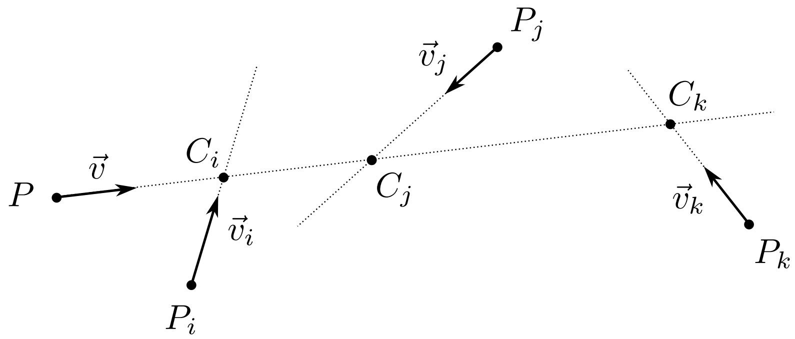

For all i, we have:

$$C_i = P_i + t_i\vec{v}_i = P + t_i\vec{v} \text{ and therefore: } t_i(\vec{v}_i - \vec{v}) + P_i - P = \vec{0}$$

Hence for all (i, j, k):

$$\begin{cases} t_i(\vec{v}_i - \vec{v}) + P_i - P = \vec{0} & : \quad (1) \\ t_j(\vec{v}_j - \vec{v}) + P_j - P = \vec{0} & : \quad (2) \\ t_k(\vec{v}_k - \vec{v}) + P_k - P = \vec{0} & : \quad (3) \end{cases}$$

Which gives:

$$\begin{cases} (2)-(1): \quad (t_i - t_j)\vec{v} + t_j\vec{v}_j - t_i\vec{v}_i + P_j - P_i = \vec{0} & : \quad (4) \\ (3)-(1): \quad (t_i - t_k)\vec{v} + t_k\vec{v}_k - t_i\vec{v}_i + P_k - P_i = \vec{0} & : \quad (5) \end{cases}$$

Which in turn, with $(t_i - t_k) \times (4) - (t_i - t_j) \times (5)$, implies:

$$t_it_j(\vec{v}_j - \vec{v}_i) + t_jt_k(\vec{v}_k - \vec{v}_j) + t_it_k(\vec{v}_i - \vec{v}_k) + t_i(P_j - P_k) + t_j(P_k - P_i) + t_k(P_i - P_j) = \vec{0} : \quad (6)$$

For all i, we note:

$$P_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \text{ and } \vec{v}_i = \begin{bmatrix} v_{xi} \\ v_{yi} \\ v_{zi} \end{bmatrix}$$

Furthermore, for all (i, j), we note

$$P_j - P_i = \begin{bmatrix} x_j - x_i \\ y_j - y_i \\ z_j - z_i \end{bmatrix} = \begin{bmatrix} \delta x_{ij} \\ \delta y_{ij} \\ \delta z_{ij} \end{bmatrix} \text{ and } \vec{v}_j - \vec{v}_i = \begin{bmatrix} v_{xj} - v_{xi} \\ v_{yj} - v_{yi} \\ v_{zj} - v_{zi} \end{bmatrix} = \begin{bmatrix} \delta v_{xij} \\ \delta v_{yij} \\ \delta v_{zij} \end{bmatrix}$$

We can now write (6) as:

$$\begin{cases} \delta v_{xij}t_it_j + \delta v_{xjk}t_jt_k + \delta v_{xki}t_it_k + \delta x_{kj}t_i + \delta x_{ik}t_j + \delta x_{ji}t_k = 0 & : \quad (7) \\ \delta v_{yij}t_it_j + \delta v_{yjk}t_jt_k + \delta v_{yki}t_it_k + \delta y_{kj}t_i + \delta y_{ik}t_j + \delta y_{ji}t_k = 0 & : \quad (8) \\ \delta v_{zij}t_it_j + \delta v_{zjk}t_jt_k + \delta v_{zki}t_it_k + \delta z_{kj}t_i + \delta z_{ik}t_j + \delta z_{ji}t_k = 0 & : \quad (9) \end{cases}$$

With $\delta v_{yij} \times (7) - \delta v_{xij} \times (8)$ we get:

$$\begin{aligned} &\delta v_{yij}\left[\delta v_{xjk}t_jt_k + \delta v_{xki}t_it_k + \delta x_{kj}t_i + \delta x_{ik}t_j + \delta x_{ji}t_k\right] \\ &-\delta v_{xij}\left[\delta v_{yjk}t_jt_k + \delta v_{yki}t_it_k + \delta y_{kj}t_i + \delta y_{ik}t_j + \delta y_{ji}t_k\right] \\ &= (\delta v_{yij}\delta v_{xjk} - \delta v_{xij}\delta v_{yjk})t_jt_k \\ &+(\delta v_{yij}\delta v_{xki} - \delta v_{xij}\delta v_{yki})t_it_k \end{aligned}$$