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0.1 Question 9 (5 pts)

Write a function `summation(n)` that uses vectorization in Numpy to evaluate the following summation for $n \geq 1$:

$$\sum_{i=1}^{n} (i^3 + 3i^2)$$

Note: You should **NOT** use ANY `for` loops in your solution. You may find `np.arange` helpful for this question!

In [ ]:
```
def summation(n):
    """Compute the summation i^3 + 3 * i^2 for 1 <= i <= n."""
    return np.sum(((np.arange(1, n + 1)) ** 3) + (3 * (np.arange(1, n + 1)) ** 2)) #np.arange v

#Do not change the following cells:
print("summation(5) =  ", summation(5))
print("summation(200) = ", summation(200))
```

In [ ]:
```
grader.check("q9")
```

Question 10 Solution) **Type your answer to Question 10** in this cell (You can copy the LaTeX from the first 2 lines above as a start and then complete the rest of the problem). Show all of your steps and fully justify your answer. Do not add any additional cells to this part.

$$
\begin{aligned}
&\int_0^\infty \lambda e^{-\lambda x}\, dx && (1)\\
&= \lim_{b\to\infty} \int_0^b \lambda e^{-\lambda x}\, dx && (2)\\
&= \lim_{b\to\infty} \int_0^b \lambda \frac{e^u}{-\lambda} = \lim_{b\to\infty} -e^u && (u = -\lambda x\ ,\ du = -\lambda dx,\ dx = \tfrac{1}{-\lambda} du)\\
&= \lim_{b\to\infty} -e^{-\lambda x}\Big|_0^b && \text{(substitute back x)}\\
&= \lim_{b\to\infty} -e^{-\lambda b} + e^0 && (3)\\
&= \lim_{b\to\infty} \frac{1}{-e^{\lambda b}} + e^0 && (4)\\
&= 0 + 1 = 1 && \text{(evaluated the tag)}
\end{aligned}
$$

Question 11abc Solutions) Use LaTeX (not code) in the cells below to show all of your steps for parts 11a, 11b and 11c and fully justify your answers. Do not add any additional cells to this part.

Enter your answer for part 11a) in this cell (double click on this cell and write all steps using Markdown and LaTex):

##

a)

The definition of conditional probability is the probability that an event will happen knowing that something else happened. Mathematically, this is represented by:

$$P(A|B) =$$

$$\frac{P(A \wedge B)}{B}.$$

There are other ways we can manipulate that equation but essentially it can be written like so. Here, we are saying that the probability that of an event A given B is the probability that both events happen over the probability of B.

Enter your answer for part 11b) in this cell (double click on this cell and write all steps using Markdown and LaTex):

So, first, we need to define the conditional probability in this case:

$$P(B|A) = \frac{P(B)}{P(A)}$$

In this case, $B =$ The two flips both yield heads, while $A =$ at least one of the flips is yields heads. (notice that the numerator is suppoed to be $P(A \wedge B)$ but in this case it is just P(B) because B itself overrides A because B has both of them as heads and A is asking that at least one of them is heads.)

A:

To calculate P(B), we can say that when you flip two coins, there is four possible choices: HT, HH, TT, TH

Because B is concerned with HH which only occurs once in the above possible choices, we can easily calculate the probability by saying: $P(B) = 1/4$.

To calculate P(A), we have to analyze this further: P(A) = P(Only one of the coins yields heads) + P(Both coins yield heads). From the possible choices given above, we can say (let's call the first probability $P(A_1)$ and the second $P(A_2)$):

$$
\begin{aligned}
P(A) &= P(A_1) + P(A_2) && (5)\\
&= 2/4 + 1/4 && (6)\\
&= 3/4 && (7)\\
& && (8)
\end{aligned}
$$

Now that we have both of the probabilities, we can calculate P(B | A) easily by:

$$P(B|A) = \frac{P(B)}{P(A)}$$

$$P(B|A) = \frac{1/4}{3/4}$$

$$P(B|A) = \frac{1}{3}$$

Enter your answer for part 11c) in this cell (double click on this cell and write all steps using Markdown and LaTex):

##

Part i)

First of all, we need to define how many permutations we can have with the eight marbles we have. Notice that in the question, order really matters here so this is a permutation and not a combination.

Second, notice that replacement is not allowed. This means that we need to use the following equation used for permutation with no replacement:

$$\frac{n!}{(n-r)!}$$

Since we have 8 total marbles and we are trying to arrange them in pairs, then the total numbers of ways we can arrange them is:

$$\frac{8!}{(8-2)!}$$

$$\frac{8!}{(6)!}$$

$$8 \cdot 7 = 56$$

There are 56 ways we can arrange those 8 marbles when replacement is not allowed and when order matters.

Now, the question asks us to find the following: $P(B|A) = \frac{P(A \wedge B)}{P(A)}$.

Now, we need to find P(A) and P(A and B). Assuming that A = the first marble is red and B = the second marble is red.

There is two ways that the first marble can be red: the first marble is red and the second one is red as well, and then the first one is red but the second is blue.

For the first case, when only considering the red marbles and trying to find the different permutation of trying to group them in two groups where order matters but with no replacement. We can use the same exact permuation equation we used above but instead here $n = 5$ and $r = 2$. Notice ofcourse that r is still the same because still want to group them in two:

$$\frac{5!}{(5-2)!}$$

$$\frac{5!}{(3)!}$$

$$5 \cdot 4 = 20$$

There are 20 ways to combine two red marbles together. Now, in the second case, where the first marble is red and the second blue. Here, we can say that if you take one red marble and with that marble combine it with the rest of the blue marbles, you will have 3 ways to order them that way where we have a red marble first then the blue. But now, multiply $3 * 5$ (since we are doing it for all red marbles), and you will 15 ways where you can get the red marble first then the blue.

Thus, $P(A) = \frac{20+15}{56} = \frac{35}{56}$.

Now, for $P(A \wedge B)$, we are saying that the first one has to be red but also the second one has to be red as well. You can see that we already calculated the permuations for that above where we had 20 ways to combine marbles in a way that both are reds.

Thus, $P(A \wedge B) = \frac{20}{56}$.

Therefore,

$$
\begin{align}
P(B|A) &= \frac{P(A \wedge B)}{P(A)} \tag{9}\\
&= \frac{\frac{20}{56}}{\frac{35}{56}} \tag{10}\\
&= \frac{20}{35} \tag{11}\\
&= \frac{4 \cdot 5}{7 \cdot 5} \tag{12}\\
&= \frac{4}{7} \tag{13}\\
\tag{14}
\end{align}
$$

##

Part ii)

In here, we are only asked to find the probability that both of the marbles are blue.

We already know the number of permutations to group the 5 red marbles and 3 blue marbles into two groups where order matters and replacement not allowed which is 56.

We can use the same equation we used above to find how many ways we can group two blue marbles together. Since we have 3 blue marbles and we have to group them into two groups, we have:

$$\frac{3!}{(3-2)!}$$

$$\frac{3!}{(1)!}$$

$$3 \cdot 2 = 6$$

There are 6 ways to have two blue marbles together.

Thus, P(Both are blue marbles) =

$$\frac{6}{56}$$