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Optimal Assignment (Assignment 5)

$$Q1. \text{Entropy } H(Y) = - \sum_{i=1}^K p(Y=y_i) \log_2 p(Y=y_i)$$

Finding the entropy of taste

$$p(\text{taste} = \text{Meh}) = \frac{5}{10}$$

$$p(\text{taste} = \text{Yummy}) = \frac{5}{10}$$

hence

$$H(Y) = -\frac{5}{10} \log_2 \left(\frac{5}{10}\right) - \frac{5}{10} \log_2 \left(\frac{5}{10}\right)$$

$$= \underline{\underline{1.0}}$$

Q2:

Information gain given visual defects as root of decision tree.

$$\text{Information gain } IG(\text{visual defects}) = H(\text{taste}) - H(\text{taste} | \text{visual defect})$$

Visual defects

Some - 3, all 3 are meh

None - 4 $\begin{cases} 2 - \text{meh} \\ 2 - \text{yummy} \end{cases}$

many = 3, 3 are yummy

$$H(\text{taste} | \text{visual defect}) =$$

$$\frac{3}{10} (-\frac{3}{3} \log_2 \frac{3}{3}) + \frac{4}{10} (-\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4}) +$$

$$\frac{3}{10} (-\frac{3}{3} \log_2 \frac{3}{3}) = \boxed{0.4}$$

$$\text{Information gain} \Rightarrow H(\text{taste}) - 0.4$$

$$= 1.0 - 0.4$$

$$= \boxed{0.6}$$

Question 3

entropy

④ (Taste | Visual defect == some)

Visual defects

some - 3 $\left\{ \begin{array}{l} \text{meh} = 3 \\ \text{yummy} = 0 \end{array} \right.$

$$\begin{aligned} \Rightarrow & -\frac{3}{3} \log_2 \left(\frac{3}{3} \right) - 0 \log_2 (0) \\ = & -1 \log_2 (1) - 0 \log_2 (0) \\ = & \boxed{0} \end{aligned}$$

⑤ visual defects :

(Taste | Visual defect == none)

Visual defects

none - 4 $\left\{ \begin{array}{l} \text{meh} = 2 \\ \text{yummy} = 2 \end{array} \right.$

$$\begin{aligned} \Rightarrow & -\frac{2}{4} \log_2 \left(\frac{2}{4} \right) - \frac{2}{4} \log_2 \left(\frac{2}{4} \right) \\ = & -\frac{1}{2} - \frac{1}{2} = \boxed{1} \end{aligned}$$