Section "2"

* Rotation about line parallel To axis & passing Through point

Solution Method

Translate Point To The line To orgin.

1) Translate To Given point (-)
2) Rotation about Given axis
3) Translate To Given point (+)

undo Translate

Ex Rotate about line parallel to X-axis & passing Through (3, 2, 1) using $\theta = 90°$

1) Translate To (-3, -2, -1)

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2) Rotate about X-axis with $\theta = 90$

Cos 90 = 0 Sin 90 = 1

$$R_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

3) Translate To Cen (3, 2, 1)

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Final Matrix: $(T_2 * R_x) * T_1$

$$= \left[\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right] * \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

الضرب هيتم كالآتي:

1) هنضرب $T_2 * R_x$ وناخد الناتج

2) هنضرب الناتج في T_1

(2)

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Ex

1) Rotate about line parallel To y-axis & passing Through (3, 5, 7) With $\theta = 45°$

2) Rotate about line parallel To Z-axis & passing Through (2, 4, 6) With $\theta = 90°$

Examples By using any polygons

(1) Rotate The Following polygon around The line parallel To X-axis & Passing Through (3, 2, 1) With angle 90°

Answer

(1) Translate To Given Point (-)

$$T_1 = \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(2) Rotate about X-axis With $\theta = 90^\circ$

$$R_X = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos 90 & -\sin 90 & 0 \\ 0 & \sin 90 & \cos 90 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$\cos 90^\circ = 0 \quad \sin 90^\circ = 1$

[3] Translate To Given Point (+)

$$T_2 = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

F_x (Final Matrix) $= (T_2 * R_x) * T_1$

1) T_2 $\quad$ R_x

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2)

$$= \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$T_2 * R_x$ $\quad$ T_1

$$F_x = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

* طبقنا فى المسألة كيف شكل معين ذى كانت عدد أضلاعه طبق على ال Rotation.

Polygon: (10, 60, 30), (55, 40, 20)
(65, 60, 40), (75, 45, 10), (35, 100, 20)

1) كنحول النقط دى لـ Matrix

$$\text{Polygon} = \begin{bmatrix} 10 & 55 & 65 & 75 & 35 \\ 60 & 40 & 60 & 45 & 100 \\ 30 & 20 & 40 & 10 & 20 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Matrix (4 * 5) ← عدد أضلاع الشكل

ج) لتطبيق F_x على الـ Polygon

F_x * Polygon

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 10 & 55 & 65 & 75 & 35 \\ 60 & 40 & 60 & 45 & 100 \\ 30 & 20 & 40 & 10 & 20 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 55 & 65 & 75 & 35 \\ -27 & -17 & -37 & -7 & -27 \\ 59 & 39 & 59 & 44 & 99 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

Polygon Points after Rotation:

= (10, -27, 59), (55, -17, 39)
(65, -37, 59), (75, -7, 44)
(35, -27, 99).

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① Rotate The Following polygon around line parallel To Y-axis passing Through (2, 3, 4) With $\theta = 45^\circ$

polygon: (5, 55, 25), (50, 45, 15) (60, 55, 35), (70, 40, 5).

② Rotate The Following polygon around line parallel To Z-axis passing Through (1, 4, 2) with $\theta = 90^\circ$

polygon = (45, 40, 10), (55, 50, 30) (65, 35, 1)