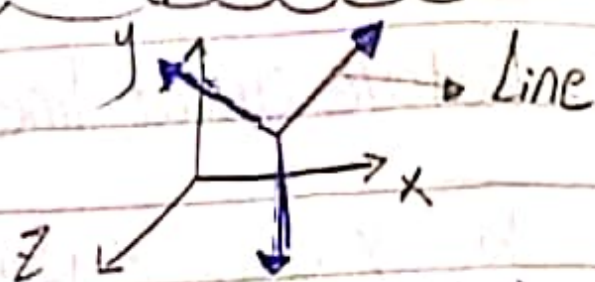


Section 13

①

* Rotation around line in the space

Method



- 1) Translate To Given point (-)
- 2) Generate New system (u, v, w) on old system using line Direction

u v w - if The Direction are Given Directly
↓ ↓ ↓ Assume u or v or w
x y z The Direction

- if we have 2 points

* Direction = $P_2 - P_1$

① Normalize $V(a, b, c)$

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}}, \frac{b}{\sqrt{a^2 + b^2 + c^2}}, \frac{c}{\sqrt{a^2 + b^2 + c^2}}$$

2

- $u = (a, b, c)$

- $v = (-b, a, 0)$

- $w = u \times v$

Ex:

$$u = (2, 1, 1)$$

$$v = (-1, 2, 0)$$

2

$$u \cdot v = 0$$

Dot product
(ضرب عادي)

$$u = (a, b, c)$$

$$v = (-b, a, 0)$$

$$-ab + ab + 0 = 0$$

///

3

$$w = u \times v \quad \text{Cross product}$$

$$w = \begin{vmatrix} u & v \\ a & b & c \\ -b & a & 0 \end{vmatrix}$$

علامة زيب قيمة عنصر وينذف العنصر الاخير
ونضرب العمودين الباقيين مقص (Cross)

Ex

$$\begin{vmatrix} 1 & 2 & 1 \\ -2 & 1 & 0 \end{vmatrix} = (-1, 2, 5)$$

3

$$u = (1, 2, 1)$$

$$v = (-2, 1, 0)$$

$$\begin{bmatrix} 1 & 2 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

$$W = u \times v \quad \text{Cross product}$$

لنحذف الأول عمود

$$(2 \times 0) - (1 \times 1) = -1$$

لنحذف العمود الثاني

$$(1 \times 0) - (1 \times -2) = 2$$

القيمة اى تكونه سالب

$$(+ \times -) = - \quad + \quad \text{لو الناتج مبالغ}$$

$$(- \times -) = + \quad - \quad \text{لو مبالغ}$$

لنحذف العمود الثالث

$$(1 \times 1) - (2 \times -2) = 1 + 4 = 5$$

$$W = (-1, 2, 5)$$

(9)

Ex: $u = (3, 1, 5)$

$v = (-1, 3, 0)$

$w = u \times v$

$$\begin{bmatrix} 3 & 1 & 5 \\ -1 & 3 & 0 \end{bmatrix}$$

لنحذف العمود الأول

$$(1 \times 0) - (3 \times 5) = -15$$

لنحذف العمود الثاني

$$(3 \times 0) - (-1 \times 5) = 5$$

يبقى (-5)

لنحذف العمود الثالث

$$(3 \times 3) - (-1 \times 1) = 10$$

$$w = (-15, -5, 10)$$

3) Rotate New system to old system

$$R_1 = \begin{bmatrix} u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ w_x & w_y & w_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4) if we consider the line U Rotate about X -axis

if $v \rightarrow$ Rotate about Y -axis

if $w \rightarrow$ Rotate about Z -axis

R_2

5) Rotate from old system to New system

$$R_3 = R_1^T \rightarrow \text{Transpose}$$

دوران
المعكوسة

6) Translate to Given point (+)

6

Final Matrix $((T_2 * R_3 * R_2) * R_1) * T_1$

Ex Rotate about line having The Direction $(2, 1, 1)$ & passing through $(4, 1, 6)$

1) Translate To $(-4, -1, -6)$

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -4 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2) Generate new system (u, v, w)

Note let $u = (2, 1, 1)$ $\left| \begin{array}{l} u = (a, b, c) \\ v = (-b, a, 0) \end{array} \right.$
 $u \cdot v = 0$ so $v = (-1, 2, 0)$

$$W = u \times v$$

$$W = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \times \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$$

$$W = (-2, -1, 5)$$

- Normalize u, v, w

$$\star u = (2, 1, 1)$$

$$\frac{a}{\sqrt{a^2 + b^2 + c^2}}$$

$$\frac{2}{\sqrt{2^2 + 1^2 + 1^2}} = \frac{2}{\sqrt{6}}$$

$$\frac{1}{\sqrt{2^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{6}}$$

$$\frac{1}{\sqrt{2^2 + 1^2 + 1^2}} = \frac{1}{\sqrt{6}}$$

$$\therefore \left(\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}} \right)$$

$$\star v = (-1, 2, 0) \quad \left| \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right.$$

(8)

$$\hat{u} = \left(\frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}, 0 \right)$$

$$\star W = (-2, -1, 5)$$

$$\frac{-2}{\sqrt{2^2 + (-1)^2 + 5^2}} = \frac{-2}{\sqrt{30}}, \quad \left(\frac{-1}{\sqrt{30}}, \frac{5}{\sqrt{30}} \right)$$

$$\hat{u} = \left(\frac{-2}{\sqrt{30}}, \frac{-1}{\sqrt{30}}, \frac{5}{\sqrt{30}} \right)$$

Rotate to New System.

$$R = \begin{matrix} & \begin{matrix} x & y & z & 1 \end{matrix} \\ \begin{matrix} x \\ y \\ z \\ 1 \end{matrix} & \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & 0 \\ \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{5}} & 0 & 0 \\ \frac{-2}{\sqrt{30}} & \frac{-1}{\sqrt{30}} & \frac{5}{\sqrt{30}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{matrix}$$

Note (3) we let U Direction
The Rotate about X-axis.

3. g_0 $\cos g_0 = 0$ $\sin g_0 = 1$

$$R_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

4 Rotate from old To new system

$R_3: R_1^T$ (مذور المصفوفات)

المصفوفة عكس

المصفوفة عكس

$$R_3 = \begin{bmatrix} \frac{2}{\sqrt{6}} & \frac{-1}{\sqrt{5}} & \frac{2}{\sqrt{30}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{2}{\sqrt{5}} & \frac{-1}{\sqrt{30}} & 0 \\ \frac{1}{\sqrt{6}} & 0 & \frac{5}{\sqrt{30}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

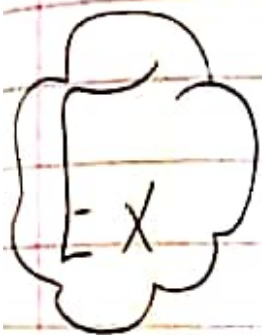
5 Translate To (4, 1, 0)

10

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Matrix:

$$(((T_2 * R_3) * R_2) * R_1) * T_1$$



Ex Rotate about line having the
Direction $(1, 2, 3)$ & passing
Through $(4, 5, 6) \rightarrow (\theta = 45^\circ)$



Rotate about line passing Through
 $(3, 2, 1)$ & $(5, 6, 7)$ ($\theta = 90^\circ$)

1) Translate about one of Two points
 $(-3, -2, -1)$

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2) Create New system

Direction: $P_2 - P_1$
 النقطة الأولى \rightarrow النقطة الثانية

$$\begin{aligned} \text{Direction} &= (5, 6, 7) - (3, 2, 1) \\ &= (2, 4, 6) \end{aligned}$$

(12)

$$\text{let } u = (2, 4, 6)$$

$$v = (-4, 2, 0)$$

$$w = \begin{bmatrix} 2 & 4 & 6 \\ -4 & 2 & 0 \end{bmatrix} = (-12, -24, 20)$$

* Normalize u, v, w

$$u = \left(\frac{2}{\sqrt{56}}, \frac{4}{\sqrt{56}}, \frac{6}{\sqrt{56}} \right)$$

$$v = \left(\frac{-4}{\sqrt{20}}, \frac{2}{\sqrt{20}}, 0 \right)$$

$$w = \left(\frac{12}{\sqrt{1120}}, \frac{-24}{\sqrt{1120}}, \frac{-20}{\sqrt{1120}} \right)$$

* Rotate To New System

$$u, v, w$$

$$R_1 = \begin{bmatrix} \frac{2}{\sqrt{50}} & \frac{4}{\sqrt{50}} & \frac{6}{\sqrt{50}} & 0 \\ \frac{-4}{\sqrt{20}} & \frac{2}{\sqrt{20}} & 0 & 0 \\ \frac{12}{\sqrt{1120}} & \frac{24}{\sqrt{1120}} & \frac{-20}{\sqrt{1120}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

③ We let $U \rightarrow$ Direction so Rotate about X-axis

$$\cos \theta_0 = 0 \quad \sin \theta_0 = 1$$

$$R_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

④ Rotate from old To New System

$$R_3 = R^T$$

$$P_3 = \begin{bmatrix} \frac{2}{\sqrt{56}} & \frac{-4}{\sqrt{20}} & \frac{12}{\sqrt{1120}} & 0 \\ \frac{4}{\sqrt{56}} & \frac{2}{\sqrt{20}} & \frac{-24}{\sqrt{1120}} & 0 \\ \frac{6}{\sqrt{56}} & 0 & \frac{-20}{\sqrt{1120}} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

⑤ Translate To $(3, 2, 1)$

$$\overline{T}_2 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Matrix: $\overline{T}_2 * P_3 * P_2 * P_1 *$

Ex Rotate about line passing through.

$$(2, 4, 6), (3, 5, 7) \quad (\theta = 45^\circ)$$

★ Rotate The following polygon about line passing through $(3, 2, 1)$ with $\theta = 90^\circ$

$$\text{Polygon} = (5, 55, 25), (50, 45, 15)$$

$$(60, 55, 35), (70, 40, 5)$$