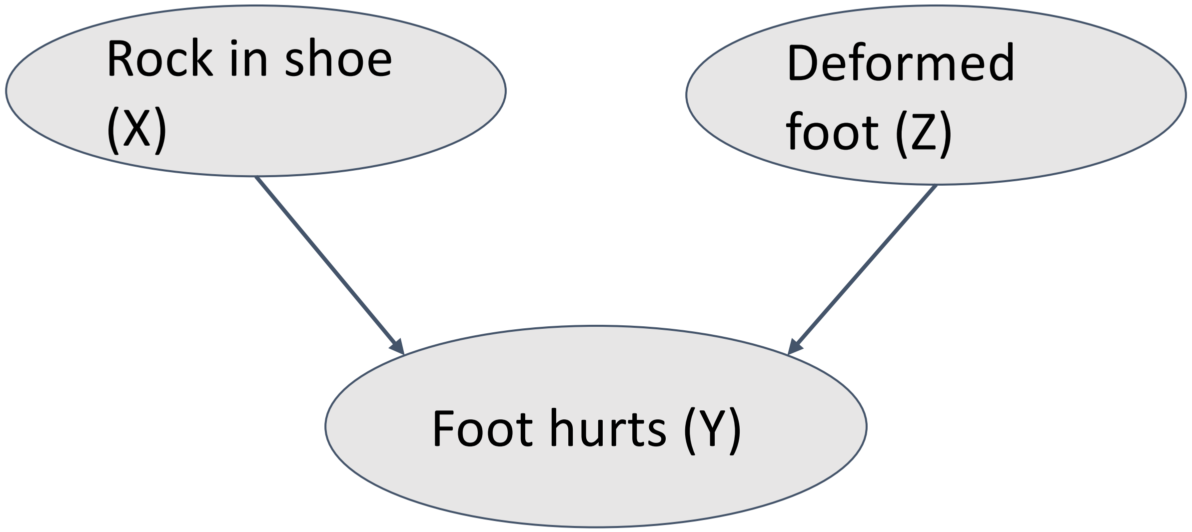
This is a canonical case is called a **causal chain**



Bayesian Networks: Canonical Cases

Common effect is the third canonical case.

→ One effect, two possible causes



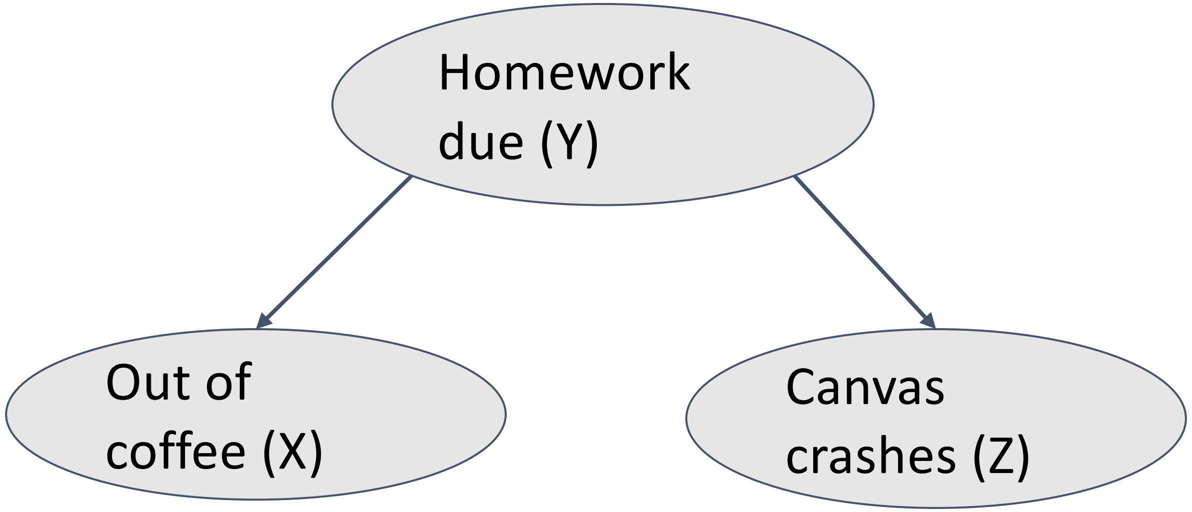
Example: Are X and Z independent?

Are X and Z independent ***given*** Y?

Bayesian Networks: Canonical Cases

Common cause is another canonical case.

→ Two effects, from the same cause

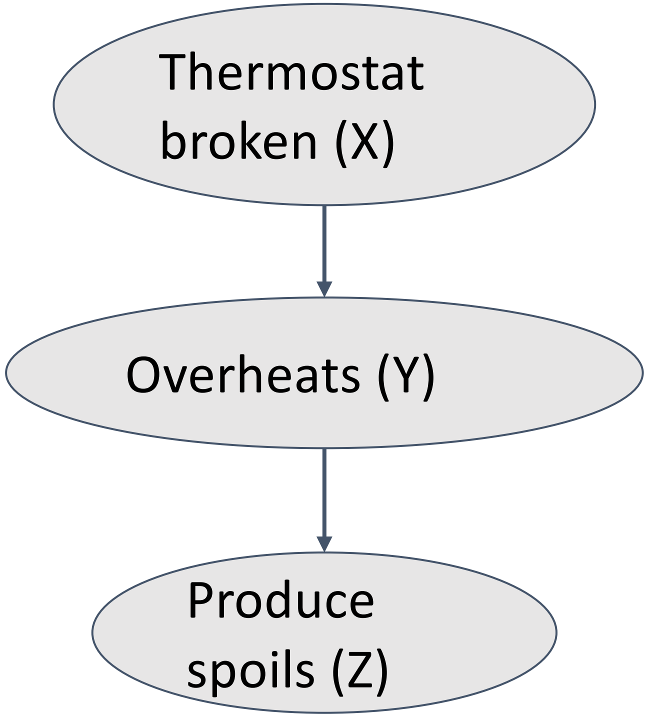


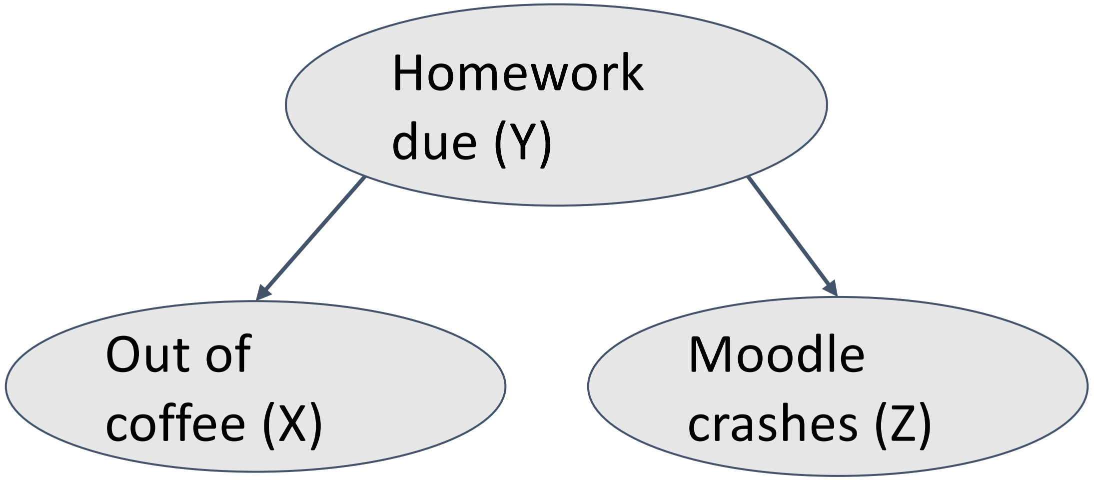
Example: Are X and Z independent?

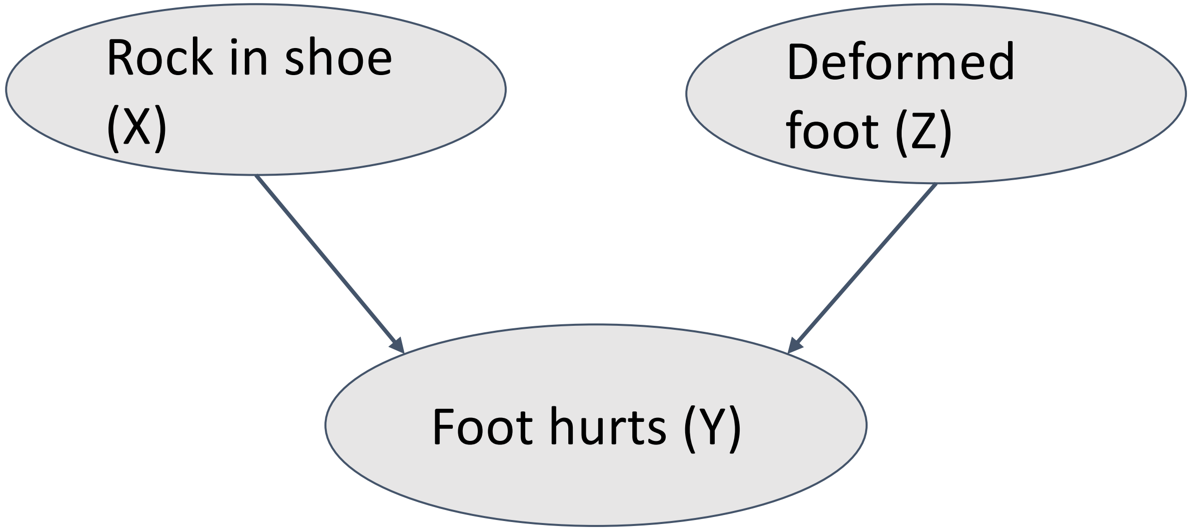
Are X and Z independent ***given*** Y?

Causal vs Diagnostic Modeling

Causal:







Diagnostic: observing an effect leads to competition between possible causes

→ *diagnose* which is most likely

Causal vs Diagnostic Modeling

Causal:

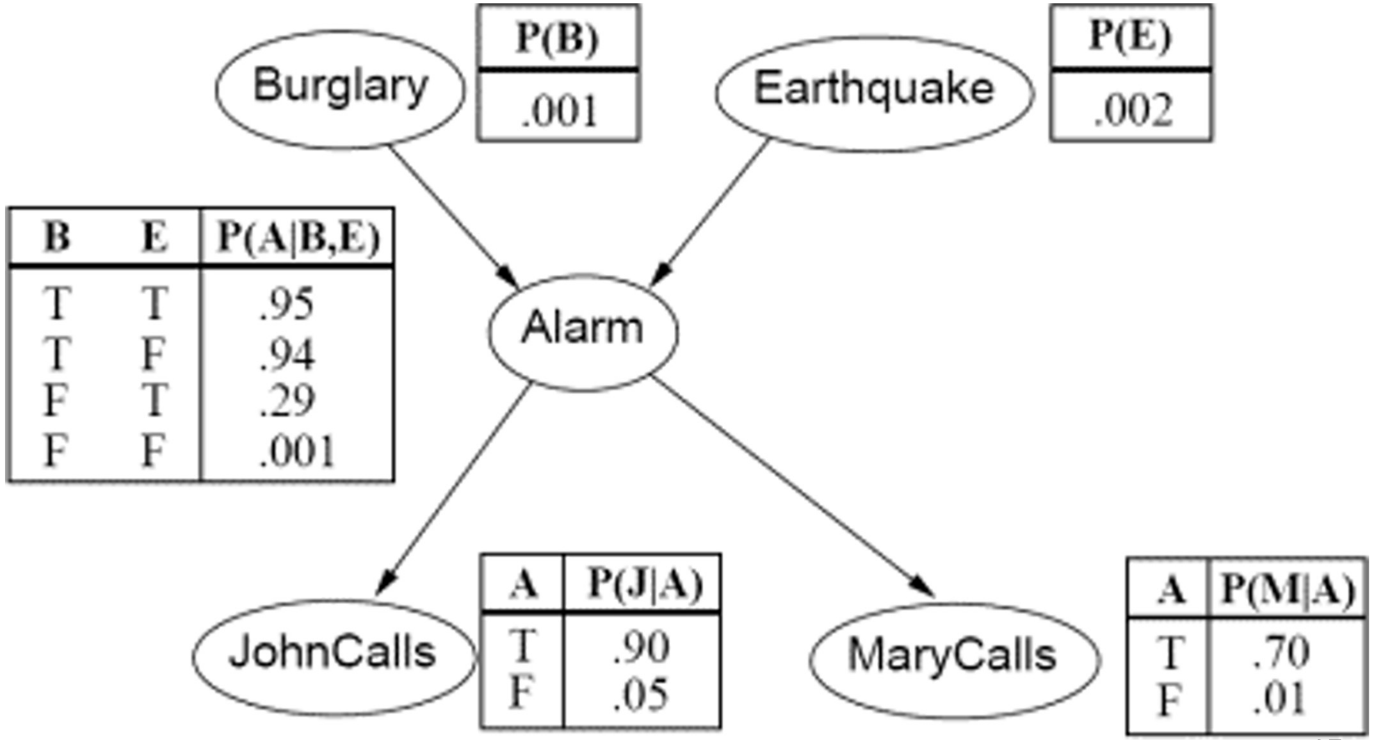
Diagnostic: observing an effect leads to competition between possible causes

→ *diagnose* which is most likely

Bayesian Networks: “Explaining Away”

Suppose we know that the alarm has gone off.

Suppose we find out later that we have been robbed



P(B)
.001

P(E)
.002

B	E	P(A|B,E)
T	T	.95
T	F	.94
F	T	.29
F	F	.001

A	P(J|A)
T	.90
F	.05

A	P(M|A)
T	.70
F	.01

Bayesian Networks – basic structure

Bayesian Networks

Bayes nets implicitly encode joint distributions as a product of the local conditional distributions:

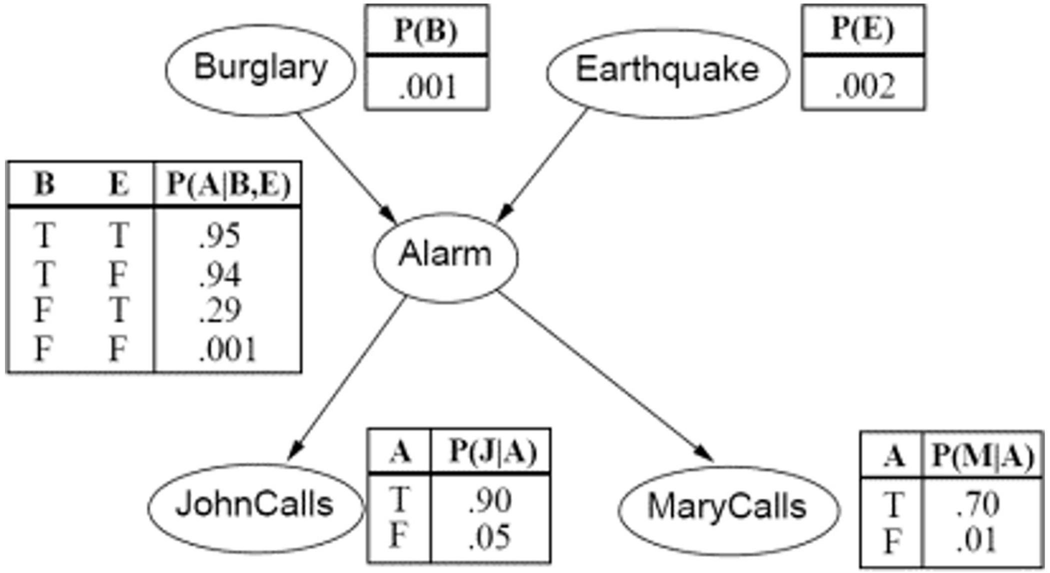
$P(x_1, x_2, \ldots, x_n) = ?$

$$
\begin{aligned}
P(x_1, x_2, \ldots, x_n) &= P(x_n \mid x_{n-1}, x_{n-2}, \ldots, x_2, x_1) P(x_{n-1}, x_{n-2}, \ldots, x_2, x_1) \\
&= P(x_n \mid x_{n-1}, x_{n-2}, \ldots, x_2, x_1) P(x_{n-1} \mid x_{n-2}, \ldots, x_2, x_1) P(x_{n-2}, \ldots, x_2, x_1) \\
&= \ldots \\
&= P(x_n \mid x_{n-1}, x_{n-2}, \ldots, x_2, x_1) P(x_{n-1} \mid x_{n-2}, \ldots, x_2, x_1) \ldots P(x_3 \mid x_2, x_1) P(x_2 \mid x_1) P(x_1) \\
&= \prod_{i=1}^{n} P(x_i \mid x_{i-1}, x_{i-2}, \ldots, x_2, x_1)
\end{aligned}
$$

Bayesian Networks – example

Bayes nets implicitly encode joint distributions as a product of the local conditional distributions:

$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$$



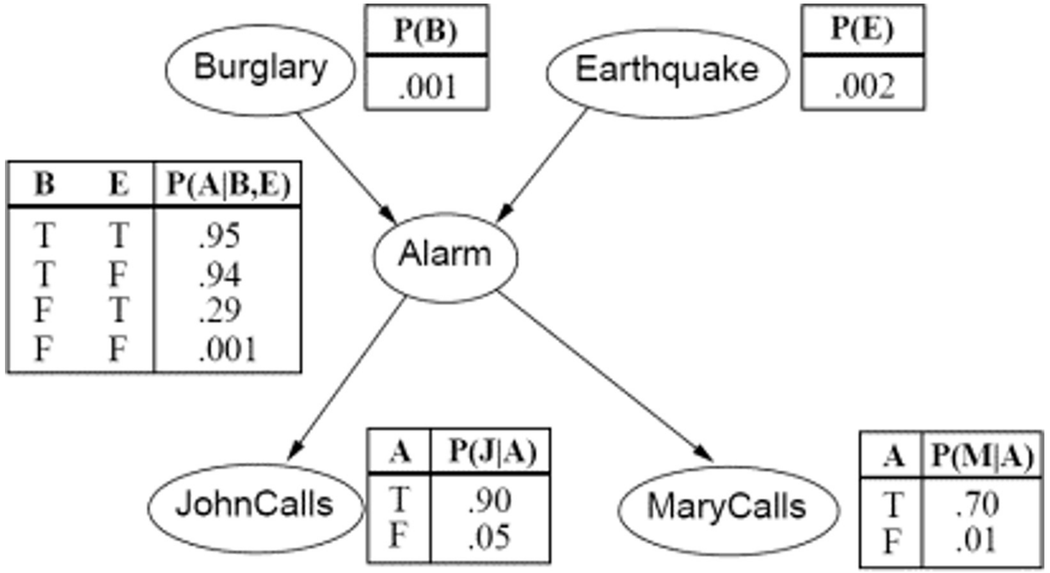
P(a) =

Bayesian Networks – example

Bayes nets implicitly encode joint distributions as a product of the local conditional distributions:

$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$$

P(j) =

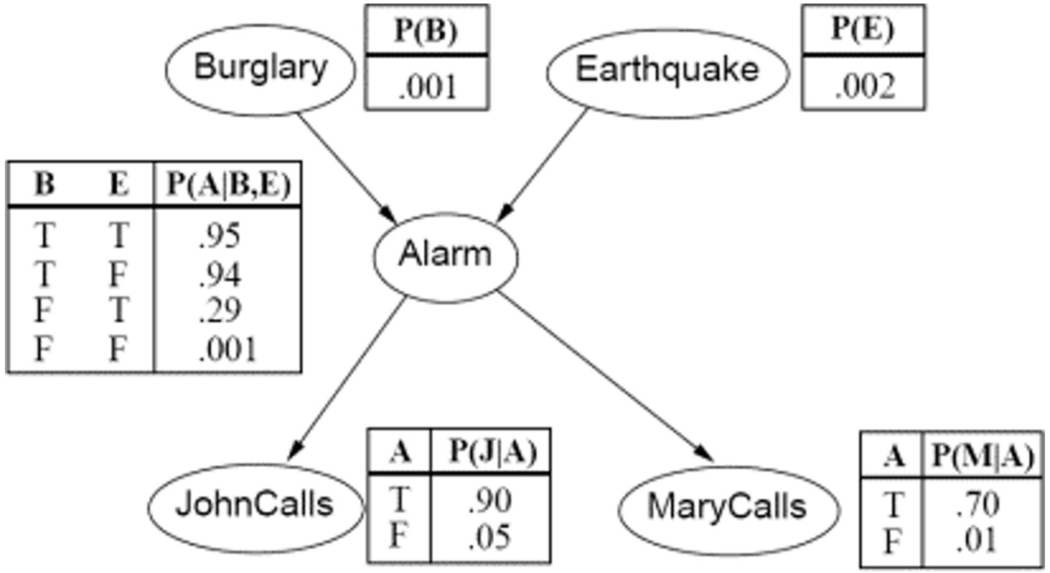


Bayesian Networks – example

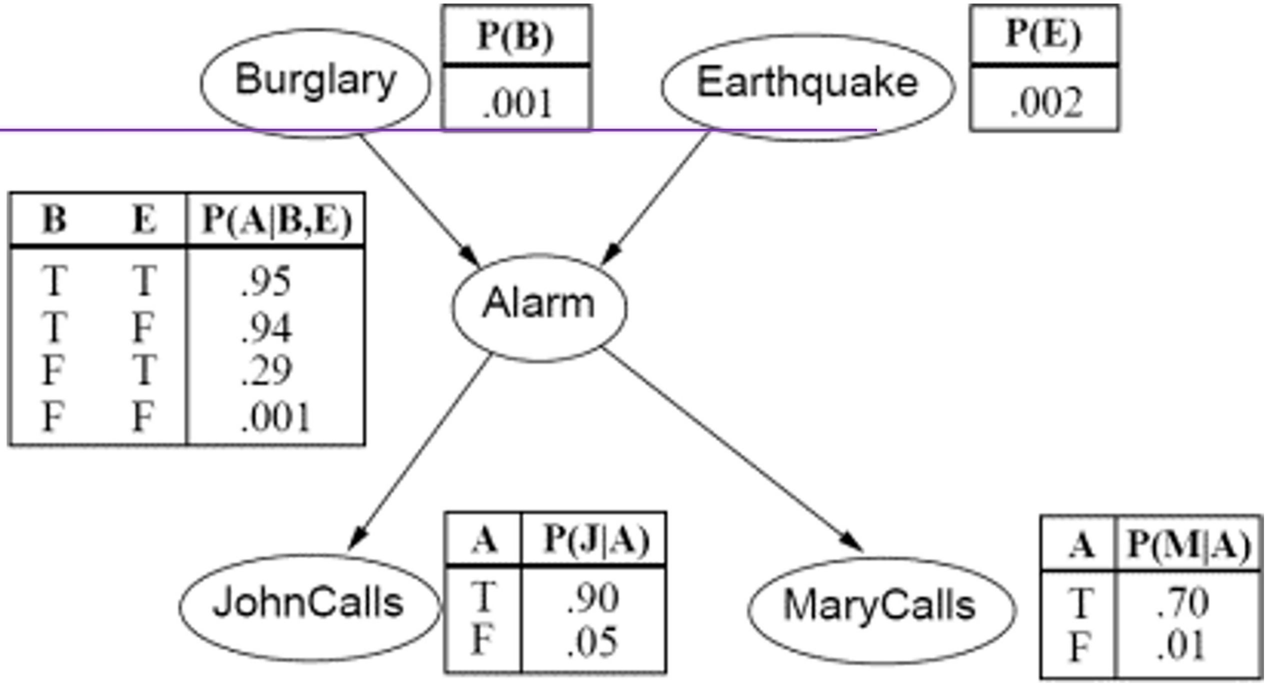
Bayes nets implicitly encode joint distributions as a product of the local conditional distributions:

$$P(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$$

P(b | j) =



Bayesian Networks



Example: *S'pose* we know the alarm has gone off, and both John and Mary have called to warn us. What is the probability that we have been burgled?

→ *P*(+b | +j, +m) = ?

Query variables: what we want the **posterior** probability of, **given** some **evidence**

X = B

Evidence variables: the variables we are given an assignment of (the **data**)

E = [+j, +m]

Hidden variables: the non-evidence, non-query variables

y = [E, A]

Bayesian Networks

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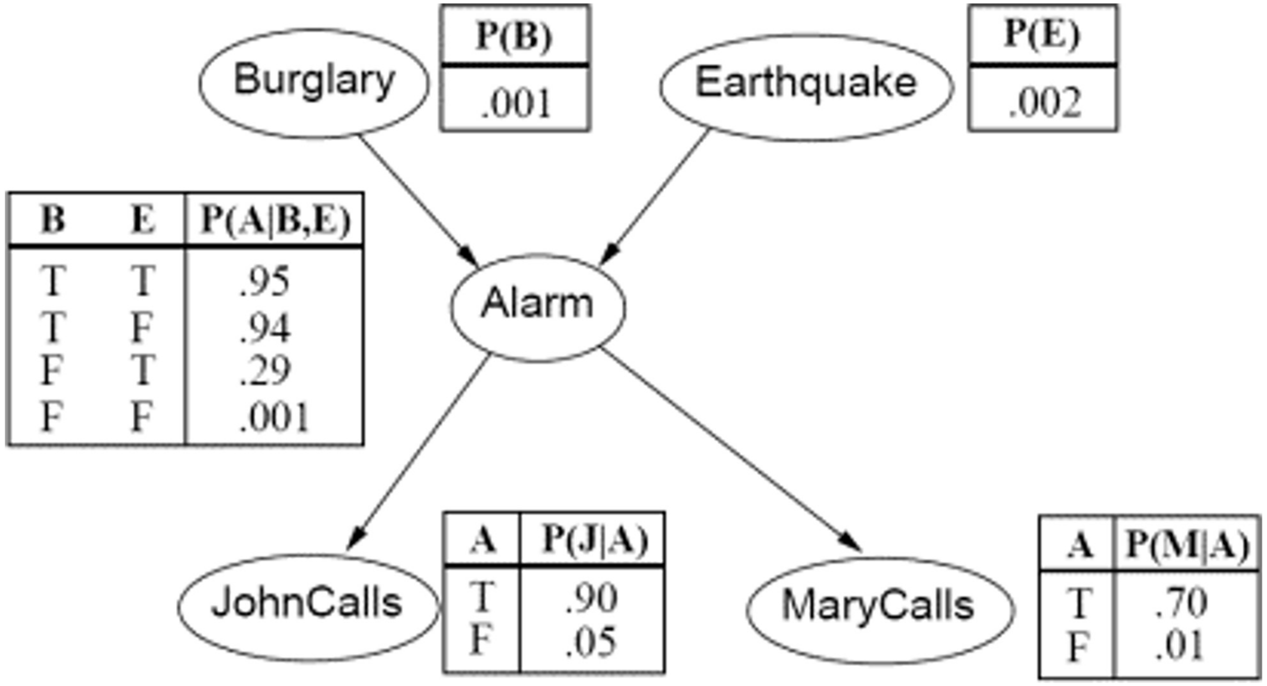

Calculation by **enumeration:**

Bayesian Networks

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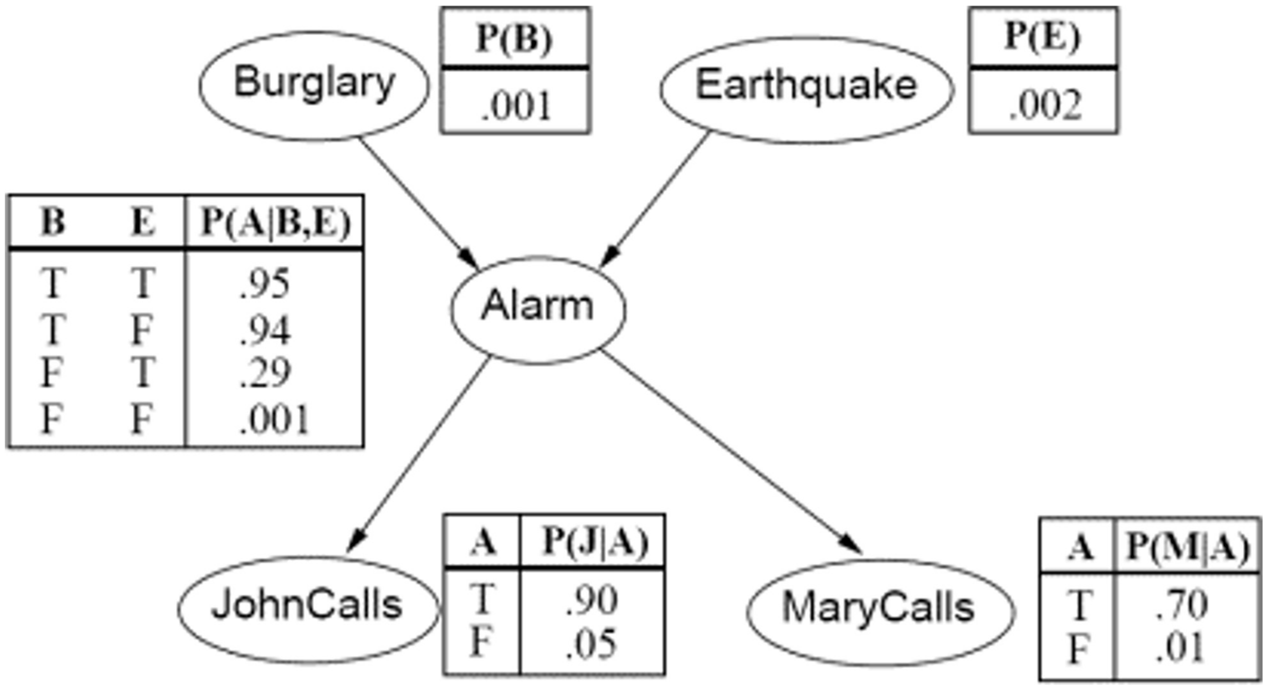

Calculation by **enumeration:**

$$P(B \mid j, m) = \frac{P(B, j, m)}{P(j, m)} \quad \Rightarrow \quad \alpha = \frac{1}{P(j, m)} \quad \Rightarrow \quad P(B \mid j, m) = \alpha P(B, j, m)$$

We'll do our thing, then figure out the normalizing constant α later

Bayesian Networks

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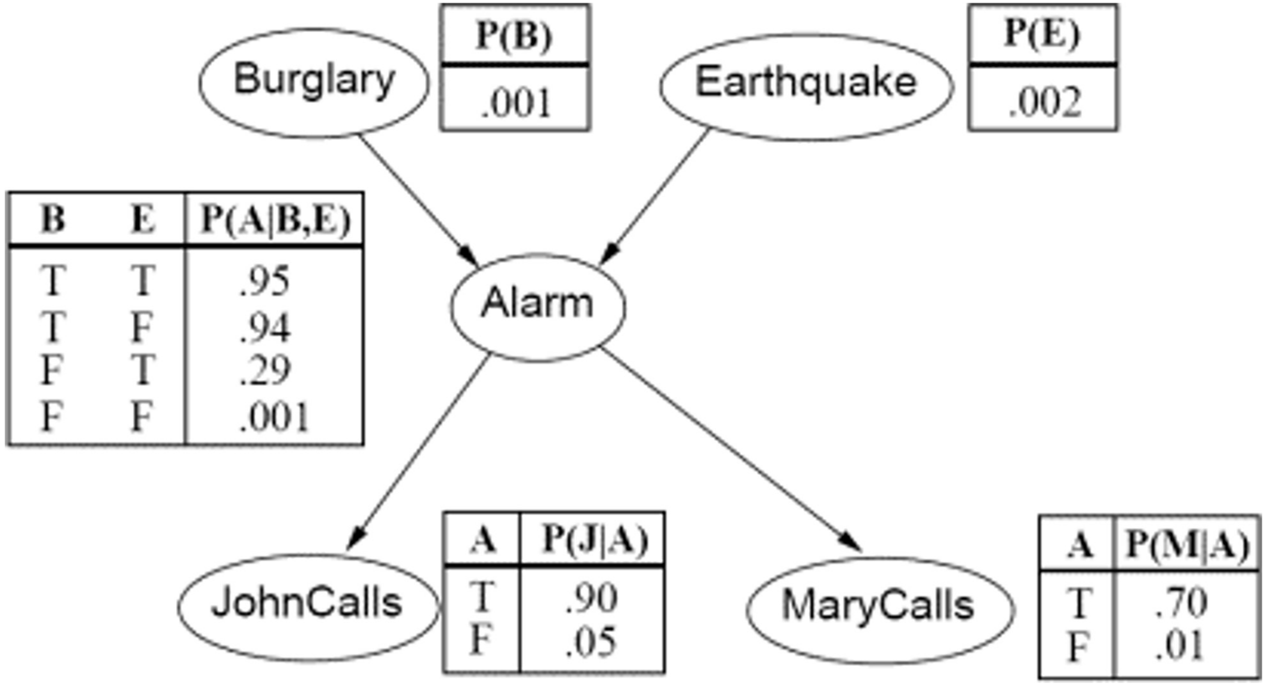


Calculation by **enumeration:**

$$P(B \mid j, m) = \alpha P(B, j, m)$$

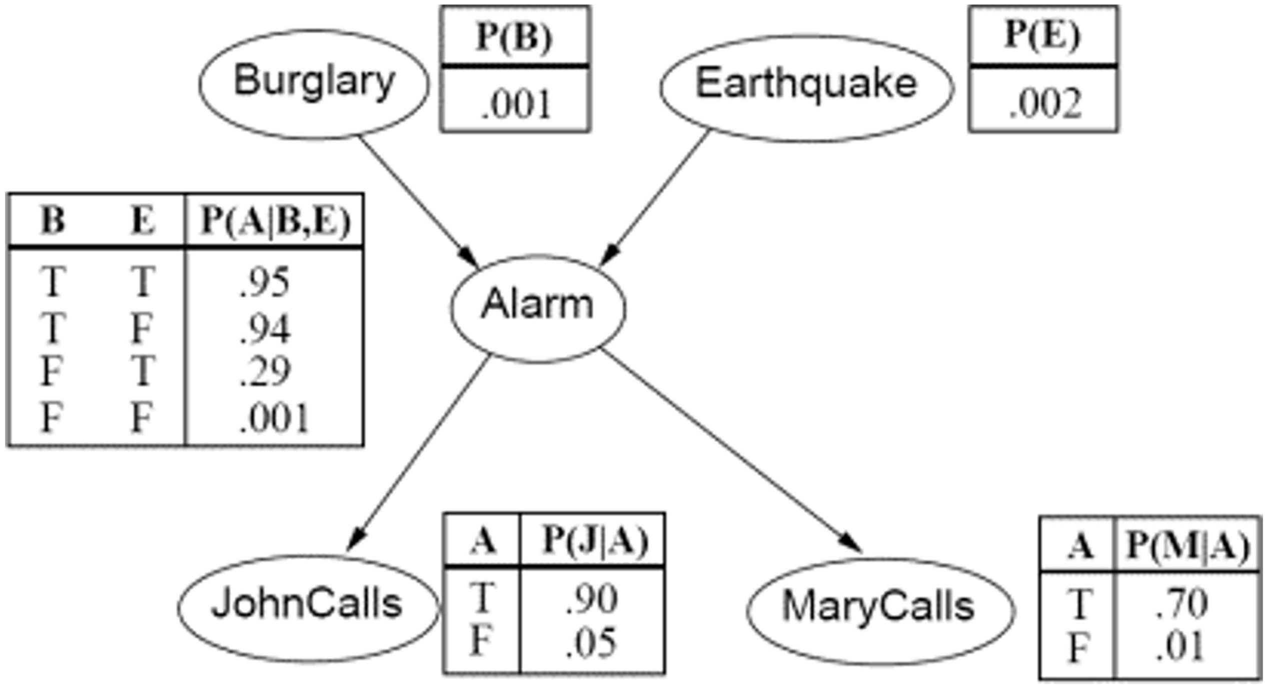
$$= \alpha \sum_a P(B, j, m \mid a) P(a)$$

$$= \alpha \sum_a P(B, j, m, a)$$

Bayesian Networks

Example: Suppose we know the alarm has gone off, and both John and Mary have called to warn us. What is the probability that we have been burgled?

→ *P*(+b | +j, +m) = ?



Calculation by **enumeration:**

$$P(B \mid j, m) = \alpha \sum_a P(B, j, m, a)$$

$$= \alpha \sum_e \sum_a P(B, j, m, a \mid e) P(e)$$

$$= \alpha \sum_e \sum_a P(B, j, m, a, e)$$

Bayesian Networks

Example: Suppose we know the alarm has gone off, and both John and Mary have called to warn us. What is the probability that we have been burgled?

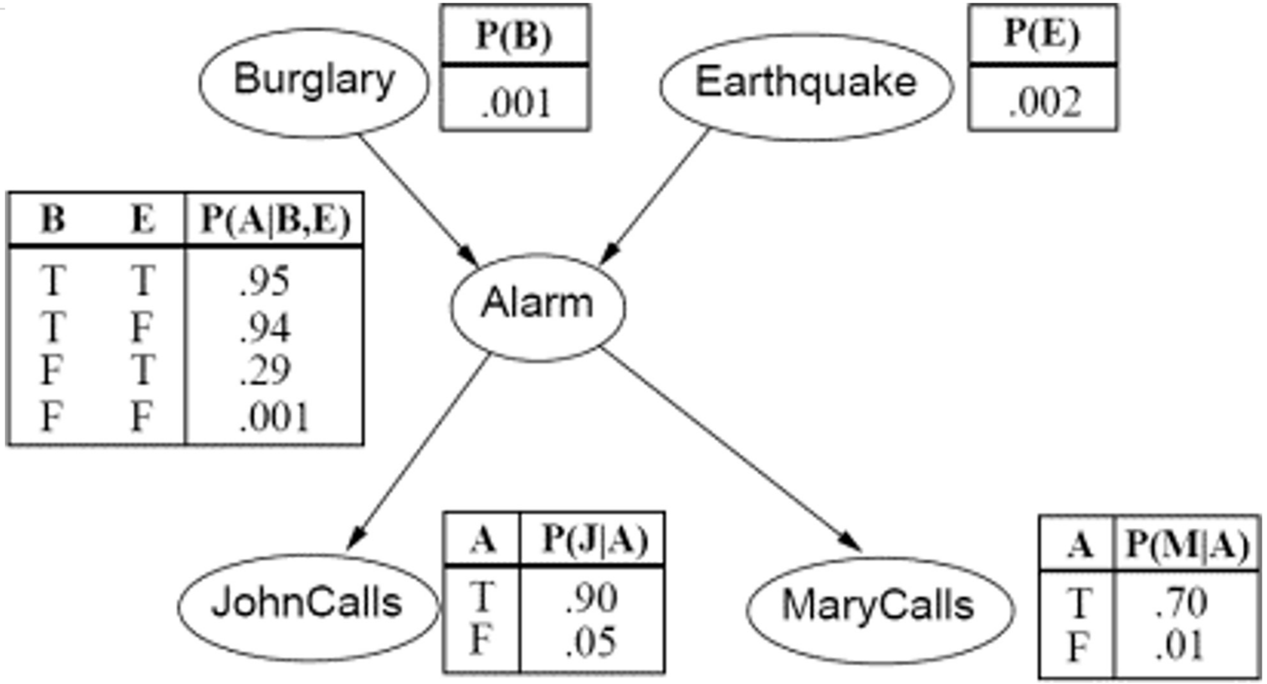
→ $P(+b \mid +j, +m) = ?$

FINALLY we have:

$$P(B \mid j,m) = \alpha \sum_e \sum_a P(B,j,m,a,e)$$

From the conditional independence of the Bayes net:

$$P(B \mid j,m) = \alpha \sum_e \sum_a \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$$



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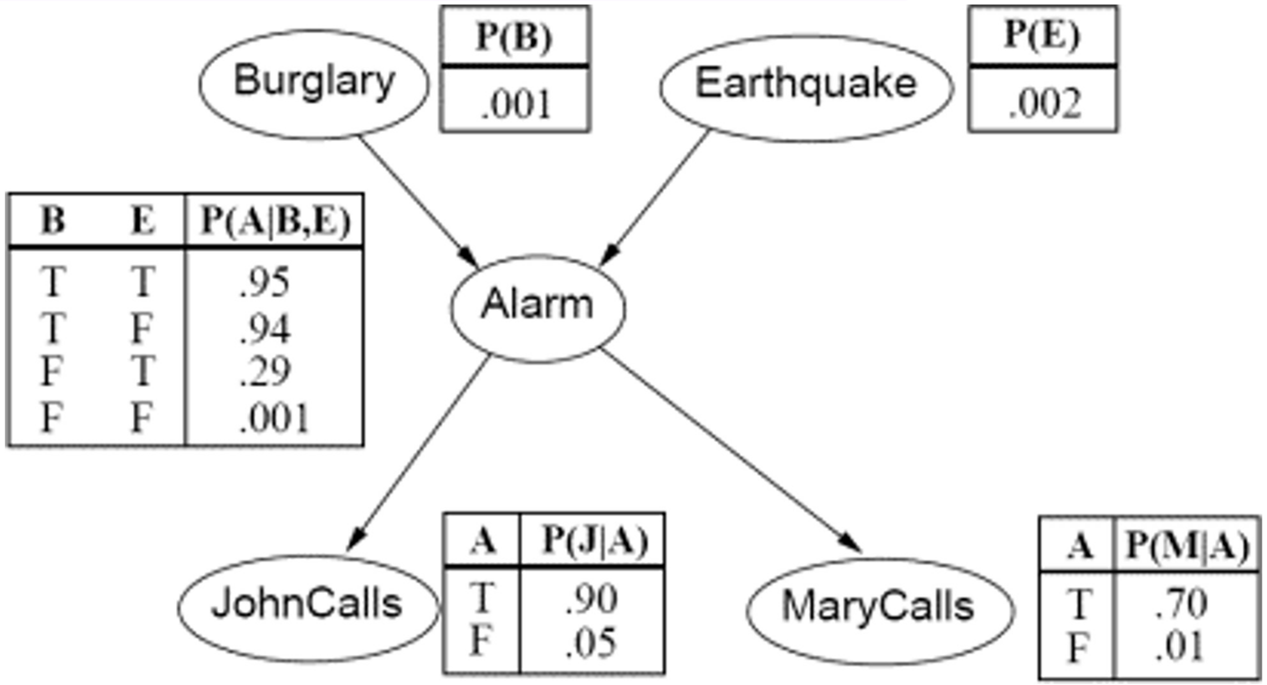


So for this problem…

$$P(B \mid j, m) = \alpha \sum_e \sum_a \prod_{i=1}^{n} P(x_i \mid \text{parents}(X_i))$$

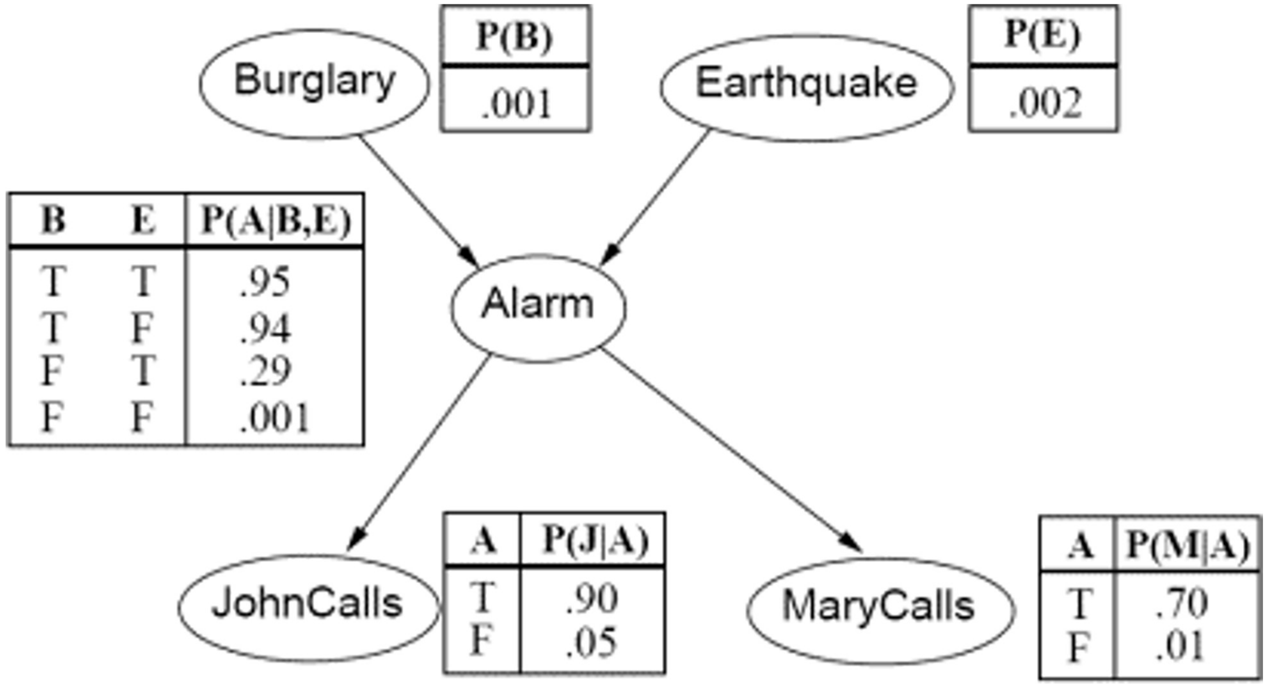
$$= \alpha \sum_e \sum_a P(B)P(e)P(a \mid B, e)P(j \mid a)P(m \mid a)$$

$$= \alpha P(B) \sum_e P(e) \sum_a P(a \mid B, e)P(j \mid a)P(m \mid a)$$

Bayesian Networks

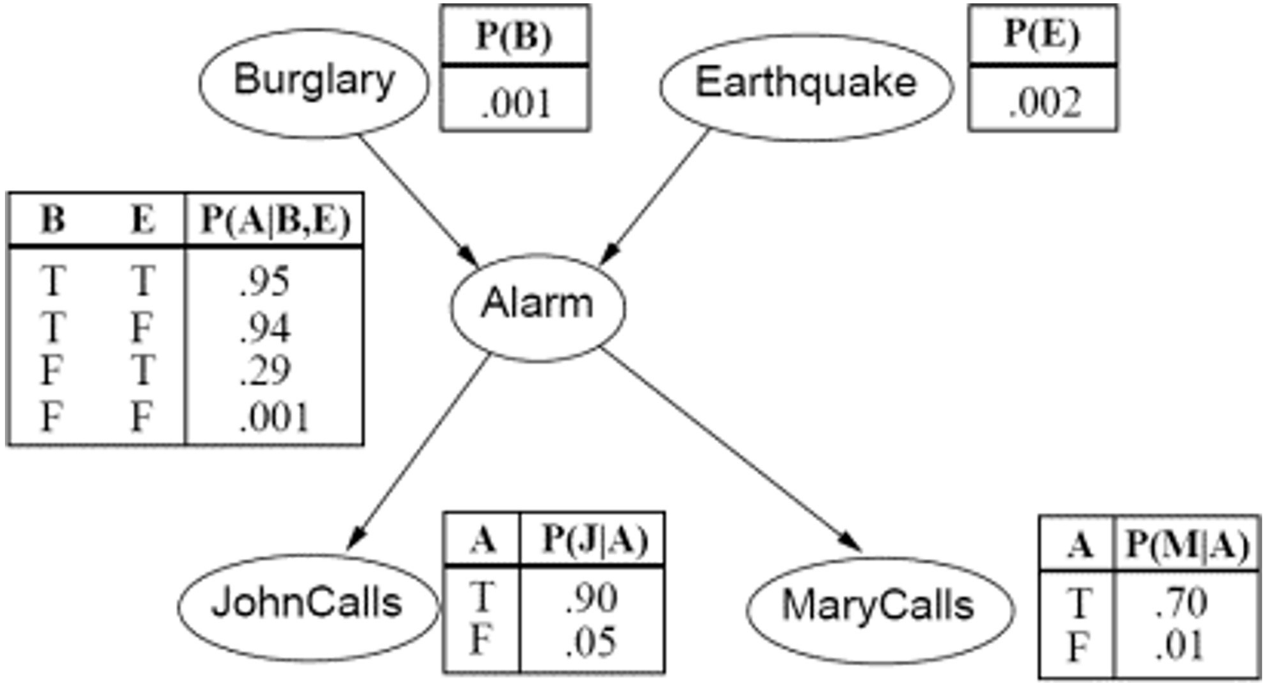
Example:

$$P(B \mid j, m) = \alpha P(B) \sum_e P(e) \sum_a P(a \mid B, e) P(j \mid a) P(m \mid a)$$



Bayesian Networks

$$P(B \mid j, m) = \alpha P(B) \sum_{e} P(e) \sum_{a} P(a \mid B, e) P(j \mid a) P(m \mid a)$$



Bayesian Networks

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