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To: A. M. Perry

From: T. W. Kerlin

Subject: Preliminary Dynamics Model of the MSBR

Introduction

This memo describes a model being used in the dynamic analysis of the MSBR. This information should assist in future up-dating of the model and in comparing results of this study with other MSBR dynamics studies.

System

The system currently being modeled in this study contains one of the earlier MSBR modular core designs. The hexagonal core fuel assemblies contain three upflow fuel channels with a diameter of 7/8 in. The fuel downflow is in a single 1 1/2-in. channel. The fertile salt flows through channels between adjacent hexagonal graphite blocks.

Each module produces 556 Mw(t) and 250 Mw(e).

Dynamic Model

The dynamic model will eventually include all components in the system. Currently, the model includes: (a) neutronics, (b) core heat transfer, (c) fuel salt heat exchanger, (d) blanket salt heat exchanger, and (e) salt side of boiler and reheater.

The neutronics model is a point kinetics model with six delayed neutron groups and explicit accounting for the delayed neutron losses in external loops.

The core heat transfer is handled by breaking the core into nine lumps or nodes. These give three graphite lumps, two fuel upflow lumps, two fuel downflow lumps, and two blanket stream lumps.

The fuel salt heat exchanger is also modeled with a lumped approximation. It contains eleven lumps. These give two tube metal lumps, two fuel downflow lumps, a fuel mixing plenum lump, a fuel upflow lump, two coolant salt downflow lumps, and two coolant salt upflow lumps. The coolant salt inlet temperature is either constant or a system perturbation, but is not currently a dependent system variable.

The fertile salt heat exchanger model contains five lumps. This gives one tube metal lump, two fertile salt lumps, and two coolant salt lumps.

The transport of fluids from one component to another is modeled with pure time delays. The following delays are included:

1. Fuel salt (core → heat exchanger).
2. Fuel salt (heat exchanger → core).
3. Fertile salt (core → heat exchanger).
4. Fertile salt (heat exchanger → core).
5. Coolant salt (fuel salt heat exchanger → fertile salt heat exchanger).
6. Coolant salt (fertile salt heat exchanger → boiler).
7. Coolant salt (fertile salt heat exchanger → reheater).
8. Coolant salt (boiler → fuel salt heat exchanger).
9. Coolant salt (reheater → fuel salt heat exchanger).

The overall model for system heat transfer is shown in Fig. 1.

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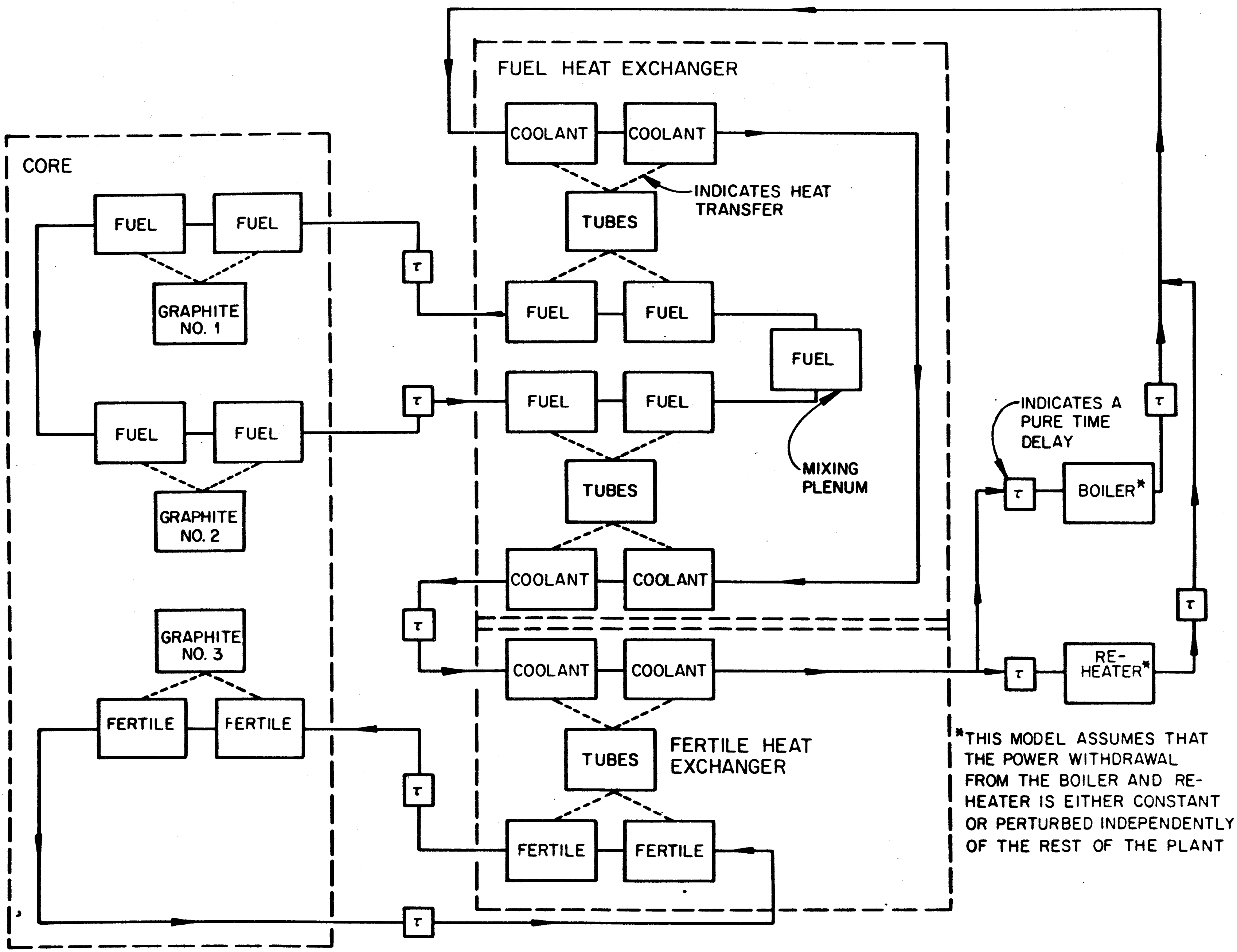


Fig. 1. Schematic of MSBR Dynamic Model

Neutronics

The neutronics model is a standard point model with explicit accounting for delayed neutron losses in external loops. The appropriate equations are (see page 52 of "System Analysis of Reactor Dynamics" by Weaver):

$$\frac{dn}{dt} = \left(\frac{\rho - \beta}{\Lambda}\right) n + \sum_{i=1}^{6} \lambda_i C_i \ ,$$

$$\frac{dC_i}{dt} = \frac{\beta_i n}{\Lambda} - \lambda_i C_i + \frac{C_i(t - \tau_L)\, e^{-\lambda_i \tau_L}}{\tau_c} - \frac{C_i(t)}{\tau_c} \ ,$$

where all of the symbols have their usual meaning. The term, C_i, is the concentration of precursor i in the core, and τ_L and τ_c represent the transit time of the fuel in the external loop and in the core respectively.

The reactivity required for steady-state is not zero as for static-fuel reactors. This can be seen by setting the derivatives equal to zero and solving for ρ_o. This gives

$$\rho_o = \beta_T - \sum_{i=1}^{6} \frac{\beta_i}{1 + \frac{1}{\lambda_i \tau_c}\left(1 - e^{-\lambda_i \tau_L}\right)} \ .$$

The dependent variables in the equations are replaced by the sum of a steady state and a perturbation quantity:

$$n = n_o + \delta n \ ,$$

$$C_i = C_{io} + \delta C_i \ .$$

These expressions are substituted for n and C_i in the equations, the steady-state terms drop out, and the nonlinear term is eliminated. The resulting linearized equations are:

$$\frac{d\delta n}{dt} = \left(\frac{\rho_o - \beta_T}{\Lambda}\right) \delta n + \sum_{i=1}^{n} {}_iC_i + \frac{n_o \delta\rho_{ex}}{\Lambda} + \frac{n_o \delta\rho_{fb}}{\Lambda} \ ,$$

$$\frac{d\delta C_i}{dt} = \frac{\beta_i \delta n}{\Lambda} - \lambda_i \delta C_i + \frac{\delta C_i(t - \tau_L)\, e^{-\lambda_i \tau_L}}{\tau_c} - \frac{\delta C_i(t)}{\tau_c} \ ,$$

where $\delta\rho_{ex}$ is the external reactivity input and $\delta\rho_{fb}$ is the feedback reactivity caused by temperature changes.

The form of the feedback reactivity term is

$$\delta\rho_{fb} = \alpha_F \delta T_F + \alpha_G \delta T_G + \alpha_B \delta T_B ,$$

where the α's represent the temperature coefficients and F, G, and B designate fuel, graphite, and fertile. Subsequent sections will show that the lumped parameter model of the core contains four fuel lumps, three graphite lumps, and two fertile lumps. In this model, all fuel and fertile lumps were given equal reactivity worths. Thus, the temperature coefficient for each fuel lump is one-fourth the total fuel temperature coefficient, and the temperature coefficient for each fertile lump is one-half the total fertile temperature coefficient. For the graphite, I estimated that 89% of the power generated in the graphite is removed by the fuel stream and 11% by the fertile stream. I weighted each graphite lump with the power removed from that lump. Thus, the two larger lumps had a temperature coefficient equal to 44.5% of the total temperature coefficient and the smaller lump had a temperature coefficient equal to 11% of the total temperature coefficient.

Preliminary estimates of the temperature coefficients and the generation time were used in the early calculations. These were:

$$\alpha_F = -4.6 \times 10^{-5} \ \delta\rho/°F$$

$$\alpha_G = +1.43 \times 10^{-5} \ \delta\rho/°F$$

$$\alpha_B = +5.1 \times 10^{-6} \ \delta\rho/°F$$

$$\Lambda = 3.3 \times 10^{-4} \ \text{sec} .$$

More reliable estimates for the temperature coefficients (obtained by Lee Smith and Cecil Thomas) have recently been calculated. These are:

$$\alpha_F = -4.54 \times 10^{-5} \ \delta\rho/°F$$

$$\alpha_G = +1.12 \times 10^{-5} \ \delta\rho/°F$$

$$\alpha_B = +9.2 \times 10^{-6} \ \delta\rho/°F .$$

These newer temperature coefficients and the original generation time estimate are being used in current studies.

The core and external loop transit times are 3.28 sec and 5.85 sec respectively.

The delayed neutron data are shown below (see page 17 of ANL-6600).

Group	λ_i (sec^{-1})	β_i (^{233}U)	β_i (^{235}U)	β_i (MSBR)
1	0.0126	0.00023	0.000215	0.000229
2	0.0337	0.00079	0.00142	0.000832
3	0.139	0.00067	0.00127	0.000710
4	0.325	0.00073	0.00257	0.000852
5	1.13	0.00013	0.00075	0.000171
6	2.50	0.00009	0.00027	0.000102
		0.00264	0.00650	0.002896

The term, β_i (MSBR), is an average β_i for the composition of the MSBR. The weighting is according to the number of fissions in each isotope. The values are 93.4% in ^{233}U and 6.6% in ^{235}U.

The value of ρ_o obtained with these data is:

$$\rho_o = 0.00138 .$$

This represents the reactivity change in going from stationary to circulating fuel.

The resulting equations are:

$$\frac{d\delta n}{dt} = -4.306\ \delta n + 0.0126\ \delta C_1 + 0.0337\ \delta C_2 + 0.139\ \delta C_3 + 0.325\ \delta C_4 + 1.13\ \delta C_5 + 2.5\ \delta C_6 - 19.12\ \delta T_{F_1} - 19.12\ \delta T_{F_2} + 8.40\ \delta T_{G_1} - 19.12\ \delta T_{F_3} - 19.12\ \delta T_{F_4} + 8.40\ \delta T_{G_2} + 7.75\ \delta T_{B_1} + 7.75\ \delta T_{B_2} + 2.08\ \delta T_{G_3} ,$$

$$\frac{d\delta C_1}{dt} = 0.694\ \delta n - 0.317\ \delta C_1 + 0.283\ \delta C_1(t - 5.85) ,$$

$$\frac{d\delta C_2}{dt} = 2.52\ \delta n - 0.339\ \delta C_2 + 0.250\ \delta C_2(t - 5.85) ,$$

$$\frac{d\delta C_3}{dt} = 2.15\ \delta n - 0.444\ \delta C_3 + 0.135\ \delta C_3(t - 5.85) ,$$

$$\frac{d\delta C_4}{dt} = 2.58\ \delta n - 0.630\ \delta C_4 + 0.0455\ \delta C_4(t - 5.85)\ ,$$

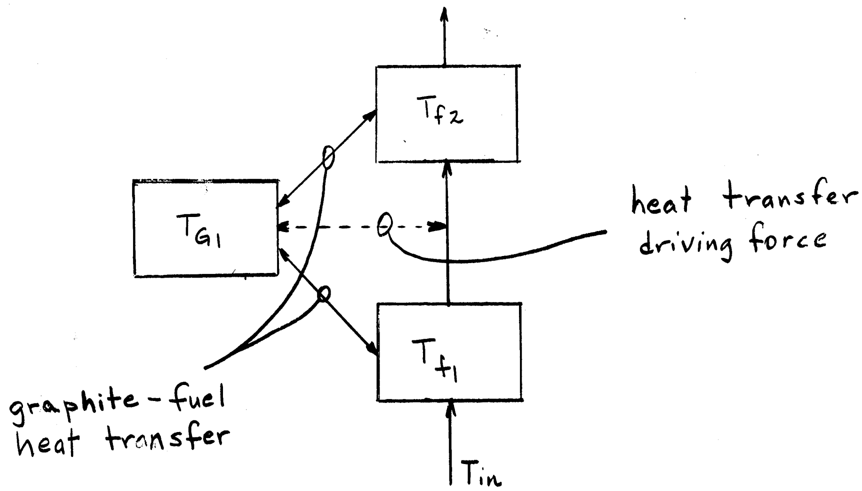
$$\frac{d\delta C_5}{dt} = 0.518\ \delta n - 1.435\ \delta C_5 + 0.000410\ \delta C_5(t - 5.85)\ ,$$

$$\frac{d\delta C_6}{dt} = 0.309\ \delta n - 2.805\ \delta C_6 + 0.000000136\ \delta C_6(t - 5.85)\ .$$

Core Heat Transfer

1. Core Upflow

(a) Model



(b) Equations

$$(MC_p)_{G_1} \frac{dT_{G_1}}{dt} = (hA_{fg})(T_{f_1} - T_G) + K_{G_1} P \ ,$$

$$(MC_p)_{f_1} \frac{dT_{f_1}}{dt} = (WC_p)(T_{in} - T_{f_1}) + K_{f_1} P + K_1 (hA_{fg})(T_{G_1} - T_{f_1}) \ ,$$

$$(MC_p)_{f_2} \frac{dT_{f_2}}{dt} = (WC_p)(T_{f_1} - T_{f_2}) + K_{f_2} P + K_2 (hA_{fg})(T_{G_1} - T_{f_1}) \ ,$$

where

M = mass of material in a lump,

C_p = specific heat,

hA_{fg} = (fuel-to-graphite heat transfer coefficient) x (heat transfer area),

K_{G_1} = fraction of total power generated in the graphite lump,

K_1 = fraction of the heat transferred from the graphite which goes to the first fuel lump,

K_2 = fraction of the heat transferred from the graphite which goes to the second fuel lump,

K_{f_1} = fraction of total power generated in lump f_1,

K_{f_2} = fraction of total power generated in lump f_2,

W = flow rate of fuel,

T_{G_1} = temperature of graphite in lump G_1,

T_{f_1} = temperature of fuel in lump f_1,

T_{f_2} = temperature of fuel in lump f_2.

The equations are divided by the coefficient on the left-hand side to give

$$\frac{dT_{G_1}}{dt} = \frac{(hA)_{fg}}{(MC_p)_{G_1}} (T_{f_1} - T_{G_1}) + \frac{K_{G_1}}{(MC_p)_{G_1}} P ,$$

$$\frac{dT_{f_1}}{dt} = \frac{1}{\tau_1}(T_{in} - T_{f_1}) + \frac{K_{f_1}}{(MC_p)_{f_1}} P + \frac{K_1(hA)_{fg}}{(MC_p)_{f_1}}(T_{G_1} - T_{f_1}) ,$$

$$\frac{dT_{f_2}}{dt} = \frac{1}{\tau_2}(T_{f_1} - T_{f_2}) + \frac{K_{f_2}}{(MC_p)_{f_2}} P + \frac{K_2(hA)_{fg}}{(MC_p)_{f_2}}(T_{G_1} - T_{f_1}) .$$

(c) Values of coefficients

1. $(hA)_{fg} = 0.962$ Mw/°F.

Basis for value:

$h = 1438$ Btu/hr-ft^2-°F , (Calculated by G. H. Llewellyn)

$A = 2285$ ft^2

$= \pi \times$ tube diameter $\times$ tube length $\times$ number of tubes

$$= \frac{\pi \times 0.866 \times 120 \times 1008}{144} = 2285 \ ,$$

$$hA = 3.286 \times 10^6 \text{ Btu/hr-°F} \times 2.928 \times 10^{-7} \text{ Mwhr/Btu} = 0.96 \text{ Mw/°F.}$$

2. $(MC_p)_{G_1} = 7.8$ Mwsec/°F.

Mass = (core volume)(graphite volume fraction)(density of graphite)

$$= (502.65 \text{ ft}^3)(0.777)(106 \text{ lb/ft}^3)$$

$$= 4.14 \times 10^4 \text{ lb} \ ,$$

$C_p = 0.4$ Btu/lb-°F at 1000°F (Glasstone and Sesonske, p.813)

$$= 0.4 \text{ Btu/lb-°F} \times 2.929 \times 10^{-7} \text{ Mwhr/Btu} \times 3600 \text{ sec/hr}$$

$$= 4.23 \times 10^{-4} \text{ Mwsec/lb-°F} \ ,$$

$$(MC_p)_{G_1} = 4.14 \times 10^4 \times 4.23 \times 10^{-4} = 17.6 \text{ Mwsec/°F}$$

for all of the graphite. This heat capacity is allocated among the various graphite lumps according to the assumed fraction of the power generated in the graphite which is removed by each stream. The values assumed were:

fraction of power generated in graphite which is removed by fuel stream = 0.89,

fraction of power generated in graphite which is removed by fertile stream = 0.11.

Since there are two graphite lumps transferring heat with the fuel stream,

$$(MC_p)_{G_1} = 17.6 \text{ Mwsec/°F} \times \frac{0.89}{2} = 7.8 \text{ Mwsec/°F} \ .$$

3. $K_{G_1} = 0.033$.

K_{G_1} = (fraction of power generated in graphite) × (fraction of graphite power generated in G_1)

$$= 0.074 \times \frac{0.89}{2} = 0.033 \ .$$

4. $\tau_1 = \tau_2 = 0.84$ sec.

$$\tau_1 = \frac{(\frac{1}{2}\text{ core height})}{(\text{fuel velocity})}$$

$$= \frac{5\text{ ft}}{6\text{ ft/sec}} = 0.84\text{ sec}\ .$$

5. $K_{f_1} = K_{f_2} = 0.221.$

(fraction of heat generated in core fuel) =

1 − (fraction generated in graphite)

− (fraction generated in fertile salt)

− (fraction generated in blanket and annulus)

$= 1 - 0.074 - 0.017 - 0.025 = 0.884$.

Since each fuel lump contains one-fourth of the core fuel,

$$K_{f_1} = \frac{0.884}{4} = 0.221\ .$$

6. $(MC_p)_{f_1} = (MC_p)_{f_2} = 1.53$ Mwsec/°F.

Mass = (core volume)(fuel volume fraction)(density of fuel)

$= (502.65\text{ ft}^3)(0.165)(127\text{ lb/ft}^3)$

$= 1053$ lb ,

$C_p = 0.55\text{ Btu/lb- F} \times 2.928 \times 10^{-7}\text{ Mwhr/Btu} \times 3600\text{ sec/hr}$

$= 5.80 \times 10^{-4}$ Mwsec/lb-°F ,

$(MC_p) = 1.053 \times 10^3\text{ lb} \times 5.80 \times 10^{-4}$ Mwsec/lb-°F

$= 6.11 \times 10^{-1}$ Mwsec/°F

for whole core.

Since each fuel node contains one-fourth of the total core fuel,

$$(MC_p)_{f_1} = \frac{0.611}{4}$$

$$= 1.53 \text{ Mwsec/°F} .$$

7. $K_1 = K_2 = 0.5$. (One-half of heat transferred from graphite goes to each fuel lump)

(d) Final equations

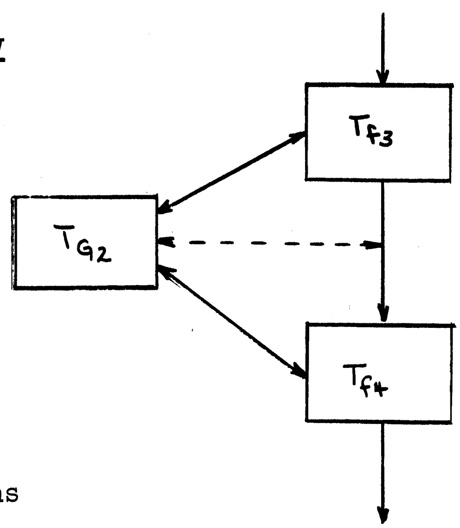
$$\frac{dT_{G_1}}{dt} = 0.00423\ P + 0.123\ T_{f_1} - 0.123\ T_{G_1} ,$$

$$\frac{dT_{f_1}}{dt} = 0.144\ P - 1.504\ T_{f_1} + 0.314\ T_{G_1} + 1.190\ T_{in} ,$$

$$\frac{dT_{f_2}}{dt} = 0.144\ P + 0.876\ T_{f_1} - 1.190\ T_{f_2} + 0.314\ T_{G_1} .$$

2. Core Downflow

(a) Model



(b) Equations

$$\frac{dT_{G_2}}{dt} = \frac{(hA)_{fg}}{(MC_p)_{G_2}} (T_{f_3} - T_G) + \frac{K_{G_2} P}{(MC_p)_{G_2}} ,$$

$$\frac{dT_{f_3}}{dt} = \frac{1}{\tau_3}(T_{f_2} - T_{f_3}) + \frac{K_3(hA)_{fg}}{(MC_p)_{f_3}}(T_{G_2} - T_{f_3}) ,$$

$$\frac{dT_{f_4}}{dt} = \frac{1}{\tau_4}(T_{f_3} - T_{f_4}) + \frac{K_4(hA)_{fg}}{(MC_p)_{f_4}}(T_{G_2} - T_{f_3}) .$$

(c) Values of coefficients

1. $(hA)_{fg} = 0.624$ Mw/°F.

Basis for value

$h = 1616$ Btu/hr-ft²-°F , (Calculated by G. H. Llewellyn)

$A = 1319$ ft³ ,

$A = \pi \times$ tube diameter $\times$ tube length $\times$ number of tubes

$$= \frac{\pi \times 1.5 \times 120 \times 336}{144} = 1319 ,$$

$hA = 2.132 \times 10^6$ Btu/hr-°F $\times$ 2.928×10^7 Mwhr/Btu

$= 0.624$ Mw/°F .

2. $(MC_p)_{G_1} = 7.8$ Mwsec/°F. (same as upflow)

3. $K_{G_1} = 0.33$. (same as upflow)

4. $\tau_3 = \tau_4 = 0.84$ sec. (same as upflow)

5. $K_{f_3} = K_{f_4} = 0.221$. (same as upflow)

6. $(MC_p)_{f_3} = (MC_p)_{f_4} = 1.53$ Mwsec/°F. (same as upflow)

7. $K_1 = K_2 = 0.5$. (same as upflow)

(d) Final equations

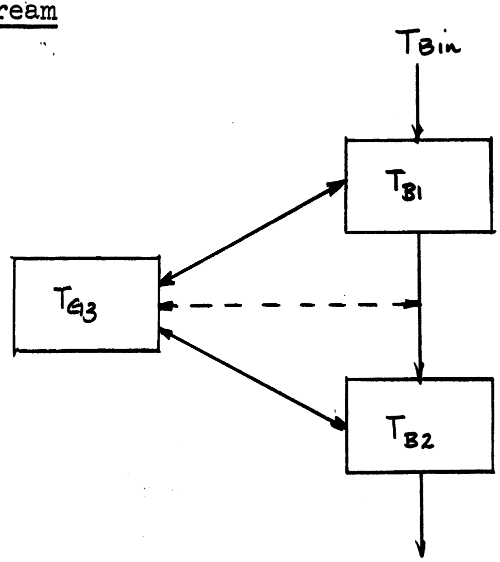
$$\frac{dT_{G_2}}{dt} = 0.00423\ P + 0.080\ T_{f_3} - 0.080\ T_{G_2}\ ,$$

$$\frac{dT_{f_3}}{dt} = 0.144\ P + 1.190\ T_{f_2} - 1.394\ T_{f_3} + 0.204\ T_{G_2}\ ,$$

$$\frac{dT_{f_4}}{dt} = 0.144\ P + 0.986\ T_{f_3} - 1.190\ T_{f_4} + 0.204\ T_{G_2}\ .$$

3. Fertile Stream

(a) Model



(b) Equations

$$\frac{dT_{G_3}}{dt} = \frac{(hA)_{bg}}{(MC_p)_{G_3}}\,(T_{B_1} - T_{G_3}) + \frac{K_{G_3}}{(MC_p)_{G_3}}\,P\ ,$$

$$\frac{dT_{B_1}}{dt} = \frac{1}{\tau_{B_1}}(T_{B_{in}} - T_{B_1}) + \frac{K_{B_1}}{(MC_p)_{B_1}}\,P + \frac{K_1(hA)_{bg}}{(MC_p)_{B_1}}(T_{G_3} - T_{B_1})\ ,$$

$$\frac{dT_{B_2}}{dt} = \frac{1}{B_2}(T_{B_1} - T_{B_2}) + \frac{K_{B_2}}{(MC_p)_{B_2}} P + \frac{K_1(hA)_{bg}}{(MC_p)_{B_2}}(T_{G_3} - T_{B_1}) .$$

(c) Values of coefficients

1. $(hA)_{bg} = 0.586$.

Llewellyn estimates that h could range from 0 to 1000 Btu/hr-ft^2-°F. For this preliminary study, a value of 500 was arbitrarily selected:

$$A \cong 4000 \text{ ft}^2 ,$$

$$hA = 2 \times 10^6 \text{ Btu/hr-°F} ,$$

$$hA = 2 \times 10^6 \text{ Btu/hr-°F} \times 2.928 \times 10^{-7} \text{ Mwhr/Btu} = 0.586 \text{ Mw/°F}.$$

2. $(MC_p)_{G_3} = 1.936$ Mwsec/°F.

$(MC_p)_{G_3}$ = (total graphite heat capacity)(fraction transferring heat to fertile stream)

= 17.6 Mwsec/°F × 0.11

= 1.936 Mwsec/°F .

3. $K_{G_3} = 0.00814$.

K_{G_3} = (fraction of power generated in graphite) × (fraction of graphite power generated in G_3)

= 0.074 × 0.11 = 0.00814 .

4. $\tau_{B_1} = \tau_{B_2} = 7.0$ sec.

This value was obtained by assuming that the fertile stream flow is designed to give an inlet temperature of 1150°F and an outlet temperature of 1250°F. The blanket salt carried out about 2.5% of the power or about 13.75 Mw. Thus

$$Q = WC_p \Delta T ,$$

$$C_p = 0.22 \text{ Btu/lb-°F} \times 2.928 \times 10^{-7} \text{ Mwhr/Btu} \times 3600 \text{ sec/hr}$$

$$= 2.32 \times 10^{-4} \text{ Mwsec/lb-°F} ,$$

$$W = \frac{Q}{C_p T} = \frac{13.75 \text{ Mw}}{2.32 \times 10^{-4} \text{ Mwsec/lb-°F} \times 100\text{°F}}$$

$$= 593 \text{ lb/sec.}$$

The mass of fertile salt in the core is

M = (core volume)(fertile stream volume fraction)(fertile salt density) ,

$$M = (502.65 \text{ ft}^3)(0.058)(277 \text{ lb/ft}^3) = 8300 \text{ lb} .$$

The residence time of fertile salt in a lump is one-half the total fertile salt residence time:

$$\tau_{B_1} = \tau_{B_2} = \frac{1}{2} \frac{8300 \text{ lb}}{593 \text{ lb/sec}} = 7 \text{ sec.}$$

5. $K_{B_1} = K_{B_2} = 0.0085.$

$$K_{B_1} = \frac{1}{2}(\text{fraction of heat generated in total fertile salt})$$

$$= \frac{1}{2} \times 0.017 = 0.0085 .$$

6. $(MC_p)_{B_1} = (MC_p)_{B_2} = 0.97$ Mwsec/°F .

$$(MC_p)_{B_1} = \frac{1}{2} \text{ total in-core fertile salt heat capacity}$$

$$= \frac{1}{2} (8300 \times 2.32 \times 10^{-4})$$

$$= 0.97 \text{ Mwsec/°F} .$$

7. $K_1 = K_2 = 0.5.$

(d) Final equations

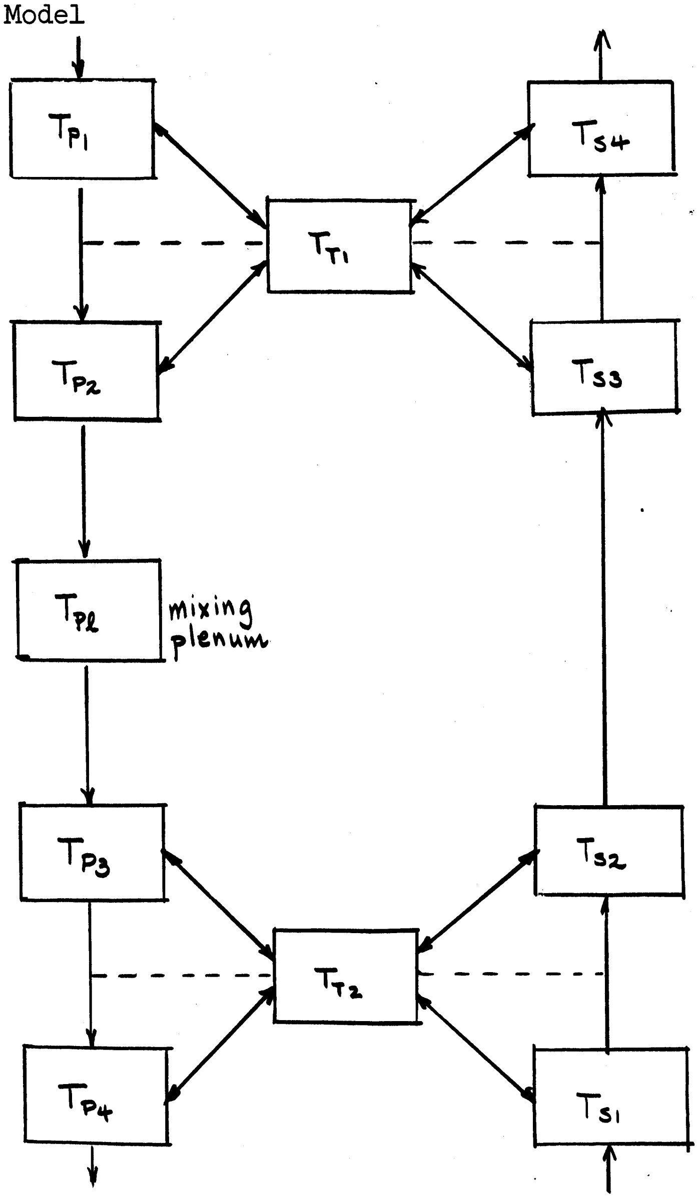
$$\frac{dT_{G_3}}{dt} = 0.00423\ P + 0.3027\ T_{B_1} - 0.3027\ T_{G_3}\ ,$$

$$\frac{dT_{B_1}}{dt} = 0.0088\ P - 0.445\ T_{B_1} + 0.3021\ T_{G_3} + 0.1429\ T_{B_{in}}\ ,$$

$$\frac{dT_{B_2}}{dt} = 0.0088\ P - 0.1592\ T_{B_1} - 0.1429\ T_{B_2} + 0.3021\ T_{G_3}\ .$$

Primary Heat Exchanger

(a) Model



(b) Equations

$$\frac{dT_{p_1}}{dt} = \frac{1}{\tau_{p_1}}(T_{p_{in}} - T_{p_1}) - \frac{(hA)_{p_1}}{(MC_p)_{p_1}}(T_{p_1} - T_{T_1}) ,$$

$$\frac{dT_{p_2}}{dt} = \frac{1}{\tau_{p_2}}(T_{p_1} - T_{p_2}) - \frac{(hA)_{p_2}}{(MC_p)_{p_2}}(T_{p_1} - T_{T_1}) ,$$

$$\frac{dT_{p\ell}}{dt} = \frac{1}{\tau_{p\ell}}(T_{p_2} - T_{p\ell}) ,$$

$$\frac{dT_{p_3}}{dt} = \frac{1}{\tau_{p_3}}(T_{p\ell} - T_{p_3}) - \frac{(hA)_{p_3}}{(MC_p)_{p_3}}(T_{p_3} - T_{T_2}) ,$$

$$\frac{dT_{p_4}}{dt} = \frac{1}{\tau_{p_4}}(T_{p_3} - T_{p_4}) - \frac{(hA)_{p_4}}{(MC_p)_{p_4}}(T_{p_3} - T_{p_2}) ,$$

$$\frac{dT_{T_1}}{dt} = \frac{2(hA)_{p_1}}{(MC_p)_{T_1}}(T_{p_1} - T_{T_1}) - \frac{2(hA)_{s_3}}{(MC_p)_{T_1}}(T_{T_1} - T_{s_3}) ,$$

$$\frac{dT_{T_2}}{dt} = \frac{2(hA)_{p_3}}{(MC_p)_{T_2}}(T_{p_3} - T_{T_2}) - \frac{2(hA)_{s_1}}{(MC_p)_{T_2}}(T_{T_2} - T_{s_1}) ,$$

$$\frac{dT_{s_1}}{dt} = \frac{1}{\tau_{s_1}}(T_{sin} - T_{s_1}) - \frac{(hA)_{s_1}}{(MC_p)_{s_1}}(T_{s_1} - T_{T_2}) ,$$

$$\frac{dT_{s_2}}{dt} = \frac{1}{\tau_{s_2}}(T_{s_1} - T_{s_2}) - \frac{(hA)_{s_2}}{(MC_p)_{s_2}}(T_{s_1} - T_{T_2}) ,$$

$$\frac{dT_{s_3}}{dt} = \frac{1}{\tau_{s_3}}(T_{s_2} - T_{s_3}) - \frac{(hA)_{s_3}}{(MC_p)_{s_3}}(T_{s_3} - T_{T_1}) ,$$

$$\frac{dT_{s_4}}{dt} = \frac{1}{s_4}(T_{s_3} - T_{s_4}) - \frac{(hA)_{s_3}}{(MC_p)_{s_3}} (T_{s_3} - T_{T_1}) \ ,$$

where

M = mass of material in a lump,

C_p = specific heat,

hA = heat transfer coefficient,

τ = residence time in a lump,

T_{p_j} = temperature of the fuel salt in lump i of the primary heat exchanger,

T_{s_i} = temperature of the coolant salt in lump i of the primary heat exchanger,

$T_{p\ell}$ = temperature of the fuel salt in the mixing plenum,

T_{pin} = temperature of fuel salt entering the heat exchanger,

T_{sin} = temperature of coolant salt entering the heat exchanger.

(c) Values of the coefficients

1. $\tau_{p_1} = \tau_{p_2} = \tau_{p_3} = \tau_{p_4} = 0.5$ sec.

There are 52.6 ft^3 of fuel salt in the heat exchanger tubes and the flow is 25 ft^3/sec. This gives a total residence time in the tubes of about 2 sec, giving 0.5 sec in each of the four fuel lumps.

2. $(hA)_{p_1} = (hA)_{p_2} = (hA)_{p_3} = (hA)_{p_4} = 1.65$ Mw/°F.

J. K. Jones provided the following data:

Coolant side h (inner region)	=	3700 Btu/hr-ft^2-°F
Coolant side h (outer region)	=	5124 Btu/hr-ft^2-°F
Tube wall conductance	=	4114 Btu/hr-ft^2-°F

The overall heat transfer coefficient, u, given on page 71 of ORNL-3996 is 1110 Btu/hr-ft^2-°F. This value is an average of the coefficients for the two sections. For the purposes of the dynamic analysis, it will be assumed constant in each region. This permits a solution for the fuel side h using (ignoring differences in inside area and outside area)

$$u = \cfrac{1}{\cfrac{1}{h(\text{coolant})} + \cfrac{1}{h(\text{fuel})} + \cfrac{1}{(\text{tube conductance})}} \; .$$

This gave values for coolant side coefficients of

Fuel side h (inner region) = 2590 Btu/hr-ft^2-°F
Fuel side h (outer region) = 2160 Btu/hr-ft^2-°F

For this study, average coefficients were used to apply in both regions:

Fuel side h = 2300 Btu/hr-ft^2-°F
Coolant side h = 4500 Btu/hr-ft^2-°F

The heat transfer area in each region is about 5000 ft^2. This gives

$$(hA) = 2300 \text{ Btu/hr-ft}^2\text{-°F} \times 5000 \text{ ft}^2 = 1.15 \times 10^7 \text{ Btu/hr-°F}$$

$$= 1.15 \times 10^7 \text{ Btu/hr-°F} \times 2.928 \times 10^7 \text{ Mwhr/Btu}$$

$$= 3.3 \text{ Mw/°F} .$$

Since each region contains two lumps,

$$(hA)_p = 1.65 \text{ Mw/°F} .$$

3. $(MC_p)_{p_1} = (MC_p)_{p_2} = (MC_p)_{p_3} = (MC_p)_{p_4} = 0.87$ Mwsec/°F.

$$C_p = 5.8 \times 10^{-4} \text{ Mwsec/lb-°F} ,$$

$$M = (\text{flow rate})(\text{density})(\text{residence time})$$

$$= (25 \text{ ft}^3\text{/sec})(127 \text{ lb/ft}^3)(0.5 \text{ sec})$$

$$= 1.5 \times 10^3 \text{ lb} .$$

$$MC_p = (1.5 \times 10^3 \text{ lb})(5.8 \times 10^{-4} \text{ Mwsec/lb-°F})$$

$$= 0.87 \text{ Mwsec/°F} .$$

4. $\tau_{p\ell} = 1.0$ sec.

The mixing plenum contains about 25 ft^3 of fuel salt and the flow rate is 25 ft^3/sec.

5. $(MC_p)_{T_1} = (MC_p)_{T_2} = 1.2$ Mwsec/°F.

$C_p(\text{INOR}) = 0.14$ Btu/lb-°F

$= 0.14$ Btu/lb-°F $\times 2.928 \times 10^{-7}$ Mwhr/Btu $\times 3600$ sec/hr

$= 1.48 \times 10^{-4}$ Mwsec/lb-°F ,

$M = \frac{\pi}{4}$ [(OD)^2] − (ID)^2] [(number of center section tubes)(length of center section tubes) + (number of outer section tubes)(length of outer section tubes] [INOR density] ,

$$M = \frac{\pi}{4} \frac{[(0.375 \text{ in.})^2 - (0.305 \text{ in.})^2}{144 \text{ in.}^2/\text{ft}^2} [4347 \times 13.5 \text{ ft}$$

$$+ 3794 \times 14.5 \text{ ft}] [548 \text{ lb/ft}^3] = 1.62 \times 10^4 \text{ lb} ,$$

$$MC_p = (1.62 \times 10^4 \text{ lb})(1.48 \times 10^{-4} \text{ Mwsec/lb-°F}) = 2.4 \text{ Mwsec/°F} .$$

Since each section contains about half of the tube mass,

$$(MC_p)_{T_1} = (MC_p)_{T_2} = 1.2 \text{ Mwsec/°F} .$$

6. $(hA)_{s_1} = (hA)_{s_2} = (hA)_{s_3} = (hA)_{s_4} = 3.3$ Mw/°F.

(See item 2 in this section.)

7. $\tau_{s_1} = \tau_{s_2} = \tau_{s_3} = \tau_{s_4} = 1.9$ sec.

Coolant salt volume in the heat exchanger = 280 ft^3 ,

Coolant salt flow rate = 37.2 ft^3/sec ,

$$\tau(\text{total}) = \frac{280}{37.2} = 7.5 \text{ sec} ,$$

$$\tau(\text{each lump}) = \frac{7.5}{4} = 1.9 .$$

8. $(MC_p)_{s_1} = (MC_p)_{s_2} = (MC_p)_{s_3} = (MC_p)_{s_4} = 3.75$ Mwsec/°F.

$$M = \left(\frac{\text{coolant salt volume in the heat exchanger}}{4}\right)(\text{density})$$

$$= \left(\frac{280\ ft^3}{4}\right)(125\ lb/ft^3) = 8.75 \times 10^3\ lb\ ,$$

$$C_p = 0.41\ Btu/lb\text{-}°F \times 2.928 \times 10^{-7}\ Mwhr/Btu \times 3600\ sec/hr$$

$$= 4.32 \times 10^{-4}\ Mwsec/lb\text{-}°F\ ,$$

$$MC_p = (0.875 \times 10^4\ lb)(4.32 \times 10^{-4}\ Mwsec/lb\text{-}°F) = 3.75\ .$$

(d) Final equations

$$\frac{dT_{p_1}}{dt} = -\ 3.90\ T_{p_1} + 1.90\ T_{T_1} + 2.0\ T_{pin}\ ,$$

$$\frac{dT_{p_2}}{dt} = 0.1\ T_{p_1} - 2.0\ T_{p_2} + 1.90\ T_{T_1}\ ,$$

$$\frac{dT_{p\ell}}{dt} = T_{p_2} - T_{p\ell}\ ,$$

$$\frac{dT_{p_3}}{dt} = 2.0\ T_{p\ell} - 3.90\ T_{p_3} + 1.90\ T_{T_2}\ ,$$

$$\frac{dT_{p_4}}{dt} = 0.1\ T_{p_3} - 2.0\ T_{p_4} + 1.90\ T_{T_2}\ ,$$

$$\frac{dT_{T_1}}{dt} = 2.75\ T_{p_1} + 5.50\ T_{s_3} - 8.25\ T_{T_1}\ ,$$

$$\frac{dT_{T_2}}{dt} = 2.75\ T_{p_3} + 5.50\ T_{s_1} - 8.25\ T_{T_2}\ ,$$

$$\frac{dT_{s_1}}{dt} = 0.88\ T_{T_2} - 1.41\ T_{s_1} + 0.53\ T_{sin}\ ,$$

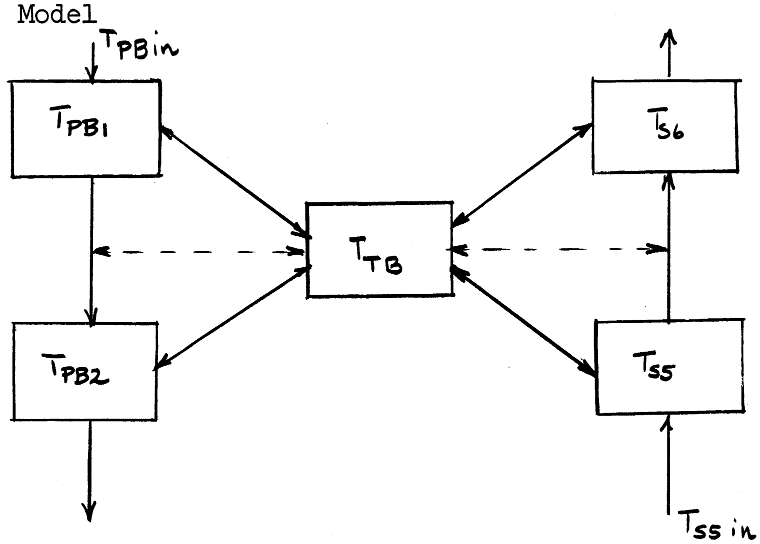
$$\frac{dT_{s_2}}{dt} = -\ 0.35\ T_{s_1} + 0.88\ T_{T_2} - 0.53\ T_{s_2}\ ,$$

$$\frac{dT_{s_3}}{dt} = 0.53\ T_{s_2} + 0.88\ T_{T_1} - 1.41\ T_{s_3}\ ,$$

$$\frac{dT_{s_4}}{dt} = -\ 0.35\ T_{s_3} + 0.88\ T_{T_1} - 0.53\ T_{s_4}\ .$$

Fertile Heat Exchanger

(a) Model



(b) Equations

$$\frac{dT_{PB_1}}{dt} = \frac{1}{\tau_{PB_1}}(T_{PBin} - T_{PB_1}) - \frac{(hA)_{PB_1}}{(WC_p)_{PB_1}}(T_{PB_1} - T_{TB}) ,$$

$$\frac{dT_{PB_2}}{dt} = \frac{1}{\tau_{PB_2}}(T_{PB_1} - T_{PB_2}) - \frac{(hA)_{PB_2}}{(WC_p)_{PB_2}}(T_{PB_1} - T_{TB}) ,$$

$$\frac{dT_{TB}}{dt} = \frac{2(hA)_{PB}}{(MC_p)_{TB}}(T_{TB_1} - T_{TB}) - \frac{2(hA)_s}{(MC_p)_{TB}}(T_{TB} - T_{s_5}) ,$$

$$\frac{dT_{s_5}}{dt} = \frac{1}{\tau_{s_5}}(T_{s_{5in}} - T_{s_5}) + \frac{(hA)_{s_5}}{(MC_p)_{s_5}}(T_{TB} - T_{s_5}) ,$$

$$\frac{dT_{s_6}}{dt} = \frac{1}{\tau_{s_6}}(T_{s_5} - T_{s_6}) + \frac{(hA)_{s_6}}{(MC_p)_{s_6}} (T_{TB} - T_{s_5}) \ .$$

(c) Values of coefficients

1. $\tau_{PB_1} = \tau_{PB_2} = 0.8$ sec.

$$\tau_{PB} = \frac{\text{(mass of blanket salt in a lump)}}{\text{(blanket salt flow rate)}} ,$$

$$\text{(Mass in both lumps)} = \frac{\pi}{4}(\text{tube ID})^2 \text{ (number of tubes)(length of}$$

$$\text{tubes)(salt density)} = \frac{\pi}{4} \frac{(0.305 \text{ in.})^2(1641)(8.25 \text{ ft})(277 \text{ lb/ft}^3)}{144 \text{ in.}^2/\text{ft}^2}$$

$$= 1900 \text{ lb} ,$$

$$\text{Flow} = 1.19 \times 10^3 \text{ lb/sec} ,$$

$$\tau_{PB} = \frac{950}{1190} = 0.8 \text{ sec} .$$

2. $(hA)_{PB_1} = (hA)_{PB_2} = (hA)_{s_5} = (hA)_{s_6} = 0.4$ Mw/°F.

A value of U of 1020 Btu/hr-ft^2 was obtained from page 76 of ORNL-3996. A rough approximation for the coefficients is obtained by ignoring tube conductance and assuming equal coefficients on each side of the tubes. This gives

$h = 2040$ Btu/hr-ft^2 ,

$A = 1330$ ft^2 (p. 75 of ORNL-3996)

$hA = 2040 \text{ Btu/hr-ft}^2 \times 1330 \text{ ft}^2 = 2.71 \times 10^6 \text{ Btu/hr-°F}$

$= 2.71 \times 10^6 \text{ Btu/hr-°F} \times 2.928 \times 10^{-7} \text{ Mwhr/Btu} = 0.8 \text{ Mw/°F}$,

For one lump, $hA = 0.4$ Mw/°F.

3. $(MC_p)_{PB_1} = (MC_p)_{PB_2} = 0.22.$

$M = 950$ lb , (see item 1 of this section)

$C_p = 2.32 \times 10^{-4}$ Mwsec/lb-°F ,

$MC_p = 9.5 \times 10^2 \times 2.32 \times 10^{-4} = 0.22$ Mwsec/°F .

4. $(MC_p)_T = 0.29$ Mwsec/°F.

$M = \frac{\pi}{4}$ [(OD)^2 − (ID)^2][number of tubes][tube length]

[density of INOR]

$$= \frac{\pi}{4} \frac{[(0.375 \text{ in.})^2 - (0.305)^2][1641][8.25 \text{ ft}][548 \text{ lb/ft}^3]}{144 \text{ in.}^2/\text{ft}^2}$$

= 1930 lb ,

$C_p = 1.48 \times 10^{-4}$ Mwsec/lb-°F ,

$(MC_p)_T = (1.93 \times 10^3 \text{ lb})(1.48 \times 10^{-4} \text{ Mwsec/lb-°F})$

$= 0.29$ Mwsec/°F .

5. $(MC_p)_{s_5} = (MC_p)_{s_6} = 1.5$ Mwsec/°F.

M = (volume of coolant salt)(density)

$= \frac{1}{2}$ (internal volume of heat exchanger − fertile salt volume − tube volume)(density) ,

$M = \frac{1}{2} (64.8 - 3.5 - 6.9 \text{ ft}^3)(125 \text{ lb/ft}^3) = 3.4 \times 10^3$ lb ,

$C_p = 0.43 \times 10^{-3}$ Mwsec/lb-°F ,

$(MC_p)_s = (3.4 \times 10^3 \text{ lb})(0.43 \times 10^{-3} \text{ Mwsec/lb-°F}) = 1.5$ Mwsec/°F .

6. $\tau_{s_5} = \tau_{s_6} = 0.74$ sec.

$$\tau = \frac{(\text{volume of lump})}{(\text{flow})} = \frac{27.7 \text{ ft}^3}{(37.2 \text{ ft}^3/\text{sec})} = 0.74 \text{ sec} .$$

(d) Final equations

$$\frac{dT_{PB_1}}{dt} = - 3.07\, T_{PB_1} + 1.82\, T_{TB} + 1.25\, T_{PBin} ,$$

$$\frac{dT_{PB_2}}{dt} = 1.25\, T_{PB_1} - 3.07\, T_{PB_2} + 1.82\, T_{TB} ,$$

$$\frac{dT_{TB}}{dt} = 2.76\, T_{PB_1} + 2.76\, T_{s_5} - 5.52\, T_{TB} ,$$

$$\frac{dT_{s_5}}{dt} = 0.27\, T_{TB} - 1.64\, T_{s_5} + 1.37\, T_{s_{5in}} ,$$

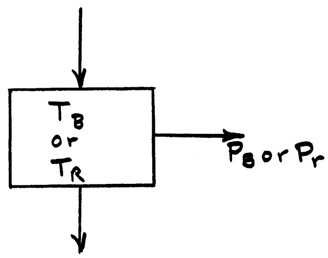
$$\frac{dT_{s_6}}{dt} = 1.08\, T_{s_5} + 0.27\, T_{TB} - 1.35\, T_{s_6} .$$

Boiler and Reheater

(a) Model

The dynamic model of the MSBR does not yet include equations for the steam system. The model includes the salt side of the boiler and reheater. The heat transfer to the steam is currently approximated by a power removal term. This power removal can be taken as constant in studying the effect of disturbances in the reactor or may be taken as the system input disturbance if an approximate analysis of the system response to load disturbances is desired. A preliminary dynamic analysis of the system is being made using this model prior to completion of the model for the final analysis.

The system model for the boiler and the reheater is:



(b) Equations

$$\frac{dT_B}{dt} = \frac{1}{T_B}(T_{B_{in}} - T_B) - \frac{P_B}{M_B C_{p_B}} ,$$

$$\frac{dT_R}{dt} = \frac{1}{\tau_R}(T_{R_{in}} - T_R) - \frac{P_R}{M_R C_{p_R}} ,$$

where

T_B = temperature of salt in the boiler,

T_R = temperature of salt in the reheater,

τ_B = residence time of salt in the boiler,

τ_R = residence time of salt in the reheater,

M_B = mass of salt in the boiler,

M_R = mass of salt in the reheater,

C_{p_B} = specific heat of salt in the boiler,

C_{p_R} = specific heat of salt in the reheater,

P_B = power removed from salt in the boiler,

P_R = power remover from salt in the reheater,

$T_{B_{in}}$ = salt temperature at boiler inlet,

$T_{R_{in}}$ = salt temperature at reheater inlet.

(c) Values of coefficients

1. $(MC_p)_B = 18.3$ Mwsec/°F .

The salt is on the shell side of a boiler.

$$\text{Total volume in boiler} = \pi(\text{shell radius})^2(\text{shell length})$$

$$= \pi(0.76 \text{ ft})^2(63.8 \text{ ft})$$

$$= 115 \text{ ft}^3 ,$$

$$\text{Volume of tubes} = (\text{number of tubes})\ \pi(\text{tube radius})^2(\text{tube length})$$

$$= \pi(349)(0.0208 \text{ ft})^2(63.8 \text{ ft}) = 30 \text{ ft}^3 ,$$

$$\text{Volume of salt} = 115 - 30 = 85 \text{ ft}^3 ,$$

$$\text{Total mass in 4 boilers} = 4(\text{volume in one boiler}) \times (\text{density})$$

$$= 4 \times 85 \text{ ft}^3 \times 126 \text{ lb/ft}^3 = 42{,}400 \text{ lb} ,$$

$$C_p = 4.32 \times 10^{-4} \text{ Mwsec/lb-°F} ,$$

$$(MC_p)_B = 4.24 \times 10^4 \text{ lb} \times 4.32 \times 10^{-4} \text{ Mwsec/lb-°F} = 18.3 \text{ Mwsec/lb-°F} .$$

2. $\tau_B = 10.4$ sec.

$$\tau_B = \frac{\text{Mass in a boiler}}{\text{Mass flow rate through boiler}}$$

$$= \frac{\frac{1}{4} \times 42{,}400 \text{ lb}}{3.66 \times 10^6 \text{ lb/hr}} \times 3600 \text{ sec/hr} = 10.4 \text{ sec} .$$

3. $P_B = 0.87$ P, where P = total power.

ORNL-3996 indicates that 87% of the power is removed by the boiler at steady state.

4. $(MC_p)_R = 5.80$ Mwsec/°F.

The salt is on the shell side of a reheater.

Total volume in reheater = π(shell radius)^2(shell length)

$$= \pi(1.16 \text{ ft})^2(22.9 \text{ ft}) = 98 \text{ ft}^3 ,$$

Volume of tubes = (number of tubes) π(tube radius)^2(tube length)

$$= \pi\ 628(0.031 \text{ ft})^2(22.9 \text{ ft}) = 44 \text{ ft}^3 ,$$

Volume of salt = $98 - 44 = 54$ ft^3 ,

Total mass in 2 reheaters = 2(volume in one reheater)(density)

$$= 2 \times (54 \text{ ft}^3)(125 \text{ lb/ft}^3) = 13{,}500 \text{ lb} ,$$

$C_p = 4.32 \times 10^{-4}$ Mwsec/lb-°F ,

$$(MC_p)_R = 1.35 \times 10^4 \text{ lb} \times 4.32 \times 10^{-4} \text{ Mwsec/lb-°F} = 5.8 \text{ Mwsec/°F} .$$

5. $\tau_R = 22$ sec.

$$\tau = \frac{\text{Mass in a reheater}}{\text{Mass flow through reheater}} = \frac{\frac{1}{2}(13{,}500 \text{ lb})}{(1.1 \times 10^6 \text{ lb/hr})} (3600 \text{ sec/hr})$$

$$= 22 \text{ sec} .$$

6. $P_R = 0.13$ P.

Component Couplings

The various subsystems in the MSBR are coupled by fluid transport. These were treated as pure transport delays in the model. For example, the fuel salt takes 1.32 sec to travel from the core to the heat exchanger. The equation derived earlier for this heat exchanger node was

$$\frac{dT_{p_1}}{dt} = - 3.90\ T_{p_1} + 1.90\ T_{T_1} + 2.0\ T_{p_{in}} .$$

The term, $T_{p_{in}}$, is given by

$$T_{p_{in}}(t) = T_{f_4}(t - 1.32) ,$$

where

T_{f_4} = temperature of the fuel salt in the last (fourth) core node.

The transit times were obtained from the expression:

$$\tau = \frac{\text{Flow path length (ft)}}{\text{Fluid linear flow rate (ft/sec)}} .$$

The flow paths were taken from the plant layouts (furnished mostly by W. Terry). The resulting delay times are given below:

Flow Path	Delay Time (sec)
1. Fuel heat exchanger → core	1.22
2. Core → fuel heat exchanger	1.32
3. Fuel heat exchanger → fertile heat exchanger	7.88
4. Fertile heat exchanger → core	7.0
5. Core → fertile heat exchanger	7.0
6. Fertile heat exchanger → boiler	13.5
7. Fertile heat exchanger → reheater	17.3
8. Boiler → fuel heat exchanger	4.2
9. Reheater → fuel heat exchanger	11.1

Distribution

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