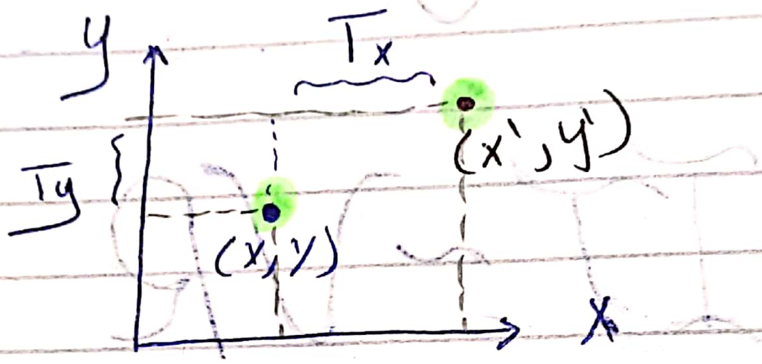
Section 1

3D Transformation

1 Translation: (Moving Things)



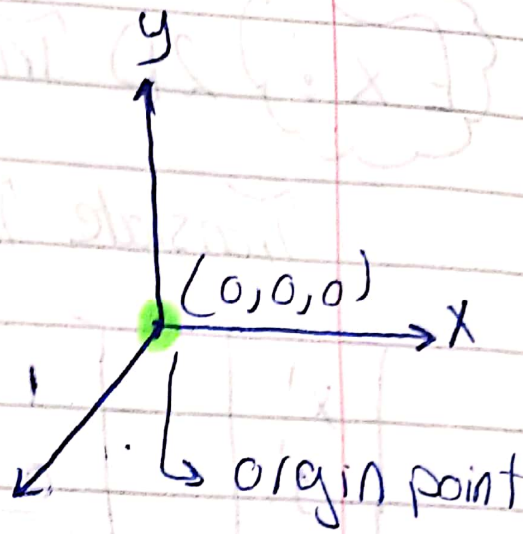
$$x' = x + T_x \qquad y' = y + T_y$$

2D: Translation Matrix.

$$T = \begin{array}{c} \\ x' \\ y' \\ 1 \end{array}\begin{array}{c} \begin{array}{ccc} x & y & 1 \end{array} \\ \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

$$\therefore \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & T_x \\ 0 & 1 & T_y \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

3D Translation.



∘ $x' = x + T_x$

$y' = y + T_y$

$z' = z + T_z$

∘ Matrix:

$$T = \begin{array}{c} \\ x' \\ y' \\ z' \\ 1 \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & 1 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

∘

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{array}{c} \\ x' \\ y' \\ z' \\ 1 \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & 1 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & T_x \\ 0 & 1 & 0 & T_y \\ 0 & 0 & 1 & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array} * \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Ex: 2D Translation:

Translate This point (2,6) with $T_x = 3$ $T_y = 4$.

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 \\ 6 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 10 \\ 1 \end{bmatrix}$$

Ex: 3D Translation:

Translate (1,1,3) with $T_x = 2$ $T_y = 1$ $T_z = 3$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 1 \\ 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 2 \\ 6 \\ 1 \end{bmatrix}$$

X :

1 Translate (2,6) (3,4) with $T_x = 2$, $T_y = 4$.

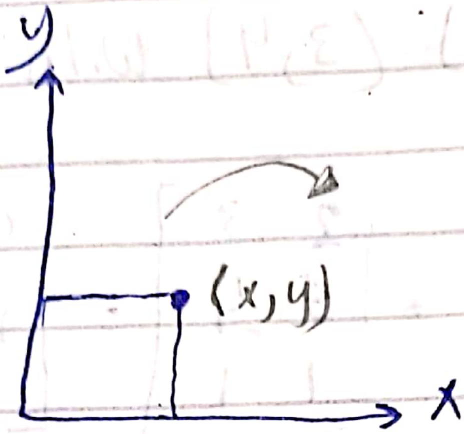
$$\begin{bmatrix} x'_1 & x'_2 \\ y'_1 & y'_2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 2 & 3 \\ 6 & 4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 10 & 8 \\ 1 & 1 \end{bmatrix}$$

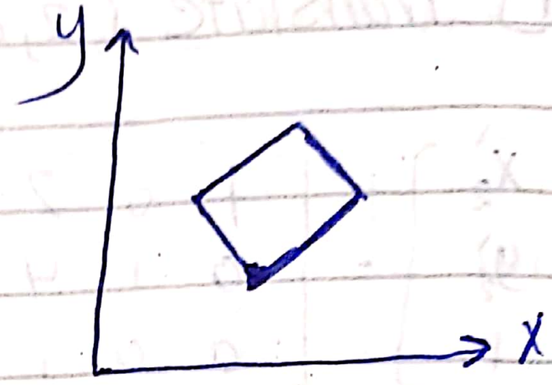
Line

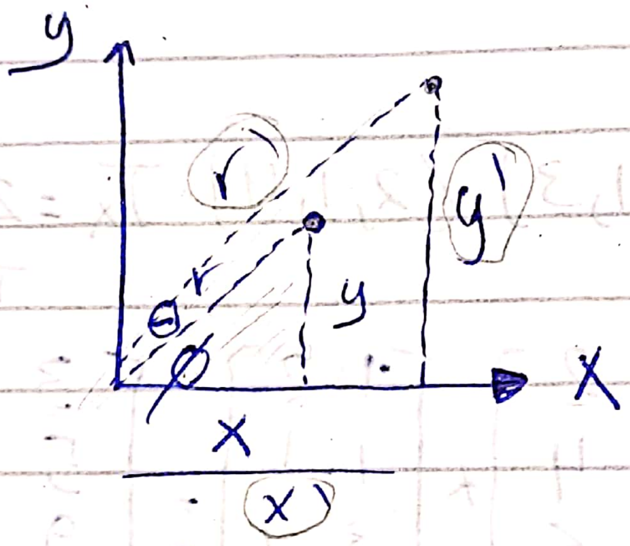
2 Translate (1,1,3) (2,1,4) $T_x = 2$ $T_y = 4$ $T_z = 6$

$$\begin{bmatrix} x'_1 & x'_2 \\ y'_1 & y'_2 \\ z'_1 & z'_2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 4 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 3 & 4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 5 & 5 \\ 9 & 10 \\ 1 & 1 \end{bmatrix}$$

[2] RoTaTion: (Moving about point by a given Angle)







Note

$\sin\theta = \dfrac{\text{المقابل}}{\text{الوتر}}$

$\cos\theta = \dfrac{\text{المجاور}}{\text{الوتر}}$

$$\cos\phi = \frac{x}{r} \Rightarrow x = r\cos\phi$$

$$\sin\phi = \frac{y}{r} \Rightarrow y = r\sin\phi$$

$$\therefore \cos(\phi+\theta) = \frac{x'}{r}$$

$$\therefore \sin(\phi+\theta) = \frac{y'}{r}$$

$$x' = r\cos(\theta+\phi)$$

$$\therefore\ x' = r\left[\cos\theta\cos\phi - \sin\theta\sin\phi\right]$$

$$x' = r\cos\theta\cos\phi - r\sin\theta\sin\phi$$

$$\therefore\ \boxed{x' = x\cos\theta - y\sin\theta}$$

$$\therefore\ y' = r\sin(\theta+\phi)$$

$$y' = r\sin\theta\cos\phi + r\cos\theta\sin\phi$$

$$\therefore\ \boxed{y' = x\sin\theta + y\cos\theta}$$

$$R_x = \begin{array}{c} \\ x' \\ y' \\ 1 \end{array}\begin{array}{c} \begin{array}{ccc} x & y & 1 \end{array} \\ \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array}$$

Ex : Rotate This point $(3,4)$ with $\Theta = 90^\circ$

$\sin 90^\circ = 1$
$\cos 90^\circ = 0$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$$

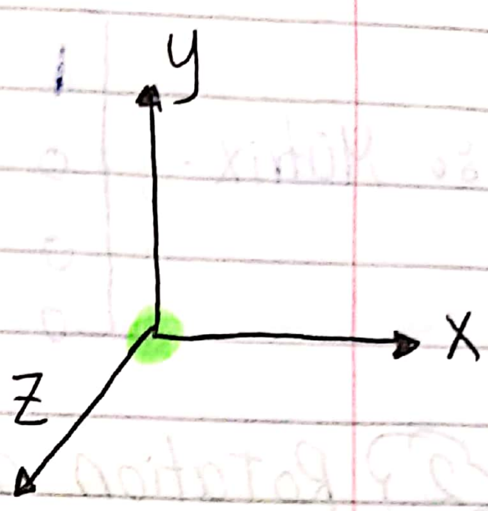
$$= \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \\ 1 \end{bmatrix}$$

Ex : Rotate $(3,4)$ $(5,6)$ with $\Theta = 90^\circ$

$$\begin{bmatrix} x_1' & x_2' \\ y_1' & y_2' \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 3 & 5 \\ 4 & 6 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4 & -6 \\ 3 & 5 \\ 1 & 1 \end{bmatrix}$$

3D Rotation:



$\therefore x' = x\cos\theta - y\sin\theta$

$\therefore y' = x\sin\theta + y\cos\theta$

(1) Rotation about X-axis

Note:

١) المحور الا هنعمل دوران حوله بنثبت قيمته وبنغير قيمه المحاور الآخرى

٢) لابد من مراعاه الترتيب في المحاور:

X - aixs

$\therefore x' = x$

$\therefore y' = y\cos\theta - z\sin\theta$

$\therefore z' = y\sin\theta + z\cos\theta$

$$\text{Matrix} = \begin{array}{c} \\ x' \\ y' \\ z' \\ 1 \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & 1 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta & 0 \\ 0 & \sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

(2) Rotation about Y-axis

$y' = y$

$z' = z\cos\theta - x\sin\theta$

$x' = z\sin\theta + x\cos\theta$

$$\text{Matrix} = \begin{array}{c} \\ x' \\ y' \\ z' \\ 1 \end{array} \begin{array}{c} \begin{array}{cccc} x & y & z & 1 \end{array} \\ \begin{bmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

3 Rotation about Z-axis

∴ $Z' = Z$

∴ $X' = X\cos\theta - y\sin\theta$

∴ $y' = y\cos\theta + x\sin\theta$

Matrix:

$$\begin{array}{c|cccc} & x & y & z & 1 \\ \hline x' & \cos\theta & -\sin\theta & 0 & 0 \\ y' & \sin\theta & \cos\theta & 0 & 0 \\ z' & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{array}$$

EX: Rotate (1, 3, 2) about X-axis with $\theta = 90°$

$\cos 90 = 0$

$\sin 90 = 1$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 3 \\ 2 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -2 \\ 3 \\ 1 \end{bmatrix}$$

Ex: Rotate (1,3,2) (1,5,6) about X-axis with $\theta = 90^\circ$

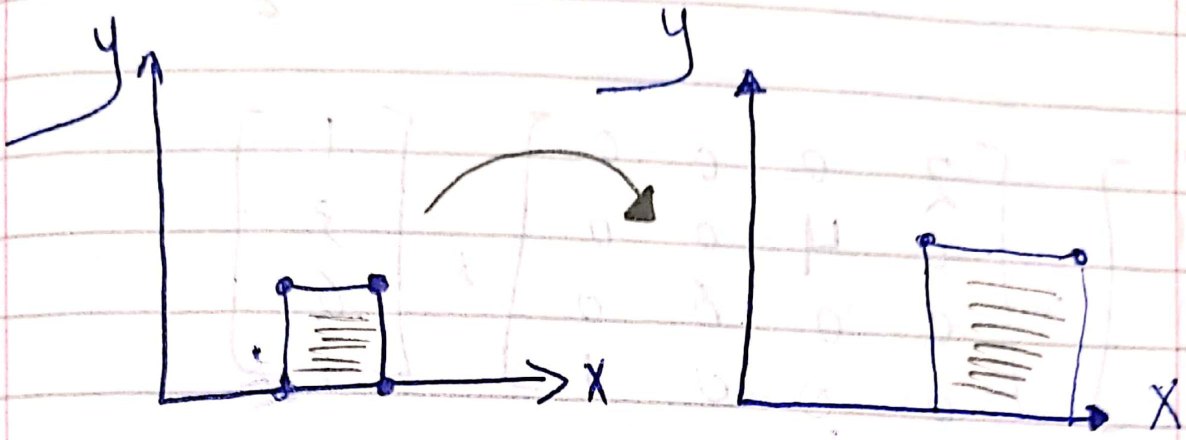
$$\begin{bmatrix} x_1' & x_2' \\ y_1' & y_2' \\ z_1' & z_2' \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 1 \\ 3 & 5 \\ 2 & 6 \\ 1 & 1 \end{bmatrix}$$

$$\therefore = \begin{bmatrix} 1 & 1 \\ -2 & -6 \\ 3 & 5 \\ 1 & 1 \end{bmatrix}$$

* بالمثل حول Y-axis و Z-axis

باختلاف ال Matrix في كل حالة.

3 Scaling: (change size of Things).



$$x' = x S_x$$

$$y' = y S_y$$

$$z' = z S_z$$

∴ Matrix
$$\begin{array}{c|cccc} & x & y & z & \\ \hline x' & S_x & 0 & 0 & 0 \\ y' & 0 & S_y & 0 & 0 \\ z' & 0 & 0 & S_z & 0 \\ 1 & 0 & 0 & 0 & 1 \end{array}$$

Ex: Scale (3,4) with $S_x = 2$ $S_y = 4$

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} = \begin{bmatrix} 6 \\ 16 \\ 1 \end{bmatrix}$$

Ex: Scale (1, 3, 6) with $S_x = 2$ $S_y = 4$ $S_z = 6$

$$\begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 \\ 3 \\ 6 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 12 \\ 36 \\ 1 \end{bmatrix}$$

Ex: Scale (1, 3, 5) (2, 4, 6) with $S_x = 1$ $S_y = 2$ $S_z = 3$

$$\begin{bmatrix} x_1' & x_2' \\ y_1' & y_2' \\ z_1' & z_2' \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \\ 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 \\ 6 & 8 \\ 15 & 18 \\ 1 & 1 \end{bmatrix}$$