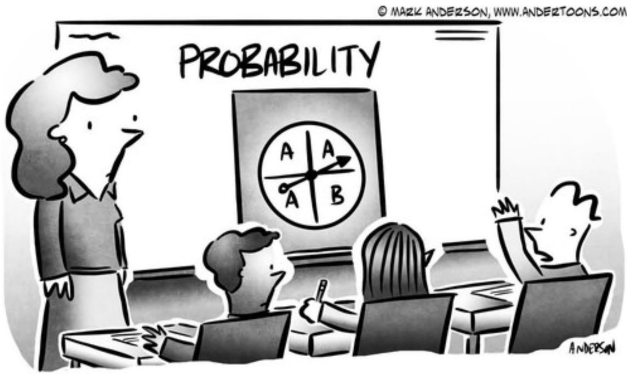

"I know mathematically that A is more likely, but I gotta say, I feel like B wants it more."

LECTURE 6

Probability, Part 1

Basic Rules of Probability Used in CSCI 3022

CSCI 3022

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Course Logistics: Your Third Week At A Glance

Mon 1/29	Tues 1/30	Wed 1/31	Thurs 2/1	Fri 2/2	Sat 2/3	
	(Optional): Attend Notebook Discussion with our TA (5-6pm Zoom)	Attend & Participate in Class	HW 3 Due: 11:59pm via Gradescope	Quiz 2: Scope - Lessons: Prerequisites & L1-L3 (HW 2, nb2) Attend & Participate in Class		
			Graded HW 2 posted	HW 4 Released Discussion NB 4 released		

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Roadmap

Lesson 6, CSCI 3022

- **Finish Lesson 5: Visualization**
- Probability: Key Concepts from Discrete Structures used in CSCI 3022

Probability

Lesson 6, CSCI 3022

- Finishing Lesson 5: Visualization
- **Probability: Key Concepts from Discrete Structures used in CSCI 3022**

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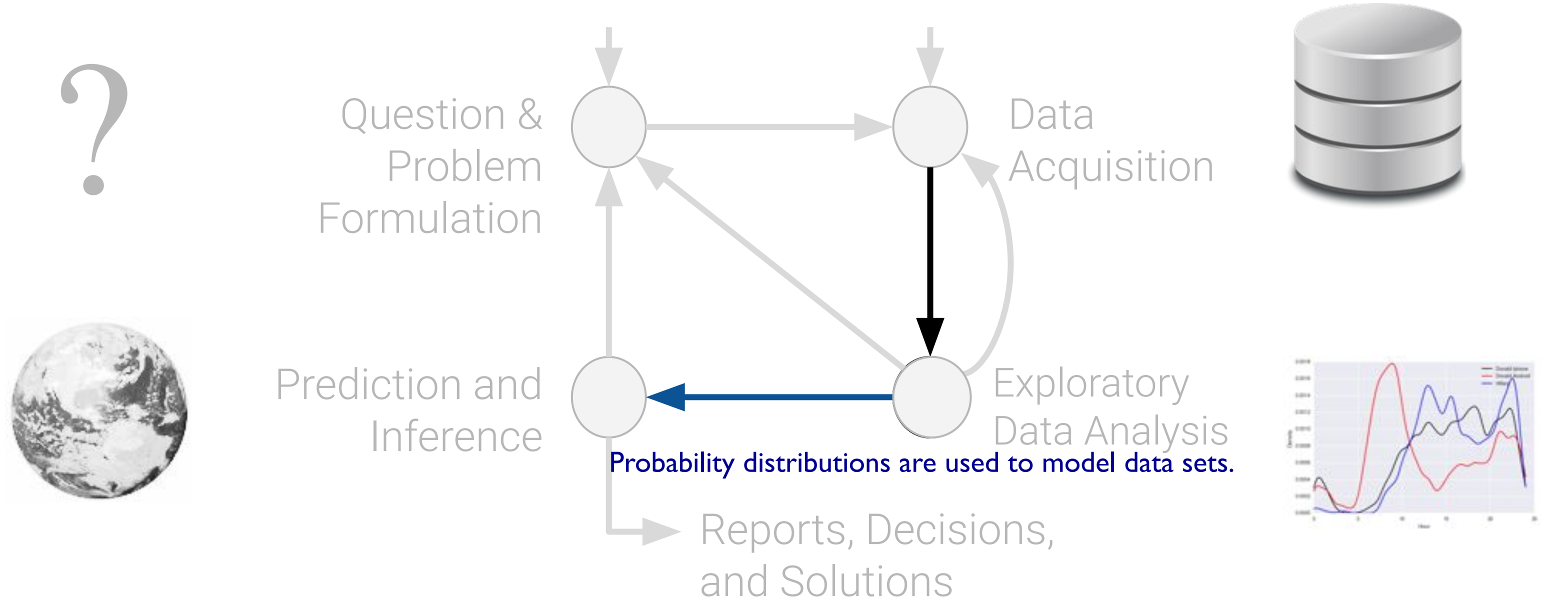
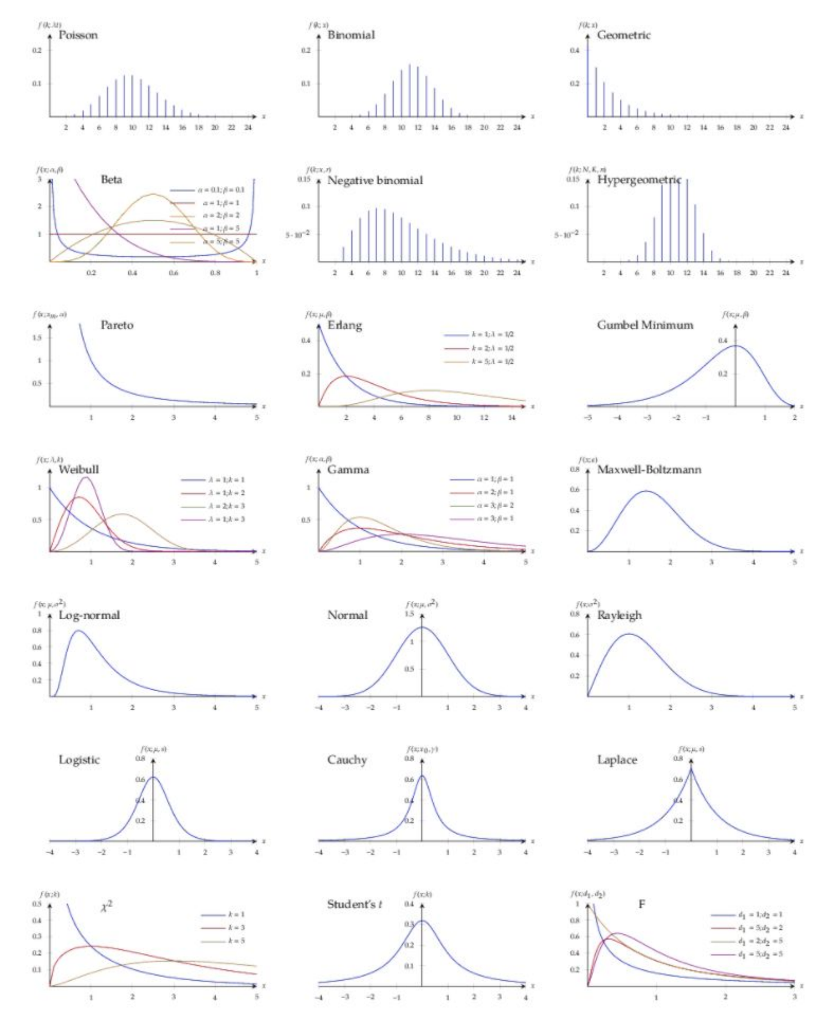
Plan for next 3 weeks



(Weeks 3 -5)

Probability & Probability Distributions



- Probability and more specifically probability distributions are used to model data sets.

- We can study and/or ‘assign’ a probability distribution to a set of data in an attempt to discover patterns and relationships.

Probability Terminology

Experiment: A procedure that can be repeated, involves and element of chance, and has well defined outcomes.

Outcome: Result of an experiment

Sample Space Ω: The set of all possible outcomes

Cardinality of Ω (denoted $|\Omega|$): The number of elements in the sample space set.

Event: Subset of Ω

	Example 1	Example 2
Experiment	Flipping a coin	Flipping a coin twice
Sample Space Ω		
$\|\Omega\|$		
Example of an event	Getting tails:	Getting at least one tail:

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Key Probability Recap from Discrete Structures

A **probability function, *P*,** assigns a value in $[0, 1]$ to each outcome or event in the sample space Ω, such that:

- $P(\Omega) = 1$
- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

Complement Rule: $P(A') = 1 - P(A)$

In general, the conditional probability of an event A, given that an event B has occurred, is equal to:

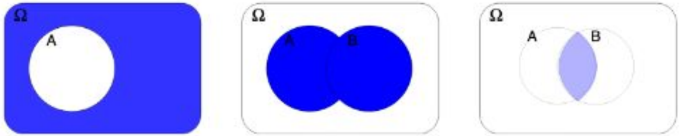
$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

provided $P(B) > 0$.

this leads us to the multiplication rule:

The multiplication rule:

Basic Set Operations



Complement; A^C; "Not"

Union; $A \cup B$; "Or"

Intersection; $A \cap B$; "And"

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Philosophy of Probability

Probability of equally likely outcomes:
If Ω is a finite nonempty sample space of ***equally likely* outcomes**, and E is an event, that is, a subset of Ω, then the probability of E *is*

$$P(E) = \frac{|E|}{|\Omega|}$$

- Applies easily to fair coins/die/card examples
- Doesn't handle situations when outcomes are not equally likely
- Doesn't handle situations when the # of possible outcomes is infinite.

What does a meteorologist mean when they say there's a 10% probability that rain will occur somewhere in the area for which the forecast is being prepared?

Objective (Frequentist view) Interpretation:
Defines probabilities as relative frequencies.
So, what occurs in the long run is the probability.

Track all days on which it is forecast to rain with probability 10%. Over long period of time, percent of days it actually does rain gets closer and closer to 10%

Subjective (sometimes called Bayesian) Interpretation:
Defines probabilities as subjective degree of belief.

Someone or some group (i.e. people who constructed weather model) believe with 10% confidence it will rain today.

If you're interested in exploring this more check out **STAT4700: Philosophy of Statistics**

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Probability Practice

I have 3 cards: Ace of Hearts, King of Diamonds and Queen of Spades.

I shuffle them and draw one card (without replacement) and then draw a 2nd card.

What is the chance that I get the Queen followed by the King?

Multiplication Rule

Chance that two events *A* and *B* both happen= P(*A* happens) x P(*B* happens given that *A* has happened)

- The answer is *less than or equal to* each of the two chances being multiplied
- The more conditions you have to satisfy, the less likely you are to satisfy them all

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Probability Practice

I have 3 cards: Ace of Hearts, King of Diamonds and Queen of Spades.

I shuffle them and draw one card (without replacement) and then draw a 2nd card.

What is the chance that one of the cards I draw is a King and the other is a Queen?

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Probability Practice

What is the probability of getting at least one head in 3 coin tosses?

General Problem Solving Technique For Calculating Probabilities Involving Trials of Events:

Ask yourself what event must happen on the first trial:

- If there’s a clear answer (e.g. “not a six”) whose probability you know, you can most likely use the **multiplication rule**.

- If there’s no clear answer (e.g. “could be King or Queen, but then the next one would have to be Queen or King …”), list all the **distinct ways** your event could occur and **add up their chances**.

- If the list above is long and complicated, look at the **complement**. If the complement is simpler (e.g. the complement of “at least one” is “none”), you can find its chance and subtract that from 1.

Discussion Question

A population has 100 people, including Rick and Morty.

We sample two people at random without replacement.

(a) P(both Rick and Morty are in the sample)

(b) P(neither Rick nor Morty is in the sample)

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