Transition

X_{t-1}	X_t	$P(X_t \mid X_{t-1})$
h	h	0.7
h	f	0.3
f	h	0.4
f	f	0.6

Emission

X_t	E_t	$P(E_t \mid X_t)$
H	Dizzy	0.1
	Cold	0.4
	Normal	0.5
F	Dizzy	0.6
	Cold	0.3
	Norm	0.1

$$X_0 = [0.6, 0.4]$$

$$\begin{array}{ccccc} X_0 & \rightarrow & X_1 & \rightarrow & X_2 \cdots \\ & & \downarrow & & \downarrow \\ & & E_1 & & E_2 \end{array}$$

$$P(X_t) = \cancel{P(X_t)} = \sum_{X_{t-1}} P(\underset{t=1}{X_t} \mid \underset{t-1=0}{X_{t-1}}) P(\underset{t-1=0}{X_{t-1}})$$

$$P(X_1) = \sum P(X_1 \mid X_0) P(X_0)$$

Evidence: $P(X_t \mid e_{1:t}) = \alpha P(e_{1\to t} \mid X_t) P(X_t)$

By Bayes $= \alpha \dfrac{P(e_t \mid X_t) \cdot P(X_t)}{P(e_{1\to t})}$

↑

unknown.

Day 1: $X_0 \to [0.6, 0.4]$

$$P(X_1) = \sum_{X_0} P(X_1 \mid X_0) \cdot P(X_0)$$

$$= [\underbrace{h \to h, f \to h}_{\text{health}},\ \underbrace{h \to f + f \to f}_{\text{fever}}]$$

$$= [0.7 * 0.6 + 0.4 \times 0.4,\ 0.3 * 0.6 + 0.6 \cdot 0.4]$$

$$= [0.58, 0.42]$$

Evidence $P(X_1 | normal) = \alpha P(Normal | X_1) \cdot P(X_1)$

$$= \cancel{\alpha}[0.5, 0.1][0.58, 0.42]$$

$$= \alpha [0.5 * 0.58, \ 0.1 * 0.42]$$

$$= \alpha [0.29, 0.042] \qquad \alpha = 0.29 + 0.42$$

$$= 0.332$$

Normalizing $\left[\frac{0.29}{\alpha}, \frac{0.42}{\alpha}\right]$

$$= [0.87, 0.13]$$

$$P(X_1 = health | normal) = \frac{P(Normal | healthy) \cdot P(health)}{P(Normal)}$$

$$= \frac{0.5 \cdot \cancel{0.48} 0.58}{0.332} = 0.87$$

$P(X_2 | E_{1:t} = normal, dizzy)$

$$P(X_2 | normal) = \sum_{X_1} P(X_2 | X_1) \cdot P(X_1 | normal)$$

$$= [\underbrace{0.7 * 0.57 + 0.4 * 0.13}_{heath}, \ \underbrace{0.3 * 0.87 + 0.6 * 0.13}_{fever}]$$

$$= [0.661, \ 0.339]$$

Evidence: $P(X_2 | normal, dizzy) =$

$= \alpha\, P(d_{k}34_{2} \mid X_2)\, P(X_2 \mid \text{normal})$

$= \alpha\, P[0.1 * 0.661,\ 0.6 * 0.339]$

$= \alpha\, [0.0661,\ 0.2034]$

$\alpha = 0.0661 + 0.2034] = 0.2695$

$P(X_2 = \text{health} \mid n, d) = \dfrac{0.1 * 0.661}{0.2695} = 0.245$

$P(X_2 = \text{Fever} \mid n, d) = \dfrac{0.6 * 0.339}{0.2695} = 0.755$

$P(X_2 \mid n, d) = [0.245,\ \cancel{0.27}\ 0.755]$

$P(X_3 \mid n, d, n)$

$P(X_3 \mid n, d) = \sum P(X_3 \mid X_2) \cdot P(X_2 \mid n, d).$

$= [0.7 * 0.245 + 0.4 * 0.755,\ 0.3 * 0.755 + 0.6 * 0.245]$

$= [0.47,\ 0.53]$

Evidence

$P(X_3 \mid n, d, n) = \alpha\, P(n \mid X_3) \cdot P(X_3 \mid n, d)$

$= [0.5 * 0.47,\ 0.1 * 0.53]$

$\alpha = 0.24 + 0.053 = 0.293$

$= \left[\frac{0.24}{0.293}, \frac{0.053}{0.293}\right]$

$= [0.82, 0.18]$

$P(X_3 = H \mid n, d, n)$ $\quad$ $P(X_3 = F \mid n, d, n)$

Part 4 Equation to incorporate evidence in $P(X_4 \mid n, d, n, c)$

$P(X_4 \mid n, d, n, c) = \alpha\, P(cold \mid X_4) \cdot P(X_4 \mid n, d, n)$

$= \alpha\, P(cold \mid X_4) \sum_{x_3} P(X_4 \mid X_3) P(X_3 \mid n, d, n)$

What is your belief in X_4 without evidence?

$P(X_4 \mid n, d, n) = \sum P(X_4 \mid X_3) \cdot P(X_3 \mid n, d, n)$