

Section 2

* Rotation about line parallel to axis & passing through point

Solution Method.

Translate point to the line to origin.

- ①) Translate to Given point (-)
2) Rotation about Given axis
3) Translate to Given point (+)
②

Ex Rotate about line parallel to X-axis & passing through (3, 2, 1) using $\theta = 90^\circ$

- 1) Translate to (-3, -2, -1)

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- 2) Rotate about X-axis with $\theta = 90^\circ$

$$\cos 90^\circ = 0 \quad \sin 90^\circ = 1$$

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) Translate to Cent (3, 2, 1)

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Matrix: $(T_2 * R_x) * T_1$

$$= \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

المضروب ههنا كالتالي:
 (1) المضروب $T_2 * R_x$ ونطلع الناتج
 (2) المضروب الناتج في T_1

Sheet



①

Rotate about line parallel

to y-axis & passing through

(3, 5, 7) with $\theta = 45^\circ$

② Rotate about line parallel to

z-axis & passing through

(2, 4, 6) with $\theta = 90^\circ$

3

(4)

Examples By using: any polygons

- ① Rotate the following polygon around the line parallel to x -axis & passing through $(3, 2, 1)$ with angle 90° .

Answer:

- ① Translate To Given point (-)

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

- ② Rotate about x -axis with $\theta = 90^\circ$

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 & 0 \\ 0 & \sin 90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\cos 90^\circ = 0 \quad \sin 90^\circ = 1$$

⑤

③ Translate To Given Point (+)

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F_x (\text{Final Matrix}) = (T_2 * R_x) * T_1$$

1) $\overleftarrow{T_2}$ $\rightarrow R_x$

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2)

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

\downarrow \downarrow

$T_2 * R_x$ T_1

$$F_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

★ طريقاً في المسألة كيف يمكن شكل معين ذي
كانت عدد أضلاعها طبق على
Rotation.

polygon: (10, 60, 30), (55, 40, 20)
(65, 60, 40), (75, 45, 10), (35, 100, 20)

Matrix: (1) كنترول النقطة دي

$$\text{polygon:} \begin{bmatrix} 10 & 55 & 65 & 75 & 35 \\ 60 & 40 & 60 & 45 & 100 \\ 30 & 20 & 40 & 10 & 20 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Matrix (4*5) ← عدد أضلاع الشكل

(c) تطبيق T_x على polygon

$$T_x * \text{polygon}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 10 & 55 & 65 & 75 & 35 \\ 60 & 40 & 60 & 45 & 100 \\ 30 & 20 & 40 & 10 & 20 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 55 & 65 & 75 & 35 \\ -27 & -17 & -37 & -7 & -27 \\ 59 & 39 & 59 & 44 & 99 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Polygon points after Rotation

$$= (10, -27, 59), (55, -17, 39) \\ (65, -37, 59), (75, -7, 44) \\ (35, -27, 99).$$

Sheet

8

① Rotate The Following Polygon
around line parallel To

Y -axis Passing Through $(2, 3, 4)$

With $\theta = 45^\circ$

polygon: $(5, 55, 25), (50, 45, 15)$

$(60, 55, 35), (70, 40, 5)$

② Rotate The Following Polygon around
line parallel To Z -axis passing through

$(1, 4, 2)$ With $\theta = 90^\circ$

polygon = $(45, 40, 10), (55, 50, 30)$

$(65, 35, 1)$