Probabilistic Quantification of Power Distribution System Operational Resilience

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***Abstract*—High-impact, low-probability (HILP) events resulting from extreme weather conditions have a significant impact on the aging power distribution infrastructures. It is of growing concern to minimize the impacts of such catastrophic events on critical infrastructures by appropriately hardening the infrastructure and implementing new operational procedures. This calls for a quantitative assessment of distribution system resilience that can not only predict the impacts of the future events but also be used to evaluate different planning measures taken to minimize the impacts of extreme events. In this paper, we propose a probabilistic metric to quantify the operational resilience of the distribution grid. The metric is based on Conditional Value-at-Risk ($CVaR$) measure where resilience is defined as the conditional expectation of loss of energy in MWh for events beyond a prespecified risk threshold. A simulation-based framework to evaluate the proposed metric for resilience under different weather scenarios is presented. The impacts of restorative actions, specifically using distributed generators and remote-controlled switches (RCS) and the impacts of infrastructure hardening on resilience metric is also investigated. Numerical simulations are performed on the IEEE 123-bus test system.**

***Index Terms*—Distribution system resilience, Resilience metric, Value at Risk (VaR), Conditional Value at Risk (CVaR),**

I. Introduction

In recent years, weather-related outages have severely affected the performance of aging electric power distribution systems. Recently, when hurricanes Harvey, Irma, and Maria hit the United States, a total of around 7.5 million customers in Texas, Florida and Puerto Rico were left without electricity [1]. Due to climate change, the frequency and severity of the extreme weather-related outages are expected to increase in near future [2]. This calls for proactive threat management paradigm that is driven by high-impact low-probability (HILP) events rather than persistent costs.

The operational resilience of the distribution system is characterized as the system's ability to respond to and recover from an HILP event. Planning for resilience requires a metric that can not only quantify the impacts of a future event on the grid but also help evaluate/compare different planning alternatives for their contribution to improving resilience. Traditionally, the performance of the power distribution system is measured using post-event reliability metrics such as SAIDI, SAIFI, MAIFI that provide an evidence-based performance indication on how well a specific distribution grid responded to the normal chance failure events/outages. Unlike routine outages, adequately anticipating and responding to HILP events is inherently difficult as they are rare. Therefore, the impacts of HILP events cannot be properly quantified using reliability metrics calling for further considerations for beyond the classical reliability-oriented view [3].

Resiliency and robustness of the power system modeled as a complex network have been studied in [4], [5]; however, there exist a few indicators to compare the effectiveness of different proactive measures on power grid resilience. Numerous optimization-based restoration methods as proposed in [6], [7] quantify the resilience based on the amount and duration of critical loads restored. A few authors developed the metric to quantify the resilience by measuring "reduced consequences from failures" or "deviation of system performance" [8], [9]. While a significant body of literature exists in defining and quantifying resilience, no formal grid resilience metric is universally accepted. It is desired that the resilience metric provides an indication on the *potential impacts* of a *future HILP event*. In addition, the metric should also measure the *expected performance* of the system during an extreme event and quantify the effectiveness of *potential resilience enhancement strategies and investments*.

Along these lines, a few have suggested using risk-based characterization for resilience [10]. For example, [11] proposes a method to assess resilience at the critical infrastructure sector level for risk-based resource decision making. A probabilistic approach to assess and evaluate the time-dependent resilience metric in transmission network is proposed [12]. However, specific risk-based metrics and a framework to evaluate the same for power distribution grid has not been developed. Moreover, in existing literature little to no effort has been made on a detailed resilience quantification framework that can help compare improvements in distribution grid resilience due to alternate planning measures.

With these considerations, we propose a framework to evaluate the resilience of the distribution grid using a risk-based quantitative measure called conditional-value-at risk ($CVaR$). $CVaR$ metric has been extensively used in risk-averse financial planning to manage the impacts of low-probability high-risk financial investments. Similar considerations apply when managing the impacts of HILP events making it a suitable metric to quantify not only the potential impacts of HILP events but also for comparative evaluation of the potential

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benefits received from alternate planning investments. Notice that advanced grid applications that leverage upon distribution automation and other smart grid technologies can help improve operational resilience. (GridAPPS-D is a platform under development that aims to support such advanced grid applications [13]). The specific contributions are summarized below:

1) A $CVaR$ metric is defined and a framework is developed to quantify the operational resilience of the power distribution system when impacted by $(1-\alpha)\%$ highest impact events.
2) Monte-Carlo simulations are performed to understand the impacts of risk and uncertainty of catastrophic events on distribution grid resilience with the help of $CVaR$ metric. Different levels of damages in the distribution grid are taken into consideration by modeling different probabilities of component failure when exposed to an extreme event.
3) The proposed metric is also used to quantify the effects of potential threat-management solutions that a utility can employ for improving system resilience. This is achieved by developing system-specific models for the functional dependence between the event impacts and system loss and recovery functions under specified resource allocations.

II. Probabilistic Distribution Grid Resilience

In this section, we describe our approach for probabilistic quantification of system resilience when subjected to HILP events. The resilience metric is computed based on the loss in system resilience caused by probabilistic threat events.

A. Definitions

1) Probabilistic event characterization: An event is characterized by two parameters - intensity of the impact (I) and probability of its occurrence ($p(I)$). The intensity affects the failure probability of the system equipment which will affect the system performance loss function.

2) System Loss Function, $U(I)$: Resilience quantifies changes in system performance over time when impacted by an event. System performance is represented as a non-linear function of loss of load, $L(I)$ and total time taken to recover the system back to an acceptable level of performance, $t(I)$ [3]. Loss function is defined as the area under the performance curve. To simplify the analysis, the non-linear performance curve is approximated using a triangular function (see Fig. 3) [14]. With this approximation, the decrease in system resilience, or loss function, $U(I)$, when impacted by event I is given by (1). Note that the loss function is impacted by proactive planning measures such as advanced restoration methods (active islanding) and hardening of distribution lines (making them underground). These effects can be modeled by appropriately representing the system performance curve.

$$U(I) = 0.5 \times (L(I)\ t(I)),\ \textit{in MWh} \tag{1}$$

3) Resilience Metrics: We define two risk-based resilience metrics: Value at Risk (VaR) and Conditional-Value-at-Risk $CVaR$. VaR calculates the maximum loss expected over a given time period and given a specified degree of confidence. Simply, VaR refers to the lowest amount ζ such that with probability α the loss will not exceed ζ. In case of resilience quantification we measure the decrease in resilience or system loss function, $U(I)$, to quantify VaR metric.

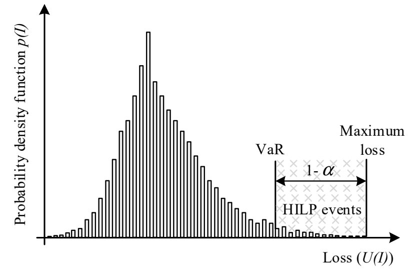

Fig. 1. VaR and CVaR assessment for a probabilistic weather event. HILP events identified as the top $(1-\alpha)\%$ high impact threats.

The probability of system loss, $U(I)$, when impacted by event I not exceeding a threshold ζ is given by (2).

$$\psi(\zeta) = \int_{U(I)\leq\zeta} p(I)dI \tag{2}$$

where, ψ is the cumulative distribution function for the loss which determines the behavior of random event I. By definition, with respect to a specified probability level α in $(0,1)$, VaR_α is given by (3).

$$VaR_\alpha = \min\{\zeta \in \mathbb{R} : \psi(\zeta) \geq \alpha\} \tag{3}$$

Next, $CVaR$ metric is defined to calculate the expected resilience loss due to probabilistic threat events, conditioned on the events being HILP. In general, $CVaR_\alpha$ is defined as the conditional expectation of the loss greater than those associated with VaR_α. For resilience quantification, the metric $CVaR_\alpha$ is computed based on the decrease in system resilience caused by those probabilistic threat events I that cause highest impacts [15]. In other words, the expected resilience loss (MWh) due to the top $(1-\alpha)\%$ of high impact events is characterized by the metric $CVaR_\alpha$. Thus, a $CVaR_\alpha$ metric finds the expected resilience loss due to threat events, conditioned on the events being HILP (i.e., in the tail of $U(I)$ distribution (See Fig. 1)), as detailed in (4).

$$CVaR_\alpha = (1-\alpha)^{-1} \int_{U(I)\geq VaR_\alpha} U(I)\ p(I)\ dI. \tag{4}$$

B. Probabilistic Event and Impact Model: Regional Wind Profiles and Fragility Modeling

In this paper, we specifically develop resilience metric to evaluate the effect of wind related weather phenomenon. The weather events are characterized using probability density functions (PDF) associated with their severity. For example, Fig. 2a shows PDFs of regional wind speed for three different regions observing extreme, high, and normal wind speeds.

The next task is to model the impact of weather event on distribution system. Component-level fragility curves can be used to effectively model the impacts of hurricanes or other high-wind events on on distribution system components. These curves express the failure probability of power system components as a function of the intensity of weather parameter (e.g., wind speed) [12]. Specifically, a fragility function maps

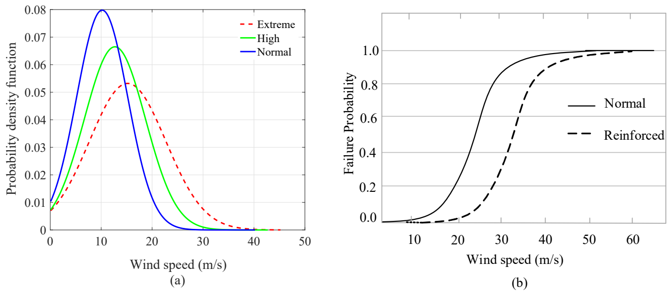

Fig. 2. Fragility curve and regional wind profile.

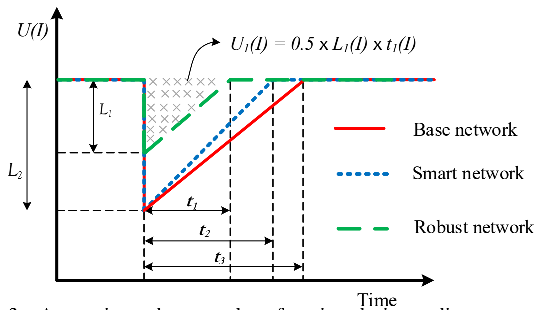

Fig. 3. Approximated system loss function during a disaster scenario.

the probability of failure of distribution system components (distribution lines, poles/towers) conditioned on the intensity of the hazard (e.g., a wind speed). An example of the line fragility curve is shown in Fig. 2b, which relates the failure probability of lines to wind speed and expressed as following.

$$P_l(\omega) = \begin{cases} P_l^n, & \text{if } \omega < \omega_{critical} \\ P_l(\omega), & \text{if } \omega_{critical} < \omega < \omega_{collapse} \\ 1, , & \text{if } \omega > \omega_{collapse} \end{cases}$$

where, $P_l(\omega)$ is the failure probability of line as a function of wind speed (ω), P_l^n is the failure rate for normal weather condition, $\omega_{critical}$ is the wind speed at which the line's failure probability picks up the value and has negligible probability of survival at $\omega_{collapse}$. All these values were selected randomly for simulation purposes. However, empirical statistical data, if available, can be used to adjust the parameters in the fragility curves. Note that Fig. 2(b) also shows another fragility curve in which the components are hardened for higher intensities of weather events. This represents an example of resilience enhancement case, for example, undergrounding the distribution lines to reduce the impacts of wind-related phenomenon.

C. Characterize Loss Function (MWh)

The component level fragility curves are used to generate system loss function. Since weather intensity and its impact on equipment are not deterministic, a Monte-Carlo simulation method, largely used in bulk-system reliability calculation, is employed to evaluate the probabilistic impacts of weather event (in this case wind) on distribution system. Specifically, multiple random Monte-Carlo runs are simulated by sampling weather event from PDF in Fig 2a and corresponding component failure probability from Fig 2b. The component fails or remains operational based on a random number generated using component failure probability obtained from fragility curve for specific weather-event corresponding to a specific Monte-Carlo simulation. After the operational state of all components in the distribution system are obtained for a specific Monte-Carlo simulation, the loss of load for the system is measured. The process is repeated for several Monte-Carlo simulations and system loss function is obtained.

Next, we present the approach to include recovery/restoration process in defining system loss function. Once an extreme event strikes a distribution network, utility starts the recovery process by restoring the power to outaged loads. The time to recovery depends on several factors including disaster impact, and the presence of smart grid technologies that can be used of restoration such as grid-forming DERs and remote-controlled switches (RCS) to provide alternate restoration paths. In this paper, to capture the effect of advanced restoration on enhancing resilience, we formulate the restoration process as a mixed-integer linear program (MILP) with the objective of maximizing the load restored with the help of all available resources. Note that grid-forming DERs can help maintain the basic level of services to critical loads in the distribution system by forming small islands thus improving the resilience. Please refer to [7] for the detailed formulation on restoration problem that includes DERs.

Fig. 3 shows the approximated system loss functions represented using loss-of-load (L) and recovery time (t). If L or t equals zero, then there are no performance loss and the system is assumed to be well operated with sufficient resilience. From an operational standpoint, a decision maker can improve resilience by allocating resources to either lessen the average impact (L) or to decrease the time to recovery (t) or both. For example, reinforcing distribution lines can lessen the impact of disaster while advanced restoration methods using active islanding and using RCS can help reduce recovery time (Fig. 3). These alternatives are included while characterizing the system loss function thus the metric can help compare alternate planning measures towards improving resilience.

D. Compute Risk-based Resilience Metric (CVaR and VaR)

The loss function, $U(I)$, is calculated by randomly sampling the weather event using Monte-Carlo simulations. The system loss computed for different wind speeds is mapped onto the PDF of wind profile. This results in a probabilistic loss function for the system under consideration due to event I which in this case is high wind speeds. The curve for probabilistic loss function obtained for a given weather event is used to compute VaR_α and $CVaR_\alpha$ metric using (3) and (4). For a given confidence level, α $CVaR_\alpha$ measures the conditional expectation of observing a system loss (energy loss in MWh) due to $(1-\alpha)\%$ of highest impact events.

III. Proposed Methodology

The proposed methodology for resilience quantification of a given distribution grid is detailed. The overall methodology is shown in Fig. 4 and is broadly divided into three steps:

Step 1: The fragility curve of each line is obtained from the given weather data and distribution line parameters. Fragility

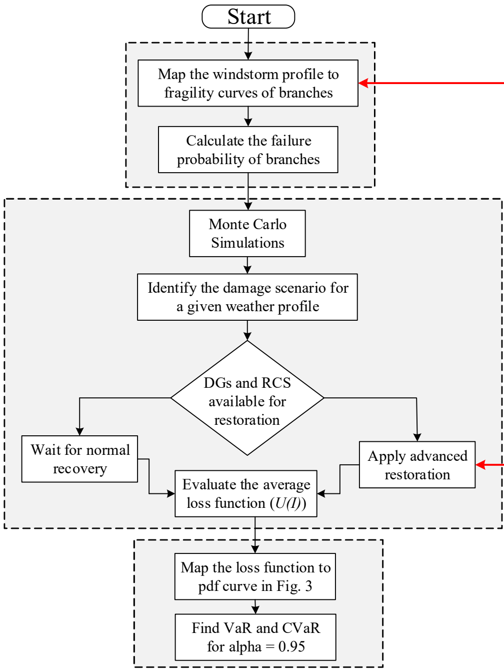

Fig. 4. Algorithm to calculate probabilistic resilience metric

curve can be derived empirically, experimentally or analytically using expert judgments [16]. The probability density function of the regional wind profile for the extreme case can be obtained from meteorological data collected using weather sensors. By mapping the wind speed PDF (Fig.2a) to the component fragility curves (Fig. 2b), the event-dependent failure probabilities for distribution components are obtained. Here, we assume that the weather intensity across the distribution grid is same as it covers a small geographical area.

Step 2: Monte-Carlo simulations are done to identify the impact of each damage scenario on the overall distribution system. The wind-dependent operational state of distribution lines as obtained from Monte-Carlo simulation is used to generate the corresponding damage scenario. The damage scenario is then fed into the restoration problem. The loss in resilience is evaluated using the loss function defined in (1). Two different cases are considered for the recovery of the network from the damage scenario. First, the distribution grid is assumed to be smart with improved response and restoration using DERs and RCS. Second, a conventional distribution grid is assumed that recovers normally without any smart actions.

Step 3: The loss function is defined to integrate both the impact and recovery time after an extreme event such that resilience metric is made multidimensional. The obtained loss function is mapped into the PDF curve in Step 1 using which VaR_α and $CVaR_\alpha$ are computed following (3) and (4).

IV. Results and Discussions

In this study, IEEE 123-node test case is used to validate the effectiveness of the proposed approach. The total demand on the feeder is 3490 kW and 1920 kVAr. Two tie switches, 54-94 and 151-300 are included in the model for creating different restoration scenarios. In addition, to fully utilize the performance of tie switch 54-94, line 93-94 is ugraded to three phase. Three grid-forming DERs are located in the feeder to assist the restoration process (See Fig. 5).

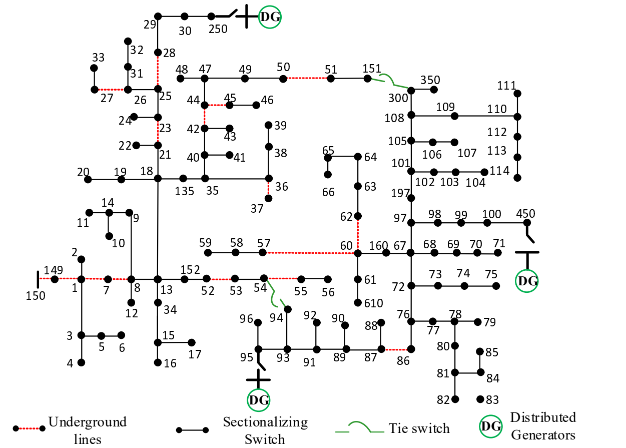

Fig. 5. IEEE 123-node distribution feeder.

A. Parameters and Environment Setup

First, several Monte-Carlo simulations are done to obtain the required number of trials for the convergence of system loss function for a given damage scenario. Each weather event (wind speed) is sampled from Fig. 2a to obtain the failure probabilities of distribution lines. This is done to ensure that the resilience is quantified for several intensity of weather parameter to model significant uncertainties of catastrophic events. 1000 trials of Monte-Carlo simulations are done for each sampled event. For each trial, a loss function is obtained by solving a restoration problem using CPLEX 12. The simulation is carried out on a PC of Intel Core i7-6700 @ 3.4 GHz processor with 16 GB RAM. The loss function as obtained from the restoration solutions for each Monte-Carlo run are averaged. It is observed that 1000 trials are enough to simulate for a sampled weather event to obtain the loss function. Note that $\alpha = 0.95$ is chosen for the simulation purpose in this paper.

TABLE I
VaR and CVaR for Different Weather Scenarios With $\alpha = 95\%$

S.No	Weather Profile	VaR_α (MWh)	$CVaR_\alpha$(MWh)
1	Extreme	14.35	19.40
2	High	3.588	4.850
3	Normal	0.574	0.776

B. CVaR computation

1) Base Network: In this case, a base case network is assumed without robustness and without any smart features of restoration. The loss function for each possible damage scenario in a particular region following a regional wind profile is obtained and plotted as a PDF (See corresponding black x-axes in Fig. 6). The vertical dashed line in figure represents the VaR_α which forms a baseline for evaluation of expected resilience loss in HILP event. Table I reports the VaR_α and $CVaR_\alpha$ for all weather scenarios. It is observed that the expected resilience loss (MWh) beyond a threshold (defined by VaR_α) is higher for an extreme wind profile and lower for normal wind loading condition. This illustrates that the proposed metric can effectively quantify the resilience for any intensity of weather event.

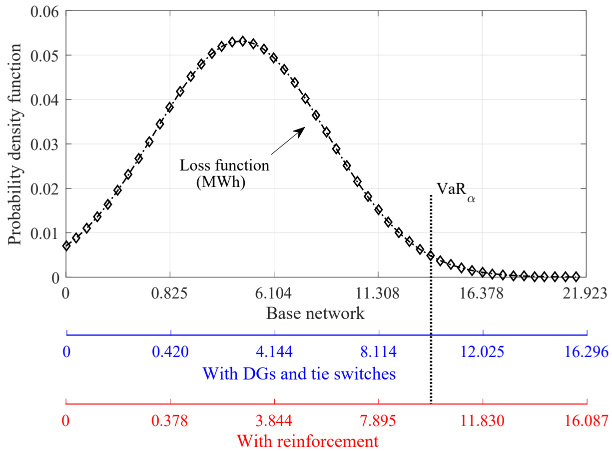

Fig. 6. Loss function (MWh) for extreme wind profile for different cases.

2) Proactive Planning Measures: In this case, two different planning measures are implemented for the test feeder and resilience improvements is analyzed for the system when impacted by the wind event corresponding to extreme weather profile. First, the distribution network is assumed to have improved response and smart restoration actions with DERs and tie switches (see Fig. 5). In this case, for any damage scenario, the system recovers with smart restoration actions utilizing the grid-forming DER technology and tie switches. The loss function for each each wind speed is calculated using Monte-Calro simulations and mapped onto the PDF for wind intensity. The system loss is plotted as blue x-axis in Fig. 6. In second case the distribution network is made robust via reinforcing/hardening a few distribution lines. Specifically, 16 random distribution lines are undergrounded and hence are not affected by windstorms (See Fig. 5). With this, the loss function is computed and mapped into PDF as shown in the red x-axis of Fig. 6. Notice that in both case, the black x-axis represents the probabilistic loss function for the base case.

TABLE II
VaR and CVaR For Different Cases With $\alpha = 95\%$

S.No	Test-case	VaR_α(MWh)	$CVaR_\alpha$(MWh)
1	Original System	14.35	19.40
2	Smart network	10.46	14.34
3	Robust network	10.27	14.15

Once the PDF for system loss is generated, $\alpha = 0.95$ is used to capture the HILP or an extreme case of regional wind profile. VaR_α and $CVaR_\alpha$ are calculated as shown in Table II. Note that the numerical values in the table reports the maximum (VaR_α) and expected loss ($CVaR_\alpha$) of energy loss (in MWh) due to $(1-\alpha)\%$ of highest impact events. It can be observed that both VaR_α and $CVaR_\alpha$ values are reduced for the network with improved restoration plan and the network with added reinforcement measures. The proposed metric, therefore, quantifies the operational resilience of a power distribution network and can be used as an effective tool for comparing different strategies for boosting grid resilience.

V. Conclusions

In this paper, a probabilistic metric for quantifying resilience of distribution grid is proposed based on the concept of conditional-value-at-risk. It is observed that the expected resilience loss due to HILP events like windstorm, especially the (1-α)% events are characterized by $CVaR_\alpha$ metric. Thus, the worst outcomes of any extreme event occurring with low probability are effectively captured by this metric. Effects of smart measures and reinforcement measures on resilience enhancement are also quantified using the proposed resilience metric; making it suitable for comparing different resilience enhancement strategies and assist with the cost-benefit and planning analysis for resilience-driven investments. Although the proposed formulation is mainly focused on windstorms in this paper, it can easily be modified for other extreme events.

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