

International Macroeconomics

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[Chapters 12-16 are preliminary and incomplete.]

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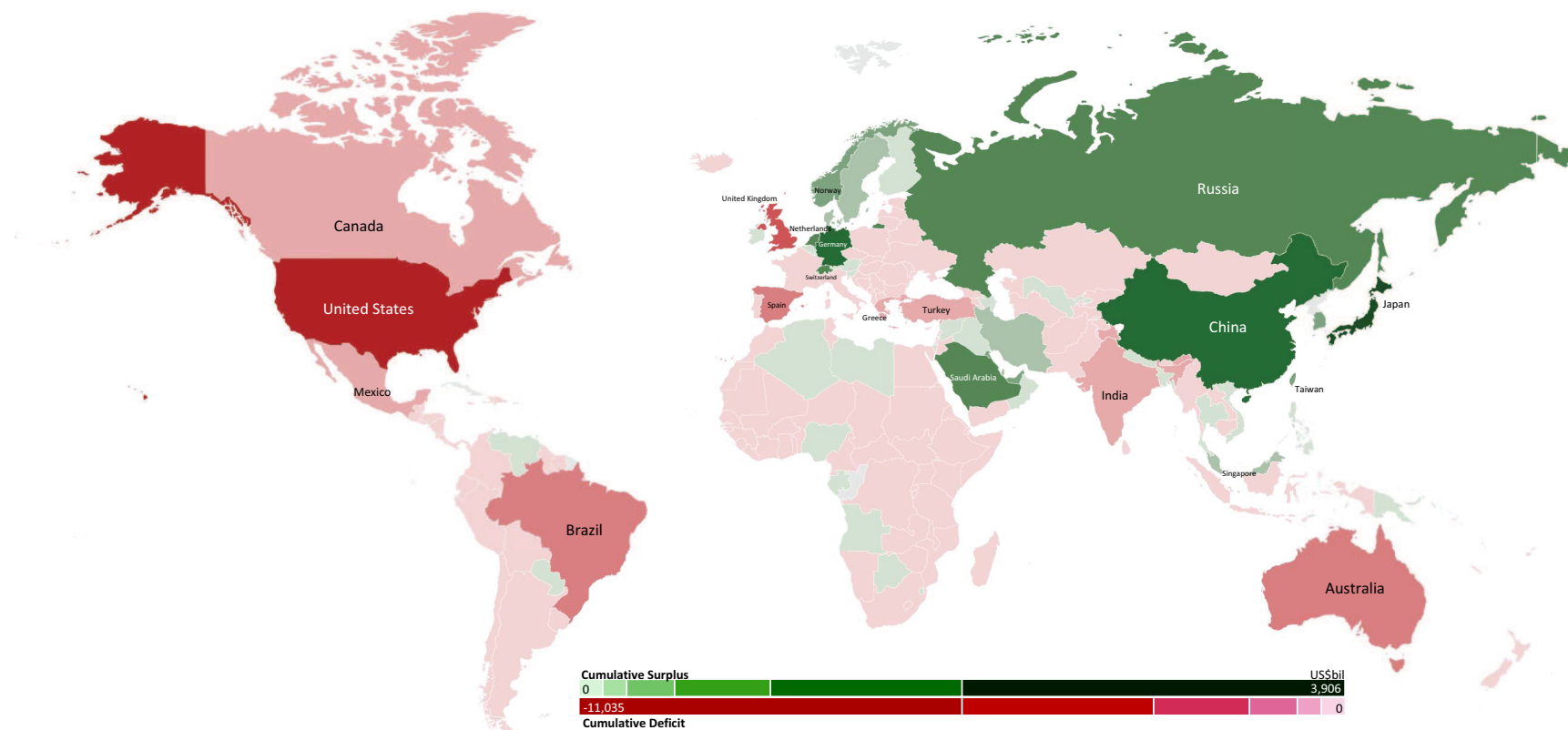
Chapter 1

Global Imbalances

In the past three decades, the world has witnessed the emergence of large external debt positions in some countries and large external asset positions in others. The United States became the largest external debtor in the world in the late 1980s and has maintained this position ever since. At the same time, China, Japan, and Germany hold large asset positions against the rest of the world. This phenomenon has come to be known as global imbalances.

The map in figure [1.1](#) presents the accumulated current account balances from 1980 to 2017 for 212 countries. As we will explain in more detail later in this chapter, to a first approximation the current account measures the change in a country's net foreign asset position. Current account surpluses increase a country's asset position and current account deficits decrease it. By accumulating the current account balances of each country over time, we can obtain an idea of which countries have been playing the role of lenders and which the role of borrowers. Cumulative surpluses appear in green and cumulative deficits in red. Darker tones correspond to larger cumulative

Figure 1.1: Cumulative Current Account Balances Around the World: 1980-2017



Note. The map shows for each country the sum of current account balances (in billions of U.S. dollars) between 1980 and 2017. Cumulative current account surpluses appear in green and cumulative deficits in red. There are six shades of red and green corresponding, respectively, at least one half, one fourth, one eighth, one sixteenth, and one thirty-second of the maximum cumulative current account deficit (U.S.: -\$11,035bn) and the maximum cumulative current account surplus (Japan: \$3,906bn). The data source is Philip R. Lane and Gian Maria Milesi-Ferretti (2017), "International Financial Integration in the Aftermath of the Global Financial Crisis," IMF Working Paper 17/115. Data for former Soviet Union countries start in 1992. Countries for which no data are available appear in gray. Country names are displayed for the countries with the top 10 largest cumulated current account surpluses and deficits.

deficits or surpluses. If the long-run cumulative current accounts of all countries were more or less balanced, then the map should be filled in with only light colors. The fact that the map has several dark green and dark red colors is therefore an indication that some countries have been consistently borrowing and others consistently lending over the past thirty eight years.

The United States appears in dark red and China in dark green, reflecting the fact that the former is the world's largest external debtor and the latter one of the world's largest creditors. More generally, the pattern that emerges is that over the past four decades, the lenders of the world have been Japan, China, Germany, and oil- and gas-exporting countries (Russia, Norway, Saudi Arabia, Kuwait, United Arab Emirates, and Qatar). The rest of the world has been borrowing from these countries.

This chapter traces global imbalances across time and across its three main components, international trade in goods and services, cross-border asset flows, and changes in the market value of foreign asset positions. It begins by introducing some basic concepts related to a country's external accounts.

1.1 Balance-of-Payments Accounting

A country's international transactions are recorded in the balance-of-payments accounts. In the United States this data is produced by the Bureau of Economic Analysis (www.bea.gov) and called International Transactions Accounts (ITA). A country's balance of payments has two main components: the *current account* and the *financial account*. The current account records

exports and imports of goods and services and international receipts or payments of income. Exports and income receipts enter with a plus and imports and income payments enter with a minus. For example, if a U.S. resident buys a smartphone from South Korea for \$500, then the U.S. current account goes down by \$500. This is because this transaction represents an import of goods worth \$500.

The financial account keeps record of transactions in financial assets between residents and nonresidents. Sales of assets to nonresidents represent an export of an asset and are given a positive sign in the financial account. Purchases of assets from nonresidents represent an import of a financial asset and enter the financial account with a negative sign. For example, in the case of the import of the smartphone, suppose the U.S. resident pays for the phone with U.S. currency, then this represents a sale (export) of U.S. financial assets (currency) to a South Korean resident (Samsung Co., say) in the amount of \$500. Accordingly, the U.S. financial account records a positive entry of \$500.

The smartphone example illustrates a fundamental principle of balance-of-payments accounting known as *double-entry bookkeeping*. Each transaction enters the balance of payments twice, once with a positive sign and once with a negative sign. To illustrate this principle with another example, suppose that an Italian friend of yours comes to visit you in New York and stays at the Lucerne Hotel. He pays \$400 for his lodging with his Italian VISA card. In this case, the U.S. is exporting a service (hotel accommodation), so the current account increases by \$400. At the same time, the Lucerne Hotel (a U.S. resident) purchases (imports) a financial asset worth \$400 (the

promise of VISA-Italy, a nonresident, to pay \$400), which decreases the U.S. financial account by \$400. (Can you figure out how this transaction would be recorded in the Italian balance of payments accounts?)

An implication of the double-entry bookkeeping methodology is that any change in the current account must be reflected in an equivalent change in the country's financial account, that is, the current account equals the difference between a country's purchases of assets from foreigners and its sales of assets to them, which is the financial account preceded by a minus sign. This relationship is known as the *fundamental balance-of-payments identity*. Formally,

$$\text{Current Account Balance} = -\text{Financial Account Balance} \quad (1.1)$$

There is a third component of the Balance of Payments (and thus a third term in the balance-of-payments identity), called the capital account. It keeps record of international transfers of financial capital. The major types of entries in the capital account are debt forgiveness and migrants' transfers (goods and financial assets accompanying migrants as they leave or enter the country). Although insignificant in the United States, movements in the capital account can be important in other countries. For instance, in July 2007 the U.S. Treasury Department announced that the United States, Germany, and Russia will provide debt relief to Afghanistan for more than 11 billion dollars. This is a significant amount for the balance of payments accounts of Afghanistan, representing about 99 percent of its foreign debt obligations. But the amount involved in this debt relief operation is a small

figure for the balance of payments of the three donor countries. The capital account also records payments associated with foreign insurance contracts. For example, in the fourth quarter of 2012 net capital account receipts were \$7.2 billion reflecting receipts from foreign insurance companies for losses resulting from Hurricane Sandy. Because the capital account is quantitatively irrelevant for the balance of payments of most countries, we will ignore it in the remainder of the book and will focus on the current account and the financial account.

Let's now take a closer look at each side of the fundamental balance-of-payments identity (1.1). A more detailed decomposition of the current account is given by

$$\begin{aligned} \text{Current Account Balance} &= \text{Trade Balance} \\ &+ \text{Income Balance} \\ &+ \text{Net Unilateral Transfers.} \end{aligned}$$

In turn, the trade and income balances each include two components as follows

$$\begin{aligned} \text{Trade Balance} &= \text{Merchandise Trade Balance} \\ &+ \text{Services Balance} \end{aligned}$$

and

$$\begin{aligned}\text{Income Balance} &= \text{Net Investment Income} \\ &+ \text{Net International Compensation to Employees.}\end{aligned}$$

The Trade Balance, or Balance on Goods and Services, is the difference between exports and imports of goods and services. The Merchandise Trade Balance, or Balance on Goods, keeps record of net exports of goods and the Services Balance keeps record of net exports of services, such as transportation, travel expenditures, and legal assistance.

In the Income Balance, net investment income is given by the difference between income receipts on U.S.-owned assets abroad and income payments on foreign-owned assets in the United States. Net investment income includes items such as international interest and dividend payments and earnings of domestically-owned firms operating abroad. In the United States, net investment income is by far the most important component of the income balance.

The second component of the Income Balance, Net International Compensation to Employees, includes, as positive entries compensation receipts from (1) earnings of U.S. residents employed temporarily abroad, (2) earnings of U.S. residents employed by foreign governments in the United States, and (3) earnings of U.S. residents employed by international organizations located in the United States, such as the United Nations, the International Monetary Fund, and the International Bank for Reconstruction and Development. The third category represents the largest source of such receipts.

Negative entries to Net International Compensation to Employees include U.S. compensation payments to (1) Foreign workers (mostly from Canada and Mexico) who commute to work in the United States, (2) foreign students studying in the United States (yes, some students do get paid to study! Mostly at the graduate level, though), (3) foreign professionals temporarily residing in the United States, and (4) foreign temporary workers in the United States. In the United States, however, Net International Compensation to Employees is so small that the Income Balance is basically equal to Net Investment Income. In most of what follows we will ignore Net International Compensation to Employees because such transactions are quantitatively minor and hence will equate the Income Balance with Net Investment Income.

The third component of the current account, Net Unilateral Transfers (also called secondary income in the ITA accounts), keeps record of the difference between gifts, that is, payments that do not correspond to purchases of any good, service, or asset, received from the rest of the world and gifts made by the United States to foreign countries. One big item in this category is private remittances. For example, payments by a U.S. resident to relatives residing in Mexico would enter with a minus in Net Unilateral Transfers. Another prominent type of unilateral transfer is U.S. Government Grants, which represent transfers of real resources or financial assets to foreigners for which no repayment is expected.

The Financial Account, which, as mentioned earlier, measures the difference between sales of assets to foreigners and purchases of assets from

foreigners, has two main components as follows:

$$\begin{aligned}\text{Financial Account} &= \text{Increase in Foreign-owned assets in the United States} \\ &\quad - \text{Increase in U.S.-owned assets abroad.}\end{aligned}$$

Foreign-owned assets in the United States includes U.S. securities held by foreign residents, U.S. currency held by foreign residents, U.S. borrowing from foreign banks, and foreign direct investment in the United States. U.S.-owned assets abroad includes foreign securities, U.S. bank lending to foreigners, and U.S. direct investment abroad.

An international transaction does not necessarily have to give rise to one entry in the current account and one entry in the financial account. It can be the case that it gives rise to two offsetting entries in the financial account or two offsetting entries in the current account. The two examples given earlier, namely, importing a smartphone and paying for it with cash or the Italian tourist paying the New York hotel with a credit card, both give rise to one entry in the current account and one entry in the financial account. International transactions that involve the exchange of financial assets generate two entries in the financial account and no entry in the current account. For example, if a U.S. resident purchases shares from FIAT Italy paying with dollars, then the financial account receives both a positive entry (the sale, or export, of dollars to Italy) and a negative entry (the purchase, or import, of equity shares from Italy). It can also be the case that a transaction generates two offsetting entries in the current account and no entry in the financial account. Suppose the U.S. donates

Table 1.1: The Current Account of the United States in 2018

Item	Billions of dollars	Percentage of GDP
Current Account	-488.5	-2.4
Trade Balance	-622.1	-3.0
Balance on Goods	-891.3	-4.3
Balance on Services	269.2	1.3
Income Balance	244.3	1.2
Net Investment Income	258.1	1.3
Compensation of Employees	-13.8	-0.1
Net Unilateral Transfers	-110.7	-0.5
Private Transfers	-94.4	-0.5
U.S. Government Transfers	-16.2	-0.1

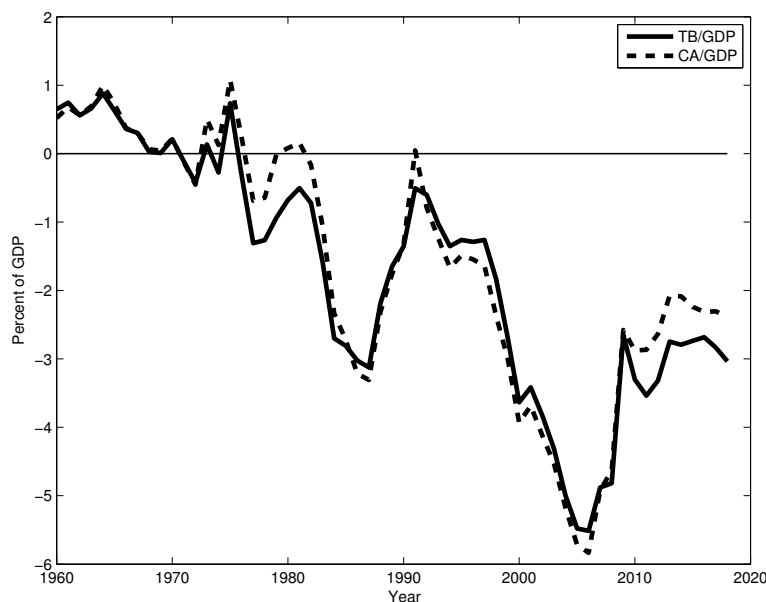
Data Source: Authors' calculations based on data from ITA Tables 1.1 and 5.1. and NIPA Table 1.1.5. of the Bureau of Economic Analysis.

medications worth \$10 million to an African country afflicted by Malaria. This gift gives rise to two entries in the current account. The export of the malaria medication would be recorded in the merchandise trade balance, which is part of the current account, with +\$10 million. As the U.S. does not receive an item of economic value in return, the offset is an entry of -\$10 million in net unilateral transfers, which is also part of the current account.

1.2 The Trade Balance and the Current Account

What does the U.S. current account look like? Take a look at table 1.1, which displays the U.S. international transactions recorded in the current account for 2018. In that year, the United States experienced a large current account deficit of \$488.5 billion or 2.4 percent of Gross Domestic Product (GDP) and also a large trade deficit of \$622.1 billion, or 3.0 percent of GDP.

Figure 1.2: The U.S. Trade Balance and Current Account As Percentages of GDP: 1960-2018



Notes. *TB* and *CA* stand for trade balance and current account, respectively. Data source: ITA Tables 1.1 and NIPA Table 1.1.5. <http://www.bea.gov>

Current-account and trade-balance deficits are frequently observed. In fact, as shown in figure 1.2, the U.S. trade and current-account balances have been in deficit ever since the 1980s. The plot covers the years 1960 to 2018. Over this period, the current-account and trade-balance deficits have been roughly equal to each other.

Looking inside the trade balance, we note from table 1.1 that in 2018 the United States was a net importer of goods, with a deficit in the trade of goods of 4.3% of GDP, and, at the same time, a net exporter of services, with a service balance surplus of 1.3% of GDP. The U.S. has a comparative advantage in the production of human-capital-intensive services, such

as professional consulting, higher education, research and development, and health care. At the same time, the U.S. imports basic goods, such as primary commodities (e.g., crude oil), consumer durables (e.g., electronics), and textiles.

The fact that in the United States the trade balance and the current account have been broadly equal to each other in magnitude for the past sixty years means that the sum of the other two components of the current account, the income balance and net unilateral transfers, was small in most years during that period. Since 2008, however, the difference between the current account and the trade balance has become larger. For example, in 2018 the difference between the current account and the trade balance was \$133.6 billion, or about 20 percent of the trade balance. This was accounted for mainly by an increase in the income balance.

The negative entry for net unilateral transfers in table 1.1 means that in 2018 the United States made more gifts to other nations than it received. This has been the case in most prior years. A large fraction of these international gifts are remittances of foreign workers residing in the U.S. to relatives in their countries of origin. Typically foreign workers residing in the U.S. send much larger remittances abroad than U.S. workers residing abroad send back to the United States. In fact, the latter figure is so small that it is often not reported separately in the International Transactions Accounts. A special report by the Bureau of Economic Analysis in 2011 shows that in 2009 personal transfers of U.S. immigrants to foreign residents were \$38 billion but personal transfers from U.S. emigrants living abroad to U.S. res-

idents were less than \$1 billion.¹ Thus, net private remittances were almost the same as gross private remittances.

Overall, net remittances are a small fraction of the U.S. current account. But, for some countries, they can represent a substantial source of income. For example, in 2016 Honduras received remittances for \$3.9 billion, almost exclusively coming from the United States. This figure represents 18.4 percent of Honduras' GDP, but only 0.02 percent of the United States'. The same is true for other small countries in Central America. For El Salvador, for example, the flow of dollars coming from the United States has been so large that in 2001 its government decided to adopt the U.S. dollar as legal tender. Even for much larger economies remittances can represent a nonnegligible source of income. For example, in 2016 Mexico received \$28.7 billion in remittances amounting to 2.7 percent of its GDP. As in the cases of Honduras and El Salvador, virtually all of the remittances received by Mexico originated in the United States, for whom they represented only 0.15 percent of GDP.

U.S. net unilateral transfers have been negative ever since the end of World War II, with one exception. In 1991, net unilateral transfers were positive because of the payments the U.S. received from its allies in compensation for the expenses incurred during the Gulf war.

¹The data source is www.bea.gov, filename, Private Remittances.pdf, page 64, Table 16.

1.3 Trade Balances and Current-Account Balances Across Countries

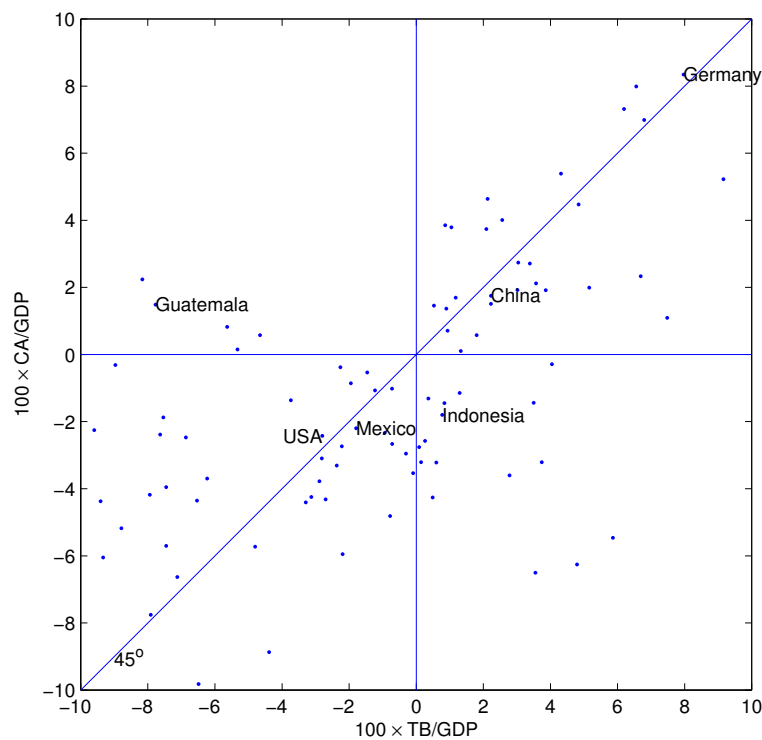
Although in the United States current account balances and trade balances typically have the same sign and similar sizes, this need not be the case for every country. The current account can be larger or smaller than the trade balance. Furthermore, the trade balance and the current account can both be positive or both be negative or can even have opposite signs.

Figure 1.3 illustrates this point. It displays the trade balance and the current account as percentages of GDP, denoted TB/GDP and CA/GDP , respectively, in 2016 for 88 countries. The space $(TB/GDP, CA/GDP)$ is divided into six regions, depending on the signs of the trade balance and the current account and on their relative magnitudes. It is evident from figure 1.3 that most $(TB/GDP, CA/GDP)$ pairs fall around the 45-degree line. This means that for many countries the trade balance and the current account are of the same sign and of roughly the same magnitude. The clustering around the 45-degree line suggests that, as in the United States, in many countries, the trade balance is the dominant component of the current account.

But, there are deviations from this pattern. Figure 1.3 and table 1.2 highlight six countries with all six possible configurations of the pair $(TB/GDP, CA/GDP)$.

China is an example of a country that in 2016 ran surpluses in both the trade balance and the current account with the trade balance exceeding the current account (a dot in the first quadrant and below the 45-degree line). The trade balance surplus was larger than the current account because China

Figure 1.3: Trade Balances and Current Account Balances Across Countries in 2016



Notes. TB denotes the trade balance and CA denotes the current account balance. The data source is World Development Indicators (WDI), available at databank.worldbank.org. There are 88 countries included in the figure. Countries in the WDI database with trade balances or current account balances in excess of ± 10 percent of GDP were excluded.

Table 1.2: Trade Balance and Current Account as Percentages of GDP in 2016 for Selected Countries

Country	TB/GDP	CA/GDP
China	2.2	1.8
Germany	8.0	8.3
Guatemala	-7.8	1.5
United States	-2.8	-2.4
Mexico	-1.8	-2.2
Indonesia	0.8	-1.8

Notes. TB denotes trade balance and CA denotes current account. Data source. World Development Indicators, available online at databank.worldbank.org.

ran a deficit in the income balance, and, in particular, a negative balance in net investment income. This is a bit surprising, because China is a large net creditor to the rest of the world, so one would expect that its net investment income (such as net interest, dividend, and earnings income) is positive. The reason why it is negative is that China saves in safe low-return assets such as U.S. government bonds, while foreign investment in China is predominantly in the form of high-return assets, mostly foreign direct investment. This is not a peculiarity of the year 2016. China has been running a deficit on its net investment income account in spite of holding a positive net foreign asset position for the past decade. We expound this point in section 1.7.3.

Like China, Germany displays both a current-account and a trade-balance surplus. However, unlike China, the German current-account surplus is larger than its trade-balance surplus (a dot in the first quadrant and above the 45-degree line). This difference can be explained by the fact that Ger-

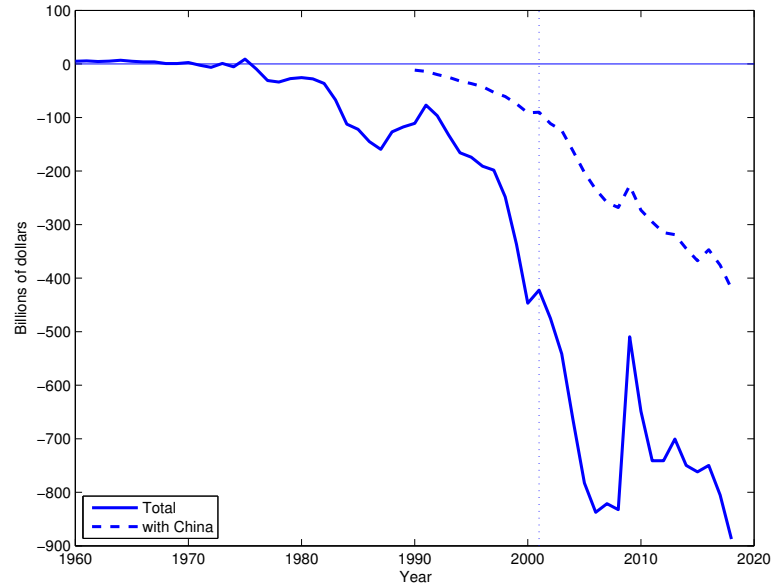
many, unlike China, receives positive net investment income on its positive net foreign asset position. Germany is a safe place to invest and therefore can borrow at low rates. At the same time, German investors hold relatively more risky and hence higher return foreign assets.

Guatemala provides an example of a country with a current account surplus in spite of a sizable trade-balance deficit (a dot in the second quadrant). The positive current account balance is the consequence of large personal remittances received (amounting to more than 10 percent of GDP in 2016) from Guatemalans residing abroad, mostly in the United States.

Mexico, the United States, and Indonesia all experienced current-account deficits in 2016. In the case of Mexico and the United States, the current-account deficits were associated with trade deficits of about equal sizes. However, unlike in the United States, in Mexico the current account deficit was slightly larger than the trade deficit (a dot on the third quadrant and below the 45-degree line). This is because Mexico has an external debt that generates large negative net investment income. In spite of these large income payments, the difference between the trade balance and the current account ends up being small because of large positive net unilateral transfers stemming from personal remittances received from Mexicans working in the United States.

Finally, Indonesia displays a negative current account in spite of running trade balance surpluses (a dot in the fourth quadrant) and being a net receiver of remittances. The reason is that this country has a large negative foreign asset position of close to 50 percent of GDP, whose service requires large interest payments to external creditors.

Figure 1.4: The U.S. Merchandise Trade Balance with China



Notes. The data source for the U.S. merchandise trade balance is ITA Table 1.1. The data source for the bilateral merchandise trade balance between the United States and China is the OECD, <http://stats.oecd.org> for the period 1990 to 2002 and ITA Table 1.3 for the period 2003 to 2018. The vertical line marks the year 2001 when China became a member of the World Trade Organization.

1.4 Imbalances in Merchandise Trade

Figure 1.4 displays the U.S. merchandise trade balance since 1960 and its bilateral merchandise trade balance with China since 1990. The starting date for China is dictated by data availability. Most likely, however, the bilateral trade balance prior to 1990 was as small as or even smaller than in 1990 because of legal and political impediments to Sino-American trade. During the 1960s, trade was limited by an existing embargo. Despite the fact that, following his famous trip to China, President Nixon lifted the U.S. trade embargo on China in 1971, and despite the fact that the U.S. Congress

passed a trade agreement conferring contingent Most Favored Nation status on China in 1980, trade impediments persisted because of existing laws linking trade benefits with human right policies of communist countries.

Figure 1.4 shows that the U.S. merchandise trade deficit with China has widened over time, in particular, since China became a member of the World Trade Organization (WTO) in December 2001. When a country joins the WTO, it gains improved access to global markets and, in return, must grant other countries better access to its domestic market. For example, in the case of China the WTO agreement obliged this country to cut import tariffs and give foreign businesses greater access to domestic insurance, banking and telecommunications markets. Between 2001 and 2018, the deficit on the U.S. bilateral merchandise trade balance with China increased from \$90 billion to \$420 billion accounting for 70 percent of the overall deterioration of the U.S. merchandise trade balance over this period. Prior to 2001 the U.S. merchandise trade deficit with China accounted for about 20 percent of its overall deficit. After 2001, this share started climbing and reached close to 50 percent by 2018. The figure thus suggests that U.S. trade imbalances with China have widened after China joined the WTO not only in absolute terms but also in relative terms. At the same time, the figure also shows that imbalances in U.S. merchandise trade are not limited to trade with China as half of the overall merchandise trade deficit of the United States continues to stem from imbalanced trade with countries other than China.

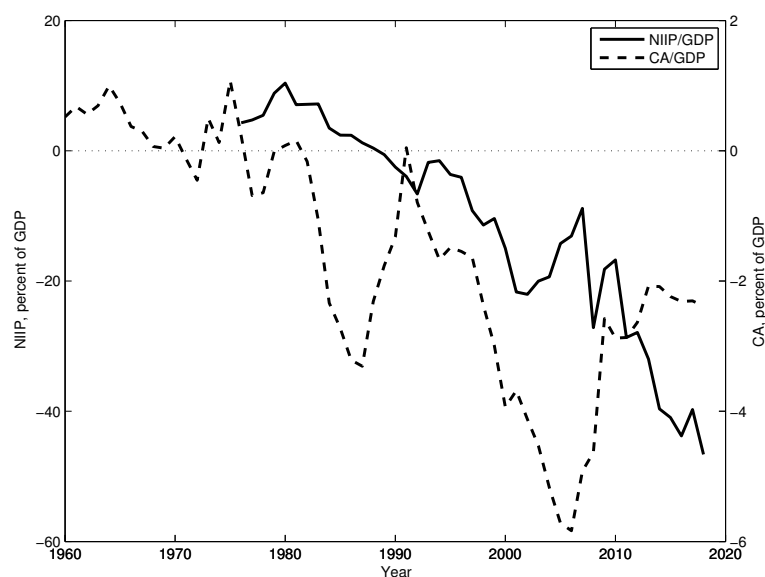
1.5 The Current Account and the Net International Investment Position

One reason why the concept of current account balance is economically important is that it reflects a country's net borrowing needs. For example, as we saw earlier, in 2018 the United States ran a current account deficit of \$488.5 billion. To pay for this deficit, the country must have either reduced part of its international asset position or increased its international liability position or both. In this way, the current account is related to changes in a country's *net international investment position* (NIIP). The term net international investment position is used to refer to a country's net foreign wealth, that is, the difference between the value of foreign assets owned by the country's residents and the value of the country's assets owned by foreign residents. The net international investment position is a stock, while the current account is a flow.

Figure 1.5 shows the U.S. current account for the period 1960 to 2018 and net international investment position (expressed in percent of GDP) for the period 1976 to 2018.² The U.S. net international investment position was positive at the beginning of the sample. However, in the early 1980s, the United States began running large current account deficits. By 1989 these deficits had eroded the net foreign wealth of the United States and she became a net debtor to the rest of the world for the first time since World War I.

²The starting date of the net international investment position series is determined by data availability.

Figure 1.5: The U.S. Current Account and Net International Investment Position



Notes. *CA*, *NIIP*, and *GDP* stand for current account, net international investment position, and gross domestic product, respectively. The sample period for *CA* is 1960 to 2018 and for *NIIP* 1976 to 2018. The data source is <http://www.bea.gov>.

The U.S. current account deficits of the 1980s did not turn out to be temporary. As a consequence, by the end of the 1990s, the United States had become the world's largest external debtor. Moreover, the current account deficits continued to rise for twenty five years. Only shortly before the onset of the Great Recession of 2008, did the downward trend stop and current account deficits stabilized. In fact, they became smaller in magnitude.

By the end of 2018, the net international investment position of the United States stood at -9.6 trillion dollars or -47 percent of GDP. This is a big number, and many economist wonder whether the observed downward trend in the net international investment position will be sustainable over time.³ The concern stems from the fact that countries that accumulated large external debt to GDP ratios in the past, such as many Latin American countries in the 1980s, Southeast Asian countries in the 1990s, and more recently peripheral European countries, have experienced sudden reversals in international capital flows that were followed by costly financial and economic crises. The 2008 financial meltdown in the United States brought this issue to the fore.

1.6 Valuation Changes and the Net International Investment Position

The current account is not the only source of changes in a country's net international investment position. It can also change due to variations in the prices of the financial instruments that compose a country's international

³Chapter 2 analyzes this concern in detail.

asset and liability positions. So we have that

$$\Delta NIIP = CA + \text{valuation changes}, \quad (1.2)$$

where $\Delta NIIP$ denotes the change in the net international investment position.

To understand how valuation changes can alter a country's net international investment position, consider the following hypothetical example. Suppose a country's international asset position, denoted A , consists of 25 shares in the Italian company Fiat. Suppose the price of each Fiat share is 2 euros. Then we have that the foreign asset position measured in euros is $25 \times 2 = 50$ euros. Suppose that the country's international liabilities, denoted L , consist of 80 units of bonds issued by the local government and held by foreigners. Suppose further that the price of local bonds is 1 dollar per unit, where the dollar is the local currency. Then we have that total foreign liabilities are $L = 80 \times 1 = 80$ dollars. Assume that the exchange rate is 2 dollars per euro. Then, the country's foreign asset position measured in dollars is $A = 50 \times 2 = 100$. The country's net international investment position is given by the difference between its international asset position, A , and its international liability position, L , or $NIIP = A - L = 100 - 80 = 20$. Suppose now that the euro suffers a significant depreciation, losing half of its value relative to the dollar. The new exchange rate is therefore 1 dollar per euro. Since the country's international asset position is denominated in euros, its value in dollars automatically falls. Specifically, its new value is $A' = 50 \times 1 = 50$ dollars. The country's international liability position

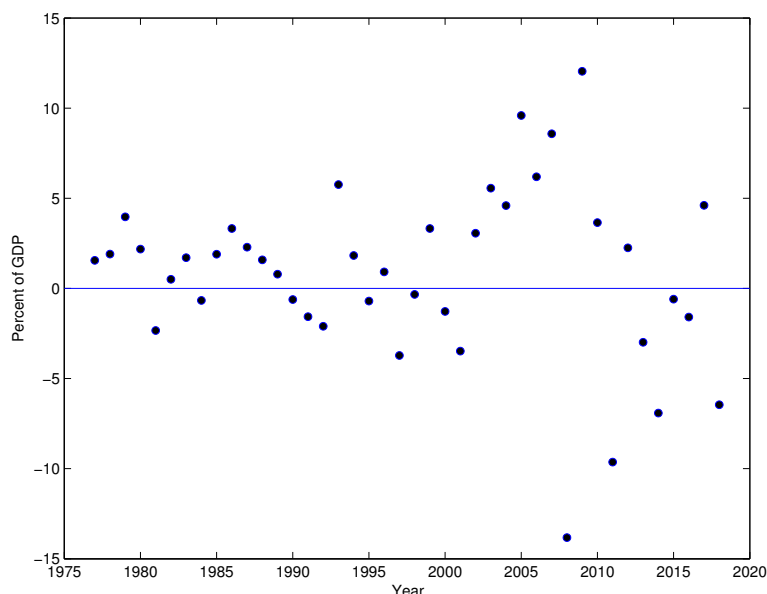
measured in dollars does not change, because it is composed of instruments denominated in dollars. As a result, the country's new net international investment position is $NIIP' = A' - L = 50 - 80 = -30$. It follows that just because of a movement in the exchange rate, the country went from being a net creditor of the rest of the world to being a net debtor. This example illustrates that an appreciation of the domestic currency can reduce the net foreign asset position.

Consider now the effect an increase in foreign stock prices on the net international investment position of the domestic country. Specifically, suppose that the price of the Fiat stock jumps up from 2 to 7 euros. This price change increases the value of the country's asset position to $25 \times 7 = 175$ euros, or at an exchange rate of 1 dollar per euro to 175 dollars. The country's international liabilities do not change in value. The net international investment position then turns positive again and equals $175 - 80 = 95$ dollars. This example shows that an increase in foreign stock prices can improve a country's net international investment position.

Finally, suppose that, because of a successful fiscal reform in the domestic country, the price of local government bonds increases from 1 to 1.5 dollars. In this case, the country's gross foreign asset position remains unchanged at 175 dollars, but its international liability position jumps up to $80 \times 1.5 = 120$ dollars. As a consequence, the net international investment position falls from 95 to 55 dollars.

The above hypothetical examples illustrate how a country's net international investment position can display large swings because of movements in asset prices or exchange rates. This is indeed the case in actual data as

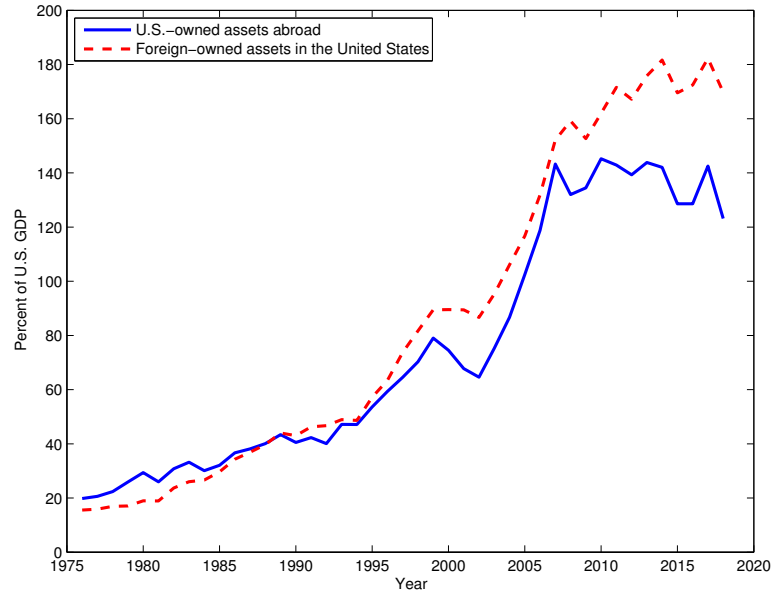
Figure 1.6: Valuation Changes in the U.S. Net International Investment Position, 1977-2018



Note. The figure shows year-over-year changes in the U.S. net international investment position arising from valuation changes expressed in percent of GDP. The data source is <http://www.bea.gov>.

well. Valuation changes have been an important source of movements in the net international investment position of the United States, especially in the past two decades. Figure 1.6 displays valuation changes between 1977 and 2018. The figure reveals a number of noticeable characteristics of valuation changes. First, valuation changes can be large, reaching ± 15 percent of GDP in some years. Second, large valuation changes are a recent phenomenon. Until 2003, the typical valuation change was between -1 and 2 percent of GDP. Third, the past 15 years have also been characterized by higher volatility in valuation changes, as both increases and decreases in valuation became larger. Fourth, over the period 1977 to 2018, the United States experienced

Figure 1.7: U.S.-Owned Assets Abroad and Foreign-Owned Assets in the United States



valuation gains more often than valuation losses, 25 versus 17 times.

Why have valuation changes become so large lately? One reason is that gross international asset and liability positions have exploded since the 2000s, as shown in figure 1.7. Gross positions grew from about 80 to about 180 percent of GDP between 2000 and 2018. When gross positions are large relative to net positions, just a small change in the price of an asset that is asymmetrically represented in assets and liabilities can result in large changes in the value of the net position. For example, most of the United States' international liabilities are denominated in dollars, whereas most of its international asset position is denominated in foreign currency. As a result, a small appreciation of the dollar vis-à-vis other currencies can cause a significant deterioration of the net international investment position.

Valuation changes played a dominant role for the evolution of the U.S. net international investment position in the run-up to the Great Recession of 2007 to 2012. The period 2002-2007 exhibited the largest current account deficits since 1976, which is the beginning of our sample. In each of these years, the current account deficit exceeded 4 percent of GDP, with a cumulative deficit of 3.9 trillion dollars, or 32 percent of GDP. Nevertheless, the net international investment position actually improved by 80 billion dollars. The discrepancy of almost \$4 trillion between the accumulated current account balances and the change in the net international investment position was the result of increases in the market value of U.S.-owned foreign assets relative to foreign-owned U.S. assets.

What caused these large changes in the value of assets in favor of the United States? Milesi-Ferretti, of the International Monetary Fund, identifies two main factors.⁴ First, the U.S. dollar depreciated relative to other currencies by about 20 percent. This is a relevant factor because, as we mentioned earlier, the currency denomination of the U.S. foreign asset and liability positions is asymmetric. The asset side is composed mostly of foreign-currency denominated financial instruments, while the liability side is mostly composed of dollar-denominated instruments. As a result, a depreciation of the U.S. dollar increases the dollar value of U.S.-owned assets, while leaving more or less unchanged the dollar value of foreign-owned assets, thereby strengthening the U.S. net international investment position. Second, the stock markets in foreign countries significantly outperformed the U.S. stock

⁴See Gian Maria Milesi-Ferretti, “A \$2 Trillion Question,” VOX, January 28, 2009, available online at <http://www.voxeu.org>

market. Specifically, a dollar invested in foreign stock markets in 2002 returned 2.90 dollars by the end of 2007. By contrast, a dollar invested in the U.S. market in 2002, yielded only 1.90 dollars at the end of 2007. These gains in foreign equity resulted in an increase in the net equity position of the U.S. from an insignificant level of \$40 billion in 2002 to \$3 trillion in 2007.

The large valuation changes observed in the period 2002-2007, which allowed the United States to run unprecedented current account deficits without a concomitant deterioration of its net international investment position, came to an abrupt end in 2008. Look at the dot corresponding to 2008 in figure 1.6, which shows that valuation losses in that year were almost 15 percent of GDP. The source of this drop in value was primarily the stock market. In 2008 stock markets around the world plummeted. Because the net equity position of the U.S. had gotten so large by the beginning of 2008 the decline in stock prices outside of the U.S. inflicted large losses on the value of the U.S. equity portfolio.

Another way to visualize the importance of valuation changes is to compare the actual net international investment position with a hypothetical one that results from removing valuation changes. To compute a time series for this hypothetical net international investment position, start by setting its initial value equal to the actual value. Our sample starts in 1976, so we set

$$\text{Hypothetical } NIIP_{1976} = NIIP_{1976}.$$

Now, according to identity (1.2), after removing valuation changes in 1977,

we have that the change in the hypothetical net international investment position between 1976 and 1977, is equal to the current account in 1977, that is,

$$\text{Hypothetical } NIIP_{1977} = NIIP_{1976} + CA_{1977},$$

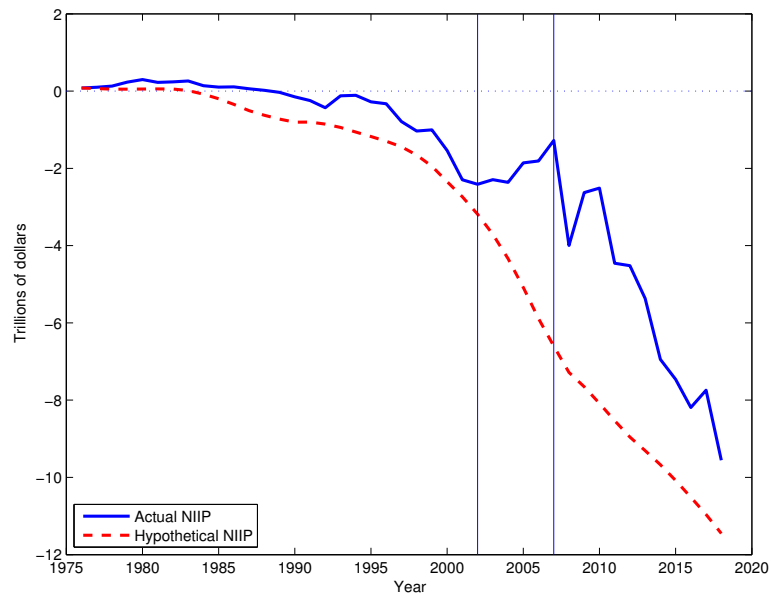
where CA_{1977} is the actual current account in 1977. Combining this expression with identity (1.2) we have that the hypothetical net international investment position in 1978 is given by the net international investment position in 1976 plus the accumulated current accounts from 1977 to 1978, that is,

$$\text{Hypothetical } NIIP_{1978} = NIIP_{1976} + CA_{1977} + CA_{1978}.$$

In general, for any year $t > 1978$, the hypothetical net international investment position is given by the actual net international investment position in 1976 plus the accumulated current accounts between 1977 and t . Formally,

$$\text{Hypothetical } NIIP_t = NIIP_{1976} + CA_{1977} + CA_{1978} + \cdots + CA_t.$$

Figure 1.8 plots the actual and hypothetical net international investment positions over the period 1976 to 2018 in percent of GDP. Until 2002, the actual and hypothetical net international investment positions were not that different from each other, implying that valuation changes were not sizable. In 2002, however, the hypothetical net international investment position started to fall at a much faster pace than its actual counterpart. This means that after 2002 the United States started to benefit from sizable valuation

Figure 1.8: Actual and Hypothetical U.S. *NIIP*s: 1976 to 2018

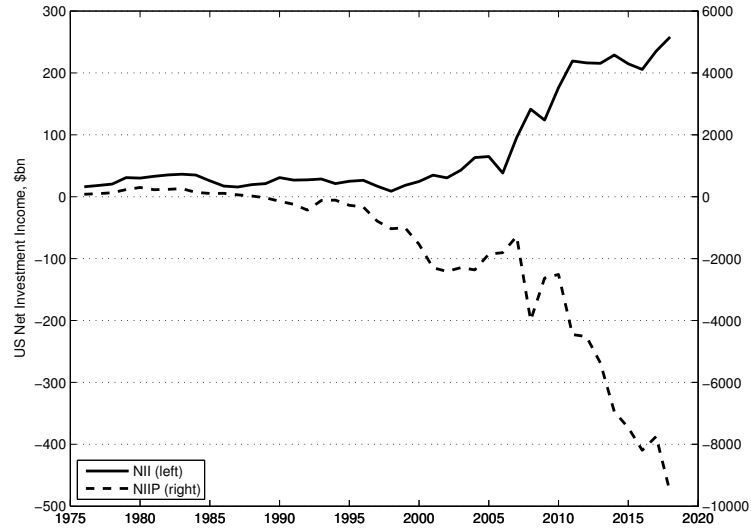
Notes. The hypothetical *NIIP* for a given year is computed as the sum of the *NIIP* in 1976 and the cumulative sum of current account balances from 1977 to the year in question. The vertical lines indicate the year 2002 and 2007, respectively. Source. Own calculations based on data from the Bureau of Economic Analysis.

gains. By the end of 2007, the gap between the actual and hypothetical net international investment positions had widened from 7 percent to 37 percent of GDP. Without this lucky strike, all other things equal, the U.S. net foreign asset position in 2007 would have been an external debt of 46 percent of GDP instead of the actual 9 percent. The reversal of fortune that came with the Great Recession of 2008 is evident from the narrowing of the gap between the two net international investment positions. By 2018 this gap stood at only 9 percent of GDP, a figure not significantly different from the ones observed prior to the exuberant 2002-2007 quinquennial.

1.7 The Negative *NIIP* And Positive *NII* Paradox: Dark Matter?

We have documented that for the past thirty years, the United States has had a negative net international investment position ($NIIP < 0$). This means that the United States has been a net debtor to the rest of the world. One would therefore expect that during this period the U.S. paid more interest and dividends to the rest of the world than it received. In other words, we would expect that the net investment income (*NII*) component of the current account be negative ($NII < 0$). This is, however, not observed in the data. Take a look at figure 1.9. It shows net investment income and the net international investment position since 1976. *NII* is positive throughout the sample, whereas net international investment position has been negative since 1989. How could it be that a debtor country, instead of having to make payments on its debt, receives income on it? Here are two

Figure 1.9: Net Investment Income and the Net International Investment Position, United States 1976 to 2018



Data Source: ITA, Table 1.1.

explanations.

1.7.1 Dark Matter

One explanation of the $NIIP - NII$ paradox, proposed by Ricardo Hausmann and Federico Sturzenegger, is that the Bureau of Economic Analysis may underestimate the net foreign asset holdings of the United States.⁵

One source of underestimation could be that U.S. foreign direct investment contains intangible human capital, such as entrepreneurial capital and brand capital, whose value is not correctly reflected in the official balance of payments. At the same time, the argument goes, this human capital in-

⁵See Hausmann, Ricardo and Federico Sturzenegger, “U.S. and Global Imbalances: Can Dark Matter Prevent a Big Bang?,” working paper CID (Center For International Development), Harvard University, 2005.

vested abroad may generate income for the U.S., which may be appropriately recorded. It thus becomes possible that the U.S. could display a negative net foreign asset position and at the same time positive net investment income. Hausmann and Sturzenegger refer to the unrecorded U.S.-owned foreign assets as dark matter.

To illustrate the dark matter argument, consider a McDonald's restaurant functioning in Moscow. This foreign direct investment will show in the U.S. foreign asset position with a dollar amount equivalent to the amount McDonald's invested in items such as land, the building, cooking equipment, and restaurant furniture. However, the market value of this investment may exceed the actual amount of dollars invested. The reason is that the brand McDonald's provides extra value to the goods (burgers) the restaurant produces. It follows that in this case the balance of payments, by not taking into account the intangible brand component of McDonald's foreign direct investment, would underestimate the U.S. international asset position. On the other hand, the profits generated by the Moscow branch of McDonald's are observable and recorded, so they make their way into the income account of the balance of payments.

How much dark matter was there in 2018? Let $TNIIP$ denote the 'true' net international investment position and $NIIP$ the recorded one. Then we have that

$$TNIIP = NIIP + \text{Dark Matter.}$$

Let r denote the interest rate on net foreign assets. Then, net investment income equals the return on the country's net international investment po-

sition,

$$NII = r \times TNIIP.$$

In this expression, we use $TNIIP$ and not $NIIP$ to calculate NII because, according to the dark-matter hypothesis, the recorded level of NII appropriately reflects the return on the true level of net international investment. In 2018, NII was 258.1 billion dollars (see table 1.1). Suppose that r is equal to 5 percent per year. Then, we have that $TNIIP = 258.1/0.05 = 5.2$ trillion dollars. The recorded $NIIP$ in 2018 was -9.6 trillion dollars. This means that dark matter in 2018 was 14.8 trillion dollars, or 72 percent of U.S. GDP. This seems like a big number to go under the radar of the Bureau of Economic Analysis. So it is of interest to consider in addition a competing explanation of the negative NIIP and positive NII paradox.

1.7.2 Return Differentials

An alternative explanation for the paradoxical combination of a negative net international investment position and positive net investment income is that the United States earns a higher interest rate on its foreign asset holdings than foreigners earn on their U.S. asset holdings. The rationale behind this explanation is the observation that the U.S. international assets and liabilities are composed of different types of financial instruments. Foreign investors typically hold low-risk U.S. assets, such as Treasury Bills. These assets carry a low interest rate. At the same time, American investors tend to purchase more risky foreign assets, such as foreign stocks and foreign direct investment, which earn relatively high returns.

How big does the spread between the interest rate on U.S.-owned foreign assets and the interest rate on foreign-owned U.S. assets have to be to explain the paradox? Let A denote the U.S. gross foreign asset position and L the U.S. gross foreign liability position. Further, let r^A denote the interest rate on A and r^L the interest rate on L . Then, we have that

$$NII = r^A A - r^L L. \quad (1.3)$$

Let's put some numbers in this expression. According to the BEA, in 2018 A was \$25.3 trillion and L was \$34.8 trillion. From table 1.1, we have that in 2018 NII was 258.1 billion dollars. Suppose we set r^L equal to the return on U.S. Treasury securities. This makes sense because most of the United States' foreign liabilities are in the form of government bonds. In 2018, the rate of return on one-year U.S. Treasuries was 2.25 percent per year, so we set $r^L = 0.0225$. Now let's plug these numbers into expression (1.3) to get

$$0.2581 = r^A \times 25.3 - 0.0225 \times 34.8,$$

which yields $r^A = 4.12\%$. That is, we need an interest rate spread of 187 basis points ($r^A - r^L = 1.87\%$) to explain the paradox. This figure seems more empirically plausible than 14.8 trillion dollars of dark matter.

1.7.3 The Flip Side of the $NIIP - NII$ Paradox

If we divide the world into two groups, the United States and the rest of the world, then the rest of the world should display the anti paradox, that is, a

positive net foreign asset position and negative net investment income. The reason is that what is an asset of the United States is a liability for the rest of the world and vice versa. The same is true for net investment income. International income receipts by the United States are international income payments by the rest of the world. So we have that

$$NIIP^{US} = A^{US} - L^{US} = L^{RW} - A^{RW} = -NIIP^{RW}$$

and

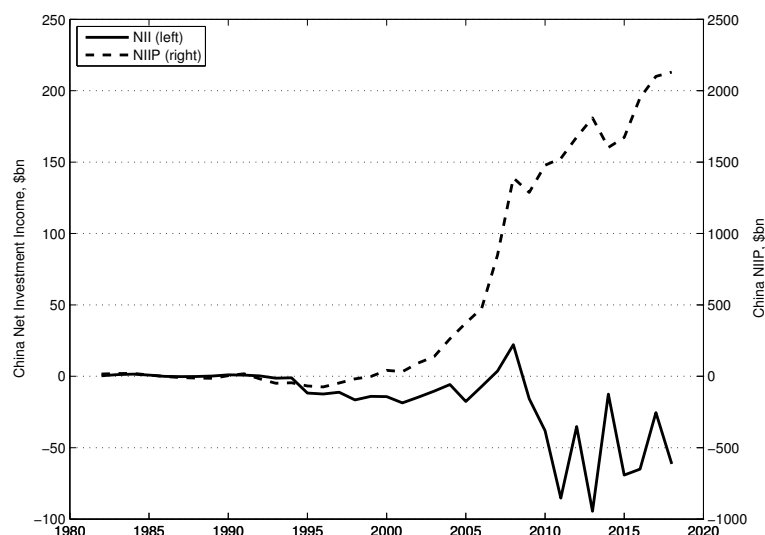
$$NII^{US} = r^A A^{US} - r^L L^{US} = r^A L^{RW} - r^L A^{RW} = -NII^{RW},$$

where the superscripts US and RW refer to the United States and the rest of the world.

This means that at least one country in the rest of the world must display the anti paradox. One candidate is China for two reasons: first, as we observed when discussing global imbalances (see the map in figure 1.1), China has been accumulating large current account surpluses for the past quarter century, so it is a likely candidate to have a positive $NIIP$. Second, in table 1.1 we showed that in 2016 the Chinese trade balance surplus was larger than the current account surplus, and pointed out this was due to a negative NII .

Figure 1.10 confirms our conjecture. It displays the $NIIP$ and NII of China for the period 1982 to 2018. It shows that starting from a small negative value in 1999, China's net foreign asset position grew rapidly reaching

Figure 1.10: Net Investment Income and the Net International Investment Position, China 1982 to 2018



Notes. The figure illustrates that the $NIIP - NII$ paradox in the United States must have a flip side in the rest of the world. Since 2000, with the exception of the Great Contraction years (2007 and 2008), China has displayed a positive $NIIP$ and a negative NII . Data Sources: $NIIP$ for 1982 to 2007 is from Lane and Milesi-Ferretti (2017, for details see the notes to figure 1.1) and for 2008 to 2018 from IFS. NII is from IFS.

\$1 trillion by 2008 and \$2 trillion by 2018. However, China's net investment income has been negative during this period, at around -30 billion dollars.

China does not represent the entire flip side of the U.S. $NIIP - NII$ paradox, because the sizes of its $NIIP$ and NII are much smaller than those of the United States. This means that there must be other countries displaying the anti paradox.

1.8 Summing Up

Let's take stock of what we have learned in this chapter.

- The current account keeps record of a country's net exports of goods and services and net international income receipts.
- It has three components, the trade balance, the income balance, and net unilateral transfers.
- For most countries, including the United States, the trade balance is the largest component of the current account.
- In the United States, the trade balance and the current account move closely together over time.
- The United States have been running large current account deficits since the early 1980s.
- Current account deficits deteriorate a country's net international investment position, which is the difference between a country's international asset position and its international liability position.

- Due to its large current account deficits, the United States turned from being a net external creditor to being the world's largest debtor.
- A second source of changes in a country's net international asset position is valuation changes, originating from changes in exchange rates and in the price of the financial instruments that compose a country's asset and liability positions. In the United States, valuation changes became large in the early 2000s, reaching values as high as plus or minus 15 percent of GDP in a single year.
- Paradoxically, the United States has a negative net international investment position $NIIP < 0$ and positive net investment income $NII > 0$. Two stories aim to explain the $NIIP - NII$ paradox: the dark matter hypothesis and the rate-of-return-differential hypothesis. The $NIIP - NII$ paradox in the United States must have a flipped paradox in the rest of the world. We documented that China has had a positive $NIIP$ and negative NII since the 2000.
- Worldwide, the distribution of external debts and credits is not even. Some countries, like the United States, are large debtors and some, like China, are large creditors.

Many wonder whether these global imbalances can continue to widen over time. Will the large current account deficits of the United States prove unsustainable? We take up this issue in the next chapter.

1.9 Exercises

Exercise 1.1 (Balance of Payments Accounting) Describe how each of the following transactions affects the U.S. Balance of Payments. (Recall that each transaction gives rise to two entries in the Balance-of-Payments Accounts.)

1. Jorge Ramírez, a landscape architect residing in Monterrey, Mexico, works for three months in Durham, NC, creating an indoor garden for a newly built museum and receives wages for \$35,000.
2. Jinill Park's mother, a resident of South Korea, pays her son's tuition to Columbia University via a direct deposit.
3. Columbia University buys several park benches from Spain and pays with a \$ 120,000 check.
4. Floyd Townsend, of Tampa Florida, buys 5,000 dollars worth of British Airlines stock from Citibank New York, paying with U.S. dollars.
5. A French consumer imports American blue jeans and pays with a check drawn on J.P. Morgan Chase Bank in New York City.
6. An American company sells a subsidiary in the United States and with the proceeds buys a French company.
7. A group of American friends travels to Costa Rica and rents a vacation home for \$2,500. They pay with a U.S. credit card.
8. The U.S. dollar depreciates by 10 percent vis-à-vis the euro.

9. The United States sends medicine, blankets, tents, and nonperishable food worth 400 million dollars to victims of an earthquake in a foreign country.
10. Olga Rublev, a billionaire from Russia, enters the United States on an immigrant visa (that is, upon entering the United States she becomes a permanent resident of the United States.) Her wealth in Russia is estimated to be about 2 billion U.S. dollars.
11. The United States forgives debt of \$500,000 to Nicaragua.

Exercise 1.2 Find the most recent data on the U.S. current account and its components. Present your answer in a form similar to table 1.1, that is, show figures in both current dollars and as a percentage of GDP. For current account and GDP data visit the Bureau of Economic Analysis' website. Compare your table with table 1.1.

Exercise 1.3 Suppose Columbia University, a U.S. resident, acquires \$100,000 worth of shares of Deutsche Telekom from a German resident. How does this transaction affect the U.S. Balance of Payments Accounts and the U.S. Net International Investment Position in each of the following three scenarios. Be sure to list the entries in the U.S. current account and the U.S. financial account separately.

1. Columbia pays for the shares with U.S. dollar bills.
2. Columbia pays for the shares with an apartment it owns in midtown.

3. The German resident attends Columbia College and settles the tuition bill with the Deutsche Telekom shares.
4. Do all three scenarios have the same effects on the U.S. current account and on the U.S. NIIP?

Exercise 1.4 (The Effects of the 2017 Tax Cuts and Jobs Act (TCJA) on Components of the International Transactions Accounts)

1. Read “Apple, Capitalizing on New Tax Law, Plans to Bring Billions in Cash Back to U.S.,” which appeared online in the New York Times on January 17, 2018 and is available at <https://nyti.ms/2EPQet6>. We will use this article to learn what the TCJA is about and to see if we can follow the algebra given in the article.
 - (a) Based on the information given in the article, how much corporate cash held abroad is Apple repatriating?
 - (b) What is the tax saving from the repatriation under the new one-time lower tax relative to the pre-reform tax rate?
 - (c) What is the potential maximum tax saving for Apple from the repatriation under the new one-time lower tax rate relative to the post-reform tax rate of foreign corporate cash holdings. That is, how high is the incentive to repatriate the cash, taking as given that the tax law changed.
 - (d) The article states that ‘By shifting the money under the new terms, Apple has saved \$43 billion in taxes.’ Given the informa-

tion provided in the article, do you agree or disagree with this figure?

2. Assume that Apple Ireland is owned by Apple USA and that 2018 earnings of Apple Ireland are 0. Assume that nevertheless Apple Ireland paid Apple USA cash dividends in the amount of \$100 billion. Read the short BEA FAQ article “How are the International Transactions affected by an increase in direct investment dividend receipts,” June 20, 2018, available at <https://www.bea.gov/help/faq/166> and then answer the following questions.

- (a) How will this cash repatriation enter the U.S. current account.
(If you find it helpful, discuss which line in ITA Table 1.2 would be affected.)
- (b) How will this cash repatriation enter the U.S. financial account?
(Again, if you find it helpful, discuss which lines in ITA Table 1.2 will be affected.)
- (c) Finally, discuss whether the repatriation of earnings of foreign subsidiaries of U.S. companies will or will not improve the U.S. current account deficit.

Exercise 1.5 (Net Foreign Asset Positions Around the World) Download data on current accounts and net foreign asset positions from the External Wealth of Nations Database put together by Philip Lane and Gian Maria Milesi-Ferretti. For each country that has current account and net foreign asset position data starting in 1980 sum the current account balances from

1980 to the latest date available and find the change in NFA over the corresponding period. Then plot the change in the net foreign asset position against the cumulated current account balances. Discuss your results. In particular, comment on whether cumulative current account balances represent a good measure of global imbalances (refer to figure 1.1). What does your graph suggest, if anything, about the quantitative importance of valuation changes for the majority of countries in your sample?

Exercise 1.6 (Bigger Debtor Nation) On July 4, 1989 the New York Times reported, under the headline ‘U.S. is Bigger Debtor Nation’, that *‘The United States, already the world’s largest debtor, sank an additional \$154.2 billion into the red last year as foreign money poured in to plug the nation’s balance-of-payments gap. The increasing debt is likely to mean that American living standards will rise a bit more slowly than they otherwise would, as interest and dividend payments to foreigners siphon off an increasing share of the United States’ output of goods and services.’* With the benefit of hindsight, critically evaluate the last statement.

Exercise 1.7 Indicate whether the statements are true, false, or uncertain and explain why.

1. The net international investment position of South Africa was -70.5 billion USD in 2010 and -19.7 billion USD in 2011. The current account in 2011 was -10.1 billion USD. There must be an error in the official numbers. The correct figure should be a net international investment position of -80.6 billion USD in 2011.

2. The fact that the United States made large valuation gains on average over the past 40 years means that the rest of the world as a whole made equally large valuation losses. After all, this is a zero sum game.
3. The United States has large unrecorded foreign asset holdings.
4. Saving is a stock variable.

Exercise 1.8 This question is about the balance of payments of a country named Outland. The currency of Outland is the dollar.

1. Outland starts a given year with holdings of 100 shares of the German car company Volkswagen. These securities are denominated in euros. The rest of the world holds 200 units of dollar-denominated bonds issued by the Outlandian government. At the beginning of the year, the price of each Volkswagen share is 1 euro and the price of each unit of Outlandian bond is 2 dollars. The exchange rate is 1.5 dollars per euro. Compute the net international investment position (NIIP) of Outland at the beginning of the year.
2. During the year, Outland exports toys for 7 dollars and imports shirts for 9 euros. The dividend payments on the Volkswagen shares were 0.05 euros per share and the coupon payment on Outlandian bonds was 0.02 dollars per bond. Residents of Outland received money from relatives living abroad for a total of 3 euros and the government of Outland gave 4 dollars to a hospital in Guyana. Calculate the Outlandian trade balance, net investment income, and net unilateral transfers in

that year. What was the current account in that year? What is the Outlandian NIIP at the end of the year.

3. Suppose that at the end of the year, Outland holds 110 Volkswagen shares. How many units of Outlandian government bonds are held in the rest of the world? Assume that during the year, all financial transactions were performed at beginning-of-year prices and exchange rates.
4. To answer this question, start with the international asset and liability positions calculated in item 3. Suppose that at the end of the year, the price of a Volkswagen share falls by 20 percent and the dollar appreciates by 10 percent. Calculate the end-of-year NIIP of Outland.

Exercise 1.9 Suppose the world consists of two countries, country A and country C.

1. Let $NIIP^A$ denote the net foreign asset position of country A. Find the net foreign asset position of country C.
2. Let CA^A denote the current account balance of country A. Find the current account balance of country C.
3. Let A^A denote foreign assets owned by residents of country A and L^A denote country A's assets owned by residents of country C. Find the foreign asset and liability positions of country C denoted A^C and L^C , respectively.
4. Assume that the value of country A's foreign liabilities increases by 20

percent. Find the change in the net foreign asset position of country A and country C. Find the valuation changes for country A and C. To which extend are valuation changes a zero sum game?

Exercise 1.10 In section 1.6, we showed that over the past 40 years the *NIIP* of the United States greatly benefited from valuation changes. In this question, you are asked to analyze how valuation changes affected the *NIIP* of China between 1981 and 2017.

1. Download data on China's current account, net foreign asset position, and gross domestic product from the External Wealth of Nations Database put together by Philip Lane and Gian Maria Milesi-Ferretti. Use these time series to construct the Hypothetical *NIIP* of China and then, using a software like Matlab or Excel, plot the actual *NIIP* and the Hypothetical *NIIP* for China both expressed in terms of GDP. Your plot should be a version of figure 1.8 but using Chinese instead of U.S. data. Compare and contrast your findings to those obtained for the United States.
2. Then construct a time series for valuation changes in China's net foreign asset position. Plot valuation changes as a share of GDP since 1981. Use the same scale for the vertical axis as that of figure 1.6. Then compare and contrast the valuation changes experienced by China with those experienced by the United States. What may account for the observed differences.

Exercise 1.11 (Dark Matter Over Time) Use data from the BEA to

construct a time series of dark matter using the methodology explained in subsection 1.7.1. Construct a time series as long as the available data permits. Discuss the plausibility of the dark matter hypothesis based not on its size, but on its volatility over time.

Exercise 1.12 (An $NII - NIIP$ Paradox) A country exhibits the paradoxical situation of having negative net investment income (NII) of -100 and a positive net international investment position ($NIIP$) of 1000. Economists' opinions about this are divided. Group A thinks that the explanation lies in the fact that, because of the bad reputation of the country in world financial markets, foreign investors charge a higher interest rate when they lend to this country, relative to the interest rate the country receives on its investments abroad. Group B believes that domestic investors inflate their gross international asset positions to look like big players in the world market.

1. Calculate the interest-rate premium that would explain the paradox under group A's hypothesis, assuming that the interest rate on assets invested abroad is 5 percent and that the country's gross international asset position is 4000.
2. Calculate the amount by which domestic investors inflate their gross foreign asset positions under group B's hypothesis assuming that the interest rate on assets and liabilities is 5 percent.

Chapter 2

Current Account

Sustainability

A natural question that arises from our description of the recent history of the U.S. external accounts is whether the observed trade and current account deficits are sustainable in the long run. In this chapter, we develop a framework to address this question.

2.1 Can a Country Run a Perpetual Trade Balance Deficit?

The answer to this question depends on the sign of a country's initial net international investment position. A negative net international investment position means that the country as a whole is a debtor to the rest of the world. Thus, the country must generate trade balance surpluses either currently or at some point in the future in order to service its foreign debt.

Similarly, a positive net international investment position means that the country is a net creditor of the rest of the world. The country can therefore afford to run trade balance deficits forever and finance them with the interest revenue generated by its credit position with the rest of the world.

Let's analyze this idea more formally. Consider an economy that lasts for two periods, period 1 and period 2. Let TB_1 denote the trade balance in period 1, CA_1 the current account balance in period 1, and B_1^* the country's net international investment position (or net foreign asset position) at the end of period 1. If $B_1^* > 0$, then the country is a creditor to the rest of the world in period 1 and if $B_1^* < 0$, then the country is a debtor. For example, if the country in question was the United States and period 1 was meant to be the year 2016, then $CA_1 = -451.7$ billion, $TB_1 = -504.8$, and $B_1^* = -\$8.3$ trillion (see table 1.1 and section 1.5 in chapter 1).

Let r denote the interest rate paid on assets held for one period and B_0^* denote the net foreign asset position at the end of period 0. Then, the country's net investment income in period 1 is given by

$$\text{Net investment income in period 1} = rB_0^*.$$

This expression says that net investment income in period 1 is equal to the return on net foreign assets held by the country's residents between periods 0 and 1.

In what follows, we ignore net international payments to employees, net unilateral transfers, and valuation changes, by assuming that they are always equal to zero. Given these assumptions, the current account equals the sum

of net investment income and the trade balance, that is,

$$CA_1 = rB_0^* + TB_1 \quad (2.1)$$

and the change in the net international asset position equals the current account

$$B_1^* - B_0^* = CA_1. \quad (2.2)$$

Combining equations (2.1) and (2.2) to eliminate CA_1 yields:

$$B_1^* = (1 + r)B_0^* + TB_1.$$

A relation similar to this one must also hold in period 2. So we have that

$$B_2^* = (1 + r)B_1^* + TB_2.$$

Combining the last two equations to eliminate B_1^* we obtain

$$(1 + r)B_0^* = \frac{B_2^*}{(1 + r)} - TB_1 - \frac{TB_2}{(1 + r)}. \quad (2.3)$$

Now consider the possible values that the net foreign asset position at the end of period 2, B_2^* , can take. If B_2^* is negative ($B_2^* < 0$), it means that in period 2 the country is holding debt maturing in period 3. However, in period 3 nobody will be around to collect the debt because the world ends in period 2. Thus, the rest of the world will not be willing to lend to our country in period 2. This means that B_2^* cannot be negative, or that $B_2^* \geq 0$.

must satisfy

$$B_2^* \geq 0.$$

This terminal restriction on asset holdings is known as the no-Ponzi-game constraint.¹ Can B_2^* be strictly positive? The answer is no. A positive value of B_2^* means that the country is lending to the rest of the world in period 2. But the country will be unable to collect this debt in period 3 because, again, the world ends in period 2. Thus, the country will never choose to hold a positive net foreign asset position at the end of period 2, that is, it would always choose $B_2^* \leq 0$. If B_2^* can be neither positive nor negative, then it must be equal to zero,

$$B_2^* = 0.$$

This condition is known as the transversality condition. Using the transversality condition in (2.3), we obtain

$$(1+r)B_0^* = -TB_1 - \frac{TB_2}{(1+r)}. \quad (2.4)$$

This equation states that a country's initial net foreign asset position (including interest) must equal the present discounted value of its future trade

¹It is named after Charles K. Ponzi, who introduced pyramid schemes in the 1920s in Massachusetts. To learn more about the remarkable criminal career of Ponzi, visit <http://www.mark-knutson.com>. A recent example of a Ponzi scheme is given by Bernard L. Madoff's fraudulent squandering of investments valued around \$64 billion in 2008. For more than 20 years Madoff's scheme consisted in paying steady returns slightly above market to a large variety of clients ranging from hedge funds to university endowments to low-income retirees. When Madoff's own investments failed to produce such returns, the scheme required the acquisition of new clients to survive. In the financial crisis of 2008 the flow of new clients dried up and his scheme imploded overnight. In June 2009, Bernard Madoff, then 71 years old, was sentenced to 150 years in prison.

deficits. Our earlier claim that a negative initial net foreign wealth position implies that the country must generate trade balance surpluses, either currently or at some point in the future, can now be easily verified using equation (2.4). Suppose that the country is a net debtor to the rest of the world ($B_0^* < 0$). Clearly, if it never runs a trade balance surplus ($TB_1 \leq 0$ and $TB_2 \leq 0$), then the left-hand side of (2.4) is negative while the right-hand side is nonnegative, so (2.4) would be violated. In this case, the country would be running a Ponzi scheme against the rest of the world.

Now suppose that the country's initial asset position is positive ($B_0^* > 0$). This means that initially the rest of the world owes a debt to our country. Then, the left-hand side of equation (2.4) is positive. If the country runs trade deficits in periods 1 and 2, then the right hand side of (2.4) is also positive, which implies no inconsistency. Thus, the answer to the question of whether a country can run a perpetual trade balance deficit is yes, provided the country's initial net foreign asset position is positive. Of course, the country cannot run arbitrarily large trade deficits. Equation (2.4) states that their present discounted sum is bounded above by the country's initial net asset position gross of interest.

Because the United States is currently a net foreign debtor to the rest of the world, it follows from our analysis that it will have to run trade balance surpluses at some point in the future. This result extends to economies that last for any number of periods, not just two. Indeed, the appendix to this chapter shows that the result holds for economies that last forever (infinite-horizon economies).

2.2 Can a Country Run a Perpetual Current Account Deficit?

In a finite-horizon economy like the two-period world we are studying here, the answer to this question is, again, yes, provided the country's initial net foreign asset position is positive. To see why, note that an expression similar to (2.2) must also hold in period 2, that is,

$$CA_2 = B_2^* - B_1^*.$$

Combining this expression with equation (2.2) to eliminate B_1^* , we obtain

$$B_0^* = -CA_1 - CA_2 + B_2^*.$$

Imposing the transversality condition, $B_2^* = 0$, it follows that

$$B_0^* = -CA_1 - CA_2. \tag{2.5}$$

This equation says that a country's initial net foreign asset position must be equal to the sum of its present and future current account deficits. Suppose the country's initial net foreign asset position is negative, that is, $B_0^* < 0$. Then for this country to satisfy equation (2.5) the sum of its current account surpluses must be positive ($CA_1 + CA_2 > 0$), that is, the country must run a current account surplus in at least one period. However, if the country's initial asset position is positive, that is, if $B_0^* > 0$, then the country can run a current account deficit in both periods, which in the present two-period

economy is tantamount to a perpetual current account deficit.

The result that a debtor country cannot run a perpetual current account deficit but a net creditor can is valid for any finite horizon. This means that the result applies not only to economies that last for only two periods, as in the present example, but also to economies that last for any finite number of periods, even if this number is very large, say 1 million years. However, the appendix shows that in an infinite horizon economy, a negative initial net foreign asset position does not preclude an economy from running perpetual current account deficits. The requirement for an infinite-horizon economy not to engage in a Ponzi scheme is that it pay periodically part of the interest accrued on its net foreign debt to ensure that the foreign debt grows at a rate less than the interest rate. In this way, the present discounted value of the country's debt would be zero, which is to say that in present-discounted-value terms the country would pay its debt. Because in this situation the country's net foreign debt is growing over time, the economy must devote an ever larger amount of resources (i.e., it must generate larger and larger trade surpluses) to servicing part of its interest obligations with the rest of the world. The need to run increasing trade surpluses over time requires domestic output to also grow over time. For if output did not grow, the required trade balance surpluses would eventually exceed GDP, which is impossible.

2.3 Savings, Investment, and the Current Account

In this section, we show how to link, using accounting identities, the current account to a number of familiar macroeconomic aggregates, such as national savings, investment, gross domestic product, and domestic absorption. These accounting identities allow us to view current account deficits from a number of perspectives and will be of use when studying the determination of the current account in a general equilibrium model.

2.3.1 Current Account Deficits As Declines in the Net International Investment Position

Recall that in the absence of valuation changes the current account measures the change in the net international investment position of a country, that is,

$$CA_t = B_t^* - B_{t-1}^*,$$

where CA_t denotes the country's current account in period t and B_t^* the country's net international investment position at the end of period t . If the current account is in deficit, $CA_t < 0$, then the net international investment position falls, $B_t^* - B_{t-1}^* < 0$. Similarly, if the current account displays a surplus, $CA_t > 0$, then the net international investment position improves, $B_t^* - B_{t-1}^* > 0$.

2.3.2 Current Account Deficits As Reflections of Trade Deficits

All other things equal, larger trade imbalances, or a larger gap between imports and exports, are reflected in larger current account deficits. This follows from the definition of the current account as the sum of the trade balance and net investment income (again, we are ignoring net international compensation to employees and net unilateral transfers),

$$CA_t = TB_t + rB_{t-1}^*, \quad (2.6)$$

where TB_t denotes the trade balance in period t , and r denotes the interest rate. Figure 1.2 of chapter 1 shows that in the United States, the trade balance and the current account move closely together.

2.3.3 The Current Account As The Gap Between Savings and Investment

The current account is in deficit when investment exceeds savings. To see this, begin by recalling, from chapter 1, that the trade balance equals the difference between exports and imports of goods and services. Letting X_t denote exports in period t and IM_t denote imports in period t , we then have that

$$TB_t = X_t - IM_t.$$

Let Q_t denote the amount of final goods and services produced domestically in period t . This measure of output is known as gross domestic product, or GDP. Let C_t denote the amount of goods and services consumed domesti-

cally by the private sector in period t , G_t government consumption in period t , and I_t the amount of goods and services used for domestic investment (in plants, infrastructure, etc.) in period t . We will refer to C_t , G_t , and I_t simply as consumption, government spending, and investment in period t , respectively. Then we have that

$$Q_t + IM_t = C_t + I_t + G_t + X_t.$$

This familiar national accounting identity states that the aggregate supply of goods, given by the sum of GDP and imports, can be used in four ways, private consumption, investment, public consumption, or exports. Combining the above two equations and rearranging, we obtain

$$TB_t = Q_t - C_t - I_t - G_t. \quad (2.7)$$

Plugging this relation into equation (2.6) yields

$$CA_t = rB_{t-1}^* + Q_t - C_t - I_t - G_t.$$

The sum of GDP and net investment income, $Q_t + rB_{t-1}^*$, is called national income, or gross national product (GNP). We denote national income in period t by Y_t , that is,

$$Y_t = Q_t + rB_{t-1}^*.$$

Combining the last two expressions results in the following representation of the current account

$$CA_t = Y_t - C_t - I_t - G_t. \quad (2.8)$$

National savings in period t , which we denote by S_t , is defined as the difference between national income and the sum of private and government consumption, that is,

$$S_t = Y_t - C_t - G_t. \quad (2.9)$$

It then follows from this expression and equation (2.8) that the current account is equal to savings minus investment,

$$CA_t = S_t - I_t. \quad (2.10)$$

According to this relation, a deficit in the current account occurs when a country's investment exceeds its savings. Conversely, a current account surplus obtains when a country's investment falls short of its savings.

2.3.4 The Current Account As the Gap Between National Income and Domestic Absorption

A country's absorption, which we denote by A_t , is defined as the sum of private consumption, government consumption, and investment,

$$A_t = C_t + I_t + G_t.$$

Combining this definition with equation (2.8), the current account can be expressed as the difference between income and absorption:

$$CA_t = Y_t - A_t. \quad (2.11)$$

Thus, the current account is in deficit when domestic absorption of goods and services exceeds national income.

2.4 Summing Up

In this chapter, we have studied conditions under which paths of the trade balance and the current account are sustainable. We derived a number of results:

- A country that is a net external debtor cannot run a perpetual trade balance deficit.
- A country that is a net external debtor cannot run a perpetual deficit in the current account. This result applies to economies that last for any finite number of periods. For infinite-horizon economies, perpetual current account deficits are possible even if the country is an external debtor, if the economy is growing and dedicates a growing amount of resources to pay interest on the debt.

- We derived four alternative expressions for the current account:

$$CA_t = B_t^* - B_{t-1}^*$$

$$CA_t = rB_{t-1}^* + TB_t$$

$$CA_t = S_t - I_t$$

$$CA_t = Y_t - A_t$$

which emphasize the relationship between the current account and a number of relevant macroeconomic aggregates, namely, net foreign assets, the trade balance, national savings, investment, national income, and domestic absorption.

You should always keep in mind that all four of the above expressions represent accounting identities that must be satisfied at all times in any economy. They do not provide any explanation, or theory, of the determinants of the current account. For example, the identity $CA_t = S_t - I_t$, by itself, does not provide support to the pessimistic view that the U.S. current account is in deficit because Americans save too little. Nor does it lend support to the optimistic view that the U.S. current account is in deficit because American firms invest vigorously in physical capital.

To understand what determines the current account we need a model, that is, a story of the economic behavior of households, firms, governments, and foreign residents. This is the focus of the following chapters.

2.5 Appendix: Perpetual Trade-Balance and Current-Account Deficits in Infinite-Horizon Economies

In a world that lasts for only 2 periods, forever means for periods 1 and 2. Therefore, in such a world a country runs a perpetual trade deficit if the trade balance is negative in periods 1 and 2. Similarly, in a two-period world a country runs a perpetual current account deficit if it experiences a negative current account balance in periods 1 and 2. In the body of this chapter, we showed that a two-period economy can run a perpetual trade balance deficit only if it starts with a positive net international investment position. A similar condition holds for the current account: a two-period country can run a perpetual current account deficit only if its initial net international asset position is positive. In this appendix we study how these results change in a setting in which the economy lasts for an infinite number of periods.

Suppose that the economy starts in period 1 and lasts forever. The net foreign asset position at the end of period 1 is given by

$$B_1^* = (1 + r)B_0^* + TB_1.$$

Solve for B_0^* to obtain

$$B_0^* = \frac{B_1^*}{1 + r} - \frac{TB_1}{1 + r}. \quad (2.12)$$

Now shift this expression one period forward to obtain

$$B_1^* = \frac{B_2^*}{1 + r} - \frac{TB_2}{1 + r}.$$

Use this formula to eliminate B_1^* from equation (2.12), which yields

$$B_0^* = \frac{B_2^*}{(1+r)^2} - \frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2}.$$

Shifting (2.12) two periods forward yields

$$B_2^* = \frac{B_3^*}{1+r} - \frac{TB_3}{1+r}.$$

Combining this expression with the one right above it, we obtain

$$B_0^* = \frac{B_3^*}{(1+r)^3} - \frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2} - \frac{TB_3}{(1+r)^3}.$$

Repeating this iterative procedure T times results in the relationship

$$B_0^* = \frac{B_T^*}{(1+r)^T} - \frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2} - \cdots - \frac{TB_T}{(1+r)^T}. \quad (2.13)$$

In an infinite-horizon economy, the no-Ponzi-game constraint becomes

$$\lim_{T \rightarrow \infty} \frac{B_T}{(1+r)^T} \geq 0.$$

This expression says that the net foreign debt of a country must grow at a rate lower than the interest rate. A debt trajectory that grows at the rate r or higher is indeed a scheme in which the principal and the interest accrued on the debt are perpetually rolled over. That is, it is a scheme whereby the debt is never paid off. The no-Ponzi-game constraint precludes this type of situations.

At the same time, the country will not want to have a net credit with the

rest of the world growing at a rate r or higher, because that would mean that the rest of the world forever rolls over its debt and interest with the country in question, never paying either one. Thus the path of net international investment position must satisfy

$$\lim_{T \rightarrow \infty} \frac{B_T}{(1+r)^T} \leq 0.$$

This restriction and the no-Ponzi-game constraint can be simultaneously satisfied only if the following transversality condition holds:

$$\lim_{T \rightarrow \infty} \frac{B_T}{(1+r)^T} = 0.$$

According to this expression, a country's net foreign asset position must converge to zero in present discounted value. Letting T go to infinity and using this transversality condition, equation (2.13) becomes

$$B_0^* = -\frac{TB_1}{1+r} - \frac{TB_2}{(1+r)^2} - \dots$$

This expression states that the initial net foreign asset position of a country must equal the present discounted value of the stream of current and future expected trade deficits. If the initial foreign asset position of the country is negative ($B_0^* < 0$), then the country must run trade balance surpluses at some point. We conclude that regardless of whether we consider a finite horizon economy or an infinite horizon economy, a country that starts with a negative net foreign asset position cannot perpetually run trade balance deficits.

We next revisit the question of whether a country can run perpetual current account deficits. We can write the evolution of the country's net foreign asset position in a generic period $t = 1, 2, 3, \dots$ as

$$B_t^* = (1 + r)B_{t-1}^* + TB_t.$$

Suppose that the initial net foreign asset position of the country is negative ($B_0^* < 0$). That is, the country starts out as a net debtor to the rest of the world. Consider an example in which each period the country generates a trade balance surplus sufficient to pay a fraction α of its interest obligations. That is,

$$TB_t = -\alpha r B_{t-1}^*,$$

where the factor α is between 0 and 1. According to this expression, whenever the country is a net debtor to the rest of the world, i.e., whenever $B_{t-1}^* < 0$, it generates a trade balance surplus. We are assuming that the interest rate is positive, or $r > 0$. Combining the last two expressions to eliminate TB_t , we obtain

$$B_t^* = (1 + r - \alpha r)B_{t-1}^*. \quad (2.14)$$

Because by assumption B_0^* is negative and $1 + r - \alpha r$ is positive, we have that the net foreign asset position of the country will be forever negative. Furthermore, each period the country runs a current account deficit. To see this, recall that the current account is given by $CA_t = rB_{t-1}^* + TB_t$, which,

given the assumed debt-servicing policy, results in

$$CA_t = r(1 - \alpha)B_{t-1}^* < 0.$$

Thus the country runs a perpetual current account deficit. To determine whether these current account deficits are sustainable, the key question is whether the country satisfies the transversality condition. If the transversality condition is not satisfied, then the trajectory of current account deficits is not sustainable, because in that case either the country would be playing a Ponzi game on the rest of the world or vice versa. The law of motion of B_t^* given equation (2.14) implies that

$$B_t^* = (1 + r - \alpha r)^t B_0^*.$$

It follows that

$$\frac{B_t^*}{(1 + r)^t} = \left[\frac{1 + r(1 - \alpha)}{1 + r} \right]^t B_0^*,$$

which converges to zero as t becomes large because $1 + r > 1 + r(1 - \alpha)$.

Notice that under the assumed policy the trade balance evolves according to

$$TB_t = -\alpha r [1 + r(1 - \alpha)]^{t-1} B_0^*.$$

That is, the trade balance is positive and grows unboundedly over time at the rate $r(1 - \alpha)$. In order for a country to be able to generate this path of trade balance surpluses, its GDP must be growing over time at a rate equal or greater than $r(1 - \alpha)$. If this condition is satisfied, then the repayment

policy described in this example would support perpetual current account deficits even if the initial net foreign asset position is negative.

2.6 Exercises

Exercise 2.1 Indicate whether the following statements are true, false, or uncertain and explain why.

1. An economy that starts with a positive net international investment position will run a trade balance deficit at some point.
2. A country has been having trade balance deficits for 45 years. Four decades ago, the country was a net creditor, but after so many trade deficits it became a debtor. Clearly, this economy will have to run trade surpluses at some point.
3. A two-period economy runs trade surpluses in both periods. It follows that the current account in period 1 can have either sign (depending on the magnitude of TB_1), but the current account in period 2 must be positive.
4. When the world interest rate is negative a two-period economy can run perpetual trade deficits even if its initial net foreign asset position is negative.
5. When the world interest rate is negative a two-period economy cannot run perpetual current account deficits if its initial net foreign asset position is negative.
6. A country starts 2017 as a net creditor. The interest rate on its net asset position is 10 percent. That year, it runs a current account deficit. It follows that the trade balance in 2017 was also negative.

7. The fact that over the past quarter century the United States has run larger and larger current account deficits is proof that American household savings have been shrinking.

Exercise 2.2 Consider a two-period economy that has at the beginning of period 1 a net foreign asset position of -100. In period 1, the country runs a current account deficit of 5 percent of GDP, and GDP in both periods is 120. Assume the interest rate in periods 1 and 2 is 10 percent.

1. Find the trade balance in period 1 (TB_1), the current account balance in period 1 (CA_1), and the country's net foreign asset position at the beginning of period 2 (B_1^*).
2. Is the country living beyond its means? To answer this question find the country's current account balance in period 2 and the associated trade balance in period 2. Is this value for the trade balance feasible? [Hint: Keep in mind that the trade balance cannot exceed GDP.]
3. Now assume that in period 1, the country runs instead a much larger current account deficit of 10 percent of GDP. Find the country's net foreign asset position at the end of period 1, B_1^* . Is the country living beyond its means? If so, show why.

Chapter 3

An Intertemporal Theory of the Current Account

Why do some countries borrow and others lend? Why do some countries run trade balance deficits and others trade balance surpluses? This chapter addresses these and other related questions by building a model of an open economy to study the determinants of the trade balance and the current account. At the heart of this theory is the optimal intertemporal allocation of expenditure. Countries will borrow or lend to smooth consumption over time in the face of uneven output streams.

This chapter analyzes the determination of the current account in a small open economy. Chapter [7](#) extends the theory to a large open economy. We say that an economy is open when it trades in goods and financial assets with the rest of the world. We say that an economy is small when world prices and interest rates are independent of domestic economic conditions.

Most countries in the world are small open economies. Examples of developed small open economies are the Netherlands, Switzerland, Austria, New Zealand, Australia, Canada, and Norway. Examples of emerging small open economies are Chile, Peru, Bolivia, Greece, Portugal, Estonia, Latvia, and Thailand. Examples of large developed open economies are the United States, Japan, Germany, and the United Kingdom. China and India are examples of large emerging open economies. There are not many examples of completely closed economies. Perhaps the closest examples are North Korea, Cuba, Iran, and in the past few years, Venezuela.

The economic size of a country may not be related to its geographic size. For example, Australia and Canada are geographically large, but economically small. On the other hand, Japan, Germany, the United Kingdom, and France are geographically small, but economically large. Also, demographic and economic size may not be correlated. For example, Indonesia is demographically large, but remains economically small.

3.1 The Intertemporal Budget Constraint

Consider an economy in which people live for two periods, 1 and 2, and are endowed with Q_1 units of goods in period 1 and Q_2 units in period 2. Suppose that goods are perishable in the sense that they cannot be stored from one period to the next. Think of fresh food in a tropical island without refrigeration. Although in the present economy households are unable to store goods, they can reallocate resources between periods 1 and 2 via the international financial market. Specifically, assume that in period 1 each

household is endowed with B_0^* units of a bond. These bond holdings generate interest income in the amount of $r_0 B_0^*$ in period 1, where r_0 denotes the interest rate. In period 1, the household's income is therefore given by the sum of interest income, $r_0 B_0^*$, and the endowment of goods, Q_1 , that is, period-1 income is equal to $r_0 B_0^* + Q_1$.

The household can allocate its income to two alternative uses, purchases of consumption goods, which we denote by C_1 , and purchases (or sales) of bonds, $B_1^* - B_0^*$, where B_1^* denotes bond holdings at the end of period 1. Thus, in period 1 the household faces the following budget constraint:

$$C_1 + B_1^* - B_0^* = r_0 B_0^* + Q_1. \quad (3.1)$$

Similarly, in period 2 the household faces a budget constraint stating that consumption expenditure plus bond purchases must equal income,

$$C_2 + B_2^* - B_1^* = r_1 B_1^* + Q_2, \quad (3.2)$$

where C_2 denotes consumption in period 2, r_1 denotes the interest rate on bonds held between periods 1 and 2, and B_2^* denotes bond holdings at the end of period 2.

As explained in chapter 2, by the no-Ponzi-game constraint households are not allowed to leave any debt at the end of period 2, that is, B_2^* must be greater than or equal to zero. Also, because the world is assumed to last for only 2 periods, agents will choose not to hold any positive amount of assets at the end of period 2, as they will not be around in period 3 to spend those

savings on consumption goods. So B_2^* must be less than or equal to zero. Thus, asset holdings at the end of period 2 must be exactly equal to 0,

$$B_2^* = 0. \quad (3.3)$$

In chapter 2 we referred to this terminal restriction as the transversality condition.

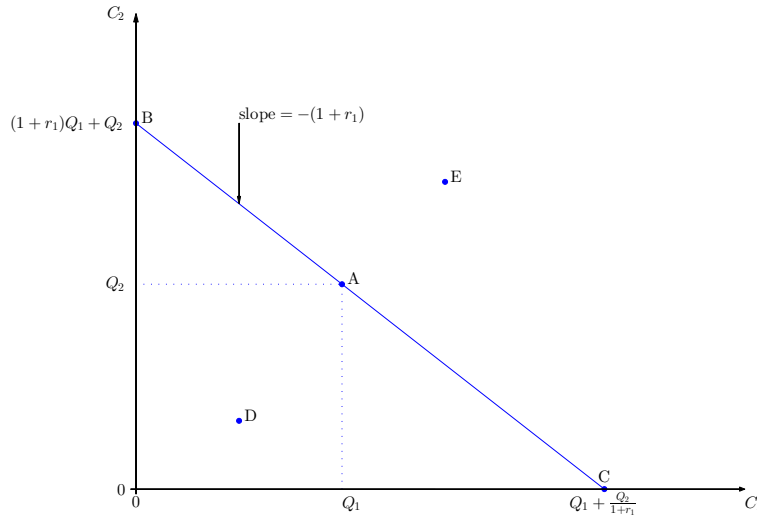
Combining the period budget constraints (3.1) and (3.2) with the transversality condition (3.3) to eliminate B_1^* and B_2^* gives rise to the intertemporal budget constraint of the household

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}. \quad (3.4)$$

The intertemporal budget constraint states that the present discounted value of consumption (the left-hand side) must be equal to the initial stock of wealth plus the present discounted value of the endowment stream (the right-hand side). The household chooses consumption in periods 1 and 2, C_1 and C_2 , taking as given all other variables appearing in the intertemporal budget constraint (3.4), namely, r_0 , r_1 , B_0^* , Q_1 , and Q_2 .

Figure 3.1 displays the pairs (C_1, C_2) that satisfy the household's intertemporal budget constraint (3.4). For simplicity, we assume for the remainder of this section that the household's initial asset position is zero, that is, we assume that $B_0^* = 0$. Then, clearly, the consumption path $C_1 = Q_1$ and $C_2 = Q_2$ (point A in the figure) satisfies the intertemporal budget constraint (3.4). In words, it is feasible for the household to consume its

Figure 3.1: The intertemporal budget constraint



Notes: The downward-sloping line represents the consumption paths (C_1, C_2) that satisfy the intertemporal budget constraint (3.4). The figure is drawn under the assumption that the household's initial asset position is zero, $B_0^* = 0$.

endowment in each period. But the household's choices are not limited to this particular consumption path. In period 1 the household can consume more or less than its endowment by borrowing or saving.

For example, if the household chooses to allocate all of its lifetime income to consumption in period 2, it can do so by saving all of its period-1 endowment. In period 2, its income is the period-1 savings including interest, $(1+r_1)Q_1$, plus its period-2 endowment, Q_2 . Then, C_2 equals $(1+r_1)Q_1 + Q_2$ and C_1 is, of course, nil. This consumption path is located at the intersection of the intertemporal budget constraint with the vertical axis (point B in the figure). At the opposite extreme, if the household chooses to allocate its entire lifetime income to consumption in period 1, it has to borrow $Q_2/(1+r_1)$ units of goods in period 1. Then, C_1 equals the period-1 endow-

ment plus the loan, $Q_1 + Q_2/(1 + r_1)$. In period 2, the household must pay the principal of the loan, $Q_2/(1 + r_1)$, plus interest, $r_1 Q_2/(1 + r_1)$, or a total of Q_2 , to cancel its debt. As a result, period-2 consumption is zero, $C_2 = 0$, since all of the period-2 endowment must be used to retire the debt. This consumption path corresponds to the intersection of the intertemporal budget constraint with the horizontal axis (point C in the figure). All points on the straight line that connects points B and C belong to the intertemporal budget constraint.

The intertemporal budget constraint dictates that if the household wants to increase consumption in one period, it must reduce consumption in the other period. Specifically, for each additional unit of consumption in period 1, the household has to give up $1 + r_1$ units of consumption in period 2. This is because the alternative to consuming an extra unit in period 1 is to save that unit in an interest-bearing asset, which yields $1 + r_1$ units the next period. Graphically, this tradeoff between consumption in period 1 and consumption in period 2 is reflected in the intertemporal budget constraint being downward sloping, with a slope equal to $-(1 + r_1)$.

The intertemporal budget constraint depicted in figure 3.1 allows us to see which consumption paths entail saving or borrowing in period 1. Savings is defined as the difference between income and consumption. Thus, letting S_1 denote savings in period 1, we have $S_1 = r_0 B_0^* + Q_1 - C_1$. Consumption paths on the budget constraint located southeast of point A (all points between A and B), are associated with negative savings ($S_1 < 0$), that is, with borrowing. Because in the figure we are assuming that the household starts with zero assets ($B_0^* = 0$), we have that all points on the budget

constraint located southeast of point A are associated with $B_1^* < 0$, that is, with the household starting period 2 as a debtor. Similarly, points on the budget constraint located northwest of point A (all points between A and C) are associated with positive savings in period 1 ($S_1 > 0$), or a positive asset position at the beginning of period 2 ($B_1^* > 0$).

Consumption paths (C_1, C_2) located below and to the left of the intertemporal budget constraint (such as point D in figure 3.1) are feasible, but do not exhaust the household's lifetime income. The household could consume more in one period without sacrificing consumption in the other period or could consume more in both periods. Such consumption paths violate the transversality condition (3.3), as the household leaves resources on the table at the end of period 2 ($B_2^* > 0$).

Consumption paths located above and to the right of the intertemporal budget constraint (such as point E in the figure) are not feasible, as they have a present discounted value larger than the household's lifetime income. These consumption paths violate the no-Ponzi-game constraint, equation (3.3), because they imply that the household ends period 2 with unpaid debts ($B_2^* < 0$).

3.2 The Lifetime Utility Function

Which consumption path (C_1, C_2) on the intertemporal budget constraint the household will choose depends on its preferences for current and future consumption. We assume that these preferences can be described by the

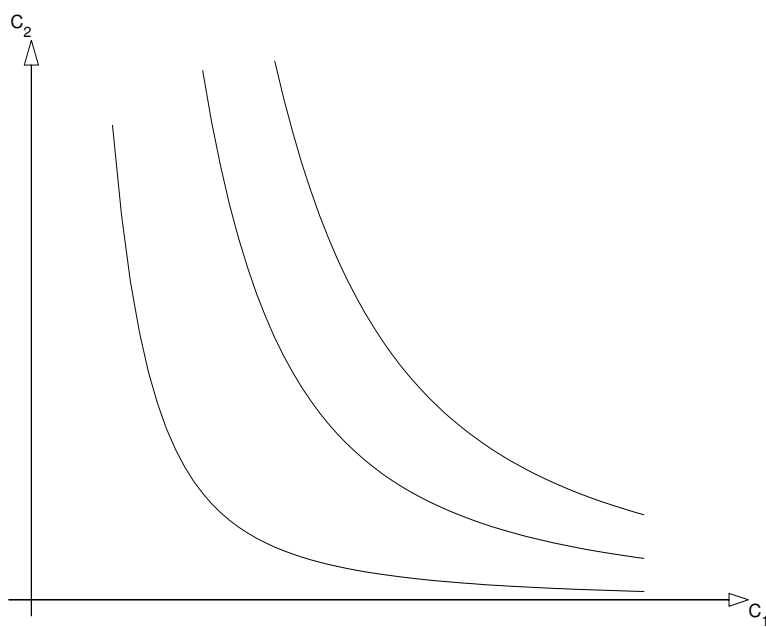
function

$$U(C_1) + \beta U(C_2). \quad (3.5)$$

This function is known as the *lifetime utility function*. It indicates the level of satisfaction (or felicity) derived by the household from different consumption paths (C_1, C_2) . The function $U(\cdot)$ is known as the period utility function and the parameter β is known as the *subjective discount factor*. We assume that consumption in periods 1 and 2, C_1 and C_2 , are both goods, that is, items for which more is preferred to less. In other words, we assume that households enjoy consuming goods in periods 1 and 2. This means that the period utility function is increasing and that the subjective discount factor is positive, $\beta > 0$. The subjective discount factor is a measure of impatience. The smaller is β , the more impatient the consumer will be. In the extreme case in which $\beta = 0$, consumers care only about present consumption. Typically, β is set to a value greater than 0 and less than or equal to 1, which implies that households do not care more about future than present consumption.

Figure 3.2 displays the household's indifference curves. All consumption paths (C_1, C_2) on a given indifference curve provide the same level of utility. Because consumption in both periods are goods, as one moves northeast in figure 3.2, utility increases. This means that indifference curves are downward sloping. An increase in period-1 consumption requires a decrease in period-2 consumption if the household is to remain indifferent. In the figure, we drew only three indifference curves, but the positive quadrant is densely populated with indifference curves. In fact, every point in the posi-

Figure 3.2: Indifference curves



Notes: The level of the lifetime utility associated with each of the three indifference curves is L_1 , L_2 , and L_3 , respectively. All consumption paths (C_1, C_2) on a given indifference curve provide the same level of lifetime utility. Across indifference curves, the lifetime utility increases as one moves northeast in the figure, that is, $L_1 < L_2 < L_3$.

tive quadrant—i.e., every consumption path (C_1, C_2) —belongs to one (and only one) indifference curve.

An important property of the indifference curves drawn in figure 3.2 is that they are convex toward the origin, so that at low levels of C_1 relative to C_2 the indifference curves are steeper than at relatively high levels of C_1 . Intuitively, at low levels of consumption in period 1 relative to consumption in period 2, the household is willing to give up relatively many units of period-2 consumption for an additional unit of period-1 consumption. Similarly, if period-1 consumption is high relative to period-2 consumption, then the household will not be willing to sacrifice much period-2 consumption for an additional unit of period-1 consumption. Put differently, as consumption in period 1 decreases, the household will demand larger increments of consumption in period 2 to keep its level of utility unchanged.

To obtain the slope of an indifference curve, proceed as follows. Fix the level of lifetime utility at some constant, say L . Then the indifference curve associated with a level of lifetime utility L is given by all the paths (C_1, C_2) satisfying

$$U(C_1) + \beta U(C_2) = L.$$

Now differentiate this expression with respect to C_1 and C_2 to obtain

$$U'(C_1)dC_1 + \beta U'(C_2)dC_2 = 0,$$

where a prime denotes the derivative of a function with respect to its argument. The objects $U'(C_1)$ and $U'(C_2)$ are known as the *marginal utility of consumption* in periods 1 and 2, respectively. The marginal utility of con-

sumption in period 1 indicates the increase in lifetime utility derived from consuming one more unit in period 1, and $\beta U'(C_2)$ indicates the increase in lifetime utility derived from consuming one more unit in period 2. Rearranging, we obtain that the slope of the indifference curve, $\frac{dC_2}{dC_1}$, is given by

$$\text{slope of the indifference curve} = -\frac{U'(C_1)}{\beta U'(C_2)}.$$

The convexity of the indifference curves requires that this slope becomes smaller in absolute value as C_1 increases. Now, when C_1 increases, C_2 decreases, because we are moving along a given indifference curve. The only way the indifference curve can become flatter as C_1 increases and C_2 decreases is if $U'(C_1)$ is decreasing in C_1 , that is, if $U''(C_1)$ is negative, or $U(C_1)$ is concave. We have therefore arrived at the result that the convexity of the indifference curve requires that the period utility function $U(\cdot)$ be concave.

An example of a lifetime utility function that satisfies all of the properties discussed thus far is the time-separable logarithmic lifetime utility function without time discounting, which is given by

$$\ln C_1 + \ln C_2, \tag{3.6}$$

where \ln denotes the natural logarithm. In this example, $U(C_i) = \ln C_i$, for $i = 1, 2$, and $\beta = 1$. Because the natural logarithm is an increasing and concave function, this lifetime utility function is increasing in consumption in both periods, and its associated indifference curves are convex toward the origin. To see that the indifference curves implied by these preferences

are downward sloping and convex, fix the level of lifetime utility at some arbitrary level, say 3. Then, the indifference curve corresponding to this utility level is given by all the consumption paths (C_1, C_2) satisfying $\ln C_1 + \ln C_2 = 3$. Solving for C_2 , we obtain $C_2 = \frac{20.1}{C_1}$. This expression says that along the indifference curve associated with a level of lifetime utility of 3, C_2 is a decreasing and convex function of C_1 .

The negative of the slope of the indifference curve is known as the *intertemporal marginal rate of substitution* of C_2 for C_1 . The intertemporal marginal rate of substitution indicates how many units of period-2 consumption the household is willing to give up for one additional unit of period-1 consumption if its lifetime utility is to remain unchanged. The assumption of convexity of the indifference curves can then be stated as saying that along a given indifference curve, the intertemporal marginal rate of substitution decreases with C_1 . For example, in the logarithmic lifetime utility function given above, the intertemporal marginal rate of substitution associated with a level of lifetime utility of 3 is equal to $20.1/C_1^2$, which is decreasing in C_1 .

Before moving on, it is important to point out that what matters for all of the analysis that follows is that the indifference curves be decreasing and convex. For this property to obtain, it is not necessary that the lifetime utility function be concave in (C_1, C_2) . Any increasing monotonic transformation of the lifetime utility function (3.5) will deliver the same map of indifference curves. For example, the lifetime utility function $C_1 C_2$ is not concave, but delivers the same indifference curves as the logarithmic lifetime utility function given in equation (3.6). This is because $C_1 C_2 = \exp(\ln C_1 + \ln C_2)$ and because the exponential function is strictly increasing. Can you show,

for example, that the indifference curve that crosses the consumption path $C_1 = 3$ and $C_2 = 2$, is the same under both lifetime utility functions?

3.3 The Optimal Intertemporal Allocation of Consumption

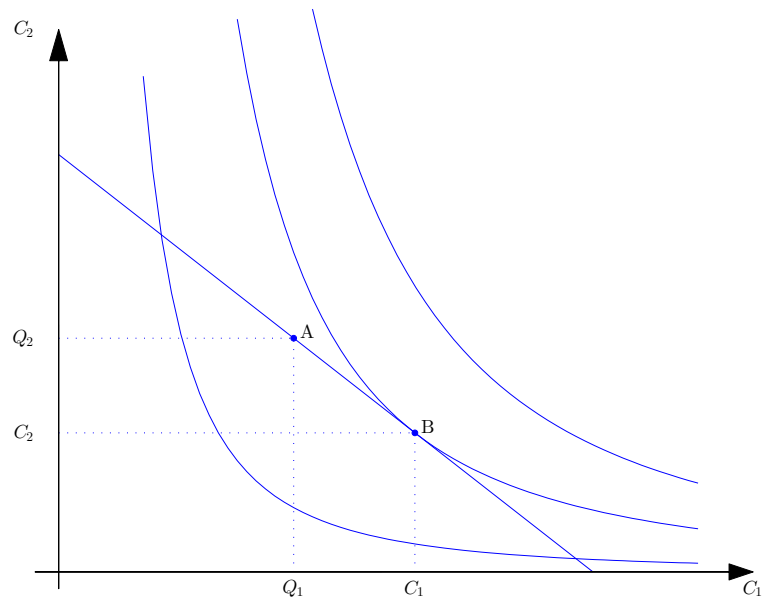
The household chooses C_1 and C_2 to maximize the lifetime utility function (3.5) subject to the intertemporal budget constraint (3.4) taking as given the initial wealth, $(1 + r_0)B_0^*$, the endowments, Q_1 and Q_2 , and the interest rate, r_1 .

The graphical approach to solving this problem is to superimpose the graph of the intertemporal budget constraint (figure 3.1) on the graph of the indifference curves (figure 3.2). This results in figure 3.3. To maximize utility, the household chooses a consumption path (C_1, C_2) that is on the intertemporal budget constraint and on an indifference curve that provides the highest level of lifetime utility, that is, an indifference curve that is as far northeast as possible. At the feasible consumption path that maximizes the household's utility, the indifference curve is tangent to the budget constraint (point B in figure 3.3).

The analytical approach to characterizing the optimal intertemporal allocation of consumption is as follows. Solve the intertemporal budget constraint (3.4) for C_2 to obtain

$$C_2 = (1 + r_1) \left[(1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r_1} - C_1 \right].$$

Figure 3.3: Equilibrium in the endowment economy



Notes. The figure displays the equilibrium in a small open economy with free capital mobility. The equilibrium is at point B, where an indifference curve is tangent to the intertemporal budget constraint. Because of free capital mobility, the slope of the intertemporal budget constraint is determined by the world interest rate and equal to $-(1 + r^*)$. As the figure is drawn, the country runs trade balance and current account deficits in period 1. The figure is drawn under the assumption of a zero initial net foreign asset position, $B_0^* = 0$.

Define

$$\bar{Y} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r_1}.$$

Here, \bar{Y} represents the household's lifetime wealth, which is composed of its initial asset holdings and the present discounted value of the stream of income (Q_1, Q_2) . The household takes \bar{Y} as given. We can then rewrite the intertemporal budget constraint as

$$C_2 = (1 + r_1)(\bar{Y} - C_1). \quad (3.7)$$

Use this expression to eliminate C_2 from the lifetime utility function (3.5) to obtain

$$U(C_1) + \beta U((1 + r_1)(\bar{Y} - C_1)). \quad (3.8)$$

The optimization problem of the household then reduces to choosing C_1 to maximize (3.8), taking as given \bar{Y} and r_1 . The first-order optimality condition associated with this maximization problem results from taking the derivative of this expression with respect to C_1 and setting it equal to zero. Performing this operation and rearranging yields

$$U'(C_1) = (1 + r_1)\beta U'(C_2), \quad (3.9)$$

where we replaced $(1 + r_1)(\bar{Y} - C_1)$ by C_2 . The optimality condition (3.9) is known as the consumption *Euler equation*. It is quite intuitive. Suppose that the household sacrifices one unit of consumption in period 1. This reduces its utility by $U'(C_1)$. Thus the left-hand side of the Euler equation represents

the utility cost of reducing period-1 consumption by one unit. Suppose further that the household saves this unit of consumption in a bond paying the interest rate r_1 in period 2. In period 2, the household receives $1 + r_1$ units of consumption, each of which increases its lifetime utility by $\beta U'(C_2)$. Thus the right-hand side of the Euler equation represents the utility gain of sacrificing one unit of period-1 consumption. If the left-hand side of the Euler equation is greater than the right-hand side, then the household can increase its lifetime utility by saving less (and hence consuming more) in period 1. Conversely, if the left-hand side of the Euler equation is less than the right-hand side, then the household will be better off saving more (and consuming less) in period 1. At the optimal allocation, the left- and right-hand sides of the Euler equation must be equal to each other, so that at the margin the household is indifferent between consuming an extra unit in period 1 and consuming $1 + r_1$ extra units in period 2.

To see that the Euler equation (3.9) is equivalent to the requirement that at the optimum (point B in figure 3.3) the indifference curve be tangent to the budget constraint, divide the left- and right-hand sides of the Euler equation by $-\beta U'(C_2)$ to obtain

$$-\frac{U'(C_1)}{\beta U'(C_2)} = -(1 + r_1).$$

The left-hand side of this expression, $-\frac{U'(C_1)}{\beta U'(C_2)}$, is the negative of the marginal rate of intertemporal substitution of C_2 for C_1 at the consumption path (C_1, C_2) , which, as shown above is the slope of the indifference curve. The right-hand side of the expression, $-(1 + r_1)$, is the negative of the slope of

the budget constraint.

3.4 The Interest Rate Parity Condition

We assume that households have unrestricted access to international financial markets. This assumption is known as *free capital mobility*. It means that the country can borrow from or lend to the rest of the world without any impediments. Free capital mobility tends to eliminate differences between the domestic interest rate, r_1 , and the interest rate prevailing in the rest of the world, which we denote by r^* . This is because if the domestic interest rate were higher than the world interest rate ($r_1 > r^*$), then a pure *arbitrage opportunity* would arise whereby financial investors could make infinite profits by borrowing from abroad at the rate r^* and lending domestically at the rate r_1 . This would drive down the domestic interest rate, as the country would be flooded with credit. Conversely, if the domestic interest rate were lower than the world interest rate, then an arbitrage opportunity would allow financial investors to make infinite profits by borrowing domestically at the rate r_1 and lending internationally at the rate $r^* > r_1$. This would drive up the domestic interest rate, as all funds would move abroad. Arbitrage opportunities disappear when the domestic interest rate equals the world interest rate, that is, when

$$r_1 = r^*.$$

This expression is known as the *interest rate parity condition*. Chapter 10 studies empirically the extent to which financial capital can move across countries. For many countries, especially in the developed world, free capital

mobility is not an unrealistic description of the functioning of their financial markets. For other countries, especially in the emerging world, large deviations from free capital mobility are often observed. In later chapters, we will study how the present economy works when financial capital is not free to move across borders, so that the interest rate parity conditions does not hold. For the time being, we assume that the interest rate parity condition does hold.

3.5 Equilibrium in the Small Open Economy

We assume that all households are identical. Thus, by studying the behavior of an individual household, we are also learning about the behavior of the country as a whole. For this reason, we will not distinguish between the behavior of an individual household and that of the country as a whole.

The country is assumed to be sufficiently small so that its savings decisions do not affect the world interest rate. Because all households are identical, at any point in time all domestic residents will make identical saving decisions. This implies that domestic households will never borrow or lend from one another and that all borrowing or lending takes the form of purchases or sales of foreign assets. Thus, we can interpret B_t^* ($t = 0, 1, 2$) as the country's net foreign asset position, or net international investment position (*NIIP*), at the end of period t .

Furthermore, the assumption that all households are identical implies that the intertemporal budget constraint of an individual household, given by equation (3.4), can be interpreted as the country's *intertemporal resource*

constraint.

An equilibrium then is a consumption path (C_1, C_2) and an interest rate r_1 that satisfy the country's intertemporal resource constraint, the consumption Euler equation, and the interest rate parity condition, that is,

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r_1}, \quad (3.10)$$

$$U'(C_1) = (1 + r_1)\beta U'(C_2), \quad (3.11)$$

and

$$r_1 = r^*,$$

given the exogenous variables r_0 , B_0^* , Q_1 , Q_2 , and r^* . Here, the term *exogenous* refers to variables whose values are determined outside of the model. For instance, the initial net foreign asset position B_0^* is determined in period 0, before the beginning of the economy. The world interest rate, r^* , is determined in world financial markets, which our economy cannot affect because it is too small. And the endowments, Q_1 and Q_2 , represent manna-type receipts of goods whose quantity and timing lies outside of the control of the household.

3.6 The Trade Balance and the Current Account

In the present economy the trade balance in period 1 equals the difference between the endowment of goods in period 1, Q_1 , and consumption of goods in period 1, C_1 , that is,

$$TB_1 = Q_1 - C_1. \quad (3.12)$$

Similarly, the trade balance in period 2 is given by

$$TB_2 = Q_2 - C_2.$$

The current account is equal to the sum of net investment income and the trade balance. Thus in period 1 the current account is given by

$$CA_1 = r_0 B_0^* + TB_1, \tag{3.13}$$

and the current account in period 2 is given by

$$CA_2 = r^* B_1^* + TB_2.$$

Combining the period-1 budget constraint (3.1) with the above expressions for the trade balance and the current account, equations (3.12) and (3.13), we can alternatively express the current account in period 1 as the change in the country's net foreign asset position,

$$CA_1 = B_1^* - B_0^*.$$

A similar expression holds in period 2, except that, because period 2 is the last period of the economy, B_2^* is nil. So we have that

$$CA_2 = -B_1^*,$$

which says that in the last period the country retires any outstanding debt if $B_1^* < 0$ or spends any asset holdings if $B_1^* > 0$.

Look again at figure 3.3. As we saw earlier, the equilibrium is at point B. Because B is located southeast of the endowment point A, consumption in period 1 exceeds the endowment, $C_1 > Q_1$. Equation (3.12) then implies that the country runs a trade deficit in period 1, that is, $TB_1 < 0$. Also, recalling that the figure is drawn under the assumption that foreign asset holdings in period 0 are nil ($B_0^* = 0$), by equation (3.13), the current account in period 1 equals the trade balance in that period, $CA_1 = TB_1 < 0$. Thus, in equilibrium, the current account is in deficit in period 1. In turn, the current account deficit in period 1 implies that the country starts period 2 as a net debtor to the rest of the world ($B_1^* < 0$). As a result, in period 2 the country must generate a trade surplus to repay the debt plus interest, that is, $TB_2 = -(1 + r^*)B_1^* > 0$.

In general, the equilibrium need not feature trade and current account deficits in period 1. In figure 3.3, the endowments, preferences, and the world interest rate are such that point B lies southeast of point A. But under different preferences, endowments, or world interest rates, the equilibrium may lie northwest of the endowment point A. In that case, the country would run trade balance and current account surpluses in period 1.

3.7 Adjustment to Temporary and Permanent Output Shocks

What is the effect of an increase in output on the current account? It turns out that this question, as formulated, is incomplete, and, as a result, does not have a clear answer. The reason is that in a world in which agents

make decisions based on current and future expected changes in the economic environment, one needs to specify not only what the current change in the environment is, but also what the future expected changes are. The information that current output increases does not tell us in what direction, if any, future output is expected to move. Consider the following example. A self-employed income earner of a family falls ill and is unable to work full time. How should the members of the household adjust their consumption expenditures in response to this exogenous shock? It really depends on the severity of the illness affecting the head of the household. If the illness is transitory (a cold, say), then the income earner will be expected to be back on a full-time schedule in a short period of time (within a week, say). In this case, although the family is making less income for one week, there is no reason to implement drastic adjustments in spending patterns. Consumption can go on more or less as usual. The gap between spending and income during the week in which the bread winner of the family is out of commission can be covered with savings accumulated in the past or, if no savings are available, by borrowing a little against future earnings. Future consumption should not be much affected either. For, due to the fact that the period during which income was reduced was short, the interest cost of the borrowing (or decumulation of wealth) that took place during that time is small relative to the level of regular income. However, if the affliction is of a more permanent nature (a chronic back injury, say), then one should expect that the reduction in the work week will be of a more permanent nature. In this case, the members of the household should expect not only current but also future income to go down. As a result consumption must

be permanently adjusted downward by cutting, for instance, items that are not fully necessary, such as extra school activities or restaurant meals.

The general principle that the above example illustrates is that forward-looking, optimizing individuals will behave differently in response to an income shock depending on whether it is temporary or permanent. They will tend to finance temporary income shocks, by increasing savings if the temporary shock is positive or by dissaving if the temporary shock is negative. On the other hand, they will adjust in response to permanent income shocks, by cutting consumption if the permanent shock is negative or by increasing consumption if the permanent shock is positive. This same principle can be applied to countries as a whole. In this section, we develop this principle more formally in the context of the model of current account determination laid out above.

3.7.1 Adjustment to Temporary Output Shocks

Consider the adjustment of a small open economy to a temporary variation in output. For example, suppose that Ecuador loses 20 percent of its banana crop due to a drought. Suppose further that this decline in output is temporary, in the sense that it is expected that next year the banana crop will be back at its normal level. How would such a shock affect consumption, the trade balance, and the current account? Intuitively, Ecuadorian households will cope with the negative income shock by running down their savings or borrowing against their future income levels, which are unaffected by the drought. In this way, they can smooth consumption over time by not having to cut current spending by as much as the decline in current output.

Figure 3.4: A temporary decline in output and the intertemporal budget constraint

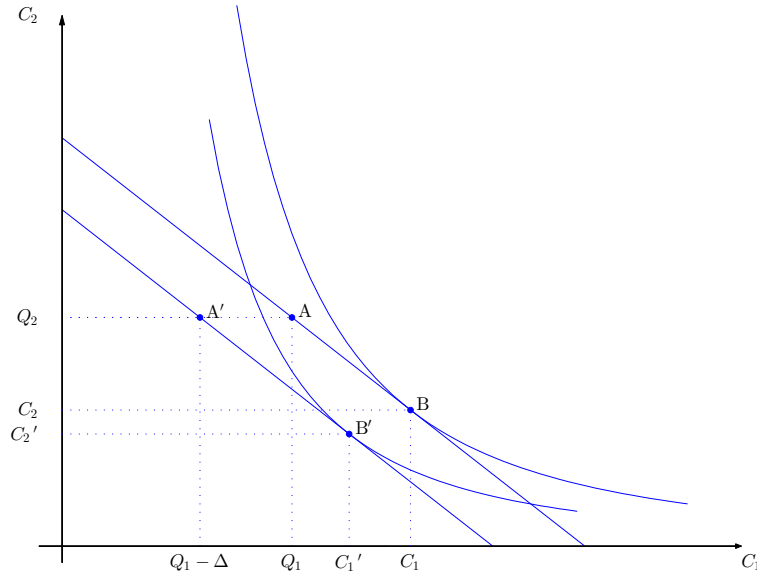


Notes. In response to a decline in Q_1 equal to Δ the the intertemporal budget constraint shifts to the left by Δ . Point A indicates the endowment point prior to the output decline and point A' indicates the endowment point after the decline.

It follows that the temporary drought will induce a deterioration of the trade balance and the current account.

Formally, assume that the negative shock produces a decline in output in period 1 from Q_1 to $Q_1 - \Delta < Q_1$, but leaves output in period 2 unchanged. The situation is illustrated in figure 3.4, where A indicates the endowment point before the shock (Q_1, Q_2) and A' the endowment point after the shock $(Q_1 - \Delta, Q_2)$. Note that because Q_2 is unchanged points A and A' can be connected by a horizontal line. As a consequence of the decline in Q_1 , the budget constraint shifts toward the origin. The new budget constraint is parallel to the old one because the world interest rate is unchanged. The

Figure 3.5: Adjustment to a temporary decline in output



Notes. The figure depicts the adjustment of the economy to a decline in the period-1 endowment equal to Δ . The endowment point shifts left from point A to point A' and the optimal consumption path shifts from point B to point B' . Period-1 consumption declines by less than Δ . The period-1 trade balance becomes more negative, $Q_1 - \Delta - C_1' < Q_1 - C_1$.

household could adjust to the output shock by reducing consumption in period 1 by exactly the amount of the output decline, Δ , thus leaving consumption in period 2 unchanged. However, if both C_1 and C_2 are *normal goods* (i.e., goods whose consumption increases with income), the household will choose to smooth consumption by reducing both C_1 and C_2 . Figure 3.5 depicts the economy's response to the temporary output shock. As a result of the shock, the new optimal consumption path, B' , is located southwest of the pre-shock consumption allocation, B . In smoothing consumption over time, the country runs a larger trade deficit in period 1 (recall that it was running a trade deficit even in the absence of the shock) and finances it by

acquiring additional foreign debt. Thus, the current account deteriorates. In period 2, the country must generate a larger trade surplus than the one it would have produced in the absence of the shock in order to pay back the additional debt acquired in period 1.

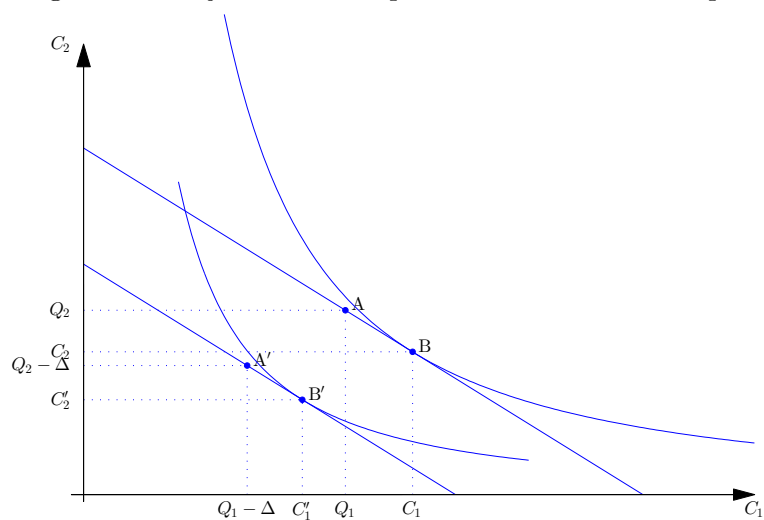
The important principle to take away from this example is that temporary negative income shocks are smoothed out by borrowing from the rest of the world rather than by fully adjusting current consumption by the size of the shock. A similar principle applies for positive temporary income shocks. In this case, the trade balance and the current account improve, as households save part of the increase in income for future consumption.

3.7.2 Adjustment to Permanent Output Shocks

The pattern of adjustment to changes in income is quite different when the income shock is of a more permanent nature. To continue with the example of the drought in Ecuador, suppose that the drought is not just a one-year event, but is expected to last for many years due to global climate changes. In this case, it would not be optimal for households to borrow against future income, because future income is expected to be as low as current income. Instead, Ecuadorian consumers will have to adjust to the new climatic conditions by cutting consumption in all periods by roughly the size of the decline in the value of the banana harvest.

Formally, consider a permanent negative output shock that reduces both Q_1 and Q_2 by Δ . Figure 3.6 illustrates the situation. As a result of the decline in endowments, the budget constraint shifts to the left in a parallel fashion. The new budget constraint crosses the point $(Q_1 - \Delta, Q_2 - \Delta)$. As

Figure 3.6: Adjustment to a permanent decline in output



Notes. The figure depicts the adjustment to a decline in Q_1 and Q_2 equal to Δ . The endowment point A shifts down and to the left to point A' . The intertemporal budget constraint shifts down in a parallel fashion. The optimal consumption path (C_1, C_2) shifts from point B to point B' . The figure is drawn for the case $B_0^* = 0$. The period-1 trade balance is little changed.

in the case of a temporary output decline, consumption-smoothing agents will adjust by reducing consumption in both periods. If consumption in each period fell by exactly Δ , then the trade balance would be unaffected in both periods. In general the decline in consumption should be expected to be close to Δ , implying that a permanent output shock has little consequences for the trade balance and the current account.

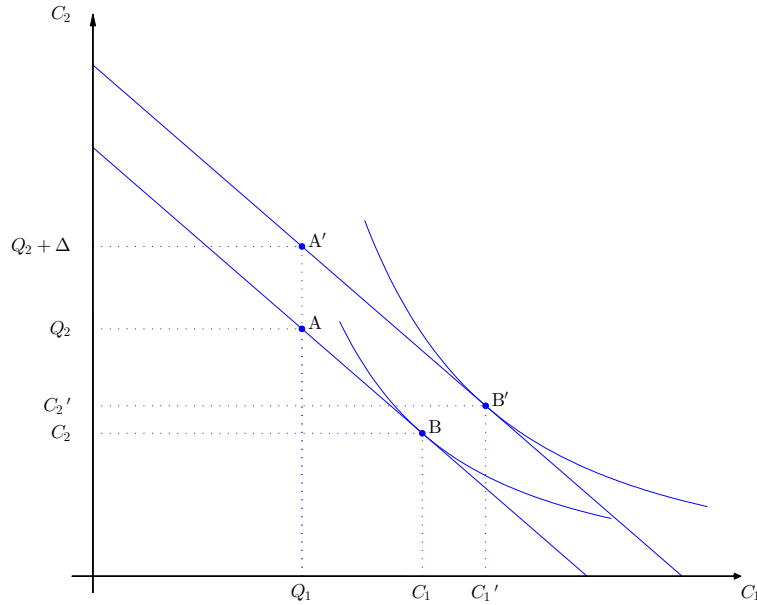
Comparing the effects of temporary and permanent output shocks on the current account, the following general principle emerges: *Economies will tend to finance temporary shocks (by borrowing or lending on international capital markets) and adjust to permanent ones (by varying consumption in both periods up or down).* Thus, temporary shocks tend to produce large movements in the current account whereas permanent shocks tend to leave the current account largely unchanged.

3.8 Anticipated Income Shocks

Suppose households learn that the endowment will be higher next period. For example, suppose that in an island with banana trees, the weather forecast anticipates an increase in rainfall next year, and that more rain will make the banana crop more abundant. Suppose further that current climatic conditions did not change, so that the present banana harvest is unchanged.

How does the economy adjust an anticipated output shock of this type? Intuitively, households feel richer, as the present discounted value of their endowment stream went up. As a result, they will wish to consume more

Figure 3.7: Adjustment to an Anticipated Increase in Output



Notes. The figure depicts the adjustment to an anticipated increase in Q_2 equal to $\Delta > 0$. The initial net foreign asset position is assumed to be zero, $B_0^* = 0$. The anticipated increase in Q_2 shifts the intertemporal budget constraint up by Δ . The increase in the period-2 endowment causes an increase in period-1 consumption from C_1 to C_1' . Because the endowment in period 1 is unchanged, the period-1 trade balance deteriorates.

in both periods. But because the endowment in period 1 is unchanged, the increase in consumption causes the trade balance to deteriorate. The effect of an anticipated increase in income is illustrated in figure 3.7. The initial endowment path is (Q_1, Q_2) and corresponds to point A. The initial consumption path, (C_1, C_2) , is point B. The indifference curve that crosses point B is tangent to the intertemporal budget constraint. At point B, the economy runs a trade balance deficit in period 1, as C_1 is larger than Q_1 . The figure is drawn under the assumption that the initial net international investment position, B_0^* , is nil. Consequently, the current account equals

the trade balance, and is therefore also negative.

Suppose now that in period 1 everybody learns that the endowment in period 2 will increase from Q_2 to $Q_2 + \Delta$, with $\Delta > 0$, and that the endowment in period 1 remains constant at Q_1 . The new endowment path is given by point A' , located exactly above point A . The expanded endowment in period 2 produces a shift up in the intertemporal budget constraint. The vertical distance between the new and the old intertemporal budget constraints equals the anticipated output change Δ . The two intertemporal budget constraints are parallel because the interest rate did not change. Since households are richer, they want to consume more in both periods. In the figure, the new consumption path is marked by point B' , located north-east of point B . To expand consumption in period 1, the country must run a larger trade balance deficit (recall that Q_1 is unchanged). The current account deficit increases one for one with the trade balance.

In sum, the model predicts that an anticipated increase in output causes an expansion in consumption and a deterioration of the trade balance and the current account.

3.9 An Economy with Logarithmic Preferences

We now illustrate, by means of an algebraic example, the results obtained thus far. Let the utility function be of the log-linear type without discounting given in equation (3.6), that is,

$$U(C_1) + \beta U(C_2) = \ln C_1 + \ln C_2.$$

Households choose the consumption path (C_1, C_2) to maximize this lifetime utility function subject to the intertemporal budget constraint

$$C_1 + \frac{C_2}{1 + r_1} = \bar{Y},$$

where, as before, $\bar{Y} = (1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1+r_1}$ is the household's lifetime wealth. Solving the intertemporal budget constraint for C_2 and using the result to eliminate C_2 from the lifetime utility function, we have that the household's optimization problem reduces to choosing C_1 to maximize

$$\ln(C_1) + \ln((1 + r_1)(\bar{Y} - C_1)).$$

The first-order condition associated with this problem is

$$\frac{1}{C_1} - \frac{1}{\bar{Y} - C_1} = 0.$$

Solving for C_1 yields

$$C_1 = \frac{1}{2}\bar{Y}.$$

This result says that households find it optimal to consume half of their lifetime wealth in the first half of their lives.

Using the definition of \bar{Y} and the fact that under free capital mobility the domestic interest rate must equal the world interest rate, or $r_1 = r^*$, we have that C_1 , C_2 , TB_1 , and CA_1 are given by

$$C_1 = \frac{1}{2} \left[(1 + r_0)B_0^* + Q_1 + \frac{Q_2}{1 + r^*} \right] \quad (3.14)$$

$$C_2 = \frac{1}{2}(1+r^*) \left[(1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r^*} \right] \quad (3.15)$$

$$TB_1 = \frac{1}{2} \left[Q_1 - (1+r_0)B_0^* - \frac{Q_2}{1+r^*} \right] \quad (3.16)$$

$$CA_1 = r_0B_0^* + \frac{1}{2} \left[Q_1 - (1+r_0)B_0^* - \frac{Q_2}{1+r^*} \right] \quad (3.17)$$

Consider now the effects of temporary and permanent output shocks on consumption, the trade balance, and the current account. Assume first that income falls temporarily by one unit, that is, Q_1 decreases by one and Q_2 is unchanged. From equation (3.14), we see that consumption falls by $1/2$ in period 1 and by $(1+r^*)/2$ in period 2. Intuitively, households smooth the effect of the negative endowment shock by reducing consumption in both periods by roughly the same amount. Because consumption in period 1 falls by less than the fall in the endowment, the trade balance must deteriorate. In turn, the deterioration of the trade balance requires an increase in international borrowing, or a current account deterioration. These effects can be confirmed by inspecting equations (3.16) and (3.17), which indicate that a unit fall in the period-1 endowment causes a fall in TB_1 and CA_1 of $1/2$.

Suppose next that income falls permanently by one unit, that is, Q_1 and Q_2 both fall by one. Then the trade balance and the current account decline by $\frac{1}{2} \frac{r^*}{1+r^*}$. Consumption in period 1 falls by $\frac{1}{2} \frac{2+r^*}{1+r^*}$. For realistic values of r^* , the predicted deterioration in the trade balance and current account in response to the assumed permanent negative income shock is close to zero and in particular much smaller than the deterioration associated with

the temporary negative income shock. For example, assume that the world interest rate is 10 percent, $r^* = 0.1$. Then, both the trade balance and the current account in period 1 fall by 0.046 in response to the permanent output shock and by 0.5 in response to the temporary shock. That is, the current account deterioration is 10 times larger under a temporary shock than under a permanent one. Intuitively, if income falls by one unit in both periods, cutting consumption by roughly one unit in both periods leaves the consumption path as smooth as before the shock. So households do not need to use the financial market (the current account) to smooth consumption.

Finally, consider an anticipated unit increase in the period-2 endowment, with the period-1 endowment unchanged. By equations (3.14)-(3.14) we have that consumption in period 1 increases by $\frac{1}{2(1+r^*)}$ and the trade balance and the current account both deteriorate by the same amount. The intuition is clear. The increase in the future endowment makes households richer, inducing them to increase consumption in both periods. With Q_1 unchanged, the increase in current consumption causes an increase in the trade deficit, which must be financed by external borrowing, that is, by a current account deterioration. Thus, good news about the future causes a deterioration of the current account. This shows that current account deficits are not necessarily an indication of a weak economy.

3.10 Summing Up

This chapter presents an intertemporal model of the current account. The main building blocks of the intertemporal model are:

- Households face an intertemporal budget constraint that allows them to consume more than their current income by borrowing against future income or to consume less than their current income by lending to the rest of the world.

- Households have preferences over present and future consumption. Their preferences are described by indifference curves that are downward sloping and convex towards the origin.

- Households choose a consumption path that maximizes lifetime utility subject to the intertemporal budget constraint. At the optimal consumption path the intertemporal budget constraint is tangent to an indifference curve.

- Free capital mobility implies that the domestic interest rate must equal the world interest rate.

The intertemporal model of the current account delivers the following key insight:

- In response to temporary income shocks, countries use the current account to smooth consumption over time. Positive temporary shocks cause an improvement in the current account and negative temporary shocks cause a deterioration. In response to permanent income shocks, countries adjust consumption without much movement in the current account.

- Finally, a second important prediction of the intertemporal model is that in response to an anticipated increase in future income, the trade balance and the current account deteriorate, as forward-looking, consumption-smoothing households borrow against their higher future expected income to expand current spending.

3.11 Exercises

Exercise 3.1

Countries A and B are identical in all respects, except that the initial net international asset position (B_0^*) of country A is lower than that of country B. Indicate whether the following statements are true, false, or uncertain and explain why.

1. It must be the case that consumption in country A is lower than consumption in country B.
2. It must be the case that the trade balance in country A in period 1 is higher than in country B.
3. It must be the case that the current account in country A in period 2 is higher than in country B.
4. All of the above statements are true.

Exercise 3.2 (Endowment Shocks) Consider a two-period lived household, whose preferences for consumption are described by the lifetime utility function

$$-C_1^{-1} - C_2^{-1},$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively.

1. Do these preferences give rise to indifference curves that are downward sloping and convex? Show your work.

2. Suppose that the household starts period 1 with financial wealth equal to $(1 + r_0)B_0^*$, where B_0^* is an inherited stock of bonds and r_0 is the interest rate on assets held between periods 0 and 1. In addition, the household receives endowments of goods in the amounts Q_1 and Q_2 in periods 1 and 2, respectively. In period 1, the household can borrow or lend at the interest rate $r_1 > 0$ via a bond denoted B_1^* . Find the optimal levels of consumption in periods 1 and 2 as functions of the household's lifetime wealth, $\bar{Y} \equiv (1 + r_0)B_0^* + Q_1 + Q_2/(1 + r_1)$ and the interest rate r_1 .
3. Find the responses of consumption in period 1, ΔC_1 , the trade balance in period 1, ΔTB_1 , and the current account in period 1, ΔCA_1 , to a temporary increase in the endowment, $\Delta Q_1 > 0$ and $\Delta Q_2 = 0$.
4. Find the responses of consumption in period 1, ΔC_1 , the trade balance in period 1, ΔTB_1 , and the current account in period 1, ΔCA_1 , to a permanent increase in the endowment, $\Delta Q_1 = \Delta Q_2 > 0$.
5. Compare your findings to those obtained under log-preferences as presented in section 3.9.

Exercise 3.3 (An Anticipated Output Shock I) Consider a two-period small open endowment economy populated by a large number of households with preferences described by the lifetime utility function

$$C_1^{\frac{1}{10}} C_2^{\frac{1}{11}},$$

where C_1 and C_2 denote, respectively, consumption in periods 1 and 2.

Suppose that households receive exogenous endowments of goods given by $Q_1 = Q_2 = 10$ in periods 1 and 2, respectively. Every household enters period 1 with some debt, denoted B_0^* , inherited from the past. Let B_0^* be equal to -5. The interest rate on these liabilities, denoted r_0 , is 20 percent. Finally, suppose that the country enjoys free capital mobility and that the world interest rate on assets held between periods 1 and 2, denoted r^* , is 10 percent.

1. Compute the equilibrium levels of consumption, the trade balance, and the current account in periods 1 and 2.
2. Assume now that the endowment in period 2 is expected to increase from 10 to 15. Calculate the effect of this anticipated output increase on consumption, the trade balance, and the current account in both periods. Provide intuition.

Exercise 3.4 (An Anticipated Output Shock II) Consider a two-period small open endowment economy. The lifetime utility functions of households takes the form, $C_1^{1/5} C_2^{2/5}$. In period 1 households receive an endowment of Q_1 and in period 2 they receive Q_2 . Assume that the world interest rate is 33.3% ($r^* = 1/3$) and that the initial net foreign asset position is zero ($B_0^* = 0$). Find the change in the current account in periods 1 and 2 in response to news in period 1 that the period-2 endowment will increase by ΔQ_2 . Provide an intuitive explanation for your findings.

Exercise 3.5 (Debt Forgiveness) Consider a small open economy where

households live for two periods and have logarithmic preferences,

$$\ln C_1 + \beta \ln C_2,$$

where the subjective discount factor β equals $\frac{10}{11}$. Suppose that households receive a constant endowment over time equal to 10, $Q_1 = Q_2 = 10$. Suppose that households start period 1 with debt including interest equal to 5, $(1 + r_0)B_0^* = -5$ and that $r_0 = 0.1$. Finally, assume that the country enjoys free capital mobility and that the world interest rate is 10 percent, $r^* = 0.1$.

1. Calculate the equilibrium values of consumption, the trade balance, and the current account in period 1.
2. Suppose now that foreign lenders decide to forgive all of the country's initial external debt including interest. Calculate the effect of this external gift on consumption, the trade balance, and the current account in period 1. Provide an intuitive explanation of your findings.

Exercise 3.6 (A Three-Period Open Economy) Consider a three-period small open endowment economy populated by a large number of households with preferences given by the lifetime utility function

$$\ln C_1 + \ln C_2 + \ln C_3,$$

where C_1 , C_2 , and C_3 denote, respectively, consumption in periods 1, 2, and 3. Suppose that households receive exogenous endowments of goods given by Q_1 , Q_2 , and Q_3 in periods 1, 2, and 3, respectively. Every household enters

period 1 with an asset position, including interest, equal to $(1 + r_0)B_0^*$, where r_0 denotes the interest rate prevailing in period 0. Finally, suppose that the country enjoys free capital mobility and that the world interest rate is constant over time and equal to r^* .

1. Write the household's budget constraint in periods 1, 2, and 3.
2. Write the no-Ponzi game constraint.
3. Derive the intertemporal budget constraint.
4. Compute the equilibrium levels of consumption, the trade balance, and the current account in periods 1, 2, and 3.
5. Assume that in period 1 the economy receives a temporary increase in the endowment equal to $\Delta Q > 0$, that is, assume that Q_1 increases by ΔQ and that Q_2 and Q_3 remain unchanged. Calculate the changes in consumption, the trade balance, and the current account in period 1, denoted ΔC_1 , ΔTB_1 , and ΔCA_1 , respectively.
6. Now assume that the endowment shock is permanent, that is, the endowments in periods 1, 2, and 3 all increase by $\Delta Q > 0$. Calculate the changes in consumption, the trade balance, and the current account in period 1.
7. Compare your answers to the ones obtained in the two-period economy.
8. Answer questions 5 and 6 in the general case of a T -period economy, where T is any integer larger than 2.

Exercise 3.7 (Durability and the Countercyclicalities of the Trade Balance)

Consider a two-period, small, open, endowment economy with durable consumption goods. Purchases of durable consumption goods in period 1, denoted C_1 , continue to provide utility in period 2. The service flow households receive from the stock of durables in period 2 depends on new purchases of durables in period 2, C_2 , and on the un-depreciated stock of durables purchased in period 1. Durable consumption goods are assumed to depreciate at the rate $\delta \in [0, 1]$. Preferences are described by the following utility function

$$\ln(C_1) + \ln(C_2 + (1 - \delta)C_1).$$

Assume that the world interest rate, r^* , is 10 percent per year, that the endowment in period one, denoted Q_1 is 1, and that the endowment in period 2, denoted Q_2 , is equal to 1.1. Finally assume that the initial asset position, B_0^* , is zero.

1. State the household's budget constraints in periods 1 and 2.
2. Characterize the equilibrium allocation under free capital mobility. At this point do not use numerical values. Express the equilibrium levels of consumption in terms of the exogenous variables, Q_1 , Q_2 , r^* and the parameter δ .
3. Assume now that $\delta = 1$. Find the equilibrium values of consumption and the trade balance in periods 1 and 2.
4. Suppose that in period 1 the country experiences a persistent increase in output. Specifically, assume that output increases by 1 in period 1

and by $\rho \in (0, 1)$ in period 2. Continue to assume that $\delta = 1$, that is, that consumption is nondurable. Is the trade balance in period 1 countercyclical, that is, does the change in the trade balance have the opposite sign as the change in Q_1 ? Why or why not. Find the change in the trade balance in period 1 and provide intuition for your answer.

5. Continue to assume that the economy experiences a positive and persistent output shocks, that is, Q_1 increases by 1 and Q_2 increases by $\rho \in (0, 1)$. But now do not impose that $\delta = 1$. Find the pairs (δ, ρ) such that the response of the trade balance in period 1 is countercyclical, i.e., negative, and consumption purchases, C_1 and C_2 , are positive. Provide intuition for your answer.

Exercise 3.8 (Habit Formation and the Trade Balance) Consider a two-period, small open economy populated by a large number of identical households with preferences described by the utility function

$$\ln C_1 + \ln(C_2 - \alpha C_1),$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively, and $\alpha \in (0, 1)$ is a parameter measuring the degree of habit formation. This preference specification nests the standard case of no habits when α is zero. The reason why these preferences capture consumption habits is that current consumption influences the marginal utility of future consumption. Specifically, the marginal utility of period-2 consumption is given by $1/(C_2 - \alpha C_1)$, which is higher for $\alpha > 0$ than for $\alpha = 0$. Intuitively, the more the household

eats in period 1, the hungrier it will wake up in period 2.

Households are endowed with $Q > 0$ units of consumption goods each period and can borrow or lend at the world interest rate, r^* , which, for simplicity, we assume is equal to zero. Households start period 1 with no assets or debts from the past ($B_0^* = 0$).

1. Derive the household's intertemporal budget constraint.
2. Calculate the equilibrium levels of consumption and the trade balance in period 1 as functions of the structural parameters of the model, α and Q . Compare the answer to the one that would have obtained in the absence of habits and provide intuition.

Exercise 3.9 (External Habit Formation) The specification of habits considered in exercise 3.8 is known as internal habit formation, because the individual consumer internalizes the fact that his choice for current consumption, C_1 , affects his marginal utility of consumption in the next period. Now assume instead that the utility function is of the form

$$\ln C_1 + \ln(C_2 - \alpha \tilde{C}_1),$$

where \tilde{C}_1 denotes the cross sectional average level of consumption in period

1. This preference specification is known as external habits or catching up with the Joneses, because the individual consumer's happiness is affected by the consumption of others.

1. Calculate the equilibrium values of consumption and the trade balance in period 1 as functions of the structural parameters of the model, α

and Q . Hint: in taking first-order conditions, consider \tilde{C}_1 as a parameter. This is right, because the individual consumer regards the average level of consumption in the economy as out of his control. However, after you have derived the first-order conditions, you have to take into account that, because all households are identical, in equilibrium \tilde{C}_1 is equal to C_1 .

2. Compare the answers you obtained in question 1 with the ones obtained in the case of internal habits studied in exercise 3.8 and provide intuition.
3. Discuss to what extent the economy with external habits under- or oversaves relative to the economy with internal habits. To this end, compute and compare the current account in period 1 under internal and external habits. Provide meaning to the terms oversaving or undersaving, by computing the level of lifetime welfare under internal and external habits. Provide intuition for your findings.

Chapter 4

The Terms of Trade, the Interest Rate, and the Current Account

In chapter 3, we studied an economy with a single good. We had in mind, for example, an island with banana trees. Sometimes households choose to consume fewer bananas than the trees produce. In these periods, the country exports bananas. Sometimes households wish to consume more bananas than the trees produce, and therefore the country imports bananas. In the real world, however, not all goods that households consume are domestically produced. In general, the type of goods a country exports may be different from the type of goods a country imports. For instance, some countries in the Middle East are highly specialized in the production of oil. These countries export most of their oil production and import most of the goods

they consume (food, electronics, clothing, etc.). To capture this aspect of the real world, in this chapter we study a model of current account determination that allows for differences in the type of goods imported and exported. The relative price of exports in terms of imports is known as *the terms of trade*. We will pay special attention to how movements in the terms of trade affect the trade balance and the current account.

Another simplification of the open economy model of chapter 3 is that the world interest rate is constant. In reality, however, the interest rate prevailing in world financial markets, such as the U.S. Treasury Bill rate, moves over time and is a critical determinant of the current account and aggregate activity in open economies around the world. With this motivation in mind, in this chapter we study the adjustment of the current account in response to world interest rate shocks.

4.1 Terms-of-Trade Shocks

In the model of chapter 3, the type of goods households are endowed with are the same as the types of goods households like to consume. For example, households are endowed with wheat and also like to consume wheat. Let us now modify the model by assuming that the good households like to consume, say wheat, is different from the good they are endowed with, say oil. In such an economy, both C_1 and C_2 must be imported, while Q_1 and Q_2 must be exported. Let P_1^M and P_1^X denote the prices of imports and exports in period 1, respectively. A country's terms of trade in period 1, denoted TT_1 , is the ratio of the price of its exports to the price of its imports,

that is,

$$TT_1 \equiv \frac{P_1^X}{P_1^M}.$$

Continuing with the example, if the price of oil is 90 dollars per barrel and the price of wheat is 10 dollars per bushel, then $P_1^X = 90$, $P_1^M = 10$, and terms of trade is 9, or $TT_1 = 9$. Here, TT_1 represents the price of oil in terms of wheat and indicates the amount of wheat that the country can buy from the sale of one barrel of oil. Put differently, $TT_1 = 9$ means that with one barrel of oil one can buy 9 bushels of wheat. In general, we have that Q_1 units of endowment are worth $TT_1 Q_1$ units of consumption goods.

The household's budget constraint in period 1 is then given by

$$C_1 + B_1^* - B_0^* = r_0 B_0^* + TT_1 Q_1.$$

This formulation assumes that foreign assets are expressed in units of consumption goods. In terms of the example, in period 1 the bond costs one bushel of wheat and pays $1 + r_1$ bushels of wheat in period 2. Similarly, the budget constraint in period 2 is

$$C_2 + B_2^* - B_1^* = r_1 B_1^* + TT_2 Q_2.$$

The above two budget constraints are identical to the budget constraints of the one-good economy of chapter 3, given in (3.1) and (3.2), except for the fact that the terms of trade are multiplying the endowments.

Using the terminal condition $B_2^* = 0$, the above two period budget constraints can be combined to obtain the following intertemporal budget con-

straint:

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^* + TT_1Q_1 + \frac{TT_2Q_2}{1 + r_1}.$$

Comparing this intertemporal budget constraint with the one pertaining to the one-good model presented in chapter 3, equation (3.4), it is clear that terms-of-trade shocks are just like output shocks. For the household, it makes no difference whether its income in a given period changes because of a change in the terms of trade or because of a change in the endowment. What matters is the level of income in terms of the consumption good, TT_1Q_1 and TT_2Q_2 , but not the breakdown into price and quantity. Consequently, the adjustment to terms of trade shocks is identical to the adjustment to endowment shocks. In response to a transitory deterioration of the terms of trade, that is, a fall in TT_1 with TT_2 , Q_1 , and Q_2 unchanged, the economy will not adjust consumption much and instead will borrow on the international capital market, which will result in a trade balance deterioration and current account deficit. Similarly, in response to a permanent terms of trade decline (a fall in both TT_1 and TT_2 , with Q_1 and Q_2 unchanged), the country will adjust consumption in both periods down, with little change in the trade balance or the current account.

We conclude that the key prediction of the one-good economy also applies in the two-good economy: The country finances temporary changes in the terms of trade by increasing the current account in response to positive shocks and decreasing the current account in response to negative shocks. And the country adjusts to permanent terms-of-trade shocks by cutting consumption in response to negative shocks and increasing consumption in

response to positive ones, with little movement in the current account.

4.2 Terms-of-Trade Shocks, Imperfect Information, and the Current Account

The central prediction of the intertemporal theory of the current account is that consumption, the trade balance, and the current account react differently to temporary and permanent shocks. In reality, however, agents have imperfect information. When a shock hits the economy, it is not always easy to tell whether the shock is permanent or temporary. Agents must form expectations about the duration of the shock, which may or may not be validated by future developments. When expectations are not fulfilled, the behavior of the economy may ex-post look at odds with the predictions of the intertemporal theory of current account determination. The following example illustrates this point.

Consider an economy in which initially $TT_1 = TT_2 = TT$ and $Q_1 = Q_2 = Q$. Suppose now that in period 1 the terms of trade appreciate by Δ , where $\Delta > 0$, and that in period 2 they increase by 2Δ , that is, the terms of trade increase in period 1 and increase by even more in period 2. How does the current account adjust to this development? The answer depends on what people in period 1 expect TT_2 will be. If the expectation is that the terms-of-trade improvement in period 1 is temporary, that is, if they think that $TT_2 = TT$, then the current account in period 1 will improve, as agents will save some of the period-1 windfall for future consumption. However, if households correctly anticipate that the terms of trade will rise in period 2

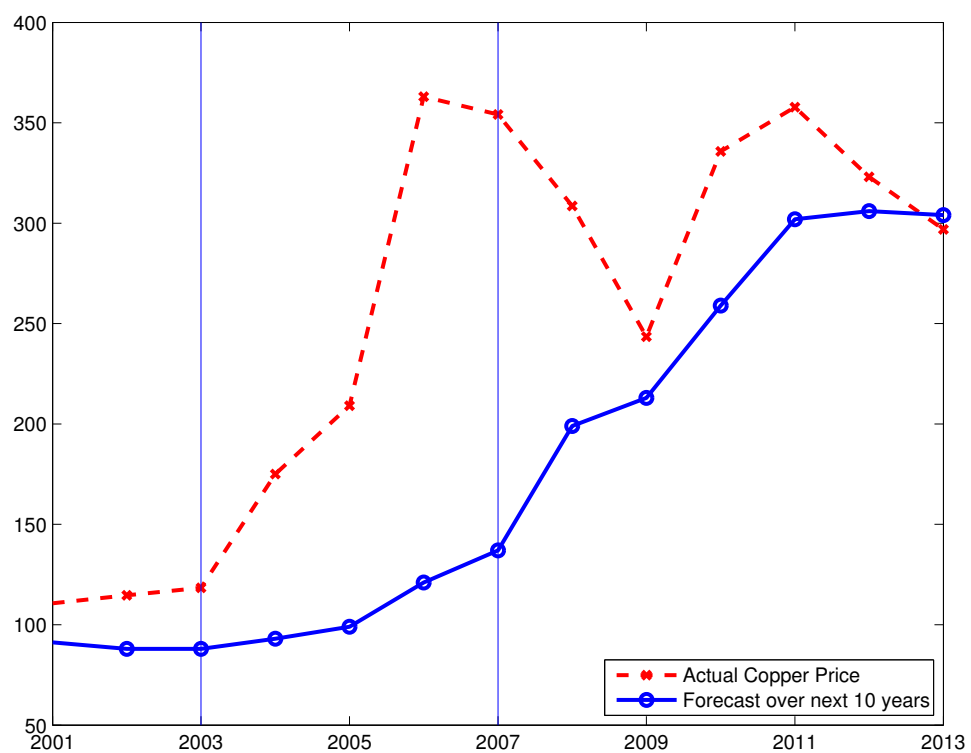
to $TT + 2\Delta$, then the current account in period 1 will deteriorate, as they will borrow against their higher expected future income.

The take away from this hypothetical example is that what matters for the determination of the current account is not the actual path of income, but the expected path of income. This point is important for analyzing actual historical episodes, as the following empirical example illustrates.

The empirical example has to do with the behavior of the Chilean current account and the price of copper in the 2000s.¹ Copper is the main export product of Chile, representing more than 50 percent of that country's exports. Consequently, the Chilean terms-of-trade are driven to a large extent by movements in the price of copper. After two decades of stable levels, the price of copper began to grow vigorously in the early 2000s. This development turned out to be long lasting. As shown in figure 4.1, in 2013, the price of copper (crossed broken line) was 150 percent higher than in 2003. If in 2003 Chilean households had had perfect foresight about the future path of the copper price, that is, if they had correctly anticipated that the price would continue to rise, then according to the model, the current account should have deteriorated. Households would have felt richer and increased their demand for consumption goods, causing the current account to deteriorate as a way to finance the expansion in aggregate demand. In turn, the current account deficit would have been paid for by the expected future increases in the price of copper. But this prediction did not materialize.

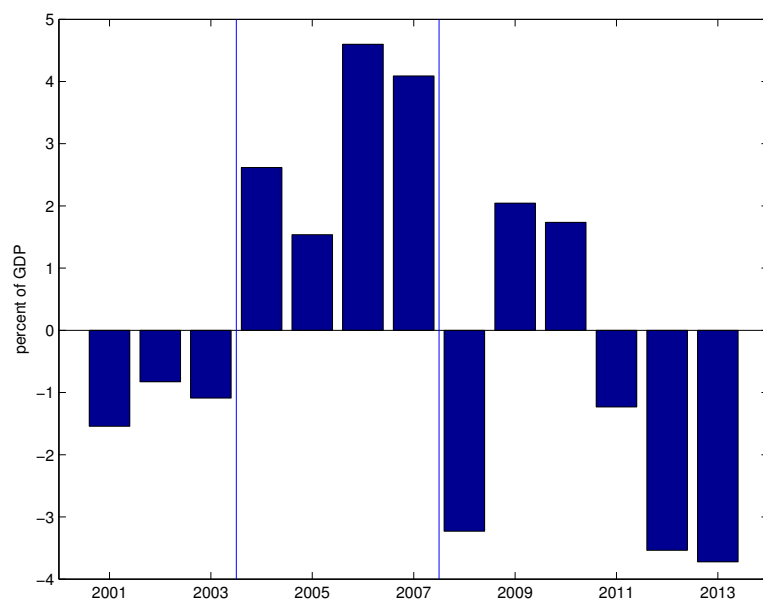
¹It is taken from Jorge Fornero and Markus Kirchner, "Learning About Commodity Cycles and Saving-Investment Dynamics in a Commodity-Exporting Economy," *International Journal of Central Banking* 14, March 2018, 205-262. We thank Markus Kirchner for sharing the data shown in figures 4.1 and 4.2.

Figure 4.1: Forecast versus Actual Real Price of Copper, Chile, 2001-2013



Notes. The crossed broken line shows the actual real price of copper. The circled solid line shows the forecast of the average real price of copper over the next ten years. The boom in the price of copper between 2003 and 2007 was expected to be temporary. The forecast of the average copper price over the next ten years rose but by only by a small fraction of the rise in the actual price. Data Source: Central Bank of Chile.

Figure 4.2: The Current Account, Chile, 2001-2013



Notes. The graph shows that between 2003 and 2007 the Chilean current account improved from a deficit of around 1 percent of GDP to a surplus of about 3 percent of GDP. This behavior of the current account is in line with view that the boom in the copper price that took place during this period was expected to be temporary. Data Source: Central Bank of Chile.

Instead of deteriorating, the current account actually improved. As can be seen from figure 4.2, the Chilean current account experienced a significant improvement between 2003 and 2007 from deficits of around one percent of GDP to surpluses of about three percent of GDP.

Concluding that the intertemporal theory of current account determination fails to explain this evidence, however, requires to assume that when the price of copper started to increase agents expected the improvement would last for a long period of time (as it did). Figure 4.1 shows that this assumption is misplaced. The figure displays the actual real price of copper (the crossed broken line) and the forecast of the average real price of copper over the next ten years produced by Chilean experts (the circled solid line). Despite the fact that the actual price of copper grew rapidly between 2003 and 2007, the experts did not expect the price to be much higher for the next ten years. Indeed, until 2007 they expected that over the coming ten years it would be only slightly higher than at the beginning of the decade. That is, experts expected the improvement in the price of copper to be temporary. Only in the second half of the 2000s did forecasters begin to raise their predictions for the average price of copper over the next ten years.

In light of these expectations, the behavior of the current account is no longer in conflict with the predictions of the theoretical model. For the theoretical model predicts that in response to an improvement in the terms of trade that is expected to be short-lived, the current account should improve, which is what indeed happened.

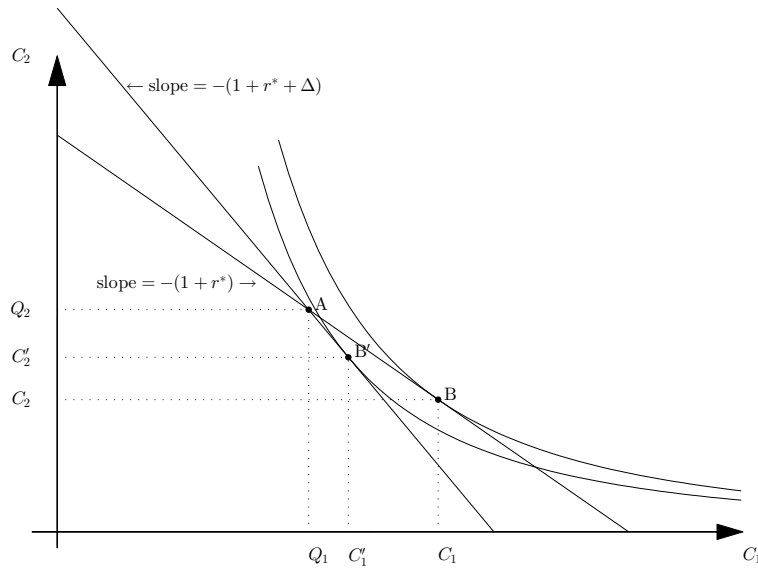
4.3 World Interest Rate Shocks

What happens in an open economy when the world interest rate changes? This question is important because sizable changes in the world interest rate occur frequently and are considered to be an important factor driving business cycles and the external accounts in economies that are open to trade in goods and financial assets.

An increase in the world interest rate, r^* , has two potentially opposing effects on consumption in period 1. On the one hand, an increase in the interest rate makes savings more attractive because the rate of return on foreign assets is higher. This effect is referred to as the *substitution effect*, because it induces people to substitute future for present consumption through saving. By the substitution effect, a rise in the interest rate causes consumption in period 1 to decline and therefore the current account to improve. On the other hand, an increase in the interest rate makes debtors poorer and creditors richer. This effect is called the *income effect*. By the income effect, an increase in the interest rate leads to a decrease in consumption in period 1 if the country is a debtor, reinforcing the substitution effect, and to an increase in consumption if the country is a creditor, offsetting (at least in part) the substitution effect. We will assume that the substitution effect is stronger than the income effect, so that savings increases in response to an increase in interest rates. Therefore, an increase in the world interest rate, r^* , induces a decline in C_1 and thus an improvement in the trade balance and the current account in period 1.

Figure 4.3 describes the case of an increase in the world interest rate

Figure 4.3: Adjustment to an increase in the world interest rate



Notes. Prior to the interest rate increase, the optimal consumption path is point B . Then the world interest rate increases from r^* to $r^* + \Delta$. This causes the intertemporal budget constraint to rotate clockwise around the endowment point A . The new optimal consumption path is point B' . The increase in the interest rate causes period-1 consumption to fall from C_1 to C'_1 and period-2 consumption to increase from C_2 to C'_2 . The figure is drawn under the assumption that $B_0^* = 0$.

from r^* to $r^* + \Delta$. We deduced in section 3.1 of chapter 3 that the slope of the intertemporal budget constraint is given by $-(1+r^*)$. Thus, an increase in the world interest rate makes the budget constraint steeper. The figure is drawn under the assumption that the household starts period zero with no debts or assets, $B_0^* = 0$. Thus, the endowment point, point A in the figure, lies on both the old and the new budget constraints. This means that as the world interest rate increases from r^* to $r^* + \Delta$ the budget constraint rotates clockwise around point A. The initial optimal consumption point is given by point B, where in period 1 consumption is larger than the endowment and therefore the household is borrowing. The new optimal consumption path is point B', which is located west of point B. The increase in the world interest rate is associated with a decline in C_1 and thus an improvement in the trade balance and the current account in period 1. Because the household was initially borrowing, the income and substitution effects triggered by the rise in the interest rate reinforce each other, so savings increase unambiguously.

Let us now consider the effect of an increase in the world interest rate in the economy with log preferences studied in section 3.9 of chapter 3. There, we deduced that in equilibrium the economy consumes half of its lifetime wealth,

$$C_1 = \frac{1}{2} \left[(1+r_0)B_0^* + Q_1 + \frac{Q_2}{1+r^*} \right]$$

and therefore the trade balance, $TB_1 = Q_1 - C_1$, and the current account, $CA_1 = r_0B_0^* + TB_1$, are given by

$$TB_1 = \frac{1}{2} \left[-(1+r_0)B_0^* + Q_1 - \frac{Q_2}{1+r^*} \right]$$

and

$$CA_1 = \frac{1}{2} \left[(-1 + r_0)B_0^* + Q_1 - \frac{Q_2}{1 + r^*} \right],$$

respectively. Clearly, in response to an increase in r^* , in period 1 consumption falls and both the trade balance and the current account improve. Note that the decline in consumption in period 1 is independent of whether prior to the increase in r^* the country is a net foreign borrower or a net foreign lender in period 1. This is because for the particular preference specification considered in this example (log-linear preferences), the substitution effect always dominates the income effect.

4.4 Summing Up

This chapter analyzes the effects of terms-of-trade shocks and interest-rate shocks in the context of the intertemporal model of the current account developed in chapter 3.

- The terms of trade is the relative price of export goods in terms of import goods.
- Terms-of-trade shocks have the same effects as endowment shocks: the economy uses the current account to smooth consumption over time in response to temporary terms of trade shocks, and adjusts consumption with little movement in the current account in response to permanent terms-of-trade shocks.
- Interest rate shocks have a substitution and an income effect.
- By the substitution effect, an increase in the interest rate discourages current consumption and incentivizes savings causing the current account

and the trade balance to improve.

- The income effect associated with an increase in the interest rate depends on whether households are borrowing or lending. If households are borrowing, an increase in the interest rate has a negative income effect, as it makes borrowers poorer. As a result, consumption falls and the trade balance and the current account improve. In this case, the income and substitution effect go in the same direction. If households are lending, the income effect associated with an increase in the interest rate is positive and leads to higher consumption and a deterioration in the trade balance and the current account. In this case, the income and substitution effects go in the opposite direction, partially offsetting each other. Under log-preferences the substitution effect dominates.

4.5 Exercises

Exercise 4.1 (The Terms of Trade and the Current Account) Consider the following chart showing commodity prices in world markets:

Price		
Commodity	Period 1	Period2
Wheat	1	1
Oil	1	2

In the table, prices of oil are expressed in dollars per barrel and prices of wheat are expressed in dollars per bushel. Kuwait is a two-period economy that produces oil and consumes wheat. Consumers have preferences described by the lifetime utility function

$$C_1 C_2,$$

where C_1 and C_2 denote, respectively, consumption of wheat in periods 1 and 2, measured in bushels. Kuwait's per-capita endowments of oil are 5 barrels in each period. The country starts period 1 with net financial assets carried over from period 0, including interest of 10 percent, worth 1.1 bushels of wheat (i.e., $(1 + r_0)B_0^* = 1.1$). The country enjoys free capital mobility and the world interest rate is 10 percent. Financial assets are denominated in units of wheat.

1. What are the terms of trade faced by Kuwait in periods 1 and 2?
2. Calculate consumption, the trade balance, the current account and savings in periods 1 and 2.

3. Answer the previous question assuming that the price of oil in the second period is not 2 but 1 dollar per barrel. Provide intuition.

Exercise 4.2 (Anticipated Terms-of-Trade Shock) Consider a two-period small open endowment economy populated by a large number of households with preferences given by the lifetime utility function

$$\sqrt{C_1 C_2},$$

where C_1 and C_2 denote consumption of food in periods 1 and 2, respectively. Suppose that households receive exogenous endowments of copper given by $Q_1 = Q_2 = 10$ in periods 1 and 2, respectively. The terms of trade in periods 1 and 2 are $TT_1 = TT_2 = 1$. Every household enters period 1 with no assets or liabilities inherited from the past, $B_0^* = 0$. Finally, suppose that the country enjoys free capital mobility and that the world interest rate on assets held between periods 1 and 2, denoted r^* , is 5 percent.

1. Compute the equilibrium levels of consumption, the trade balance, and the current account in periods 1 and 2.
2. Assume now that the terms of trade in period 2 are expected to increase by 50 percent. Calculate the effect of this anticipated terms of trade improvement on consumption, the trade balance, and the current account in periods 1 and 2. Provide intuition.
3. Relate your findings to those discussed in the case study of the copper price appreciation experience by Chile in the early 2000s presented in section 4.2. In particular, explain why the results obtained in items

1 and 3 make the behavior of the Chilean current account in the period 2003-2007 puzzling under the naive view that in the early 2000s Chileans had perfect foresight about the future path of the price of copper.

Exercise 4.3 (Oil Discovery and Extraction Costs) Consider an island populated by households with preferences given by

$$-C_1^{-1} - C_2^{-1},$$

where C_1 denotes consumption of food when young and C_2 denotes consumption of food when old. Households are endowed with 8 units of food when young and 5 units when old. Households are born with assets worth 2 units of food ($(1 + r_0)B_0^* = 2$), where $r_0 = 5\%$ denotes the interest rate in period 0 and B_0^* denotes the inherited stock of bonds. Suppose that households have access to the international financial market, where the interest rate, denoted r^* is 10 percent.

1. Calculate the equilibrium levels of consumption, the trade balance, and the current account when young and when old.
2. Now assume that when young, each household finds 20 barrels of oil reserves in its backyard. Households will be able to sell their oil in the international market only when they are old. The price of a barrel of oil in terms of units of food is expected to be 0.2 in period 2. Calculate the equilibrium levels of consumption, the trade balance, and the current account when young and old.

3. Now assume that extracting the oil requires investment in period 1 equal in value to 3.5 units of food. The investment must be made when young in order for the oil to be available for sale when old. Does it pay to make the investment? Calculate consumption and the trade balance when young.
4. Now assume that households revise their expectations and now believe that the price of oil will be 0.1 when old. Is it still profitable to extract the oil? Show your work.

Exercise 4.4 (Unfulfilled Expectations) Consider a two-period small open economy populated by households whose preferences are given by

$$\ln C_1 + \ln C_2,$$

where C_1 and C_2 denote consumption of food in periods 1 and 2, respectively. Households are endowed with 1 ton of copper in each period and start period 1 with a zero net asset position. The relative price of copper in terms of food is 1 in both periods, and the world interest rate is zero.

1. What is consumption and the trade balance in periods 1 and 2?
2. Suppose now that in period 1 the relative price of copper continues to be 1, but that the expected relative price of copper in period 2 increases to 1.5. Calculate consumption and the trade balance in both periods.
3. Finally, continue to assume that in period 1 the relative price of copper

is 1 and households are 100 percent sure that the relative price of copper in period 2 is going to be 1.5. However, assume that when period 2 arrives, expectations are not fulfilled, and the price remains at 1. What is consumption and the trade balance in periods 1 and 2? Provide intuition.

Exercise 4.5 (A World Interest Rate Shock) Consider an individual who lives for two periods, denoted 1 and 2. Her preferences for consumption in each period are described by the lifetime utility function

$$-C_1^{-1} - C_2^{-1},$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively.

1. Suppose that the consumer starts period 1 with financial wealth equal to $(1 + r_0)B_0^*$, where B_0^* is an inherited stock of bonds and r_0 is the interest rate on bonds held between periods 0 and 1. Suppose further, that the individual receives endowments of goods in the amounts Q_1 and Q_2 in periods 1 and 2, respectively. In period 1, the individual can borrow or lend at the interest rate $r_1 > 0$ via a bond denoted B_1^* . Find the optimal levels of consumption in periods 1 and 2 as functions of the individual's lifetime wealth, $\bar{Y} \equiv (1 + r_0)B_0^* + Q_1 + Q_2/(1 + r_1)$ and the interest rate r_1 . Find the responses of consumption in period 1, ΔC_1 , the trade balance in period 1, ΔTB_1 , and the current account in period 1, ΔCA_1 to an increase in the world interest rate equal to $\Delta r^* > 0$.

Chapter 5

Current Account Determination in a Production Economy

Thus far, we have studied open economy models without investment, so that the current account, which is given by the difference between savings and investment, was simply determined by savings. Investment, which consists of spending in goods such as machines, new structures, equipment, and inventories, however, is an important component of aggregate demand amounting to around 20 percent of GDP in most countries. Further, investment is the most volatile component of aggregate demand, and, as such, it is important for understanding movements in the current account. In this chapter, we extend our theory by studying the determination of the current account in a production economy with investment in physical capital. In

this environment, factors affecting the firm's investment decision will have a direct effect on the current account even if savings were unchanged. In equilibrium, factors that affect investment will in general also affect households' savings decisions through, for example, income effects, and thus will indirectly also affect the current account.

5.1 The Investment Decision of Firms

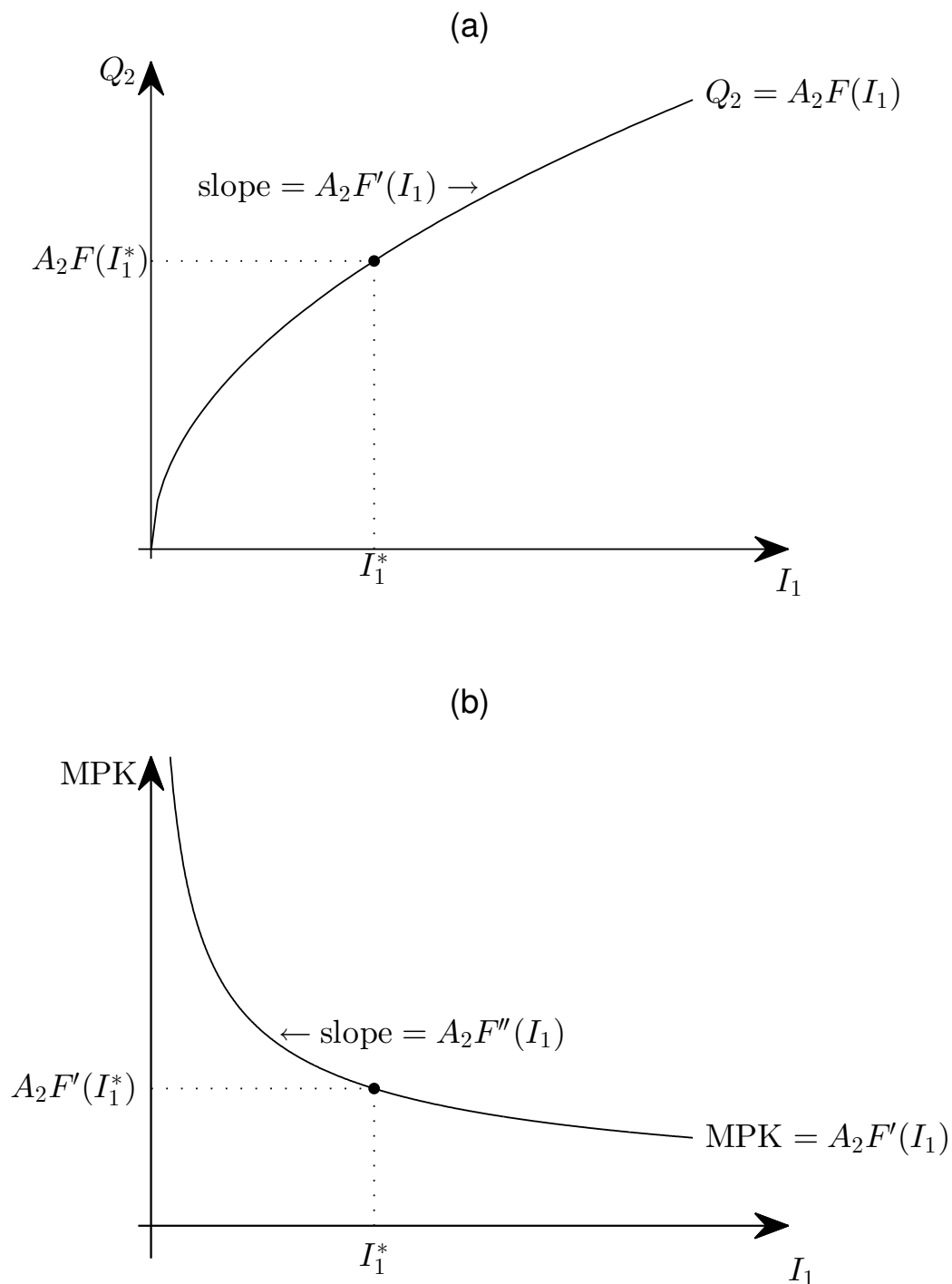
Consider an economy populated by a large number of firms and households. As before, the economy lasts for two periods, denoted period 1 and period 2. In this economy, however, output is no longer an endowment but is produced by firms.

Suppose that in period 1 firms invest in physical capital, which they use in period 2 to produce goods. Specifically, output in period 2, denoted Q_2 , is produced according to the *production function*

$$Q_2 = A_2 F(I_1),$$

where $A_2 > 0$ is an efficiency parameter capturing factors such as the state of *technology*, $F(I_1)$ is a function, and I_1 denotes *investment in physical capital* in period 1, which becomes productive in period 2. The production function $A_2 F(I_1)$ describes a technological relation specifying the amount of output obtained for each level of capital input. Panel (a) of figure 5.1 plots the production function. We impose a number of properties on the production technology. First, we assume that output is zero when investment is zero,

Figure 5.1: The production function and the marginal product of capital



Notes. Panel (a) displays the production function. It depicts output in period 2, Q_2 , as an increasing and concave function of capital invested in period 1, I_1 , with a zero intercept. Panel (b) displays the marginal product of capital, MPK , as a positive and decreasing function of I_1 . Panels (a) and (b) are related by the fact that at any given level of capital, say I_1^* , the slope of the production function equals the level of the marginal product of capital.

$A_2F(0) = 0$. Second, we assume that output is increasing in the amount of physical capital, $A_2F'(I_1) > 0$, where $F'(I_1)$ denotes the derivative of $F(I_1)$. Another way of stating this assumption is to say that the *marginal product of capital* (or MPK) is positive. The marginal product of capital is the amount by which output increases when the capital stock is increased by one unit,

$$\text{MPK} = A_2F'(I_1).$$

Finally, we assume that the production function is concave in capital, $A_2F''(I_1) < 0$, where $F''(I_1)$ denotes the second derivative of $F(I_1)$. When the production function is concave, output increases with capital at a decreasing rate. It means, for example, that increasing the number of tractors from 1 to 2 in a one-hundred acre farm yields more additional output than increasing the number of tractors from 20 to 21. This property of the production function is known as *diminishing marginal product of capital*.

Panel (b) of figure 5.1 displays the marginal product of capital as a function of the level of capital. Panels (a) and (b) are related by the fact that the slope of the production function at a given level of investment equals the level of the marginal product of capital at the same level of investment. For example, in the figure, the slope of the production function when the capital stock equals I_1^* equals the level of the marginal product of capital when the capital stock also equals I_1^* . The marginal product of capital schedule is downward sloping, reflecting the assumption of diminishing marginal product.

As an example, consider the production function

$$Q_2 = \sqrt{I_1}.$$

In this case, $A_2 = 1$ and $F(I_1)$ is the square root function. According to this technology, output is nil when the capital stock, I_1 , is zero, and is increasing in the stock of capital. The marginal product of capital is given by

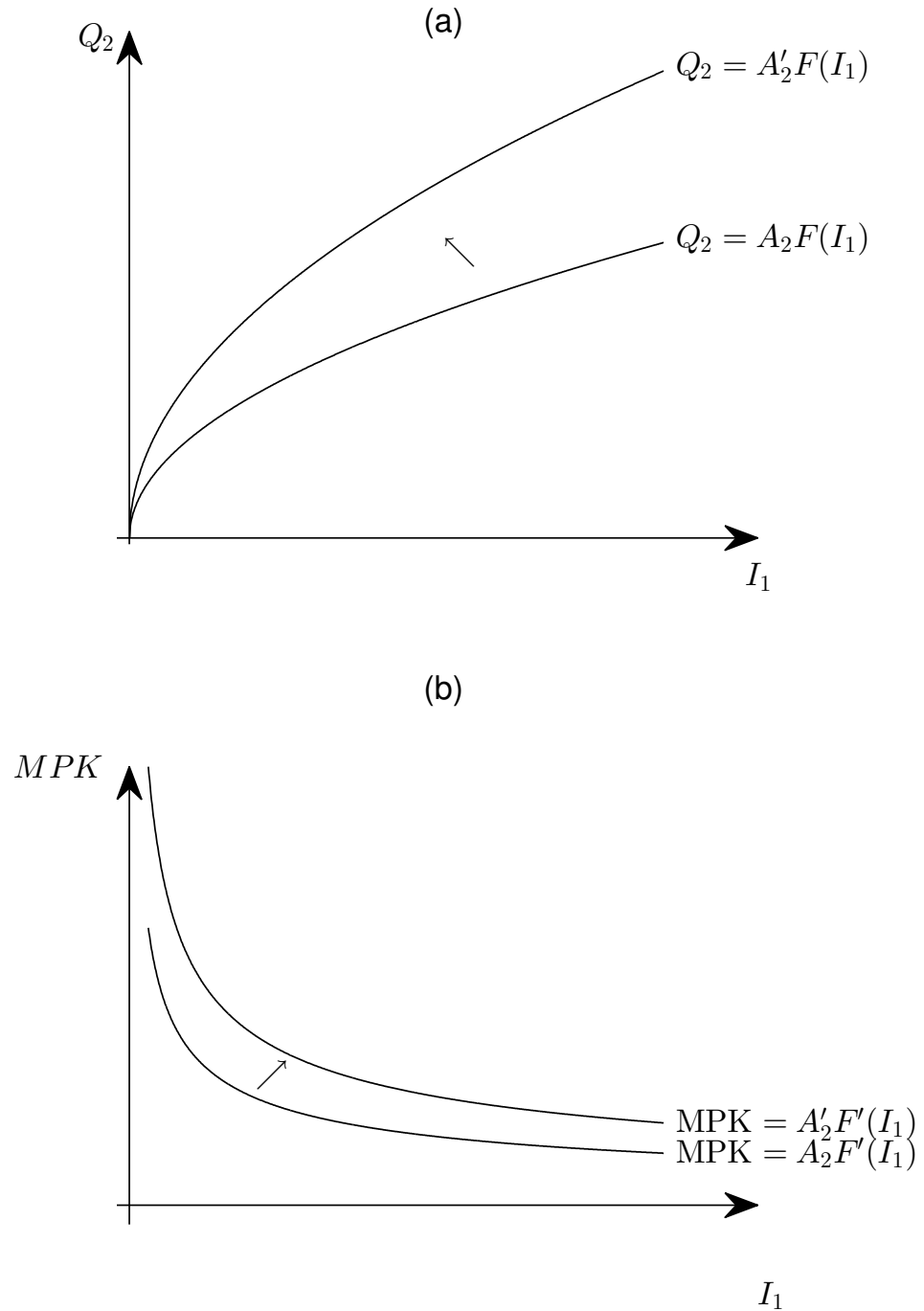
$$\text{MPK} = \frac{1}{2\sqrt{I_1}},$$

which is decreasing in the level of capital.

Consider now the effect of a productivity improvement on the production function and the MPK schedule. Suppose specifically that the efficiency parameter increases from A_2 to $A'_2 > A_2$. After the productivity improvement, the firm can produce more output at every level of capital. Figure 5.2 displays the effect of this productivity shock on the production function and the marginal product of capital. Panel (a) shows that the production function shifts upward, rotating counterclockwise around the origin. This means that the production function becomes steeper for a given level of investment. In other words, the technological improvement causes the marginal product of capital to be higher for a given level of investment. This effect is shown in panel (b) as a shift up and to the right of the marginal product of capital schedule.

Changes in productivity can be permanent or temporary. Permanent productivity changes typically stem from technological improvements. For

Figure 5.2: Effect of an increase in productivity on the production function and the marginal product of capital schedule



Note. A positive productivity shock that increases the level of technology from A_2 to $A'_2 > A_2$ rotates the production function counterclockwise around the origin and shifts the marginal product of capital schedule up and to the right.

example, the introduction of the assembly line by Henry Ford in 1913 reduced the time to build a car from 12 hours to two and a half hours. In the U.S. farming sector, over the past fifty years, the average cow increased its milk production from five thousand pounds to eighteen thousand pounds, due to improvements in breeding, health care, and feeding techniques. An example of temporary productivity changes is weather conditions in the farming sector, which can alter yields from one year to the next.

Thus far we have discussed purely technological relationships involving no economic choices on the part of the firm. We now turn to the optimal investment and production decisions of firms. We assume that firms borrow in period 1 to finance purchases of investment goods, such as new machines and structures. Let D_1^f denote the amount of debt assumed by the firm in period 1. We then have that

$$D_1^f = I_1. \quad (5.1)$$

Firms repay the loan in period 2. Let the interest rate on debt held from period 1 to period 2 be r_1 . The total repayment in period 2, including interest, is then given by $(1 + r_1)D_1^f$. The firm's profits in period 2, denoted Π_2 , are given by the difference between revenues from the sale of output and repayment of the investment loan, that is,

$$\Pi_2 = A_2 F(I_1) - (1 + r_1)D_1^f. \quad (5.2)$$

Using equation (5.1) to eliminate debt from this expression, we can express

period-2 profits simply in terms of investment and the interest rate

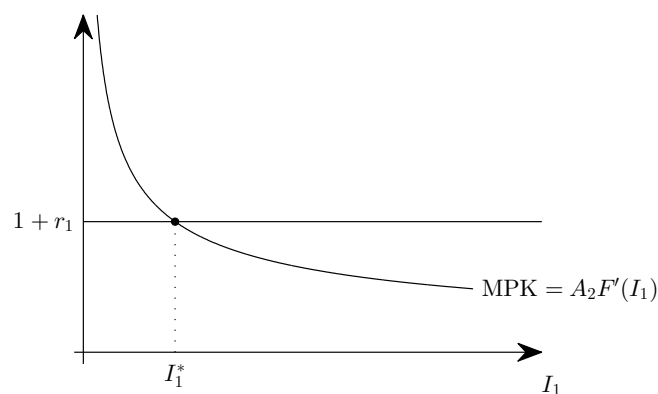
$$\Pi_2 = A_2 F(I_1) - (1 + r_1)I_1. \quad (5.3)$$

Firms choose I_1 to maximize profits, taking as given the interest rate r_1 and the productivity factor A_2 . The first-order condition associated with this profit maximization problem is the derivative of the right-hand side of (5.3) with respect to I_1 equated to zero. Performing this operation yields, after a slight rearrangement,

$$A_2 F'(I_1) = 1 + r_1. \quad (5.4)$$

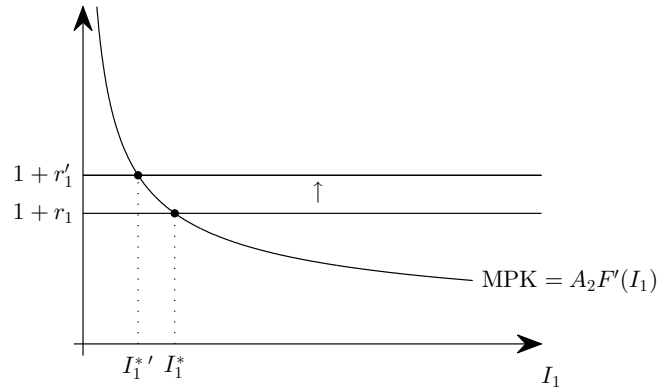
This optimality condition is quite intuitive. Figure 5.3 displays the left- and right-hand sides of (5.3) as functions of I_1 . The left-hand side is the marginal product of capital, $A_2 F'(I_1)$, which, as discussed before, is a decreasing function of the level of capital, I_1 . The right-hand side is the marginal cost of capital, $1 + r_1$, and is a horizontal line because it is independent of the level of investment. Each additional unit of capital costs the firm $1 + r_1$ in period 2. This is because for each unit of capital used in period 2, the firm must take a loan in the amount of one unit in period 1 and must pay back this loan plus interest, $1 + r_1$, in period 2. For low values of capital investment is highly productive so that the marginal product of capital, $A_2 F'(I_1)$, exceeds the marginal cost of capital, $1 + r_1$. In this case, the firm can increase profits by buying an additional unit of capital in period 1. The firm will continue to buy additional units of capital as long as the marginal

Figure 5.3: The optimal level of investment



Notes. The downward sloping line is the marginal product of capital schedule. The horizontal line depicts the marginal cost of capital schedule, which is equal to the gross interest rate, $1 + r_1$, for any level of investment. Firms invest up to the point where the marginal product of capital equals the marginal cost of capital, I_1^* . Profits are given by the triangle below the marginal product of capital schedule and above the marginal cost of capital.

Figure 5.4: Effect of an increase in the interest rate on investment



Notes. An increase in the interest rate from r_1 to $r'_1 > r_1$ shifts the marginal cost schedule up. As a result, the optimal level of investment falls from I_1^* to $I_1^{*'}$.

product exceeds the marginal cost. As investment increases, however, the marginal productivity of capital diminishes. For sufficiently large levels of investment, the marginal product of capital falls below the marginal cost of capital. In this range, that is, for $I_1 > I_1^*$ in the figure, an additional unit of capital reduces profits, as the amount of additional output it generates, $A_2F'(I_1)$, is less than its cost, $(1 + r_1)$. Consequently, the firm can increase profits by reducing I_1 . The optimal level of investment is reached when the marginal product of investment equals its marginal cost, that is, when equation (5.4) holds. At the optimal level of investment, that is, at I_1^* in the figure, profits of the firm are given by the area below the marginal product of capital curve and above the marginal cost of capital curve.

Figure 5.4 illustrates the effect on investment of an increase in the interest rate from r_1 to $r'_1 > r_1$. When the interest rate is r_1 , the optimal level of

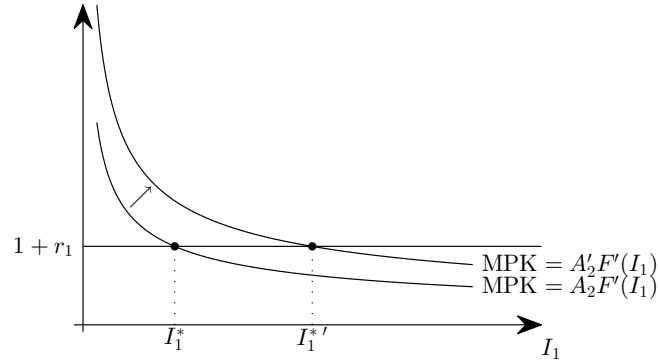
investment is I_1^* . As the interest rate increases, the horizontal marginal cost function shifts upward. At I_1^* , the marginal product of capital is now below the marginal cost of capital, $1 + r_1'$, so it pays for the firm to reduce investment. The firm will cut investment up to the point at which the marginal product of capital meets the higher cost of capital, $1 + r_1'$, which occurs at $I_1^{*'}$. It follows that investment is a decreasing function of the interest rate.

Profits also fall as the interest rate increases. This can be seen graphically from the fact that the triangular area below the marginal product of capital schedule and above the marginal cost of capital schedule gets smaller as the latter shifts up. Thus, all else constant, profits are a decreasing function of the interest rate.

Consider next the effect of a productivity shock. Assume that the efficiency factor of the production function increases from A_2 to $A_2' > A_2$. A positive productivity shock causes the marginal product of capital schedule to shift up and to the right, as shown in figure 5.5. Because capital is now more productive, all of the investment projects that were profitable before become even more profitable. In addition, investment opportunities that were not profitable before the increase in productivity now became profitable. As a result, it is optimal for firms to increase investment. In the figure, investment increases from I_1^* to $I_1^{*'}$ as the level of technology increases from A_2 to A_2' . It follows that, all other things equal, investment is an increasing function of the technology factor A_2 .

Period-2 profits increase by an amount equal to the expansion of the triangular area below the new marginal product of capital schedule and above the marginal cost of capital schedule. Thus, profits, like investment,

Figure 5.5: Effect of an increase in productivity on investment



Notes. A positive productivity shock shifts the MPK schedule up and to the right. The firm expands investment until the new MPK schedule meets the marginal cost of capital schedule. Investment increases from I_1^* to $I_1^{*'}$. Profits increase, because the area below the MPK schedule and above the marginal cost schedule gets larger.

are an increasing function of the technology factor A_2 . Combining this result with the one discussed earlier that profits are a decreasing function of the interest rate, we have that

$$\Pi_2 = \Pi_2 \underset{-}{r_1}, \underset{+}{A_2},$$

where the function $\Pi_2(\cdot, \cdot)$ is decreasing in its first argument, the interest rate, and increasing in its second argument, the productivity factor.

You might have noticed that we have concentrated on the profit maximization problem of the firm in period 2, and have said nothing about profits in period 1. The reason for this omission is that profits in period 1 are determined by investment decisions made before period 1. As a result, there is nothing the firm can do in period 1 to alter its profitability. Specifically,

profits in period 1, denoted Π_1 , are given by

$$\Pi_1 = A_1 F(I_0) - (1 + r_0) D_0^f, \quad (5.5)$$

with

$$D_0^f = I_0. \quad (5.6)$$

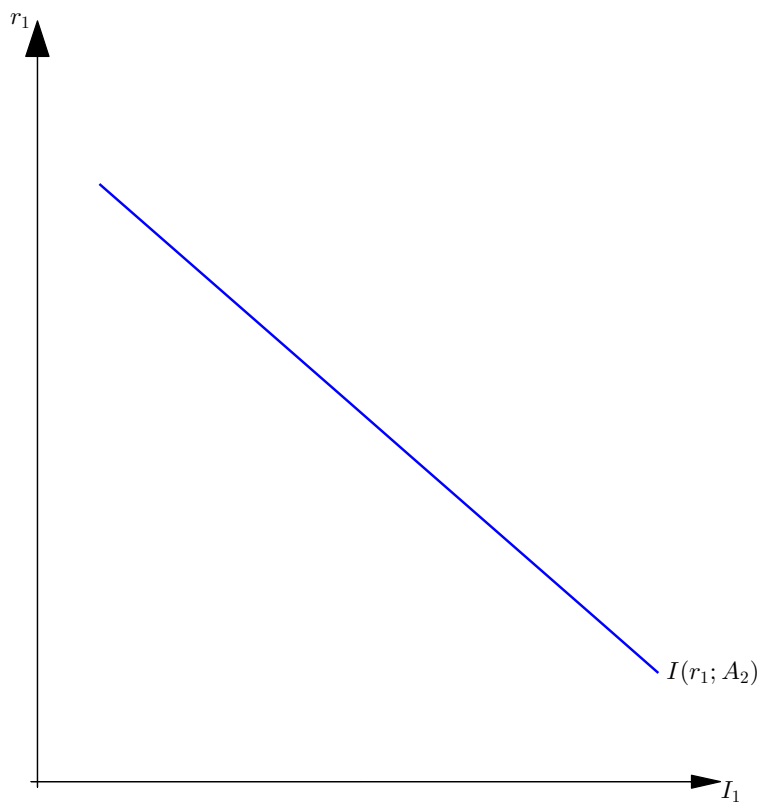
The variables I_0 , D_0^f , and r_0 are all predetermined in period 1 and are therefore taken as exogenous by the firm in period 1. The level of productivity, A_1 , is indeed determined in period 1, but is out of the control of the firm. As a result, period-1 profits are determined in period 1, but are not affected by any decision made by the firm in that period. Higher interest rates, r_0 , lower profits as firms have to make higher interest payments to its creditors and a higher productivity factor in period 1, A_1 , raises profits as it increases the amount of output that the firm can produce with the predetermined level of investment, I_0 . Thus, we can again express profits as a decreasing function of the interest rate and an increasing function of the period-1 productivity factor:

$$\Pi_1 = \Pi_1(\underset{-}{r_0}, \underset{+}{A_1}).$$

5.2 The Investment Schedule

Aggregate investment is the sum of the investment decisions of individual firms. If we assume that all firms have the same technology and face the same interest rate, we have that all firms will make the same investment decisions. As a result, total investment in the economy will behave just like

Figure 5.6: The investment schedule



Notes. The investment schedule relates the aggregate level of investment and the interest rate, given the productivity factor A_2 . The investment schedule slopes downward because the profit maximizing level of investment is decreasing in the marginal cost of capital.

investment at the firm level. We then have that aggregate investment is a function of the interest rate and the productivity factor and write

$$I_1 = I(r_1; A_2), \quad (5.7)$$

where now I_1 denotes aggregate investment in period 1. We will refer to this function as the *investment schedule*. It is decreasing in the interest rate and increasing in the level of productivity.

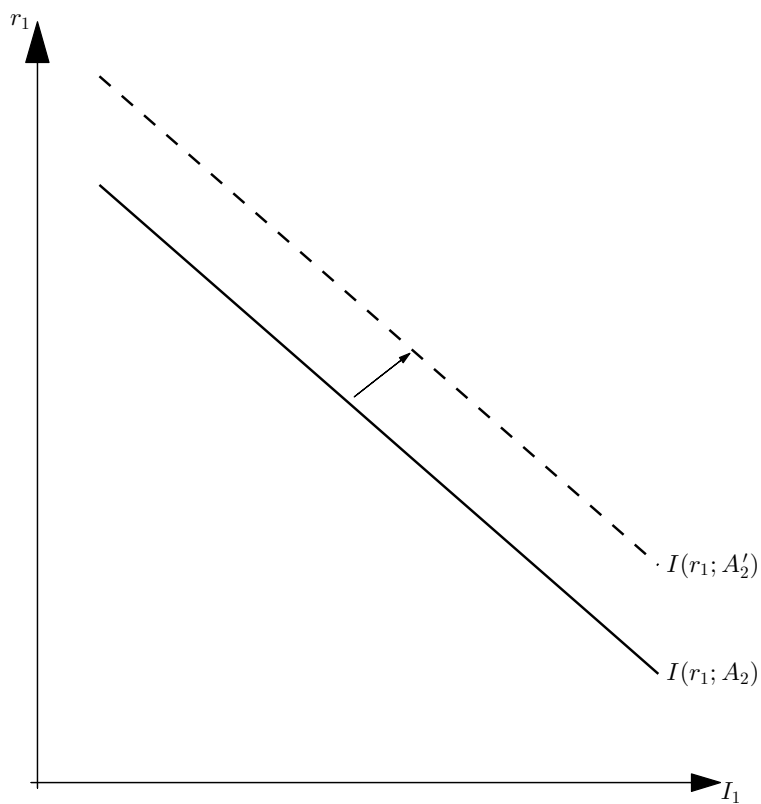
Figure 5.6 depicts the investment schedule in the space (I_1, r_1) for a given level of A_2 . As the interest rate in period 1 increases, all other things equal, aggregate investment falls.

Suppose now that a technological improvement causes the efficiency parameter of the production function to increase from A_2 to A'_2 . As we discussed earlier, firms now have an incentive to increase investment for every level of the interest rate. This means that the investment schedule shifts up and to the right in response to the positive productivity shock, as shown in figure 5.7.

5.3 The Consumption Decision of Households

Households in the present economy are quite similar to those in the endowment economy studied in chapter 3. The only difference is that now households are the owners of firms. Consequently, instead of receiving an endowment each period, households receive profit payments from firms, $\Pi_1(r_0, A_1)$ in period 1 and $\Pi_2(r_1, A_2)$ in period 2. We write profits as a function of the

Figure 5.7: The effect of an increase in productivity on the investment schedule



Notes. The figure depicts the effect of an increase in the productivity parameter from A_2 to A'_2 . The positive productivity shock shifts the investment schedule up and to the right because for every level of the interest rate, the profit-maximizing level of investment is now higher.

interest rate and the efficiency factor as a reminder that they are decreasing in the former and increasing in the latter.

At the beginning of period 1, the household is endowed with B_0^h units of bonds, which yield the interest income $r_0 B_0^h$, where r_0 is the interest rate on bonds held between periods 0 and 1. We use the superscript h to refer to bonds held by households and will continue to reserve the superscript $*$ to refer to the country's net foreign asset position. In previous chapters, the only agent in the economy holding liability or asset positions was the household. Now firms also participate in the financial market. As a result, the net asset position of households may be different from that of the country as a whole.

Total household income in period 1 equals $\Pi_1(r_0, A_1) + r_0 B_0^h$. The household uses its income for consumption and additions to its stock of bonds. The budget constraint of the household in period 1 is then given by

$$C_1 + B_1^h - B_0^h = \Pi_1(r_0, A_1) + r_0 B_0^h. \quad (5.8)$$

Similarly, the household's budget constraint in period 2 takes the form

$$C_2 + B_2^h - B_1^h = \Pi_2(r_1, A_2) + r_1 B_1^h, \quad (5.9)$$

where B_2^h denotes the stock of bonds the household holds at the end of period 2. Because period 2 is the last period of life, the household will not want to hold any positive amount of assets maturing after that period. Consequently, the household will always find it optimal to choose $B_2^h \leq 0$. At the same

time, the household is not allowed to end period 2 with unpaid debts (the no-Ponzi-game condition), so that $B_2^h \geq 0$. Therefore, household's financial wealth at the end of period 2 must be equal to zero

$$B_2^h = 0.$$

Using this terminal condition, the budget constraint (5.9) becomes

$$C_2 = (1 + r_1)B_1^h + \Pi_2(r_1, A_2). \quad (5.10)$$

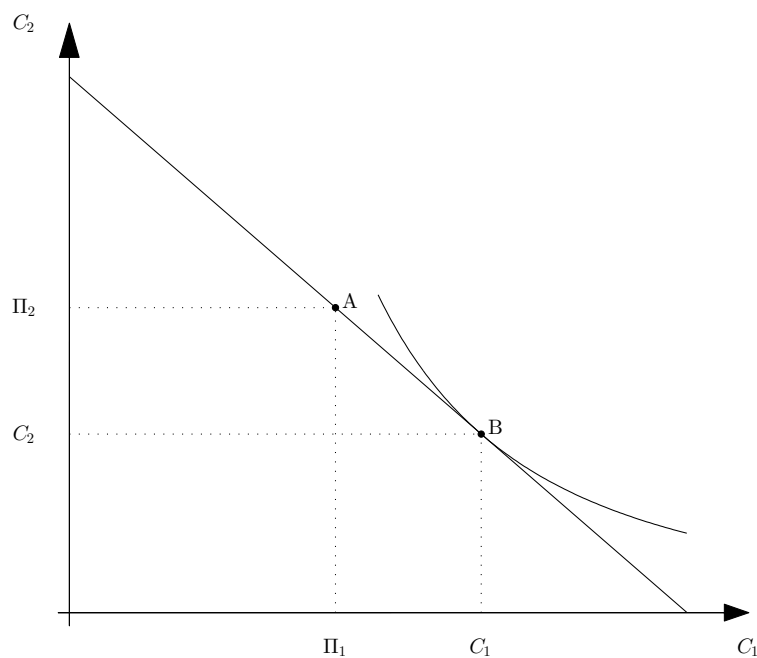
Combining (5.8) and (5.10) to eliminate B_1^h yields the household's intertemporal budget constraint,

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^h + \Pi_1(r_0, A_1) + \frac{\Pi_2(r_1, A_2)}{1 + r_1}. \quad (5.11)$$

This expression is similar to the intertemporal budget constraint corresponding to the endowment economy of chapter 3, equation (3.4), with the only difference that the present discounted value of endowments, $Q_1 + Q_2/(1 + r_1)$, is replaced by the present discounted value of profits $\Pi_1(r_0, A_1) + \Pi_2(r_1, A_2)/(1 + r_1)$. In particular, the intertemporal budget constraint continues to be a downward sloping straight line with slope equal to $-(1 + r_1)$.

Figure 5.8 depicts the household's intertemporal budget constraint in the space (C_1, C_2) . The figure is drawn under the assumption that the household starts period 1 with no assets or liabilities ($B_0^h = 0$). As a result, the path of profits $(\Pi_1(r_0, A_1), \Pi_2(r_1, A_2))$ is on the intertemporal budget constraint. Note that the household takes the entire right-hand side of the intertemporal

Figure 5.8: The optimal intertemporal consumption choice in the production economy



Notes. The figure depicts the optimal choice of consumption in periods 1 and 2. The intertemporal budget constraint is the straight downward sloping line. It crosses the profit path (Π_1, Π_2) at point A and has a slope equal to $-(1+r_1)$. The optimal consumption path (C_1, C_2) is at point B, where an indifference curve is tangent to the intertemporal budget constraint. The figure is drawn under the assumption that the household starts period 1 with a zero net asset position, $B_0^h = 0$.

budget constraint (5.11) as given, since it has no control over the path of profits, $(\Pi_1(r_0, A_1), \Pi_2(r_1, A_2))$, its initial wealth, $(1 + r_0)B_0^h$, or the interest rate r_1 . Thus, we can define $\bar{Y} = (1 + r_0)B_0^h + \Pi_1(r_0, A_1) + \frac{\Pi_2(r_1, A_2)}{1 + r_1}$ to be the household's lifetime wealth, just as we did in the endowment economy of chapter 3. The intertemporal budget constraint can then be written exactly like equation (3.7), which we reproduce here for convenience,

$$C_2 = (1 + r_1)(\bar{Y} - C_1). \quad (\text{R3.7})$$

Households are assumed to derive utility from consumption in periods 1 and 2. Their preferences are described by the utility function given in equation (3.5) of chapter 3, which we also reproduce here,

$$U(C_1) + \beta U(C_2). \quad (\text{R3.5})$$

The household chooses C_1 and C_2 to maximize its utility function subject to the intertemporal budget constraint (R3.7) taking as given \bar{Y} and r_1 . The household optimization problem is then identical to that of the endowment economy.

Figure 5.8 depicts the household's optimal intertemporal consumption choice. The optimal consumption path is at point B on the intertemporal budget constraint. At point B the intertemporal budget constraint is tangent to an indifference curve. This means that, as before, at the optimal consumption path the slope of the indifference curve is equal to $-(1 + r_1)$. Recalling that the slope of the indifference curve is equal to the marginal

rate of substitution, we have that the optimal consumption path must satisfy the Euler equation

$$\frac{U'(C_1)}{\beta U'(C_2)} = 1 + r_1. \quad (5.12)$$

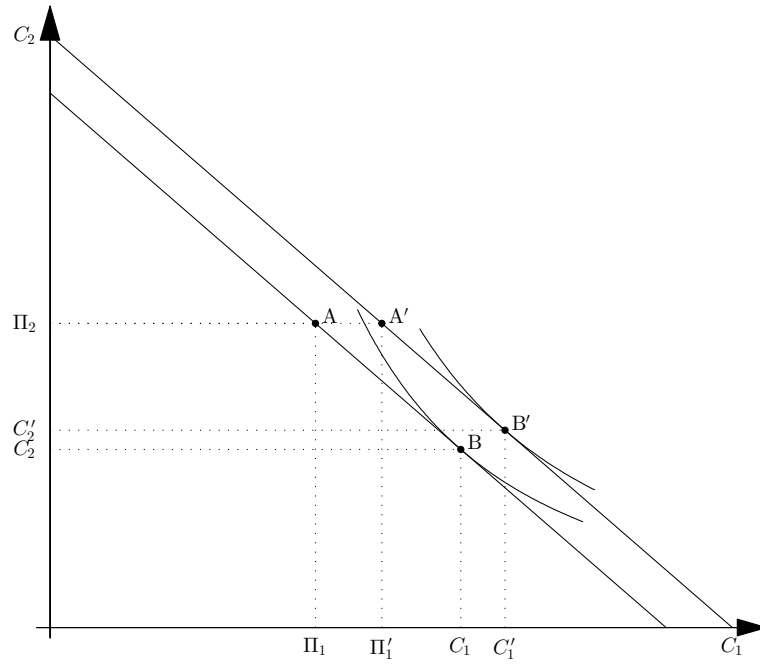
Again, this optimality condition is identical to the one pertaining to the endowment economy (see equation (3.9)).

5.3.1 Effect of a Temporary Increase in Productivity on Consumption

Suppose now that unexpectedly the economy experiences an increase in the productivity of capital in period 1. Specifically, suppose that the efficiency parameter of the production function increases from A_1 to $A'_1 > A_1$. Assume further that the productivity shock is transitory, so that A_2 remains unchanged. The increase in A_1 could reflect, for example, an improvement in weather conditions in the farming sector that is not expected to last beyond the present harvest. The increase in A_1 raises firms' profits from $\Pi_1(r_0, A_1)$ to $\Pi_1(r_0, A'_1)$. The capital stock in period 1, I_0 , does not change because it is predetermined. Profits in period 2 are unchanged, because the productivity shock is temporary.

The effect on the optimal consumption path is shown in figure 5.9. Before the shock, the profit path is at point A. The new profit path is point A' located directly to the right of the original profit path. The increase in period-1 profits shifts the intertemporal budget constraint in a parallel fashion out and to the right. The slope of the intertemporal budget constraint is unchanged because the interest rate, r_1 , is constant. The increase in

Figure 5.9: Effect of a temporary increase in productivity on consumption



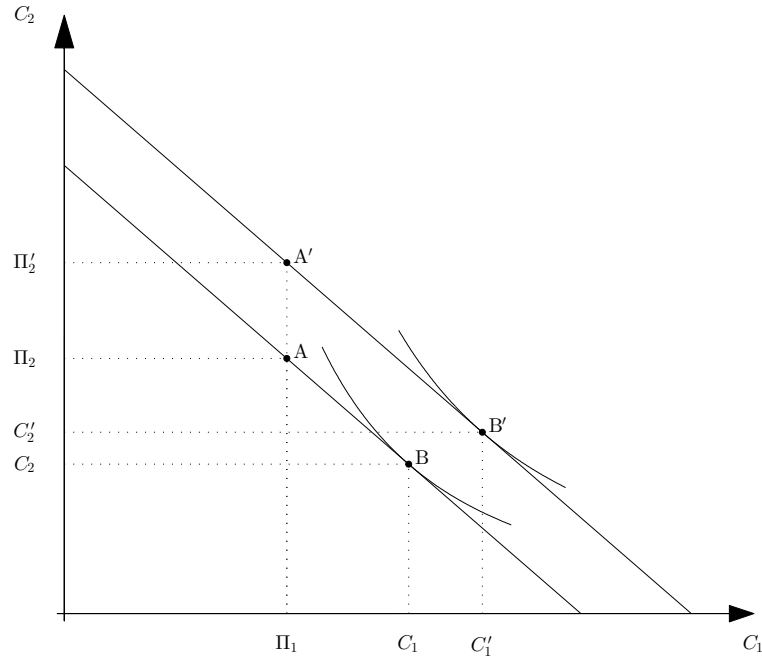
Notes. The figure depicts the adjustment of consumption to an increase in the productivity of capital in period 1 from A_1 to A'_1 holding constant the productivity of capital in period 2, A_2 . Prior to the productivity shock, the profit path is at point A and the optimal consumption path is at point B. When productivity increases, period-1 profits increase from Π_1 to Π'_1 , and period-2 profits remain unchanged. The new profit path is at point A'. The intertemporal budget constraint shifts in a parallel fashion out and to the right. The new consumption path is at point B' and features higher consumption in both periods. Period-1 consumption increases by less than period-1 profits, so household saving increases. The figure is drawn under the assumption that the household starts period 1 with a zero net asset position, $B_0^h = 0$.

profit income induces households to consume more in both periods. The new consumption path, (C'_1, C'_2) is at point B', located northeast of the initial consumption path, point B. The increase in consumption in period 1 is smaller than the increase in profit income, because households save part of the increase in period-1 income for future consumption. As a result, the temporary productivity shock causes an increase in household savings.

5.3.2 Effect of an Anticipated Future Productivity Increase on Consumption

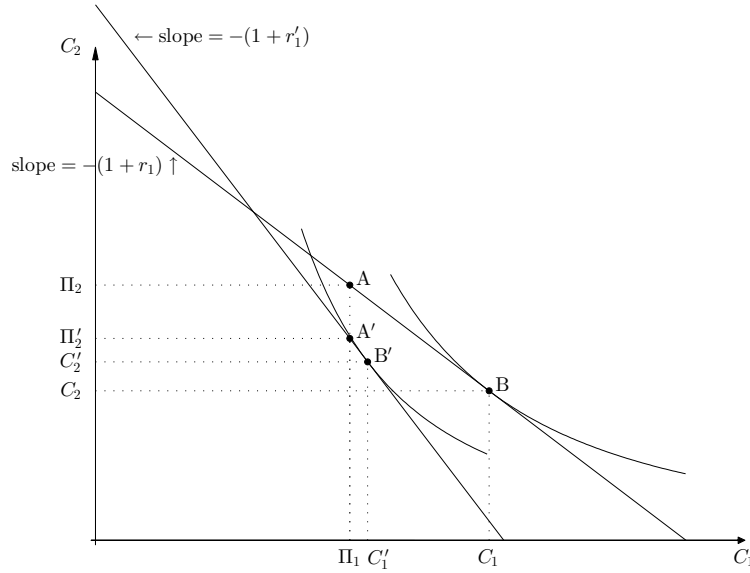
Next, let us analyze the effect of an anticipated increase in the productivity of capital. Specifically, suppose that in period 1 households learn that because of a technological improvement the efficiency parameter of the production function will increase in period 2 from A_2 to A'_2 . Assume further that the productivity of capital in period 1 is unchanged. We showed earlier that an increase in A_2 raises period-2 profits. Period-1 profits are unchanged as A_1 is assumed to stay the same. Figure 5.10 displays the adjustment of consumption to the anticipated productivity shock. The initial profit path is at point A and the new one is at point A', located straight north of point A. As a result, the intertemporal budget constraint shifts out and to the right in a parallel fashion. As in the case of an increase in period-1 profits, the increase in period-2 profits makes households richer and because consumption in both periods is a normal good consumption rises in both periods. The new consumption path is at point B' located northeast of the original consumption path, point B. Because period-1 profits are unchanged, the household must finance the expansion in period-1 consumption by borrowing. Thus,

Figure 5.10: Effect of an anticipated future productivity increase on consumption



Notes. The figure depicts the effect of an increase in the productivity of capital in period 2 from A_2 to A_2' , holding the productivity of capital in period 1, A_1 , constant. The initial path of profits is at point A, and the initial optimal consumption path is at point B. The anticipated positive productivity shock increases period-2 profits. The new path of profits is at point A'. The intertemporal budget constraint shifts out and to the right in a parallel fashion. The new consumption path is at point B' and features higher consumption in both periods. The increase in period-1 consumption is financed by an increase in borrowing. The figure is drawn under the assumption that the household starts period 1 with a zero net asset position, $B_0^h = 0$.

Figure 5.11: Effect of an increase in the interest rate on consumption



Notes. The initial path of profits is at point A on the intertemporal budget constraint, and the initial optimal consumption path is at point B where the intertemporal budget constraint is tangent to an indifference curve. The figure is drawn under the assumption of zero initial assets ($B_0^h = 0$). The increase in the interest rate causes a reduction in profits in period 2, so the new path of profits, point A', is directly below the original one, point A. In addition, the increase in the interest rate makes the intertemporal budget constraint steeper. The new optimal consumption path, (C_1', C_2') , is point B' and features lower consumption in period 1.

the anticipated positive productivity shock causes a fall in household saving.

5.3.3 Effect of an Increase in the Interest Rate on Consumption

Finally, consider the effects of an increase in the interest rate from r_1 to $r_1' > r_1$. The situation is depicted in figure 5.11. The initial profit path is at point A and the associated optimal consumption path is at point B. As before, the figure is drawn under the assumption of zero initial assets ($B_0^h = 0$), so the

intertemporal budget constraint passes through point A. The household is initially borrowing the amount $C_1 - \Pi_1 > 0$. As in the endowment economy, the increase in the interest rate makes the intertemporal budget constraint steeper, because now consuming an extra unit in period 1 requires sacrificing $r'_1 - r_1 > 0$ extra units of consumption in period 2. But in the production economy the increase in the interest rate has an additional effect on the intertemporal budget constraint that was absent in the endowment economy. Specifically, at the higher interest rate firms make less profits in period 2, which reduces the household's income in that period. Recall that period 2 profits are a decreasing function of the interest rate r_1 . Profits in period 1 are unaffected by the increase in r_1 . As shown in the figure, the new profit path, given by point A', is located directly below the original profit path (point A). Thus, the increase in the interest rate causes two changes in the position of the intertemporal budget constraint: it rotates it clockwise and at the same time it shifts it downward.

The new consumption path is point B' and features lower consumption in period 1 ($C'_1 < C_1$). The reduction in period-1 consumption is due to three effects. Two of them are the familiar substitution and wealth effects studied in the endowment economy of chapter 3. As we discussed there, when the interest rate increases, the substitution effect always induces households to consume less in the current period, as interest-bearing assets become a more attractive option. The wealth effect is also negative in this example because the household was initially borrowing so that the increase in the interest rate makes it poorer. The third and novel effect is a negative income effect stemming from the reduction in profit income in period 2. Consequently, we

have that the increase in the interest rate causes a fall in borrowing (from $C_1 - \Pi_1$ to $C'_1 - \Pi_1$), or equivalently an increase in household saving.

5.4 The Saving Schedule

In chapter 2 we saw that national saving, S_1 , is the difference between national income, Y_1 , and private and government consumption, $C_1 + G_1$, see equation (2.9). In the present economy there is no government, so that $G_1 = 0$ and national saving is simply the difference between national income and private consumption,

$$S_1 = Y_1 - C_1.$$

From the analysis of the previous section we have that private consumption in period 1 is a decreasing function of the interest rate and an increasing function of current and expected future productivity. Thus we can write

$$C_1 = C(\underset{-}{r_1}, \underset{+}{A_1}, \underset{+}{A_2}). \quad (5.13)$$

At the same time, national income is an increasing function of current productivity, A_1 , but is independent of both the interest rate and future expected productivity. To see this, recall that national income is the sum of net investment income and output,

$$Y_1 = r_0 B_0^* + Q_1,$$

where B_0^* is the country's net foreign asset position at the beginning of

period 1, and Q_1 is output in period 1. The country's net foreign asset position at the beginning of period 1 is the sum of the household's net asset position, B_0^h , and the firm's net asset position, $-D_0^f$, that is,

$$B_0^* = B_0^h - D_0^f. \quad (5.14)$$

Since B_0^h , D_0^f , and r_0 are all determined prior to period 1, we can regard net investment income as exogenously given. Output in period 1 is given by

$$Q_1 = A_1 F(I_0).$$

The stock of capital in period 1, I_0 , is determined prior to period 1 and the productivity parameter A_1 is determined in period 1, but is exogenous. Thus, we can regard period-1 output also as exogenously given. We can therefore write national income as

$$Y_1 = Y(A_1)_+. \quad (5.15)$$

Combining expressions (5.13) and (5.15) we have

$$S_1 = Y(A_1)_+ - C(r_1, A_1, A_2)_{-+}.$$

It is clear from this expression that national saving is increasing in the interest rate, r_1 , and decreasing in the future expected level of productivity, A_2 . But what about A_1 ? On the surface, a temporary productivity shock in period 1, i.e., an increase in A_1 , has an ambiguous effect on saving, as it

causes an increase in both national income and consumption. However, we saw in the previous section that a positive temporary productivity shock induces an increase in consumption that is smaller than the increase in income, as consumption-smoothing households prefer to save part of the increase in profit income generated by the productivity shock for future consumption. Thus, national saving increases unambiguously with an increase in A_1 .

We can summarize the results derived in this section by writing national saving as

$$S_1 = S(r_1; A_1, A_2). \quad (5.16)$$

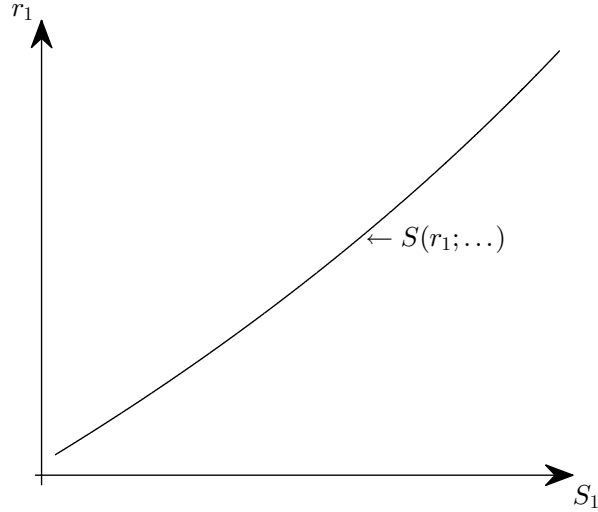
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This expression is the *saving schedule* and is depicted in figure 5.12. The saving schedule is upward sloping, because an increase in the interest rate leaves national income unchanged and discourages consumption.

Consider the effect of a temporary productivity shock that increases A_1 , with A_2 unchanged. Graphically, a temporary productivity shock shifts the saving schedule down and to the right, as shown in figure 5.13. The shift in the saving schedule reflects the fact that the temporary increase in productivity induces households to save more at every level of the interest rate.

What about an anticipated increase in productivity in period 2, that is, an increase in A_2 holding A_1 constant? An anticipated increase in productivity causes the saving schedule to shift up and to the left as shown in figure 5.14. The intuition should be familiar by now. At every given level of the interest rate, the increase in future productivity makes households richer, as they expect an elevation in future profit income. As a conse-

Figure 5.12: The saving schedule



Notes. The saving schedule relates national saving to the interest rate. It slopes upward because an increase in the interest rate induces households to postpone current consumption and increase their holdings of interest bearing assets.

quence, consumption in period 1 increases. However, period-1 income is not affected by the future increase in productivity. Thus, the expansion in current consumption is financed entirely by a fall in saving.

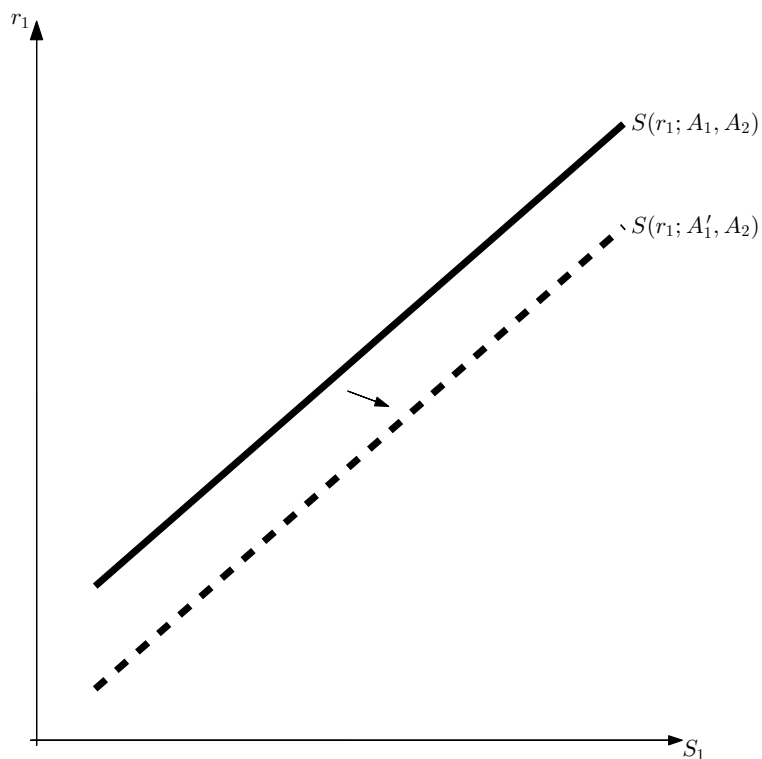
5.5 The Current Account Schedule

The current account is the difference between saving and investment. Using the saving and investment schedules given in equations (5.16) and (5.7), respectively, we can write

$$CA_1 = S_1 - I_1 = S(r_1; A_1, A_2) - I(r_1; A_2).$$

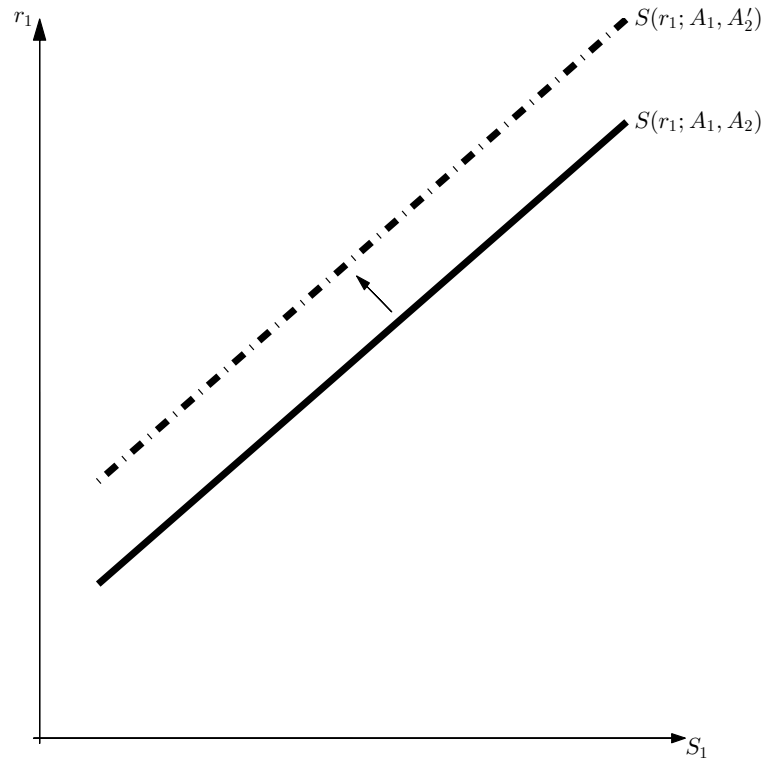
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Figure 5.13: Effect of a temporary productivity increase on the saving schedule



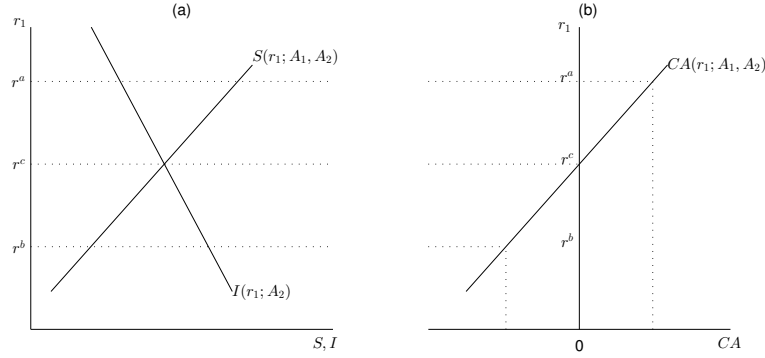
Notes. The figure depicts the effect of an increase in the productivity parameter in period 1 from A_1 to $A'_1 > A_1$ holding A_2 constant. A positive temporary productivity shock shifts the savings schedule down and to the right, because at every level of the interest rate households save part of the additional profit income generated by the increase in productivity for future consumption.

Figure 5.14: Effect of an anticipated future productivity increase on the saving schedule



Notes. The figure depicts the effect of an increase in the productivity parameter from A_2 to $A'_2 > A_2$ on the savings schedule. The anticipated productivity shock shifts the saving schedule up and to the left. The increase in future productivity leaves current income unchanged but increases future income. In period 1, households consume more in anticipation of the future increase in profit income. Consequently, at every given level of the interest rate saving declines.

Figure 5.15: Saving, investment and the current account



Notes. This figure presents a graphical derivation of the current account schedule. Panel (a) depicts the saving and investment schedule. Panel (b) depicts the current account schedule, which is the horizontal difference between the saving and investment schedule.

This expression implies that the current account is an increasing function of the interest rate. It also implies that the current account is increasing in the level of productivity in period 1 and decreasing in the level of productivity in period 2. We can therefore write

$$CA_1 = CA(r_1; A_1, A_2). \quad (5.17)$$

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This equation represents the *current account schedule*, which expresses the current account as an increasing function of the interest rate with productivity as shifters of this function. Figure 5.15 presents a graphical derivation of the current account schedule. Panel (a) plots the investment and saving schedules. The vertical axes measure the interest rate, r_1 , and the horizontal measures either investment, I_1 , national savings, S_1 , or the current account,

CA_1 .¹ The plot of the investment schedule reproduces figure 5.6 and the plot of the saving schedule reproduces figure 5.12. Panel (b) plots the horizontal difference between the saving and investment schedules, which is the current account schedule. Suppose that the interest rate is r^a . From panel (a) we see that at this level of the interest rate saving exceeds investment. Accordingly, panel (b) shows that when $r_1 = r^a$ the current account is in surplus. If the interest rate is equal to r^c , then investment equals savings and the current account is zero. The interest rate r^c is the one that would prevail in a closed economy, that is, in an economy that does not have access to international capital markets. For interest rates below r^c , such as r^b , investment is larger than savings so that the country runs a current account deficit. In general, as the interest rate decreases, the current account deteriorates, therefore, as shown in panel (b), the current account is an increasing function of the interest rate. With the help of this graphical apparatus, it is now straightforward to analyze the equilibrium determination of saving, investment, and the current account in the production economy, as well as the effect of various aggregate shocks of interest.

¹The saving, investment, and current account graph shown in figure 5.15 is known as the Metzler diagram after Lloyd A. Metzler (1913-1980). It was presented as a lecture in 1960 and published in 1968 as Metzler, Lloyd A., *The Process of International Adjustment under Conditions of Full Employment: A Keynesian View*, in Richard E. Caves and Harry G. Johnson, Editors, *Readings in International Economics*, Volume XI, American Economic Association, Richard D. Irwin, Homewood IL, 1968, pages 465-486.

5.6 Equilibrium in the Production Economy

As we discussed in chapter 3, in an open economy with free capital mobility, the domestic interest rate must equal the world interest rate, r^* , that is,

$$r_1 = r^*. \quad (5.18)$$

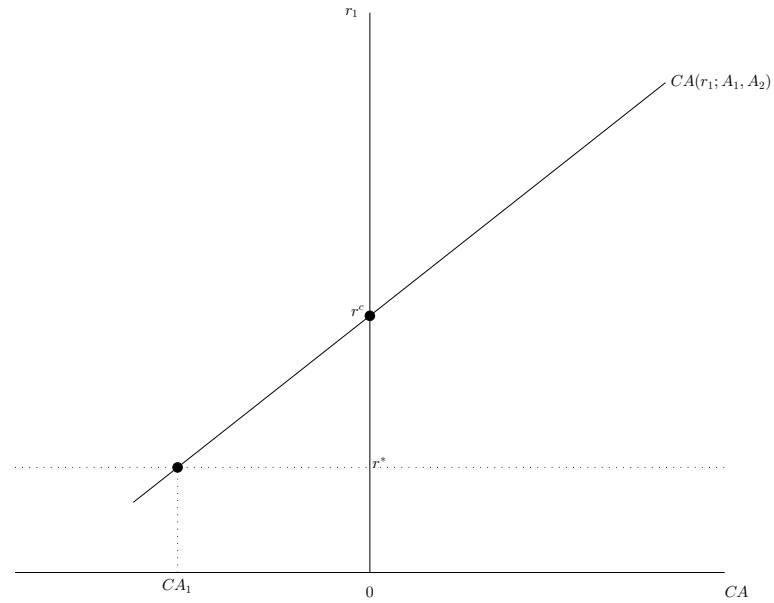
The world interest rate r^* is exogenous not only to households and firms but also to the country as a whole, because we are assuming that the domestic economy is too small to affect international asset prices.² Thus we can find the equilibrium level of the current account by simply evaluating the current account schedule at $r_1 = r^*$ as shown in figure 5.16. In the case depicted in the figure the current account balance is negative in equilibrium. If the economy were closed to trade in goods and financial assets, then the current account would always be nil, and the equilibrium value of the domestic interest rate would be determined by the intersection of the current account schedule with the vertical axis, r^c in the figure.

5.6.1 Adjustment of the Current Account to Changes in the World Interest Rate

Suppose the world interest rate increases from r^* to $r^{*'} > r^*$. Figure 5.17 depicts the adjustment of the current account to this external shock. At the initial interest rate r^* , the country runs a current account deficit equal to CA_1 . The increase in the world interest rate does not change the position

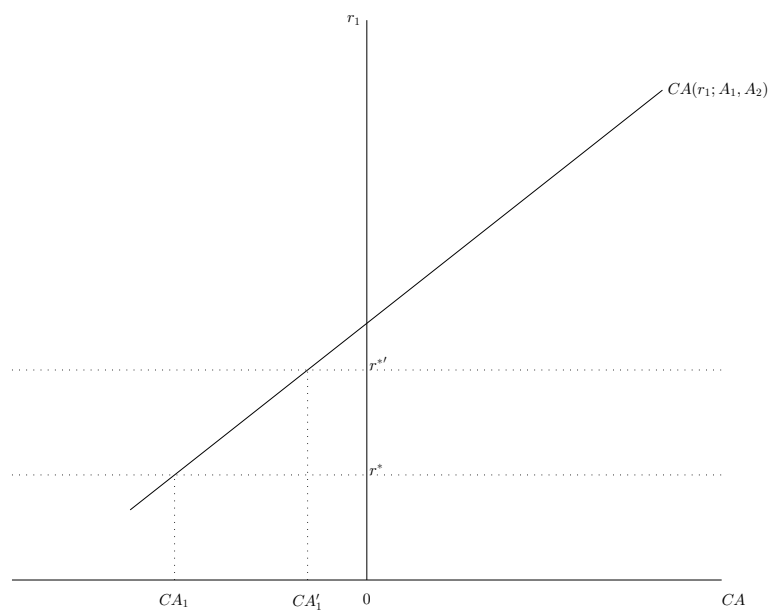
²In chapter 7, we relax this assumption and analyze current account determination in a large open economy.

Figure 5.16: Current Account Determination in the Production Economy



Notes. This figure displays the current account schedule and the world interest rate in the space (CA, r_1) . Under free capital mobility the current account is determined by the intersection of the current account schedule and the world interest rate, r^* . In a closed economy the current account is nil, and the domestic interest rate is r^c and is determined by the intersection of the current account schedule with the vertical axis.

Figure 5.17: Current account adjustment to an increase in the world interest rate



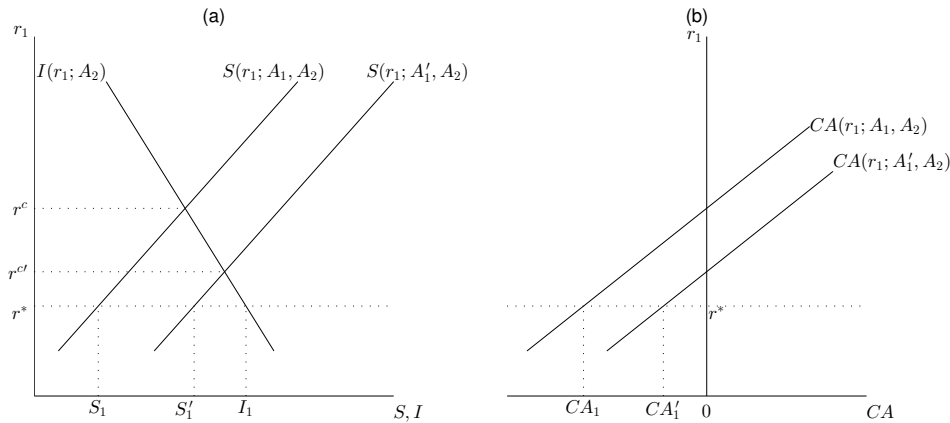
Notes. The initial world interest rate is r^* and the equilibrium current account is CA_1 . The world increases from r^* to $r^{*'}$. The higher interest rate leads to an improvement in the current account from CA_1 to CA'_1 .

of the current account schedule, but represents a movement along it. The new equilibrium value of the current account is given by CA'_1 , where the current account schedule intersects the new higher world interest rate, $r^{*'}.$ The economy thus experiences an improvement in the current account. The higher interest rate encourages domestic saving, as bonds become more attractive, and discourages firms' investment in physical capital, because the interest cost of financing spending in capital goods goes up. If the economy were closed, it would be isolated from world financial markets, and, as a result, no domestic variable will be affected by the change in the world interest rate. The first two columns of table 5.1 summarize these results.

5.6.2 Adjustment of the Current Account to a Temporary Increase in Productivity

Consider next the effect of a temporary increase in productivity, that is, an increase in A_1 holding A_2 constant. Specifically, suppose that the productivity parameter of the production function increases from A_1 to $A'_1 > A_1$. Figure 5.18 depicts the adjustment of saving, investment, and the current account. Prior to the productivity change, saving equals S_1 , investment equals I_1 , and the current account equals CA_1 . In the figure, at the world interest rate r^* , saving is lower than investment and therefore the economy runs a current account deficit, $CA_1 < 0$. The temporary increase in productivity shifts the saving schedule down and to the right, because, for any given level of the interest rate, households save part of the increase in profit income for future consumption. The investment schedule does not shift because investment is not affected by temporary changes in productivity. The rightward

Figure 5.18: Current account adjustment to a temporary increase in productivity



Notes. Productivity in period 1 increases from A_1 to $A'_1 > A_1$. The increase in A_1 shifts the saving schedule down and to the right. The investment schedule is unchanged. The current account schedule shifts down and to the right. In the new equilibrium the current account improves to CA'_1 , saving increases to S'_1 , and investment remains unchanged. In the closed economy, the increase in A_1 causes a fall in the domestic interest rate from r^c to $r^{c'}$ and higher saving and investment.

shift in the saving schedule implies that at any given interest rate the difference between saving and investment is larger than before the increase in productivity. As a result, the current account schedule shifts down and to the right. At the world interest rate r^* , which is unchanged, saving equals $S'_1 > S_1$, investment continues to be I_1 , and the current account equals $CA'_1 > CA_1$. Thus, we have that a temporary increase in productivity produces an increase in saving, no change in investment, and an improvement in the current account.

The adjustment in a closed economy is quite different. The equilibrium is always at the intersection of the saving and investment schedules and the current account is always nil. The rightward shift in the saving schedule causes the equilibrium interest rate to fall from r^c to $r^{c'}$. The intuition behind the decline in the equilibrium interest rate in the closed economy is that households wish to save part of the increase in profit income. However, firms do not demand more funds at the original interest rate r^c . For the loan market to clear, the interest rate must fall. In the new equilibrium both saving and investment increase. However, the increase in saving is smaller than in the open economy because the fall in the interest rate partially offsets the positive effect of A_1 . The middle two columns of table 5.1 summarize the adjustment of the open and closed economies to a temporary increase in productivity.

Table 5.1: Adjustment of the Production Economy to Changes in the World Interest rate and Productivity in Open and Closed Economies

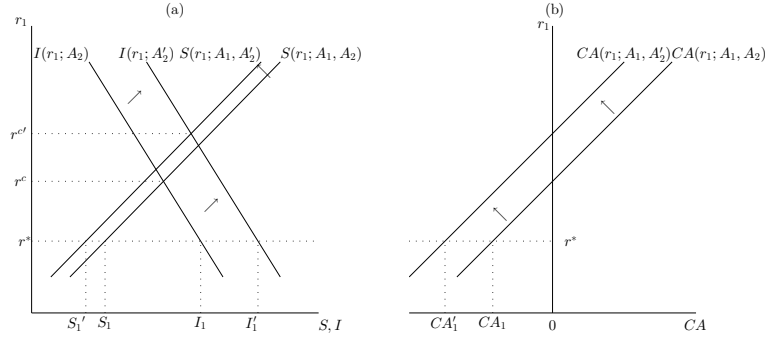
	$r^* \uparrow$		$A_1 \uparrow$		$A_2 \uparrow$	
	Open	Closed	Open	Closed	Open	Closed
S_1	\uparrow	—	\uparrow	\uparrow	\downarrow	\uparrow
I_1	\downarrow	—	—	\uparrow	\uparrow	\uparrow
CA_1	\uparrow	—	\uparrow	—	\downarrow	—
r_1	\uparrow	—	—	\downarrow	—	\uparrow

Notes. This table summarizes the effect of three different shocks on saving (S_1), investment (I_1), the current account (CA_1), and the domestic interest rate (r_1). The shocks considered are an increase in the world interest rate ($r^* \uparrow$), a temporary increase in productivity ($A_1 \uparrow$), and a future expected increase in productivity ($A_2 \uparrow$). Two different economic environments are considered: free capital mobility (Open) and a closed economy (Closed).

5.6.3 Adjustment of the Current Account to an Anticipated Future Productivity Increase

Suppose that in period 1 agents learn that in period 2 the productivity of capital will increase. Specifically, suppose that the efficiency parameter of the period-2 production function is expected to increase from A_2 to $A'_2 > A_2$. This type of news triggers an investment surge, as firms choose to increase investment in period 1 for any given level of the interest rate. Consequently, the investment schedule shifts up and to the right, as depicted in panel (a) of figure 5.19. The new equilibrium level of investment increases from I_1 to I'_1 . The expected increase in productivity also affects the saving schedule, $S(r_1; A_1, A_2)$, through its effect on expected future profit income. In particular, the saving schedule shifts up and to the left, as, at every level of the interest rate, households borrow more against future profit income

Figure 5.19: Current account adjustment to an expected future increase in productivity



Notes. The productivity parameter is expected to increase from A_2 to A'_2 . The investment schedule shifts to the right and the saving schedule shifts to the left, although by less than the shift in the investment schedule. The current account schedule shifts to the left. In the open economy, investment increases from I_1 to I'_1 , saving falls from S_1 to S'_1 , and the current account deteriorates from CA_1 to CA'_1 . In the closed economy, the interest rate increases from r^c to $r^{c'}$.

to finance higher current consumption. The equilibrium level of saving falls from S_1 to S'_1 . These shifts in the investment and saving schedules imply that the current account schedule shifts up and to the left as shown in panel (b). Consequently, the current account deteriorates from CA_1 to CA'_1 .

If the economy were closed, the investment surge would trigger a rise in the domestic interest rate from r^c to $r^{c'}$ and thus investment would increase by less than in the open economy. The last two columns of table 5.1 collect these results. Note that the result that investment and saving increase in the closed economy need not hold. It depends on the assumption that the horizontal shift in the saving schedule is smaller than that of the investment schedule. But this does not always have to be the case. Depending on preferences and technologies the shift in the saving schedule may be larger or smaller than the shift in the investment schedule.

5.7 Equilibrium in the Production Economy: An Algebraic Approach

In section 5.6 we used a graphical approach to characterizing the equilibrium determination of the current account and other macroeconomic indicators in a small open economy with production. Here, we perform the characterization of equilibrium using an algebraic approach. We begin by deriving the economy's intertemporal resource constraint.

In section 5.3, we saw that the household's intertemporal budget constraint is given by

$$C_1 + \frac{C_2}{1+r_1} = (1+r_0)B_0^h + \Pi_1 + \frac{\Pi_2}{1+r_1}.$$

Using the definitions of profits in periods 1 and 2 given by equations (5.3) and (5.5) to eliminate Π_1 and Π_2 and equation (5.14) to eliminate B_0^h and D_0^f , we can rewrite this expression as

$$C_1 + \frac{C_2}{1+r_1} + I_1 = (1+r_0)B_0^* + A_1F(I_0) + \frac{A_2F(I_1)}{1+r_1}. \quad (5.19)$$

This equation is the economy's intertemporal resource constraint. The left-hand side is the present discounted value of domestic absorption (consumption plus investment). The right-hand side is the sum of initial wealth and the present discounted value of output.

An equilibrium in the production economy with free capital mobility is an allocation $\{C_1, C_2, I_1, r_1\}$ satisfying the firm's optimality condition (5.4), the household's Euler equation (5.12), the interest parity condition (5.18),

and the economy's intertemporal resource constraint (5.19). The complete set of equilibrium conditions is therefore given by

$$A_2 F'(I_1) = 1 + r_1, \quad (\text{R5.4})$$

$$\frac{U'(C_1)}{\beta U'(C_2)} = 1 + r_1, \quad (\text{R5.12})$$

$$r_1 = r^*, \quad (\text{R5.18})$$

and

$$C_1 + \frac{C_2}{1 + r_1} + I_1 = (1 + r_0)B_0^* + A_1 F(I_0) + \frac{A_2 F(I_1)}{1 + r_1}, \quad (\text{R5.19})$$

given the initial capital stock, I_0 , the initial net foreign asset position including interest, $(1 + r_0)B_0^*$, the world interest rate r^* , and the levels of productivity in periods 1 and 2, A_1 and A_2 .

Equilibrium conditions (5.4), (5.12), (5.18), and (5.19) represent a system of four equations that can be solved for the four unknowns C_1 , C_2 , I_1 , and r_1 . Consider the following example. Suppose that the period utility function is the square root function,

$$U(C) = \sqrt{C},$$

and that technology is a power function of capital,

$$F(I) = I^\alpha,$$

where $\alpha \in (0, 1)$ is a parameter governing the rate at which the marginal product of capital diminishes with the stock of capital itself. With this production function, equilibrium condition (5.4) becomes

$$\alpha A_2 I_1^{\alpha-1} = 1 + r_1.$$

The left hand side is the marginal product of capital, given by the derivative of the production function with respect to I_1 . Solving for I_1 yields the following investment schedule

$$I_1 = \left(\frac{\alpha A_2}{1 + r_1} \right)^{\frac{1}{1-\alpha}}. \quad (5.20)$$

As expected, investment is a decreasing function of the interest rate, r_1 , and an increasing function of the expected level of productivity, A_2 . Under the assumed functional form for the period utility function, the Euler equation (5.12) becomes

$$\frac{\sqrt{C_2}}{\beta \sqrt{C_1}} = 1 + r_1. \quad (5.21)$$

Using the interest-rate parity condition (5.18) to eliminate r_1 , the investment schedule (5.20) to eliminate I_1 , and the Euler equation (5.21) to eliminate C_2 from the economy's intertemporal resource constraint (5.19) yields the following expression for the equilibrium level of consumption in period 1:

$$C_1 = \frac{1}{1 + \beta^2(1 + r^*)} \left[(1 + r_0)B_0^* + A_1 I_0^\alpha + \left(\frac{1 - \alpha}{\alpha} \right) \left(\frac{\alpha A_2}{1 + r^*} \right)^{\frac{1}{1-\alpha}} \right]. \quad (5.22)$$

Intuitively, this expression says that in equilibrium current consumption is

a decreasing function of the world interest rate, r^* , an increasing function of productivity in periods 1 and 2, A_1 and A_2 , and an increasing function of the country's initial net international investment position, $(1 + r_0)B_0^*$. Also, current consumption is decreasing in the subjective discount factor β . Recall that this parameter measures the degree of impatience, so it makes sense that as households become more impatient they devote more resources to consumption early in their lives.

We saw before that saving in period 1 is given by the difference between national income, $r_0B_0^* + Q_1$, and consumption,

$$S_1 = r_0B_0^* + Q_1 - C_1,$$

where $Q_1 = A_1I_0^\alpha$ denotes output in period 1. The current account in period 1 is given by the difference between saving and investment, $CA_1 = S_1 - I_1$, or

$$CA_1 = r_0B_0^* + Q_1 - C_1 - I_1.$$

Using this expression and the equilibrium values of I_1 and C_1 given in equations (5.20) and (5.22), we can write the current account as

$$CA_1 = \underbrace{r_0B_0^* + A_1I_0^\alpha}_{\text{national income}} - \underbrace{\frac{1}{1 + \beta^2(1 + r^*)} \left[(1 + r_0)B_0^* + A_1I_0^\alpha + \left(\frac{1 - \alpha}{\alpha} \right) \left(\frac{\alpha A_2}{1 + r^*} \right)^{\frac{1}{1 - \alpha}} \right]}_{\text{consumption}} - \underbrace{\left(\frac{\alpha A_2}{1 + r^*} \right)}_{\text{investment}} \quad (5.23)$$

This expression for the current account looks complicated. But as will become clear in our discussion of how various disturbances affect the current account, it is indeed quite intuitive.

5.7.1 Adjustment to an Increase in the World Interest Rate

Suppose that the world interest rate increases, that is, r^* increases. As is clear from (5.23), national income in period 1 is unchanged, and consumption and investment both fall. As a result, the increase in r^* causes an improvement in the current account. The increase in the world interest rate depresses domestic spending in consumption and capital goods, freeing up resources that are allocated to purchases of foreign assets. This result is in line with the result we obtained graphically in subsection 5.6.1.

5.7.2 Adjustment to a Temporary Increase in Productivity

Suppose now that the economy experiences a positive temporary productivity improvement whereby A_1 increases by $\Delta A_1 > 0$. From equation (5.23) we have that the change in the current account in period 1 is given by

$$\Delta CA_1 = \Delta A_1 I_0^\alpha - \frac{1}{1 + \beta^2(1 + r^*)} \Delta A_1 I_0^\alpha > 0,$$

which, in accordance with the results obtained graphically in subsection 5.6.2, says that the current account improves in response to a temporary increase in productivity. The intuition is straightforward. The first term on the right-hand side of the above expression is the increase in period-1 income caused by the improvement in productivity. The second term is the increase in consumption, which is a fraction $1/[1 + \beta^2(1 + r^*)] < 1$ of the increase in income. Households consume only a fraction of the increase in period-1 income because they prefer to leave part of the windfall for future consumption.

How would the closed economy adjust to the increase in A_1 ? Take another look at equation (5.23). Set the left-hand side to zero, since in the closed economy the current account is always nil. Also, replace r^* with r_1 , since in the closed economy the domestic interest rate has to adjust to make the current account zero. This yields

$$0 = \underbrace{r_0 B_0^* + A_1 I_0^\alpha}_{\text{national income}} - \underbrace{\frac{1}{1 + \beta^2(1 + r_1)} \left[(1 + r_0) B_0^* + A_1 I_0^\alpha + \left(\frac{1 - \alpha}{\alpha} \right) \left(\frac{\alpha A_2}{1 + r_1} \right)^{\frac{1}{1 - \alpha}} \right]}_{\text{consumption}} - \underbrace{\left(\frac{\alpha A_2}{1 + r_1} \right)^{\frac{1}{1 - \alpha}}}_{\text{investment}} \quad (5.24)$$

This expression says that in the closed economy saving must equal investment. All other things equal, as we just discussed, the increase in A_1 produces an increase in the right-hand side of (5.24), as national income increases by more than consumption. This would violate equation (5.24), as saving would exceed investment. The excess supply of funds would put downward pressure on the domestic interest rate r_1 , which stimulates consumption and investment, restoring the equality of saving and investment. Thus, in the closed economy a temporary increase in productivity causes an expansion in consumption, investment, and saving, and a decline in the interest rate.

5.7.3 Adjustment to an Anticipated Future Increase in Productivity

Finally, consider the effect of an anticipated increase in productivity. Specifically, suppose that in period 1 agents learn that the productivity of capital will increase in period 2. It is clear from equation (5.23) that an increase

in A_2 causes a deterioration in the current account. The reason is that consumption and investment are both stimulated by the expected increase in productivity, while national income is unchanged. This result is consistent with the graphical analysis of subsection 5.6.3.

What happens in the closed economy? The increase in A_2 produces an expansion in consumption and investment, which, all other things equal would violate equation (5.24), because investment would be larger than saving. The excess demand in the loan market will push the interest rate up, depressing consumption and investment until saving and investment are again equal. Note that $1 + r_1$ increases but proportionally less than A_2 . To see this, note that if $1 + r_1$ were to increase by the same proportion as A_2 , investment would remain unchanged but consumption would fall, which would imply an excess of saving over investment. We therefore have that in the closed economy saving, investment, and the interest rate all increase in response to an anticipated increase in productivity. It is noteworthy that in the closed economy consumption in period 1 falls despite the good news. This prediction is quite different from what happens in the open economy, where consumption expenditure increases unambiguously.

5.8 The Terms of Trade in the Production Economy

In chapter 4, we arrived at the conclusion that terms of trade shocks are just like endowment shocks. This result extends to the production economy. Specifically, in this section we show that in the production economy terms

of trade shocks have the same effects as productivity shocks.

Suppose, as we did in chapter 4, that the good households like to consume is different from the good the economy produces. For example, suppose that households have preferences over the consumption of food and that the economy produces oil. Profits in periods 1 and 2 are now given by

$$\Pi_1 = TT_1 A_1 F(I_0) - (1 + r_0)I_0$$

and

$$\Pi_2 = TT_2 A_2 F(I_1) - (1 + r_1)I_1,$$

where TT_1 is the terms of trade in period 1, defined as the relative price of the export good (oil) in terms of the import good (food). A similar definition applies to TT_2 . Here, we are assuming that capital is imported and that the relative price of capital in terms of food is unity.³ Notice that the terms of trade appear always multiplying the productivity parameter, that is, where we used to have A_1 we now have $TT_1 A_1$ and where we used to have A_2 we now have $TT_2 A_2$.

In period 1, the firm chooses I_1 to maximize Π_2 taking A_2 , TT_2 , and r_1 as given. The profit maximization condition of the firm is

$$TT_2 A_2 F'(I_1) = 1 + r_1.$$

This optimality condition is exactly like the one obtained in the one-good

³In exercise 5.3, we consider the case that the relative price of capital in terms of food is not equal to unity, and ask you to characterise the adjustment of the current account in response to a change in this relative price.

economy, equation (5.4), except that instead of A_2 it features TT_2A_2 . It follows that a change in the terms of trade in period 2 has exactly the same effect on investment as a change in productivity in period 2. Thus, we can write the investment schedule as

$$I_1 = I(r_1; \underset{-}{TT_2A_2}).$$

The household's intertemporal budget constraint continues to be (5.11), with the only difference that now profits in periods 1 and 2 depend, respectively, on TT_1A_1 and TT_2A_2 instead of A_1 and A_2 . We can therefore write

$$C_1 = C(r_1, \underset{+}{TT_1A_1}, \underset{+}{TT_2A_2}).$$

National income equals $r_0B_0^* + Q_1 = r_0B_0^* + TT_1A_1F(I_0)$, which also depends on the product of TT_1 and A_1 . We therefore have that saving, the difference between national income and consumption, behaves exactly as in the one-good economy except that, again, TT_1A_1 and TT_2A_2 take the place of A_1 and A_2 , respectively. Thus, we write

$$S_1 = S(r_1; \underset{+}{TT_1A_1}, \underset{+}{TT_2A_2}, \underset{-}{TT_2A_2}).$$

Finally, the same principle applies to the current account schedule, since it is the difference between saving and investment,

$$CA_1 = CA(r_1; \underset{+}{TT_1A_1}, \underset{+}{TT_2A_2}, \underset{-}{TT_2A_2}).$$

It follows that the current account adjustment to terms of trade shocks can be read off table 5.1 by replacing A_1 by TT_1 and A_2 by TT_2 . In particular, a temporary terms of trade improvement (an increase in TT_1) produces an increase in saving, an improvement in the current account, and no change in investment. And an anticipated future improvement in the terms of trade (an increase in TT_2) causes a fall in saving, an expansion in investment, and a deterioration of the current account.

5.9 Giant Oil Discoveries, Saving, Investment, and the Current Account

Are the predictions of the open economy model with production studied in this chapter empirically compelling? To answer this question, we examine a number of natural experiments realized in different countries and at different points in time. The natural experiments are giant oil discoveries. In the context of the model, the news of a giant oil discovery can be interpreted as an anticipated increase in the productivity of capital, that is, as an anticipated increase in A_2 . The reason why a discovery is an anticipated productivity shock is that it takes time and extensive investment in oil production facilities to extract the oil and bring it to market. The average delay from discovery to production is estimated to be between 4 and 6 years.

The macroeconomic effects of a giant oil discovery can be analyzed with the help of figure 5.19. Upon the news of the discovery, the investment schedule shifts up and to the right and the saving schedule shifts up and to the left. The current account schedule shifts up and to the left. The

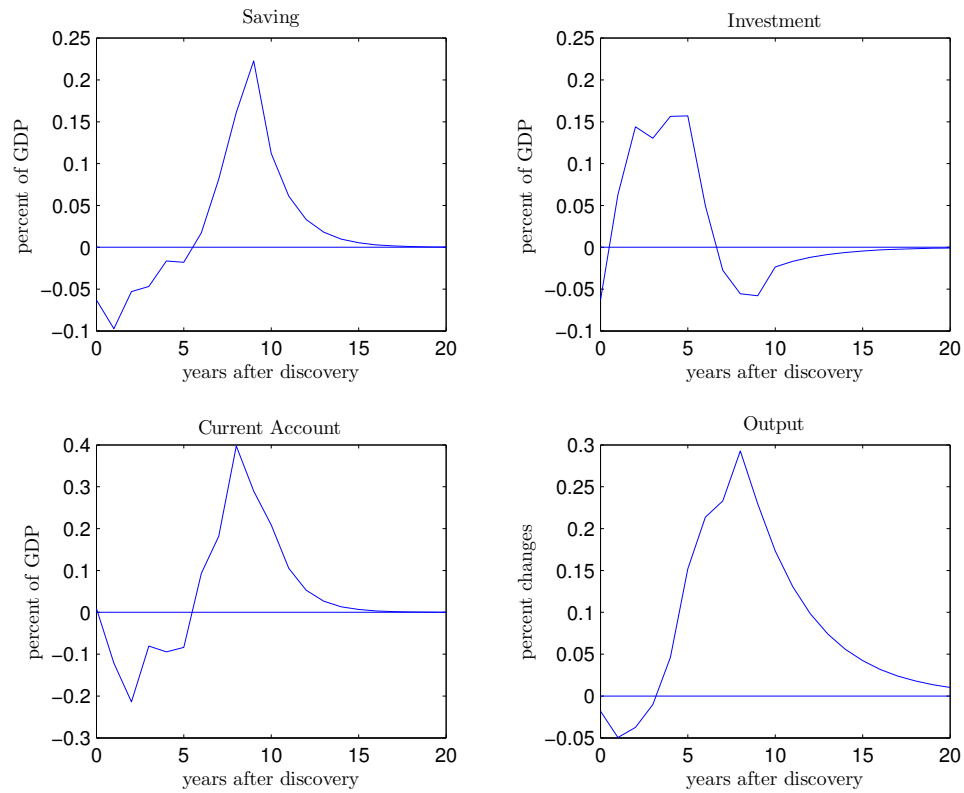
world interest rate does not change. Thus, the model predicts that after an oil discovery a country experiences an investment boom, a decline in saving, and a deterioration in the current account. Once the oil is brought to market (period 2 in the model, and 4 to 6 years post discovery in reality), output (oil production) increases, investment falls, saving increases (to pay back the debt accumulated in period 1 for the construction of oil facilities and for consumption), and the current account improves. Are these predictions of the model borne out in the data?

Rabah Arezki, Valerie Ramey, and Liugang Sheng of the International Monetary Fund, the University of California at San Diego, and the Chinese University of Hong Kong, respectively, analyze the effect of giant oil discoveries in 180 countries from 1970 to 2012.⁴ A giant oil discovery is defined as a discovery of an oil and/or gas field that contains at least 500 million barrels of ultimately recoverable oil equivalent. In turn, ultimately recoverable oil equivalent is the amount that is technically recoverable given existing technology. The sample contains in total 371 giant oil discoveries, which took place in 64 different countries (so 116 of the 180 countries experienced no giant oil discoveries over the sample period). The peak period of giant oil discoveries was the 1970s and, not surprisingly, the region with the largest number of oil discoveries is the Middle East and North Africa. Giant oil discoveries are really big, with a median value of 9 percent of a year's GDP.

Figure 5.20 displays the dynamic response to a giant oil discovery of saving, investment, the current account, and output. The size of the oil

⁴Arezki, Rabah, Valerie A. Ramey, and Liugang Sheng, "News Shocks in Open Economies: Evidence from Giant Oil Discoveries," *Quarterly Journal of Economics* 132, February 2017, 103-155.

Figure 5.20: Dynamic Effect of a Giant Oil Discovery



Notes. The figure displays the dynamic effect of an oil discovery on saving, investment, the current account, and output. The size of the oil discovery is 9 percent of GDP. Saving, investment, and the current account are expressed in percent of GDP. Output is expressed in percent deviation from trend. Data source: Arezki, Rabah, Valerie A. Ramey, and Liugang Sheng, “News Shocks in Open Economies: Evidence from Giant Oil Discoveries,” *Quarterly Journal of Economics* 132, February 2017, 103-155, online appendix, table D.I.

discovery is 9 percent of GDP, the typical size in the data.⁵ Upon news of the giant oil discovery, investment experiences a boom that lasts for about 5 years. Saving declines and stays below normal for about 5 years, before rising sharply for several years. The current account deteriorates for 5 years and then experiences a reversal with a peak in year 8. Finally, output is relatively stable until the fifth year, and then experiences a boom. Remarkably, the investment boom and the fall in saving and the current account last for the entire delay between discovery to production that is typical in the oil industry, which is about 8 years. This result is noteworthy because it is not built into the econometric estimation of the dynamic responses. All of the empirical responses are consistent with the predictions of the theoretical model: The giant oil discovery induces oil companies to invest in the construction of drilling platforms. In addition, households anticipate higher profit income from the future exports of oil and as a result increase consumption and cut saving. Both the expansion in investment and the contraction in saving contribute to current account deficits in the initial years following the discovery.

We conclude that the observed macroeconomic dynamics triggered by giant oil discoveries give credence to the intertemporal model of the current account studied in this chapter. This result is important because the model developed in this chapter is the backbone of many models in international macroeconomics.

⁵The dynamic responses are estimated using an econometric technique called dynamic panel model estimation with distributed lags. Essentially, one runs a regression of the variable of interest, say the current account, onto its own lag, and current and lagged values of the giant oil discovery, and other control variables, such as a constant and time and country fixed effects.

5.10 Summing Up

In this chapter, we studied the workings of an open economy with production and investment.

- Firms borrow in period 1 to purchase capital goods. In period 2, firms use capital to produce final goods, pay back the loan including interest, and distribute profits to households.
- Firms invest up to a point at which the marginal product of capital equals the gross interest rate.
- The optimal level of investment is a decreasing function of the interest rate and an increasing function of the expected level of productivity in period 2.
- The negative relationship between investment and the interest rate is the investment schedule. In the space (I_1, r_1) the investment schedule is downward sloping. An expected increase in the productivity of capital shifts the investment schedule up and to the right.
- Households maximize their lifetime utility subject to an intertemporal budget constraint. On the income side, this constraint includes the present discounted value of profits received from firms.
- The optimal level of saving is an increasing function of the interest rate and the current productivity of capital and a decreasing function of the future expected level of productivity.
- The positive relationship between the interest rate and saving is the saving schedule. In the space (S_1, r_1) , the saving schedule is upward sloping. An increase in the current productivity of capital shifts the saving schedule

down and to the right. An expected increase in future productivity shifts the saving schedule up and to the left.

- The current account schedule is the horizontal difference between the saving and the investment schedules. In the space (CA_1, r_1) the current account schedule is upward sloping. An increase in the current productivity of capital shifts the current account schedule down and to the right. An expected increase in future productivity shifts the current account schedule up and to the left.

- An increase in the world interest rate causes an improvement in the current account, an increase in saving, and a decrease in investment.

- A temporary increase in the productivity of capital produces an improvement in the current account, an increase in saving, and no change in investment.

- An expected future increase in the productivity of capital causes a deterioration in the current account, a fall in saving, and an expansion in investment.

- Terms of trade shocks are just like productivity shocks. So their effects can be read off the last two bullets.

- The observed dynamics of saving, investment, and the current account triggered by giant oil discoveries around the world over the period 1970 to 2012 are consistent with the predictions of the intertemporal model of current account determination developed in this chapter.

5.11 Exercises

Exercise 5.1 (TFU) Indicate whether the following statements are true, false, or uncertain and explain why.

1. Good news about future productivity leads to a trade deficit today.
2. A deterioration in the terms of trade causes a fall in consumption and a deterioration in the current account.
3. Countries A and B are identical in all respects, except that the initial net international asset position (B_0^*) of country A is larger than that of country B. It must be the case that:
 - (a) Consumption in country A is higher than consumption in country B in all periods.
 - (b) Investment in country A is higher than investment in country B.
 - (c) The trade balance in country A is higher than in country B.
 - (d) None of the above.
4. A country populated by more impatient households (i.e., a country with a lower β) will consume more and invest less in period 1.
5. An improvement in productivity in period 1 (higher A_1) causes an increase in net investment income in period 2.
6. An increase in the world interest rate (higher r^*) causes an increase in the current account in period 1 and a decrease in the current account in period 2.

Exercise 5.2 (A change in the initial interest rate) Suppose in period 1, unexpectedly the initial interest rate increases from r_0 to $r'_0 > r_0 > 0$. Present a graphical analysis of the effects of this change in initial conditions on consumption, saving, investment, and the current account in period 1. Distinguish the case in which the country is initially a net creditor of the rest of the world ($B_0^* > 0$) from the case in which the country is initially a net debtor ($B_0^* < 0$). Provide intuition.

Exercise 5.3 (The relative price of investment and the current account)

In section 5.8, we studied an economy in which households consume a good different from the good produced by firms (the example we gave was food and oil). There, we analyzed the effects of changes in the terms of trade. An assumption of that economy was that consumption and investment were both imported and that the relative price of the investment good in terms of the consumption good was constant and equal to one. In this exercise, you are asked to modify that economy as follows. Assume that the domestic economy produces only consumption goods. Domestic output can either be domestically consumed or exported. Investment goods are not domestically produced and must be imported. The price of investment goods in terms of consumption goods is equal to PK_1 . The economy is small in the sense that it cannot affect the price PK_1 . The production function of domestic firms is equal to $A_t I_{t-1}^\alpha$, for $t = 1, 2$ and $\alpha \in (0, 1)$, where I_t denotes investment measured in units of investment goods.

1. Characterize the schedule for expenditures on investment in period 1 expressed in units of consumption goods. That is, derive an expression

like (5.7), where now on the left hand side are expenditures on investment goods (as opposed to the quantity of investment goods) and the arguments on the right hand side are not only r_1 and A_2 but also PK_1 . How would an increase in PK_1 shift this investment schedule?

2. Characterize the saving schedule in period 1. That is, derive an expression like (5.16) for the present economy. Does an increase in PK_1 shift the saving schedule left or right or does it leave it unchanged?
3. Characterize the current account schedule in period 1. That is, derive an expression similar to (5.17) for the present economy. Does an increase in PK_1 shift the current account schedule left or right or does it leave it unchanged?
4. Suppose that the economy is small and open to international trade in goods and assets and that free capital mobility prevails. What is the effect of an increase in PK_1 on the equilibrium levels of saving, investment, and the current account?
5. How does the answer to the previous question change if the economy is assumed to be closed to trade in financial assets but open to trade in goods?

Exercise 5.4 (An Open Economy With Investment) Consider a two-period model of a small open economy with a single good each period. Let preferences of the representative household be described by the utility function

$$\ln C_1 + \ln C_2,$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively. In periods 1 and 2, the household receives profits from the firms it owns, denoted Π_1 and Π_2 , respectively. Households and firms have access to financial markets where they can borrow or lend at the interest rate r_1 . The production technologies in periods 1 and 2 are given by

$$Q_1 = A_1 I_0^\alpha$$

and

$$Q_2 = A_2 I_1^\alpha,$$

where Q_1 and Q_2 denote output in periods 1 and 2, I_0 and I_1 denote the capital stock in periods 1 and 2, A_1 and A_2 denote the productivity factors in periods 1 and 2, and α is a parameter. Assume that $I_0 = 16$, $A_1 = 3\frac{1}{3}$, $A_2 = 3.2$, and $\alpha = \frac{3}{4}$. At the beginning of period 1 households have $B_0^h = 8$ bonds. The interest rate on bonds held from period 0 to period 1 is $r_0 = 0.25$. In period 1, firms borrow the amount D_1^f to purchase investment goods that become productive capital in period 2, I_1 . Assume that there exists free international capital mobility and that the world interest rate, denoted r^* , is 20 percent.

1. Compute output and profits in period 1.
2. Compute the optimal levels of investment in period 1 and output and profits in period 2.
3. Solve for the optimal levels of consumption in periods 1 and 2.

4. Find the country's net foreign asset position at the end of period 1, denoted B_1^* , saving, S_1 , the trade balance, TB_1 , and the current account, CA_1 .
5. Now consider an interest-rate hike in period 1. Specifically, assume that as a result of turmoil in international financial markets, the world interest rate increases from 20 percent to 50 percent in period 1. Find the equilibrium levels of saving, investment, the trade balance, the current account, and the country's net foreign asset position in period 1. Provide intuition.
6. Suppose that the interest rate is 20 percent, and that A_1 increases to 4. Calculate the equilibrium values of consumption, saving, investment, and the current account in period 1. Explain.
7. Suppose that the interest rate is 20 percent, that $A_1 = 3\frac{1}{3}$, and that A_2 increases from 3.2 to 4. Calculate the equilibrium values of consumption, saving, investment, and the current account in period 1. Explain.
8. Answer questions 2-4 and 6-7 under the assumption that the economy is closed, and that $B_0^* = 0$ and $I_0 = 16$. First use a graphical approach to analyze the qualitative differences relative to the open economy for each case. Then find the numerical solution. Discuss the extent to which your numerical solutions conform (or does not conform) with the qualitative solution given in table 5.1.

Exercise 5.5 [An Investment Subsidy] Consider a two period small

open production economy. In period 1, households are endowed with $Q_1 = 2$ units of goods and initial wealth of zero. Households are owners of firms. In period 2, households receive profit income of firms in the amount of Π_2 and pay lump-sum taxes, denoted T_2 , to the government. Household's preferences over period-one and period-two consumption, C_1 and C_2 , respectively, are given by $C_1^{\frac{1}{2}} + C_2^{\frac{1}{2}}$. The country enjoys free capital mobility and the world interest rate is zero. Firms invest in period 1 in order to be able to produce in period 2. Investment in physical capital in period 1 is denoted I_1 and the production function for output in period 2 is $F(I_1) = 3I_1^{1/3}$. Assume that the government subsidizes investment by offering firms to borrow at the gross interest rate $1 + \tilde{r}$, such that $1 + \tilde{r} = (1 + r_1)(1 - \tau)$, where r_1 denotes the domestic interest rates on loans between periods 1 and 2 and $\tau \geq 0$ denotes the investment subsidy. The government finances the investment subsidy by levying a lump-sum tax on households in period 2, T_2 .

1. Find the equilibrium value of the firm's profit in period 2, Π_2 , for the case that $\tau = 0$ and for the case that $\tau = 0.5$. Interpret your findings.
2. Find the equilibrium value of lump-sum taxes in period 2, T_2 , for the case that $\tau = 0$ and for the case that $\tau = 1/2$.
3. Find the value of the household's profit income net of lump-sum taxes in period 2, $\Pi_2 - T_2$, for the case that $\tau = 0$ and for the case that $\tau = 1/2$. Provide intuition for your finding.
4. Find the equilibrium level of period-1 consumption, C_1 , for the case that $\tau = 0$ and for the case that $\tau = 1/2$. Provide an intuitive explanation.

nation of your findings.

5. Find the equilibrium value of the trade balance in period 1, TB_1 , for the case that $\tau = 0$ and for the case that $\tau = 1/2$. Interpret your findings.

Chapter 6

Uncertainty and the Current Account

Thus far, we have studied the response of the current account to changes in fundamentals that are known with certainty. The real world, however, is an uncertain place. Some periods display higher macroeconomic volatility than others. A natural question, therefore, is how the overall level of uncertainty affects the macroeconomy, and, in particular, the external accounts. This chapter is devoted to addressing this question. It begins by documenting that between the mid 1980s and the mid 2000s the United States experienced a period of remarkable aggregate stability, known as the Great Moderation, and that the Great Moderation period coincided with the emergence of large current account deficits. The chapter then expands the open economy model of chapter 3 to introduce uncertainty. This modification allows us to understand the effect of changes in the aggregate level of uncertainty on

consumption, saving, the trade balance, and the current account.

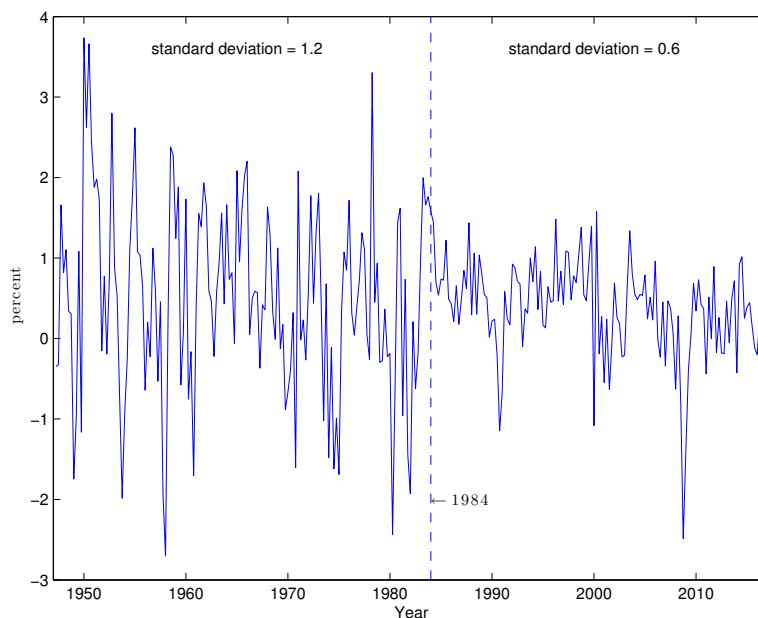
6.1 The Great Moderation

A number of researchers have documented that the volatility of U.S. output declined significantly starting in the early 1980s. This phenomenon has become known as *the Great Moderation*.¹ Figure 6.1 depicts the quarterly growth rate of real per capita gross domestic product in the United States over the period 1947:Q2 to 2017:Q4. It also shows, with a vertical line, the beginning of the Great Moderation in 1984. It is evident from the figure that output growth is much smoother in the post 1984 subsample than it is in the pre-1984 subsample.

The most commonly used measure of *volatility* in macroeconomic data is the standard deviation. According to this statistic, postwar U.S. output growth became half as volatile after 1983. Specifically, the standard deviation of quarter-to-quarter real per capita output growth was 1.2 percent over the period 1947 to 1983, but only 0.6 percent over the period 1984 to 2017. Some economists believe that the Great Moderation ended in 2007, just before the onset of the Global Financial Crisis, while others argue that the Great Moderation is still ongoing, as the volatility of output has returned to pre-crisis levels. The standard deviation of output growth is less than 0.1 percentage points lower over the period 1984 to 2006 than it is over the period 1984 to 2017. The discussion that follows includes the entire

¹Early studies documenting the Great Moderation are Kim and Nelson (1999) and McConnell and Perez-Quiróz (2000). Stock and Watson (2002) present a survey of this literature.

Figure 6.1: Quarterly real per capita GDP growth in the United States from 1947Q2 to 2017Q4



Notes. The figure shows that the growth rate of real GDP per capita in the United States has been less volatile since the beginning of the Great Moderation in 1984. Data Source: <http://www.bea.gov>.

post-1984 period in the definition of the Great Moderation.

6.2 Causes of the Great Moderation

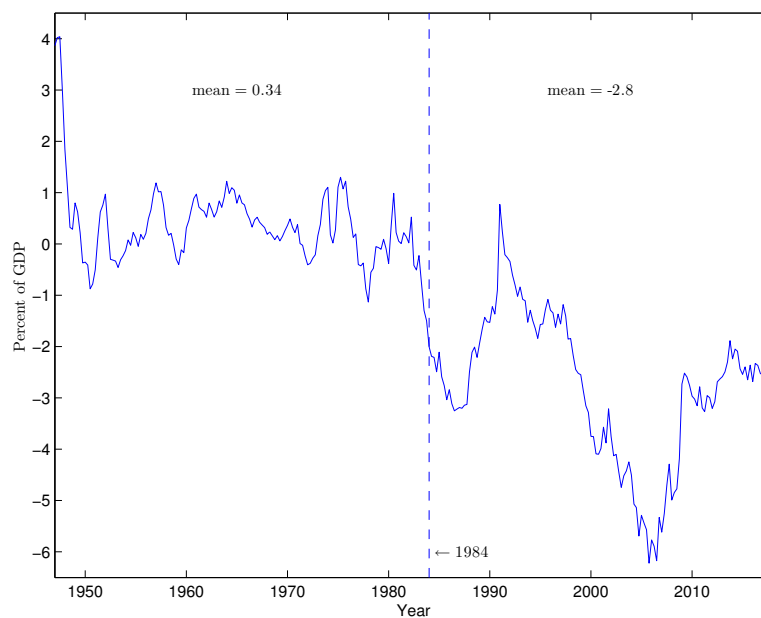
Researchers have put forward three alternative explanations of the Great Moderation: good luck, good policy, and structural change. The *good-luck hypothesis* states that by chance, starting in the early 1980s the U.S. economy has been blessed with smaller shocks. The *good policy hypothesis* maintains that starting with former Fed chairman Paul Volker's aggressive monetary

policy that brought to an end the high inflation of the 1970s and continuing with the low inflation policy of Volker's successor Alan Greenspan, the United States experienced a period of extraordinary macroeconomic stability. Good regulatory policy has also been cited as a cause of the Great Moderation. Specifically, the early 1980s witnessed the demise of regulation Q (or Reg Q), which imposed a ceiling on the interest rate that banks could pay on deposits. It became law in 1933, and its objective was to make banks more stable. Competition for deposits was thought to increase costs for banks and to force them into making riskier loans with higher expected returns. Thus allowing banks to pay interest on deposits was believed to contribute to bank failures.² However, Regulation Q introduced a financial distortion. Because of the ceiling on the interest rate on deposits, when expected inflation goes up (as it did in the 1970s) the real interest rate on deposits, given by the difference between the interest rate on deposits and expected inflation, falls and can even become negative, inducing depositors to withdraw their funds from banks. As a consequence, banks are forced to reduce the volume of loans generating a credit-crunch-induced recession.

Lastly, the *structural change hypothesis* maintains that the Great Moderation was in part caused by structural change, particularly in inventory management and in the financial sector. These technological developments, the argument goes, allowed firms to display smoother flows of sales, production, and employment, thereby reducing the amplitude of the business cycle.

²For more information on Reg Q see R. Alton Gilbert, "Requiem for Regulation Q: What It Did and Why It Passed Away," Federal Reserve Bank of St. Louis Review, February 1986, 68(2), pp. 22-37.

Figure 6.2: The current-account-to-GDP ratio in the United States from 1947Q1 to 2017Q4



Notes. The figure shows that the emergence of persistent current account deficits in the United States coincided with the beginning of the Great Moderation in 1984. Data Source: <http://www.bea.gov>.

We will not dwell on which of the proposed explanations of the Great Moderation has more merit. Instead, our interest is in possible connections between the Great Moderation and the significant current account deterioration observed in the United States over the post-1984 period.

6.3 The Great Moderation and the Emergence of Current Account Imbalances

The beginning of the Great Moderation coincided with a significant change in the sign and absolute size of the U.S. current account. Figure 6.2 displays the ratio of the current account to GDP in the United States over the period 1947Q1-2017Q4. The behavior of the current account is familiar from chapter 1. During the period 1947-1983 the United States experienced on average positive current account balances of 0.34 percent of GDP. Starting in the early 1980s, large current account deficits averaging 2.8 percent of GDP opened up.

Is the timing of the Great Moderation and the emergence of protracted current account deficits pure coincidence, or is there a causal connection between the two? To address this issue, we will explore the effects of changes in output uncertainty on the trade balance and the current account in the context of our theoretical framework of current account determination.

6.4 An Open Economy With Uncertainty

In the economy studied in chapter 3, the endowments Q_1 and Q_2 are known with certainty. What would be the effect of making the future endowment, Q_2 , uncertain? How would households adjust their consumption and saving decisions in period 1 if they knew that the endowment in period 2 could be either high or low with some probability? Intuitively, we should expect the emergence of *precautionary savings* in period 1. That is, an increase in

saving in period 1 to hedge against a bad income realization in period 2. The desired increase in savings in period 1 must be brought about by a reduction in consumption in that period. With the period-1 endowment unchanged and consumption lower, the trade balance must improve. By the same token, a decline in income uncertainty, like the one observed in the United States since the early 1980s, should be associated with a deterioration in the trade balance.

To formalize these ideas, consider initially an economy in which the stream of output is known with certainty and is constant over time. Specifically suppose that $Q_1 = Q_2 = Q$. Assume further that preferences are logarithmic with a unit subjective discount factor ($\beta = 1$), that is, the life-time utility function is given by

$$\ln C_1 + \ln C_2.$$

To simplify the analysis, assume that initial asset holdings are nil, that is, $B_0^* = 0$, and that the world interest rate is zero, or $r^* = 0$. In this case, the intertemporal budget constraint of the representative household is given by $C_2 = 2Q - C_1$. Using this expression to eliminate C_2 from the utility function, we have that the household's utility maximization problem consists in choosing C_1 so as to maximize $\ln C_1 + \ln(2Q - C_1)$. The first-order condition associated with this maximization problem is the derivative of this expression with respect to C_1 equated to zero, or $\frac{1}{C_1} - \frac{1}{2Q - C_1} = 0$. Solving for C_1 , we obtain $C_1 = Q$. It follows that the trade balance in period 1, given by $Q_1 - C_1$, is zero. That is, $TB_1 = 0$. In this economy,

households do not need to save or borrow in order to smooth consumption over time because the endowment stream is already perfectly smooth and the subjective discount rate and the interest rate are equal to each other.

Suppose now that the endowment in period 1 continues to be equal to Q , but that Q_2 is not known with certainty in period 1. Specifically, assume that with probability $1/2$ the household receives a positive endowment shock in period 2 equal to $\sigma > 0$, and that with equal probability the household receives a negative endowment shock in the amount of $-\sigma$. That is,

$$Q_2 = \begin{cases} Q + \sigma & \text{with probability } 1/2 \\ Q - \sigma & \text{with probability } 1/2 \end{cases}.$$

This is a mean-preserving increase in uncertainty in the sense that the expected value of the endowment in period 2, given by $\frac{1}{2}(Q + \sigma) + \frac{1}{2}(Q - \sigma)$ equals Q , which equals the endowment that the household receives in period 2 in the economy without uncertainty.

The standard deviation of the endowment in period 2 is given by σ . To see this, recall that the standard deviation is the square root of the variance and that, in turn, the variance is the expected value of squared deviations of output from its mean. The deviation of output from its mean is $Q + \sigma - Q = \sigma$ in the high-output state and $Q - \sigma - Q = -\sigma$ in the low-output state. Therefore, the variance of output in period 2 is given by $\frac{1}{2} \times \sigma^2 + \frac{1}{2} \times (-\sigma)^2 = \sigma^2$. The standard deviation of period-2 output is then given by $\sqrt{\sigma^2} = \sigma$. It follows that the larger σ is, the more volatile the period-2 endowment will be.

We must specify how households value uncertain consumption paths. We will assume that households care about the expected value of utility. Specifically, the lifetime utility function is now given by

$$\ln C_1 + E \ln C_2, \quad (6.1)$$

where E denotes expected value. Note that this preference formulation encompasses the preference specification we used in the absence of uncertainty. This is because when C_2 is known with certainty, then $E \ln C_2 = \ln C_2$.

The budget constraint of the household in period 2 is given by $C_2 = 2Q + \sigma - C_1$ in the good state of the world and by $C_2 = 2Q - \sigma - C_1$ in the bad state of the world. Therefore, expected lifetime utility is given by

$$\ln C_1 + \frac{1}{2} \ln(2Q + \sigma - C_1) + \frac{1}{2} \ln(2Q - \sigma - C_1).$$

The household chooses C_1 to maximize this expression. The first-order optimality condition associated with this problem is

$$\frac{1}{C_1} = \frac{1}{2} \left[\frac{1}{2Q + \sigma - C_1} + \frac{1}{2Q - \sigma - C_1} \right]. \quad (6.2)$$

The left-hand side of this expression is the marginal utility of consumption in period 1. The right-hand side is the expected marginal utility of consumption in period 2. This means that the optimal consumption choice equates the marginal utility of consumption in period 1 to the expected marginal utility of consumption in period 2.

Consider first whether the optimal consumption choice associated with

the problem without uncertainty, given by $C_1 = Q$, represents a solution in the case with uncertainty. If this were the case, then it would have to be true that

$$\frac{1}{Q} = \frac{1}{2} \left[\frac{1}{2Q + \sigma - Q} + \frac{1}{2Q - \sigma - Q} \right].$$

This expression can be simplified to

$$\frac{1}{Q} = \frac{1}{2} \left[\frac{1}{Q + \sigma} + \frac{1}{Q - \sigma} \right].$$

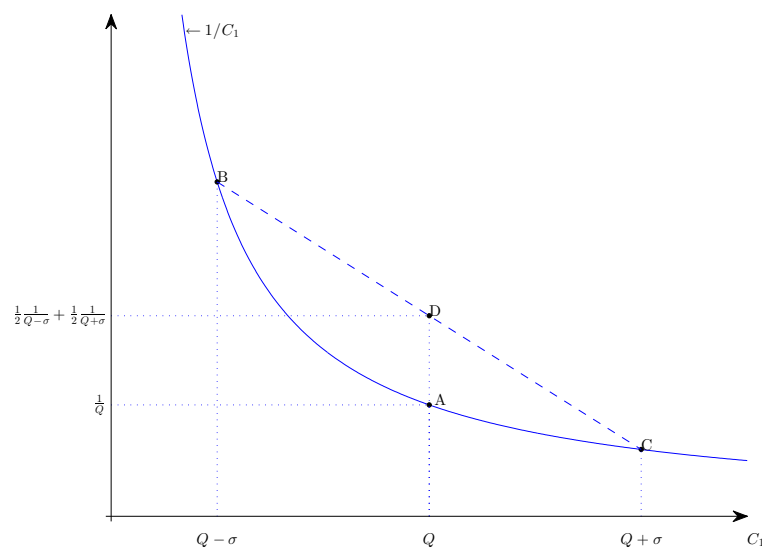
Further simplifying, we obtain

$$1 = \frac{Q^2}{Q^2 - \sigma^2},$$

which is impossible, given that $\sigma > 0$. We have thus shown that $C_1 \neq Q$. That is, a mean preserving increase in uncertainty induces households to choose a different level of period-1 consumption than the one they would choose under certainty.

Figure 6.3 provides a graphical representation of this result. It plots with a solid line the marginal utility of period-1 consumption as a function of C_1 (the left-hand side of equation 6.2). Suppose $C_1 = Q$. Then the marginal utility of period-1 consumption is equal to $1/Q$ (point A in the figure). In this case consumption in period 2 is either $(Q - \sigma)$ or $(Q + \sigma)$ and the marginal utility in period 2 is either $1/(Q - \sigma)$ or $1/(Q + \sigma)$ (points B or C, respectively). The expected marginal utility in period 2 is then given by $\frac{1}{2} \frac{1}{Q - \sigma} + \frac{1}{2} \frac{1}{Q + \sigma}$ (point D in the figure). Point D is above point A (because the marginal utility is convex), therefore when $C_1 = Q$ the marginal utility

Figure 6.3: Uncertainty and Precautionary Savings



Notes. The solid line plots the marginal utility of consumption in period 1, $1/C_1$, as a function of C_1 . In the case that $C_1 = Q$, the marginal utility of consumption in period 1, point A, lies below the expected marginal utility of consumption in period 2, given by point D.

of period-1 consumption is below the expected marginal utility of period-2 consumption. Therefore, we have shown that if we set $C_1 = Q$, then the left-hand side of optimality condition (6.2) is less than the right-hand side. In other words, if the consumer chose $C_1 = Q$, then the marginal utility of consumption in period 1 would be smaller than the expected marginal utility of consumption in period 2.

Now notice that the left-hand side of optimality condition (6.2) is decreasing in C_1 , whereas the right-hand side is increasing in C_1 . It follows that the value of C_1 that satisfies optimality condition (6.2) must be less than Q ,

$$C_1 < Q.$$

This means that an increase in uncertainty induces households to consume less and save more. By saving more in period 1, households avoid having to cut consumption by too much in the bad state of period 2. This type of saving is known as precautionary saving.

The trade balance in period 1 equals $Q - C_1$. It then follows that an increase in uncertainty causes an improvement in the trade balance. The current account equals $TB_1 + r_0 B_0^*$. Since we assume that the economy starts with zero debt ($B_0^* = 0$), we have that the current account equals the trade balance. Thus, we have that an increase in uncertainty leads to an improvement in the current account. Intuitively, in response to an increase in uncertainty, households use the current account as a vehicle to save in period 1.

Viewed through the lens of this model, the reduction in output volatil-

ity that came with the Great Moderation should have contributed to the observed concurrent deterioration of the U.S. current account.

It is evident from figure 6.3 that the positive relationship between uncertainty and precautionary saving depends on the convexity of the marginal utility of consumption. In other words, it is important that the third derivative of the period utility function $U(C)$ be positive. This requirement is satisfied in the economy analyzed here, where households have logarithmic period utility functions. Exercise 6.1 considers the case of a linear period-2 utility function. In this environment households are said to display *risk neutrality*. Exercise 6.2 considers the case of quadratic utility, which delivers *certainty equivalence*. In both cases the marginal utility of consumption ceases to be convex and therefore the connection between uncertainty and precautionary saving breaks down.

In the next section, we will analyze an environment in which precautionary saving disappears regardless of whether the marginal utility of consumption is convex or not.

6.5 Complete Asset Markets and the Current Account

In the model economy analyzed thus far, households face uninsurable income risk. This is because the only financial instrument available to them is one whose period-2 payoff is the same in the good and bad states. In principle, households would like to buy a portfolio of assets that pays more in the state in which the endowment is low than in the state in which the endowment

is high. Here, we introduce such possibility by assuming the existence of *state contingent claims*. In this environment, households do not need to rely on precautionary saving to cover themselves against the occurrence of the low-endowment state.

Formally, suppose that in period 1 the household can buy an asset that pays one unit of good in period 2 if the state of nature is good and zero units if the state is bad. Assume also that the household can buy an asset that pays one unit of good if the state of nature is bad and zero otherwise. Let P^g and P^b denote the prices of the assets that pay in the good and bad states, respectively, and B^g and B^b the quantity of each asset purchased by the household in period 1. This economy is said to feature *complete asset markets*, because households can buy asset portfolios with any payoff pattern across states in period 2. For example, if the household wishes to have a portfolio that pays x units of goods in the good state and y units in the bad state, then it must simply purchase x units of the asset that pays in the good state ($B^g = x$) and y units of the asset that pays in the bad state ($B^b = y$). This portfolio costs $P^g x + P^b y$ in period 1. By contrast, the single-bond economy studied in section 6.4 has *incomplete asset markets*, because households are restricted to buy asset portfolios with the same payoff in all states of nature in period 2.

The quantities B^g and B^b could be positive or negative. This means that we allow households to maintain *long positions* or *short positions* in different state-contingent claims. For example, the household could sell contingent claims that pay in the good state ($B^g < 0$) and buy contingent claims that pay in the bad state ($B^b > 0$).

The budget constraint of the household in period 1 is

$$C_1 + P^g B^g + P^b B^b = Q. \quad (6.3)$$

In period 2, there are two budget constraints, one for the good state and one for the bad state:

$$C_2^g = Q + \sigma + B^g \quad \text{and} \quad C_2^b = Q - \sigma + B^b, \quad (6.4)$$

where C_2^g and C_2^b denote consumption in period 2 in the good and bad states, respectively. Using these period budget constraints to eliminate C_1 , C_2^g , and C_2^b from the lifetime utility function, we can state the household's maximization problem as choosing an asset portfolio $\{B^g, B^b\}$ to maximize

$$\ln(Q - P^g B^g - P^b B^b) + \frac{1}{2} \ln(Q + \sigma + B^g) + \frac{1}{2} \ln(Q - \sigma + B^b).$$

The first-order optimality conditions associated with this utility maximization problem are

$$\frac{P^g}{C_1} = \frac{1}{2} \frac{1}{C_2^g} \quad (6.5)$$

and

$$\frac{P^b}{C_1} = \frac{1}{2} \frac{1}{C_2^b}. \quad (6.6)$$

These expressions equate the marginal rate of substitution between consumption in period 1 and consumption in a specific state of period 2 to the relative price of consumption in that particular state and present consumption. For example, in the first equation, the marginal rate of sub-

stitution between consumption in period 1 and consumption in the good state in period 2 is equal to $\frac{1}{2} \frac{1/C_2^g}{1/C_1}$ and the relative price of goods in the good state in period 2 in terms of goods in period 1 is P^g . The second optimality condition has a similar interpretation. Comparing the optimality conditions of the household's problem in this economy with the one in the incomplete-asset market model, given by equation (6.2), we see that now we have one first-order condition per state of nature in period 2, whereas in the incomplete-asset-market economy we had just one. The reason is that under complete markets the household can buy as many independent assets as there are states whereas in the incomplete-market economy the household has access to fewer assets than states (one asset in an environment with two states).

A risk-free bond is an asset that costs one unit of good in period 1 and pays $1 + r_1$ units of good in every state of period 2, where r_1 is the risk-free interest rate. Consider now constructing a portfolio of contingent claims that has the same payoff as the risk-free bond, that is, a portfolio that pays $1 + r_1$ in every state of period 2. This portfolio must contain $1 + r_1$ units of each of the two contingent claims. The price of this portfolio in period 1 is $(P^g + P^b)(1 + r_1)$. This price must equal the price of the risk free bond, namely 1, otherwise a pure arbitrage opportunity would allow agents to become infinitely rich. So we have that

$$1 + r_1 = \frac{1}{P^g + P^b}.$$

Thus, the gross risk-free interest rate is the inverse of the price of a portfolio

that pays one unit of good in every state of period 2.

As before, we assume that there is free capital mobility. This means that the domestic prices of the state-contingent claims must be equal to the corresponding world prices. Letting P^g and P^b denote the world prices of the state-contingent claims that pay in the good and bad states, respectively, we have that under free capital mobility

$$P^g = P^{g*}$$

and

$$P^b = P^{b*}.$$

The gross world interest rate, denoted $1 + r^*$, is given by

$$1 + r^* = \frac{1}{P^{g*} + P^{b*}}.$$

We assume that foreign investors make zero profits on average.³ The revenue of a foreign investor who sells B^g and B^b units of state contingent claims in period 1 is $P^{g*}B^g + P^{b*}B^b$. Suppose that the foreign investor uses these funds to buy a risk-free bond, which pays the interest rate r^* . In period 2, the foreign investor receives from this investment $(1 + r^*)(P^{g*}B^g + P^{b*}B^b)$. In period 2 the foreign investor must pay to the agents who bought the

³Zero expected profits can arise if the international capital market is competitive and foreign lenders are risk neutral. The no-expected-profit condition can also be the result of a competitive environment in which foreign investors are risk averse but hold a highly diversified portfolio. Think for example of a world populated by a continuum of small open economies identical to the one we are analyzing in which the endowment process is independent across countries. If foreign investors can invest in all countries, their profits become deterministic and equal to the expected value of profits in each individual country.

contingent claims B^g if the state is good and B^b if the state is bad. It follows that the profits of the foreign investor are state contingent and given by

$$\text{profit of foreign investor} = \begin{cases} (1 + r^*)(P^{g*}B^g + P^{b*}B^b) - B^g & \text{with probability } 1/2 \\ (1 + r^*)(P^{g*}B^g + P^{b*}B^b) - B^b & \text{with probability } 1/2 \end{cases}$$

Expected profits are then given by

$$\text{expected profit of foreign investor} = [(1+r^*)P^{g*} - \frac{1}{2}]B^g + [(1+r^*)P^{b*} - \frac{1}{2}]B^b.$$

The assumption that the foreign investor makes zero expected profits on any portfolio $\{B^g, B^b\}$ implies that

$$(1 + r^*)P^{g*} = \frac{1}{2}$$

and

$$(1 + r^*)P^{b*} = \frac{1}{2}.$$

Finally, we assume, as we did in the economy with incomplete asset markets studied in section 6.4, that the world interest rate is zero, $r^* = 0$, so that

$$P^{g*} = P^{b*} = \frac{1}{2}.$$

Using these prices to replace P^g and P^b in the first-order conditions of the

household, given in equations (6.5) and (6.6), we obtain

$$C_1 = C_2^g = C_2^b.$$

This means that the presence of complete asset markets allows households to completely smooth consumption across states of nature, $C_2^g = C_2^b$. In addition, because the subjective discount factor, β , and the pecuniary discount factor, $1/(1+r^*)$, are equal to each other, it is optimal for households to smooth consumption across time, $C_1 = C_2$.

Combining the result that $C_1 = C_2^g = C_2^b$ with the budget constraints in periods 1 and 2, given in equations (6.3) and (6.4), respectively, yields

$$B^g = -\sigma,$$

$$B^b = \sigma,$$

and

$$C_1 = C_2^g = C_2^b = Q.$$

The first two expressions say that the household takes a short position in contingent claims that pay in the good state and a long position in claims that pay in the bad state. In this way, households can transfer resources from the good state to the bad state in period 2, which allows them to smooth consumption across states. Notice the difference with the single-bond economy of section 6.4. There, households cannot transfer resources across states, because the financial instruments to do so are unavailable.

Instead, to self-insure in period 1 households must engage in precautionary saving to transfer resources to both states in period 2 through the single bond traded in the market. This is an inferior option because it forces them to transfer resources from period 1 to the good state in period 2, where they are not needed.

The trade balance in period 1 is given by

$$TB_1 = Q - C_1 = 0.$$

Because households start period 1 with zero net assets, the current account equals the trade balance and is therefore also equal to zero,

$$CA_1 = 0.$$

This result obtains regardless of the amount of uncertainty in the economy, that is, independently of the value of σ . We have therefore established that under complete financial markets precautionary saving is zero and the link between the level of uncertainty and the current account disappears.

Consequently, the main result of section 6.4, namely, that a decrease in uncertainty leads to a deterioration in the current account relies on the assumption that financial markets are incomplete. It is therefore natural to ask whether reality is better approximated by the assumption of complete markets or by the assumption of incomplete markets. Domestic financial markets could be incomplete for various reasons. For example, it could be the case that world financial markets themselves are incomplete. This

is not an unreasonable scenario. In the real world there are many more states of nature than just good and bad, as assumed here. Disturbances of many natures, including those stemming from policymaking, weather, natural catastrophes, and technological innovations, create a large, possibly infinite number of states of nature. In this context, it is not unconceivable that the number of available assets is insufficient to span all possible states of nature. Furthermore, even if international financial markets were complete, domestic financial markets may be incomplete if policymakers restrict access to a subset of international financial markets. For example, the government may prohibit trade in derivatives or in short term assets. To the extent that asset markets are not complete, uncertainty will continue to induce precautionary saving and hence variations in uncertainty will lead to variations in the current account.

6.6 Summing Up

In this chapter we studied the effect of uncertainty on the current account.

A summary of the main results are:

- In the postwar United States GDP growth was about half as volatile after 1984 than before. This phenomenon is known as the Great Moderation.
- Three main explanations of the Great Moderation have been proposed: good luck, good policy, and structural change.
- The Great Moderation coincided with the beginning of sizable and persistent current account deficits in the United States.
- A model of an open economy with uncertainty about future realizations

of income predicts that an increase in income volatility causes an increase in precautionary saving. In turn, the increase in precautionary saving leads to an improvement in the current account. Thus, the model predicts that the Great Moderation should have been associated with a worsening of the external accounts.

- The prediction that an increase in uncertainty results in elevated precautionary saving depends on the assumption that financial markets are incomplete. Under complete markets, households are able to insure against output volatility without resorting to precautionary saving. Consequently, under complete markets the positive relationship between the level of uncertainty and the current account disappears.

6.7 Exercises

Exercise 6.1 (Risk Neutrality) Redo the analysis in section 6.4 assuming that households are risk neutral in period 2. Specifically, assume that their preferences are logarithmic in period-1 but linear in period-2 consumption, $\ln C_1 + EC_2$. Assume that $Q = 1$.

1. Assume that $\sigma = 0$. Find the equilibrium values of C_1 and B_1^* .
2. Now assume that $\sigma > 0$. Find the equilibrium value of B_1^* . What is the predicted effect of the Great Moderation on the current account? Explain.

Exercise 6.2 (Certainty Equivalence) Consider a two-period, small, open, endowment economy populated by households with preferences described by the lifetime utility function

$$-\frac{1}{2}(C_1 - \bar{C})^2 - \frac{1}{2}E(C_2 - \bar{C})^2,$$

where \bar{C} represents a satiation level of consumption, and E denotes the mathematical expectations operator. In period 1, households receive an endowment $Q_1 = 1$ and have no assets or liabilities carried over from the past ($B_0^* = 0$). Households can borrow or lend in the international financial market at the world interest rate $r^* = 0$. Compute consumption and the current account in periods 1 and 2 under the following two assumptions regarding the endowment in period 2, denoted Q_2 :

1. Q_2 equals 1 and

2. Q_2 is random and takes the values 0.5 with probability 1/2 or 1.5 with probability 1/2.

Provide intuition for your findings.

Exercise 6.3 (The Current Account As Insurance Against Catastrophic Events)

Consider a two-period endowment economy populated by identical households with preferences defined over consumption in period 1, C_1 and consumption in period 2, C_2 , and described by the utility function

$$\ln C_1 + E \ln C_2,$$

where C_1 denotes consumption in period 1, C_2 denotes consumption in period 2, and E denotes the expected value operator. Each period, the economy receives an endowment of 10 units of food. Households start period 1 carrying no assets or debts from the past ($B_0^* = 0$). Financial markets are incomplete. There is a single internationally traded bond that pays the interest rate $r^* = 0$.

1. Compute consumption, the trade balance, the current account, and national saving in period 1.
2. Assume now that the endowment in period 1 continues to be 10, but that the economy is prone to severe natural disasters in period 2. Suppose that these negative events are very rare, but have catastrophic effects on the country's output. Specifically, assume that with probability 0.01 the economy suffers an earthquake in period 2 that causes the endowment to drop by 90 percent with respect to period 1. With

probability 0.99, the endowment in period 2 is 111/11. What is the expected endowment in period 2? How does it compare to that of period 1?

3. What percent of period-1 endowment will the country export? Compare this answer to what happens under certainty and provide intuition.
4. Suppose that the probability of the catastrophic event increases to 0.02, all other things equal. Compute the mean and standard deviation of the endowment in period 2. Is the change in probability mean preserving?
5. Calculate the equilibrium levels of consumption and the trade balance in period 1.
6. Compare your results with those pertaining to the case of 0.01 probability for the catastrophic event. Provide interpretation.

Exercise 6.4 (Interest-Rate Uncertainty) Consider a two-period economy inhabited by a large number of identical households with preferences described by the utility function

$$\ln C_1 + \ln C_2,$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively. Households are endowed with $Q > 0$ units of consumption goods each period, and start period 1 with no assets or debt carried over from the past

($B_0^* = 0$). In period 1, households can borrow or lend by means of a bond, denoted B , that pays the world interest rate, denoted r^* . Assume that $r^* = 0$.

1. Derive the optimal levels of consumption in periods 1 and 2, as functions of exogenous parameters only. Derive the equilibrium levels of the trade balance and the current account.

Now assume that the world interest rate is not known with certainty in period 1, that is, the one-period bond carries a floating rate. Specifically, assume that r^* is given by

$$r^* = \begin{cases} \sigma & \text{with probability } 1/2 \\ -\sigma & \text{with probability } 1/2 \end{cases},$$

where $\sigma \in (0, 1)$ is a parameter. In this economy, financial markets are incomplete, because agents have access to a single bond in period 1. Preferences are described by the utility function

$$\ln C_1 + E \ln C_2,$$

where E denotes the expectations operator. The present economy nests the no-uncertainty economy described above as a special case in which $\sigma = 0$.

2. Write down the household's budget constraint in periods 1 and 2. To this end, let C_2^1 and C_2^2 denote consumption in period 2 when the world interest rate is σ and $-\sigma$, respectively. Note that the budget constraint in period 2 is state contingent.

3. Derive the optimality conditions associated with the household's problem.
4. Show whether the equilibrium level of consumption in period 1 is greater than, less than, or equal to the one that arises when $\sigma = 0$.
5. Find the sign of the trade balance in equilibrium. Compare your answer to the one for the case $\sigma = 0$ and provide intuition. In particular, discuss why a mean preserving increase in interest-rate uncertainty affects the trade balance in period 1 the way it does.
6. Are the results obtained above due to the particular (logarithmic) preference specification considered? To address this question, show that all of the results obtained above continue to obtain under a more general class of preferences, namely, the class of *CRRA preferences*

$$\frac{C_1^{1-\gamma} - 1}{1-\gamma} + E \frac{C_2^{1-\gamma} - 1}{1-\gamma},$$

for $\gamma > 0$, which encompasses the log specification as a special case when $\gamma \rightarrow 1$.

7. Finally, show that interest rate uncertainty does have real effects when the desired asset position in the absence of uncertainty is nonzero. To this end, return to the log preference specification and assume that the endowment in period 1 is zero and that the endowment in period 2 is $Q > 0$. How does the trade balance in period 1 compare under no uncertainty ($\sigma = 0$) and under uncertainty ($\sigma > 0$)?

Exercise 6.5 (Complete Asset Markets and the Risk-Free Interest Rate)

The model with complete financial markets studied in section 6.5 assumes that the risk-free world interest rate is nil, $r^* = 0$. Derive the results of that section under the assumption that r^* is positive and then answer the following questions:

1. Do households continue to perfectly smooth consumption across states?
Explain.
2. Do households continue to perfectly smooth consumption across time?
Explain.
3. Find the sign of the current account in period 1 and provide intuition.
4. Does the model continue to predict that the level of uncertainty has no effect on the current account? Why or why not?

Chapter 7

Large Open Economies

Thus far, we have analyzed the determination of the current account in a small open economy. A defining feature of a small open economy is that even if the country borrows or lends a large sum relative to its output in international financial markets, it will not affect the world interest rate. The reason is that the economy is too small to make a dent in the world supply or demand of funds. The story is different when the country is large. Suppose, for example, that, due to an expected increase in future output, the United States at current world interest rates wishes to increase its current account deficit by 5 percent of GDP. In this case, the world financial market will face an additional demand for funds of about 1 trillion dollars. This development is likely to move interest rates up, which would have repercussions both in the United States and in the rest of the world.

In this chapter, we present a framework suitable for analyzing the determination of the current account, the world interest rate, and other macroeconomic indicators in large open economies. The model will build on the

microfoundations derived in previous chapters. This makes sense, because the problem of a household or a firm should not be different whether the household or the firm is located in a large or small economy.

7.1 A Two-Country Economy

Let's divide the world into two regions, the United States (US) and the rest of the world (RW). Because a U.S. current account deficit represents the current account surplus of the rest of the world and conversely, a U.S. current account surplus is a current account deficit of the rest of the world, it follows that the world current account must always be equal to zero; that is,

$$CA^{US} + CA^{RW} = 0,$$

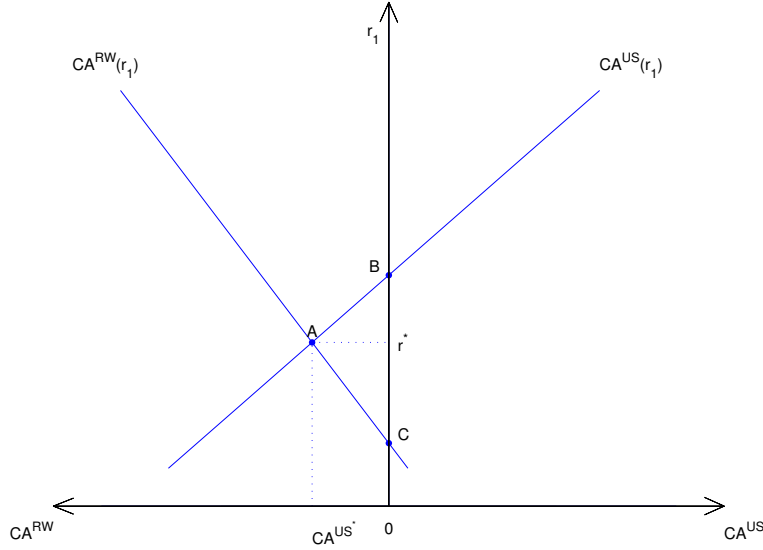
where CA^{US} and CA^{RW} denote, respectively, the current account balances of the United States and the rest of the world.

As we saw in section 5.5 of chapter 5, the current account schedule is an increasing function of the interest rate and other variables. So we can write

$$CA_1^{US} = CA_+^{US}(r_1),$$

where CA_1^{US} denotes the current account balance in the United States in period 1, r_1 is the interest rate in period 1, and $CA_+^{US}(\cdot)$ is an increasing function. Intuitively, an increase in the interest rate induces U.S. households to increase savings in period 1 and U.S. firms to cut investment in the same period. As a result, the current account of the United States improves as

Figure 7.1: Current account determination in a large open economy



Notes. The figure depicts the current account schedules of the United States, $CA^{US}(r_1)$, and the rest of the world, $CA^{RW}(r_1)$. The horizontal axis measures from left to right the current account balance of the United States and from right to left the current account balance of the rest of the world. Equilibrium occurs at the intersection of the current account schedules of the United States and the rest of the world and is marked by point A. In equilibrium, the current account is $CA^{US*} < 0$ in the United States and $-CA^{US*} > 0$ in the rest of the world. The equilibrium interest rate is r^* . If the two economies were closed, the equilibrium would be at point B in the United States and at point C in the rest of the world.

the interest rate increases.

Similarly, the current account of the rest of the world is an increasing function of the interest rate,

$$CA_1^{RW} = CA^{RW}(r_1),$$

where CA_1^{RW} denotes the current account balance of the rest of the world in period 1, and $CA^{RW}(\cdot)$ is an increasing function.

Figure 7.1 shows the current account schedules of the United States and the rest of the world. The current account of the United States is measured from left to right, so the current account schedule of the United States is upward sloping in the graph. The current account of the rest of the world is measured from right to left, so the current account schedule of the rest of the world is downward sloping in the graph. To the left of 0 the rest of the world runs a current account surplus and the United States a current account deficit, whereas to the right of 0, the United States runs a current account surplus and the rest of the world a current account deficit.

Equilibrium in the world capital markets is given by the intersection of the current account schedules of the United States and the rest of the world. In the figure, the equilibrium is marked by point A, at which the current account of the United States is $CA^{US*} < 0$ (a deficit), the current account of the rest of the world is $-CA^{US*} > 0$ (a surplus), and the world interest rate is r^* .

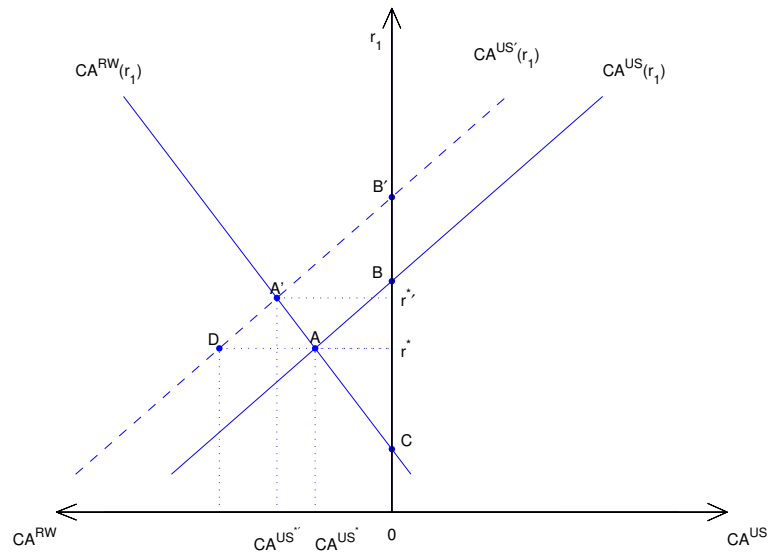
If the United States were a closed economy, its equilibrium would be at point B, where the U.S. current account is nil. The U.S. interest rate would be larger than r^* . This makes sense, because at r^* the desired current account balance in the United States is negative, so a larger interest rate is required to induce households to save more and firms to invest less. If the United States were a closed economy, the equilibrium in the rest of the world would be at point C. Of course, like in the United States, the current account in the rest of the world would be zero. Unlike in the United States, however, the equilibrium interest rate in the rest of the world would be below r^* . This is because at r^* the desired current account in the rest of the world

is positive, so a fall in the interest rate is required to induce households to increase spending and cut savings in period 1 and to induce firms to expand investment in physical capital.

7.2 An Investment Surge in the United States

Suppose that in period 1 firms in the United States learn that capital will be more productive in period 2. This could happen, for example, because of a technological improvement, such as fracking, discovered in period 1 that is expected to be in place in period 2, or because of a forecast of better weather conditions in period 2 that will enhance the productivity of farm land. As we analyzed in section 5.6.3 of chapter 5, in response to this positive news, U.S. firms desire to increase investment at any given interest rate. Thus, the U.S. investment schedule shifts up and to the right. Also, U.S. households, in anticipation of higher future incomes generated by the investment boom, reduce current savings at any given interest rate, so that the U.S. saving schedule shifts up and to the left (see figure 5.19 in chapter 5). This means that the current account schedule of the United States, which is the difference between the saving and investment schedules, shifts up and to the left as shown in figure 7.2. In the figure, the original U.S. current account schedule is $CA^{US}(r_1)$ and is shown with a solid line, and the new current account schedule is $CA^{US'}(r_1)$ and is shown with a broken line. A shift up and to the left in the current account schedule happens independently of the size of the U.S. economy. The current account schedule of the rest of the world, $CA^{RW}(r_1)$, is unaffected by the investment surge in the United

Figure 7.2: Current Account Adjustment to an Investment Surge in the United States



Notes. The figure depicts the effects of an investment surge in the United States on the world interest rate and the current account balances of the United States and the rest of the world. The investment surge shifts the current account schedule of the United States up and to the left as shown with a broken line. The equilibrium before the investment surge is at point A, and the equilibrium after the investment surge is at point A', where the world interest rate is higher, the current account deficit of the United States is larger, and the current account surplus of the rest of the world is higher.

States.

The equilibrium prior to the investment surge is a point A, where the U.S. current account equals CA^{US*} , the current account of the rest of the world equals $-CA^{US*}$, and the world interest rate equals r^* . After the investment surge, the equilibrium is given by point A', where the schedule $CA^{US'}(r_1)$ and the schedule $CA^{RW}(r_1)$ intersect. In the new equilibrium, the world interest rate is higher and equal to $r^{*'} > r^*$. This is because the higher demand for international funds caused by the investment surge would result in an excess demand for funds at the original interest rate r^* . In the figure, this excess demand is given by the horizontal distance between points D and A. The increase in the world interest rate from r^* to $r^{*'}$ induces the demand for funds by the United States to shrink and the supply of funds by the rest of the world to increase, thereby eliminating the excess demand for funds in the international financial market. In the new equilibrium, the current account of the United States deteriorates to $CA^{US*'} < CA^{US*}$ and the current account of the rest of the world improves to $-CA^{US*'} > -CA^{US*}$. In sum, because the U.S. is a large open economy, the investment surge produces a large increase in the demand for loans, which drives world interest rates up. As a result, the deterioration in the U.S. current account is not as pronounced as the one that would have resulted if the interest rate had remained unchanged (point D in the figure).

Note further that the increase in the U.S. interest rate is smaller than the one that would have occurred if the U.S. economy was closed, given by the vertical distance between points B and B'.

7.3 Microfoundations of the Two-Country Model

In the previous two sections, the starting point of the analysis was the current account schedule of each country. In this section, we derive the equilibrium levels of the current account and the world interest rate starting from the decisions of individual households. The analysis will provide insight on how different features of the economy, such as current and future endowments in the domestic economy and the rest of the world affect the world interest rate and global imbalances.

Consider a two-period economy composed of two countries, the United States and the rest of the world. Suppose that each country is populated by a large number of households. In both countries, households derive utility from the consumption of a tradable perishable good. Preferences of households in the United States and the rest of the world are given by, respectively,

$$\ln C_1^{US} + \ln C_2^{US} \tag{7.1}$$

and

$$\ln C_1^{RW} + \ln C_2^{RW},$$

where C_t^{US} and C_t^{RW} denote consumption in period $t = 1, 2$ in the United States and the rest of the world.

Let the endowment of households in the United States and the rest of the world be given by Q_t^{US} and Q_t^{RW} for $t = 1, 2$. Households can borrow or lend at the interest rate r_1 . There is free capital mobility in the world, so households in both countries trade financial assets at the same interest rate.

Let B_t^{US} and B_t^{RW} for $t = 0, 1, 2$, denote the amount of bonds held in period t by households in the United States and the rest of the world, respectively. Suppose that households start with no debt or assets at the beginning of period 1, that is, $B_0^{US} = B_0^{RW} = 0$. Also, since no one is alive in period 3, asset holdings at the end of period 2 must be zero in both countries, that is, the transversality condition $B_2^{US} = B_2^{RW} = 0$ must hold.

In period 1 the budget constraint of U.S. households is given by

$$C_1^{US} + B_1^{US} = Q_1^{US}. \quad (7.2)$$

According to this expression, households allocate their period-1 endowment to consumption or savings. In period 2, the budget constraint of the U.S. household is

$$C_2^{US} = Q_2^{US} + (1 + r_1)B_1^{US}, \quad (7.3)$$

which says that in period 2 households consume their endowment plus the principal and interest of assets accumulated in period 1.

The U.S. household's problem is to maximize its lifetime utility subject to the budget constraints in each period. The problem of a household in a large open economy is the same as that of a household in a small open economy. Thus, we can proceed in the same way as we did in chapter 3, to characterize the household's optimal consumption-saving choice. Use the budget constraint in period 1 to eliminate B_1^{US} from the period-2 budget constraint, to obtain the present value budget constraint:

$$C_1^{US} + \frac{C_2^{US}}{1 + r_1} = Q_1^{US} + \frac{Q_2^{US}}{1 + r_1}. \quad (7.4)$$

Solve this present value budget constraint for C_2^{US} and use it to eliminate C_2^{US} from the utility function (7.1). This yields:

$$\ln C_1^{US} + \ln [(1 + r_1)(Q_1^{US} - C_1^{US}) + Q_2^{US}].$$

Taking the derivative of this expression with respect to C_1^{US} , equating it to 0, and rearranging, we obtain the optimal level of consumption in period 1,

$$C_1^{US} = \frac{1}{2} \left(Q_1^{US} + \frac{Q_2^{US}}{1 + r_1} \right). \quad (7.5)$$

This expression is familiar from chapter 3. Consumption in period 1 is increasing in both endowments as both make the household richer. Also, period-1 consumption is decreasing in the interest rate, as an increase in the interest rate makes savings more attractive, or, equivalently, makes current consumption relatively more expensive than future consumption.

The current account of the United States in period 1 is given by the change in the country's net foreign asset position, that is,

$$CA_1^{US} = B_1^{US} - B_0^{US}.$$

Recalling that $B_0^{US} = 0$, we have that

$$CA_1^{US} = B_1^{US}. \quad (7.6)$$

Using the period-1 budget constraint (7.2) to get rid of B_1^{US} , we have that

$$CA_1^{US} = Q_1^{US} - C_1^{US},$$

which says that the current account in period 1 equals the trade balance in period 1. This is the case because of the maintained assumption of no initial assets, which implies that net investment income in period 1 is nil. Finally, replacing C_1^{US} by its optimal level given in equation (7.5), we obtain the current account schedule of the United States,

$$CA^{US}(r_1) = \frac{1}{2}Q_1^{US} - \frac{1}{2} \frac{Q_2^{US}}{1+r_1}. \quad (7.7)$$

The current account schedule is increasing in the interest rate r_1 . This is because as the interest rate increases households become more attracted to saving. Also, the current account is increasing in the current endowment, Q_1^{US} , and decreasing in the future endowment, Q_2^{US} . This is because households like to smooth consumption over time, so an increase in the period-1 endowment induces them to save part of it for the future, and, similarly, an increase in the period-2 endowment induces them to borrow against part of it to increase period-1 consumption.

Households in the rest of the world are identical to U.S. households except for their endowments. In particular, they have the same preferences, start with zero assets in period 1, must end period 2 with zero assets, and face the same interest rate. This means that the optimal level of consumption in period 1 by households in the rest of the world is given by equa-

tion (7.5), with the superscript US replaced by the superscript RW , that is,

$$C_1^{RW} = \frac{1}{2} \left(Q_1^{RW} + \frac{Q_2^{RW}}{1+r_1} \right).$$

And the current account schedule of the rest of the world is identical to that of the United States, equation (7.7), with the corresponding change of superscripts,

$$CA^{RW}(r_1) = \frac{1}{2} Q_1^{RW} - \frac{1}{2} \frac{Q_2^{RW}}{1+r_1}. \quad (7.8)$$

Like the current account schedule of the United States, the current account schedule of the rest of the world is increasing in the interest rate and in the period-1 endowment and decreasing in the period-2 endowment. If one were to plot the current account schedules of the United States and the rest of the world, equations (7.7) and (7.8), they would look qualitatively like those shown in figure 7.1.

The equilibrium world interest rate, r^* , is the interest rate that guarantees that the world current account is zero, that is, it is the interest rate that satisfies

$$CA^{US}(r^*) + CA^{RW}(r^*) = 0. \quad (7.9)$$

Using equations (7.7) and (7.8) to replace $CA^{US}(r^*)$ and $CA^{RW}(r^*)$ from this equilibrium condition and rearranging, we obtain

$$r^* = \frac{Q_2^{US} + Q_2^{RW}}{Q_1^{US} + Q_1^{RW}} - 1. \quad (7.10)$$

This expression says that the equilibrium world interest rate is increasing in the growth rate of the world endowment. It is intuitive that if the world

endowment in period 2, $Q_2^{US} + Q_2^{RW}$, is larger than the world endowment in period 1, $Q_1^{US} + Q_1^{RW}$, the world interest rate is high, as, on average, households would like to borrow against the more abundant period-2 endowment. The increase in the interest rate is necessary to dissuade the world as whole from borrowing, as this is impossible. Note that the interest rate is independent of how the endowments are distributed across countries. For example, it does not matter whether one country is richer than the other or whether one country is rich in period 1 and poor in period 2 and the reverse in the other country.

7.4 International Transmission of Country-Specific Shocks

An important difference between a small and a large economy is that aggregate shocks that originate in the former do not affect the rest of the world, whereas aggregate shocks that originate in the latter impact the rest of the world through changes in international prices, such as the world interest rate.

Consider, for example, how an increase in the period-1 endowment of the United States, Q_1^{US} , transmits to other countries. By equilibrium condition (7.10), the world interest rate falls. This is because households in the United States, to smooth consumption over time, wish to save part of their increased period-1 endowment, elevating the world supply of funds. A fall in the world interest rate eliminates the excess supply of funds in international capital markets by inducing households in the rest of the world and

in the United States to increase current spending. In the United States, both the increase in the period-1 endowment and the fall in the interest rate induce households to increase period-1 consumption. The rest of the world experiences only a fall in the interest rate (no change in endowments), which stimulates consumption in period 1. Thus, the increase in the current endowment in the United States causes an increase in consumption not only domestically but also abroad.

The cross-country comovement of period-1 consumption is quite different when the shock takes the form of future expected movements in endowments. Consider an expected increase in the U.S. endowment in period 2, Q_2^{US} . By equation (7.10), the world interest rate increases, as U.S. households wish to borrow against their now higher future income. In the rest of the world, the increase in the interest rate depresses period-1 consumption. Since the world endowment in period 1 did not change, the fall in period-1 consumption in the rest of the world means that period-1 consumption in the United States must increase. Thus, period-1 consumption moves in opposite directions across countries.

7.5 Country Size and the International Transmission Mechanism

An implicit assumption of the preceding analysis is that the United States and the rest of the world have equal populations. This can be seen from the fact that we equated the bond holdings of the United States with the bond holdings of the individual household in the United States and, similarly, we

equated the bond holdings of the rest of the world with the bond holdings of the individual household in the rest of the world. That is, we assumed that B_1^{US} stands for both the individual and aggregate bond holdings in the United States, and that B_1^{RW} stands for both the individual and aggregate bond holdings of the rest of the world. This means that we assumed that both countries had a population of one household. Here, one household could stand for one thousand households, or one million households, or, in general, any equal number of households in both countries. But what if the number of households were different in both countries? How would the world interest rate be affected by differences in country size? How would the international transmission of domestic shocks be affected by the size of the country?

To address these questions, let's assume that the United States is populated by N^{US} identical households and the rest of the world by N^{RW} identical households. We continue to assume that households in both countries differ only in their endowments. The U.S. net foreign asset position in period 1 is then given by $N^{US}B_1^{US}$, where, as before, B_1^{US} denotes the bond holdings of the individual U.S. household in period 1. Then, the current account of the United States is given by

$$CA_1^{US} = N^{US}B_1^{US},$$

which is a generalization of equation (7.6) when the country is populated by N^{US} identical households. (Recall that initial assets, B_0^{US} , are assumed to be zero.) Use the period-1 budget constraint of the household, equa-

tion (7.2), to eliminate B_1^{US} . This yields

$$CA_1^{US} = N^{US}(Q_1^{US} - C_1^{US}).$$

Now replace C_1^{US} by its optimal level, given in equation (7.5), to obtain

$$CA^{US}(r_1) = \frac{N^{US}}{2} \left(Q_1^{US} - \frac{Q_2^{US}}{1 + r_1} \right). \quad (7.11)$$

Since the rest of the world is identical to the United States except for the population size and the endowments, we have that its current account schedule is given by

$$CA^{RW}(r_1) = \frac{N^{RW}}{2} \left(Q_1^{RW} - \frac{Q_2^{RW}}{1 + r_1} \right). \quad (7.12)$$

Combine (7.11) and (7.12) with the market clearing condition in the world financial market, given by equation (7.9), to obtain

$$\frac{N^{US}}{2} \left(Q_1^{US} - \frac{Q_2^{US}}{1 + r^*} \right) + \frac{N^{RW}}{2} \left(Q_1^{RW} - \frac{Q_2^{RW}}{1 + r^*} \right) = 0. \quad (7.13)$$

This is one equation in one unknown, the equilibrium world interest rate r^* .

Solving for r^* we obtain

$$1 + r^* = \frac{N^{US}Q_2^{US} + N^{RW}Q_2^{RW}}{N^{US}Q_1^{US} + N^{RW}Q_1^{RW}}.$$

Let $\alpha \equiv N^{US}/(N^{US} + N^{RW})$ denote the share of the U.S. population in the

world population. Then we can express the world interest rate as

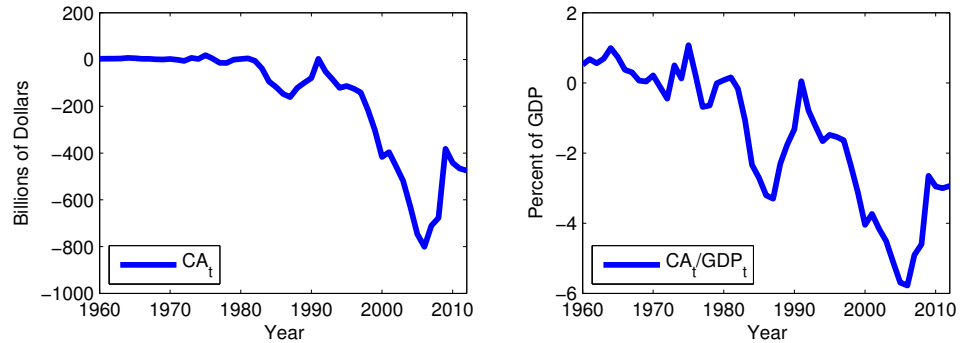
$$r^* = \frac{\alpha Q_2^{US} + (1 - \alpha) Q_2^{RW}}{\alpha Q_1^{US} + (1 - \alpha) Q_1^{RW}} - 1. \quad (7.14)$$

This expression shows that the larger is the U.S. economy, that is, the larger is α , the more important U.S. endowment shocks will be for the determination of the world interest rate. In the limiting case in which the rest of the world is a small economy so that $1 - \alpha$ approaches zero, the world interest rate is unaffected by shocks in the rest of the world. In particular, as $1 - \alpha$ approaches zero, the world interest rate becomes

$$r^* = \frac{Q_2^{US}}{Q_1^{US}} - 1.$$

This result provides a justification for the assumption maintained in previous chapters that a small open economy takes the world interest rate as exogenously given. Furthermore, the above expression shows that shocks in the large economy are transmitted to the small economy through variations in the world interest rate. In particular, a temporary output increase in the United States, that is, an increase in Q_1^{US} lowers the world interest and stimulates current consumption spending in the rest of the world resulting in a deterioration of its current account, see equation (7.12). Also a future expected increase in U.S. output, that is, an expected increase in Q_2^{US} , causes an increase in the world interest rate, a contraction in consumption in the rest of the world, and an improvement of the current account of the rest of the world.

Figure 7.3: The U.S. Current Account Balance: 1960-2012



Data Source: Bureau of Economic Analysis.

7.6 The Global Saving Glut Hypothesis

In the decade preceding the great recession of 2007, the U.S. current account deficit experienced a dramatic increase from about \$200 to \$800 billion dollars. (See the left panel of figure 7.3.) This \$600 billion dollar increase brought the deficit from a relatively modest level of 1.5 percent of GDP in 1996 to 6 percent of GDP by 2006. (See the right panel of figure 7.3.) With the onset of the great recession of 2007, the ballooning of the current account deficits came to an abrupt stop. By 2009, the current account deficit had shrunk back to less than 3 percent of GDP. What factors are responsible for these large swings in the U.S. current account? In particular, we wish to know whether the recent rise and fall in the current account deficit was primarily driven by domestic or external factors.

7.6.1 Two Competing Hypotheses

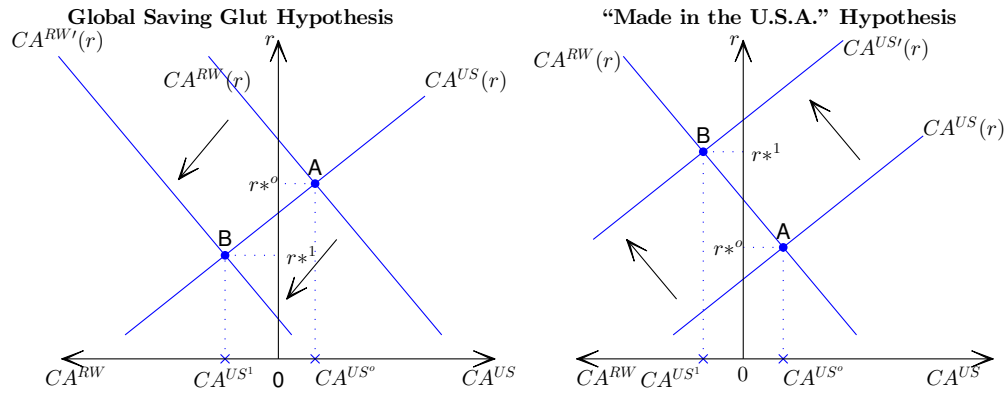
In 2005 Ben Bernanke, then a governor of the Federal Reserve Board, delivered a speech in which he argued that the deterioration in the U.S. current account deficit was caused by external factors.¹ He coined the term ‘global saving glut’ to refer to these external factors. In particular, Bernanke argued that the rest of the world experienced a heightened desire to save but did not have incentives to increase domestic capital formation in a commensurate way. As a result, the current account surpluses of the rest of the world had to be absorbed by current account deficits in the United States.

Much of the increase in the desired current account surpluses in the rest of the world during this period originated in higher desired savings in emerging market economies. In particular, Bernanke attributes the increase in the desire of emerging countries to save to two factors. One factor is an increase in foreign reserve accumulation to avoid or be better prepared to face future external crises of the type that had afflicted emerging countries in the 1990s. The second factor was government induced foreign currency depreciation aimed at promoting export-led growth. Bernanke also cites an external factor originating in developed countries, namely, an increase in saving rates in preparation for an aging population.

The global saving glut hypothesis was unconventional at the time. The more standard view was that the large U.S. current account deficits were the results of economic developments inside the United States and unrelated to external factors. In particular, it was argued that financial innovation in

¹Bernanke, Ben S., “The Global Saving Glut and the U.S. Current Account Deficit,” Homer Jones Lecture, St. Louis, Missouri, April 14, 2005.

Figure 7.4: U.S. Current Account Deterioration: Global Saving Glut or “Made in the U.S.A.”



the United States had induced low private savings rates and over-investment in residential housing. Bernanke refers to this alternative hypothesis as the “Made in the U.S.A.” view.

How can we tell which view is right, the global saving glut hypothesis or the “Made in the U.S.A.” hypothesis? To address this question, we use the graphical tools developed in section 7.1.

The left panel of figure 7.4 illustrates the effect of a desired increase in savings in the rest of the world. The initial position of the economy, point A, is at the intersection of the CA^{US} and CA^{RW} schedules. In the initial equilibrium, the U.S. current account equals CA^{US^0} and the world interest rate equals r^{*0} . The increase in the desired savings of the rest of the world shifts the current account schedule of the rest of the world down and to the left as depicted by the schedule CA'^{RW} . The new equilibrium, point B, features a deterioration in the current account of the United States from CA^{US^0} to CA^{US^1} and a fall in the world interest rate from r^{*0} to r^{*1} .

Intuitively, the United States will borrow more from the rest of the world only if it becomes cheaper to do so, that is, only if the interest rate falls. This prediction of the model implies that if the global saving glut hypothesis is valid, then we should have observed a decline in the interest rate.

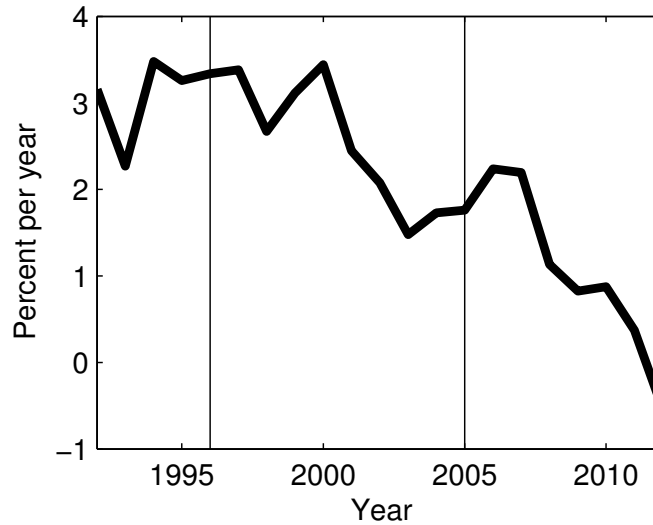
The “Made in the U.S.A.” hypothesis is illustrated in the right hand panel of figure 7.4. Again, in the initial equilibrium, point A , the U.S. current account equals CA^{US^0} and the world interest rate equals r^{*0} . Under this view, the current account schedule of the rest of the world is unchanged. Instead the current account schedule of the United States shifts up and to the left, as depicted by the schedule CA^{US^1} . The new equilibrium, point B , features a deterioration in the current account of the United States from CA^{US^0} to CA^{US^1} and a rise in the world interest rate from r^{*0} to $r^{*1} > r^{*0}$.

Both hypotheses can explain a deterioration in the U.S. current account. However, the global saving glut hypothesis implies that the current account deterioration should have been accompanied by a decline in world interest rates, whereas the “Made in the U.S.A.” hypothesis implies that world interest rates should have gone up. Hence we can use data on the behavior of interest rates to find out which hypothesis is right.

Figure 7.5 plots the world interest rate from 1992 to 2012.² It shows that the large current account deterioration in the United States was associated with a significant fall in the interest rate, giving credence to the global saving glut hypothesis.

²The world interest rate is computed as the difference between the 10-year constant maturity Treasury rate and expected inflation. Expected inflation in turn is measured as the median CPI-inflation forecast over the next 10 years and is taken from the Survey of Professional Forecasters.

Figure 7.5: The World Interest Rate: 1992-2012



Notes. The world interest rate is measured as the difference between the rate on 10-year U.S. Treasury securities and expected inflation.

7.6.2 The ‘Made in the U.S.A.’ Hypothesis Strikes Back

With the onset of the global financial crisis of 2007, the trajectory of the U.S. current account displayed a sharp reversal. Specifically, as can be seen in figure 7.3, the current account deficit shrunk from a peak of 6 percent of GDP in 2006 to less than 3 percent of GDP in 2009. Can the global saving glut hypothesis also explain this development?

Under the global saving glut hypothesis, the reversal in the current account deficit would be attributed to a decline in desired savings in the rest of the world. Again we can use the graphical tools developed earlier in this chapter to evaluate the plausibility of this view. Consider the left panel of figure 7.4. Assume that right before the beginning of the global financial crisis the economy is at point B, where the world interest rate is equal to r^{*1}

and the U.S. current account deficit is equal to CA^{US^1} . A decline in desired savings in the rest of the world shifts the current account schedule of the rest of the world up and to the right. For simplicity, assume that the current account schedule of the rest of the world goes back to its original position, given by CA^{RW} . The current account schedule of the United States does not change position. The new equilibrium is given by point A . The shift in the current account schedule of the rest of the world causes the U.S. current account to improve from CA^{US^1} to CA^{US^0} and the interest rate to rise from r^{*1} to r^{*0} . It follows that under the global saving glut hypothesis, the V-shaped U.S. current account balance observed around the global financial crisis should have been accompanied by a V-shaped pattern of the interest rate. However, figure 7.5 shows that the interest rate does not display such a pattern. In fact, during the crisis the interest rate declined further rejecting the global saving glut hypothesis as the dominant factor of the U.S. current account reversal.

We conclude that the global saving glut hypothesis presents a plausible explanation for the observed developments in the U.S. current account deficit over the pre-crisis period. At the same time, the behavior of the interest rate, suggests that the dynamics of the U.S. current account during the crisis were not primarily driven by external factors.

So what drove the current account improvements during the crisis? Many academic and policy observers have argued that the bursting of a bubble in the U.S. housing market led to a switch in the spending pattern of U.S. households and firms away from current consumption and investment and in favor of savings. In terms of the graphical apparatus developed in this

chapter, this expenditure switch shifts the current account schedule of the United States from its position just before the beginning of the global financial crisis down and to the right. In the right panel of figure 7.4, the position of the economy before the beginning of the crisis is represented by point B . The contraction in domestic spending shifts the U.S. current account schedule down and to the right from $CA^{US'}(r)$ to $CA^{US}(r)$. The current account schedule of the rest of the world does not change position. The new equilibrium is at point A , where the U.S. current account improves and the world interest rate is even lower than it was at the beginning of the crisis. The improvement in the U.S. current account balance in the context of low and declining interest rates is in line with the observed behavior of these two variables in the aftermath of the financial crisis. This analysis suggests that domestic factors might have played a dominant role in explaining the U.S. current account dynamics during the global financial crisis.

7.7 Summing Up

Let's take stock of what we have learned in this chapter.

- We analyze the determination of the current account in a world with large open economies.
- The world interest rate responds to factors affecting savings and investment in large economies.
- A temporary output increase in a large country depresses the world interest rate.

- An expected future increase in output in a large economy drives the world interest rate up.
- The world interest rate is determined by the growth rate of global output. The larger the expected growth in global output is, the higher the world interest rate will be.
- The theoretical framework developed in this chapter, suggests that the large increase in U.S. current account deficits between the mid-1990s and the onset of the global financial crisis of 2007 was predominantly driven by an increase in the global supply of savings, known as the ‘global savings glut.’
- The theoretical framework also suggests that the sharp reduction of U.S. current account deficits in the aftermath of the global financial crisis was primarily caused by an increase in U.S. savings and a reduction in U.S. investment triggered by the bursting of the U.S. housing bubble. This explanation is known as the ‘Made in the U.S.A. hypothesis.’

7.8 Exercises

Exercise 7.1 Indicate whether the following statements are true, false, or uncertain and explain why.

1. In a large open economy, a shock that affects only the savings schedule can give rise to a positive comovement between savings and investment.
2. The larger a country is, the larger the international effects of domestic disturbances will be.
3. Small economies are unaffected by shocks that are internal to a large economy.
4. News about a future increase in productivity has a larger effect on investment in a small economy than in a large economy.
5. Suppose that in period 1 all small open economies in the world experience an increase in their current endowment, Q_1 . This development should have no effect on the large economies of the world.
6. A large closed economy experiences an increase in its period-1 endowment. This should have no effect on the world interest rate, and therefore no effect on other countries in the world.
7. According to Ben Bernanke's 'Global Saving Glut' hypothesis, over-borrowing of U.S. households during the housing boom of the late 1990s and early 2000s was a primary driver of the deterioration of the U.S. current account.

Exercise 7.2 Suppose that, due to exceptionally good weather in the Mid West, the United States experiences an increase in grain production in period 1. Use a graphical approach to analyze the effect of this shock on the current account, savings, investment, and the world interest rate in the United States and in a small open economy such as El Salvador.

Exercise 7.3 Suppose that a protectionist initiative in Congress succeeds in passing a bill closing the U.S. economy to international trade in goods and financial assets. What would be the effect of this policy change on the world interest rate and on savings and investment in the United States and in the rest of the world.

Exercise 7.4 (A Two-Country Economy) Consider a two-period, two-country, endowment economy. Let one of the countries be the United States (U) and the other Europe (E). Households in the United States have preferences described by the utility function

$$\ln C_1^U + \ln C_2^U,$$

where C_1^U and C_2^U denote consumption of U.S. households in periods 1 and 2, respectively. Europeans have identical preferences, given by

$$\ln C_1^E + \ln C_2^E,$$

where C_1^E and C_2^E denote consumption of European households in periods 1 and 2, respectively. Let Q_1^U and Q_2^U denote the U.S. endowments of goods in periods 1 and 2, respectively. Similarly, let Q_1^E and Q_2^E denote the European

endowments of goods in periods 1 and 2, respectively. Assume further that the endowments are nonstorable, that the U.S. and Europe are of equal size, and that there is free capital mobility between the two economies. The United States starts period 1 with a zero net foreign asset position.

1. **Symmetric Equilibrium** Suppose that $Q_1^U = Q_2^U = Q_1^E = Q_2^E = 10$. Calculate the equilibrium world interest rate, and the current accounts in the United States and Europe in period 1.
2. **US-Originated Contraction # 1** Suppose that a contraction originates in the United States. Specifically, assume that Q_1^U drops from 10 to 8. All other endowments (Q_2^U , Q_1^E , and Q_2^E) remain unchanged at 10. This contraction in output has two characteristics: First, it originates in the United States (the European endowments are unchanged.) Second, it is temporary (the U.S. endowment is expected to return to its normal value of 10 after one period). Calculate the equilibrium interest rate and the current accounts of the United States and Europe in period 1. Provide intuition.
3. **US-Originated Contraction # 2** Consider now a second type of contraction in which the U.S. endowment falls from 10 to 8 in the first period and is expected to continue to fall to 6 in the second period ($Q_1^U = 8$ and $Q_2^U = 6$). The endowments in Europe remain unchanged at 10 each period ($Q_1^E = Q_2^E = 10$). Like the one described in the previous item, this contraction originates in the United States. However, it differs from the one described in the previous item in that it is more protracted. Calculate again the equilibrium interest rate

and the two current accounts in period 1. Point out differences in the effects of the two types of contraction and provide intuition.

4. At the beginning of the great contraction of 2008, interest rates fell sharply around the world. What does the model above say about people's expectations around 2008 regarding the future path of real activity.

Exercise 7.5 (World Wide Recession) Consider the same economy as described in exercise 7.4. First solve question 1 of exercise 7.4. Then answer the following questions:

1. **World Wide Contraction #1** Consider a global contraction. Specifically, assume that Q_1^U and Q_1^E fall from 10 to 8 and that Q_2^U and Q_2^E are unchanged. Calculate the equilibrium world interest rate.
2. **World Wide Contraction #2** Consider next a persistent global contraction. Specifically, assume that Q_1^U and Q_1^E fall from 10 to 8 and that Q_2^U and Q_2^E also fall from 10 to 8. Calculate the equilibrium world interest rate.
3. **World Wide Contraction #3** Finally, consider a global contraction that is expected to get worse over time. Specifically, assume that Q_1^U and Q_1^E fall from 10 to 8 and that Q_2^U and Q_2^E fall from 10 to 6. Calculate the equilibrium world interest rate.
4. Suppose you are a member of a team of economic analysts at an investment bank and observe that both Europe and the United States

fall into recession. You would like to know what market participants are expecting about future output. In particular, a third of your team believes that the world wide output decline is temporary, a third believes that this is the new normal, that is, output will not recover to its original level, and the rest of the team believes that the current recession is just the beginning of a deeper recession to come. How can you use data available in period 1 to tell which view is right.

Exercise 7.6 (Global Uncertainty) Suppose that, for a host of reasons, part of the world suddenly becomes more uncertain (think of wars, political instability, economic crises, etc.). Refer to this group of more uncertain countries as UC. Assume that the increase in uncertainty is manifested in a higher standard deviation of future output. Refer to the rest of the world as RW. Analyze the effect of this increase in uncertainty on the world interest rate and on consumption, savings, and the current account in the UC and the RW. You might want to accompany your explanation with one or more graphs.

Exercise 7.7 (Effects of World Shocks on Small Economies) Consider a small open economy, say Ecuador. Suppose that the United States experiences an investment surge. Analyze the effects of this development on the Ecuadorian economy. In particular, discuss the effects on the current account, savings, investment, and the domestic interest rate. Consider the following alternative scenarios:

1. All countries in the world are open.

2. Ecuador is a closed economy.
3. The United States is a closed economy.
4. All countries in the world other than the United States and Ecuador are closed economies.

Chapter 8

The Real Exchange Rate and Purchasing Power Parity

You might have noticed that sometimes Europe seems much cheaper than the United States and sometimes it is the other way around. In the first case, Americans have incentives to visit Europe and to import European goods and services. In the second case, European tourists come in larger numbers to the United States and the United States has an easier time exporting goods and services to Europe. The real exchange rate measures how expensive a foreign country is relative to the home country. It tracks the evolution over time of the price of a basket of goods abroad in terms of baskets of goods at home. When prices expressed in the same currency are equalized at home and abroad, the real exchange rate is unity. In this case, the purchasing power of the domestic currency is the same at home and abroad, in the sense that it can buy the same amount of goods in

both countries. When this happens, we say that purchasing power parity holds. An important empirical question in international macroeconomics is how large and persistent are price differences across countries, that is, how large and persistent are movements in the real exchange rate, or how large and persistent are deviations from purchasing power parity. Equally important is the question of what factors determine how cheap or expensive a country is relative to other countries, that is, what the determinants of the real exchange rate or of deviations from purchasing power parity are. This chapter studies these questions.

8.1 The Law of One Price

When a good costs the same abroad and at home, we say that the Law of One Price (LOOP) holds. Let P denote the domestic-currency price of a particular good in the domestic country, P^* the foreign-currency price of the same good in the foreign country, and \mathcal{E} the nominal exchange rate, defined as the domestic-currency price of one unit of foreign currency. The LOOP holds if

$$P = \mathcal{E}P^*.$$

The good is more expensive in the foreign economy if $\mathcal{E}P^* > P$, and less expensive if $\mathcal{E}P^* < P$.

Why should the law of one price hold? Imagine that a can of coke costs 2 dollars in country A and 1 dollar in country B. In a frictionless world, one could become infinitely rich by buying coke cans in country B and selling them in country A. This arbitrage opportunity would cause the price of

coke to fall in country A and to increase in country B. This tendency would continue until the price is equalized across countries.

The world, however, is not a frictionless environment. For example, if countries A and B are far apart from each other, transportation costs would have to be taken into account. Also, bringing the can from its port of entry in country A to a convenience store involves distribution costs, such as loading and unloading, additional transportation, storage, advertising, and retail services. If making the can of coke of country B available to customers in country A costs more than one dollar, it will not pay for an entrepreneur to exploit the cross-country price difference of one dollar. Thus, one should expect deviations from the law of one price to persist over time.

For some goods, the law of one price is a better approximation than for others. For example, prices are similar across countries for highly traded commodities, such as gold, oil, soy beans, and wheat. The same is true for luxury consumer goods, such as Rolex watches, Hermes neckties, and Montblanc fountain pens. On the other hand, large differences in prices across countries are observed for personal services, such as health care, education, restaurant meals, domestic services, and personal care (haircuts is a prototypical example). Similarly, prices of local goods, such as housing, transportation, and utilities can display large variation across countries or regions.

How large are observed deviations from the law of one price? To answer this question for a particular good, one would need to collect data on its price in different countries and then express all prices in the same currency using the corresponding nominal exchange rate. One easy product to start

with is McDonald's Big Mac, because *The Economist* has been collecting data on the price of Big Macs around the world since 1986. The Big Mac is also a good example because it is made pretty much the same way all over the world, so we are sure we are comparing the price of the same good across countries.

Let P^{BigMac} denote the dollar price of a Big Mac in the United States and P^{BigMac^*} the foreign-currency price of a Big Mac in a foreign country. Then we can construct a measure of how many U.S. Big Macs it takes to buy one Big Mac abroad. This measure is called the Big Mac real exchange rate, and we denote it by e^{BigMac} . Formally, e^{BigMac} is given by

$$e^{\text{BigMac}} = \frac{\mathcal{E} P^{\text{BigMac}^*}}{P^{\text{BigMac}}}.$$

If $e^{\text{BigMac}} > 1$, then the Big Mac is more expensive abroad. In this case, if you exchange the dollar value of one Big Mac in the United States into foreign currency, you would not have enough money to buy one Big Mac abroad. We say that the law of one price holds for Big Macs when the Big Mac real exchange rate is unity,

$$\text{LOOP holds when } e^{\text{BigMac}} = 1.$$

Table 8.1 presents Big Mac real exchange rates for 40 countries measured in January 2019. The table shows that the law of one price does not hold well for the Big Mac. For example, in Russia the least expensive country in the sample, a Big Mac sells for the equivalent of \$1.65, whereas in the

Table 8.1: The Big-Mac Real Exchange Rate, January 2019

Country	$P^{\text{BigMac*}}$	\mathcal{E}	$\mathcal{E}P^{\text{BigMac*}}$	e^{BigMac}	$\mathcal{E}^{\text{BigMac PPP}}$
Switzerland	6.50	1.02	6.62	1.19	0.86
Norway	50.00	0.12	5.86	1.05	0.11
Sweden	52.00	0.11	5.84	1.05	0.11
United States	5.58	1	5.58	1	1
Canada	6.77	0.75	5.08	0.91	0.82
Euro area	4.05	1.15	4.64	0.83	1.38
Denmark	30.00	0.15	4.60	0.83	0.19
Brazil	16.90	0.27	4.55	0.81	0.33
Australia	6.10	0.71	4.35	0.78	0.91
Uruguay	140.00	0.03	4.31	0.77	0.04
Singapore	5.80	0.74	4.28	0.77	0.96
New Zealand	6.20	0.68	4.19	0.75	0.90
Britain	3.19	1.28	4.07	0.73	1.75
South Korea	4500.00	0.00	4.02	0.72	0.00
Chile	2640.00	0.00	3.89	0.70	0.00
United Arab Emirates	14.00	0.27	3.81	0.68	0.40
Czech Republic	85.00	0.04	3.81	0.68	0.07
Costa Rica	2290.00	0.00	3.77	0.68	0.00
Colombia	11900.00	0.00	3.73	0.67	0.00
Thailand	119.00	0.03	3.72	0.67	0.05
Japan	390.00	0.01	3.60	0.64	0.01
Pakistan	460.00	0.01	3.31	0.59	0.01
Saudi Arabia	12.00	0.27	3.20	0.57	0.47
Peru	10.50	0.30	3.14	0.56	0.53
China	20.90	0.15	3.05	0.55	0.27
Hungary	850.00	0.00	3.03	0.54	0.01
Poland	10.50	0.27	2.80	0.50	0.53
Philippines	140	0.02	2.67	0.48	0.04
Hong Kong	20	0.13	2.55	0.46	0.28
India	178	0.01	2.55	0.46	0.03
Mexico	49	0.05	2.54	0.45	0.11
Indonesia	33000.00	0.00	2.34	0.42	0.00
Taiwan	69.00	0.03	2.24	0.40	0.08
South Africa	31.00	0.07	2.24	0.40	0.18
Egypt	40.00	0.06	2.23	0.40	0.14
Malaysia	9.05	0.24	2.20	0.39	0.62
Argentina	75.00	0.03	2.00	0.36	0.07
Turkey	10.75	0.19	2.00	0.36	0.52
Ukraine	54.00	0.04	1.94	0.35	0.10
Russia	110.17	0.01	1.65	0.30	0.05

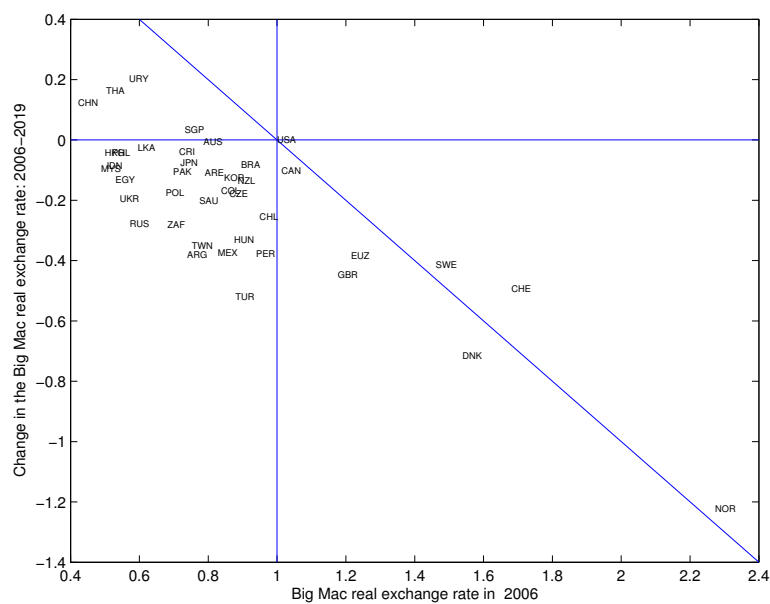
Notes. $P^{\text{BigMac*}}$ denotes the price of a Big Mac in the country's currency; \mathcal{E} denotes the dollar exchange rate, defined as the dollar price of one unit of the country's currency; $e^{\text{BigMac}} = \mathcal{E}P^{\text{BigMac*}}/P^{\text{BigMac}US}$ denotes the Big-Mac real exchange rate; $P^{\text{BigMac}US}$ denotes the price of the Big Mac in the United States; and $\mathcal{E}^{\text{BigMac PPP}}$ denotes the Big-Mac PPP exchange rate discussed in section 8.3. Own calculations based on data from *The Economist*.

United States it sells for \$5.58. Thus for the price of one Big Mac in Russia one can buy only 0.3 Big Macs in the United States, that is, the Big Mac real exchange rate is 0.3. In Switzerland, the most expensive country in the sample, a Big Mac sells for the equivalent of \$6.62. Thus, one Big Mac in Switzerland costs the same as 1.19 Big Macs in the United States, or, equivalently, the Big Mac real exchange rate is 1.19.

Why is the Big Mac so expensive in some countries and so cheap in others? Perhaps the most important factor determining the observed price differences is the international tradability of the different items that compose the cost of producing the Big Mac. Tradable components of a Big Mac include grain (wheat and sesame seeds), meat, and dairy (the cheese on top of the burger). The prices of these items tend to be similar across countries. However, these components combined represent a small fraction of the total cost of producing and serving a Big Mac. Most of the production costs stem from local components, such as labor, rent, electricity, and water. These items are not easily tradable across countries, and, as a result, their prices can display significant cross-sectional variation. For example, a worker preparing burgers in Indonesia makes only a fraction of the wage of his or her American counterpart. It follows that we should expect the Big Mac to be more expensive in countries where the aforementioned nontradable components are also more expensive. We will show in [section 8.3.2](#) that nontradable goods and services tend to be relatively more expensive in richer countries.

[Table 8.1](#) presents a static picture of deviations from the law of one price across countries. It tells us what these deviations looked like at one point

Figure 8.1: Changes in Big Mac Real Exchange Rates from 2006 to 2019



Notes. The figure plots the change in the Big Mac real exchange rate, $e^{\text{BigMac}} = \mathcal{E}P^{\text{BigMac}*}/P^{\text{BigMac}}$, between 2006 and 2019 against the Big Mac real exchange rate in 2006 for 40 countries. Because, by definition, the Big Mac real exchange rate is always equal to one for the United States, this country is located at coordinate (1,0). Countries below the horizontal line became relatively cheaper and countries above the horizontal line became relatively more expensive. The figure shows that most countries were cheaper than the United States in 2006 and that many of them became even cheaper by 2019. Country names are shown using ISO abbreviations. Source: Own calculations based on data from *The Economist*.

in time, namely, the year 2019. One might wonder whether deviations from the law of one price change over time. In particular, do deviations from the law of one price tend to disappear with time, that is, have countries that were more expensive than the United States in the past become relatively less expensive. Similarly, have countries that were cheaper than the United States in the past become relatively more expensive. Figure 8.1 provides an answer to these questions for the period 2006 to 2019.¹ It plots the change in the Big Mac real exchange rate between 2006 and 2019 against the Big Mac real exchange rate in 2006 for forty countries. Because the Big Mac real exchange rate is the ratio of the dollar price of a Big Mac in a given country to the dollar price of a Big Mac in the United States, by construction, the coordinate of the United States is $(1, 0)$ (the level of the U.S. Big Mac real exchange rate is always 1 and never changes). Countries located below the horizontal line became cheaper relative to the United States over the period 2006 to 2019 and countries located above the horizontal line became relatively more expensive. The figure also displays a downward sloping 45-degree line crossing the U.S. position $(1, 0)$. Countries on the 45-degree line converged to the law of one price by 2019.

The figure shows that deviations from the law of one price are persistent. Most countries were cheaper than the United States in 2006 and continue to be cheaper in 2019 (points to the left of the vertical line and below the 45-degree line). In fact, most countries (28 out of the 40) were cheaper than the United States in 2006 and became even cheaper over the period 2006 to

¹We picked the year 2006 as the starting date because it is the first year in which the database contains Big Mac prices for at least forty countries.

2019 (points located to the left of the vertical line and below the horizontal line). For these countries, deviations from the law of one price became larger rather than smaller.

The figure suggests that there is convergence to the law of one price for rich developed countries, such as Canada, Great Britain, Sweden, Switzerland, Denmark, Norway, and the euro area. In these countries the dollar price of the Big Mac was higher than in the United States in 2006, but over the subsequent 13 years the price difference has narrowed significantly.

8.2 Purchasing Power Parity

Purchasing power parity (PPP) is the generalization of the idea of the law of one price for broad baskets of goods representative of households' actual consumption, as opposed to a single good. Let P denote the domestic-currency price of such a basket of goods and P^* the foreign-currency price of a basket of goods in the foreign country. The real exchange rate, denoted e , is defined as

$$e = \frac{\mathcal{E}P^*}{P}.$$

The real exchange rate indicates the relative price of a consumption basket in the foreign country in terms of consumption baskets in the home country. When $e > 1$, the foreign country is more expensive than the domestic country and when $e < 1$, the foreign country is less expensive than the domestic country.

We say that *absolute purchasing power parity* holds when the price of the consumption basket expressed in a common currency is the same domesti-

cally and abroad, $P = \mathcal{E}P^*$, or, equivalently, when the real exchange rate is unity, $e = 1$.

Ascertaining whether absolute PPP holds requires gathering data on the price of the domestic basket, P , the price of the foreign basket, P^* , and the nominal exchange rate, \mathcal{E} . Data on price levels of large baskets of goods for many countries is produced by the World Bank's International Comparison Program (ICP).² This data has a frequency of about six years. At the writing of the current edition of this book, the most recent ICP release contains data for the year 2011. It reports price level data of more than 1000 individual goods for 199 countries, including 100 developing and emerging economies and 46 advanced economies. Data on the nominal exchange rate is readily available from many sources including the ICP database. The ICP produces a measure of the level of the real exchange rate and hence allows for testing of absolute PPP.

Table 8.2 shows the dollar real exchange rate of selected developing and advanced economies in 2011. For each country it shows the dollar price of a basket relative to the United States, $e = \mathcal{E}P^*/P^{US}$, where P^{US} denotes the price of a basket of goods in the United States, P^* denotes the price of a basket of goods in a given country, and \mathcal{E} denotes the nominal exchange rate (the dollar price of one unit of currency of the given country). If absolute PPP held, then a basket of goods that costs 100 dollars in the United States should also cost 100 dollars in every country. However, as the table shows this is far from being the case. There are wide deviations from absolute PPP

²See the report "Purchasing Power Parities and Real Expenditures of World Economies, Summary of Results and Findings of the 2011 International Comparison Program," especially Table 6.1, The World Bank, 2014.

Table 8.2: Deviations From Absolute PPP in Selected Countries: Evidence from the International Comparison Program

Country	e	\mathcal{E}	\mathcal{E}^{PPP}
Switzerland	1.63	1.13	0.69
Norway	1.60	0.18	0.11
Australia	1.56	1.03	0.66
Sweden	1.36	0.15	0.11
Japan	1.35	0.0125	0.00931
Canada	1.26	1.01	0.80
France	1.17	1.39	1.18
New Zealand	1.17	0.79	0.67
Belgium	1.17	1.39	1.19
Netherlands	1.16	1.39	1.20
Austria	1.15	1.39	1.20
Ireland	1.15	1.39	1.21
United Kingdom	1.12	1.60	1.43
Germany	1.08	1.39	1.28
Italy	1.07	1.39	1.30
United States	1	1	1
South Korea	0.7711	0.0009023	0.00117
China	0.54	0.15	0.29
Sierra Leone	0.36	0.000231	0.000644
Sri Lanka	0.35	0.01	0.03
Burundi	0.34	0.000793	0.00235
Gambia, The	0.34	0.03	0.10
Nepal	0.33	0.01	0.04
Madagascar	0.33	0.000494	0.00148
Tanzania	0.33	0.000636	0.00191
Cambodia	0.33	0.000246	0.000742
Uganda	0.33	0.000396	0.0012
Vietnam	0.33	4.88e-05	0.000149
India	0.32	0.02	0.07
Bangladesh	0.31	0.01	0.04
Ethiopia	0.29	0.06	0.20
Pakistan	0.28	0.01	0.04
Egypt	0.27	0.17	0.62

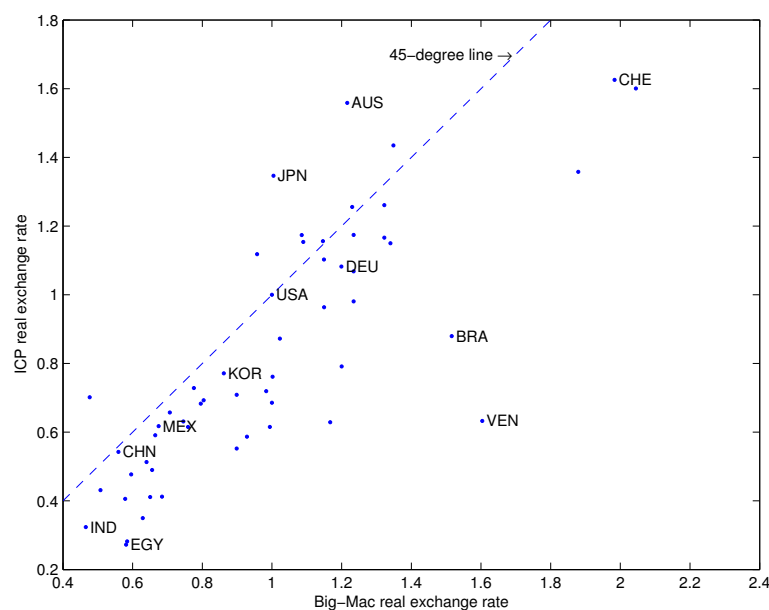
Notes. The table shows the dollar real exchange rate, $e = \mathcal{E}P^*/P^{US}$, the nominal exchange rate, \mathcal{E} (dollar price of one unit of foreign currency), and the PPP exchange rate, $\mathcal{E}^{PPP} = P^{US}/P^*$ in selected countries in 2011. The variable P^* denotes the foreign-currency price of a basket in the foreign country, and P^{US} denotes the dollar price of a basket in the United States. The table suggests that there are large deviations from absolute PPP. For example, a basket that in 2011 cost 100 dollars in the United States cost 163 dollars in Switzerland and only 27 dollars in Egypt. The PPP exchange rate will be discussed in section 8.3. Data Source: Purchasing Power Parities and Real Expenditures of World Economies, Summary of Results and Findings of the 2011 International Comparison Program, Table 6.1, The World Bank, 2014.

across countries. For example, a basket that in 2011 sold for 100 dollars in the United States sold for 163 dollars in Switzerland and for 27 dollars in Egypt. Thus, in 2011, Switzerland was 63 percent more expensive than the United States, and Egypt was 63 percent cheaper.

How does the ICP real exchange rate shown in table 8.2 compare with the Big Mac real exchange rate shown in table 8.1? As we mentioned earlier, one advantage of the real exchange rate measure produced by the World Bank's International Comparison Program (ICP) is that it covers baskets containing hundreds of goods. Its downside is that, because the collection of such a large set of prices across many countries is costly, the frequency with which the data is produced is low, about once every six years. As a result, we don't have a good idea of how the cost of living changes in different countries at higher frequency, say yearly. By contrast, the Big-Mac real exchange rate is relatively easy to construct, since the price of this meal is readily available across countries. This facilitates its publication at a much higher frequency. *The Economist*, for example, publishes this indicator at least once a year. The drawback of the Big-Mac real exchange rate is that it is based on a single good, the Big Mac, and therefore may not be representative of the expenditure structure of the overall economy. The ICP and Big-Mac real exchange rates therefore present a trade-off between coverage and frequency. For this reason, it is of interest to ask how closely the Big-Mac real exchange rate approximates the ICP real exchange rate. If the approximation is good, then the simpler measure could provide a reliable idea of how relative prices move across countries at a high frequency.

Figure 8.2 compares the Big-Mac and ICP real exchange rates in 2011

Figure 8.2: Comparing the ICP and Big-Mac Real Exchange Rates in 2011



Notes. The figure plots the ICP real exchange rate against the Big-Mac real exchange rate for 57 countries in 2011. The figure shows that the Big-Mac real exchange rate is highly correlated with the ICP counterpart. This suggests that the Big Mac real exchange rate is a good measure of how expensive different countries are relative to one another. Selected country names are indicated using ISO abbreviations. Source: See notes to tables 8.1 and 8.2.

across 57 countries. Because all real exchange rates are vis-à-vis the United States, the latter is located at the coordinate (1,1), on the 45-degree line. The figure shows that the Big-Mac and ICP real exchange rates are highly correlated. The actual correlation is 0.81, which is close to unity, the value that would obtain if the Big-Mac real exchange rate was a perfect proxy of the ICP real exchange rate. This means that the Big-Mac real exchange rate provides a reasonable approximation of how expensive countries are relative to one another. The fact that the majority of countries lie below the 45-degree line means that the Big-Mac real exchange rate exaggerates how expensive countries are relative to the United States. Overall, however, this bias is small, about 5 percent. We therefore conclude, that as a quick measure of how expensive countries are, the Big-Mac real exchange rate does a pretty good job.

8.3 PPP Exchange Rates

Suppose you work in New York and get a job offer in Mumbai. You want to know whether the offer represents an increase or a decrease in your income. One way to compare the two options is to convert the Mumbai offer from rupees to dollars using the market exchange rate. The problem with this comparison is that prices of goods and services in Mumbai might be quite different from those in New York. A better way to compare income or expenditure across countries is to convert amounts expressed in local currency to dollars using the *PPP exchange rate*, which takes into account possible price differences. The resulting amount is said to be PPP adjusted.

The PPP exchange rate is defined as the nominal exchange rate that makes the consumption basket in two countries equally expensive. Put differently, the PPP exchange rate is the nominal exchange rate that would make PPP hold. Formally, letting \mathcal{E}^{PPP} denote the PPP exchange rate, we have that

$$\mathcal{E}^{PPP} P^* = P,$$

where, as before, P denotes the price level in the domestic country and P^* denotes the price level in the foreign country. If the PPP exchange rate is higher than the market exchange rate, $\mathcal{E}^{PPP} > \mathcal{E}$, then the domestic country is more expensive than the foreign country, $P > \mathcal{E}P^*$. In this case, it is said that according to this measure the domestic currency is overvalued (and the foreign currency is undervalued). If $\mathcal{E}^{PPP} < \mathcal{E}$, then the domestic economy is cheaper than the foreign economy, $P < \mathcal{E}P^*$, and we say that according to this measure, the domestic currency is undervalued (and the foreign currency is overvalued).

The last column of table 8.1 shows the PPP exchange rate for Big Macs, which, following the above definition, is given by

$$\mathcal{E}^{\text{BigMac PPP}} = \frac{P^{\text{BigMac US}}}{P^{\text{BigMac}^*}}.$$

For example, the Big Mac PPP exchange rate for Switzerland is 0.86 dollars per Swiss franc, while the market exchange rate is 1.02 dollars per Swiss franc. This means that according to this measure the Swiss franc is overvalued. If one believes that in the long run the law of one price should hold for Big Macs, then one would expect the Swiss franc to depreciate against the

dollar by 15.6 percent. By contrast, in the case of India, the Big Mac PPP exchange rate is 0.031 dollars per rupee, whereas the market exchange rate is 0.014 dollars per rupee. This means that according to the Big Mac PPP exchange rate, the rupee is 120 percent undervalued. Again, if one thinks that in the long run the law of one price should hold, the rupee should appreciate by 120 percent.

A number of policymakers and observers have suggested that China has put in place policies conducive to an under valuation of the yuan with the intention to boost China's competitiveness in international trade. Let's see what the Big Mac PPP exchange rate has to say about the undervaluation of the yuan. The Big Mac PPP exchange rate is 0.27 dollars per yuan, and the market exchange rate is 0.15. Thus, according to the Big Mac PPP exchange rate, the yuan is 80 percent undervalued. This result appears to give credence to the critics of China's exchange-rate policy. However, in section 8.3.2, we will show that the observed level of undervaluation of the yuan is likely to have more to do with China's income per capita than with a deliberate exchange-rate manipulation.

Consider now PPP exchange rates for baskets of goods. The last two columns of table 8.2 display, respectively, market and PPP exchange rates in 2011 for 33 selected countries. PPP exchange rates are constructed using the price level data from the 2011 ICP program. As we mentioned earlier, the ICP reports prices of baskets containing hundreds of goods. The table suggests a similar pattern of over- and undervaluation of different currencies vis-à-vis the dollar as the one that emerges from Big Mac PPP exchange rates, which should not be surprising given the comparison of ICP and Big

Mac real exchange rates presented in figure 8.2. For example, the market exchange rate of the Swiss franc is 1.13 dollars per franc, whereas the PPP exchange rate is 0.69 dollars per franc. It follows that according to the PPP exchange rate, the Swiss franc is 38.9 percent overvalued. In other words, if the market exchange rate were expected to converge to the PPP exchange rate, then the Swiss franc would be expected to depreciate by 38.9 percent. In India, the market exchange rate is 0.021 dollars per rupee whereas the PPP exchange rate is 0.066 dollars per rupee, suggesting an undervaluation of the rupee vis-à-vis the dollar of 214 percent. Finally, the yuan also appears to be greatly undervalued when one uses ICP prices, as the market and PPP exchange rates are 0.15 and 0.29 dollars per yuan, respectively.

8.3.1 PPP Exchange Rates and Standard of Living Comparisons

Comparing standards of living across countries is complicated by the fact that prices of similar goods vary widely across borders. For example, as shown in table 8.3, in 2011 GDP per capita was 49,782 dollars in the United States but only 1,533 dollars in India. According to this measure, the average American is 32 times richer than the average Indian. This measure, however, does not take into account the possibility that when expressed in dollars, goods and services might be cheaper in India than in the United States. In this case, a given dollar amount would buy a larger amount of goods in India than in the United States and Indians would not be as poor as suggested by the ratio of dollar per capita GDPs. Consider, for example, measuring per capita incomes in units of Big Macs. According to table 8.1, a Big

Mac costs 5.58 dollars in the United States but only 2.55 dollars in India. This implies that one U.S. per capita GDP buys 8,922 Big Macs and one Indian per capita GDP buys 601 Big Macs. According to this measure, the average American is 15 times richer than the average Indian. This is still a big income gap, but not as large as the one suggested by the simple ratio of dollar GDPs.

But the Big Mac is not the only good that is cheaper in India than in the United States. Other items, especially services, such as haircuts, domestic services, transportation, and health, are also cheaper in India. We should therefore expect to arrive at a similar conclusion, namely, that the gap in standards of living are not as pronounced as suggested by dollar GDP ratios, when incomes are measured in terms of broader baskets of goods, such as those used in the World Bank's International Comparison Program.

Let GDP^I denote the GDP per capita in India expressed in Indian rupees. Let P^I denote the rupee price of one basket of goods in India. Then, GDP^I/P^I is per capita GDP in India measured in units of baskets of goods. Similarly, letting GDP^{US} and P^{US} denote GDP in the United States and the price of one basket of goods in the United States, both measured in dollars, we have that GDP^{US}/P^{US} represents per capita GDP in the United States measured in units of baskets of goods. The ratio of per capita GDP in the United States to per capita GDP in India measured in units of baskets

of goods is then given by

$$\begin{aligned}\text{Ratio of Incomes in Baskets of Goods} &= \frac{GDP^{US}/P^{US}}{GDP^I/P^I} \\ &= \frac{1}{P^{US}/P^I} \frac{GDP^{US}}{GDP^I}\end{aligned}$$

Now, recall that P^{US}/P^I is the dollar-rupee PPP exchange rate, which we denote by $\mathcal{E}^{PPP,I}$. Then we have that

$$\text{Ratio of Incomes in Baskets of Goods} = \frac{GDP^{US}}{\mathcal{E}^{PPP,I} GDP^I}.$$

The product $\mathcal{E}^{PPP,I} GDP^I$ is called per capita *GDP* at *PPP* exchange rates, and we denote it by $GDP^{PPP,I}$. It represents per capita GDP in India when goods are priced in dollar prices of the United States. This measure of GDP per capita is now comparable across countries since in all countries goods are priced at U.S. prices. Since, of course, all goods in the United States are priced at dollar prices of the United States, we have that $GDP^{PPP,US} = GDP^{US}$. Then we can write

$$\text{Ratio of Incomes in Baskets of Goods} = \frac{GDP^{US}}{GDP^{PPP,I}}.$$

Table 8.3 displays dollar GDP per capita at market exchange rates and at PPP exchange rates for 33 countries in 2011. In India, GDP per capita was 1,533 dollars when measured at market exchange rates but 4,735 dollars when measured at PPP exchange rates. Comparing these figures to per capita GDP in the United States, which in 2011 was 49,782 dollars, we have

Table 8.3: GDP Per Capita in U.S. Dollars at Market and PPP Exchange Rates in 2011

Country	GDP	GDP ^{PPP}	$\frac{GDP^{US}}{GDP}$	$\frac{GDP^{US}}{GDP^{PPP}}$
Norway	99035	61879	0.50	0.80
Switzerland	83854	51582	0.59	0.97
Australia	65464	42000	0.76	1.19
Sweden	56704	41761	0.88	1.19
Canada	51572	41069	0.97	1.21
Netherlands	49888	43150	1.00	1.15
United States	49782	49782	1	1
Austria	49590	42978	1.00	1.16
Ireland	49383	42942	1.01	1.16
Belgium	46759	40093	1.06	1.24
Japan	46131	34262	1.08	1.45
Germany	44365	40990	1.12	1.21
France	42728	36391	1.17	1.37
United Kingdom	39241	35091	1.27	1.42
New Zealand	36591	31172	1.36	1.60
Italy	36180	33870	1.38	1.47
South Korea	22388	29035	2.22	1.71
China	5456	10057	9.12	4.95
Egypt	2888	10599	17.24	4.70
Sri Lanka	2836	8111	17.56	6.14
Vietnam	1543	4717	32.26	10.55
India	1533	4735	32.47	10.51
Pakistan	1255	4450	39.68	11.19
Cambodia	902	2717	55.20	18.32
Bangladesh	874	2800	56.95	17.78
Nepal	739	2221	67.35	22.41
Uganda	528	1597	94.33	31.17
Tanzania	517	1554	96.37	32.03
Gambia, The	508	1507	97.94	33.04
Sierra Leone	490	1369	101.50	36.36
Madagascar	470	1412	105.98	35.26
Ethiopia	353	1214	140.85	41.00
Burundi	240	712	207.06	69.91

Notes. GDP stands for GDP per capita at market exchange rates and GDP^{PPP} stands for GDP per capita at PPP exchange rates. Date Source: See table 8.2.

that the average American is 32 times as rich as the average Indian when GDP is converted into dollars at market exchange rates, but 11 times as rich when GDP is converted at PPP exchange rates. This comparison suggests that failing to adjust for price differences across countries can result in large under- or overestimation of living standards across countries.

8.3.2 Rich Countries are More Expensive than Poor Countries

The comparison of living standards in the United States and India we just performed shows that on average prices in India are lower than in the United States. It is natural to wonder whether this result belongs to a general pattern whereby poor countries tend to be cheaper than rich countries.

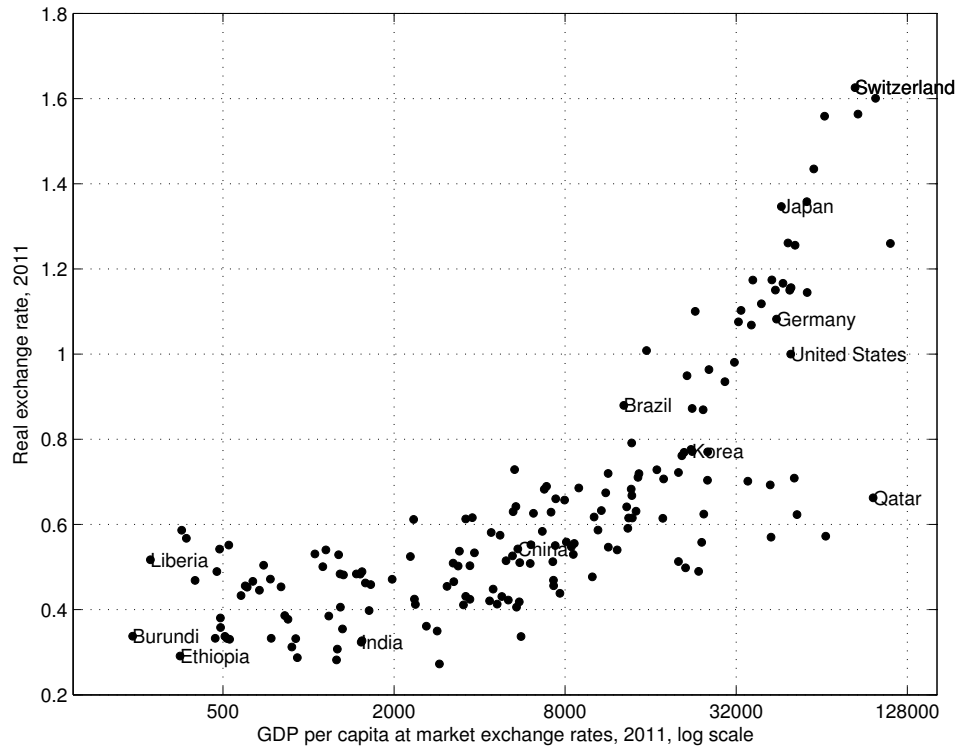
Figure 8.3 shows that this is indeed the case. It plots the dollar real exchange rate, $\mathcal{E}P^*/P^{US}$, against GDP per capita for 177 countries in 2011. The real exchange rate is an appropriate measure of the relative cost of living, because it represents the relative price of baskets of goods in two countries. In the figure, each dot represents one country.

The cloud of points is clearly upward sloping, and indeed it appears to be convex. Countries that are much poorer than the United States, such as Burundi, Liberia, and Ethiopia, are also significantly cheaper. By contrast, countries that are as or more developed than the United States, such as Switzerland, are also more expensive.³

Note that, consistent with this pattern, China is both poorer and cheaper

³The positive relationship between standards of living and costs of living shown in figure 8.3 is robust to measuring GDP per capita at PPP exchange rates. Exercise 8.2 asks you to establish this result.

Figure 8.3: Higher Prices in Rich Countries



Notes. The graph plots the dollar real exchange rate, $e = \mathcal{E}P^*/P^{US}$, against per capita GDP at market exchange rates in 2011 for 177 countries. The figure shows that countries with higher per capita incomes tend to be more expensive. The data source is the 2011 ICP. See notes to table 8.2 for details.

than the United States. Going back to the claim often made by economic observers that the Chinese yuan is kept artificially undervalued to boost China's competitiveness in export markets, we observe that if this view explained all of the yuan's undervaluation, in the figure China should be an outlier located below the cloud of points, which is clearly not the case.

8.4 Relative Purchasing Power Parity

Most studies of purchasing power parity focus on changes in the real exchange rate, rather than on the level of the real exchange rate. The advantage of focusing on the change in the real exchange rate is that one does not need information on the level of the price of baskets of goods, and can instead work with consumer price index data, which is available at higher frequency and for longer periods of time. Price indices, such as the consumer price index, provide information about how prices of large baskets of goods change over time, but not on the absolute price level. This is reflected in the index having a base year at which it takes an arbitrary value, typically 100.

We say that *relative PPP* holds if the real exchange rate does not change over time, that is,

$$\text{relative PPP holds if } \Delta e_t \equiv \Delta \frac{\mathcal{E}_t P_t^*}{P_t} = 0,$$

where e_t denotes the real exchange rate at time t , \mathcal{E}_t denotes the nominal exchange rate at time t , P_t denotes the domestic consumer price index at

time t , P_t^* denotes the foreign consumer price index at time t . The symbol Δ denotes change over time, so, for example, $\Delta e_t = e_t - e_{t-1}$. The notation now introduces time subscripts because relative PPP is all about changes in the real exchange rate over time.

If relative PPP holds, then the same basket of goods need not fetch the same price in two countries, but its price, expressed in a common currency, must change at the same rate over time in both countries. When Δe_t is negative, we say that the real exchange appreciates. In this case the domestic country becomes more expensive over time relative to the foreign country. And when Δe_t is positive, we say that the real exchange rate depreciates. In this case, the domestic country becomes less expensive over time relative to the foreign country.

The empirical question of whether relative PPP holds can be divided into two parts: Does relative PPP hold in the long run? And, does relative PPP hold in the short run? Let's begin with the first part.

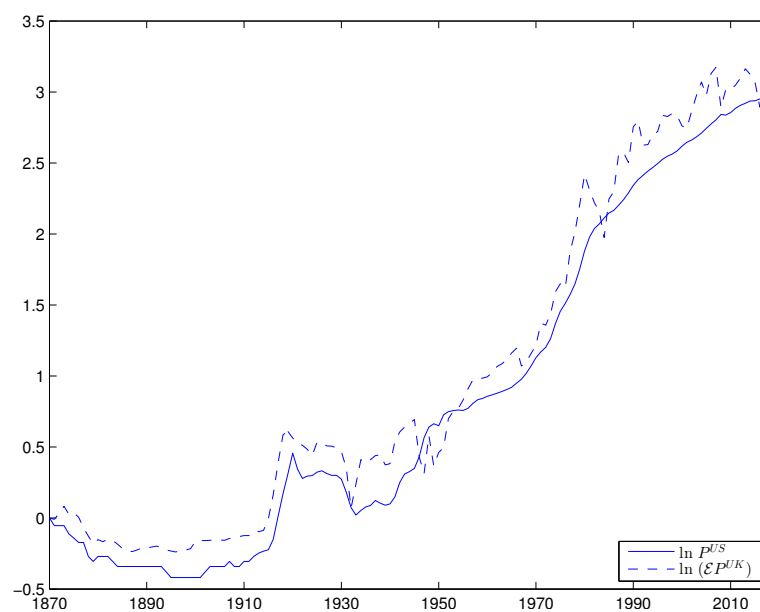
8.4.1 Does Relative PPP Hold in the Long Run?

Consider the dollar-pound real exchange rate at time t , which is given by

$$e_t = \frac{\mathcal{E}_t P_t^{UK}}{P_t^{US}},$$

where \mathcal{E}_t denotes the dollar-pound nominal exchange rate at time t , defined as the dollar price of one pound, P_t^{UK} denotes the consumer price index in the United Kingdom at time t , and P_t^{US} denotes the consumer price index in the United States at time t . Because consumer price indices are arbitrarily

Figure 8.4: U.S. and U.K. Consumer Price Indices in Dollars: 1870-2018



Notes. Both price indices are normalized to 1 in 1870 and are expressed in logarithms. The fact that the two lines keep close to each other over 148 years suggests that relative PPP holds in the long run. Data Source: Until 2013, Òscar Jordà, Moritz Schularick, and Alan M. Taylor. 2017. “Macrofinancial History and the New Business Cycle Facts,” in NBER Macroeconomics Annual 2016, Volume 31, edited by Martin Eichenbaum and Jonathan A. Parker. Chicago: University of Chicago Press. After 2013, IFS, FRED, and U.K. Office of National Statistics.

normalized at a base year, the level of e_t , as defined here, is meaningless. However, variations in e_t over time, Δe_t , provide information about changes in the relative costs of living in the two countries, which is precisely why the concept of relative PPP is useful.

Figure 8.4 shows with a solid line the natural logarithm of P_t^{US} and with a broken line the natural logarithm of $\mathcal{E}_t P_t^{UK}$ over the period 1870–2018. To facilitate interpretation, we normalize both P_t^{US} and $\mathcal{E}_t P_t^{UK}$ to one in 1870 (i.e., at $t = 1870$), so that their logarithms are zero in that year. The figure shows that over the long run, P_t^{US} and $\mathcal{E}_t P_t^{UK}$ move in tandem. In other words, over the past 148 years the United States did not become systematically cheaper or more expensive than the United Kingdom. This suggests that relative PPP holds in the long run between these two countries.

Let's now see whether relative PPP also holds in the long run for other country pairs. To this end, for a given country, let P_t denote its consumer price index at time t , let \mathcal{E}_t denote its dollar exchange rate at time t , defined as the price of one dollar in terms of the country's currency, and P_t^{US} the U.S. consumer price index at time t . If relative PPP holds between this country and the United States in the long run, then it must be the case that the real exchange rate does not change in the long run. At any time t , the real exchange rate, e_t , is given by

$$e_t = \frac{\mathcal{E}_t P_t^{US}}{P_t}.$$

The rate of change of the real exchange rate, which we denote by ϵ_t^r , is called

the *real depreciation rate* and satisfies

$$1 + \epsilon_t^r = \frac{e_t}{e_{t-1}} = \frac{(\mathcal{E}_t/\mathcal{E}_{t-1})(P_t^{US}/P_{t-1}^{US})}{P_t/P_{t-1}}.$$

Let $\epsilon_t = \mathcal{E}_t/\mathcal{E}_{t-1} - 1$ be the rate of change of the nominal exchange rate. This variable is called the *nominal depreciation rate*. Also, let $\pi_t = P_t/P_{t-1} - 1$ and $\pi_t^{US} = P_t^{US}/P_{t-1}^{US} - 1$ be the inflation rates in the country considered and in the United States, respectively. Then, the real depreciation rate can be written as

$$1 + \epsilon_t^r = \frac{(1 + \epsilon_t)(1 + \pi_t^{US})}{(1 + \pi_t)}. \quad (8.1)$$

Taking the natural logarithm of the left- and right-hand sides of the above expression and using the approximation $\ln(1 + x) \approx x$ for any x , we obtain

$$\epsilon_t^r = \epsilon_t + \pi_t^{US} - \pi_t.$$

Relative *PPP* holds if the real exchange rate does not change over time, which means that the real depreciation rate is nil, $\epsilon_t^r = 0$. So we can write

$$\text{Relative PPP holds if } \epsilon_t = \pi_t - \pi_t^{US}.$$

In words, relative PPP holds if the rate of depreciation of the country's currency against the dollar, ϵ_t , is equal to the inflation differential between the country considered and the United States, $\pi_t - \pi_t^{US}$.

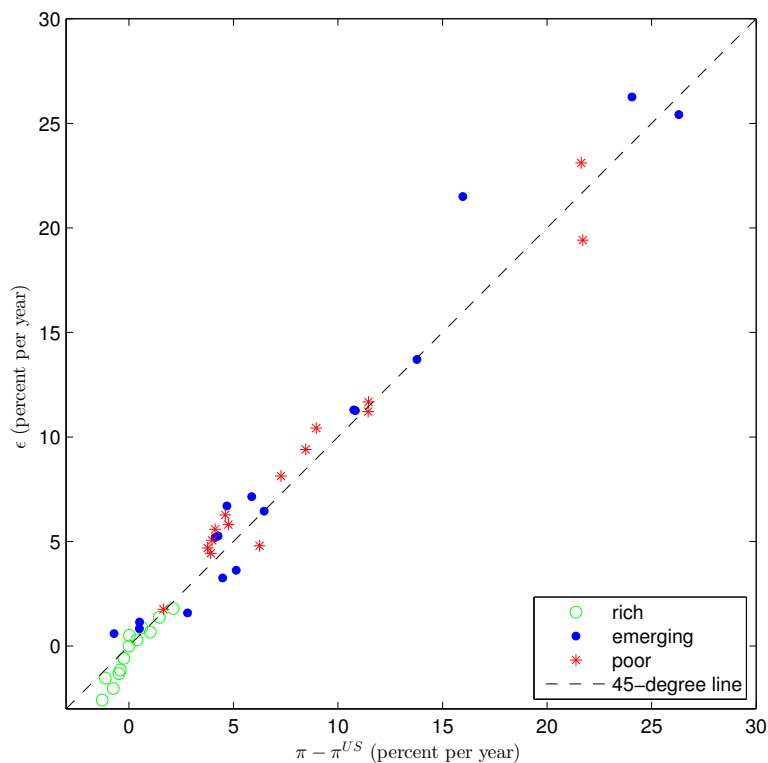
We say that relative PPP holds in the long run if ϵ_t^r is zero on average over a long period of time. Let ϵ , π , and π^{US} denote the average values of

ϵ_t , π_t , and π_t^{US} . Then we can write

$$\text{Relative PPP holds in the long run if } \epsilon = \pi - \pi^{US}.$$

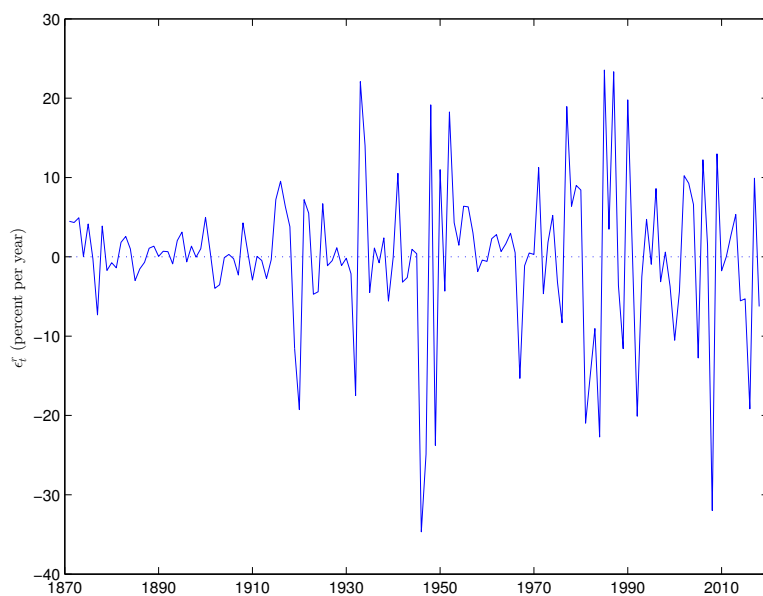
Figure 8.5 plots the average depreciation rate, ϵ , against the average inflation differential, $\pi - \pi^{US}$, for 45 countries. Averages are taken over the period 1960 to 2017. The figure includes 15 poor countries, shown with stars, 17 emerging countries, shown with bullets, and 13 rich countries, shown with circles.⁴ The fact that most observations line up along the 45-degree line suggests that the condition $\epsilon = \pi - \pi^{US}$ holds relatively well. This is particularly the case for rich countries. There are some observations lying relatively far from the 45-degree line. These observations correspond mostly to countries that suffered high inflation during the sample period, such as Chile, Hungary, Poland, and Sudan. In these cases the weakening of the relationship between the inflation differential and the depreciation rate might be due to difficulties measuring prices accurately in high-inflation environments. Overall, however, the clustering of observations close to the 45-degree line is remarkable, which suggests that relative PPP holds fairly well for many countries over the long run.

Figure 8.5: Inflation Differentials and Depreciation Rates, 1960 to 2017 Averages



Notes. Each marker represents a country. There are 45 countries in total; 13 rich, 17 emerging, and 15 poor. For a given country, ϵ denotes the average depreciation rate against the U.S. dollar and π denotes the average inflation rate. The variable π^{US} denotes the average U.S. inflation rate. The observations line up close to the 45-degree line, indicating that relative PPP holds well in the long run. Data sources. World Development Indicators and FRED. Countries with populations less than five million, with average inflation rates in excess of 30 percent, or with average depreciation rates in excess of 30 percent, or with less than 40 consecutive years of data were excluded.

Figure 8.6: Year-Over-Year Percent Change in the Dollar-Pound Real Exchange Rate: 1870-2018



Notes. The figure shows that the dollar-pound real exchange rate changes significantly from one year to the next, suggesting that relative PPP does not hold in the short run. Data Source: See Figure 8.4.

8.4.2 Does Relative PPP Hold in the Short Run?

Take another look at figure 8.4. As discussed before, over the past 150 years prices in the United States and the United Kingdom expressed in the same currency changed by about the same proportion. At the same time, the figure shows that on a period-by-period basis the difference between the two prices displays significant changes. Sometimes the two lines get closer to each other and sometimes they move further apart. This means that sometimes the United States becomes cheaper and sometimes more expensive relative to the United Kingdom. In other words, the real exchange rate changes in the short run.

This fact is shown more clearly in figure 8.6, which displays the year-over-year real depreciation rate, ϵ_t^r , expressed in percent per year, which corresponds the change in the distance between the solid and the broken lines in figure 8.4. If the real exchange rate were constant over time, figure 8.6 would display a flat line at zero. But this is far from being the case. The figure shows that the real depreciation rate moves around quite a bit. The standard deviation of ϵ_t^r is 9.3 percent. This means that typically, from one year to the next, the United States becomes almost 10 percent more expensive or cheaper than the United Kingdom. Thus, you should not be surprised if one year you visit the United Kingdom and find it cheap and

⁴The rich countries are: Australia, Austria, Belgium, Canada, France, Germany, Italy, Japan, Netherlands, Sweden, Switzerland, United Kingdom, and the United States. The emerging countries are: Algeria, Chile, Colombia, Egypt, Greece, Hungary, Iran, South Korea, Malaysia, Mexico, Morocco, Poland, Portugal, South Africa, Spain, Thailand, and Turkey. The poor countries are: Cameroon, Ethiopia, Ghana, India, Indonesia, Kenya, Madagascar, Myanmar, Nepal, Nigeria, Pakistan, Philippines, Sri Lanka, Sudan, and Tanzania. The classification of countries is taken from Uribe and Schmitt-Grohé, *Open Economy Macroeconomics*, Princeton University Press, 2017.

just a couple of years later you go back and find it quite expensive. The fact that ϵ_t^r is quite volatile means that relative PPP does not hold in the short run.

8.5 How Wide Is the Border?

We just documented that relative PPP does not hold in the short run. Relative costs of living can change significantly across countries from one year to the next. A natural question is whether these time varying deviations from purchasing power parity are due to the existence of a country border or hold more broadly across different geographic locations. One factor that could explain the failure of relative PPP is transportation costs. It might pay for New York households to shop in Newark to exploit relatively small differences in prices between the two cities. However, it would take much larger price differences to induce New York households to shop in Philadelphia. Therefore, differences in prices between New York and Philadelphia are likely to be larger than differences in prices between New York and Newark. Moreover, if transportation costs change over time (say because of changes in the cost of gasoline), price differences are also likely to change over time.

But the mere existence of an international border between two locations could by itself introduce impediments for prices of baskets of goods to equalize across locations. Reasons why an international border might matter for the size of deviations from relative PPP include movements in nominal exchange rates in combination with rigidities in local currency prices and

trade frictions that contribute to market segmentation across countries such as tariffs, quotas, and government regulation. If these factors are sufficiently pronounced, then we could observe, for example, that even though New York is much farther away from Los Angeles than from Toronto, relative PPP fail more significantly for the NY-Toronto pair than for the NY-LA pair.

In an influential paper, Charles Engel of the University of Wisconsin and John Rogers of the Board of Governors of the Federal Reserve System set out to quantify how important the international border is for short-run deviations from relative PPP.⁵ They consider the consumer price index of 14 baskets of goods in 14 U.S. cities and 9 Canadian cities over the period September 1978 to December 1994.⁶ Let $P_{c,t}^g$ be the price index of good basket g in city c at time t . Then, the real exchange rate of basket g in the city pair $(c1, c2)$ at time t , which we denote by $e_{c1,c2,t}^g$, is given by

$$e_{c1,c2,t}^g = \frac{\mathcal{E}_{c1,c2,t} P_{c2,t}^g}{P_{c1,t}^g},$$

where $\mathcal{E}_{c1,c2,t}$ is the nominal exchange rate between cities $c1$ and $c2$ in period t defined as the price of one unit of the currency used in city $c2$ in terms of units of the currency used in city $c1$. Of course, if the two cities are located in the same country, then $\mathcal{E}_{c1,c2,t}$ is unity (the U.S.-dollar price of one U.S.

⁵Charles Engel and John H. Rogers, "How Wide is the Border?," *American Economic Review* 86, December 1996, 1112-1124.

⁶The baskets of goods are food at home; food away from home; alcoholic beverages; shelter; fuel and other utilities; household furnishings and operations; men's and boy's apparel; women's and girl's apparel; footwear; private transportation; public transportation; medical care; personal care; and entertainment. The U.S. cities included are Baltimore; Boston; Chicago; Dallas; Detroit; Houston; Los Angeles; Miami; New York; Philadelphia; Pittsburgh; San Francisco; St. Louis; and Washington, DC. The Canadian cities included are Calgary; Edmonton; Montreal; Ottawa; Quebec; Regina; Toronto; Vancouver; and Winnipeg.

dollar is one, and the Canadian-dollar price of one Canadian dollar is also one).

Let $\Delta \ln e_{c1,c2,t}^g$ denote the time difference of the logarithm of the real exchange rate of basket g between cities $c1$ and $c2$. This variable measures the percentage change in the real exchange rate over time. If relative PPP holds, then $\Delta \ln e_{c1,c2,t}^g$ should be close to zero. Further, let $\sigma_{c1,c2}^g$ be the standard deviation of $\Delta \ln e_{c1,c2,t}^g$ taken across time periods. A large value of $\sigma_{c1,c2}^g$ indicates large violations of relative PPP in the short run.

For each basket of goods g there are as many standard deviations as there are city pairs. Since the dataset contains 14 baskets of goods and 23 cities, the maximum possible number of standard deviations is 3542.

Engel and Rogers estimate the following regression:⁷

$$\sigma_{c1,c2}^g = \text{constant} + 0.00106 \ln d_{c1,c2} + 0.0119 B_{c1,c2} + \mu_{c1,c2}^g,$$

where $d_{c1,c2}$ denotes the distance in miles between cities $c1$ and $c2$, $B_{c1,c2}$ is a variable that takes the value 1 if cities $c1$ and $c2$ are separated by an international border and zero otherwise, and $\mu_{c1,c2}^g$ is a regression residual. The positive coefficients on distance and border indicate, as expected, that the volatility of changes in deviations from relative PPP are increasing in these two variables. The farther apart are cities, the larger the variations in real exchange rate changes will be. The existence of an international border between two cities elevates the volatility of real exchange rate changes, that is, for the same distance between two cities the standard deviation of changes

⁷The term labeled ‘constant’ includes an intercept and city fixed effects.

in real exchange rates is larger for U.S.-Canada city pairs than it is for U.S.-U.S. or Canada-Canada city pairs.

To quantify the impact of the international border on deviations from relative PPP, one could ask what is the required increase in distance between two cities in the same country to obtain the same volatility in real-exchange-rate changes as two cities separated by the same distance but located on different sides of the U.S.-Canada border. To answer this question, note that the existence of a border between two cities increases the standard deviation of changes in real exchange rates by 0.0119. Increasing the distance between two cities by one mile raises the standard deviation by

$$\frac{\partial \sigma_{c1,c2}^g}{\partial d_{c1,c2}} = 0.00106 \frac{\partial \ln d_{c1,c2}}{\partial d_{c1,c2}} = 0.00106 \frac{1}{d_{c1,c2}}.$$

The average distance between two cities in the Engel-Rogers dataset is about 1,100 miles. Thus, we have that the increase in volatility associated with an increase in distance of one mile is

$$\frac{\partial \sigma_{c1,c2}^g}{\partial d_{c1,c2}} = 0.00106 \frac{1}{1100} = 0.00000096364.$$

Since each mile adds 0.00000096364 to the standard deviation and the border adds 0.0119 to the standard deviation, we have that being separated by the border is equivalent to increasing the distance between two cities by 0.0119/0.00000096364, or about 12 thousand miles. This is quite a wide border!

In sum, the evidence presented here suggests that the amplitude of de-

viations from relative PPP is increasing in the distance separating two locations. In addition, the mere existence of an international border separating two locations adds significantly to this amplitude. This means that factors such as exchange rate volatility, local price rigidities, tariffs, quotas, and cross-border regulations play an important role in determining the size of changes in real exchange rates in the short run.

8.6 Nontradable Goods and Deviations from Purchasing Power Parity

In the preceding sections, we have documented the existence of large and persistent deviations from purchasing power parity. For example, we saw that in 2011 a basket of goods that cost 100 dollar in the United States cost only 32 dollars in India, that is, India was three times as cheap as the United States. One may wonder whether it wouldn't pay for U.S. consumers to import goods from India. This would tend to equalize prices across the two countries.

One reason why price differences tend to persist is that not all goods are internationally tradable. For these goods, transportation costs are too large for international trade to be profitable. For instance, few people would fly from the United States to India just to take advantage of a cheaper haircut. Goods and services with these characteristics are called nontradable goods or nontraded goods. Examples of nontradable goods include services, such as haircuts, restaurant meals, housing, some health services, and some educational services. But not all services are nontradables. For instance,

the United States exports high-level educational services, such as college, master, and doctoral education. There are also non-service goods that could be nontradable. For instance, some fresh vegetables, such as lettuce, are typically grown and consumed locally. Tradable goods include agricultural commodities, such as wheat, corn, and soybeans, metals, minerals, oil, and many manufacturing goods. In general, nontradables make up a significant share of a country's output, typically above 50 percent.

The existence of nontradables gives rise to systematic deviations from PPP. To see this, note that the consumption price level P is an average of all prices in the economy. Consequently, it includes both the prices of nontradables and the prices of tradables. The prices of nontradables are determined entirely by domestic factors, so one should not expect the law of one price to hold for this type of goods. Let P_T and P_N denote the domestic prices of tradables and nontradables, respectively, and P_T^* and P_N^* the corresponding foreign prices. Suppose that the law of one price holds for tradable goods, that is,

$$P_T = \mathcal{E}P_T^*.$$

By contrast, the law of one price does not hold for nontradable goods

$$P_N \neq \mathcal{E}P_N^*.$$

Suppose the price level, P , is some average of the prices of tradables and nontradables. We can then write

$$P = \phi(P_T, P_N),$$

where the function $\phi(\cdot, \cdot)$ is increasing in P_T and P_N and homogeneous of degree one. The homogeneity property means that if both P_T and P_N increase by the same percentage, then the price level P also increases by the same percentage. For example, if P_T and P_N increase by 5 percent, then P also increases by 5 percent.⁸ A number of functional forms satisfy the conditions imposed on $\phi(\cdot, \cdot)$. For instance, if P is a simple average of P_T and P_N , then we have that $\phi(P_T, P_N) = (P_T + P_N)/2$. If instead P is a geometric average of P_T and P_N , then $\phi(P_T, P_N) = (P_T)^\gamma (P_N)^{1-\gamma}$, with $\gamma \in (0, 1)$. Section 8.9 provides microfoundations for this functional form.

Assume that the price level in the foreign country is also constructed as some average of the prices of tradables and nontradables. For simplicity, assume the same functional form as in the domestic country, that is,

$$P^* = \phi(P_T^*, P_N^*).$$

We can then write the real exchange rate, e , as

$$\begin{aligned} e &= \frac{\mathcal{E}P^*}{P} \\ &= \frac{\mathcal{E}\phi(P_T^*, P_N^*)}{\phi(P_T, P_N)} \\ &= \frac{\mathcal{E}P_T^*\phi(1, P_N^*/P_T^*)}{P_T\phi(1, P_N/P_T)} \\ &= \frac{\phi(1, P_N^*/P_T^*)}{\phi(1, P_N/P_T)}. \end{aligned} \tag{8.2}$$

The last equality says that the real exchange rate depends on the relative

⁸Technically, homogeneity of degree one means that $\phi(\lambda P_T, \lambda P_N) = \lambda \phi(P_T, P_N)$ for any $\lambda > 0$.

price of nontradables in terms of tradables across countries. The real exchange rate is less than one, that is, the consumption basket is less expensive abroad than domestically, if the relative price of nontradables in terms of tradables is lower in the foreign country than domestically. Formally,

$$e < 1 \text{ if } \frac{P_N^*}{P_T^*} < \frac{P_N}{P_T}.$$

Going back to the example of India and the United States, this inequality says that India is cheaper than the United States because in India the relative price of nontradables in terms of tradables is lower than in the United States.

8.7 Trade Barriers and Real Exchange Rates

In the previous section, deviations from PPP occur due to the presence of nontradable goods. In this section, we investigate deviations from PPP that may arise even when all goods are traded. Specifically, we study deviations from PPP that arise because governments impose trade barriers, such as import tariffs, export subsidies, and quotas, that artificially distort relative prices across countries.

Consider, for simplicity, an economy in which all goods are internationally tradable. Suppose further that there are two types of tradable goods, importables and exportables. Importable goods are goods that are either imported or produced domestically but coexist in the domestic market with identical or highly substitutable imported goods. Exportable goods are goods that are produced domestically and sold in foreign and possibly

domestic markets. Let the world price of importables be P_M^* , and the world price of exportables be P_X^* . In the absence of trade barriers, PPP must hold for both goods, that is, the domestic prices of exportables and importables must be given by

$$P_X = \mathcal{E}P_X^*$$

and

$$P_M = \mathcal{E}P_M^*,$$

where, as before, \mathcal{E} denotes the nominal exchange rate defined as the domestic currency price of one unit of foreign currency. The domestic price level, P , is an average of P_X and P_M . Specifically, assume that P is given by

$$P = \phi(P_X, P_M),$$

where $\phi(\cdot, \cdot)$ is an increasing and homogeneous-of-degree-one function. A similar relation holds in the foreign country,

$$P^* = \phi(P_X^*, P_M^*).$$

The real exchange rate, $e = \mathcal{E}P^*/P$, can then be written as

$$e = \frac{\mathcal{E}P^*}{P} = \frac{\mathcal{E}\phi(P_X^*, P_M^*)}{\phi(P_X, P_M)} = \frac{\phi(\mathcal{E}P_X^*, \mathcal{E}P_M^*)}{\phi(P_X, P_M)} = \frac{\phi(P_X, P_M)}{\phi(P_X, P_M)} = 1,$$

where the third equality uses the fact that ϕ is homogeneous of degree one and the fourth equality uses the fact that the law of one price holds for both goods.

Consider next the consequences of imposing a tariff $\tau > 0$ on imports in the home country. Now an importer of goods pays $\mathcal{E}P_M^*$ to the foreign producer and $\tau\mathcal{E}P_M^*$ in tariffs to the local government. Consequently, the domestic price of the importable good increases by a factor of $1 + \tau$, that is,

$$P_M = (1 + \tau)\mathcal{E}P_M^*.$$

The domestic price of exportables is unaffected by the import tariff. Then the real exchange rate becomes

$$e = \frac{\mathcal{E}\phi(P_X^*, P_M^*)}{\phi(P_X, P_M)} = \frac{\phi(\mathcal{E}P_X^*, \mathcal{E}P_M^*)}{\phi(\mathcal{E}P_X^*, (1 + \tau)\mathcal{E}P_M^*)} < 1,$$

where the inequality follows from the fact that $\phi(\cdot, \cdot)$ is increasing in both arguments and that $1 + \tau > 1$. This expression shows that the imposition of import tariffs leads to an appreciation of the real exchange rate, that is, it makes the domestic consumption basket more expensive relative to the foreign consumption basket. Exercise 8.6 asks you to analyze how the imposition of an export subsidy affects the real exchange rate.

We have established that trade barriers can cause persistent deviations from PPP. According to this analysis, one should expect that protectionist trade policies, like the import tariffs imposed by the Trump administration in 2019, will cause an appreciation of the real exchange rate (a fall in e), making the United States more expensive relative to the rest of the world.

8.8 Home Bias and the Real Exchange Rate

Thus far we have seen that PPP may fail because not all goods are tradable or because of the presence of tariffs. In this section, we introduce a third reason for why PPP fails, namely, that the weights with which a particular good enters in the consumption basket is different across countries. Such differences in weights reflect primarily differences in tastes across countries. And in turn these differences in taste might reflect a preference for goods that the country specializes in. For instance, Argentines might spend a larger fraction of their income on beef than Germans. And Germans might spend a larger fraction of their income on cars than Argentines. Such a preference for domestically produced goods is called *home bias*. To see why home bias can lead to variations in the real exchange rate, suppose that beef becomes more expensive relative to cars. Since beef has a larger share in the Argentine consumption basket, the Argentine basket will become more expensive relative to the German basket. In other words, the Argentine peso will experience a real appreciation against the euro.

To formalize the way in which home bias affects the real exchange rate, suppose that in Argentina and Germany households consume only two goods, beef and cars. Let P_b denote the price of beef and P_c the price of cars in Argentina expressed in Argentine pesos. Similarly, let P_b^* and P_c^* , respectively, denote the price of beef and cars in Germany expressed in euros. Let P denote the price index in Argentina,

$$P = \phi(P_b, P_c) = (P_b)^\gamma (P_c)^{1-\gamma}$$

and P^* the price index in Germany,

$$P^* = \phi^*(P_b^*, P_c^*) = (P_b^*)^{\gamma^*} (P_c^*)^{1-\gamma^*}.$$

The parameters $\gamma \in (0, 1)$ and $\gamma^* \in (0, 1)$ capture the weights given to the price of beef in the Argentine and German consumer price indices, respectively. Now suppose that Argentines spend a lot on beef and not that much on cars and that Germans spend a lot on cars and relatively little on beef. In this case, the price of beef should have a larger weight in the Argentine price index than in the German one, that is, it should be the case that $\gamma > \gamma^*$. Suppose further that both beef and cars are freely traded internationally so that the law of one price holds for both goods, that is,

$$P_b = \mathcal{E} P_b^*$$

and

$$P_c = \mathcal{E} P_c^*,$$

where \mathcal{E} denotes the nominal exchange rate expressed as the peso price of one euro. The real exchange rate can then be written as,

$$e = \frac{\mathcal{E} P^*}{P} = \left(\frac{P_c}{P_b} \right)^{\gamma - \gamma^*}.$$

Because $\gamma > \gamma^*$, an increase in the price of beef in terms of cars causes a real appreciation of the peso (a fall in e). Intuitively, if the relative price of beef increases, then the price of the Argentine consumption basket, P ,

increases by more than the price of the German consumption basket, P^* , since beef has a larger weight in the Argentine basket than in the German basket. It follows that the Argentine consumption basket becomes relatively more expensive.

8.9 Price Indices and Standards of Living

Thus far, we have assumed that the price level, P , is given by some function $\phi(P_T, P_N)$, assumed to be increasing and homogeneous of degree one in the nominal prices of tradables, P_T , and nontradables, P_N . Intuitively, the function $\phi(P_T, P_N)$ is an average of P_T and P_N . But what type of average? What weights should the average place on P_T and P_N ? Is the price index useful to measure standards of living? For example, suppose that your money income increases by 10 percent and the price level increases by 11 percent. Are you better off or worse off? At first glance, we could say that your real income fell by 1(=10-11) percent, suggesting that you are worse off. However, suppose that all of the 11 percent increase in the price level is due to increases in the price of meat. If you happen to be a vegetarian, the price index that is relevant to you should place a weight of 0 on meat products. This price index could have shown no movement or even a decrease, in which case your real income would have increased and you would be better off.

The above example suggests that the weights assigned by the price index to different individual prices should reflect consumers' preferences. In this section, we establish this connection.

Suppose that the household values current consumption according to the

utility function

$$U(C),$$

where C denotes current consumption and $U(\cdot)$ is an increasing function. Suppose, in turn, that consumption is a composite of tradable and nontradable consumption given by the aggregator function

$$C = C_T^\gamma C_N^{1-\gamma}, \quad (8.3)$$

where C_T and C_N denote, respectively, consumption of tradable goods and consumption of nontradable goods, and γ is a parameter lying in the interval $(0, 1)$. The aggregator function can be interpreted either as a sub-utility function or as a technology that combines tradables and nontradables to produce the composite consumption good. This aggregator function is known as the *Cobb-Douglas* aggregator. It is a special case of an aggregator function called the *constant elasticity of substitution (CES) aggregator* or the *Armington aggregator*, which we introduce in exercise 8.8.

Let's define the consumer price level, P , as the minimum amount of money necessary to purchase one unit of the composite consumption good C . Formally, P is given by

$$P = \min_{\{C_T, C_N\}} \{P_T C_T + P_N C_N\}$$

subject to

$$C_T^\gamma C_N^{1-\gamma} = 1,$$

taking as given P_T and P_N . This is a constrained minimization problem in two variables, C_T and C_N . To transform it into an unconstrained problem in just one variable, solve the constraint for C_N and use the resulting expression to eliminate C_N from the objective function. Then the objective function features just one unknown, C_T . Let's follow these steps one at a time. Solving the constraint for C_N yields

$$C_N = C_T^{\frac{-\gamma}{1-\gamma}}. \quad (8.4)$$

Now using this expression to eliminate C_N from the objective function gives

$$P = \min_{\{C_T\}} \left\{ P_T C_T + P_N C_T^{\frac{-\gamma}{1-\gamma}} \right\}. \quad (8.5)$$

The first term of the objective function is increasing in C_T , reflecting the direct cost of purchasing tradable consumption goods. The second term is decreasing in C_T , because an increase in the consumption of tradables allows for a reduction in the consumption of nontradables while still keeping the amount of composite consumption at unity. The optimality condition associated with the minimization problem given in (8.5) is the derivative of the objective function set to zero, that is,

$$P_T - \frac{\gamma}{1-\gamma} P_N C_T^{\frac{-1}{1-\gamma}} = 0.$$

Solving for C_T , we obtain

$$C_T = \left[\frac{\gamma}{1-\gamma} \frac{P_N}{P_T} \right]^{1-\gamma}. \quad (8.6)$$

Now using this expression to eliminate C_T in equation (8.4), we get

$$C_N = \left[\frac{\gamma}{1-\gamma} \frac{P_N}{P_T} \right]^{-\gamma}. \quad (8.7)$$

Intuitively, the above two expressions say that as the nontradable good becomes relatively more expensive, that is, as P_N/P_T increases, there is an optimal substitution away from nontradables and toward tradables in the production of the unit of composite consumption. Finally, use (8.6) and (8.7) to eliminate C_T and C_N from the objective function (8.5) to obtain

$$P = P_T^\gamma P_N^{1-\gamma} A,$$

where $A \equiv \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)}$ is a constant independent of prices. The above formula is important because it tells us that the weights assigned to the prices of tradable and nontradable goods in the consumer price level are related to the weights assigned to the corresponding goods in the aggregator function, equation (8.3). The more important is the good in the aggregator function, the larger the weight its price will receive in the consumer price index.

With the microfoundations of the consumer price index in hand, we can now revisit the question of how changes in real income relate to welfare. If the household allocates an amount of money Y to consumption, it is clear from the fact that P represents the minimum amount of money required to

obtain 1 unit of the composite good, that C is given by

$$C = \frac{Y}{P}.$$

This means that when the construction of the price level P assigns the correct weight to the price of each of the goods that compose the consumption basket, in this case P^T and P^N , changes in real income are directly linked to changes in consumers' welfare. That is, the price level constructed here guarantees that if real income, Y/P , goes up, the household can afford a higher level of composite consumption, C . Moreover, the percent change in real income tells us the percent increase in consumption the consumer is able to afford. For instance, suppose that $\gamma = 0.25$, and that in the course of one year nominal income increased by 10 percent, the price of tradables by 12 percent, and the price of nontradables by 8 percent, i.e., $\% \Delta Y = 0.1$, $\% \Delta P_T = 0.12$, and $\% \Delta P_N = 0.08$. Is the household better off or worse off relative to the previous year? In other words, can the consumer afford more or less composite consumption in the current year relative to the previous one? Without knowing the price index, this question does not have an answer, because, although nominal income increased, and the price of nontradables increased proportionally less than income, the price of tradables increased proportionally more than income. Knowing the weights in the price level, however, the answer is straightforward. The percent increase

in the amount of consumption the consumer can enjoy this year is given by

$$\begin{aligned}
 \% \Delta C &= \% \Delta \frac{Y}{P} \\
 &= \% \Delta Y - \% \Delta P \\
 &= \% \Delta Y - \gamma \% \Delta P_T - (1 - \gamma) \% \Delta P_N \\
 &= 0.1 - 0.25 \times 0.12 - 0.75 \times 0.08 \\
 &= 1\%.
 \end{aligned}$$

This means that the consumer is better off, as he or she can afford 1% more consumption relative to the previous year. The intuition behind this increase in welfare is that the price that increases proportionally more than income corresponds to a good (the tradable good) that is not too important in the generation of composite consumption, as measured by the weight γ .

What could happen if the statistical office used a wrong value of γ in constructing the price level? To answer this question, let us redo the above exercise using a weight $\tilde{\gamma} = 0.75$. Under this incorrect weight, the change in real income is

$$\begin{aligned}
 \% \Delta \frac{Y}{P} &= 0.1 - 0.75 \times 0.12 - 0.25 \times 0.08 \\
 &= -1\%,
 \end{aligned}$$

which leads to the misleading conclusion that consumers are worse off and can afford 1% less consumption than in the previous year. The problem here is that the statistical office is assigning too much weight to the price that increased the most.

It follows from the above example that if price indices are to be informative about changes in standard of living, they must assign the correct weights to individual prices. But how can the statistical agency know what the value of γ is? Even if the government could conduct a survey asking consumers what their individual γ is, people might not know what to answer. The typical individual has never heard anything about utility functions or aggregator functions. Fortunately, there is an indirect and more practical way to infer the value of γ . It consists in observing patterns of consumer expenditure in different types of goods. Specifically, dividing (8.6) by (8.7) and solving for γ yields

$$\gamma = \frac{P_T C_T}{P_T C_T + P_N C_N}.$$

This expression says that γ equals the ratio of expenditure on tradable in total expenditure. Thus, knowing how much individuals spend on each category of goods allows us to obtain the correct weights of each individual price in the aggregate price index P . Statistical agencies periodically conduct surveys asking individuals about their expenditure behavior and use this information as an input in the construction of price indices.

8.10 Summing Up

This chapter is concerned with differences in costs of living across countries. Key concepts introduced in the chapter are the law of one price (LOOP), purchasing power parity (PPP), the real exchange rate, nontradable goods,

home bias, and price indices.

- The LOOP states that the same good must have the same price across countries or regions when expressed in a common currency.

- There exist large and systematic deviations from the LOOP.

- Absolute PPP extends the concept of the LOOP to baskets of goods.

It says that consumption baskets must have the same price across countries or regions when expressed in a common currency.

- Observed deviations from absolute PPP are large and persistent.

- The real exchange rate is the relative price of a basket of goods in a foreign country in terms of a basket of goods in the domestic country. By definition, when absolute PPP holds, the real exchange rate is one.

- Relative PPP holds when the real exchange rate does not change over time.

- Relative PPP holds over the long run, but fails over the short run.

- Rich countries are systematically more expensive than poor countries.

- The PPP exchange rate is the nominal exchange rate that makes PPP hold. Evaluating GDP per capita at PPP exchange rates allows for meaningful comparisons of standards of living.

- Deviations from PPP across locations are accounted for by a number of factors:

- ◊ Distance between the two locations, which can be a reflection of transportation costs.

- ◊ The existence of an international border separating the two locations. This can reflect the presence of nominal exchange-rate volatility, local-currency price rigidity, cross-border regulations, import and export

tariffs, and trade quotas.

◇ The existence of nontradable goods. Nontradable goods are goods that are domestically produced, but are neither importable nor exportable.

- If the weights on prices that enter the consumer price index differ across countries, then variations in relative prices can lead to variations in real exchange rates. If domestically produced goods make up a bigger share of the domestic basket than of the foreign basket and hence receive a larger weight in the home country's price index than in the foreign country's price index, then we say there is home bias in consumption.

- The imposition of import tariffs or export subsidies makes the country more expensive relative to the rest of the world, that is, it causes a real exchange rate appreciation.

- The optimal price index assigns weights to individual prices that correspond to the weight of the associated good in the consumer's utility function. When nominal income is deflated by the optimal price index, variations in real income represent variations in welfare in the same direction.

8.11 Exercises

Exercise 8.1 (PPP in China) The International Comparison Program (ICP) reported a price level index (PLI) for China of 42 in 2005 and 54 in 2011. Recall that, by construction, the PLI for the United States is always 100.

1. Find the percent change in the Yuan-dollar real exchange rate between 2005 and 2011.
2. In 2005 the size of the Chinese economy, at PPP exchange rates, was 43 percent that of the U.S. economy. Ignoring growth in physical output, find the size of the Chinese economy, at 2011 PPP exchange rates, relative to that of the U.S. economy.
3. Suppose instead that all of the observed real appreciation of the yuan was due to the imposition of import tariffs by China. Assume that in the U.S. and China the price level is given by $P = P_X^\gamma P_M^{1-\gamma}$, where $\gamma = 0.5$, P_X and P_M denote export and import prices, respectively, and that absent tariffs the law of one price holds. Find the size of the import tariff.

Exercise 8.2 (Higher Prices in Richer Countries) Figure 8.3 plots the real exchange rate against per capita GDP at market exchange rates. Redo this figure using per capita GDP at PPP exchange rates instead. Use red dots. The data is available at the source indicated in the figure. For comparison, the figure should also include, with black dots, the relationship using

GDP at market exchange rates. Discuss whether it is still the case that richer countries are more expensive. Explain why the slope changes.

Exercise 8.3 (International Comparison of Standards of Living) The following table displays (fictional) quantities of final goods produced and prices in the United States and Argentina in 2021.

Good	United States		Argentina	
	Quantity	Price	Quantity	Price
Tradable	10	5	4	50
Nontradable	20	15	8	100

U.S. prices are expressed in dollars and Argentine prices in pesos. The typical basket of goods in both countries includes 1 unit of tradable goods and 2 units of nontradable goods. Tradable goods are freely traded internationally. For this type of good PPP holds.

1. Calculate the market exchange rate, expressed as the dollar price of one peso.
2. Calculate the price levels in the United States and Argentina, expressed in dollars and pesos, respectively.
3. What is the dollar/peso real exchange rate? In which country is it cheaper to live?
4. Calculate GDP in the United States and Argentina at market prices. According to this measure, how big is Argentina relative to the United States ?

5. Calculate the PPP exchange rate.
6. Calculate GDP at PPP prices.
7. Comment on the numbers you obtained for the two measures of GDP.

Exercise 8.4 (Consumption Baskets and Comparisons of Living Standards)

Following up with the theme of exercise 8.3, suppose that the table of quantities and prices is now given by

United States			Argentina	
Good	Quantity	Price	Quantity	Price
Grain	10	1	10	1
Energy	10	2	10	2

Again, U.S. prices are expressed in dollars and Argentine prices in pesos. The typical consumption basket in the United States contains 1 unit of each good, while the typical consumption basket in Argentina contains 1.5 units of grain and 0.5 units of energy. Both goods are freely traded internationally and PPP holds for each type of good.

1. Calculate GDP in the United States and Argentina in the following alternative ways:
 - (a) At market prices.
 - (b) At PPP exchange rates using for each country its own basket.
 - (c) At PPP exchange rates using the U.S. basket.
 - (d) At PPP prices using the Argentine basket.

2. Which of the above measures give a correct idea of the relative sizes of the U.S. and Argentine GDPs?
3. Taking into account what you learned from this exercise and exercise 8.3, what methodological guidelines would you give to statistical agencies that produce international comparisons of standards of living.

Exercise 8.5 (Cost-of-Living Comparison) Suppose you work for an investment firm in New York City. You derive utility from consumption, as described by the utility function $U(C)$, where C denotes consumption and $U(\cdot)$ is an increasing function. Consumption is a composite of food and housing given by

$$C = \sqrt{C_F} \sqrt{C_H},$$

where C_F and C_H denote, respectively, consumption of food and housing. Suppose that in your current job, you make \$250,000 per year. Your company offers you a one-year position in its Bolivian branch located in La Paz. In NYC the price of food is \$50 per unit and the price of housing is \$750 per unit. Food is internationally traded, but housing is nontradable. The dollar/Bolivian peso exchange rate is 5 pesos per dollar. The price of housing in La Paz is 2,000 pesos. Suppose that all you care about is to maximize your utility and that you always spend all of your income in consumption (no savings).

1. Your boss would like to know what would be the minimum income (in dollars) you would require to be willing to work in La Paz. What would you answer? Show your work and provide intuition.

2. How many units of food and housing do you consume in NYC and how many would you consume in La Paz with your minimum required income? Provide intuition.

Exercise 8.6 (Trade Barriers and the Real Exchange Rate) In section 8.7, we deduced that the imposition of an import tariff causes an appreciation of the real exchange rate (i.e., makes the country more expensive relative to other countries). Use a similar analysis to show how the real exchange rate is affected by the following trade barriers:

1. An import subsidy.
2. An export tariff.
3. An export subsidy.

Exercise 8.7 (Trump's Trade Barriers) Evaluate the following statement: In April 2018, the United States levied a 25 percent import duty on steel and a 10 percent import duty on aluminum. This policy should make the United States more expensive relative to the rest of the world.

Exercise 8.8 (Microfounded Price Indices) Let C , C_T , and C_N denote consumption, consumption of tradables, and consumption of nontradables, respectively. Let P , P_T , and P_N denote the consumer price level, the price of tradable, and nontradable goods, respectively. Find the consumer price level under the following aggregator functions:

1. Leontief aggregator,

$$C = \min \left\{ \frac{C_T}{\gamma}, \frac{C_N}{1-\gamma} \right\},$$

where $\gamma \in (0, 1)$ is a parameter.

2. Linear aggregator,

$$C = \gamma C_T + (1 - \gamma) C_N,$$

where $\gamma \in (0, 1)$ is a parameter.

3. Constant elasticity of substitution (CES) or Armington aggregator,

$$C = \left[\gamma C_T^{1-\frac{1}{\xi}} + (1-\gamma) C_N^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}},$$

where $\gamma \in (0, 1)$ and $\xi > 0$ are parameters.

Chapter 9

Determinants of the Real Exchange Rate

In chapter 8, we documented that the real exchange rate moves substantially over time. This means that countries sometimes become cheaper than other countries and sometimes more expensive. In this chapter, we investigate what causes the real exchange rate to move over time. We will address this question from two perspectives, the short run and the long run. In the short run, the factors of production, such as labor, cannot move easily from one sector of the economy to another. For example, an accountant cannot become a farmer from one month to the next just because the wage rate went up in the agricultural sector. Similarly, a farmer cannot quickly become an accountant when the wage rate earned by accountants goes up. Technology is also fixed in the short run. It took decades for tractors to replace horses in the farming sector. Thus, in the short run, relative prices are primarily

determined by factors driving the demand for goods, such as movements in the interest rate or changes in tastes, or by factors that affect the supply of goods in the short run, such as weather conditions in the agricultural sector. Since the real exchange rate is the relative price of a basket of goods in one country in terms of baskets of goods in another country, we have that these factors will dominate movements in the real exchange rate in the short run.

By contrast, in the long run, factors of production can move more freely across sectors. The accountant can retrain herself and become a farmer, and the farmer can earn a degree in accounting. In addition, the adoption of new technologies affects the speed at which production possibilities expand in different sectors of the economy. Thus, in the long run, relative prices, and in particular the real exchange rate, are determined to a large extent by factors affecting the supply of goods.

The short- and long-run approaches to understanding the determination of the real exchange rate have given rise to two important models known as the TNT (Traded-Non-Traded) and the Balassa-Samuelson models, respectively. The present chapter is devoted to the analysis of these two models of real exchange rate determination.

9.1 The TNT Model

The TNT model is identical to the open economy model we studied in chapter 3, except that instead of a single tradable good it features two goods, one tradable and one nontradable. The tradable good can be imported or exported without restrictions. By contrast, the nontradable good is not

exchanged in international markets, and must be produced and consumed domestically. The presence of two goods introduces a new endogenous variable in the model, the relative price of nontradable goods in terms of tradable goods. As we saw in chapter 8 (see especially section 8.6), this relative price plays a key role in determining the real exchange rate. The reason is that because the law of one price holds for the tradable good, movements in its price do not introduce changes in relative consumer prices across countries. At the same time, the price of the nontradable good is not equalized across borders, so variations therein do affect relative prices across countries.

9.1.1 Households

Consider a two-period economy populated by many identical households whose preferences are described by the utility function

$$\ln C_1 + \beta \ln C_2, \quad (9.1)$$

where C_1 and C_2 denote consumption in periods 1 and 2, and $\beta \in (0, 1)$ is a parameter representing the subjective discount factor. As in section 8.9 of chapter 8, assume that consumption is a composite of tradable and nontradable goods described by the Cobb-Douglas aggregation technologies

$$C_1 = (C_1^T)^\gamma (C_1^N)^{1-\gamma}, \quad (9.2)$$

and

$$C_2 = (C_2^T)^\gamma (C_2^N)^{1-\gamma}, \quad (9.3)$$

where C_t^T and C_t^N denote consumption of tradables and nontradables in periods $t = 1, 2$, respectively, and $\gamma \in (0, 1)$ is a parameter governing the relative importance of tradable consumption in utility.

Suppose that the household is endowed with Q_t^T and Q_t^N units of tradable and nontradable goods in periods $t = 1, 2$. Households start period 1 with no debts or assets. In period 1, households can borrow or lend by means of a bond, denoted B_1 , denominated in units of tradables and paying the interest rate r in period 2. The budget constraint of the household in period 1 is then given by

$$P_1^T C_1^T + P_1^N C_1^N + P_1^T B_1 = P_1^T Q_1^T + P_1^N Q_1^N,$$

where P_t^T and P_t^N denote the prices of tradable and nontradable goods in periods $t = 1, 2$. In period 2, the household's budget constraint is given by

$$P_2^T C_2^T + P_2^N C_2^N = P_2^T Q_2^T + P_2^N Q_2^N + (1 + r)P_2^T B_1.$$

Let us now express both budget constraints in terms of tradable goods. To this end, let

$$p_t \equiv \frac{P_t^N}{P_t^T}$$

denote the relative price of the nontradable good in terms of tradable goods in periods $t = 1, 2$. Then, dividing the period-1 budget constraint by P_1^T and the period-2 budget constraint by P_2^T , we obtain

$$C_1^T + p_1 C_1^N + B_1 = Q_1^T + p_1 Q_1^N$$

and

$$C_2^T + p_2 C_2^N = Q_2^T + p_2 Q_2^N + (1+r)B_1.$$

Combining the two budget constraints to eliminate B_1 , yields the intertemporal budget constraint

$$C_2^T + p_2 C_2^N = Q_2^T + p_2 Q_2^N + (1+r)(Q_1^T + p_1 Q_1^N - C_1^T - p_1 C_1^N). \quad (9.4)$$

To save notation, let Y denote the household's lifetime income expressed in units of tradable goods in period 2, that is, let

$$Y \equiv Q_2^T + p_2 Q_2^N + (1+r)(Q_1^T + p_1 Q_1^N).$$

The intertemporal budget constraint can then be written as

$$C_2^T = Y - p_2 C_2^N - (1+r)(C_1^T + p_1 C_1^N).$$

Using the aggregator functions (9.2) and (9.3) to eliminate C_1 and C_2 from the utility function (9.1) and the above intertemporal budget constraint to eliminate C_2^T from the resulting expression, the household's optimization problem reduces to choosing C_1^T , C_1^N , and C_2^N to maximize

$$\gamma \ln C_1^T + (1-\gamma) \ln C_1^N + \beta \gamma \ln [Y - p_2 C_2^N - (1+r)(C_1^T + p_1 C_1^N)] + \beta(1-\gamma) \ln C_2^N.$$

The first-order conditions associated with this problem are obtained by taking the derivatives of this expression with respect to C_1^T , C_1^N , and C_2^N and

equating them to zero. This operation yields

$$\frac{1}{C_1^T} - \frac{\beta(1+r)}{Y - p_2 C_2^N - (1+r)(C_1^T + p_1 C_1^N)} = 0,$$

$$\frac{1-\gamma}{C_1^N} - \frac{\gamma\beta(1+r)p_1}{Y - p_2 C_2^N - (1+r)(C_1^T + p_1 C_1^N)} = 0,$$

and

$$\frac{1-\gamma}{C_2^N} - \frac{\gamma p_2}{Y - p_2 C_2^N - (1+r)(C_1^T + p_1 C_1^N)} = 0.$$

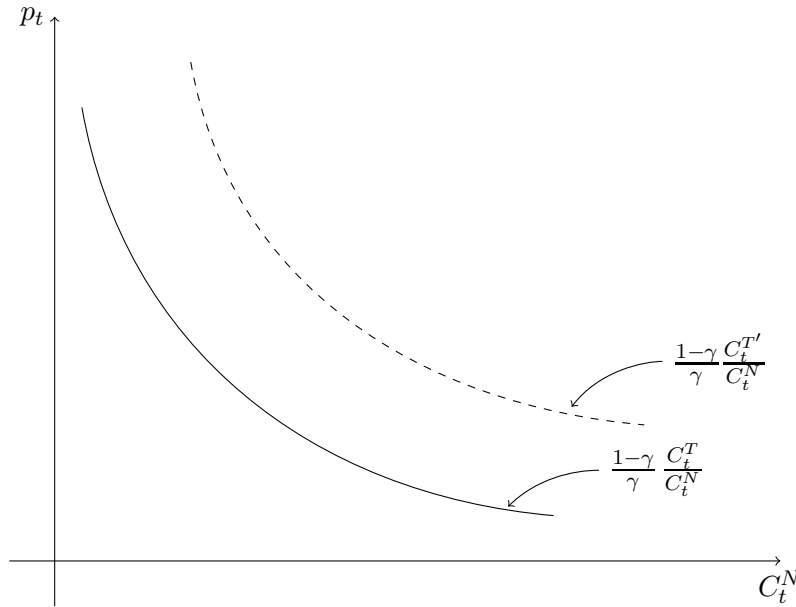
Using the fact that $C_2^T = Y - p_2 C_2^N - (1+r)(C_1^T + p_1 C_1^N)$ and rearranging, we can write these optimality conditions as

$$C_2^T = \beta(1+r)C_1^T, \quad (9.5)$$

$$C_t^N = \frac{1-\gamma}{\gamma} \frac{C_t^T}{p_t}, \quad (9.6)$$

for $t = 1, 2$. The first optimality condition is the familiar Euler equation stating that as the interest rate increases, households substitute period-2 consumption for period-1 consumption. The second optimality condition says that as nontradables become more expensive, households reduce consumption of nontradables relative to consumption of tradables. Given C_t^T , the second optimality condition represents the demand for nontradables in period t . Figure 9.1 depicts with a solid line the demand schedule (9.6) in the space (C_t^N, p_t) . Like any standard demand schedule, this one is downward sloping. An increase in the desired consumption of tradables, C_t^T , shifts the demand schedule out and to the right. In the figure, an increase in tradable consumption from C_t^T to $C_t^{T'}$ gives rise to the demand schedule depicted

Figure 9.1: The Demand Function for Nontradables



Notes. The figure depicts the demand schedule for nontradables in period t . Holding consumption of tradables, C_t^T , constant, the higher is the relative price of nontradables, p_t , the lower the demand for nontradables, C_t^N , will be. An increase in the desired consumption of tradables from C_t^T to $C_t^{T'}$ shifts the demand schedule for nontradables out and to the right.

with a broken line. Intuitively, holding the relative price, p_t , constant, the desired consumptions of tradables and nontradables move in tandem. Of course, consumption of tradables is an endogenous variable that is determined within the model. Shortly, we will see that in the present model the equilibrium level of tradable consumption is identical to its counterpart in the one-good model studied in chapter 3.

9.1.2 Equilibrium

In equilibrium, the market for nontradable goods must clear. This means that in each period consumption of nontradables must equal the endowment of nontradables. Formally,

$$C_t^N = Q_t^N, \quad (9.7)$$

for $t = 1, 2$. The country is assumed to have free capital mobility, so in equilibrium the domestic interest rate, r , is equal to the world interest rate. We maintain the assumption of free capital mobility throughout this chapter. For this reason, we refer to r interchangeably as the domestic interest rate or as the world interest rate. Using the market clearing condition (9.7) to eliminate C_1^N and C_2^N from the intertemporal budget constraint (9.4), we obtain the following intertemporal resource constraint

$$C_2^T = Q_2^T + (1 + r)(Q_1^T - C_1^T),$$

which is the same as the resource constraint of the one-good economy studied in chapter 3 (see equation (3.4), and recall that here we are assuming that $B_0 = 0$). Combining this condition with the Euler equation (9.5), yields the equilibrium level of consumption of tradables in period 1,

$$C_1^T = \frac{1}{1 + \beta} \left(Q_1^T + \frac{Q_2^T}{1 + r} \right). \quad (9.8)$$

Thus, exactly as in the one-good economy of chapter 3, consumption of tradables depends on the present discounted value of the stream of tradable endowments. In particular, the consumption of tradables is an increasing

function of the current and the future expected endowments of tradables and a decreasing function of the interest rate. We summarize this relationship by writing

$$C_1^T = C^T(r, Q_1^T, Q_2^T). \quad (9.9)$$

The economy takes all of the variables on the right hand side as exogenous. In particular, because the country is assumed to be a small player in international financial markets, it takes the world interest rate, r , as given.

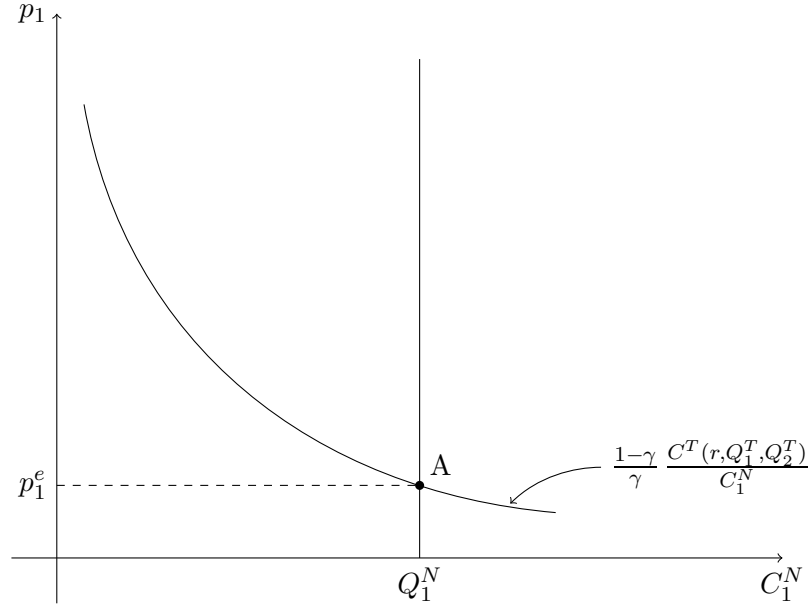
The trade balance in period 1, denoted TB_1 , is the difference between the endowment of tradables and consumption of tradables, $TB_1 = Q_1^T - C^T(r, Q_1^T, Q_2^T)$. So we can write

$$TB_1 = TB(r, Q_1^T, Q_2^T).$$

This is a familiar result. The trade balance improves in response to an increase in the interest rate or in the current endowment and deteriorates in response to an expected increase in the future endowment. The effects of changes in r and Q_2^T are straightforward. With respect to the effect of changes in Q_1^T , equilibrium condition (9.8) shows that in response to an increase in Q_1^T , C_1^T increases but by less than the increase in Q_1^T (recall that β is positive). Thus, an increase in Q_1^T causes the trade balance to improve. This result was first derived in chapter 3. Intuitively, C_1^T increases by less than Q_1^T because households save part of the income increase to smooth consumption over time.

Since the economy starts period 1 with a nil asset position, $B_0 = 0$, we have that in period 1 the current account equals the trade balance,

Figure 9.2: The Equilibrium Relative Price of Nontradables



Notes. The figure depicts the demand and supply functions of nontradables. The demand for nontradables is downward sloping and the supply schedule is a vertical line. The equilibrium relative price of nontradables is p_1^e , given by the intersection of the demand and supply schedules, point A.

$CA_1 = rB_0 + TB_1 = TB_1$. So we can write

$$CA_1 = CA(r, Q_1^T, Q_2^T). \quad (9.10)$$

Using the equilibrium level of tradable consumption given in (9.9) to eliminate C_1^T from (9.6), we obtain the following demand schedule for nontradables:

$$C_1^N = \frac{1-\gamma}{\gamma} \frac{C^T(r, Q_1^T, Q_2^T)}{p_1}, \quad (9.11)$$

which is plotted in figure 9.2. The figure also displays the supply of nontrad-

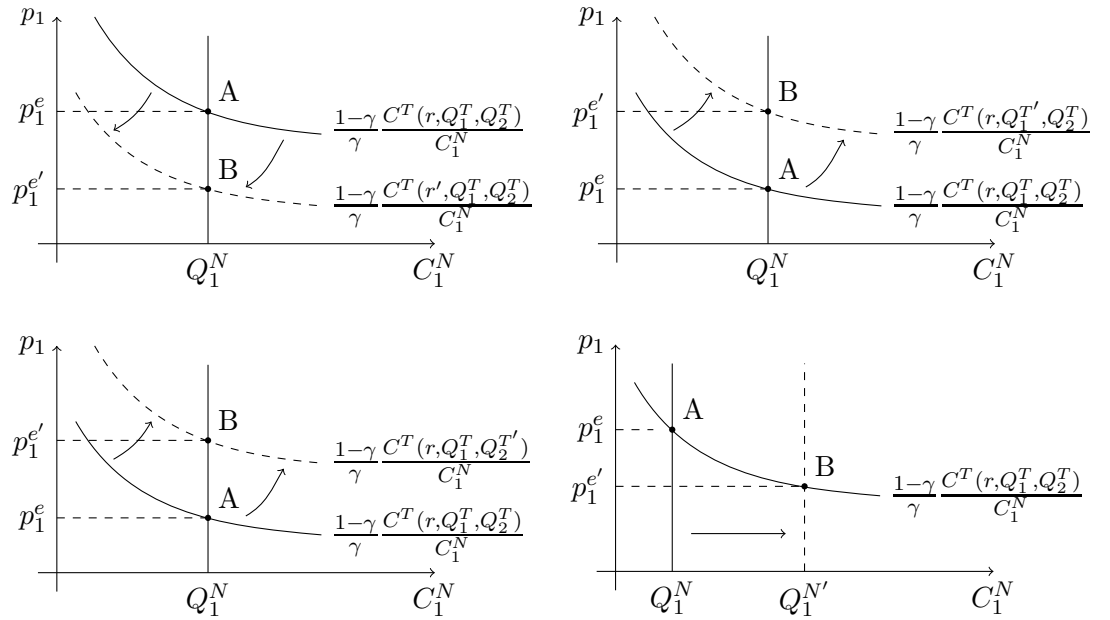
ables, Q_1^N . Since the endowment is fixed, the supply schedule is a vertical line. The equilibrium value of the relative price of nontradables, denoted p_1^e , is given by the intersection of the demand and supply schedules, point A in the figure.

9.1.3 Adjustment to Interest-Rate and Endowment Shocks

Suppose the world interest rate increases from r to $r' > r$. The situation is depicted in the top left panel of figure 9.3. The initial position is at point A. The increase in the interest rate induces households to postpone consumption and increase savings. Thus, the demand for nontradables shifts down and to the left. At the original equilibrium relative price, p_1^e , there is now an excess supply of nontradables, which induces sellers to lower prices. Prices will continue to fall until the demand for nontradables equals the endowment of nontradables. This occurs at point B, where the relative price of nontradables is $p_1^{e'} < p_1^e$. We therefore have that an increase in the world interest rate causes a fall in the equilibrium relative price of nontradables.

Suppose next that the economy experiences an increase in the endowment of tradable goods in period 1 or in period 2 or both. This case is depicted in the top right and bottom left panels of figure 9.3. The positive endowment shock could be the result, for example, of a larger harvest due to good weather conditions, or of an improvement in the country's terms of trade. The increase in tradable endowment represents a positive income effect for the household, which increases the demand for consumption. As a result, the demand for nontradables shifts up and to the right. At the old equilibrium relative price, p_1^e , there is an excess demand for nontradables,

Figure 9.3: Effects of Interest-Rate and Endowment Shocks on the Relative Price of Nontradables



Notes. The figure depicts the effect on the relative price of nontradables, p_1 , of changes in four exogenous variables. Top left panel: An increase in the interest rate from r to r' shifts the demand schedule down and to the left causing a fall in p_1 . Top right and bottom left panels: An increase in the period-1 endowment from Q_1^T to $Q_1^{T'}$, or in the period-2 endowment from Q_2^T to $Q_2^{T'}$, shifts the demand schedule for nontradables out and to the right, driving p_1 up. Bottom right panel: An increase in the endowment of nontradables from Q_1^N to $Q_1^{N'}$, shifts the supply of nontradables to the right, resulting in a fall in the equilibrium value of p_1 .

which pushes prices up until the excess demand dissipates. At the new equilibrium, point B in the figure, the relative price of nontradables is $p_1^{e'}$, which is higher than the original price p_1^e . Although the effect of an increase in the endowment of tradables on the relative price of nontradables is qualitatively the same regardless of whether the increase occurs in periods 1 or 2, or in both periods, the quantitative effect can be different. Exercise 9.1 asks you to compare the effects of temporary and permanent tradable endowment shocks.

Finally, the bottom right panel of figure 9.3 illustrates the effect of an increase in the endowment of nontradables in period 1. The increase in the nontradable endowment shifts the supply schedule to the right. At the original price, p_1^e , the economy experiences an excess supply of nontradable goods. Consequently, sellers lower prices until the market clears again. This occurs at point B in the figure. The new equilibrium price is $p_1^{e'} < p_1^e$. We conclude that increases in the endowment of nontradables cause the relative price of nontradables to decline.

Collecting the results obtained in this section, we have that the relative price of nontradables in period 1 is increasing in the endowments of tradable goods in both periods, Q_1^T and Q_2^T , and decreasing in the world interest rate, r , and the endowment of nontradables in period 1, Q_1^N , that is,

$$p_1 = p(r, Q_1^T, Q_2^T, Q_1^N). \quad (9.12)$$

9.1.4 From the relative price of nontradables to the real exchange rate

There is a tight connection between the relative price of nontradables, p_t , and the real exchange rate, e_t . Recall from chapter 8 that the real exchange rate is defined as the relative price of a basket of goods abroad in terms of domestic baskets of goods. Formally,

$$e_t = \frac{\mathcal{E}_t P_t^*}{P_t},$$

where P_t is the period- t price of a domestic consumption basket in units of domestic currency, P_t^* is the period- t price of the foreign consumption basket in units of foreign currency, and \mathcal{E}_t is the nominal exchange rate defined as the period- t price of one unit of foreign currency in terms of domestic currency.

In the present economy the consumption basket contains tradable and nontradable goods. Its price, P_t , is therefore a function of the prices of tradables and nontradables, P_t^T and P_t^N , respectively. So we can write

$$P_t = \phi(P_t^T, P_t^N),$$

where $\phi(\cdot, \cdot)$ is increasing in both arguments and homogeneous of degree one. In fact, given the assumed Cobb-Douglas aggregator functions for consumption given in equations (9.2) and (9.3), we know from the analysis of section 8.9 of chapter 8 that the price of the consumption basket is also

a Cobb-Douglas function and is given by

$$\phi(P_t^T, P_t^N) = (P_t^T)^\gamma (P_t^N)^{1-\gamma} A,$$

where $A \equiv \gamma^{-\gamma} (1-\gamma)^{-(1-\gamma)}$ is a positive constant.

Similarly, the price of the foreign consumption basket is given by $P_t^* = \phi^*(P_t^{T*}, P_t^{N*})$, where P_t^{T*} and P_t^{N*} denote the foreign prices of tradables and nontradables expressed in units of foreign currency, and $\phi^*(\cdot, \cdot)$ is an increasing and homogeneous-of-degree-one function. We can then write the real exchange rate as

$$e_t = \frac{\mathcal{E}_t \phi^*(P_t^{T*}, P_t^{N*})}{\phi(P_t^T, P_t^N)}. \quad (9.13)$$

Exploiting the fact that both $\phi(\cdot, \cdot)$ and $\phi^*(\cdot, \cdot)$ are homogeneous of degree one, we can rewrite the real exchange rate as

$$e_t = \frac{\mathcal{E}_t P_t^{T*} \phi^*(1, P_t^{N*}/P_t^{T*})}{P_t^T \phi(1, P_t^N/P_t^T)}.$$

By the assumption that the law of one price holds for tradable goods, we have that $\mathcal{E}_t P_t^{T*} = P_t^T$. The real exchange rate can then be written as

$$e_t = \frac{\phi^*(1, p_t^*)}{\phi(1, p_t)},$$

where $p_t^* \equiv P_t^{N*}/P_t^{T*}$ and $p_t \equiv P_t^N/P_t^T$ denote the relative price of nontradables in terms of tradables abroad and at home, respectively. Holding the foreign relative price of nontradables, p_t^* , constant, the real exchange rate is a decreasing function of p_t . In words, when nontradables become

more expensive relative to tradables, the domestic economy becomes more expensive relative to the rest of the world. For this reason, the relative price of nontradables in terms of tradables, p_t , is often referred to as the real exchange rate, especially in discussions involving small economies for which it is reasonable to take foreign relative prices as given.

The response of the real exchange rate to endowment and world interest rate shocks, holding constant foreign relative prices, can then be read off equation (9.12). We summarize this relationship by writing

$$e_1 = e(r, Q_1^T, Q_2^T, Q_1^N, p_1^*). \quad (9.14)$$

The real exchange rate depreciates in response to an increase in the world interest rate, appreciates in response to increases in the current or future endowment of tradables, and depreciates in response to an increase in the current endowment of nontradables.

9.1.5 The Terms of Trade and the Real Exchange Rate

Consider an economy that imports food and exports oil. How does an increase in the price of oil affect the relative price of nontradables, such as construction and wholesale and retail trade, in terms of food? In chapter 4, we established that changes in the terms of trade are like changes in the endowment of the exportable good. This result allows us to easily analyze the effects of terms of trade shocks on the relative price of nontradables and the real exchange rate.

Suppose that the tradable consumption good, C_t^T , is imported (food)

and that the tradable endowment, Q_t^T is exported (oil). Then, the value of the tradable endowment in terms of tradable consumption is given by $TOT_t Q_t^T$, where

$$TOT_t = \frac{P_t^X}{P_t^M}.$$

In this expression, P_t^X is the price of the exported endowment and P_t^M is the price of the imported consumption good.

The predictions of the TNT model carry over to the present environment except that Q_t^T must be replaced by $TOT_t Q_t^T$. In particular, the equilibrium demand for tradable consumption goods (9.9) becomes

$$C_1^T = C^T(r, \underset{+}{TOT_1 Q_1^T}, \underset{+}{TOT_2 Q_2^T}),$$

and the equilibrium demand for nontradables (9.11) becomes

$$C_1^N = \frac{1-\gamma}{\gamma} \frac{C^T(r, \underset{+}{TOT_1 Q_1^T}, \underset{+}{TOT_2 Q_2^T})}{p_1},$$

and the equilibrium real exchange rate, equation (9.14), becomes

$$e_1 = e(\underset{+}{r}, \underset{-}{TOT_1 Q_1^T}, \underset{-}{TOT_2 Q_2^T}, \underset{+}{Q_1^N}, \underset{+}{p_1^*}).$$

In the last two expressions, p_1 denotes the relative price of nontradables in terms of the imported consumption good,

$$p_1 = \frac{P_1^N}{P_1^M}.$$

It then follows immediately that an improvement in the current or future

expected terms of trade (an increase in either TOT_1 or in TOT_2) causes an increase in the equilibrium relative price of nontradables and an appreciation of the real exchange rate. The intuition is that the increase in the price of the export good makes households richer. As a result, they increase the demand for both goods, tradables (imported) and nontradables. Because the supply of nontradables is fixed, the increased demand for this type of good pushes its relative price up. In turn the increase in the relative price of nontradables makes the country more expensive relative to the rest of the world.

9.2 Sudden Stops

“It is not speed that kills, it is the sudden stop.” With this bankers’ adage, Rudiger Dornbusch, Ilan Goldfajn, and Rodrigo Valdés, baptized a particular type of macroeconomic crisis as a *sudden stop*.¹ A sudden stop occurs when foreign lenders abruptly stop extending credit to a country. This situation manifests itself by a sharp increase in the interest rate that the country faces in international financial markets. A variety of factors can be behind why international credit to a country suddenly dries up. For example, foreign lenders might become worried about the ability of a particular country to honor its external debt obligations. Or, a disruption in credit markets in the developed world might drive the world interest rate up, which foreign lenders then pass on to emerging-country debtors, as was the case during the Volcker disinflation of the early 1980s in the United States, or during

¹See, Rudiger Dornbusch, Ilan Goldfajn, and Rodrigo O. Valdés, “Currency Crises and Collapses,” *Brookings Papers on Economic Activity* 26(2), 1995, pages 219-270.

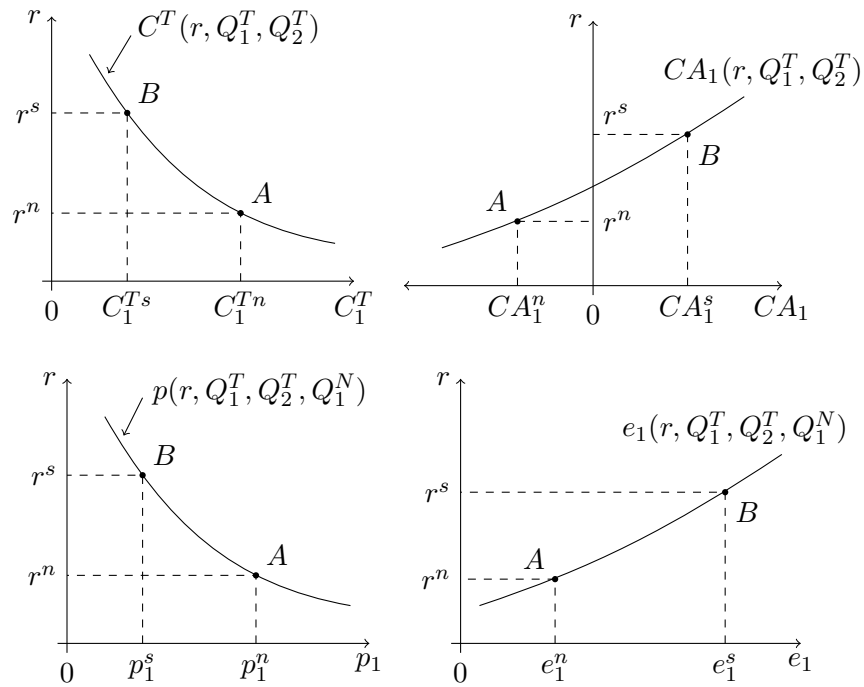
the global financial crisis of 2007-2009. Typically these two factors are concurrent. If the emerging country is highly indebted, foreign lenders might not only pass on the increase in the world interest rate, but also elevate the country interest-rate premium. Examples of sudden stops include the Latin American debt crisis of the early 1980s, the Mexican Tequila crisis of 1994, the Asian financial crisis of 1997, the Russian crisis of 1998, the Argentine crisis of 2001, and the debt crises in the periphery of Europe and Iceland in the aftermath of the global financial crisis of 2007-2009.

Three hallmark consequences of sudden stops are: (a) a *current account reversal* from a deficit to a surplus or a sizable reduction in the current account deficit; (b) a contraction in aggregate demand; and (c) a real-exchange-rate depreciation. In this section, we show that the TNT model can capture these three stylized facts, and then present two case studies, the Argentine sudden stop of 2001, and the Icelandic sudden stop of 2008.

9.2.1 A Sudden Stop Through the Lens of the TNT Model

We model a sudden stop as an increase in the world interest rate, r . Figure 9.4 summarizes the effects of an increase in the world interest rate on tradable consumption, the current account, the relative price of nontradables, and the real exchange rate as predicted by the TNT model. The top left panel plots the equilibrium level of tradable consumption as a function of the world interest rate. As indicated by equation (9.9), the consumption of tradables is an increasing function of the interest rate. Before the sudden stop the interest rate is at its ‘normal’ level, denoted r^n . At this interest rate, tradable consumption is C_1^{Tn} . When the sudden stop occurs,

Figure 9.4: Effects of a Sudden Stop as Predicted by the TNT Model



Notes. A sudden stop is modeled as an increase in the world interest rate. In the figure, the interest rate increases from a normal level, denoted r^n , before the sudden stop, to a high level, denoted r^s , after the sudden stop. The sudden stop causes a contraction in the domestic absorption of tradables, a current account reversal, a fall in the relative price of nontradables, and a real-exchange-rate depreciation.

the interest rate jumps to $r^s > r^n$. As a result, the demand for tradable goods experience a contraction from C_1^{Tn} to $C_1^{Ts} < C_1^{Tn}$. Intuitively, the increase in the interest rate creates an incentive to save in period 1, which requires postponing current spending in goods. The top right panel displays the equilibrium current account as a function of the world interest rate, equation (9.10). At the pre-crisis interest rate, the current account is negative and equal to CA_1^n . Thus, foreign lenders are extending credit to the country. The hike in the interest rate following the sudden stop causes an improvement in the current account to $CA_1^s > 0$. As mentioned above, the change of sign in the current account from deficit to surplus is known as a current account reversal. The improvement in the external balance is the result of the contraction in the absorption of tradable goods brought about by the increase in the cost of funds.

The bottom left panel of figure 9.4 displays the relative price of nontradables as a function of the world interest rate. Equation (9.12) shows that in equilibrium the relative price of nontradables is a decreasing function of the interest rate. In the figure, the sudden stop causes the relative price of nontradables to drop from p_1^n to $p_1^s < p_1^n$. This effect has a clear intuition. The increase in the interest rate induces households to cut the demand for both types of consumption goods, tradable and nontradable. The contraction in tradable consumption is met by an equivalent increase in exports (or reduction in imports). By contrast, nontradables cannot be exported, so the contraction in demand requires a fall in the relative price of nontradables to induce agents to voluntarily consume the entire endowment of this type of goods. The redirection of absorption away from tradables and toward non-

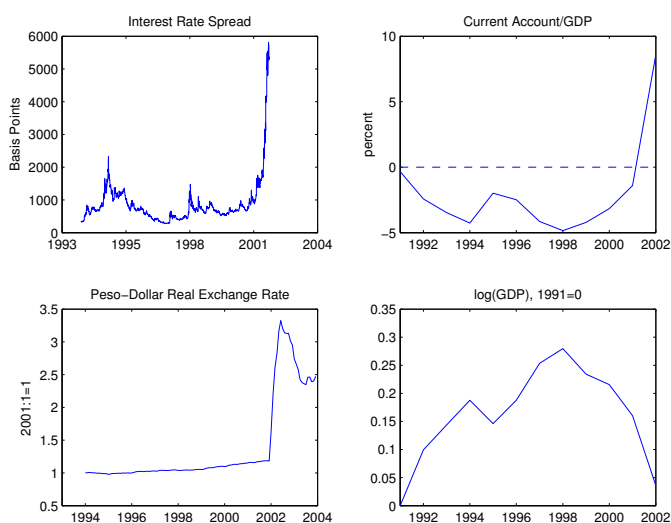
tradables facilitated by the drop in the relative price of nontradables after the sudden stop is known as an *expenditure switch*. Finally, the bottom right panel plots equation (9.14), which describes a positive relationship between the real exchange rate, e_1 , and the world interest rate, r . As we just explained, the increase in the interest rate brought about by the sudden stop lowers the price of nontradables, which makes the country cheaper relative to the rest of the world.

9.2.2 The Argentine Sudden Stop of 2001

To end a hyperinflation, in 1991 Argentina implemented an exchange-rate-based inflation stabilization plan. The plan consisted of pegging the peso to the U.S. dollar at a one-to-one parity. This exchange rate policy in fact was written into a law, called the Convertibility Law. Pegging the peso to the dollar quickly brought the inflation rate to low levels. The country was able to maintain the one-to-one parity to the U.S. dollar for an entire decade. But in 2001, Argentina fell into a crisis that culminated in default and devaluation. The default led to a cutoff from international capital markets and hence capital inflows stopped abruptly.

The top left panel of figure 9.5 displays the interest rate spread of Argentine dollar-denominated bonds over U.S. Treasuries between 1994 and 2001. Prior to 2001, spreads fluctuated around 7 percent (700 basis points). Spreads in other emerging market economies at the time were of similar size. However, in 2001 Argentine interest rate spreads exploded, reaching over 50 percent (or over 5,000 basis points) by late December. By contrast, during this time interest rate spreads of other emerging market economies

Figure 9.5: The Argentine Sudden Stop of 2001



Notes. The figure displays the behavior of the interest rate spread of Argentine dollar-denominated bonds over U.S. treasuries, the current-account-to-GDP ratio, the real exchange rate, and real per capita GDP around the Argentine sudden stop of 2001. The sudden stop was characterized by a sharp increase in the country spread, a current account reversal, a real-exchange-rate depreciation, and a large contraction in GDP.

did not increase from their prior levels. The fact that the price of foreign credit became prohibitively high meant that in effect the country was shut off from international capital markets. This suggests that in December 2001 Argentina suffered a sudden stop.

The top-right panel of the figure shows the Argentine current-account-to-GDP ratio from 1991 to 2002. From 1991 until 2000, Argentina ran current account deficits of around 3 percent of GDP on average. Thus, before the crisis the country was the recipient of sustained and sizable capital inflows. In 2002 the current account experienced a sharp and large reversal to 8 percent of GDP. This means that in spite of the fact that the country was technically in default, it transferred large amounts of resources to foreign lenders.

The exchange rate peg was abandoned in December of 2001 and the peso suffered a large devaluation plummeting from 1 peso per dollar to 3.5 pesos per dollar. In 2002 the Argentine consumer price index rose by 41 percent. At the same time, the consumer price index in the United States grew by only 2.5 percent in 2002. Recalling that the peso-dollar real exchange rate is given by $e_t = \mathcal{E}_t P_t^{US} / P_t^{AR}$, where \mathcal{E}_t denotes the peso price of one U.S. dollar, P_t^{US} the consumer price index in the United States, and P_t^{AR} the consumer price index in Argentina. We have that the percentage change in

the real exchange rate is given by

$$\begin{aligned}
 \left(\frac{e_t}{e_{t-1}} - 1 \right) \times 100 &= \left[\frac{(\mathcal{E}_t/\mathcal{E}_{t-1})(P_t^{US}/P_{t-1}^{US})}{P_t^{AR}/P_{t-1}^{AR}} - 1 \right] \times 100 \\
 &= \left(\frac{3.5 \times 1.025}{1.41} - 1 \right) \times 100 \\
 &= 154.4 \text{ percent.}
 \end{aligned}$$

This means that the peso depreciated in real terms 154 percent against the dollar. Put differently, in the course of a few months the United States relative to Argentina became 2.5 times as expensive as before the sudden stop. So visitors found Argentina quite expensive in 2001 and a bargain in 2002.

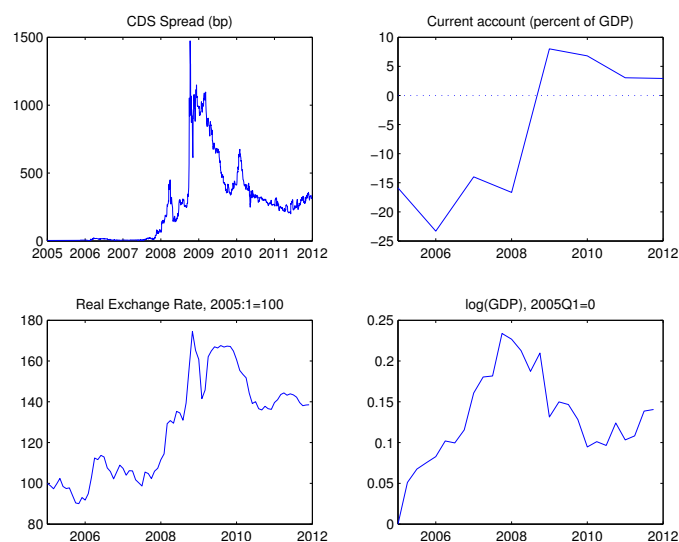
The observed behavior of the current account and the real exchange rate in the aftermath of the Argentine sudden stop are consistent with the predictions of the TNT model. The sudden stop also had significant negative consequences for aggregate activity. In 2002, real GDP per capita fell by 12.5 percent and the number of unemployed people and people working fewer hours than desired reached 35 percent of the labor force. The version of the TNT model developed in section 9.1 is mute in this respect because tradable and nontradable output are assumed to be fixed. We postpone an analysis of the effects of sudden stops on output and unemployment until chapter 13, where we will introduce market failures taking the form of nominal rigidities in the labor market.

9.2.3 The Icelandic Sudden Stop of 2008

Sudden stops are not a phenomenon that pertains only to emerging countries. During the global financial crisis of 2007-2009 a number of medium- and high-income European countries with a history of large current account deficits also suffered sudden stops. A case in point is Iceland. Between 2000 and 2008, this country ran large current account deficits, which combined increased its external debt by more than 50 percent of GDP. An immediate consequence of the disruption in financial markets in 2008 was an abrupt cut in the flow of credit to small highly indebted economies in Europe, including Iceland, Ireland, Greece, Portugal, and Spain.

Here we focus on Iceland as its sudden stop was particularly severe. The root of the Icelandic crisis was its banking sector. The combined balance sheet of the major local banks stood at more than 10 times the country's GDP on the eve of the crisis. With the global financial crisis, the balance sheets of these banks deteriorated significantly as assets lost value relative to liabilities. Fearing default, foreign lenders to the Icelandic banking sector elevated the country's cost of credit. This can be seen in the top left panel of figure 9.6, which plots Iceland's credit default swap (CDS) spreads over the period 2005 to 2011. A CDS spread is the cost of insuring against default and is measured in basis points. For example, on September 1 2008, the Icelandic CDS spread was 200 basis points. This means that to insure 100 dollars of Icelandic debt against default, one had to pay 2 dollars per year. CDS spreads represent a measure of borrowing costs on debts with default risk relative to default free debt. Data on CDS spreads is useful because it

Figure 9.6: The Icelandic Sudden Stop of 2008



Notes. The figure displays the behavior of the Icelandic CDS spread, the current-account-to-GDP ratio, the real exchange rate against the euro, and real GDP around the sudden stop of 2008. The sudden stop was characterized by a sharp increase in the CDS spread, a large current account reversal, a real-exchange-rate depreciation, and a contraction in the level of real GDP.

is available even when the debtor does not issue new debt or when there are no liquid secondary debt markets. The figure shows that CDS spreads on Icelandic debt grew rapidly from 200 basis points in early September 2008 to over 1400 basis points by mid October 2008. At this point Iceland was virtually cut off from the private international capital market. The economy was suffering a full blown sudden stop.

The large current account deficits of the pre-crisis period turned into current account surpluses virtually overnight as shown in the top right panel of the figure. Between 2008 and 2009 the current account balance went from a deficit of 17 percent of GDP to a surplus of 8 percent of GDP. Thus the Icelandic crisis conforms to a key characteristic of a sudden stop, namely, a sharp reversal of the current account.

Relative prices, shown in the bottom left panel of the figure, also experienced significant movements around the crisis in the direction predicted by theory. The krona-euro real exchange rate depreciated by 45 percent between January 2008 and January 2009, implying that Iceland became 45 percent cheaper relative to other countries in the euro zone in the aftermath of its sudden stop.

Finally, the bottom right panel of figure [9.6](#) shows that real GDP stopped growing as the sudden stop hit the country. While the Icelandic economy had grown on average by 5 percent per year between 2005 and 2007, it contracted by more than 2 percent per year between 2008 and 2011.

9.3 The TNT Model with Sectoral Production

Thus far, we have studied a version of the TNT model in which the supplies of tradable and nontradable goods are fixed. From the predictions of this version of the TNT model it is natural to conjecture that shocks affecting the economy can have consequences for the sectoral levels of output and employment. For example, we saw that an increase in the world interest rate causes a fall in the relative price of nontradables in terms of tradables. It is reasonable to expect that firms in the tradable sector will have an incentive to expand production and that firms in the nontradable sector will have an incentive to scale back production. In turn, these sectoral changes in output would require a reallocation of labor away from the nontraded sector and toward the traded sector. To see whether this intuition materializes in the context of the TNT framework, we now develop a more realistic version of the model in which tradables and nontradables are produced with labor. In this setup, sectoral output and sectoral employment adjust endogenously in response to exogenous disturbances such as changes in the world interest rate.

9.3.1 The Production Possibility Frontier

Assume that tradable and nontradable goods are produced using labor via the production technologies

$$Q_t^T = F_T(L_t^T) \tag{9.15}$$

and

$$Q_t^N = F_N(L_t^N), \quad (9.16)$$

where L_t^T and L_t^N denote labor in the traded and nontraded sectors in period $t = 1, 2$, and $F_T(\cdot)$ and $F_N(\cdot)$ are production functions. We assume that the production functions are increasing and concave, that is, $F_T' > 0$, $F_N' > 0$, $F_T'' < 0$, and $F_N'' < 0$. The assumption that the production functions are concave means that the marginal productivity of labor is decreasing in the amount of labor used.

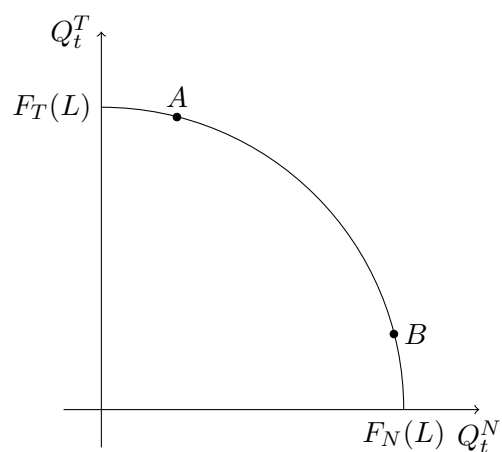
The total supply of labor in the economy is assumed to be constant and equal to L in both periods. Therefore, the allocation of labor across sectors must satisfy the following resource constraint:

$$L_t^T + L_t^N = L. \quad (9.17)$$

The two production functions along with this resource constraint can be combined into a single equation relating Q_t^T to Q_t^N . This relation is known as the economy's *production possibility frontier* (PPF). To obtain the PPF, solve (9.15) for L_t^T to obtain $L_t^T = F_T^{-1}(Q_t^T)$, where F_T^{-1} denotes the inverse function of F_T and is strictly increasing and convex. Similarly, solve (9.16) for L_t^N to obtain $L_t^N = F_N^{-1}(Q_t^N)$. Then substitute these two expressions into the resource constraint (9.17) to obtain $F_T^{-1}(Q_t^T) + F_N^{-1}(Q_t^N) = L$.

Figure 9.7 plots the PPF in the space (Q_t^N, Q_t^T) . The PPF is downward sloping because increasing nontradable output requires moving labor from the tradable sector to the nontradable sector, which causes a fall in tradable output. The PPF is concave because as employment in the nontradable sec-

Figure 9.7: The Production Possibility Frontier



Notes. The production possibility frontier (PPF) describes a negative relationship between output of nontradable goods, Q_t^N , and output of tradable goods, Q_t^T . Given the total amount of labor, L , increasing output in one sector requires reducing output in the other sector. If the sectoral production functions exhibit diminishing marginal products of labor, the PPF is concave.

tor increases, the productivity of labor in that sector decreases, which means that more labor is needed to produce each additional unit of nontradable output. This, in turn, implies that an increasing amount of tradable goods must be sacrificed to produce each additional unit of nontradables.

This is intuitive. Suppose that the tradable sector produces wheat and the nontradable sector haircuts. Consider point A in figure 9.7. At this point, most workers are employed in the traded sector. The most efficient way to increase the output of nontradables is to transfer from the tradable sector the workers with the most experience in hair dressing and with the least training in farming, such as former barbers currently working as wheat farmers. This will result in a relatively large increase in output in the nontradable sector with little sacrifice of output in the traded sector. So in the neighborhood of point A the PPF is relatively flat. As we continue to increase the output of nontradables, we must remove from the traded sector workers with increasingly less experience in cutting hair. When most workers are employed in the nontraded sector, such as at point B in figure 9.7, increasing the production of hair cuts even further requires moving away from the traded sector farmers who know how to grow wheat but are not good at cutting hair. Each of these workers will add only a small number of hair cuts and will cause a large fall in the production of wheat. Thus, in the neighborhood of point B, the PPF has a large negative slope.

Formally, the slope of the PPF can be derived as follows. Differentiate the resource constraint (9.17) to get $dL_t^T + dL_t^N = 0$, or

$$\frac{dL_t^T}{dL_t^N} = -1.$$

This expression says that, because the total amount of labor is fixed, any increase in labor input in one sector must be offset one-for-one by a reduction of labor input in the other sector. Now differentiate the production functions (9.15) and (9.16) to obtain

$$dQ_t^T = F'_T(L_t^T)dL_t^T$$

and

$$dQ_t^N = F'_N(L_t^N)dL_t^N.$$

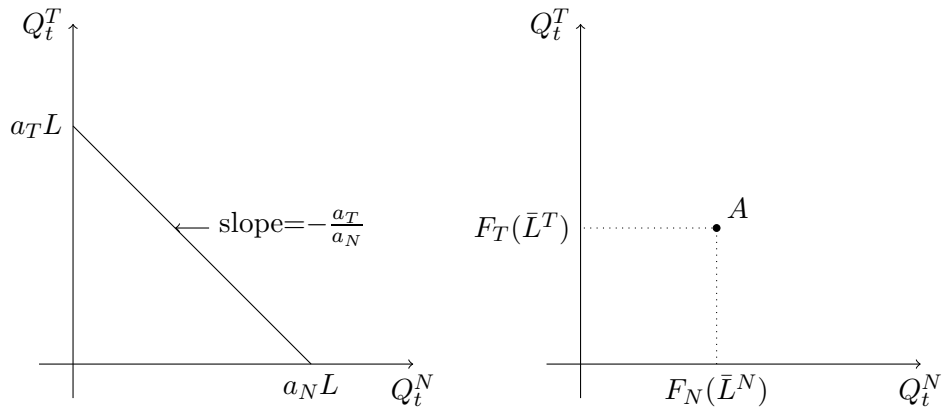
Combining the above three equations yields the following expression for the slope of the PPF:

$$\frac{dQ_t^T}{dQ_t^N} = -\frac{F'_T(L_t^T)}{F'_N(L_t^N)}.$$

Because the marginal products of labor $F'_T(L_t^T)$ and $F'_N(L_t^N)$ are both positive, we have that the slope of the PPF is negative. Also because both production functions display diminishing marginal products of labor, we have that as L_t^N increases (and hence L_t^T decreases), $F'_T(L_t^T)$ becomes larger and $F'_N(L_t^N)$ becomes smaller, so the slope of the PPF increases in absolute value. This formally establishes that if the production functions exhibit positive but diminishing marginal products of labor, the PPF is downward sloping and concave.

The curvature of the PPF depends on how quickly the marginal product of labor diminishes as employment increases. If the marginal product of labor decreases slowly, the PPF will not have a strong curvature. In the special case in which the production function is linear in labor, that is,

Figure 9.8: The PPF: Two Special Cases



Notes. The slope of the PPF depends on the degree to which the marginal product of labor diminishes with employment. In the special case of a constant marginal product of labor, as when the technologies are $Q_t^T = a_T L_t^T$ and $Q_t^N = a_N L_t^N$, the PPF is linear as shown in the left panel. When labor is fully specialized in the production of one good, as with the technologies $Q_t^T = F_T(\bar{L}^T)$ and $Q_t^N = F_N(\bar{L}^N)$, the PPF collapses to a single point, as shown in the right panel.

$Q_t^T = a_T L_t^T$ and $Q_t^N = a_N L_t^N$, where a_T and a_N are positive constants, the marginal product of labor is constant in both sectors, and the PPF becomes linear, as shown in the left panel of figure 9.8. When the PPF takes this form, the TNT model has a special name, the Balassa-Samuelson model, which we will study later in this chapter in section 9.4. On the other hand, if the marginal product of labor falls rapidly with employment, the PPF will have a strong curvature. In the special case in which each worker is fully specialized in the production of one good, so that her productivity is positive in one sector and zero in the other sector, the PPF becomes a single point, as illustrated by point A in the right panel of figure 9.8. In the figure, output in the tradable sector is $F_T(\bar{L}^T)$ and output in the nontraded sector is $F_N(\bar{L}^N)$, where \bar{L}^T and \bar{L}^N denote the number of workers specialized in the production of tradables and nontradables, respectively. This case captures the endowment version of the TNT model studied in section 9.1.

As an illustration of how the PPF is constructed, consider the following example:

$$Q_t^T = \sqrt{L_t^T}$$

and

$$Q_t^N = \sqrt{L_t^N}.$$

Both of these production technologies display positive but diminishing marginal products of labor, because the square root is an increasing and concave function. Solving both production functions for labor as functions of output yields $L_t^T = (Q_t^T)^2$ and $L_t^N = (Q_t^N)^2$. Then using these two expressions to eliminate L_t^T and L_t^N from the resource constraint (9.17), we obtain the

PPF

$$Q_t^T = \sqrt{L - (Q_t^N)^2}.$$

This expression describes a negative and concave relationship between Q_t^T and Q_t^N like the one depicted in figure 9.7. The slope of this PPF results from taking the derivative of Q_t^T with respect to Q_t^N , which yields $\frac{dQ_t^T}{dQ_t^N} = -\frac{Q_t^N}{Q_t^T}$. Clearly, the slope is negative. Furthermore, as Q_t^N increases (and hence Q_t^T decreases), the slope becomes larger in absolute value.

9.3.2 The PPF and the Real Exchange Rate

Where on the PPF the economy will operate depends on relative prices. To see this, consider the problem of a firm operating in the traded sector. Its profit, denoted Π_t^T , is given by the difference between revenues from sales of tradables, $P_t^T F_T(L_t^T)$, and the cost of production, $W_t L_t^T$, where W_t denotes the wage rate in period $t = 1, 2$,

$$\Pi_t^T = P_t^T F_T(L_t^T) - W_t L_t^T. \quad (9.18)$$

The firm chooses employment to maximize profits taking P_t^T and W_t as given. Taking the derivative of profits with respect to L_t^T and setting it to zero, we obtain the first-order optimality condition

$$P_t^T F_T'(L_t^T) - W_t = 0.$$

The intuition behind this condition is that by hiring an extra worker the firm can produce $F_T'(L_t^T)$ additional units of tradable goods. The market value

of these goods is $P_t^T F'_T(L_t^T)$, which is known as the *value of the marginal product of labor*. The cost of hiring an extra worker is the wage rate, W_t . Thus, if $P_t^T F'_T(L_t^T)$ is larger than W_t , hiring an extra worker increases the firm's profits. As employment increases, the marginal product of labor falls, and so does the increase in profits stemming from an extra worker. When $P_t^T F'_T(L_t^T)$ equals W_t , hiring an extra worker does not change the firm's profits. Beyond this level of employment, an extra worker reduces profits. So it is optimal for the firm to stop hiring when $P_t^T F'_T(L_t^T) = W_t$, that is, when the value of the marginal product of labor equals the wage rate.

Similarly, the profit of a firm operating in the nontraded sector, denoted Π_t^N , is given by

$$\Pi_t^N = P_t^N F_N(L_t^N) - W_t L_t^N. \quad (9.19)$$

The firm chooses L_t^N to maximize profits taking as given P_t^N and W_t . The resulting optimality condition is

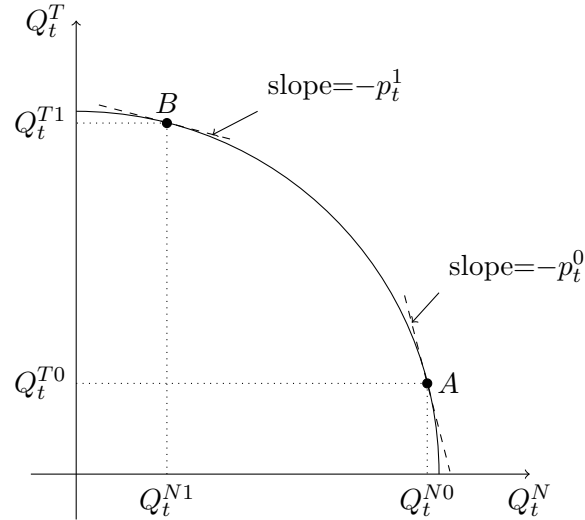
$$P_t^N F'_N(L_t^N) - W_t = 0.$$

The interpretation of this condition is the same as that of its counterpart in the traded sector. The firm hires workers until the value of the marginal product of labor equals the marginal cost of labor.

Combining the first-order conditions of the firms in the traded and nontraded sectors to eliminate W_t yields

$$\frac{F'_T(L_t^T)}{F'_N(L_t^N)} = \frac{P_t^N}{P_t^T} \equiv p_t. \quad (9.20)$$

Figure 9.9: The Optimal Production of Tradables and Nontradables



Notes. The optimal production of tradables and nontradable goods occurs at a point where (minus) the slope of the PPF equals the relative price of nontradables in terms of tradables. As the relative price of nontradables falls from p_t^0 to p_t^1 firms find it optimal to produce more tradables and less nontradables.

The left-hand side of this expression is (minus) the slope of the PPF, and the right-hand side is the relative price of nontradables in terms of tradables, p_t . Thus the economy produces tradables and nontradables in quantities such that (minus) the slope of the PPF equals the relative price of nontradables in terms of tradables.

Figure 9.9 illustrates how the optimal production allocation responds to a decline in the relative price of nontradables. Suppose that the initial position is at point A, where tradable output is Q_t^{T0} , nontradable output is Q_t^{N0} , and the relative price of nontradables in terms of tradables is p_t^0 . Consider a fall in the relative price of nontradables from p_t^0 to $p_t^1 < p_t^0$. Since in equilib-

rium (minus) the slope of the PPF equals the relative price of nontradables, production moves to a flatter part of the PPF. In the figure, the new position is at point B, where tradable output is higher and nontradable output is lower ($Q_t^{T1} > Q_t^{T0}$ and $Q_t^{N1} < Q_t^{N0}$). Since both goods are produced with labor, as the economy moves from point A to point B, employment falls in the nontraded sector and increases in the traded sector.

We established in subsection 9.1.4 that given the relative price of nontradables in the rest of the world, p_t^* , the relative price of nontradables in terms of tradables, p_t , is negatively related to the real exchange rate, $e_t = \phi^*(1, p_t^*)/\phi(1, p_t)$. It follows that when the real exchange rate depreciates (i.e., when the country becomes cheaper relative to the rest of the world) firms have an incentive to produce more tradables and less nontradables. Put differently, as one moves northwest along the PPF (as from point A to point B in figure 9.9), the real exchange rate depreciates.

9.3.3 The Income Expansion Path

We now study the consumption and saving decisions of the household. Households face the same optimization problem as in the endowment version of the TNT model presented in section 9.1, except that now their income stems not from the sale of endowments, but from labor income and from profits received from the ownership of firms. To keep the exposition of the model self-contained, we reproduce the household's utility function from section 9.1

$$\ln C_1 + \beta \ln C_2, \tag{R9.1}$$

with

$$C_1 = (C_1^T)^\gamma (C_1^N)^{1-\gamma}, \quad (\text{R9.2})$$

and

$$C_2 = (C_2^T)^\gamma (C_2^N)^{1-\gamma}, \quad (\text{R9.3})$$

where, as before, C_t , C_t^T , and C_t^N denote consumption of the composite good, consumption of tradables, and consumption of nontradables in period $t = 1, 2$.

Each period $t = 1, 2$, the household works L hours at the wage rate W_t , so its labor income is $W_t L$. In addition, households are assumed to be the owners of firms. Thus, their profit income is $\Pi_t^T + \Pi_t^N$. Households start period 1 with no debts or assets and can borrow or lend via a bond, denoted B_1 , which is denominated in units of tradable goods and pays the interest rate r . Their budget constraints in periods 1 and 2 are then given by

$$P_1^T C_1^T + P_1^N C_1^N + P_1^T B_1 = W_1 L + \Pi_1^T + \Pi_1^N, \quad (9.21)$$

and

$$P_2^T C_2^T + P_2^N C_2^N = W_2 L + \Pi_2^T + \Pi_2^N + (1+r)P_2^T B_1. \quad (9.22)$$

Let's now follow the same steps we took in subsection 9.1.1 to obtain the household's intertemporal budget constraint. That is, divide the period- t budget constraint by P_t^T , combine the resulting expressions to eliminate B_1 , and rearrange terms. This yields

$$C_2^T = Y - p_2 C_2^N - (1+r)(C_1^T + p_1 C_1^N),$$

where now the household's lifetime income expressed in units of tradable goods of period 2, Y , is given by

$$Y = \frac{W_2}{P_2^T}L + \frac{\Pi_2^T}{P_2^T} + \frac{\Pi_2^N}{P_2^T} + (1+r) \left(\frac{W_1}{P_1^T}L + \frac{\Pi_1^T}{P_1^T} + \frac{\Pi_1^N}{P_1^T} \right).$$

As in the endowment version of the TNT model, the household takes Y as given. This means that the household's utility maximization problem is identical to its counterpart in the endowment version. In particular, we have the following first-order conditions:

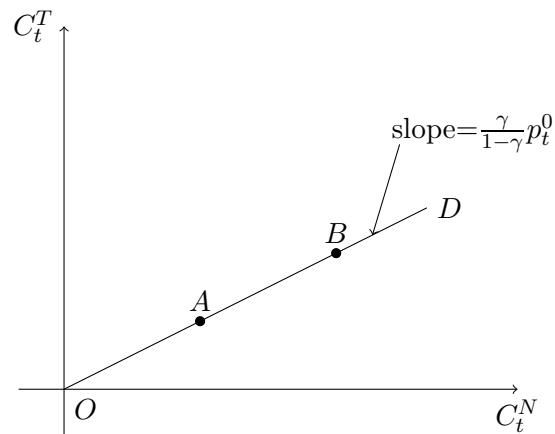
$$C_2^T = \beta(1+r)C_1^T, \quad (9.23)$$

and

$$C_t^T = \frac{\gamma}{1-\gamma} p_t C_t^N. \quad (9.24)$$

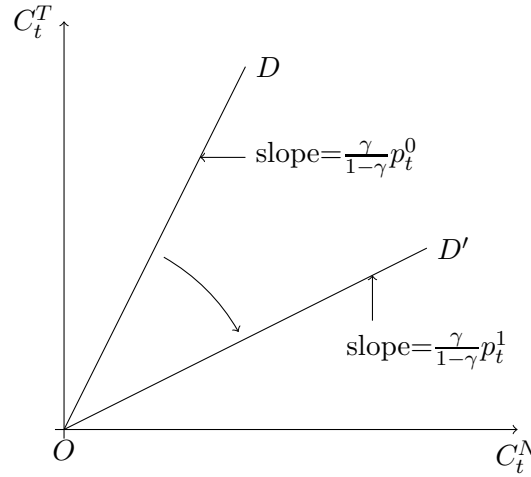
Figure 9.10 plots the optimality condition (9.24) in the space (C_t^N, C_t^T) for a given price p_t^0 as the ray \overline{OD} . This relationship is known as the *income expansion path*. Given the relative price p_t^0 , the optimal choice of C_t^T and C_t^N must lie on the income expansion path. What point on the income expansion path the household will pick depends on how much income the household allocates to consumption expenditure in period t . Given p_t^0 , the higher the amount of income allocated to consumption in period t is, the larger C_t^T and C_t^N will be, that is, the farther away from the origin the optimal consumption choice will lie. For example, consumption expenditure is higher at point B than at point A. The origin is a point on the income expansion path because if the household allocates no income to consumption spending in period t ,

Figure 9.10: The Income Expansion Path



Notes. The figure depicts the income expansion path associated with the relative price p_t^0 as the ray \overline{OD} . Given p_t^0 , the optimal combination of tradable and non-tradable consumption must lie on the income expansion path. What point on the income expansion path the household will pick depends on how much income it allocates to consumption expenditure in period t . At point B, the household allocates more income to consumption in period t than at point A. If both goods are normal, the income expansion path is upward sloping.

Figure 9.11: The Relative Price of Nontradables and the Income Expansion Path

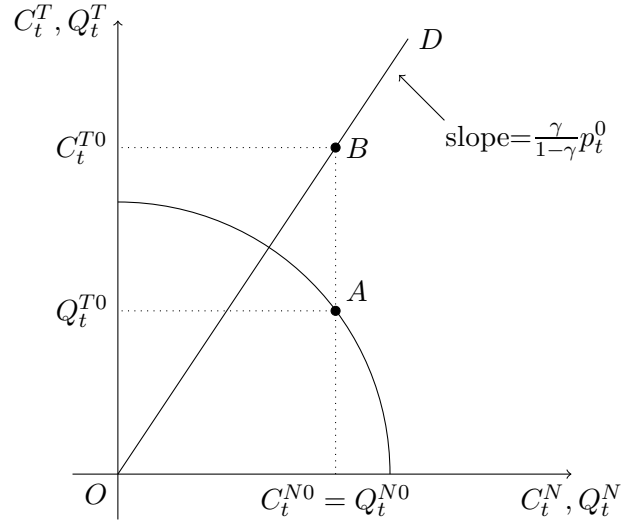


Notes. The figure displays the income expansion path associated with two values of the relative price of nontradables, p_t^0 and $p_t^1 < p_t^0$ as the rays \overline{OD} and $\overline{OD'}$, respectively. A decline in the relative price of nontradables rotates the income expansion path clockwise around the origin. This rotation is a reflection of households substituting nontradable consumption for tradable consumption as tradables become relatively more expensive.

then $C_t^T = C_t^N = 0$. The fact that the income expansion path is upward sloping means that tradables and nontradables are normal goods.

The slope of the income expansion path is increasing in the relative price of nontradables in terms of tradables, p_t . Figure 9.11 displays the income expansion paths associated with two relative prices, p_t^0 and $p_t^1 < p_t^0$ as the rays \overline{OD} and $\overline{OD'}$, respectively. As the relative price of nontradables falls from p_t^0 to p_t^1 , the income expansion path pivots clockwise around the origin. This is intuitive, because a lower price of nontradables induces households to substitute nontradables for tradables in consumption.

Figure 9.12: Partial Equilibrium



Notes. The figure displays the equilibrium for a given relative price of nontradables p_t^0 . Production is at point A, where the slope of the PPF is $-p_t^0$. Consumption is at point B. The trade balance is the vertical distance between points A and B. Because consumption of tradables, C_t^{T0} is larger than output of tradables, Q_t^{T0} , the country is running a trade deficit in period t .

9.3.4 Partial equilibrium

We now put together the first two building blocks of the model, the production possibility frontier and the income expansion path. Figure 9.12 represents the equilibrium in period t for a given relative price of nontradables, p_t^0 . Production takes place at point A, where the slope of the PPF is $-p_t^0$. Output of nontradables equals Q_t^{N0} and output of tradables equals Q_t^{T0} . The income expansion path corresponding to p_t^0 is the ray \overline{OD} . By definition, nontradable goods cannot be imported or exported. Therefore, market clearing in the nontraded sector requires that production equal con-

sumption, that is,

$$Q_t^N = C_t^N. \quad (9.25)$$

Given consumption of nontradables, the income expansion path determines uniquely the level of consumption of tradables, C_t^{T0} , at point B. The trade balance is the difference between production and consumption of tradables,

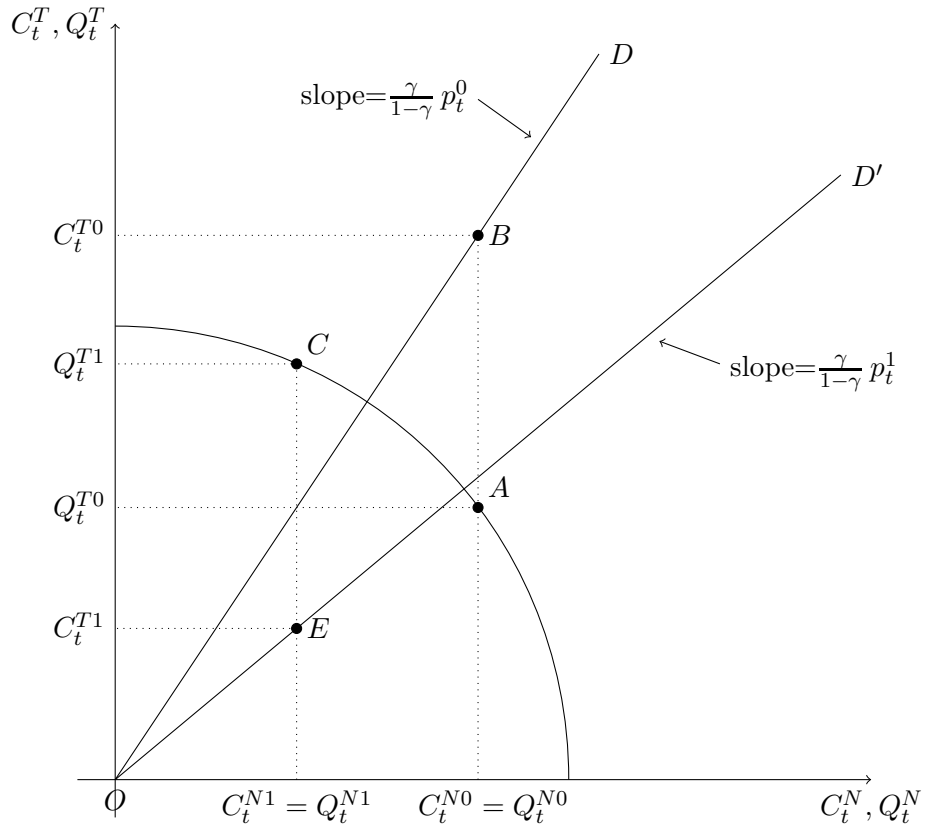
$$TB_t = Q_t^T - C_t^T. \quad (9.26)$$

In the figure, the trade balance is given by the vertical distance between points A and B. Because in the figure consumption of tradables exceeds production, the country is running a trade balance deficit.

Consider now the effect of a fall in the relative price of nontradables, from p_t^0 to $p_t^1 < p_t^0$. This represents a depreciation of the real exchange rate, so that the economy becomes cheaper relative to the rest of the world. Figure 9.13 illustrates this situation. The economy is initially producing at point A and consuming at point B. Because in equilibrium the slope of the PPF must equal the negative of the relative price of nontradables, the depreciation of the real exchange rate induces a change in the production mix to a point like C, where the PPF is flatter than at point A. This shift in the composition of production has a clear intuition: as the price of nontradables falls relative to that of tradables, firms find it profitable to reduce production of nontradable goods and increase production of tradable goods.

On the demand side of the economy, the real exchange rate depreciation causes a clockwise rotation of the income expansion path from \overline{OD} to $\overline{OD'}$ as shown in figure 9.13. Having determined the new production position

Figure 9.13: Partial Equilibrium: Adjustment to a Real Exchange Rate Depreciation



Notes. The figure displays the adjustment to a fall in the relative price of nontradables from p_t^0 to $p_t^1 < p_t^0$. Initially, production is at point A and consumption is at point B . After the real depreciation production shifts to point C and consumption to point E . Thus, production of tradables expands, production of nontradables contracts, and consumption of both tradables and nontradables contract. The new trade balance, given by the vertical distance between C and E , improves.

and the new income expansion path, we can determine the new equilibrium consumption basket (point E in the figure) and trade balance (the vertical distance between points C and E).

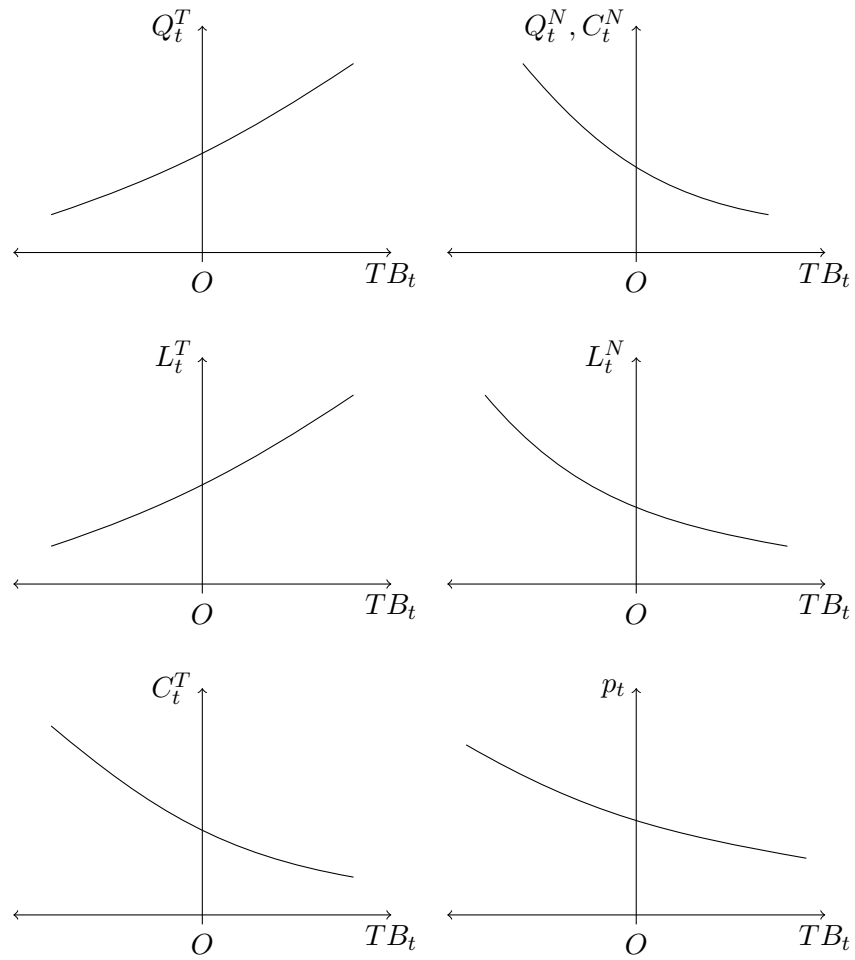
Summing up, in response to the real exchange rate depreciation, the economy produces more tradables ($Q_t^{T1} > Q_t^{T0}$) and less nontradables ($Q_t^{N1} < Q_t^{N0}$), and consumes less tradables as well as nontradables ($C_t^{T1} < C_t^{T0}$ and $C_t^{N1} < C_t^{N0}$). As a result of the expansion in the production of tradables and the contraction in consumption of tradables, the trade balance improves.

The analysis we have performed is a partial equilibrium one because we took as given the value of a variable that is endogenously determined in the model, namely, the relative price of nontradables. Equivalently, we could have taken as exogenous any other endogenous variable. For example, it is clear from the analysis we just performed that if we take as given an improvement in the trade balance, then the effect will be a real-exchange-rate depreciation, an expansion in the production of tradables, a contraction in the production of nontradables, and a contraction in the consumption of both, tradables and nontradables, as shown in figure 9.14.

9.3.5 General Equilibrium

From a formal perspective, the analysis conducted in section 9.3.4 is a partial equilibrium one because it does not make use of all equilibrium conditions of the TNT model with sectoral production. Specifically, it uses seven equations for each period $t = 1, 2$, namely, (9.15)-(9.17), (9.20), and (9.24)-(9.26), which are cast in eight endogenous variables for each period, Q_t^T , Q_t^N , L_t^T , L_t^N , C_t^T , C_t^N , p_t , and TB_t . Thus, we are short two equations.

Figure 9.14: Partial equilibrium: Endogenous Variables as Functions of the Trade Balance



Notes. The figure displays the equilibrium values of the endogenous variables of the model for different given values of the trade balance.

One of the two equilibrium conditions we have not included in the analysis is the household's Euler equation given in (9.23). In the partial equilibrium analysis we deduced that consumption of tradables in period t is a decreasing function of the trade balance in period t (see figure 9.14), so we can write

$$C_t^T = C^T(TB_t).$$

We can then rewrite the Euler equation in terms of the trade balances in periods 1 and 2 as

$$C^T(TB_2) = \beta(1+r)C^T(TB_1). \quad (9.27)$$

This expression represents a positive relationship between TB_1 and TB_2 . Figure 9.15 depicts this relationship as the locus \overline{EE} . We assume that $\beta(1+r) < 1$. Under this assumption, the locus \overline{EE} crosses the vertical axis at a positive value of TB_2 . To see this, note that if $\beta(1+r) < 1$, then, by the Euler equation (9.23), we have that $C_2^T < C_1^T$. In turn, since C_t^T is an decreasing function of TB_t , this implies that $TB_2 > TB_1$. Thus, when $TB_1 = 0$, TB_2 must be positive.

The second equilibrium condition that the partial equilibrium analysis leaves out is the economy's intertemporal resource constraint, which we derive next: Use the definitions of profits in the traded and nontraded sectors given in (9.18) and (9.19) and the market-clearing condition in the nontraded sector given in (9.25) to write the household's budget constraints in

periods 1 and 2, equations (9.21) and (9.22), as

$$C_1^T + B_1 = Q_1^T$$

and

$$C_2^T = Q_2^T + (1 + r)B_1.$$

Combining these two expressions to eliminate B_1 yields the intertemporal resource constraint,

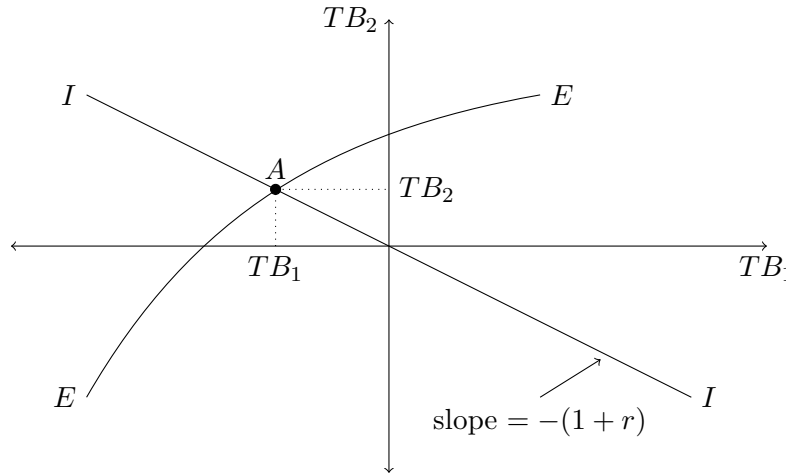
$$C_1^T + \frac{C_2^T}{1 + r} = Q_1^T + \frac{Q_2^T}{1 + r},$$

which is identical to the intertemporal resource constraint in the endowment version of the TNT model, except that now Q_1^T and Q_2^T are endogenous variables. It will prove convenient to write the intertemporal resource constraint in terms of the trade balance in periods 1 and 2. To this end, using condition (9.26), we can write

$$TB_2 = -(1 + r)TB_1. \quad (9.28)$$

Figure 9.15 plots the intertemporal resource constraint in the space (TB_1, TB_2) as the downward sloping line \overline{II} . The intertemporal resource constraint crosses the origin and has a slope equal to $-(1 + r)$. Intuitively, if the country improves its trade balance in period 1 by one unit, it can deposit these savings at the interest rate r , allowing for an increase in the trade deficit in period 2 of $1 + r$ units. The intertemporal resource constraint crosses the origin because of our maintained assumption that households start period 1

Figure 9.15: General Equilibrium Determination of the Trade Balance

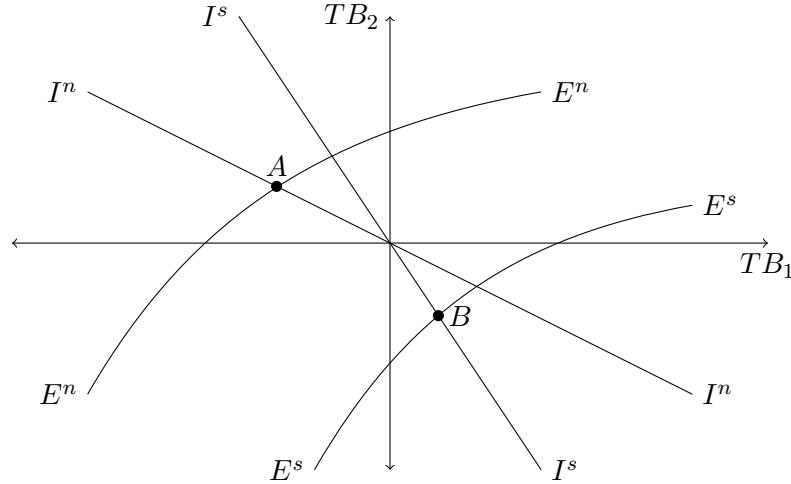


Notes. The locus \overline{EE} depicts the pairs of current and future trade balances TB_1 and TB_2 that are consistent with the household's Euler equation in equilibrium, equation (9.27). The locus \overline{II} depicts the the pairs (TB_1, TB_2) that satisfy the economy's intertemporal resource constraint, equation (9.28)).

with a zero net asset position ($B_0 = 0$). If we had assumed instead that the initial asset position is positive (negative), then the intertemporal budget constraint would cross the vertical axis below (above) the origin.

The equilibrium is at point A, where the loci \overline{EE} and \overline{II} intersect. In equilibrium, the economy runs a trade balance deficit in period 1 ($TB_1 < 0$) and a surplus in period 2 ($TB_2 > 0$). With the equilibrium values of the trade balance in hand, the equilibrium values of all other endogenous variables of the model are readily determined as explained in the partial equilibrium analysis of subsection 9.3.4.

Figure 9.16: A Sudden Stop in the TNT model with Production



Notes. The figure depicts the adjustment of the trade balance to an increase in the interest rate from r^n to $r^s > r^n$. The equilibrium before the sudden stop is at point A, where $\overline{I^n I^n}$ (the locus of pairs (TB_1, TB_2) satisfying (9.28)) intersects $\overline{E^n E^n}$ (the locus of pairs (TB_1, TB_2) satisfying (9.27)). At point A, the country runs a trade deficit in period 1. In response to the increase in the interest rate to r^s , the equilibrium moves to point B, where the loci $\overline{I^s I^s}$ and $\overline{E^s E^s}$ intersect. At point B, the country runs a trade surplus in period 1. Thus, the sudden stop leads to a trade balance reversal.

9.3.6 Sudden Stops and Sectoral Reallocations

We are now ready to analyze the effects of a sudden stop on the sectoral allocation of output and employment. Suppose that the world interest rate increases from r^n to $r^s > r^n$. To capture the effect of a large increase in the interest rate, assume that $\beta(1 + r^n) < 1$ and that $\beta(1 + r^s) > 1$. The effect of the sudden stop on the trade balance is illustrated in figure 9.16. The situation under normal times (before the sudden stop) is like the one illustrated in figure 9.15. The equilibrium is at point A, where the loci

$\overline{E^n E^n}$ and $\overline{I^n I^n}$ intersect. The country runs a trade deficit in period 1. The increase in the interest rate shifts the locus \overline{EE} down and to the right from $\overline{E^n E^n}$ to $\overline{E^s E^s}$. The intercept of $\overline{E^s E^s}$ is negative. To see this, note first that because r does not appear in any of the equations used in the partial equilibrium analysis, the function $C^T(TB_t)$ conserves its original form. Next, note that because $\beta(1 + r^s) > 1$, the Euler equation (9.27) implies that $C^T(TB_2) > C^T(TB_1)$. In turn, since $C^T(TB_t)$ is a decreasing function, we have that $TB_2 < TB_1$. Thus, for $TB_1 = 0$, TB_2 (the intercept of the locus $\overline{E^s E^s}$), must be negative. The increase in the interest rate causes the locus \overline{II} to rotate clockwise around the origin from $\overline{I^n I^n}$ to $\overline{I^s I^s}$, as is evident from equation (9.28). The equilibrium after the sudden stop is at point B, where the country runs a trade surplus in period 1. Since in this economy the current account is equal to the trade balance (recall that $B_0 = 0$), the sudden stop causes a current account reversal from a deficit to a surplus.

Having determined the effect of the sudden stop on the trade balance, the effect on all other endogenous variables can be read off directly from figure 9.14. The model predicts a contraction in aggregate demand, as both, consumption of tradables and consumption on nontradables fall. The collapse in aggregate demand causes the relative price of nontradables, p_1 , to fall, or, equivalently, the real exchange rate, e_1 , to depreciate. In turn, the fall in p_1 , induces firms to reduce output and employment in the nontraded sector and to increase output and employment in the traded sector.

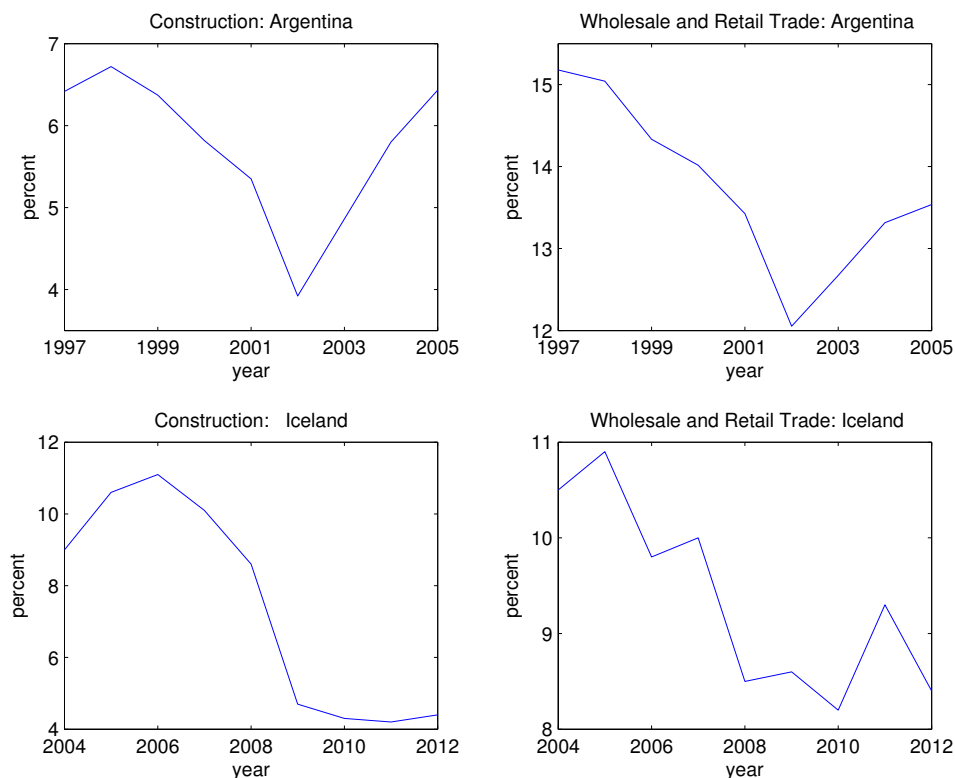
Taking stock, the effects of a sudden stop on the trade balance, the real exchange rate, and the absorption of tradable goods are the same as in the

endowment economy. The new prediction stemming from the economy with sectoral production is that the sudden stop also causes a contraction in the absorption of nontradable goods and a reallocation of output and employment away from the nontraded sector and toward the traded sector. In the model the reallocation of workers across sectors happens instantaneously. In real life, however, it is not so easy for workers to move from one sector to another. Such a transition typically involves a period of involuntary unemployment during which the workers that lost their job in the nontraded sector search for a new job. The reallocation of labor might also cause some workers to temporarily leave the labor force to acquire new skills to increase their chances of finding a new job in another sector.

The TNT model's prediction of a sectoral reallocation of production away from nontradable sectors and toward tradable sectors is borne out in the data. To document this fact, let's revisit the sudden stops of Argentina in 2001 and Iceland in 2008. Figure 9.17 plots GDP in the construction and the wholesale and retail trade sectors as shares of total GDP in an eight year window around the Argentine and Icelandic sudden stops. Construction and wholesale and retail trade are two large and labor-intensive nontradable sectors. Their combined share in GDP fell from an average of 20 percent to less than 16 percent during the Argentine sudden stop of 2001 and from an average of 20 percent to less than 15 percent during the Icelandic sudden stop of 2008.

Recalling that in 2002 total GDP in Argentina fell by 12.5 percent, it follows that the crisis in the construction and the wholesale and retail trade sectors was enormous in absolute terms. A similar pattern is observed in

Figure 9.17: GDP Shares in Construction and Wholesale and Retail Trade During Sudden Stops: Argentina 2001 and Iceland 2008



Notes. Construction and wholesale and retail trade are large, labor intensive, non-tradable sectors. Their combined share in GDP fell from an average of 20 percent to less than 16 percent during the Argentine sudden stop of 2001 and from an average of 20 percent to less than 15 percent during the Icelandic sudden stop of 2008. This pattern of sectoral reallocation of production away from nontradable sectors conforms with the predictions of the TNT model.

the Icelandic sudden stop. Because of their labor intensity, these sectors contributed greatly to the surge in involuntary unemployment suffered by Argentina and Iceland in the aftermath of their sudden stops.

9.4 Productivity Differentials and Real Exchange Rates: The Balassa-Samuelson Model

The TNT model assumes time invariant production technologies. For this reason, it is most useful for understanding short-run movements in the real exchange rate and sectoral reallocations of output and employment. In the long run, however, technological improvement generates sustained increases in productivity. In turn, sectoral differences in the speed of productivity growth can generate long-run movements in the real exchange rate and sectoral differences in output growth.

The Balassa-Samuelson model, named after its authors Bela Balassa and Paul Anthony Samuelson, is one of the most widely known theories of long-run determinants of the real exchange rate.² It predicts that persistent movements in the real exchange rate are due to cross-country differentials in relative productivities in the traded and nontraded sectors. In this section, we study a simple version of the Balassa-Samuelson model that captures this key result.

Suppose a country produces two kinds of goods, traded goods and non-

²The formulation of the model appeared in two separate publications: Bela Balassa, “The Purchasing-Power-Parity Doctrine: A Reappraisal,” *Journal of Political Economy* 72, December 1964, 584-596; and Paul Anthony Samuelson, “Theoretical Notes on Trade Problems,” *Review of Economics and Statistics* 46, March 1964, 145-154. Samuelson went on to win the Nobel Prize in Economics in 1970.

traded goods. Let Q^T and Q^N denote output in the traded and nontraded sectors, respectively. Suppose that both goods are produced with linear production technologies that take labor as the sole factor input. Labor productivity varies across sectors. Formally, the production technologies in the traded and nontraded sectors are given by

$$Q^T = a_T L^T \quad (9.29)$$

and

$$Q^N = a_N L^N, \quad (9.30)$$

where L^T and L^N denote labor input in the traded and nontraded sectors, and a_T and a_N denote labor productivity in the traded and nontraded sectors, respectively. There are two concepts of labor productivity: average and marginal labor productivity. Average labor productivity is defined as output per worker, Q/L . Marginal labor productivity is defined as the increase in output resulting from a unit increase in labor input, holding constant all other inputs. More formally, marginal labor productivity is given by the partial derivative of output with respect to labor, $\partial Q/\partial L$. For the linear technologies given in (9.29) and (9.30), average and marginal labor productivities are the same.

Workers are assumed to be able to move freely from one sector to the other. So they will always choose to work in the sector offering the higher wage. This means that if both goods are produced in positive quantities in equilibrium, the wage rate must be the same in the traded and nontraded sectors.

In the traded sector, a firm's profit is given by the difference between revenues from sales of traded goods, $P^T Q^T$, and total cost of production, WL^T , where W denotes the wage rate per worker. That is,

$$\text{profits in the traded sector} = P^T Q^T - WL^T.$$

Similarly, in the nontraded sector we have

$$\text{profits in the nontraded sector} = P^N Q^N - WL^N.$$

There is perfect competition in both sectors and no restrictions on entry of new firms. This means that as long as profits are positive, existing firms will have an incentive to increase production and new firms will have an incentive to enter, driving prices down and wages up. As a result, profits will go down. This process will continue until profits are zero in both sectors. Thus, in equilibrium it must be the case that

$$P^T Q^T = WL^T$$

and

$$P^N Q^N = WL^N.$$

Using the production functions (9.29) and (9.30) to eliminate Q^T and Q^N from the above zero-profit conditions, one obtains

$$P^T a_T = W$$

and

$$P^N a_N = W.$$

Combining these two expressions to eliminate W yields

$$\frac{P^N}{P^T} = \frac{a_T}{a_N}. \quad (9.31)$$

This expression says that in equilibrium the relative price of nontraded goods, P^N/P^T , is equal to the ratio of labor productivity in the traded sector to that in the nontraded sector, a_T/a_N . The intuition behind this condition is as follows: Suppose that a_T is greater than a_N . This means that one unit of labor produces more units of traded goods than of nontraded goods. Equivalently, producing one unit of tradable goods takes fewer units of labor than producing one unit of nontradable goods ($1/a_T < 1/a_N$). Since the wage rate is the same in both sectors, it follows that producing one unit of traded goods costs less than producing one unit of nontraded goods. Finally, since firms make zero profits, it must be the case that the good that costs less to produce sells at a lower price, $P^T < P^N$.

In the foreign country, the relative price of nontradable goods in terms of tradable goods is determined in a similar fashion, that is,

$$\frac{P^{N*}}{P^{T*}} = \frac{a_T^*}{a_N^*}, \quad (9.32)$$

where P^{N*} and P^{T*} denote the prices of nontradable and tradable goods in the foreign country, and a_N^* and a_T^* denote the labor productivities in the nontraded and traded sectors in the foreign country.

We are now ready to derive the equilibrium real exchange rate, e , which from equation (9.13) is given by

$$e = \frac{\mathcal{E}\phi^*(P^{T*}, P^{N*})}{\phi(P^T, P^N)}.$$

We assume that the law of one price holds for tradable goods, so that $P^T = \mathcal{E}P^{T*}$. Then, since the price indices $\phi(\cdot, \cdot)$ and $\phi^*(\cdot, \cdot)$ are homogeneous degree one, we can express the real exchange rate as

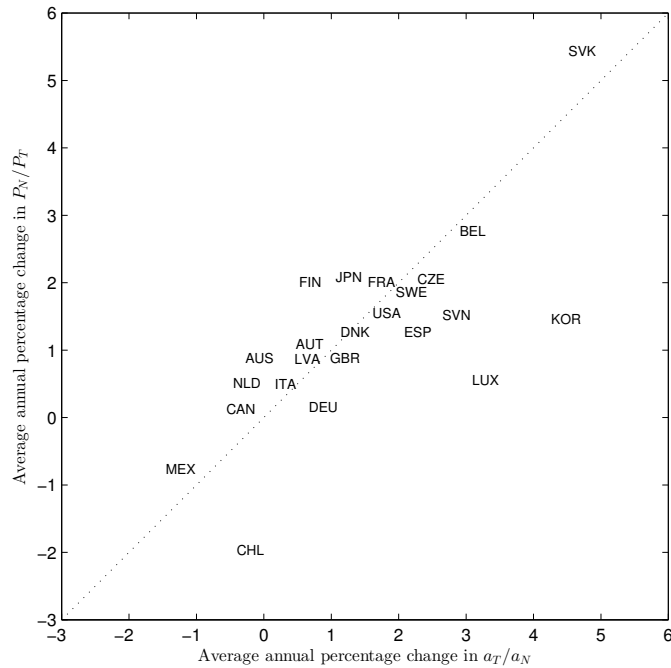
$$e = \frac{\phi^*(1, P^{N*}/P^{T*})}{\phi(1, P^N/P^T)}.$$

Using equation (9.31) to eliminate P^N/P^T and equation (9.32) to eliminate P^{N*}/P^{T*} yields

$$e = \frac{\phi^*(1, a_T^*/a_N^*)}{\phi(1, a_T/a_N)}. \quad (9.33)$$

This equation captures the main result of the Balassa-Samuelson model, namely, that deviations from PPP (i.e., deviations of e from unity) are due to differences in relative productivities across countries. In particular, if in the domestic country the relative productivity of the traded sector, a_T/a_N , grows faster than in the foreign country, then the real exchange rate will appreciate over time (i.e., e will fall over time), which means that the domestic country will become more expensive relative to the foreign country. This is because in the domestic country nontradables are becoming relatively more expensive to produce than in the foreign country, forcing the relative price of nontradables in the domestic country to grow at a faster rate than in the foreign country.

Figure 9.18: Relative Productivity Growth in the Traded and Nontraded Sectors and Changes in the Relative Price on Nontradables



Notes. The figure plots the average annual percentage change in the relative price of nontradables in terms of tradables, P^N/P^T , against the average annual percentage change in productivity in the traded sector relative to the nontraded sector, a_T/a_N , for 23 countries over the period 1996 to 2015. The strong positive relationship provides empirical support to the Balassa-Samuelson model. Source: Own calculations based on data from KLEMS and OECD STAN.

An important insight of the Balassa-Samuelson model is that in the long run the relative price of nontradable goods in terms of tradable goods, P^N/P^T , is an increasing function of the relative productivities, a_T/a_N , see equation (9.31). This prediction implies that countries in which, over a certain period of time, a_T grew faster than a_N should also exhibit a faster growth in P^N than in P^T . Is this prediction borne out in the data? Figure 9.18 plots the average annual percentage change in P^N/P^T against the average annual percentage change in a_T/a_N for 23 countries over the period 1996 to 2015.³ The figure shows that in the long run there is a strong positive relationship between sectoral differences in productivity growth and sectoral differences in price growth. This evidence provides empirical support to Balassa and Samuelson's theory of the determinants of deviations from PPP.

9.5 Summing Up

This chapter studies the determination of the real exchange rate in the short and long runs.

- The TNT model is an open economy model with tradable and nontradable goods. It is a useful framework for understanding the determinants of the real exchange rate and sectoral reallocations of output and employment in the short run.
- The TNT model predicts that in response to an increase in the world

³This figure is inspired by figure 3 of De Gregorio, José, Alberto Giovannini, and Holger C. Wolf, "International Evidence on Tradable and Nontradable Inflation," *European Economic Review* 38, June 1994, 1225-1244, which covered 14 OECD countries over the period 1970 to 1985.

interest rate the real exchange rate depreciates, the relative price of nontradable goods falls, output and employment in the nontradable sector contract, and output and employment in the tradable sector expand.

- The TNT model predicts that in response to positive shocks to the tradable endowment or improvements in the terms of trade the real exchange rate appreciates and the relative price of nontradables increases. These effects are stronger when the shocks are expected to be persistent.

- A sudden stop is a macroeconomic crisis that occurs when foreign lenders abruptly stop extending credit to a debtor nation. It manifests itself by a steep increase in the country interest rate.

- The main observed effects of a sudden stop are a current account reversal from a large deficit to near balance or even surplus, a sharp depreciation of the real exchange rate, and a reallocation of production and employment from the nontradable sector to the tradable sector.

- Two sudden stop episodes are analyzed, Argentina 2001 and Iceland 2008.

- The TNT model explains well the macroeconomic consequences of observed sudden stops.

- The Balassa-Samuelson model is a theory that explains long run movements in real exchange rates. It explains deviations from PPP as stemming from international differences in relative sectoral productivity growth.

- The Balassa-Samuelson model predicts that for a given country if productivity grows faster in the traded sector than in the nontraded sector, then the relative price of nontradables in terms of tradables grows over time. This prediction is borne out in the data. Across countries, long-run averages of

the growth rate of relative productivities in the tradable and nontradable sectors are positively correlated with long-run averages of the growth rate of the relative price on nontradables in terms of tradables.

- The Balassa-Samuelson model predicts that if in the domestic country productivity in the traded sector relative to productivity in the nontraded sector grows faster than in the foreign country, then the domestic country becomes more expensive, that is, its real exchange rate appreciates.

9.6 Exercises

Exercise 9.1 (Endowment Shocks and the Real Exchange Rate) Consider the endowment version of the TNT model studied in section 9.1. Suppose the endowment of tradables increases in period 1. Analyze the effect of this innovation on the real exchange rate depending on whether the change in the tradable endowment is temporary or permanent.

Exercise 9.2 (A Sudden Stop, I) Consider a two-period small open economy populated by a large number of households with preferences captured by the following lifetime utility function

$$\ln(C_1^T C_1^N) + \ln(C_2^T C_2^N),$$

where C_t^T and C_t^N , for $t = 1, 2$, denote consumption of tradable and nontradable goods in period t , respectively. Households are endowed with $Q_1^T = 1$ and $Q_2^T = 2$ units of tradables and $Q_1^N = Q_2^N = 1$ units of nontradables in periods 1 and 2. Households start period 1 with no assets or debts. The world interest rate is zero.

1. Calculate the equilibrium levels of the current account and the relative price of nontradables in terms of tradables in period 1, denoted CA_1 and p_1 , respectively.
2. Suppose now that suddenly the world interest rate increases from 0 to 10 percent. Calculate the new equilibrium levels of the current account and the relative price of nontradables in terms of tradables in period 1.

Exercise 9.3 (A Sudden Stop, II) Consider a two-period, small, open economy. In period 1, households receive an endowment of 6 units of tradable goods and 9 units of nontradable goods. In period 2, households receive 13.2 units of tradables and 9 units of nontradables ($Q_1^T = 6$, $Q_2^T = 13.2$, and $Q_1^N = Q_2^N = 9$). Households start period 1 with no assets or liabilities ($B_0 = 0$). The country enjoys free access to world financial markets, where the prevailing interest rate is 10 percent ($r^* = 0.1$). Suppose that the household's preferences are defined over consumption of tradable and nontradable goods in periods 1 and 2, and are described by the following utility function,

$$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

where C_t^T and C_t^N denote, respectively, consumption of tradables and nontradables in period $t = 1, 2$. Let p_1 and p_2 denote the relative prices of nontradables in terms of tradables in periods 1 and 2, respectively.

1. Write down the budget constraints of the household in periods 1 and 2.
2. Derive the household's intertemporal budget constraint. Assign this expression the number (1).
3. The household chooses consumption of tradables and nontradables in periods 1 and 2 to maximize its lifetime utility function subject to its intertemporal budget constraint. Derive the optimality conditions associated with this problem. To this end, begin by solving (1) for C_1^T and use the resulting expression to eliminate C_1^T from the lifetime util-

ity function. Take the derivatives of the resulting lifetime utility with respect to C_1^N , C_2^T , and C_2^N and sets them equal to zero. Assign the resulting three expressions the numbers (2), (3), and (4), respectively.

4. Write down the market clearing conditions in the nontradable goods market in periods 1 and 2. Assign these expressions the numbers (5) and (6) respectively.
5. Combine expressions (1) to (6) to solve for C_1^T , C_2^T , C_1^N , C_2^N , p_1 , and p_2 . Explain intuitively why the relative price of nontradables changes over time.
6. Calculate the net foreign asset position of the economy at the end of period 1, B_1 .
7. Calculate the equilibrium levels of the current account balance in periods 1 and 2 (CA_1 and CA_2).
8. Assume that the domestic consumer price index in period $t = 1, 2$, denoted P_t , is defined by $P_t = \sqrt{P_t^T P_t^N}$, where P_t^T and P_t^N denote the nominal prices of tradables and nontradables in period $t = 1, 2$, respectively. Similarly, suppose that the foreign consumer price index is given by $P_t^* = \sqrt{P_t^{T*} P_t^{N*}}$, where the superscript $*$ denotes foreign variables. Foreign nominal prices are expressed in terms of foreign currency. Assume that PPP holds for tradable goods. Finally, suppose that the foreign relative price of nontradables in terms of tradables equals unity in both periods. Compute the real exchange rate in periods 1 and 2.

9. Let us sketch a scenario like the one that took place during the Argentine sudden stop of 2001 by assuming that because of fears that the country will not repay its debts in period 2, foreign lenders refuse to extend loans to the domestic economy in period 1. Answer the questions in items 6 through 8 under these new (adverse) circumstances. Compute the equilibrium interest rate. Provide an intuitive explanation of your results.
10. Compute real GDP in period 1 under the sudden stop and no-sudden-stop scenarios. Consider two alternative measures of real GDP: GDP measured in terms of tradable goods and GDP measured in terms of the basket of goods whose price is the consumer price index P_1 . What measure is more economically sensible? Why?
11. Suppose the Inter American Development Bank (IADB) decided to implement a transfer (gift) to the country to ameliorate the effects of the sudden stop. Specifically, suppose that the IADB gives the country a transfer of F units of tradable goods in period 1. Use the utility function given above to compute the size of F that would make households as happy as in the no-sudden-stop scenario. Express F as a percentage of the country's sudden stop and no-aid GDP (in terms of tradables) in period 1.

Exercise 9.4 (The PPF) Suppose that the production functions in the traded and nontraded sectors are $Q^T = (L^T)^\alpha$ and $Q^N = (L^N)^\alpha$, where Q^i and L^i denote output and employment in sector $i = T, N$, respectively. Let L be the total number of workers in the economy.

1. Find the PPF and its slope.
2. Suppose that $\alpha = 0.25$ and that $L = 1$. Assume further that the relative price of nontradables in terms of tradables, p , equals 2. Find Q^T , Q^N , L^T , and L^N .
3. Suppose that the nominal wage, denoted W , is equal to 5. Find the nominal prices of tradables and nontradables, denoted P^T and P^N , and profits in both sectors.

Exercise 9.5 (Linear Income Expansion Paths) In section 9.3.3, we showed that when the period utility function is logarithmic, $U(C) = \ln(C)$, and the aggregator function is Cobb-Douglas, $C = (C^T)^\gamma (C^N)^{1-\gamma}$, the income expansion path is linear.

1. Show that the income expansion path is linear in the more general case in which preferences are described by a *constant relative risk aversion (CRRA) period utility function* and a CES or Armington aggregator function, that is,

$$U(C) = \frac{C^{1-\sigma} - 1}{1-\sigma}$$

and

$$C = \left[\gamma (C^T)^{1-\frac{1}{\xi}} + (1-\gamma) (C^N)^{1-\frac{1}{\xi}} \right]^{\frac{1}{1-\frac{1}{\xi}}},$$

with $\sigma, \xi > 0$, and $\gamma \in (0, 1)$.

2. Show that the logarithmic period utility function is a special case of the CRRA period utility function when $\sigma \rightarrow 1$.

3. Show that the Cobb-Douglas aggregator is a special case of the CES aggregator when $\xi \rightarrow 1$.
4. Even more generally, show that the income expansion path is linear when the period utility function is monotone and the aggregator function is homogeneous. In this case, we say that households have *homothetic preferences* for tradable and nontradable goods.

Exercise 9.6 (Equilibrium in the TNT Model with Production) Consider a two-period small open economy that produces and consumes tradable and nontradable goods. In periods $t = 1, 2$, the production possibility frontier (PPF) is of the form

$$Q_t^N = \sqrt{2 - (Q_t^T)^2},$$

where Q_t^T and Q_t^N denote, respectively, tradable and nontradable output in periods $t = 1, 2$. Preferences are described by the utility function

$$\ln(C_1^T) + \ln(C_1^N) + \ln(C_2^T) + \ln(C_2^N),$$

where C_t^T and C_t^N denote, respectively, tradable and nontradable consumption in periods $t = 1, 2$. Let $p_t \equiv P_t^N/P_t^T$ denote the relative price of nontradable goods in terms of tradable goods in periods $t = 1, 2$.

1. Suppose that p_1 is equal to 1 (i.e., one unit of nontradable goods sells for 1 unit of tradable goods). Using the information provided by the PPF, and assuming that firms producing tradables and nontradables are profit maximizers, calculate output of tradables and nontradables in period 1 (Q_1^T and Q_1^N).

2. Using the results obtained thus far, and the market clearing condition in the nontraded sector, calculate consumption of nontradables in period 1 (C_1^N).
3. Assuming that households are utility maximizers use the information given above and the results of the previous two questions to calculate consumption of tradables in period 1 (C_1^T).
4. Calculate the country's trade balance in period 1 (TB_1).
5. Suppose that the initial net foreign asset position, B_0 , is nil. Calculate the current account in period 1 (CA_1).
6. Suppose that the world interest rate, denoted r^* , equals 0 percent ($r^* = 0$). Assume further that households have access to the world financial market. Calculate consumption of tradables in period 2 (C_2^T).
7. Calculate tradable output in period 2 (Q_2^T). To this end, use the economy's intertemporal resource constraint for tradables.
8. Now calculate output of nontradables in period 2 (Q_2^N).
9. Calculate the relative price of nontradables in terms of tradables in period 2 (p_2).
10. Is the initial guess for p_1 , namely, $p_1 = 1$ an equilibrium? Be explicit about what must be verified for this to be the case.

Exercise 9.7 (The Trade Balance in the TNT Model with Sectoral Production)

State whether the following claims about the equilibrium in the TNT model

with sectoral production of section 9.3 are true, false, or uncertain and explain why.

1. If $\beta(1+r) = 1$, then the trade balance in both periods is nil, $TB_1 = TB_2 = 0$.
2. If $\beta(1+r) > 1$, then the trade balance is positive in period 1, $TB_1 > 0$.
3. If $\beta(1+r) > 1$, then an increase in the world interest rate leads to a trade balance improvement in period 1.

Exercise 9.8 (Real Exchange Rate Determination in the Balassa-Samuelson Model)

Consider two countries, say the United States and Japan. Both countries produce tradables and nontradables. Suppose that at some point in time the production technology in the United States is described by

$$Q^{TUS} = a_T^{US} L^{TUS}; \text{ with } a_T^{US} = 0.4$$

and

$$Q^{NUS} = a_N^{US} L^{NUS}; \text{ with } a_N^{US} = 0.1,$$

where Q^{TUS} and Q^{NUS} denote, respectively, output of tradables and nontradables in the U.S., a_T^{US} and a_N^{US} denote, respectively, labor productivity in the tradable and the nontradable sector, and L^{TUS} and L^{NUS} denote, respectively, the amount of labor employed in the tradable and nontradable sectors in the United States. The total supply of labor in the United States is equal to 1, so that $1 = L^{TUS} + L^{NUS}$. At the same point in time,

production possibilities in Japan are given by

$$Q_T^J = 0.2L_T^J$$

and

$$Q_N^J = 0.2L_N^J,$$

where the superscript J denotes Japan. The total supply of labor in Japan is also equal to 1. Assume that in each country wages in the traded sector equal wages in the nontraded sector. Suppose that the price index in the United States, which we denote by P^{US} , is given by

$$P^{US} = \sqrt{P_T^{US}} \sqrt{P_N^{US}},$$

where P_T^{US} and P_N^{US} denote, respectively, the dollar prices of tradables and nontradables in the United States. Similarly, the price index in Japan is given by

$$P^J = \sqrt{P_T^J} \sqrt{P_N^J},$$

where Japanese prices are expressed in yen.

1. Calculate the dollar/yen real exchange rate, defined as $e = \mathcal{E}P^J/P^{US}$, where \mathcal{E} denotes the dollar-yen exchange rate (dollar-price of one yen). The answer to this question is a number, but show your work.
2. Suppose that the U.S. labor productivity in the traded sector, a_T^{US} , grows at a 3 percent rate per year, whereas labor productivity in the nontraded sector, a_N^{US} , grows at 1 percent per year. Assume that labor

productivities in Japan are constant over time. Calculate the growth rate of the real exchange rate. Provide an intuitive explanation of your result.

Exercise 9.9 (A Two-Tradable-Good Economy with Linear Technologies)

Consider a two-country economy with two goods, A and B . Both goods are traded internationally and produced using labor and linear technologies. The production functions in the domestic economy are

$$Q^A = a_A L^A$$

and

$$Q^B = a_B L^B,$$

where Q^i , L^i and a_i denote output, labor, and labor productivity in sector $i = A, B$. Assume that labor productivity is exogenous and that the country is endowed with a fixed number of units of labor L . Analogously, in the foreign country the production functions are

$$Q^{A*} = a_{A*} L^{A*}$$

and

$$Q^{B*} = a_{B*} L^{B*},$$

and the labor endowment is L^* .

1. Show that in general, the equilibrium features production specialization in at least one country, that is, one of the two countries produces

only one good.

2. Suppose that in equilibrium the domestic country specializes in the production of good A. What relation between labor productivities and relative prices must hold in equilibrium for this to be the case.
3. Suppose that in equilibrium the domestic country specializes in the production of good A and the foreign country in the production of good B. What relation between labor productivities and relative prices must hold in equilibrium for this to be the case.

Exercise 9.10 (A Two-Sector Economy with Linear Technologies)

This exercise combines the TNT and Balassa-Samuelson models. Consider a two-period small open economy populated by a large number of identical households with preferences described by the utility function

$$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

where C_1^T and C_2^T denote consumption of tradables in periods 1 and 2, respectively, and C_1^N and C_2^N denote consumption of nontradables in periods 1 and 2. Households are born in period 1 with no debts or assets and are endowed with $L_1 = 1$ units of labor in period 1 and $L_2 = 1$ units of labor in period 2. Households offer their labor to firms, for which they get paid the wage rate w_1 in period 1 and w_2 in period 2. The wage rate is expressed in terms of tradable goods, that is, $w_t \equiv W_t/P_t^T$. Households can borrow or lend in the international financial market at the world interest rate r^* . Let p_1 and p_2 denote the relative price of nontradable goods in terms of tradable

goods in periods 1 and 2, respectively.

Firms in the traded sector produce output with the technology $Q_1^T = a_T L_1^T$ in period 1 and $Q_2^T = a_T L_2^T$ in period 2, where Q_t^T denotes output in period $t = 1, 2$ and L_t^T denotes employment in the traded sector in period $t = 1, 2$. Similarly, production in the nontraded sector in periods 1 and 2 is given by $Q_1^N = a_N L_1^N$ and $Q_2^N = a_N L_2^N$.

1. Write down the budget constraint of the household in periods 1 and 2.
2. Write down the intertemporal budget constraint of the household.
3. State the household's utility maximization problem.
4. Derive the optimality conditions associated with the household's maximization problem.
5. Derive an expression for the optimal levels of consumption of tradables and nontradables in periods 1 and 2 (C_1^T , C_1^N , C_2^T , and C_2^N) as functions of r^* , w_1 , w_2 , p_1 , and p_2 .
6. Using the zero-profit conditions on firms, derive expressions for the real wage and the relative price of nontradables (w_t and p_t , $t = 1, 2$), in terms of the parameters a_T and a_N .
7. Write down the market clearing condition for nontradables.
8. Write down the market clearing condition for labor.
9. Using the above results, derive the equilibrium levels of consumption, the trade balance, and sectoral employment (C_1^T , C_2^T , C_1^N , C_2^N , TB_1 ,

TB_2 , L_1^T , and L_2^T) in terms of the structural parameters a_T , a_N , and r^* .

10. Is there any sectoral labor reallocation over time? If so, explain the intuition behind it.

Chapter 10

International Capital Market Integration

In chapter 8, we studied whether world goods markets are integrated. We investigated whether there is a tendency for the prices of goods and services to equalize across countries. In this chapter, we study whether international capital market are integrated and investigate whether under free capital mobility there is a tendency for interest rates to equalize across countries.

Over the past few decades, the world appears to have become more financially globalized. One manifestation of this phenomenon is the explosion in gross international asset and liability positions. For example, as documented in figure 1.7 of chapter 1, in the mid 1970s U.S. gross international liabilities were only 15 percent of GDP. By 2018, they had climbed to over 170 percent of GDP. Similarly, the U.S. gross international asset position jumped from 20 percent of GDP in the mid 1970s to over 130 percent in

2018. A similar pattern of growth in gross international asset and liability positions has been observed in many countries around the world.

A number of events have contributed to this phenomenon. Significant advancements in information technologies have reduced the costs of transacting in financial markets and the costs of gathering and storing financial data. These advancements have not only allowed existing market participants to increase the volume of transactions, but have also allowed the entrance of smaller investors, making the financial marketplace more atomistic. Investment funds and other financial intermediaries have facilitated access for this type of investors to international equity and fixed income markets. The abandonment of the fixed-exchange rate system known as Bretton-Woods in the early 1970s allowed many countries and regions of the world to dismantle capital controls aimed at preventing large fluctuations in cross-border capital flows. The creation of the eurozone first generated a large area free of barriers to the movement of financial capital in the mid 1980s, and, since the inception of the euro in 1999, a common currency area. Finally, following deep market-oriented reforms in the early 1980s and accession to the WTO in the early 2000s, China has emerged as a new world economic power that significantly boosted international trade in goods and financial assets. As we saw in chapter 1 by running large current-account surpluses, China became a major supplier of funds to world capital markets.

10.1 Covered Interest-Rate Parity

In a world that enjoys perfect capital mobility, the rate of return on risk-free financial investments should be equalized across countries. Otherwise, arbitrage opportunities would arise, inducing capital to flow out of the low-return countries and into the high-return countries. This movement of capital across national borders will tend to eliminate differences in interest rates. If, on the other hand, one observes that interest rate differentials across countries persist over time, it must be the case that in some countries restrictions on international capital flows are in place.

It follows that a natural empirical test of the degree of capital market integration is to look at cross-country interest rate differentials on assets free of default risk. However, such a test is not as straightforward as it might seem. One difficulty in measuring interest rate differentials is that interest rates across countries are not directly comparable if they relate to investments in different currencies. Suppose, for example, that the interest rate on a 1-year deposit is 7 percent in the United States and 3 percent in Germany. This four-percent interest rate differential will not necessarily induce capital flows from Germany to the United States. The reason is that if the U.S. dollar depreciates sharply within the investment period (by more than 4 percent), an investor that deposited her money in Germany might end up with more dollars at the end of the period than an investor who had invested in the United States. Thus, even in the absence of capital controls, differences in interest rates might exist due to expectations of changes in the exchange rate or as a compensation for exchange rate risk. Therefore, a

meaningful measure of interest rate differentials ought to take the exchange rate factor into account.

Suppose at date t a U.S. investor has 1 U.S. dollar and is trying to decide whether to invest it domestically or abroad, say in Germany. Let i_t denote the U.S. interest rate and i_t^* the foreign (German) interest rate at time t . If in period t the investor deposits her money in the United States, then in period $t + 1$ she receives $1 + i_t$ dollars. How many dollars would she get, if instead she invested her 1 dollar in Germany? In order to invest in Germany, she must first use her dollar to buy euros. Let \mathcal{E}_t denote the spot exchange rate at date t , defined as the dollar price of 1 Euro. The investor gets $1/\mathcal{E}_t$ euros for her dollar. In period $t + 1$, she will receive $(1 + i_t^*)/\mathcal{E}_t$ euros. At this point she converts the euros back into dollars. Let \mathcal{E}_{t+1} denote the spot exchange rate prevailing in period $t + 1$. Then the $(1 + i_t^*)/\mathcal{E}_t$ euros can be converted into $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$ dollars in $t + 1$. Therefore, in deciding where to invest, the investor would like to compare the return of investing in the United States, $1 + i_t$, to the dollar return of an equivalent investment in Germany, $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$. If $1 + i_t$ is greater than $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$, then it is more profitable to invest in the United States. In fact, in this case, the investor could make unbounded profits by borrowing in Germany and investing in the United States. Similarly, if $1 + i_t$ is less than $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$, the investor could make infinite profits by borrowing in the United States and investing in Germany.

This investment strategy suffers, however, from a fundamental problem. At time t , the investor does not know \mathcal{E}_{t+1} , the exchange rate that will prevail at time $t + 1$. This means that the return associated with investing in the

United States, $1 + i_t$, and the one associated with investing in Germany, $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$, are not directly comparable because the former is known with certainty at the time the investment is made (period t), whereas the latter is uncertain at that time.

Forward exchange markets are designed precisely to allow investors to circumvent the exchange rate risk. The investor can eliminate the exchange rate uncertainty by arranging at the beginning of the investment period, the purchase of the necessary amount of U.S. dollars to be delivered at the end of the investment period for a price determined at the beginning of the period. Such a foreign currency purchase is called a *forward contract*. Let F_t denote the forward rate, that is, the dollar price at time t of 1 euro delivered and paid for at time $t + 1$. Note that when a forward contract is arranged (period t), there is no exchange of money. The exchange of money happens when the forward contract is executed (period $t + 1$). The dollar return of a one-dollar investment in Germany using the forward exchange market is $(1 + i_t^*)F_t/\mathcal{E}_t$. This return is known with certainty at time t making it comparable to the return on the domestic investment, $1 + i_t$.

The difference between the domestic return and the foreign return expressed in domestic currency by use of the forward exchange rate is known as the *covered interest rate differential*:

$$\text{Covered Interest Rate Differential} = (1 + i_t) - (1 + i_t^*)\frac{F_t}{\mathcal{E}_t}. \quad (10.1)$$

This interest rate differential is called covered because the use of the forward exchange rate covers the investor against exchange rate risk. It is also known

as the *cross-currency basis*.

When the covered interest rate differential is zero, we say that *covered interest rate parity* (CIP) holds. In the absence of barriers to capital mobility and for interest rates and forward rates that are free of default risk, a violation of CIP implies the existence of arbitrage opportunities. When an arbitrage opportunity exists there is the possibility of making unbounded profits without taking on any risk.

Consider the following example. Suppose that the annual nominal interest rate in the United States is 7% ($i_t = 0.07$), that the annual nominal interest rate in Germany is 3% ($i_t^* = 0.03$), that the spot exchange rate is \$0.50 per euro ($\mathcal{E}_t = 0.50$), and that the 1-year forward exchange rate is \$0.51 per euro ($F_t = 0.51$). In this case, the forward discount is $F_t/\mathcal{E}_t = 0.51/0.50 = 1.02$, and the covered interest rate differential is $1 + i_t - (1 + i_t^*)F_t/\mathcal{E}_t = 1.07 - 1.03 \times 1.02 = 0.0194$, or 1.94 percent. In the absence of barriers to international capital mobility, this violation of CIP implies that it is possible to make profits by borrowing in Germany, investing in the United States, and buying euros in the forward market to eliminate the exchange rate risk.

To see how one can exploit this arbitrage opportunity, consider the following sequence of trades. (1) borrow 1 euro in Germany. (2) exchange your euro in the spot market for \$0.50. (3) invest the \$0.5 in a U.S. deposit. (4) buy 1.03 euros in the forward market (you will need this amount of euros to repay your euro loan including interest). Note that buying euros in the forward market involves no payment at this point. (5) after 1 year, your U.S. investment yields $1.07 \times \$0.5 = \0.535 . (6) execute your forward

contract, that is, purchase 1.03 euros for $0.51 \times 1.03 = 0.5253$ dollars. Use this amount to repay your German loan. The difference between what you receive in (5) and what you pay in (6) is $\$0.535 - \$0.5253 = \$0.0097 > 0$. Note that this operation involved no exchange-rate risk (because you used the forward market), needed no initial capital, and yielded a pure profit of \$0.0097.

It is clear from this example that for interest rates and forward rates that are free of default risk, the covered interest rate differential should be zero if there are no barriers to international capital flows. Therefore, the existence of covered interest-rate differentials is an indication of lack of free capital mobility.

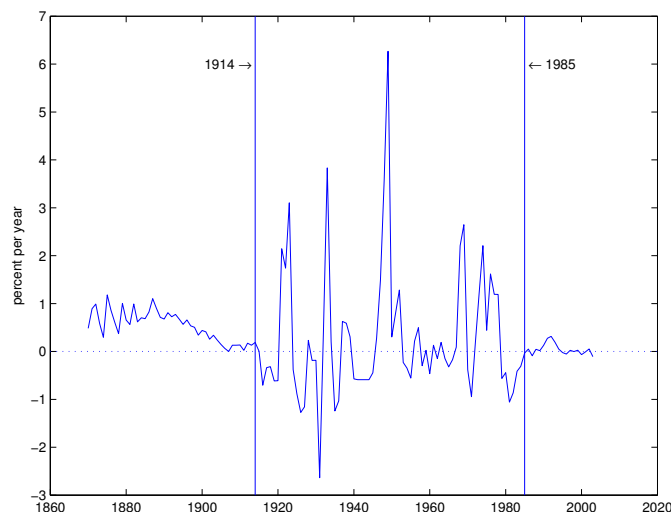
10.2 Empirical Evidence on Covered Interest-Rate Differentials

One can use data on interest rates, spot exchange rates, and forward rates to construct empirical measures of covered interest-rate differentials. This indicator can provide useful information about the evolution of international capital mobility across time. In particular, this type of empirical analysis can be used to address the question of whether the world is more globalized now than in the past, and whether globalization progresses over time or is non-monotone in nature.

Figure 10.1 displays the dollar-sterling covered interest rate differential from 1870 to 2003.¹ Covered interest rate differentials were consistently

¹We thank Alan Taylor for sharing these data. Covered interest-rate differentials before

Figure 10.1: Dollar-Pound Covered Interest Rate Differentials: 1870-2003



Notes. The figure plots annual averages of monthly dollar-pound covered interest rate differentials. Source: Obstfeld, Maurice, and Alan M. Taylor, “Globalization and Capital Markets.” In *Globalization in Historical Perspective*, edited by M. D. Bordo, A. M. Taylor and J. G. Williamson. Chicago: University of Chicago Press, 2003.

small before World War I and after 1985, suggesting a high degree of international capital-market integration during these two subperiods. The two world wars and the Great Depression threw the international financial system into disarray and led to widespread financial regulations that prevented the free flow of capital across national borders. These impediments to international capital mobility remained more or less in place until the mid 1980s, and led to high and volatile covered interest-rate differentials. Low differentials reemerged after the deregulation of financial markets undertaken by the Thatcher and Reagan administrations. A similar pattern emerges when one examines covered interest-rate differentials between the United States and Germany, with low differentials before 1914 and after the mid 1980s and high and volatile differentials in the intervening decades.²

The empirical evidence we have examined suggests that capital market integration as measured by covered interest rate differentials is not a modern phenomenon. Like in the 1990s and 2000s, financial capital flowed in a more or less unfettered fashion before World War I. Furthermore, there are no reasons to believe that the interruptions in capital mobility observed between 1914 and 1985 represent an exceptional event. As we will see shortly, the global financial crisis of 2008 brought about another wave of disruptions and government interventions in financial markets that caused an elevation in covered interest-rate differentials around the developed world, albeit less pronounced than the one that occurred in the mid decades of the twentieth

1921 are constructed using a forward exchange instrument called the long bill of exchange. Exercise 10.4 discusses how this instrument works.

²See Obstfeld, Maurice, and Alan M. Taylor, "Globalization and Capital Markets." In *Globalization in Historical Perspective*, edited by M. D. Bordo, A. M. Taylor and J. G. Williamson. Chicago: University of Chicago Press, 2003.

century.

10.3 Empirical Evidence on Offshore-Onshore Interest Rate Differentials

An alternative way to construct exchange-risk free interest-rate differentials is to use interest rates on instruments denominated in the same currency, for example, the U.S. dollar, issued in financial centers located in different countries.

For example, one can compare the interest rate on dollar time deposits in banks located in New York and London. The interest rate on the domestic instrument is called the onshore rate, and the interest rate on the foreign instrument is called the offshore rate.

Dollar deposits outside of the United States became widespread in the early 1980s. The high inflation rates observed in the 1970s together with the Federal Reserve's regulation Q, which placed a ceiling on the interest rate that U.S. banks could pay on time deposits, led to fast growth of eurocurrency markets. Eurocurrency deposits are foreign currency deposits in a market other than the home market of the currency. For example, a Eurodollar deposit is a dollar deposit outside the United States (e.g., a dollar deposit in London). The interest rate on such deposit is called the Euro dollar rate. A yen deposit at a bank in Singapore is called a Euro yen deposit and the associated interest rate is called the Euro yen rate. The biggest market for Eurocurrency deposits is London.

Letting i_t be the interest rate in period t on a dollar deposit in the United

States and i_t^* the interest rate on a dollar deposit in the foreign country, the offshore-onshore interest rate differential is

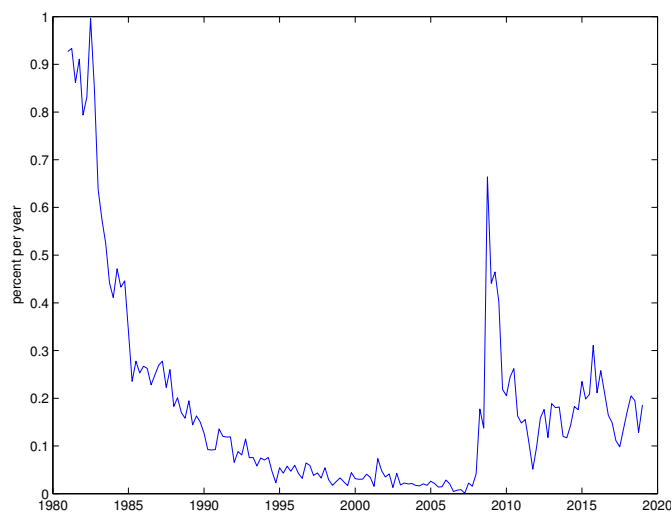
$$\text{offshore-onshore differential} = i_t^* - i_t.$$

The fact that both interest rates are on dollar deposits eliminates the exchange rate risk, thereby making them directly comparable. If both deposits are default-risk free, then, under free capital mobility, the offshore-onshore differential should be zero. Any difference between i_t and i_t^* would create a pure arbitrage opportunity that investors could exploit to make unbounded profits. This means that in the absence of default risk, nonzero offshore-onshore interest-rate differentials are an indication of lack of free capital mobility.

Figure 10.2 plots the three-month U.K.-U.S. offshore-onshore interest rate differential of the U.S. dollar over the period 1981Q1 to 2019Q1. As discussed before, until the mid 1980s, both the United States and the United Kingdom had regulations in place that hindered free international capital mobility. This is reflected in high offshore-onshore differentials during this period. The fact that during this period dollar rates are higher in the United Kingdom than in the United States indicates that investors wanted to borrow in the latter and lend in the former but could not do it to the extent they wished.

Offshore-onshore differentials fall to practically zero (below 10 basis points) by 1990 and remain at that low level until the onset of the Global Financial Crisis in 2008. In 2008, the interest-rate differential spikes briefly

Figure 10.2: Offshore-Onshore Interest Rate Differential of the U.S. Dollar: 1981Q1-2019Q1



Notes. The figure plots the average quarterly offshore-onshore interest rate differential of the U.S. dollar. The offshore rate is the 3-month Euro-Dollar deposit interest rate, Bank of England series: IUQAED3A. The onshore rate is the 3-month certificates of deposit (CD) rate for the United States, OECD MEI series: IR3TIB.

at about 60 basis points and then stabilizes at a lower level but higher than the one that prevailed prior to the crisis. The lack of convergence to pre-crisis levels can be ascribed to the adoption of prudential regulations in the United States and the United Kingdom that prevent banks from fully arbitraging these differentials away. Such regulations included money market reforms that limited participation of U.S. branches of foreign banks in U.S. CD markets, and the imposition of more stringent bank capital requirements in the United States. Although the global financial crisis had a lasting effect on interest rate differentials, deviations from parity are not nearly as large or volatile as those observed between 1914 and 1985.

The lack of convergence in cross-country interest rate differentials post global financial crisis is not limited to the U.S.-U.K. country pair, but is a fairly widespread phenomenon. Lack of convergence of interest rate differentials between, for example, the United States and Canada, the euro area, Japan, Norway, New Zealand, and Sweden have also been documented.³

10.4 Uncovered Interest Rate Parity

A much discussed concept in international finance is that of *uncovered interest rate parity* (UIP). To understand this concept, suppose first that we could see without uncertainty what the nominal exchange rate will be next period. Then, in deciding whether to invest in the domestic market or abroad, we would compare the return of one dollar invested in the domestic

³See Eugenio Cerutti, Maurice Obstfeld, and Haonan Zhou, “Covered Interest Parity Deviations: Macroeconomic Determinants,” IMF Working Paper, WP/19/14, January 2019. This study uses covered interest rate differentials as opposed to offshore-onshore interest rate differentials.

market with the return of one dollar invested in the foreign market. One dollar invested locally pays $1 + i_t$ dollars at the end of the investment period. To invest in the foreign market, we first convert 1 dollar into $1/\mathcal{E}_t$ units of foreign currency using the exchange rate \mathcal{E}_t , and then invest it at the rate i_t^* . This investment yields $(1 + i_t^*)/\mathcal{E}_t$ units of foreign currency in period $t + 1$. Converting this amount into domestic currency yields $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$ dollars. Lack of arbitrage opportunities would require that $1 + i_t$ equal $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$. However, as we discussed in section 10.1, because in period t we do not observe the nominal exchange rate that will occur in $t + 1$, it is impossible to compare the two returns. Consequently, $1 + i_t$ will in general not be equal to $(1 + i_t^*)\mathcal{E}_{t+1}/\mathcal{E}_t$ even if agents could freely arbitrage across countries.

But one might intuitively think that the two returns should be equal to each other on average. Letting E_t denote the expectations operator conditional on information available in period t , we would then have that

$$1 + i_t = (1 + i_t^*)E_t \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right). \quad (10.2)$$

This condition is known as uncovered interest rate parity. The difference between the left-hand side and the right-hand side of this equation is known as the *uncovered interest rate differential*,

$$\text{uncovered interest rate differential} = 1 + i_t - (1 + i_t^*)E_t \left(\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} \right).$$

In this section we show that the intuition that UIP should hold is not sup-

ported by theory or data. This suggests that observing sizable uncovered interest rate differentials is not an indication of the existence of impediments to free capital mobility across countries.

10.4.1 Asset Pricing in an Open Economy

Consider a small open endowment economy with free capital mobility. As in chapter 6, assume that there is no uncertainty in period 1, but that there is uncertainty about period 2. Specifically, in period 2, there are two states of the world, the good state, denoted g , which occurs with probability π , and the bad state, denoted b , which occurs with probability $1 - \pi$.

Let B_1 denote domestic currency bonds purchased in period 1. These bonds pay the nominal interest rate i when held from period 1 to period 2. Suppose that the foreign nominal interest rate is i^* . Let B_1^* denote foreign-currency-denominated bonds purchased by the domestic household in period 1 for which the household buys forward cover. That is, in period 1 the household enters into a contract that allows it to convert in period 2 $(1 + i^*)B_1^*$ units of foreign currency into domestic currency at the forward exchange rate, F_1 . Let \tilde{B}_1^* denote the quantity of foreign currency bonds the domestic household acquires in period 1 for which it does not acquire forward cover and hence is exposed to exchange rate risk.

Let \mathcal{E}_t denote the nominal exchange rate in period $t = 1, 2$, defined as the domestic-currency price of one unit of foreign currency in period t . We can then express the period-1 budget constraint of the domestic household as

$$P_1 C_1 + B_1 + \mathcal{E}_1 B_1^* + \mathcal{E}_1 \tilde{B}_1^* = P_1 Q_1, \quad (10.3)$$

where P_1 denotes the domestic price level in period 1 and Q_1 denotes the endowment of goods in period 1. Here we have assumed that the household entered period 1 without any asset holdings, $B_0 = B_0^* = \tilde{B}_0^* = 0$.

The budget constraint in the good state in period 2 is

$$P_2^g C_2^g = P_2^g Q_2^g + (1+i)B_1 + F_1(1+i^*)B_1^* + \mathcal{E}_2^g(1+i^*)\tilde{B}_1^*, \quad (10.4)$$

where P_2^g denotes the price level in the good state in period 2, C_2^g denotes the level of consumption in the good state in period 2, Q_2^g denotes the endowment in the good state in period 2, and \mathcal{E}_2^g denotes the exchange rate in the good state in period 2. Similarly, the budget constraint in the bad state in period 2 is given by

$$P_2^b C_2^b = P_2^b Q_2^b + (1+i)B_1 + F_1(1+i^*)B_1^* + \mathcal{E}_2^b(1+i^*)\tilde{B}_1^*, \quad (10.5)$$

with a notation analogous to the one for the good state.

Assume that the household's expected utility function is

$$U(C_1) + \pi U(C_2^g) + (1-\pi)U(C_2^b), \quad (10.6)$$

where $U(\cdot)$ is an increasing and concave period utility function.

The household's utility maximization problem consists in choosing C_1 , C_2^g , C_2^b , B_1 , B_1^* , and \tilde{B}_1^* to maximize (10.6) subject to the budget constraints (10.3), (10.4), and (10.5), taking as given P_1 , P_2^g , P_2^b , \mathcal{E}_1 , \mathcal{E}_2^g , \mathcal{E}_2^b , F_1 , i , i^* , Q_1 , Q_2^g , and Q_2^b . To obtain the optimality conditions associated with this problem, first solve the period-1 budget constraint for C_1 , the period-2 good-

state budget constraint for C_2^g , and the period-2 bad-state budget constraint for C_2^b . This yields

$$C_1(B_1, B_1^*, \tilde{B}_1^*) = \frac{P_1 Q_1 - B_1 - \mathcal{E}_1 B_1^* - \mathcal{E}_1 \tilde{B}_1^*}{P_1},$$

$$C_2^g(B_1, B_1^*, \tilde{B}_1^*) = \frac{P_2^g Q_2^g + (1+i)B_1 + (1+i^*)(F_1 B_1^* + \mathcal{E}_2^g B_1^*)}{P_2^g},$$

and

$$C_2^b(B_1, B_1^*, \tilde{B}_1^*) = \frac{P_2^b Q_2^b + (1+i)B_1 + (1+i^*)(F_1 B_1^* + \mathcal{E}_2^b B_1^*)}{P_2^b}.$$

Then use these three expressions to eliminate consumption from the utility function to obtain

$$U(C_1(B_1, B_1^*, \tilde{B}_1^*)) + \pi U(C_2^g(B_1, B_1^*, \tilde{B}_1^*)) + (1-\pi)U(C_2^b(B_1, B_1^*, \tilde{B}_1^*)).$$

The first-order condition with respect to B_1 is

$$U'(C_1) \frac{1}{P_1} = \pi U'(C_2^g) \frac{1+i}{P_2^g} + (1-\pi) U'(C_2^b) \frac{1+i}{P_2^b}.$$

The left-hand-side of this expression indicates the marginal utility of one unit of domestic currency in period 1: One unit of domestic currency buys $1/P_1$ units of goods, each of which provides $U'(C_1)$ units of utility. The right-hand-side also indicates the marginal utility of one unit of currency in period 1, but in this case, the one unit of currency is used to buy a domestic bond. This investment returns $(1+i)$ units of currency in period

2 regardless of the state of the economy. In the good state, this buys $1/P_2^g$ units of goods, which each provide $U'(C_2^g)$ units of utility and in the bad state this buys $1/P_2^b$ units of goods, which each provide $U'(C_2^b)$ units of utility. The right-hand side thus gives the expected marginal utility of investing one unit of domestic currency in period 1 in the domestic bond and consuming the proceeds in period 2. At the optimum, the expected utility of investing one unit of domestic currency in the domestic bond (the right-hand side) or converting it into consumption goods already in period 1 (the left-hand side) must generate the same level of utility. This optimality condition is known as the Euler equation for domestic bonds. Rewrite this Euler equation as

$$1 = (1 + i) \left[\pi \frac{U'(C_2^g)}{U'(C_1)} \frac{P_1}{P_2^g} + (1 - \pi) \frac{U'(C_2^b)}{U'(C_1)} \frac{P_1}{P_2^b} \right].$$

Notice that the expression within square brackets is an expected value. Thus, using the expectations operator E_1 , we can write

$$1 = (1 + i) E_1 \left\{ \frac{U'(C_2)}{U'(C_1)} \frac{P_1}{P_2} \right\}.$$

The object

$$M_2 \equiv \left\{ \frac{U'(C_2)}{U'(C_1)} \frac{P_1}{P_2} \right\}$$

is known as the household's pricing kernel. It is the ratio of the marginal utility of one unit of domestic currency in period 2, $U'(C_2)/P_2$, to the marginal utility of one unit of domestic currency in period 1, $U'(C_1)/P_1$. It is a pricing kernel because multiplying any nominal payment in a given state of the world in period 2 by M_2 returns the period-1 value of such payment. Using

the pricing kernel, the Euler equation for domestic bonds becomes

$$1 = (1 + i)E_1 \{M_2\}. \quad (10.7)$$

The first-order condition with respect to foreign bonds for which the household buys forward cover, B_1^* , is

$$U'(C_1)\frac{\mathcal{E}_1}{P_1} = \pi(1 + i^*)U'(C_2^g)\frac{F_1}{P_2^g} + (1 - \pi)(1 + i^*)U'(C_2^b)\frac{F_1}{P_2^b}.$$

The left-hand side of this expression is the utility of using one unit of foreign currency to purchase consumption goods in period 1. The right hand side is the expected utility of using one unit of foreign currency to purchase foreign bonds with forward cover, and spending the proceeds on consumption in period 2. This optimality condition is known as the Euler equation for foreign bonds. As we did with the Euler equation for domestic bonds, we use the pricing kernel to write the Euler equation for foreign bonds as

$$1 = (1 + i^*)\frac{F_1}{\mathcal{E}_1}E_1 \{M_2\}. \quad (10.8)$$

Finally, the household's optimality condition with respect to bonds purchased without forward cover, \tilde{B}_1^* , is

$$U'(C_1)\frac{\mathcal{E}_1}{P_1} = \pi(1 + i^*)U'(C_2^g)\frac{\mathcal{E}_2^g}{P_2^g} + (1 - \pi)(1 + i^*)U'(C_2^b)\frac{\mathcal{E}_2^b}{P_2^b}.$$

The left-hand side of this optimality condition is the utility of one unit of foreign currency used for consumption in period 1. The right-hand side is

the expected utility of investing one unit of foreign currency in the foreign bond without forward cover. Note that the domestic currency return of this investment depends on the realization of the nominal exchange rate in period 2. In other words, this investment has exposure to exchange-rate risk. Using the pricing kernel M_2 , we can rewrite this condition as

$$1 = (1 + i^*) E_1 \left\{ \left(\frac{\mathcal{E}_2}{\mathcal{E}_1} \right) M_2 \right\}. \quad (10.9)$$

10.4.2 CIP as an Equilibrium Condition

Combining the Euler equations for domestic and foreign bonds, equations (10.7) and (10.8), we obtain

$$(1 + i) = (1 + i^*) \frac{F_1}{\mathcal{E}_1}, \quad (10.10)$$

which is the CIP condition discussed in section 10.1. This result shows that CIP is not only a no arbitrage condition, as we argued in section 10.1, but also an equilibrium condition.

10.4.3 Is UIP an Equilibrium Condition?

Can we derive the UIP condition given in equation (10.2) as an equilibrium condition of the present model? As it turns out, the answer to this question is no. Optimization on the part of households implies that in general the rate of return on domestic bonds will not be equal to the expected exchange-rate adjusted rate of return on foreign bonds.

To see this, let's begin by comparing the UIP condition, equation (10.2),

to the CIP condition, equation (10.10). It is clear from this comparison that UIP holds if and only if

$$F_1 = E_1 \mathcal{E}_2, \quad (10.11)$$

that is, if and only if the forward rate, F_1 , is equal to the expected future exchange rate, $E_1 \mathcal{E}_2$.

The question of whether UIP holds in equilibrium then boils down to the question of whether the forward rate equals the expected future exchange rate. To answer this question, combine optimality conditions (10.8) and (10.9) to obtain

$$F_1 E_1 \{M_2\} = E_1 \{\mathcal{E}_2 M_2\}.$$

This expression implies that in general the forward rate, F_1 , will not equal the expected future exchange rate, $E_1 \mathcal{E}_2$,

$$F_1 \neq E_1 \mathcal{E}_2.$$

This establishes that under free capital mobility, UIP in general fails to hold. Formally, we have that, in general,

$$1 + i \neq (1 + i^*) E_1 \left(\frac{\mathcal{E}_2}{\mathcal{E}_1} \right).$$

Put differently, observing deviations from UIP in the data is not necessarily an indication of lack of free capital mobility.

Although UIP does not hold in general, it does hold in the special case in which the pricing kernel is uncorrelated with the exchange rate. To see

this, recall that for any pair of random variables a and b , their covariance conditional on information available in period 1, denoted $\text{cov}_1(a, b)$, is given by $\text{cov}_1(a, b) = E_1(ab) - E_1(a)E_1(b)$. We then can express (10.9) as

$$1 = (1 + i^*) \left[\text{cov}_1 \left(\frac{\mathcal{E}_2}{\mathcal{E}_1}, M_2 \right) + E_1 \left(\frac{\mathcal{E}_2}{\mathcal{E}_1} \right) E_1(M_2) \right].$$

If the depreciation rate, $\mathcal{E}_2/\mathcal{E}_1$, is uncorrelated with the pricing kernel, M_2 , that is, if $\text{cov}_1 \left(\frac{\mathcal{E}_2}{\mathcal{E}_1}, M_2 \right) = 0$, then equation (10.9) becomes

$$1 = (1 + i^*) E_1 \left(\frac{\mathcal{E}_2}{\mathcal{E}_1} \right) E_1(M_2).$$

Combining this expression with optimality condition (10.7), we have

$$(1 + i) = (1 + i^*) E_1 \left\{ \frac{\mathcal{E}_2}{\mathcal{E}_1} \right\},$$

which is the UIP condition. We have therefore shown that while UIP need not hold in general, it does obtain under the special case in which the pricing kernel is uncorrelated with the depreciation rate of the domestic currency.

10.4.4 Carry Trade as a Test of UIP

Suppose that UIP holds, that is, that $1 + i_t = (1 + i_t^*) E_t[\mathcal{E}_{t+1}/\mathcal{E}_t]$. It is clear from this expression that if $i_t > i_t^*$, then $E_t[\mathcal{E}_{t+1}/\mathcal{E}_t] > 1$. In words, if UIP holds, then the high interest rate currency must be expected to depreciate. This means that one should not be able to make systematic profits from borrowing at the low interest rate and lending at the high interest-rate, since exchange rate movements would exactly offset the interest rate differential

on average. Yet, this trading strategy, known as *carry trade* is widely used by practitioners, suggesting that it does indeed yield positive payoffs on average.

Empirical studies confirm that carry trade does yield positive profits on average. The payoff from carry trade is given by

$$\text{payoff from carry trade} = (1 + i_t) - (1 + i_t^*) \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t},$$

where i_t is the high interest rate currency, that is, $i_t > i_t^*$. Burnside, Eichenbaum, Kleshchelski, and Rebelo (2006) document returns to carry trade for the pound sterling against 10 currencies using monthly data covering the period 1976:1 to 2005:12.⁴ They find that the average payoffs from carry trade are positive but low, 0.0029 for one pound invested for one month. This means that to generate substantial profits, carry traders must wager large sums of money. For example, suppose a trader invests one billion pounds in carry trade, then after one month the carry trade has a payoff of 2.9 million pounds on average. The fact that the average payoff from carry trade is non-zero means that on average the uncovered interest rate differential is not zero and that UIP fails.

Carry trade does not seem to be a more risky investment than other investments such as the stock market. A commonly used measure of the risk adjusted return is the *Sharpe Ratio*, which is defined as the ratio of the

⁴See, Burnside, Craig, Martin Eichenbaum, Isaac Kleshchelski, and Sergio Rebelo, "The Returns to Currency Speculation," NBER Working paper 12489, August 2006. The study considers the payoff from carry trade between the British pound and the currencies of Belgium, Canada, France, Germany, Italy, Japan, the Netherlands, Switzerland, the United States, and the euro area.

average payoff divided by the standard deviation of the payoff, that is,

$$\text{Sharpe ratio} = \frac{\text{mean}(\text{payoff})}{\text{std}(\text{payoff})}.$$

The lower is the Sharpe ratio, the lower is the risk-adjusted return of the investment. Burnside et al. report a Sharpe ratio for carry trades that is relatively high, 0.145. This figure is similar to the Sharpe ratio of 0.14 corresponding to investing in the S&P 500 index over the same period.

Like the stock market, carry trade is subject to crash risk. Crashes in carry trade are the result of sudden large movements in exchange rates. For example, on October 6-8, 1998 there was a large surprise appreciation of the Japanese Yen against the U.S. dollar. The Yen appreciated by 14 percent (or equivalently the U.S. dollar depreciated by 14 percent). Suppose that you were a carry trader with 1 billion dollars short in Yen and long in U.S. dollars. The payoff of that carry trade in the span of 2 days was -140 million dollars. Because of this crash risk and because of its low payoff relative to the large gross positions it requires, The Economist magazine has likened carry trade to “picking up nickels in front of steamrollers.”⁵

10.4.5 The Forward Premium Puzzle

When a foreign currency is ‘more expensive’ in the forward market than in the spot market, that is, when

$$F_t > \mathcal{E}_t,$$

⁵See, “Carry on speculating,” Economic Focus, The Economist, February 24, 2007, page 90.

we say that the foreign currency is at a *premium in the forward market*, or, equivalently, that the domestic currency is at a *discount in the forward market*.

We have already established that conditional on CIP holding, UIP holds if and only if the forward rate equals the expected future spot exchange rate, that is, if and only if $F_t = E_t \mathcal{E}_{t+1}$ (see equation (10.11)). Dividing both sides by \mathcal{E}_t and rearranging, we have that

$$E_t \frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = \frac{F_t}{\mathcal{E}_t},$$

which says that conditional on CIP holding, UIP holds if and only if the domestic currency is expected to depreciate when the foreign currency trades at a premium in the forward market.

We saw in section 10.2 that CIP holds reasonably well in the data. Therefore, the above expression represents a testable implication of UIP. Consider estimating the following equation by ordinary least squares (OLS):

$$\frac{\mathcal{E}_{t+1}}{\mathcal{E}_t} = a + b \frac{F_t}{\mathcal{E}_t} + \mu_{t+1},$$

where a and b are the regression coefficients and μ_{t+1} is a regression residual. Under UIP, then the estimation should yield $a = 0$ and $b = 1$. This result, however, is strongly rejected in the data. For example, Burnside (2018) estimates this regression for the U.S. dollar against the G10 currencies using monthly observations over the period 1976:1 to 2018:3.⁶ He reports cross-

⁶See, Craig Burnside, “Exchange Rates, Interest Parity, and the Carry Trade,” Draft for publication in the Oxford Research Encyclopedia of Economics and Finance, June 2018.

country average estimates of a and b of 0.00055 and -0.75, respectively. For most countries (7 out of 10), the null hypothesis that $a = 0$ and $b = 1$ is rejected at high significance levels of 1 percent or less. This result is known as the *forward premium puzzle*.

Like the evidence on returns to carry trade analyzed in section 10.4.4, the forward premium puzzle indicates that UIP is strongly rejected by the data.

10.5 Real Interest Rate Parity

A natural question is whether free capital mobility creates a tendency for real interest rates to equalize across countries. The purpose of this section is to show that the answer of this question is no, except under special circumstances. To illustrate one such special case, consider an economy that produces and trades a single good, say apples. Let the domestic real interest rate be denoted r and the foreign real interest rate r^* . Then, if $r > r^*$, a household could borrow X apples in period t at the rate r^* in the foreign country and ship them to the home country. In the home country the household could lend out these X apples at the interest rate r . Next period the household collects $(1 + r)X$ apples on its loan. The household then ships $(1 + r^*)X$ apples abroad to repay the loan. The remainder $[(1 + r) - (1 + r^*)]X = (r - r^*)X > 0$ apples is pure profit. This investment strategy required no initial capital, did not involve any risk, and yielded a profit of $(r - r^*)X$ apples. Thus it represents a pure arbitrage opportunity

The countries included in the analysis are Australia, Canada, Denmark, Germany/euro area, Japan, New Zealand, Norway, Sweden, Switzerland, United Kingdom.

that market participants would exploit until the real interest rate differential, $r - r^*$, is nil. In this economy, therefore, free capital mobility leads to real interest rate parity. Deviations from real interest rate parity would indicate the existence of impediments to the movement of capital across borders.

In a more realistic economy, however, in which there is more than one good and in which purchasing power parity does not hold, non-zero real interest rate differentials need not imply the absence of free capital mobility. To derive this result, consider a two-period open economy with two assets. Assume that in period 1 households have access to a domestic real bond, denoted b_1 that pays $(1 + r)b_1$ units of the domestic consumption basket in period 2. The second asset is a foreign real bond, denoted b_1^* that pays $(1 + r^*)b_1^*$ units of the foreign consumption basket in period 2.

Let

$$e_t = \frac{\mathcal{E}_t P_t^*}{P_t}$$

denote the real exchange rate in period $t = 1, 2$, where P_t is the nominal price of a domestic basket of goods, P_t^* is the nominal price of a foreign basket of goods expressed in foreign currency, and \mathcal{E}_t is the nominal exchange rate, defined as the domestic-currency price of one unit of foreign currency. Think of this economy as one in which there are many goods. These goods form baskets. The household derives utility from the consumption of baskets of goods and is endowed with a number of such baskets each period. In the rest of the world, the basket of goods might have a different composition or might even contain different items. In addition, there might be tariffs

or other barriers to international trade creating differences in goods prices across countries. For all these reasons, the price of a basket of goods might be different in the domestic economy and in the rest of the world. As we saw in chapter 8, the real exchange rate, e_t , is the relative price of one foreign basket of goods in terms of domestic baskets of goods.

The budget constraint of the household in period 1 is

$$C_1 + b_1 + e_1 b_1^* = Q_1,$$

where C_1 denotes domestic consumption in period 1, and Q_1 denotes the endowment in period 1, both expressed in units of the domestic consumption basket. The remaining terms in the budget constraint are also expressed in units of domestic baskets. In particular, since the foreign real bond, b_1^* , is denominated in units of foreign baskets of goods, we have that $e_1 b_1^*$ expresses the cost of purchasing b_1^* units of the foreign bond in terms of domestic baskets of goods.

The household's budget constraint in period 2 is given by

$$C_2 = Q_2 + (1 + r)b_1 + (1 + r^*)e_2 b_1^*,$$

where C_2 denotes consumption in period 2, Q_2 denotes the endowment in period 2, and e_2 denotes the real exchange rate in period 2.

The household's utility function is assumed to be given by

$$U(C_1) + U(C_2),$$

where $U(\cdot)$ is an increasing and concave period utility function. The household chooses C_1 , C_2 , b_1 , and b_1^* to maximize this utility function subject to the budget constraints in periods 1 and 2. To derive the optimality conditions associated with this problem, solve the period-1 budget constraint for C_1 and the period-2 budget constraint for C_2 and use the resulting expressions to eliminate consumption from the utility function. This yields

$$U(Q_1 - b_1 - e_1 b_1^*) + U(Q_2 + (1 + r)b_1 + (1 + r^*)b_1^*).$$

Taking derivatives with respect to b_1 and b_1^* and equating them to zero, we obtain the following Euler equations associated with the optimal choice of domestic and foreign real bonds:

$$U'(C_1) = (1 + r)U'(C_2)$$

and

$$U'(C_1) = (1 + r^*)U'(C_2)\frac{e_2}{e_1}.$$

The left-hand side of the first Euler equation is the increase in utility of consuming one additional basket of goods in period 1. The right-hand side give the increase in utility if the household saves one basket of goods in domestic bonds. This investment yields $1 + r$ baskets of goods in period 2, each of which produces $U'(C_2)$ units of utility. At the optimum the household must be indifferent between consuming and saving the marginal basket of goods. Likewise, second Euler equation equates the utility derived from consuming an extra basket of goods in period 1 with the utility of

saving the basket in foreign bonds. One basket of goods in period 1 buys $1/e_1$ baskets of foreign goods. Invested in the foreign bond, this basket becomes $(1+r^*)/e_1$ baskets of foreign goods in period 2. In turn, with these baskets of foreign goods, the household can purchase $(1+r^*)e_2/e_1$ baskets of domestic goods in period 2.

Combining the two Euler equations we obtain

$$1+r = (1+r^*)\frac{e_2}{e_1}. \quad (10.12)$$

This expression says that r will in general not equal r^* . Even though the economy has free capital mobility, the real interest rate differential is in general different from zero. Instead, the (gross) domestic real interest rate, $1+r$, equals the (gross) foreign real interest rate adjusted by the real depreciation of the domestic currency, e_2/e_1 . Intuitively, if e_2/e_1 is bigger than 1, the foreign basket is becoming more expensive over time. So one domestic basket invested in the foreign bond yields more than $1+r^*$ units of domestic baskets in period 2. We conclude that observing nonzero real interest-rate differentials need not be indicative of restrictions to capital mobility.

This conclusion is empirically quite relevant. In section 8.4 of chapter 8, we documented that there are large deviations from relative PPP in the short run. This means that e_t/e_{t-1} changes significantly from one quarter to the next. These movements in the real exchange rate create a wedge between the domestic and the foreign real interest rates. Thus, tests of free capital mobility based real interest rate differentials should be interpreted with caution, especially during periods of large and frequent movements in

real exchange rates.

For simplicity, in this section we abstract from uncertainty. With uncertainty about the state of the world in period 2, equation (10.12) changes, as it incorporates the effect of expected comovements between the real exchange rate and consumption. However, the result that deviations from real interest rate parity are not indicative of lack of capital mobility continues to hold.⁷

10.6 Saving-Investment Correlations

In 1980 Martin Feldstein (1939-2019) and Charles Horioka published a provoking empirical paper documenting that national saving rates are highly correlated with investment rates.⁸ They examined data on average investment-to-GDP and saving-to-GDP ratios from 16 industrial countries over the period 1960-74. The data used in their study is shown in figure 10.3.

Feldstein and Horioka estimate by OLS the following linear relation between investment and saving rates:

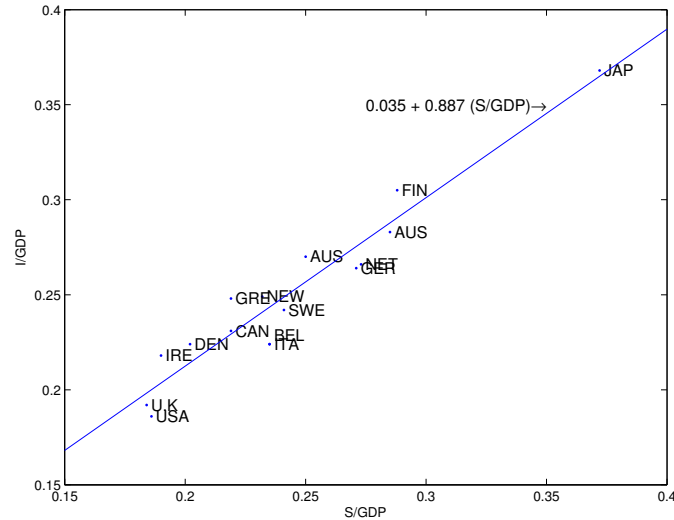
$$\left(\frac{I}{GDP}\right)_i = 0.035 + 0.887 \left(\frac{S}{GDP}\right)_i + \nu_i; \quad R^2 = 0.91,$$

where $(I/GDP)_i$ and $(S/GDP)_i$ denote, respectively, the average investment-to-GDP and saving-to-GDP ratios in country i over the period 1960-1974. Figure 10.3 shows the fitted relationship as a solid line. Feldstein and Ho-

⁷Exercise 10.6 asks you to characterize an economy like the one discussed in this section augmented with uncertainty.

⁸M. Feldstein and C. Horioka, "Domestic Saving and International Capital Flows," *Economic Journal* 90, June 1980, 314-29.

Figure 10.3: Saving and Investment Rates for 16 Industrialized Countries, 1960-1974 Averages

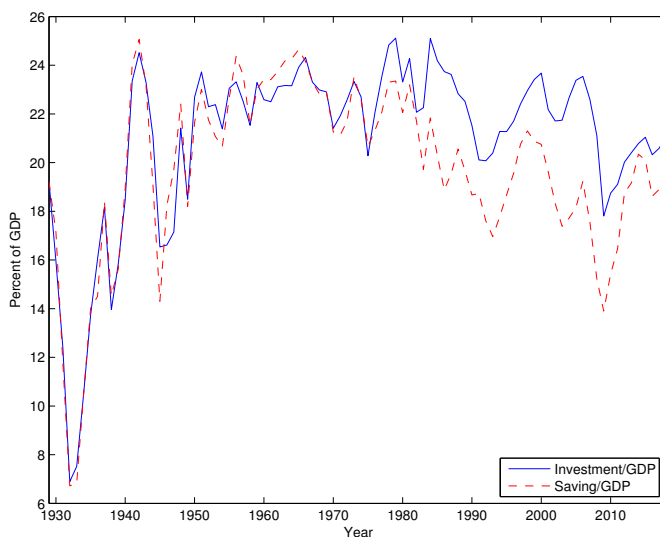


Source: M. Feldstein and C. Horioka, “Domestic Saving and International Capital Flows,” *Economic Journal* 90, June 1980, 314-29.

rioka use data on 16 OECD countries, so that their regression was based on 16 observations. The high value of the coefficient on S/GDP of 0.887 means that there is almost a one-to-one positive association between average saving rates and average investment rates. The reported R^2 statistic of 0.91 means that the estimated equation fits the data quite well, as 91 percent of the cross-country variation in I/GDP is explained by variations in S/GDP .

A positive relationship between saving and investment rates is observed not only across countries but also across time. For example, figure 10.4 shows the U.S. saving and investment rates from 1929 to 2018. The two series move closely together over time, although the comovement has weakened somewhat since the emergence of large current account deficits in the 1980s.

Figure 10.4: U.S. National Saving and Investment Rates, 1929-2018



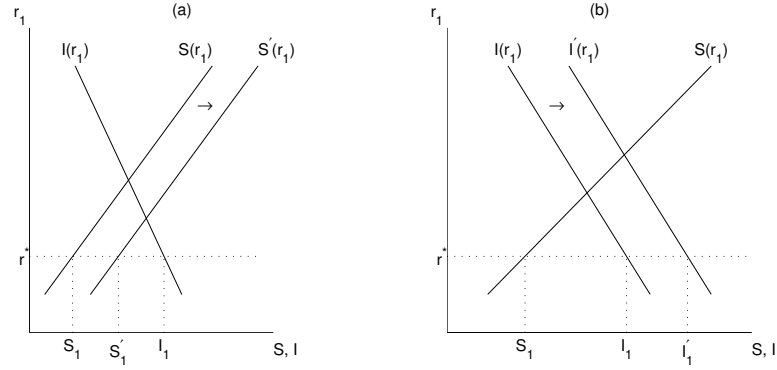
Data Source: Bureau of Economic Analysis, NIPA Table 1.1.5 and Table 5.1.

Feldstein and Horioka argued that if capital was highly mobile across countries, then the correlation between saving and investment should be close to zero, and therefore interpreted their findings as evidence of low capital mobility. The reason why Feldstein and Horioka arrived at this conclusion can be seen by considering the identity,

$$CA = S - I,$$

where CA denotes the current account balance, S denotes national saving, and I denotes investment. In a closed economy—i.e., in an economy without capital mobility—the current account is always zero, so that $S = I$ and changes in national saving are perfectly correlated with changes in investment. On the other hand, in a small open economy with perfect capital

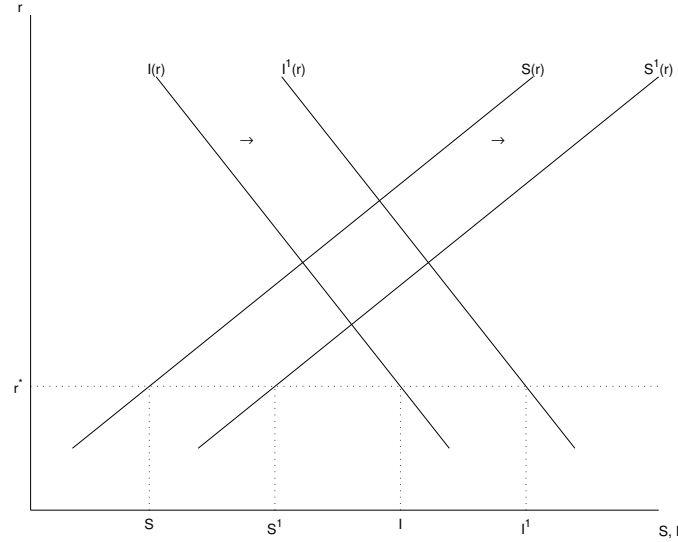
Figure 10.5: Response of S and I to independent shifts in (a) the savings schedule and (b) the investment schedule



mobility, the interest rate is exogenously given by the world interest rate, so that, if the saving and investment schedules are affected by independent factors, then the correlation between saving and investment will be zero. Figure 10.5 illustrates this point. Events that change only the saving schedule will result in changes in the equilibrium level of saving but will not affect the equilibrium level of investment (see panel (a)). Similarly, events that affect only the investment schedule will result in changes in the equilibrium level of investment but will not affect the equilibrium level of national savings (panel (b)).

But do the Feldstein-Horioka findings of high savings-investment correlations really imply imperfect capital mobility? Feldstein and Horioka's interpretation has been criticized on at least two grounds. First, even under perfect capital mobility, a positive association between savings and investment may arise because the same events might shift the savings and investment schedules. For example, suppose that, in a small open economy, the production functions in periods 1 and 2 are given by $Q_1 = A_1 F(I_0)$ and

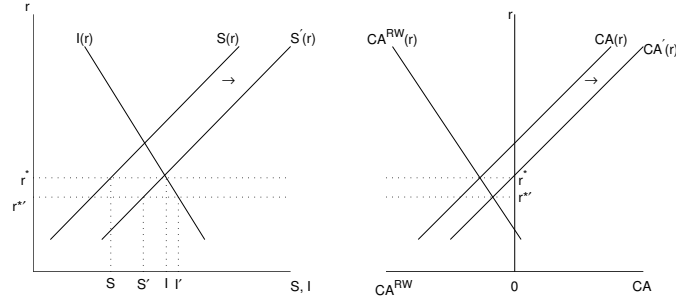
$Q_2 = A_2 F(I_1)$, respectively. Here Q_1 and Q_2 denote output in periods 1 and 2. The variable I_t , for $t = 0, 1$, denotes physical capital (like machines and structures) invested in period t that becomes productive in $t + 1$. The function $F(\cdot)$ represents the production technology and is assumed to be increasing and concave, so the higher is the capital input the higher is output. And A_1 and A_2 are exogenous efficiency parameters capturing the state of technology, the effects of weather on the productivity of capital, and so forth. Consider a persistent productivity shock. Specifically, assume that A_1 and A_2 increase and that A_1 increases by more than A_2 . This situation is illustrated in figure 10.6, where the initial situation is one in which the savings schedule is given by $S(r)$ and the investment schedule by $I(r)$. At the world interest rate r^* , the equilibrium levels of savings and investment are given by S and I . In response to the expected increase in A_2 , firms are induced to increase next period's capital stock, I_1 , to take advantage of the expected rise in productivity. Thus, I_1 goes up for every level of the interest rate. This implies that in response to the increase in A_2 , the investment schedule shifts to the right to $I^1(r)$. At the same time, the increase in A_2 produces a positive wealth effect which induces households to increase consumption and reduce savings in period 1. As a result, the increase in A_2 shifts the savings schedule to the left. Now consider the effect of the increase in A_1 . This should have no effect on desired investment because the capital stock in period 1, I_0 , is predetermined. However, the increase in A_1 produces an increase in output in period 1. Consumption-smoothing households will want to save part of the increase in Q_1 . Therefore, the effect of an increase in A_1 is a rightward shift in the savings schedule. Because we

Figure 10.6: Response of S and I to a persistent productivity shock

assumed that A_1 increases by more than A_2 , on net the savings schedule is likely to shift to the right. In the figure, the new savings schedule is given by $S^1(r)$. Because the economy is small, the interest rate is unaffected by the changes in A_1 and A_2 . Thus, both savings and investment increase to S^1 and I^1 , respectively. Thus, in this economy we would see that saving and investment are positively correlated even though the economy has free capital mobility.

A second reason why saving and investment may be positively correlated in spite of free capital mobility is the presence of large country effects. Consider, for example, an event that affects only the saving schedule in a large open economy like the one represented in figure 10.7. In response to a shock that shifts the saving schedule to the right from $S(r)$ to $S'(r)$ the current account schedule also shifts to the right from $CA(r)$ to $CA'(r)$. As a result,

Figure 10.7: Large open economy: response of S and I to a shift in the savings schedule



the world interest rate falls from r^* to $r^{*'}$. The fall in the interest rate leads to an increase in investment from I to I' . Thus, in a large open economy, a shock that affects only the saving schedule results in positive comovement between saving and investment.

We conclude that observing a positive correlation between saving and investment is not necessarily an indication of lack of capital mobility.

10.7 Summing Up

- The forward exchange rate, F_t , is the domestic currency price of one unit of foreign currency to be delivered and paid for in a future period.
- The forward discount is the ratio of the forward exchange rate to the spot exchange rate, F_t/\mathcal{E}_t . When the forward discount is greater than one, we say that the foreign currency trades at a premium and the domestic currency at a discount in the forward market.
- Covered interest rate parity (CIP) says that the domestic interest rate, i_t , must equal the foreign interest rate, i_t^* , adjusted for the forward discount,

$$1 + i_t = (1 + i_t^*)F_t/\mathcal{E}_t.$$

- The covered interest rate differential is equal to $1 + i_t - (1 + i_t^*)F_t/\mathcal{E}_t$.
- The cross-currency basis is the same as the covered interest rate differential.

• Under free capital mobility, absent default risk, covered interest rate differentials should be near zero.

• The offshore-onshore interest rate differential is the difference between the domestic nominal interest and the foreign nominal interest on domestic currency denominated assets.

• Under free capital mobility, absent default risk, offshore-onshore interest rate differentials should be near zero.

• Based on observed cross-country interest rate differentials, the developed world displayed a high degree of capital mobility between 1870 and 1914 and again after 1985. The period 1914-1985 was characterized by large disruptions in international capital market integration. This suggests that capital market integration is not a monotonic process.

• As a consequence of new financial regulations, covered interest rate differentials have displayed a slight elevation since the global financial crisis of 2008.

• Uncovered interest rate parity (UIP) says that the domestic interest rate must equal the foreign interest rate adjusted for expected depreciation,

$$1 + i_t = (1 + i_t^*)E_t\mathcal{E}_{t+1}/\mathcal{E}_t.$$

- UIP is in general not implied by an equilibrium asset pricing model.
- UIP is strongly rejected by the data.
- Deviations from real interest rate parity can arise even under free

capital mobility.

- Observing a positive correlation between saving and investment is not necessarily an indication of lack of capital mobility.

10.8 Exercises

Exercise 10.1 (TFU) Indicate whether the statement is true, false, or uncertain and explain why.

1. If there is free capital mobility between the United States and Germany, then dollar deposits in New York and Frankfurt should have the same interest rate.
2. If uncovered interest rate parity holds, then returns to carry trade must be zero not only on average but period by period.
3. The interest rate in Japan is 0 percent and the interest rate in the United States is 1.75 percent. There is clearly an arbitrage opportunity, as one can become infinitely rich without taking any risk by borrowing in yen and investing in dollars.
4. If the dollar is selling at a discount in the forward market, $F_t/\mathcal{E}_t < 1$, we should expect the dollar to depreciate, $E_t\mathcal{E}_{t+1}/\mathcal{E}_t > 1$, where \mathcal{E}_t denotes the spot exchange rate (dollar price of foreign currency), F_t is the forward exchange rate, and E_t is the expectations operator conditional on information available to speculators in period t .

Exercise 10.2 (Returns to Carry Trade) Suppose in month t the annual nominal interest rate is $i_t = 0.02$ in the United States and $i_t^* = 0.05$ in Germany. Suppose further that in month t a speculator invests 400 million dollars in carry trade for one month. Let \mathcal{E}_t be the nominal exchange rate in t , defined as the dollar price of one euro. Suppose that between t and

$t + 1$ the dollar appreciates by 2.8 percent. Did the speculator gain or lose and by how much? Express your answer in dollars.

Exercise 10.3 (Interest-Rate Differentials) Suppose that the euro-dollar spot exchange rate, \mathcal{E}_t , is 1.5 dollars per euro, that the forward exchange rate, F_t , is 2 dollars per euro, that the nominal interest rate in the United States is 3 percent, and the nominal interest rate on euro deposits in Frankfurt is 1 percent. Assume further that with probability 0.5 \mathcal{E}_{t+1} is 2 and with equal probability it equals 1.

1. Calculate the covered interest-rate differential.
2. Calculate the uncovered interest-rate differential.
3. Calculate the forward discount.
4. Suppose a carry trade investor decides to invest 1 million dollars. How much money would she make or lose under each of the two possible realizations of \mathcal{E}_{t+1} ?

Exercise 10.4 (The Long Bill of Exchange) As mentioned in the body of the chapter, prior to 1920 the covered interest rate differentials shown in figure 10.1 are constructed using data on an instrument called the long bill of exchange. The long bill consists in purchasing with domestic currency in the current period one unit of foreign currency to be delivered 90 days later. Contrary to a forward contract, in the long bill the buyer of foreign currency must make a domestic currency payment at the beginning of the investment period. Let b_t denote the long bill rate, which is defined as the

date- t dollar price in New York of £1 deliverable in London after ninety days. (Note that b_t is paid 90 days prior to the date of delivery of the £.) Let i_t^* denote the 90-day deposit rate in London, i_t the 90-day deposit rate in New York, and \mathcal{E}_t the spot exchange rate, that is, the dollar price of one British pound. Suppose you had time series data for b_t , \mathcal{E}_t , i_t and i_t^* . How can you construct a test of free capital mobility between the United States and Great Britain.

Exercise 10.5 (UIP Regression) Let $y_{t+1} = \mathcal{E}_{t+1}/\mathcal{E}_t$ and $x_t = F_t/\mathcal{E}_t$, where \mathcal{E}_t denotes the nominal exchange rate in period t and F_t denotes the forward rate in period t . Consider estimating via OLS

$$y_{t+1} = a + bx_t + \mu_{t+1}.$$

Show that if UIP holds, then it must be the case that $a = 0$ and $b = 1$.

Exercise 10.6 (Real Interest Rate Parity and Uncertainty) Consider the two-period small open endowment economy studied in section 10.4.1 and introduce two additional assets. Specifically, assume that households have access to a domestic risk-free real bond, denoted b_1 , that pays in period 2 $(1+r)b_1$ units of the domestic consumption basket. The second real asset is a foreign risk-free real bond, denoted b_1^* . The foreign real bond is denominated in units of the foreign consumption basket. It pays $(1+r^*)b_1^*$ units of the foreign consumption basket in period 2 when held from period 1 to period 2. Let $c_2^j = \mathcal{E}_2^j P_2^{*j}/P_2^j$ denote the real exchange rate in period 2 in state j for $j = g, b$. Assume that free capital mobility holds.

1. State the household's budget constraints in period 1, in the good state in period 2, and in the bad state in period 2, assuming (for simplicity) that the only assets the household has access to are real domestic and real foreign bonds.
2. Find the household's Euler equations associated with the optimal choice of b_1 and b_1^* , respectively.
3. Suppose that $cov(C_2, e_2/e_1) \neq 0$. Does real interest rate parity hold under free capital mobility?
4. Assume now that the real depreciation rate is uncorrelated with period-2 consumption, that is, assume that $cov(C_2, e_2/e_1) = 0$. Does real interest rate parity hold under free capital mobility?

Chapter 11

Capital Controls

In chapter 10, we saw that over the past 120 years international capital markets have experienced periods in which capital flowed fairly freely across countries as well as periods with significant deviations from free capital mobility. In particular, in the United States, the period starting with World War I and ending in the mid 1980s displayed low degrees of international capital market integration as a result of a number of government policies that impeded the free flow of financial capital across borders. We also documented that after a period of more than two decades of low restrictions to international capital flows, the global financial crisis of 2008 triggered new government regulations of financial markets that resulted in the reemergence of cross-country covered interest rate differentials, albeit small. Capital controls are also widespread in emerging countries. For example, in section 11.1 of this chapter, we document how Brazil's imposition of capital controls aimed at reducing the large inflows of capital stemming from low-interest-rate regions (especially United States and Europe) during the

global financial crisis, caused a significant elevation in covered interest rate differentials.

In the present chapter, we characterize the effects of capital controls on the current account, consumption, and welfare in the context of the open-economy model developed in earlier chapters. We consider controls that take the form of a tax on international financial transactions or a limit on the level of international borrowing or lending. We show that capital controls can be an effective tool to reduce current account deficits but that absent any distortions they are welfare decreasing. Thus, in these environments free capital mobility is optimal. We then study cases in which imperfections in financial markets, including externalities and market power, can provide a rationale for welfare improving capital control taxes. In these environments, free capital mobility ceases to be optimal.

11.1 Capital Controls and Interest Rate Differentials

Capital controls are restrictions imposed by a government on the flow of financial capital into or out of the country. Capital controls can take the form of quantitative limits to the amount of funds that can flow through the border or of a tax on international capital flows. The imposition of capital controls gives rise to interest rate differentials that cannot be arbitrated away.

Suppose, for example, that the country is borrowing from the rest of the world and that initially there are no capital controls. Let i be the domestic

interest rate on dollar loans (the onshore rate) and i^* the foreign interest rate on dollar loans (the offshore rate). Clearly, i cannot be lower than i^* , because if this were the case, no one would borrow internationally, since it is cheaper to borrow domestically, contradicting the assumption that the country is borrowing from the rest of the world. Also, the domestic interest rate cannot be higher than the offshore interest rate, because in this case anybody could become infinitely rich by borrowing internationally at the rate i^* and lending domestically at the rate i . Thus, the onshore interest rate must equal the offshore interest rate,

$$i = i^*.$$

Suppose now that the government imposes a tax τ per dollar borrowed internationally. Suppose further that the tax is not large enough to completely discourage international borrowing, so that the country continues to borrow from abroad in spite of the capital control tax. Now the cost of borrowing one dollar internationally is $i^* + \tau$. If the domestic banks were to offer dollar loans at a rate lower than $i^* + \tau$, then no one would borrow internationally, contradicting the assumption that the country continues to borrow from the rest of the world after the imposition of the capital control. The domestic interest rate cannot be higher than $i^* + \tau$ either, because in this case an arbitrage opportunity would arise, consisting in borrowing internationally at the cost $i^* + \tau$ and lending domestically at the rate i . So the domestic interest rate must equal the sum of the foreign interest rate and the capital

control tax rate,

$$i = i^* + \tau.$$

This establishes that when capital controls are imposed on capital inflows, that is, on international borrowing, the domestic interest rate will be higher than the foreign interest rate,

$$\text{Controls on capital inflows} \Rightarrow i > i^*.$$

The resulting interest rate differential, $i - i^*$, equals the capital control tax rate, τ . The larger the capital control tax rate is, the larger the interest rate differential will be.

When the capital control tax is imposed on capital outflows, that is, on international lending, it also creates an interest rate differential, but in the opposite direction. To see this, suppose that the country is lending to the rest of the world and that the government imposes a tax τ per unit lent internationally. In this case, the after-tax rate of return on lending one dollar abroad is $i^* - \tau$. By the same logic given in the analysis of controls on capital inflows, the domestic interest rate must equal the after-tax rate of return of lending abroad, otherwise arbitrage opportunities would arise, allowing agents to become infinitely rich. So we have that

$$i = i^* - \tau.$$

As in the case of controls on capital inflows, the imposition of controls on capital outflows creates an interest rate differential. However, unlike the

case of controls on capital inflows, controls on capital outflows cause the domestic interest rate to be lower than the world interest rate,

$$\text{Controls on capital outflows} \Rightarrow i < i^*.$$

The analysis in this section shows that the introduction of capital controls creates a wedge between the domestic and the foreign interest rate. This distortion in financial markets will in general affect the real side of the economy, because key macroeconomic indicators, such as consumption, saving, investment, and the current account depend on interest rates.

Capital Controls in Brazil: 2009-2012

In the wake of the global financial crisis of 2008 interest rates in the United States and many other developed countries fell to near zero. In response to such low rates, global investors who were looking for higher yields started sending funds to emerging market economies where interest rates were higher. One country that was a recipient of large inflows was Brazil. The Brazilian authorities were concerned that these capital inflows would destabilize their economy and enacted taxes on capital inflows. Specifically, between October 2009 and March 2012 Brazil imposed more than 10 major capital control taxes. The measures included taxes on portfolio equity inflows, taxes on fixed income inflows, and unremunerated reserve requirements. After March 2012 those restrictions were gradually removed.

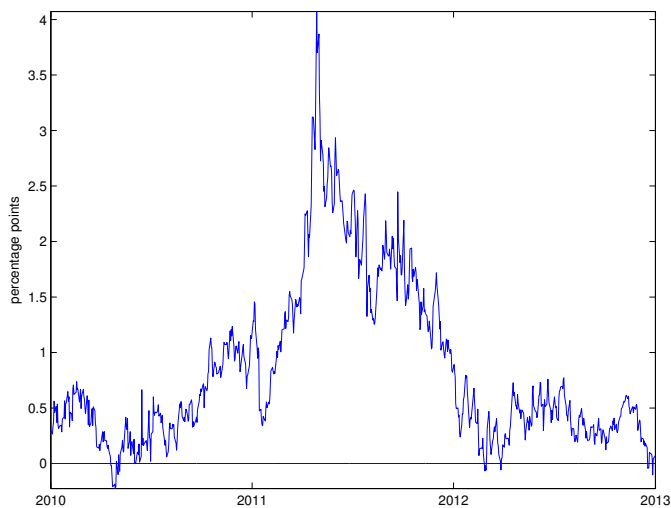
When capital inflow taxes are imposed on a specific asset or class of assets there is always the concern that market participants can find a way to

circumvent them. One way to see if in this instance the capital control taxes were effective is to look at the covered interest rate differential between the Brazilian real and the U.S. dollar. Let i_t denote the 360-day interest rate in Brazil on domestic currency deposits (reais), \mathcal{E}_t the spot exchange rate (that is, the reais price of one U.S. dollar), F_t the 360-day forward exchange rate of U.S. dollars, and i_t^* the 360-day U.S. dollar Libor rate. Then the covered interest rate differential between dollar deposits inside Brazil and outside of Brazil is given by $(1 + i_t) \frac{\mathcal{E}_t}{F_t} - (1 + i_t^*)$. The first term of this expression is called the cupom cambial, i_t^{cupom} , that is, $1 + i_t^{cupom} = (1 + i_t) \frac{\mathcal{E}_t}{F_t}$. We can then express the covered interest rate differential as

$$\text{covered interest rate differential} = i_t^{cupom} - i_t^*.$$

Figure 11.1 plots daily data for the covered interest rate differential for the period January 1, 2010 to December 31, 2012. If the inflow controls were successful, we should see that dollar interest rates inside Brazil became higher than in London, that is, that the covered interest rate differential increased. The figure shows that the covered interest rate differential was around 50 basis points until the fall of 2010. This means that the capital control measures enacted until then, which targeted mainly portfolio equity investment, were not effective in restricting arbitrage between the cupom cambial and the Libor rate. However, starting in the fall of 2010 as the government intensified capital controls, the differential starts rising and reaches a peak of 4 percentage points by April 2011, after an inflow tax of 6 percent on borrowing from abroad with maturities of less than 2 years was imposed. The

Figure 11.1: Brazilian Real-U.S. Dollar Covered Interest Rate Differentials: 2010-2012



Notes. The figure plots daily real-dollar covered interest rate differentials computed as the spread between the cupom cambial and the U.S. dollar Libor rate for the period January 1, 2010 to December 31, 2012. Data Source: Marcos Chamon and Márcio Garcia, ‘Capital Controls in Brazil: Effective?’, *Journal of International Money and Finance* 61, 2016, 163-187. We thank the authors for sharing the data.

size of the covered interest rate differentials suggest that the latter capital inflow taxes were indeed effective in the sense that they prevented interest rate equalization. By early 2012 arbitragers seem to have found ways to bypass the capital control tax as differentials return to normal levels of around 0.5 percentage points, and by June 2012 the capital control tax of 6 percent of borrowing from abroad with maturities of less than 2 years was removed.

11.2 Macroeconomic Effects of Capital Controls

Current account deficits are often viewed as bad for a country. The idea behind this view is that by running a current account deficit, the economy is living beyond its means. By accumulating external debt, the argument goes, the country imposes future economic hardship on itself in the form of reduced consumption and investment spending when the external debt becomes due. A policy recommendation sometimes offered to countries undergoing external imbalances is the imposition of capital controls, which can take the form of taxes on international capital flows or quotas on external borrowing.

11.2.1 Effects of Capital Controls on Consumption, Savings, and the Current Account

Consider a two-period economy populated by a large number of households with preferences over consumption described by the utility function

$$U(C_1) + U(C_2),$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively, and $U(\cdot)$ is an increasing and concave period utility function. Suppose that households start period 1 without any debts or assets. In period 1, they receive an endowment of Q_1 units of goods. In addition, they can borrow or lend at the interest rate i . Then the budget constraint in period 1 is given by

$$P_1 C_1 + B_1 = P_1 Q_1,$$

where P_1 denotes the price of the good in period 1 and B_1 denotes the number of bonds purchased in period 1.

In period 2, the household receives an endowment of Q_2 units of goods and a transfer from the government equal to T units of goods. In addition, in period 2 the household receives the principle and interest on its period-1 savings. Because the world ends in period 2, there is no borrowing or lending in this period. Consequently, the budget constraint in period 2 is given by

$$P_2 C_2 = P_2 Q_2 + P_2 T + (1 + i) B_1,$$

where P_2 denotes the price of the good in period 2. For simplicity, assume that the price of goods is equal to 1 in both periods, that is, $P_1 = P_2 = 1$. Combining the period-1 and period-2 budget constraints to eliminate B_1 yields the household's intertemporal budget constraint

$$C_2 = Q_2 + T + (Q_1 - C_1)(1 + i). \quad (11.1)$$

This expression says that in period 2 the household can consume its period-2

nonfinancial, disposable income, $Q_2 + T$, plus its period-1 savings including interest, $(Q_1 - C_1)(1 + i)$.

Using the period-2 budget constraint to eliminate C_2 from the utility function, we obtain

$$U(C_1) + U(Q_2 + T + (Q_1 - C_1)(1 + i)).$$

The household chooses C_1 to maximize this expression, taking Q_1 , Q_2 , T , and i as given. Taking the derivative of this expression with respect to C_1 and equating it to zero, we obtain the first-order optimality condition

$$U'(C_1) = (1 + i)U'(Q_2 + T + (Q_1 - C_1)(1 + i)).$$

Noticing that $Q_2 + T + (Q_1 - C_1)(1 + i)$ is C_2 , we can write the first-order condition as

$$\frac{U'(C_1)}{U'(C_2)} = 1 + i. \quad (11.2)$$

This is the familiar consumption Euler equation. It says that if the interest rate increases, the household has an incentive to reduce current consumption relative to future consumption. This follows from the fact that $U'(\cdot)$ is a decreasing function.

Suppose that the government imposes a capital control tax on international borrowing. Let $\tau > 0$ be the capital control tax rate. Then, by the analysis of section 11.1, we have that if the economy is borrowing in period 1, then

$$i = i^* + \tau,$$

where i^* is the world interest rate. Using the Euler equation (11.2) to eliminate i from the household's optimality condition, we obtain

$$\frac{U'(C_1)}{U'(C_2)} = 1 + i^* + \tau. \quad (11.3)$$

This expression says that a capital control tax on international borrowing creates an incentive to reduce current consumption in favor of future consumption.

Let's assume that the government returns the proceeds of the capital control tax to households. Tax revenue equals $-\tau B_1$. To understand why this expression is preceded by a minus sign, recall that if the country is borrowing, then asset holdings, B_1 , are negative, and tax revenue is positive. The amount of transfers received by each household is

$$T = -\tau B_1. \quad (11.4)$$

The household takes the transfer T as exogenously given. That is, the household does not internalize that the more capital control taxes it pays the larger its transfer will be. This type of transfer is called a lump-sum transfer. The idea is that in this economy there are many households, all of which pay capital control taxes. The government then divides total tax receipts equally among all households. So an individual household receives the same transfer regardless of how much capital control taxes it paid. Because all households are identical, it happens that in equilibrium transfers are exactly equal to the amount of taxes the household paid.

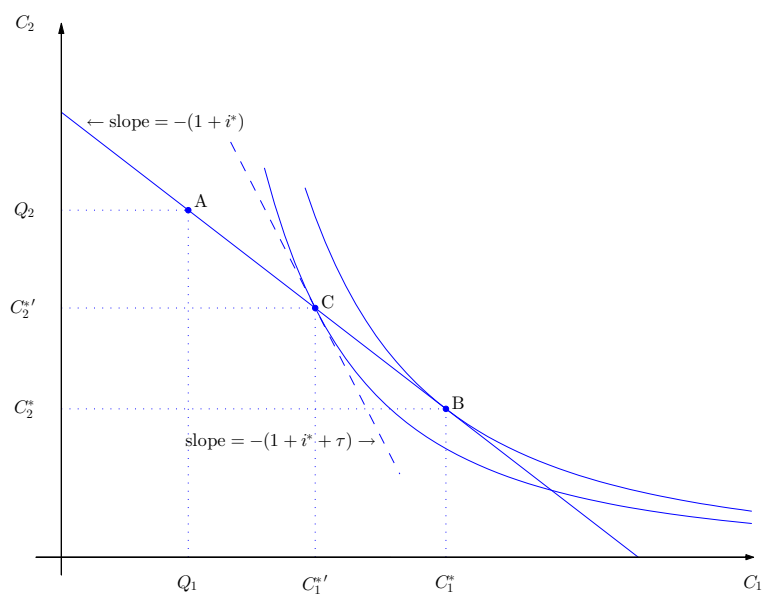
Using equation (11.4) to eliminate T from the intertemporal budget constraint (11.1), we obtain the economy's intertemporal resource constraint

$$C_2 = Q_2 + (Q_1 - C_1)(1 + i^*). \quad (11.5)$$

Note that the economy's intertemporal resource constraint is independent of the capital control tax rate, τ . This is because the government's tax revenue is returned to households, so no resources are lost as a consequence of the imposition of capital controls. This does not mean, however, that capital controls have no macroeconomic consequences. As can be seen from the optimality condition (11.3), capital controls distort the intertemporal allocation of consumption.

Figure 11.2 depicts the equilibrium effects of imposing the capital control tax τ . The downward sloping straight line is the economy's intertemporal resource constraint, given by equation (11.5). The slope of this line is $-(1 + i^*)$. It reflects the fact that if the country sacrifices one unit of consumption in period 1, in period 2 it can consume $1 + i^*$ additional units of consumption. Point A represents the endowment path, (Q_1, Q_2) . Point B represents the optimal consumption path in the absence of capital controls, $\tau = 0$. At point B, the indifference curve is tangent to the intertemporal resource constraint. In particular, the slope of the indifference curve is $-(1 + i^*)$, which equals the slope of the intertemporal resource constraint. In this example, when capital controls are zero, the economy runs a trade deficit equal to $C_1^* - Q_1 > 0$. Because the initial asset position is assumed to be nil, $B_0 = 0$, both the current account and external debt equal the trade balance. This means that

Figure 11.2: Equilibrium With and Without Capital Controls



Notes. Point A represents the endowment path, point B the equilibrium consumption path in the absence of capital controls, and point C the equilibrium consumption path with capital controls. The capital control tax on international borrowing causes a fall in period-1 consumption, an improvement in the trade balance and the current account, and a reduction in net external debt. The equilibrium under capital controls yields a lower level of welfare than the equilibrium without capital controls.

in period 1 the economy runs a current account deficit and becomes a net debtor to the rest of the world. In period 2, the household chooses a level of consumption, C_2^* , lower than its period-2 endowment, Q_2 , to allow for the repayment of the debt contracted in period 1 plus the corresponding interest.

Point C in figure 11.2 represents the equilibrium when the government imposes a capital control tax τ per unit borrowed internationally. The economy's resource constraint is unchanged. However, individually, households perceive an increase in the cost of borrowing from i^* to $i^* + \tau$. As a result, they reduce borrowing and move to a consumption bundle containing less consumption in period 1 and more in period 2 (see optimality condition (11.3)). In the figure, in response to the imposition of the capital control tax τ , consumption in period 1 falls from C_1^* to $C_1^{*'}$ and the trade and current account deficits shrink from $C_1^* - Q_1$ to $C_1^{*'} - Q_1$. Note that the slope of the indifference curve that crosses point C is steeper than the slope of the indifference curve that crosses point B. The difference in slopes is given by the capital control tax, τ .

In sum, the imposition of a capital control tax on international borrowing discourages current consumption and causes a reduction in the trade deficit, a reduction in the current account deficit, and a reduction in the country's net external debt. Next, we analyze how capital controls affect investment in physical capital.

11.2.2 Effects of Capital Controls on Investment

Suppose that in the economy we have been studying thus far there are firms that borrow in period 1 to buy physical capital, which they use in period 2 to produce goods. The setup is identical to the one introduced in chapter 5. To keep the present analysis self contained, we go over its main elements.

Specifically, suppose that output in period 2 is given by

$$Q_2 = \sqrt{I_1},$$

where I_1 is the stock of capital available in period 2. To build this stock of capital, firms invest in period 1. The investment process requires funding, which firms procure by tapping the financial market. Accordingly, in period 1 firms borrow the amount I_1 at the interest rate i . In period 2, firms must pay back these loans, including interest. Thus, profits in period 2 are given by

$$\sqrt{I_1} - (1 + i)I_1.$$

The firm chooses I_1 to maximize profits. The optimality condition associated with this maximization problem is the derivative of profits with respect to I_1 equated to zero, which slightly rearranged yields,

$$\frac{1}{2\sqrt{I_1}} = 1 + i.$$

Solving for I_1 , we obtain the optimal level of investment

$$I_1 = \left(\frac{1}{2(1 + i)} \right)^2,$$

which says that investment in physical capital is a decreasing function of the interest rate.

Recalling that we are considering an economy that borrows internationally and in which the government imposes capital controls on inflows, we have that in equilibrium the domestic interest rate must equal the world interest rate plus the capital control tax,

$$i = i^* + \tau.$$

Combining the above two expressions, we obtain

$$I_1 = \left(\frac{1}{2(1 + i^* + \tau)} \right)^2.$$

This expression says that the imposition of capital controls on inflows has a negative effect on investment, as it increases the cost of financing the purchase of physical capital. We conclude that capital control taxes distort not only the consumption-saving choice of households but also the investment choice of firms.

11.2.3 Welfare Consequences of Capital Controls

We have established that capital controls can be an effective tool to reduce external debt and current account deficits. We now ask whether these effects are desirable. To answer this question, note that the equilibrium under capital controls, given by point C in figure 11.2, is on an indifference curve located closer to the origin than the indifference curve associated with the

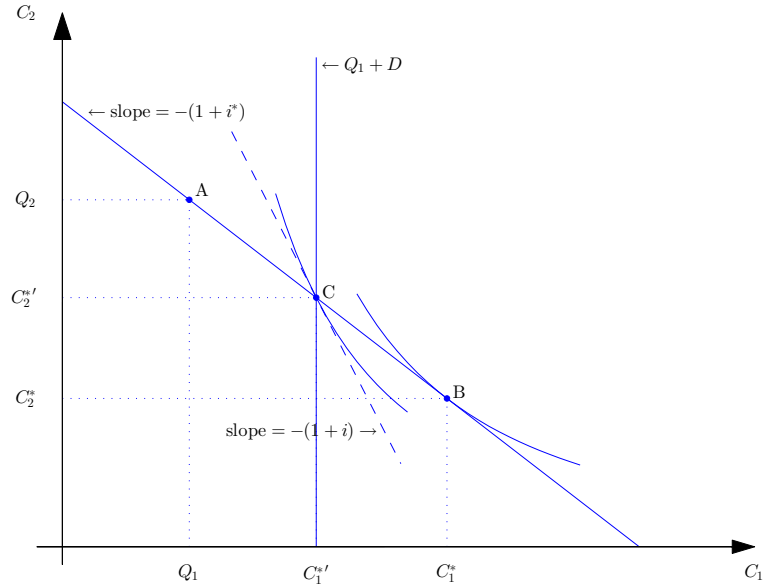
equilibrium without capital controls, point B. This means that the capital control tax is welfare decreasing. It follows that if the government wants to maximize the household's happiness, the best it can do is to set the capital control tax rate to zero. In other words, the optimal capital control tax is zero, or, equivalently, free capital mobility is optimal.

The reason why capital controls are welfare decreasing in this economy is that they create a distortion in the domestic financial market by introducing a wedge between the true cost of borrowing for the economy, given by i^* , and the perceived cost of borrowing at the individual level, $i^* + \tau > i^*$. Households borrow less than is socially optimal because the capital control tax makes them feel that the cost of borrowing is higher than it really is. In section 11.5, we will consider a large economy that has market power in world financial markets. In this environment, capital controls might be welfare increasing, as they can be used to manipulate the world interest rate in the country's favor. Before addressing this issue, however, we will consider the macroeconomic effects of capital controls that take the form of limits on the total amount of borrowing a country can undertake.

11.3 Quantitative Restrictions on Capital Flows

Capital controls can also take the form of quantitative restrictions on international borrowing and lending. In this section, we show that this form of capital controls is equivalent to those based on taxes on international flows. Suppose that the government imposes a cap on the total amount of external debt the country is allowed to have. The situation is depicted in figure 11.3.

Figure 11.3: Equilibrium under quantitative capital controls



Notes. The equilibrium under free capital mobility is at point B. At this point, period-1 consumption exceeds the endowment, and the economy borrows from the rest of the world. Quantitative capital controls forbid borrowing more than D , pushing households to consume $Q_1 + D$ in period 1, point C. The domestic interest rate under capital controls is given by the slope of the indifference curve at point C and is higher than the world interest rate i^* .

The endowment point is represented by point A and the optimal consumption path in the absence of quantitative restrictions is given by point B. The figure is drawn under the assumption that the economy starts period 1 with a zero asset position ($B_0 = 0$). In the unconstrained equilibrium, point B, households optimally choose to borrow from the rest of the world in period 1 in order to finance a level of consumption, C_1^* , that exceeds their period-1 endowment, Q_1 . As a result, in period 1 the trade balance, $Q_1 - C_1^*$, the current account, $Q_1 - C_1^*$, and the net foreign asset position, $B_1 = Q_1 - C_1^*$, are all negative.

Assume now that the government prohibits international borrowing beyond the amount D . That is, the policymaker imposes financial restriction

$$B_1 \geq -D.$$

As a result of this borrowing limit, consumption in period 1 can be at most as large as $Q_1 + D$. It is clear from figure 11.3 that any point on the intertemporal budget constraint containing less period-1 consumption than at point C (i.e., any point on the intertemporal budget constraint located northwest of C) is less preferred than point C itself. This means that when quantitative capital controls are imposed, households choose point C, and the borrowing constraint is binding, $B_1 = -D$. In the constrained equilibrium, in period 1 the household consumes the endowment plus the maximum amount of borrowing allowed, D , so $C_1^{*'} = Q_1 + D$. In period 2, the household consumes its endowment, Q_2 , net of debt obligations including interest, $(1+i^*)D$, that is, $C_2 = Q_2 - (1+i^*)D$. So we have that in response to the quantitative restrictions on capital inflows, current consumption falls from C_1^* to $C_1^{*'}$, the trade balance and the current account shrink from $Q_1 - C_1^*$ to $Q_1 - C_1^{*'} = -D$, and external debt falls from $C_1^* - Q_1$ to $C_1^{*'} - Q_1 = D$.

In the absence of quantitative restrictions, the domestic interest rate, i , equals the world interest rate i^* . Upon the imposition of the binding borrowing limit D , the domestic interest rate increases above the world interest rate. To see this, note that at the world interest rate, domestic households would like to borrow more than D . But capital controls make international

funds unavailable beyond this limit. Thus, the domestic interest rate must rise above the world interest rate to bring about equilibrium in the domestic financial market. Graphically, $1 + i$ is given by the negative of the slope of the indifference curve at point C in figure 11.3, indicated by the negative of the slope of the broken line. Only at that interest rate are households willing to consume exactly $Q_1 + D$ in period 1.

Formally, the domestic interest rate under quantitative capital controls is given by optimality condition (11.2) evaluated at the equilibrium levels of consumption, $C_1^{*'} = Q_1 + D$ and $C_2^{*'} = Q_2 - (1 + i^*)D$. That is, the domestic interest rate satisfies

$$\frac{U'(Q_1 + D)}{U'(Q_2 - (1 + i^*)D)} = 1 + i.$$

Because Q_1 , Q_2 , D , and i^* are exogenously given, this expression represents one equation in one unknown, i . Further, because $U'(\cdot)$ is a decreasing function, if D decreases, that is, if the government tightens quantitative capital controls, the numerator of the left-hand side goes up and the denominator goes down. Consequently, the more stringent capital controls are, the higher the domestic interest rate will be. We therefore have that the interest rate differential, $i - i^*$ is an increasing function of the severity of quantitative restrictions on capital inflows.

Comparing figures 11.2 and 11.3, it is clear that quantity-based and tax-based capital controls give rise to the same equilibrium, in the sense that given a capital control tax τ one can find a quantitative restriction D , such that in equilibrium, consumption, the trade balance, the current account,

the stock of external debt, and the interest rate differential are the same under both capital-control policies.

This equivalence result depends on how the quantitative restrictions are implemented. Suppose, for example, that the government allocates the entire quota of external debt to one bank. All households must channel their own borrowing through this bank. The owner of the bank earns a rent equal to $(i - i^*)$ on each domestic loan. In this case the quantitative restriction has distributional effects against households and in favor of the owner of the bank, which causes the equivalence result to break down. This type of arrangement is common in countries run by kleptocrats, where policy is contaminated by corruption and rent seeking. Alternatively, the quantitative restrictions on capital inflows could be implemented by allocating a quota of D to each household. In this case, the rent $(i - i^*)D$ is distributed equally across all households. This rent is identical to the tax rebate $T = \tau D = (i - i^*)D$ in the tax-based capital control policy studied in section 11.2. Under this modality the equivalence result obtains.

A market-based way to implement quantitative capital controls while preserving the equivalence with the tax-based form is to auction the quota D . The market price of this quota is $(i - i^*)D$, the pure rent it generates. In this case, even if all the quotas are sold to a single bank, the buyer makes zero profits. If the government distributes the proceeds of the auction to households in an egalitarian and lump-sum fashion, the resulting equilibrium is identical to the one in which each individual household is allotted a borrowing quota of D .

11.4 Borrowing Externalities and Optimal Capital Controls

The main message of sections 11.2 and 11.3 is that capital controls can be an effective instrument to curb external imbalances, but if the economy is small and has well functioning markets, they are welfare decreasing. In this section, we study an economy identical to the one considered in those two sections, except for an imperfection in its financial market. Specifically, in the present economy, foreign lenders charge an interest rate that is increasing in the country's external debt.

The assumption that the interest rate that the country faces in international markets is debt elastic is empirically plausible, especially for emerging economies. Perhaps the most compelling explanation of why international creditors charge a higher interest rate to more indebted countries is that as the level of debt grows, the probability of default increases. So a higher rate of interest is required to make the expected return of investing in the emerging country similar to that of safer alternatives. In chapter 15, we will analyze default and how default affects interest rate and other economic indicators. For now, however, we simply take it as given that the country interest rate is an increasing function of net external debt.

The debt-elastic interest rate creates an externality in the country's market for international funds. The reason is that individual households, being atomistic participants in financial markets, take the country's external debt as exogenously determined. In particular, they don't internalize the fact that their individual borrowing decisions collectively determine the level of

the interest rate. As a result, in equilibrium the economy borrows more than is socially optimal. Under these circumstances, the government has an incentive to impose capital controls as a way to make households internalize the fact that their borrowing drives the interest rate up. We show that there is an optimal level of capital controls that reduces external borrowing and is welfare increasing. We now proceed to formally establish these results.

11.4.1 An economy with a debt-elastic interest rate

Consider a two-period open economy populated by households with preferences for consumption described by the utility function

$$U(C_1) + U(C_2).$$

The household starts period 1 with no debts or assets and receives endowments of goods in the amounts Q_1 and Q_2 in periods 1 and 2, respectively. In period 1, the household can borrow or lend at the domestic interest rate i via a bond, denoted B_1 . The household's budget constraints in periods 1 and 2 are then given by

$$B_1 = Q_1 - C_1,$$

and

$$C_2 = Q_2 + (1 + i)B_1.$$

Combining these two period budget constraints to eliminate B_1 yields the familiar intertemporal budget constraint

$$C_2 = Q_2 + (1 + i)(Q_1 - C_1). \quad (11.6)$$

Using this expression to eliminate C_2 from the utility function, we obtain

$$U(C_1) + U(Q_2 + (1 + i)(Q_1 - C_1)).$$

The household chooses C_1 to maximize this function. Taking derivative with respect to C_1 and equating it to zero, we obtain the optimality condition

$$U'(C_1) - U'(Q_2 + (1 + i)(Q_1 - C_1))(1 + i) = 0.$$

Using the fact that $Q_2 + (1 + i)(Q_1 - C_1) = C_2$ and rearranging, we obtain the Euler equation

$$\frac{U'(C_1)}{U'(C_2)} = 1 + i. \quad (11.7)$$

Suppose that the country has free capital mobility. Let i^* be the interest rate charged by foreign lenders to the country in international capital markets. Then, free capital mobility implies that the onshore and offshore interest rates, i and i^* , must be equal to each other, or

$$i = i^*. \quad (11.8)$$

Suppose further that the interest rate at which the country can borrow in

international markets, i^* , is an increasing function of the external debt per capita. From the period-1 budget constraint, we have that the debt of the individual household is given by $-B_1 = C_1 - Q_1$. Let \bar{Q}_1 and \bar{C}_1 denote cross-sectional averages of output and consumption in period 1. Then the per capita level of debt in period 1 is given by $\bar{C}_1 - \bar{Q}_1$. Thus, we are assuming that i^* is an increasing function of $\bar{C}_1 - \bar{Q}_1$. Given the endowment, the higher is consumption per capita, the higher the level of debt per capita will be. In turn, the higher is the per capita debt level, the higher the interest rate will be. So we can write the debt-elastic interest rate charged by foreign lenders to the country as

$$i^* = I(\bar{C}_1),$$

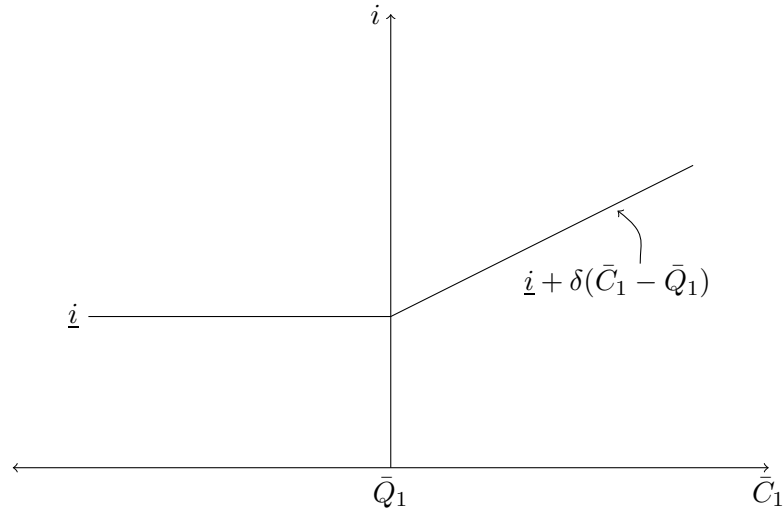
where $I(\cdot)$ is a weakly increasing function. To economize notation, we omit \bar{Q}_1 in the argument of this function. This is not a problem for the present analysis, because \bar{Q}_1 is an exogenous variable, which we will keep constant throughout. Using the fact that by free capital mobility the domestic interest rate, i , equals the interest rate the country faces in world capital markets, i^* , we can write this expression as

$$i = I(\bar{C}_1). \quad (11.9)$$

As an example of a debt-elastic interest rate, consider the function

$$i = \begin{cases} \bar{i} & \text{for } \bar{C}_1 \leq \bar{Q}_1 \\ \bar{i} + \delta(\bar{C}_1 - \bar{Q}_1) & \text{for } \bar{C}_1 > \bar{Q}_1 \end{cases}, \quad (11.10)$$

Figure 11.4: A Debt-Elastic Interest Rate



Notes. The figure displays an interest rate schedule that is weakly increasing in the level of external debt. For $\bar{C}_1 < \bar{Q}_1$, the country is a net external lender, and the interest rate is constant and equal to \underline{i} . For $\bar{C}_1 > \bar{Q}_1$, the country is a net external borrower, and the interest rate is an increasing function of the level of debt, $\bar{C}_1 - \bar{Q}_1$.

where \underline{i} and δ are positive parameters. In this example, depicted in figure 11.4, the country lends at the constant interest rate \underline{i} but borrows at an interest rate that increases linearly with the level of debt, $\bar{C}_1 - \bar{Q}_1$.

11.4.2 Competitive Equilibrium without Government Intervention

Because all households are identical in preferences and endowments, in equilibrium they all consume the same amount of goods. This means that consumption per capita equals the individual level of consumption, $\bar{C}_1 = C_1$.

So we can write the interest rate as

$$i = I(C_1).$$

It is important to understand why in deriving the household's optimality condition (11.7) we do not take into account that the interest rate depends on consumption. The reason is that the interest rate depends on aggregate per capita consumption, not on the household's individual level of consumption, and the household takes aggregate per capital consumption as given. Only in equilibrium is aggregate consumption per capita equal to individual consumption.

Use equation (11.9) to eliminate the interest rate i from the intertemporal budget constraint (11.6) and the optimality condition (11.7) to obtain

$$C_2 = Q_2 + (1 + I(C_1))(Q_1 - C_1) \quad (11.11)$$

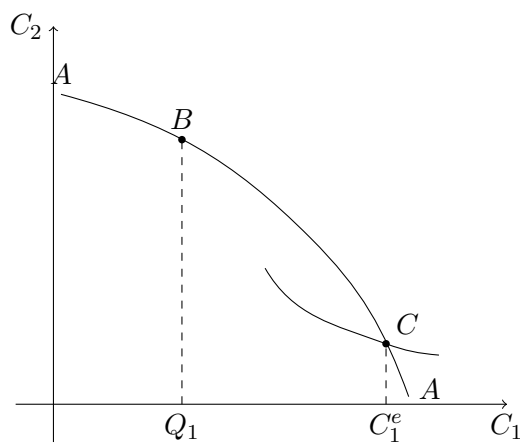
and

$$\frac{U'(C_1)}{U'(C_2)} = 1 + I(C_1). \quad (11.12)$$

These are two equations determining the equilibrium levels of consumption in periods 1 and 2.

Figure 11.5 depicts the equilibrium in the space (C_1, C_2) . Equation (11.11), shown as the locus AA , is the economy's intertemporal resource constraint. Like in the case of a constant interest rate, the resource constraint is downward sloping. Increasing consumption in period 1 requires sacrificing some consumption in period 2. The key difference with the case of a constant

Figure 11.5: Equilibrium in an Economy with Borrowing Externalities



Notes. The locus AA represents the economy's resource constraint. The endowment is at point B . The competitive equilibrium without government intervention is at point C . This equilibrium is inefficient because there are other allocations on the resource constraint that yield higher utility than point C .

interest rate is the slope. When the interest rate is constant, the slope of the resource constraint is constant and equal to minus $1 + i$. By contrast, in this economy the slope of the resource constraint is always larger than $1 + i$ in absolute value. More precisely, taking the derivative of C_2 with respect to C_1 in equation (11.11), we see that the slope of the intertemporal resource constraint is given by minus $1 + i + I'(C_1)(C_1 - Q_1)$, which is always greater than $1 + i$ if the country is a borrower, i.e., if $C_1 > Q_1$, because $I'(C_1)$ is positive. Intuitively, the reason why the slope of the intertemporal resource constraint is greater than $1 + i$ in absolute value is that if the country borrows an additional unit for consumption in period 1, in period 2 it must pay not only $1 + i$ but also the increase in the interest rate, $I'(C_1)$, caused by the increase in debt. This increase in the interest rate applies not only to the extra unit borrowed, but also to the entire debt, $C_1 - Q_1$.

The endowment is at point B, and the equilibrium is at point C where consumption in period 1 is C_1^e . Because $C_1^e > Q_1$, the country is a borrower. At point C, the indifference curve has a slope equal to minus $1 + i$, as dictated by the Euler equation (11.12). This means that at point C the indifference curve is flatter than the resource constraint, $1 + i < 1 + i + I'(C_1)(C_1 - Q_1)$. The discrepancy between the slope of the intertemporal resource constraint and that of the indifference curve renders the competitive equilibrium inefficient. It is clear from figure 11.5 that there are points on the intertemporal resource constraint that deliver higher levels of utility than point C. The inefficiency originates in the fact that private households fail to internalize the full marginal cost of current consumption. Because the interest rate does not depend on individual consumption, but on aggregate

per capita consumption, households perceive an opportunity cost of current consumption equal to $1 + i$, which is lower than the social cost of $1 + i + I'(C_1)(C_1 - Q_1)$. This induces households to consume more than is socially optimal.

11.4.3 The Efficient Allocation

Imagine a benevolent *social planner* that can allocate consumption in periods 1 and 2 to maximize households' utility subject to the economy's resource constraint (11.11). The planner gives all households the same path of consumption. The planner's optimization problem then is to maximize

$$\ln C_1 + \ln C_2.$$

subject to

$$C_2 = Q_2 + (1 + I(C_1))(Q_1 - C_1).$$

As before, use the resource constraint to get rid of C_2 in the utility function to restate the planner's problem as one of choosing C_1 to maximize

$$\ln C_1 + \ln[Q_2 + (1 + I(C_1))(Q_1 - C_1)].$$

There are two key differences between this optimization problem and that of an individual household in a market economy. First, the planner understands that C_1 is both individual and aggregate per capita consumption. Second, the household takes the interest rate as given, whereas the social planner internalizes the fact that changes in C_1 move the interest rate $I(C_1)$.

The first-order condition associated with the social planner's optimization problem is

$$U'(C_1) - U'(Q_2 + (1 + I(C_1))(Q_1 - C_1))[1 + I(C_1) + I'(C_1)(C_1 - Q_1)] = 0.$$

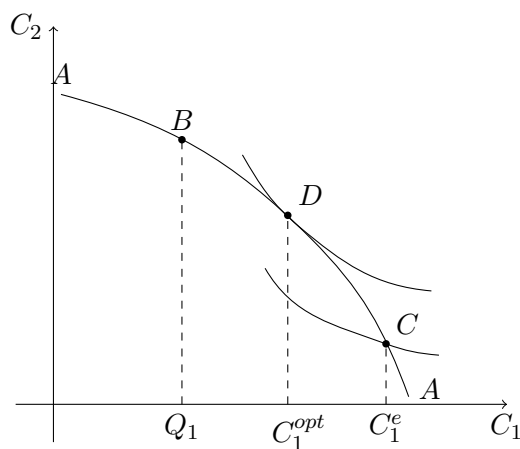
Using the fact that $C_2 = Q_2 + (1 + I(C_1))(Q_1 - C_1)$ and rearranging we can write the planner's optimality condition as

$$\frac{U'(C_1)}{U'(C_2)} = 1 + I(C_1) + I'(C_1)(C_1 - Q_1). \quad (11.13)$$

This expression says that at the efficient allocation the slope of the indifference curve equals the slope of the economy's intertemporal resource constraint. It is of interest to compare this optimality condition to its counterpart in the competitive equilibrium, given by equation (11.12). In the competitive equilibrium, the marginal rate of substitution of current for future consumption, $U'(C_1)/U'(C_2)$, is equated to the private cost of funds, $1 + I(C_1)$, whereas in the efficient allocation it is equated to the social cost of funds, $1 + I(C_1) + I'(C_1)(C_1 - Q_1)$, which is higher than the private cost.

The efficient allocation is given by the values of C_1 and C_2 that solve economy's resource constraint (11.11) and the social planner's optimality condition (11.13). Figure 11.6 provides a graphical representation of the solution to the social planner's optimality conditions. The efficient allocation is at point D. At this point, consumption satisfies the resource constraint and, in addition, the resource constraint is tangent to an indifference curve. Clearly, at point D households attain the highest possible level of welfare

Figure 11.6: The Efficient Allocation in an Economy with Borrowing Externalities



Notes. The locus AA represents the economy's resource constraint. The endowment is at point B. The competitive equilibrium without government intervention is at point C. The efficient allocation is at point D, where an indifference curve is tangent to the resource constraint. This allocation can be supported by an appropriately chosen capital control tax.

given the economy's resources. For comparison, the figure also displays the endowment, given by point B , and the competitive equilibrium, given by point C . The efficient level of consumption in period 1 is C_1^{opt} , which is higher than the endowment, Q_1 , but lower than the level of consumption attained in the competitive equilibrium, C_1^e . Accordingly, in the efficient allocation (point D), the external debt is lower than in the competitive equilibrium (point C), $C_1^{opt} - Q_1 < C_1^e - Q_1$. The excess external borrowing in the competitive equilibrium is known as *overborrowing*.

11.4.4 Optimal Capital Control Policy

Can the efficient allocation be achieved in a market economy as opposed to a centrally planned economy like the one we just studied? The answer is yes. The government can eliminate overborrowing and achieve the efficient allocation by imposing a capital control tax like the one analyzed in section 11.2. Specifically, suppose that the government imposes a tax on external borrowing at the rate τ . By the arguments given in section 11.2, the capital control tax introduces a wedge between the domestic interest rate, i , and the interest rate at which the country can borrow in international financial markets, i^* . Specifically, the interest parity condition (11.8) changes to

$$i = i^* + \tau.$$

Suppose that the government sets $\tau = I'(\bar{C}_1)(\bar{C}_1 - \bar{Q}_1)$. Because the tax rate depends on aggregate per capita variables, the household takes the tax as exogenously given, exactly as in the case studied in section 11.2. We then

have that

$$i = i^* + I'(\bar{C}_1)(\bar{C}_1 - \bar{Q}_1).$$

Since $i^* = I(\bar{C}_1)$ and since in equilibrium $\bar{C}_1 = C_1$ and $\bar{Q}_1 = Q_1$, this expression becomes

$$i = I(C_1) + I'(C_1)(C_1 - Q_1).$$

Using this expression, the household's optimality condition (11.7) can be written as

$$\frac{U'(C_1)}{U'(C_2)} = 1 + I(C_1) + I'(C_1)(C_1 - Q_1),$$

which is identical to the optimality condition of the social planner, given by (11.13). Further, if, as assumed in section 11.2, the government rebates the revenue generated by the capital control tax to the households in a lump-sum fashion, the economy's resource constraint (11.11) is unchanged. So the equilibrium values of C_1 and C_2 are determined by the solution to equations (11.11) and (11.13), which are the two equations determining the efficient allocation.

This establishes that the capital control policy $\tau = I'(\bar{C}_1)(\bar{C}_1 - \bar{Q}_1)$ supports the efficient allocation as a competitive equilibrium outcome. In other words, the proposed capital control tax places the economy at point D in figure 11.6. Intuitively, the capital control tax increases the effective cost of borrowing perceived by households, which induces them to cut consumption in period 1. Thus, the role of the capital control tax is to make households internalize that the social cost of an extra unit of consumption is not just

$1 + I(C_1)$, but $1 + I(C_1) + I'(C_1)(C_1 - Q_1)$.

Summarizing, in the presence of a borrowing externality free capital mobility ceases to be optimal. Households consume more and borrow more in period 1 than is socially optimal. Capital controls become desirable as a way to eliminate overborrowing and bring about the socially optimal allocation.

11.5 Capital Mobility in a Large Economy

When a large economy like the United States, the eurozone, or China increases its demand for international funds, the world interest rate will in general experience upward pressure. Each individual household in the large economy takes the interest rate as given, but for the country as a whole, the interest rate is an endogenous variable. This means that the government of a large economy might be able to apply policies to manipulate world interest rates in the country's favor. For example, if the country is running a current account deficit, the government could impose capital controls to curb the country's aggregate external borrowing and induce a fall in the world interest rate.

Before analyzing the effects of capital controls, however, we wish to characterize the equilibrium in a large economy under free capital mobility. This analysis will serve as a useful benchmark to evaluate national policies aimed at managing international capital flows, which we will take up in section 11.7.

The model of a large economy we present here builds on the two-country model introduced in section 7.3 of chapter 7. Consider a two-period economy composed of two countries, the home country, denoted h , and the foreign

country, denoted f . The home country receives a constant endowment in both periods. By contrast, the foreign country receives a lower endowment in period 1 than in period 2. The two economies are identical in all other respects. In particular, both have the same preferences for consumption and start period 1 with no assets or debts. If both countries had identical endowment streams, they would be content consuming their respective endowments each period and would not benefit from engaging in intertemporal trade with each other. However, because in the foreign country the endowment in period 1 is lower than in period 2, all households in the foreign country would like to borrow in period 1 against their period-2 endowment. The collective demand for funds from the foreign country will drive the world interest rate up and result in an equilibrium in which the foreign country borrows from the home country in period 1.

Assume that preferences in both countries take the form

$$\ln C_1^j + \ln C_2^j, \quad (11.14)$$

where C_1^j and C_2^j denote consumption in periods 1 and 2, respectively, in country $j = h, f$.

The budget constraint of households in country j in period 1 is given by

$$C_1^j + B_1^j = Q_1^j, \quad (11.15)$$

where B_1^j denotes bonds purchased in period 1. Because the world ends in period 2, households cannot borrow or lend in that period, so the budget

constraint in period 2 takes the form

$$C_2^j = Q_2^j + (1 + i^j)B_1^j, \quad (11.16)$$

where i^j denotes the interest rate in country j , for $j = h, f$.

Solving the budget constraint in period 1 for B_1^j to eliminate B_1^j from the period-2 budget constraint, one obtains the following intertemporal budget constraint

$$C_2^j = Q_2^j + (1 + i^j)(Q_1^j - C_1^j). \quad (11.17)$$

Using this expression to eliminate C_2^j from the utility function (11.14) yields

$$\ln C_1^j + \ln[Q_2^j + (1 + i^j)(Q_1^j - C_1^j)].$$

The objective of the household in country j is to choose C_1^j to maximize this expression, taking as given the interest rate and the endowments. Taking the derivative with respect to C_1^j and equating the resulting expression to zero, we obtain the household's optimality condition

$$\frac{1}{C_1^j} = \frac{1 + i^j}{Q_2^j + (1 + i^j)(Q_1^j - C_1^j)}.$$

Realizing that the denominator of the right-hand side is C_2^j , we can rewrite the optimality condition as the consumption Euler equation,

$$\frac{C_2^j}{C_1^j} = 1 + i^j. \quad (11.18)$$

Now solve this optimality condition for C_2^j to eliminate C_2^j from the intertemporal budget constraint (11.17). This yields the optimal level of consumption in period 1,

$$C_1^j = \frac{1}{2} \left(Q_1^j + \frac{Q_2^j}{1+i^j} \right). \quad (11.19)$$

Let us now consider the home country, that is, set $j = h$. Assume that the endowment in the home country is equal to Q in both periods. Then, by equation (11.19), consumption in period 1 in the home country satisfies

$$C_1^h = \frac{1}{2} \left(Q + \frac{Q}{1+i^h} \right). \quad (11.20)$$

This expression says that consumption is a decreasing function of the interest rate.

The trade balance, denoted TB_1^h , is the difference between the endowment and current consumption,

$$TB_1^h = \frac{Q}{2} \frac{i^h}{1+i^h}.$$

And, because the initial net asset position is assumed to be nil ($B_0^h = 0$), in period 1 the current account in the home country, denoted CA_1^h , equals the trade balance,

$$CA_1^h = \frac{Q}{2} \frac{i^h}{1+i^h}.$$

Intuitively, as the interest rate increases, households save an increasing part of their endowments, generating surpluses in the trade balance and the current account. Recalling that the current account equals the change in the

country's net foreign asset position, $CA_1^h = B_1^h - B_0^h$, and that, by assumption, the initial asset position is nil, $B_0^h = 0$, we have that the country's net foreign asset position at the end of period 1 equals the current account. So we can write

$$B_1^h = \frac{Q}{2} \frac{i^h}{1 + i^h}. \quad (11.21)$$

This expression says that the demand for bonds is an increasing function of their rate of return.

In the foreign country the endowment is $Q/2$ in period 1 and Q in period 2. Thus, from equation (11.19) we have that foreign consumption in period 1 is given by

$$C_1^f = Q \frac{3 + i^f}{4(1 + i^f)}. \quad (11.22)$$

It is straightforward to see from this expression that in period 1 consumption exceeds the endowment, $C_1^f > Q/2$, for any interest rate below 100 percent (i.e., for any $i^f < 1$). It makes sense that foreign households choose to borrow for a wide range of interest rates because their endowment in period 2 is twice as large as in period 1 (Q versus $Q/2$). So households borrow against their future endowment in order to smooth consumption over time.

Because the foreign country starts period 1 with no assets or debts, the period-1 trade balance and current account are both given by the difference between the endowment and consumption in period 1. Using the above expression for period-1 consumption we can write,

$$TB_1^f = \frac{Q}{4} \frac{i^f - 1}{(1 + i^f)},$$

and

$$CA_1^f = \frac{Q}{4} \frac{i^f - 1}{(1 + i^f)}.$$

This expression says that as long as the interest rate in the foreign country is below 100 percent ($i^f < 1$), the current account in period 1 will be negative. This is in line with the fact that households, facing an increasing path of endowments, borrow in period 1 to smooth consumption. Because the foreign country starts period 1 with no assets or debts, its net foreign asset position at the end of period 1, B_1^f , is equal to the current account. So we can write

$$B_1^f = \frac{Q}{4} \frac{i^f - 1}{(1 + i^f)}. \quad (11.23)$$

In equilibrium the world asset demand, given by the sum of the bond demands of the home and foreign countries, must be zero, that is,

$$B_1^h + B_1^f = 0. \quad (11.24)$$

Under free capital mobility, the interest rate must be the same in both countries. Let this common interest rate be denoted i^* . We refer to i^* as the world interest rate. So we have

$$i^h = i^f = i^*. \quad (11.25)$$

Use this expression to replace i^h and i^f by i^* in equations (11.21) and (11.23), respectively. Then, use the resulting expressions to eliminate B_1^h

and B_1^f from (11.24) to obtain

$$\frac{Q}{2} \frac{i^*}{1+i^*} + \frac{Q}{2} \frac{i^* - 1}{2(1+i^*)} = 0.$$

Solving for i^* , we obtain the equilibrium level of the world interest rate

$$i^* = \frac{1}{3},$$

so under free capital mobility the world interest rate is 33 percent. Because the world interest rate is less than 100 percent, we know from the bond demand of the foreign household (equation (11.23)) that the foreign country borrows internationally in period 1. In turn, if the foreign country borrows the domestic country must save in period 1.

Setting $i^h = 1/3$ in equations (11.20) and (11.21), we obtain the equilibrium levels of period-1 consumption and bond holdings in the home economy,

$$C_1^h = \frac{7}{8}Q < Q$$

and

$$B_1^h = \frac{1}{8}Q > 0.$$

In spite of having a flat path of endowments, which, if consumed, would produce a perfectly smooth path of consumption, households in the home country choose to consume less than their endowment in period 1 and to save. This is because foreign demand for funds (discussed next) drives the world interest rate up, inducing the home country to postpone consumption.

As a result, in period 2 the home country can enjoy a level of consumption higher than its endowment. This can be verified by setting $j = h$ and $i^j = 1/3$ in optimality condition (11.17), which yields

$$C_2^h = \frac{7}{6}Q > Q > C_1^h.$$

Proceeding in an analogous fashion, we obtain the following equilibrium values for the foreign country's levels of consumption and bond holdings:

$$C_1^f = \frac{5}{8}Q > \frac{1}{2}Q,$$

$$C_2^f = \frac{5}{6}Q < Q,$$

and

$$B_1^f = -\frac{1}{8}Q < 0.$$

Intuitively, facing an upward sloping path of endowments, the foreign country borrows in period 1 to smooth consumption. So it consumes above its endowment in period 1, below its endowment in period 2, and maintains a short bond position in period 1.

Welfare under free capital mobility can be found by evaluating the utility function (11.14) at the respective equilibrium consumption levels. This yields

$$\ln C_1^h + \ln C_2^h = \ln \left(\frac{49}{48} Q^2 \right) \quad (11.26)$$

for the home country, and

$$\ln C_1^f + \ln C_2^f = \ln \left(\frac{25}{48} Q^2 \right),$$

for the foreign country.

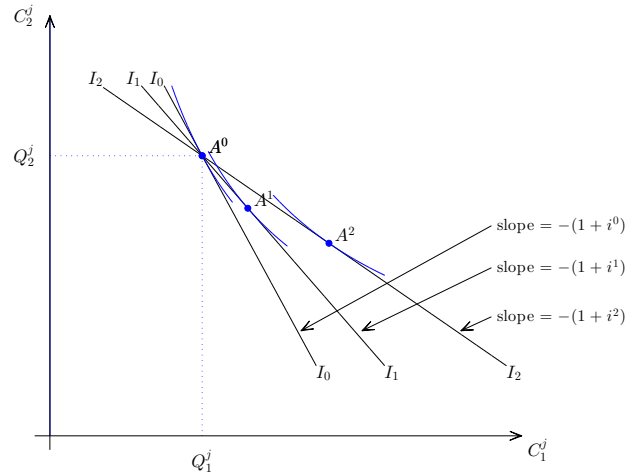
Exercise 11.9 asks to show that both countries are better off under free capital mobility than under financial autarky, which is an environment in which both economies are closed to international trade in financial assets. An implication of this result is that it does not pay for either country to impose capital controls so high that all intertemporal trade is killed. The main question we will address shortly is whether there is a capital control policy on the part of one country that induces an equilibrium in which the level of welfare of the households it represents is higher than under free capital mobility.

11.6 Graphical Analysis of Equilibrium under Free Capital Mobility

To visualize how the equilibrium under free capital mobility is determined in a two-country world as well as its implications for consumption, international borrowing, and welfare, we begin by presenting two powerful graphical objects in general equilibrium analysis, the *offer curve* and the *Edgeworth box*, both created by the Irish economist Francis Ysidro Edgeworth (1845-1926).

Consider the optimal choice of consumption in periods 1 and 2 by country $j = h, f$, C_1^j and C_2^j , for different levels of the interest rate. Figure 11.7

Figure 11.7: Optimal Intertemporal Consumption Choice at Different Interest Rates

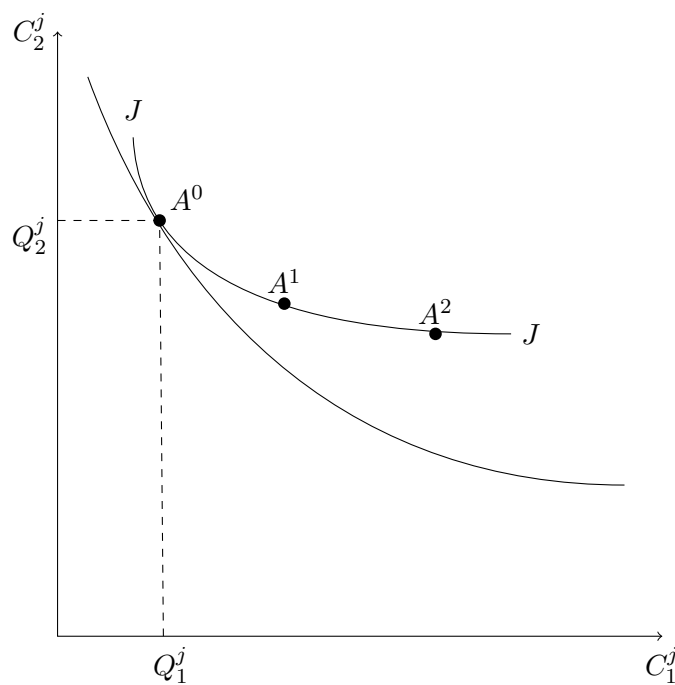


Notes. The figure displays the optimal consumption choice for three different values of the interest rate, i^0 , i^1 , and i^2 , satisfying $i^0 > i^1 > i^2$. Each interest rate is associated with a different intertemporal budget constraint. The higher the interest rate is, the steeper the intertemporal budget constraint will be. The intertemporal budget constraint $\overline{I^0 I^0}$ is induced by the highest of the three interest rates, and the intertemporal budget constraint $\overline{I^2 I^2}$ by the lowest. The associated optimal consumption path induced by the interest rate associated with budget constraint $\overline{I^0 I^0}$ is the endowment point, A^0 . The intertemporal budget constraints $\overline{I^1 I^1}$ and $\overline{I^2 I^2}$ produce optimal consumption choices given by points A^1 and A^2 , respectively. The offer curve (not shown) connects points A^0 , A^1 , A^2 .

displays these optimal choices for three different interest rates, i^0 , i^1 , and i^2 , satisfying $i^0 > i^1 > i^2$. The household's endowment, (Q_1^j, Q_2^j) , is given by point A^0 . Each of the three interest rates considered defines a different intertemporal budget constraint, $C_2^j = Q_2^j + (1+i)(Q_2^j - C_1^j)$. Each budget constraint is a downward sloping line that crosses the endowment point A^0 and has slope $-(1+i)$. The higher the interest rate is, the steeper the budget constraint will be. In the figure, the budget constraint $\overline{I^0 I^0}$ corresponds to the interest rate i^0 at which it is optimal for the household to consume its endowment. This can be seen from the fact that the indifference curve that crosses point A^0 is tangent to the budget constraint $\overline{I^0 I^0}$ at point A^0 . As the interest rate falls, the intertemporal budget constraint pivots counterclockwise around point A^0 . The budget constraint $\overline{I^1 I^1}$ corresponds to the interest rate i^1 . The optimal consumption choice at the interest rate i^1 is given by point A^1 . At this point, the household consumes more than its endowment in period 1 and less than its endowment in period 2. The household achieves this consumption allocation by borrowing in period 1. Finally, the budget constraint $\overline{I^2 I^2}$ corresponds to the interest rate i^2 . The optimal consumption choice induced by this interest rate is given by point A^2 that contains more consumption in period 1, less consumption in period 2, and more borrowing in period 1 than point A^1 .¹

¹At a sufficiently low level of the interest rate it could happen that a fall in the interest rate causes an increase in consumption in both periods. This is because when the household is indebted, a fall in the interest rate has a positive income effect, which calls for increasing consumption in both periods. If the household is highly indebted, this effect might more than offset the substitution effect, which calls for increasing consumption in period 1 and cutting consumption in period 2. Under the log-linear utility function given in (11.14), however, the substitution effect always dominates the income effect. See also the discussion of income and substitution effects associated with changes in interest rates in section 4.3 of chapter 4.

Figure 11.8: The Offer Curve



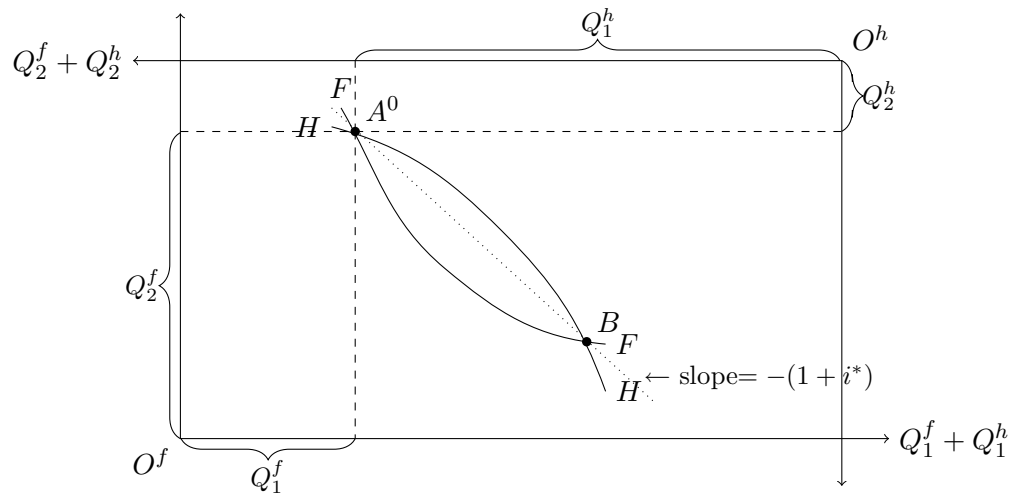
Notes. The offer curve is the locus JJ , which connects all optimal consumption allocations at different interest rates. The offer curve crosses the endowment point A^0 . The figure also shows the indifference curve that crosses the endowment point. All points on the offer curve other than the endowment point are preferred to the endowment point itself.

Now imagine doing the same not just for three interest rates, but for all possible levels of the interest rate. The offer curve is the locus that connects all the associated optimal consumption choices. Figure 11.8 displays the offer curve of country j as the locus JJ . By construction, points A^0 , A^1 and A^2 are on the offer curve. Each point on the offer curve is associated with a different value of the interest rate, i . The interest rate associated with any given point on the offer curve can be found as follows: draw the line that connects the point on the offer curve with the endowment point, A^0 . This line is the intertemporal budget constraint of the household at the interest rate i and therefore its slope equals $-(1+i)$. At the point it intersects the offer curve, this intertemporal budget constraint is tangent to an indifference curve.

In addition to the offer curve, figure 11.8 displays the indifference curve that crosses the endowment point, A^0 . All points on the offer curve other than the endowment point are strictly preferred to the endowment point itself. This is because at any interest rate, consuming the endowment is feasible.

We are now ready to analyze the equilibrium under free capital mobility. Figure 11.9 displays a box known as the Edgeworth box. The length of the horizontal side of the box is the global endowment of goods in period 1, $Q_1^h + Q_1^f$. The height of the box is the global endowment in period 2, $Q_2^h + Q_2^f$. The southwest corner of the box is the origin of the foreign country and is indicated by the symbol O^f . For the foreign country, consumption and the endowment in period 1 are measured on the horizontal axis from the origin O^f to the right, and consumption and the endowment in period 2

Figure 11.9: Equilibrium Under Free Capital Mobility in a Two-Country Model



Notes. This type of graph is known as an Edgeworth box. The origin of the foreign country is O^f and the origin of the home country is O^h . The endowment point is A^0 . The offer curve of the foreign country is the locus FF , and the offer curve of the home country is the locus HH . The equilibrium under free capital mobility is point B . The slope of the line that connects points A^0 and point B is $-(1 + i^*)$, where i^* is the equilibrium world interest rate under free capital mobility.

are measured on the vertical axis from O^f upward. The northeast corner of the box is the origin of the home country and is indicated by the symbol O^h . For this country, consumption and the endowment in period 1 are measured on the horizontal axis from O^h to the left, and consumption and the endowment in period 2 are measured on the vertical axis from O^h downward. So households in the foreign country become happier as one moves northeast in the box and households in the home country become happier as one moves southwest in the box. In the figure, the endowments of the two countries are given by point A^0 . In this example, the home country is abundant in period-1 goods, and the foreign country is abundant in period-2 goods. Any point in the box represents an allocation of consumption across time and countries that can be achieved with the existing global endowments.

The offer curve of the foreign country is the locus FF and the offer curve of the home country is the locus HH . Clearly, both offer curves must cross the endowment point A^0 . The equilibrium under free capital mobility is given by point B, where the two offer curves intersect for a second time. In equilibrium, the foreign country, which has a relatively low endowment in period 1, borrows from the home country. The equilibrium world interest rate, i^* , is determined by the slope of the line that connects points A^0 and B. This line is the intertemporal budget constraint faced by the domestic and foreign households at the equilibrium world interest rate i^* . This interest rate is lower than the domestic interest rate in the foreign country under financial autarky, which is determined by the slope of the foreign household's indifference curve at the endowment point A^0 . By the same logic, we have that the equilibrium interest rate under free capital

mobility, i^* , is higher than the domestic interest rate in the home country under financial autarky. We have therefore established that allowing for free capital mobility eliminates interest rate differentials by causing a fall in the interest rate in the borrowing country and an increase in the interest rate in the lending country.

Because in equilibrium both countries are on their respective offer curves, they are both better off than under autarky. This means that in this economy free capital mobility is welfare improving for both countries. Further, at point B the indifference curves of the home and foreign households both have a slope equal to $-(1 + i^*)$. Thus, at point B the indifference curves of the home and foreign households are tangent to each other. This implies that at no point inside the Edgeworth box can both countries be better off than at point B. In other words, any other attainable consumption allocation makes at least one country worse off relative to the allocation associated with the equilibrium under free capital mobility. When an equilibrium has this property, we say that it is Pareto optimal.²

11.7 Optimal Capital Controls in a Large Economy

Suppose that the government of the foreign country behaves strategically and manipulates capital flows by imposing capital controls to obtain a value of the world interest rate, i^* , that maximizes the welfare of its citizens. It

²This name is given after the Italian economist Vilfredo Federico Pareto (1848-1923), who first discussed this notion of efficiency.

can do so because the country is large and therefore has monopsony power in international funds markets. Unlike the country as a whole, individual households do not have market power in financial markets as they are atomistically small. Thus, exploiting the country's market power can only be achieved via government intervention.

Capital controls have two opposing macroeconomic effects. First, as we studied in section 11.1, they create a wedge between the domestic and the foreign interest rate, which is welfare decreasing because it distorts the intertemporal allocation of consumption and investment. Second, because the country is borrowing, the fall in the world interest rate caused by the imposition of capital controls is welfare increasing, for it reduces the cost of servicing the external debt. In a small open economy, the second effect is nil, because the country cannot affect the world interest rate, so, as we saw in section 11.2, the resolution of the policy trade-off calls for no capital controls. In a large economy, however, the government will in general find it optimal to impose capital controls to push the world interest rate down. The main purpose of this section is to establish this result and to study how optimal capital controls affect the current account and welfare in both countries.

Assume that in response to capital controls imposed by the foreign country, the home country does not retaliate by imposing its own capital controls. In section 11.9 we will analyze the case of retaliation. This means that the demand for international funds by the home country continues to be given

by (11.21) evaluated at the world interest rate i^* ,

$$B_1^h = \frac{Q}{2} \frac{i^*}{1+i^*}. \quad (11.27)$$

The foreign country imposes a tax τ on international borrowing. The budget constraint of the foreign household in period 1 is unchanged by this tax,

$$C_1^f + B_1^f = \frac{Q}{2}. \quad (11.28)$$

The budget constraint in period 2 is now given by

$$C_2^f = Q + T + (1+i^f)B_1^f,$$

where T denotes a lump-sum transfer received from the government.

By the analysis of section 11.1, the capital control tax creates a wedge between the world interest rate and the interest rate in the foreign country,

$$i^f = i^* + \tau. \quad (11.29)$$

Tax revenue is given by $-\tau B_1^f$. The government rebates these resources to households via lump-sum transfers,

$$T = -\tau B_1^f.$$

Thus, combining the above three expressions, the budget constraint in period 2 becomes

$$C_2^f = Q + (1+i^*)B_1^f. \quad (11.30)$$

The government internalizes that in equilibrium the international bond market must clear, that is, that $B_1^f = -B_1^h$, as indicated by the market clearing condition (11.24). The government also internalizes that the supply of funds by the home country, B_1^h , is an increasing function of the world interest rate, i^* , as shown in equation (11.27). So using (11.24) and (11.27) to express C_1^f and C_2^f in equations (11.28) and (11.30) as functions of the world interest rate, we have that

$$C_1^f = \frac{Q}{2} \frac{1 + 2i^*}{1 + i^*} \quad (11.31)$$

and

$$C_2^f = \frac{Q}{2} (2 - i^*). \quad (11.32)$$

Now use these two expressions to eliminate C_1^f and C_2^f from the utility function of the foreign household to obtain

$$\ln \left(\frac{Q}{2} \frac{1 + 2i^*}{1 + i^*} \right) + \ln \left(\frac{Q}{2} (2 - i^*) \right).$$

This object is known as (the foreign household's) *indirect utility function*. It is the lifetime utility function of the foreign household expressed in terms of the world interest rate, i^* , instead of consumption, C_1^f and C_2^f . The objective of the foreign government is to pick the world interest rate i^* to maximize the indirect utility function of the foreign household. The optimality condition associated with this maximization problem results from taking the derivative of the indirect utility function with respect to i^* and

setting it to zero. Performing this operation and rearranging yields

$$i^{*2} + 2i^* - \frac{1}{2} = 0.$$

Solving this quadratic expression for i^* , we obtain two candidate values,

$$-1 \pm \sqrt{\frac{3}{2}}.$$

We can discard the root that implies a value for the interest rate below -1, because an interest rate cannot be below -100 percent. So the only economically sensible solution is

$$i^* = -1 + \sqrt{\frac{3}{2}} = 0.22,$$

or 22 percent. This confirms the conjecture that under optimal capital controls by the foreign government the world interest rate is lower than under free capital mobility (22 versus 33 percent). Intuitively, the foreign government picks a lower interest rate to induce a positive income effect on its residents who are net borrowers in the international market. Plugging the interest rate of 0.22 into the indirect utility function of the foreign household yields a welfare of $\ln(\frac{25.2122}{48}Q^2)$ under optimal capital controls, which, compared with a welfare of $\ln(\frac{25}{48}Q^2)$ under free capital mobility, confirms that the policy is welfare increasing for foreign residents.

The foreign government induces the required fall in the world interest rate by imposing controls on capital inflows. To obtain the optimal capital control tax, τ , plug the optimal world interest rate of 0.22 into equa-

tions (11.31) and (11.32) and use the result to eliminate C_1^f and C_2^f from the household's optimality condition (11.18) to obtain

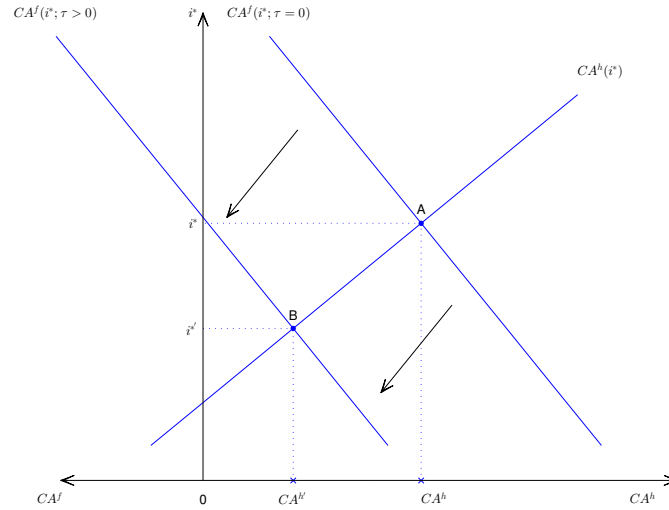
$$i^f = 0.5.$$

Then equation (11.29) gives the optimal capital control tax as the difference between the domestic interest rate in the foreign country and the world interest rate, which yields,

$$\tau = 0.28,$$

or a tax rate on capital inflows of 28 percent. By taxing international borrowing the government causes an increase in the domestic interest rate in the foreign country, i^f . A domestic interest rate in excess of the world interest rate causes an inefficient allocation of consumption over time, as households feel the cost of borrowing went up, when in reality it went down. In addition, the increase in the domestic interest rate in the foreign country, i^f , causes a negative income effect on households, as they are net debtors. How do these negative effects square with the result that capital controls are welfare increasing in the foreign country? The reason why households are better off is that the government rebates all of the capital control tax revenue to the households. As it turns out, the positive income effect resulting from this transfer more than offsets the aforementioned negative effects. The reason why, overall, households in the foreign country are better off under capital controls is that the country as a whole experiences a positive income effect: The fall in the world interest rate, i^* , implies that the foreign country as a whole transfers less resources to the home country in the form of interest

Figure 11.10: Effect of Capital Controls on the Current Account



Notes. The equilibrium without capital controls is at point A. The imposition of capital controls in the foreign country shifts the current account schedule of the foreign country, $CA^f(i^*, \tau)$ down and to the left and leaves the current account schedule of the home country, $CA^h(i^*)$, unchanged. The equilibrium with capital controls is at point B, where the world interest rate, i^* , is lower, the current account of the foreign country improves, and the current account of the home country deteriorates.

payments on the external debt.

The imposition of optimal capital controls by the foreign country causes an improvement of the current account in the foreign country and a deterioration of the current account in the home country. To see this, recall that the change in the current account is equal to the change in bond holdings (i.e., the change in B_1^h). In turn, bond holdings of the home country are increasing in the world interest rate (see equation (11.27)). Thus, when the capital controls in the foreign country push the world interest rate down from 33 to 22 percent, bond holdings and the current account in the home country

decline. By the same token, the current account in the foreign country must improve, since market clearing requires that the sum of the current accounts of the two countries be zero, $CA_1^h + CA_1^f = 0$. It follows that by imposing controls on capital inflows, the foreign government manages to improve its own current account to the detriment of the home country's.

The situation is illustrated in figure 11.10 in the space (CA, i^*) . On the horizontal axis, the current account of the home country is measured from left to right, and the current account of the foreign country from right to left. Thus, the current account schedule of the home country is upward sloping and that of the foreign country is downward sloping. The equilibrium in the absence of capital controls ($\tau = 0$), is at point A. In this equilibrium the home country runs a current account surplus and the foreign country a current account deficit.

When the foreign country imposes capital controls ($\tau > 0$), the current account schedule of the home country remains unchanged. The current account schedule of the foreign country shifts down and to the left, reflecting the fact that at each level of the world interest rate, i^* , the domestic interest rate in the foreign country increases by τ , causing a contraction in the aggregate demand for goods and an increase in the trade balance and the current account. At the original level of the world interest rate, the imposition of the capital control tax generates an excess supply of funds in the world. Restoring equilibrium in the international financial market requires a fall in the world interest rate as indicated by point B in the figure. A lower level of the world interest rate induces an expansion in the demand for goods in the home country, which causes its trade balance and current

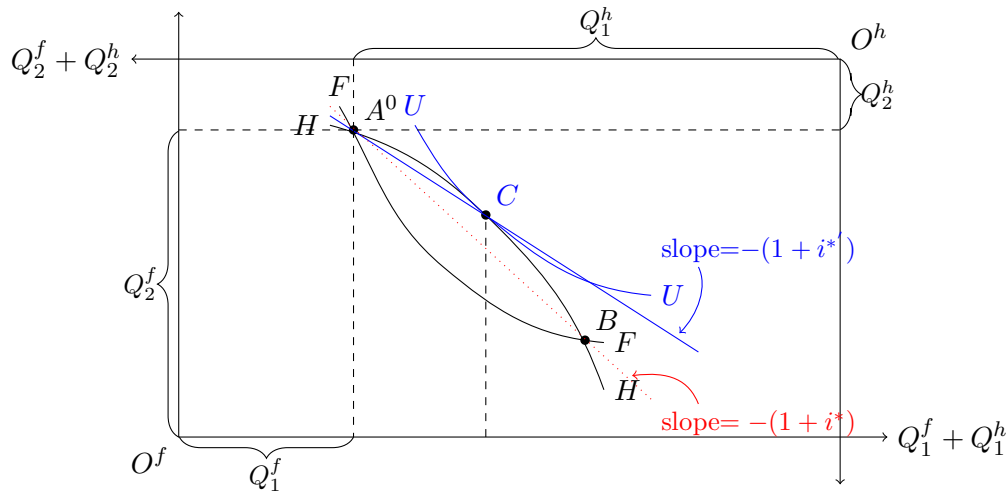
account to deteriorate. In the foreign country, the capital control tax leads to an increase in the domestic interest rate, i^f , despite the fact that the world interest rate falls. The higher domestic interest rate, in turn, causes a contraction in the aggregate absorption of goods and an improvement in the trade balance and the current account.³

Going back to the algebraic example, the imposition of optimal capital controls in the foreign country is welfare decreasing for the home country. To see this, first use the world's resource constraint to write consumption in the home country as $C_1^h = Q_1^h + Q_1^f - C_1^f$ and $C_2^h = Q_2^h + Q_2^f - C_2^f$. Then eliminate C_1^f and C_2^f by using equilibrium conditions (11.31) and (11.32) evaluated at $i^* = 0.22$ to get $C_1^h = 0.9082Q$ and $C_2^h = 1.1124Q$. Finally, plug these consumption values into the utility function of the home country to obtain $\ln C_1^h + \ln C_2^h = \ln(1.0103Q^2)$. Recalling that under free capital mobility the utility level of the home household is $\ln(1.0208Q^2)$, we conclude that the home country is hurt by the optimal capital controls in the foreign country.

Finally, the equilibrium under optimal capital controls in the foreign country is Pareto inefficient, that is, the equilibrium allocation of consumption across time and countries could be rearranged in such a way that the foreign country maintains the welfare level associated with the optimal capital control, but the home country is made better off relative to its situation under optimal capital controls. To establish this result, we resort to the

³One might be tempted to think that the capital control tax rate, τ , is given by the vertical distance from point B to the original current account schedule of the foreign country. However, this is not the case, because, as exercise 11.13 asks you to show, the current account of the foreign country does not depend on $i^* + \tau$, but on i^* and τ separately.

Figure 11.11: Optimal Capital Controls in a Large Economy



Notes. The offer curve of the home country is HH and that of the foreign country is FF . The endowment is at point A^0 . The equilibrium under free capital mobility is at point B , and the equilibrium under optimal capital controls in the foreign country is at point C . The interest rate under free capital mobility is i^* and under optimal capital controls $i^{*'}.$ The indifference curve attained by the foreign country under optimal capital controls is UU . The fact that this indifference curve is not tangent to the intertemporal budget constraint that crosses point C implies that the equilibrium with capital controls is Pareto inefficient.

graphical apparatus developed earlier in section 11.6.

11.8 Graphical Analysis of Optimal Capital Controls in a Large Economy

Figure 11.11 reproduces from figure 11.9 the offer curves of the home country and the foreign country, given by the loci HH and FF , respectively.

The endowment point is marked A^0 , and the equilibrium under free capital mobility is at point B. The world interest rate under free capital mobility, denoted i^* , is defined by the slope of the line connecting points A^0 and B. In setting capital controls, the objective of the foreign country is to attain a point on the home country's offer curve, HH , that maximizes the foreign country's utility. The foreign country is constrained to pick a point on the home country's offer curve because, by construction, only the allocations on the offer curve can be obtained as a market outcome, that is, by an appropriate choice of the world interest rate. Consequently, the allocation associated with the optimal capital control policy is one at which an indifference curve of the foreign country is tangent to the offer curve of the home country. This allocation is point C in the figure.

The world interest rate under optimal capital controls, denoted $i^{*'}$, is defined by the slope of the line that connects points A^0 and C. Clearly, this line is flatter than the one connecting points A^0 and B. This means that the imposition of optimal capital controls causes the world interest rate to fall. Also, the optimal capital control policy in the foreign country makes the foreign country better off at the expense of the home country, whose welfare goes down. To see this, recall that the equilibrium under free capital mobility is Pareto optimal, so the improvement in the foreign country's welfare must be welfare decreasing for the home country. These results echo those obtained algebraically in section [11.7](#).

We wish to show that the equilibrium under optimal capital controls in the foreign country fails to be Pareto optimal. To see this, note that at point C, the indifference curve of the home country is tangent to the

intertemporal budget constraint associated with the interest rate $i^{*'}$, whereas the indifference curve of the foreign country is tangent to the offer curve of country H (the locus HH). Since the budget constraint and HH cross each other at point C , it must be the case that the slopes of the home and foreign indifference curves at point C are not the same. This implies that the equilibrium allocation under capital controls is inefficient in the sense that the home country could be made better off without making the foreign country worse off.

11.9 Retaliation

Thus far, we have assumed that, as the foreign country imposes controls on capital inflows, the home country responds passively, without retaliating by imposing its own restrictions on capital flows. This assumption is reasonable if the home country is a small country or a region composed of small countries, each without market power in international capital markets. As we saw in section 11.2.3, the best policy for a small country in the environment studied in this chapter is to allow for free capital mobility. However, if the home country is a large economy, it will in general have an incentive to retaliate. Recall that the home country is a net lender in international markets. So it might be in its own interest to restrict the supply of funds to induce an increase in the world interest rate. In turn, the foreign country would have an incentive to readjust its capital control policy in response to the reaction of the home country. The equilibrium that will emerge under this strategic interaction depends on what type of game the two countries

play in setting capital controls. We will focus on one type of game known as *Nash equilibrium*. Essentially, in a Nash equilibrium each country sets its own capital control tax optimally taking as given the capital tax rate of the other country. An equilibrium is reached when the capital control tax that each country takes as given is indeed the tax rate that is optimal for the other country.

We saw that when the government rebates tax revenues to the public, capital control taxes entail no loss of resources, so for country $j = f, h$, the intertemporal resource constraint is the same as in the case without capital controls, that is,

$$C_1^j + \frac{C_2^j}{1+i^*} = Q_1^j + \frac{Q_2^j}{1+i^*}$$

Also, by the familiar Euler equation we have that

$$\frac{C_2^j}{C_1^j} = 1 + i^j.$$

The foreign country imposes controls on capital inflows, so its domestic interest rate is given by

$$i^f = i^* + \tau^f,$$

where τ^f denotes the capital control tax rate imposed by the foreign country.

It now carries the superscript f to distinguish it from the capital control tax rate of the home country, τ^h . Setting $j = f$ and solving for C_1^f and C_2^f , yields

$$C_1^f = \frac{Q_1^f + \frac{Q_2^f}{1+i^*}}{1 + \frac{1+i^*+\tau^f}{1+i^*}}.$$

$$C_2^f = (1 + i^* + \tau^f) \frac{Q_1^f + \frac{Q_2^f}{1+i^*}}{1 + \frac{1+i^*+\tau^f}{1+i^*}}.$$

Note that consumption depends on two endogenous variables, i^* and τ^f . So, to abbreviate notation we can write

$$C_1^f = K^f(i^*, \tau^f),$$

and

$$C_2^f = L^f(i^*, \tau^f).$$

For the home country, the derivation of the optimal levels of consumption in periods 1 and 2 is analogous, except that, because the country is a lender in the international financial market, a tax on capital outflows creates a negative differential between the domestic and the world interest rate,

$$i^h = i^* - \tau^h.$$

Combining this expression with the above intertemporal budget constraint and Euler equation and solving for consumption in periods 1 and 2, we obtain

$$C_1^h = \frac{Q_1^h + \frac{Q_2^h}{1+i^*}}{1 + \frac{1+i^*-\tau^h}{1+i^*}}$$

and

$$C_2^h = (1 + i^* - \tau^h) \frac{Q_1^h + \frac{Q_2^h}{1+i^*}}{1 + \frac{1+i^*-\tau^h}{1+i^*}}.$$

As in the case of the foreign country, consumption in both periods are functions of the world interest rate and the country's capital control tax rate.

Accordingly, we write these relations as

$$C_1^h = K^h(i^*, \tau^h)$$

and

$$C_2^h = L^h(i^*, \tau^h).$$

Market clearing in the goods market in period 1 requires that global consumption equal the global endowment,

$$K^f(i^*, \tau^f) + K^h(i^*, \tau^h) - Q_1^f - Q_1^h = 0.$$

This equation expresses the world interest rate as an implicit function of the tax rates in the home and foreign countries. We then write

$$i^* = I(\tau^f, \tau^h).$$

Using this relation to eliminate i^* from consumption in both periods in the home and foreign countries we can write

$$C_1^f = \tilde{K}^f(\tau^f, \tau^h) \equiv K^f(I(\tau^f, \tau^h), \tau^f),$$

$$C_2^f = \tilde{L}^f(\tau^f, \tau^h) \equiv L^f(I(\tau^f, \tau^h), \tau^f),$$

$$C_1^h = \tilde{K}^h(\tau^f, \tau^h) \equiv K^h(I(\tau^f, \tau^h), \tau^h),$$

$$C_2^h = \tilde{L}^h(\tau^f, \tau^h) \equiv L^h(I(\tau^f, \tau^h), \tau^h).$$

The government of the foreign country picks τ^f to maximize the utility of the foreign household taking as given the tax rate in the home country, τ^h . Thus, the objective function of the foreign country is

$$\ln \tilde{K}^f(\tau^f, \tau^h) + \ln \tilde{L}^f(\tau^f, \tau^h).$$

The first-order condition associated with the foreign government's maximization problem is the derivative of the objective function with respect to τ^f , equalized to zero. Formally,

$$\frac{\tilde{K}_1^f(\tau^f, \tau^h)}{\tilde{K}^f(\tau^f, \tau^h)} + \frac{\tilde{L}_1^f(\tau^f, \tau^h)}{\tilde{L}^f(\tau^f, \tau^h)} = 0,$$

where $\tilde{K}_1^f(\tau^f, \tau^h)$ and $\tilde{L}_1^f(\tau^f, \tau^h)$ denote, respectively, the partial derivatives of $\tilde{K}^f(\tau^f, \tau^h)$ and $\tilde{L}^f(\tau^f, \tau^h)$ with respect to the first argument, τ^f . This optimality condition implicitly defines the tax rate in the foreign country, τ^f , as a function of the tax rate in the home country, τ^h . We write the solution for τ^f as

$$\tau^f = R^f(\tau^h).$$

This relationship is called the *reaction function* of the foreign country. It represents the optimal tax response of the foreign country as a function of the tax rate in the home country.

Likewise, the objective of the home country is to choose τ^h to maximize

$$\ln \tilde{K}^h(\tau^f, \tau^h) + \ln \tilde{L}^h(\tau^f, \tau^h),$$

taking as given τ^f . The associated first-order condition is

$$\frac{\tilde{K}_1^h(\tau^f, \tau^h)}{\tilde{K}^h(\tau^f, \tau^h)} + \frac{\tilde{L}_1^h(\tau^f, \tau^h)}{\tilde{L}^h(\tau^f, \tau^h)} = 0.$$

Solving this expression for τ^h , we can write

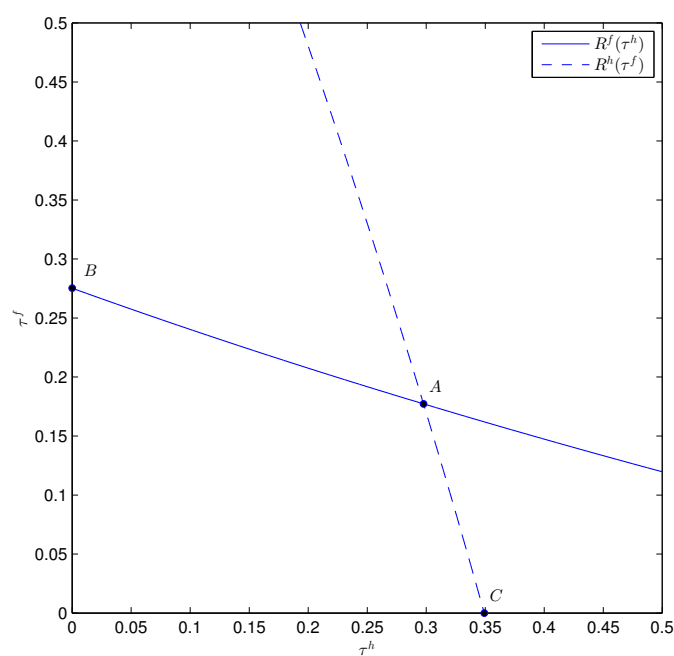
$$\tau^h = R^h(\tau^f),$$

which is the reaction function of the home country.

Figure 11.12 displays the reaction functions of the home and foreign countries in the space (τ^h, τ^f) . The Nash equilibrium is at point A, where the two reaction functions intersect. At this point, the tax rate of the foreign country maximizes the foreign government's objective function given the equilibrium tax rate in the home country, and the tax rate in the home country maximizes the objective function of the home government given the equilibrium tax rate in the foreign country. The equilibrium tax rate in the foreign country is 18 percent and in the home country it is 30 percent. This means that the retaliation of the home country is quite substantial. Also, the cross country domestic interest rate differential, $i^f - i^h = \tau^f + \tau^h$, widens from 28(=28+0) percent when the home country is passive to 48(=18+30) percent when the home country retaliates.

Point B in the figure corresponds to the case in which the foreign country behaves strategically and the home is passive, which is the case we studied in section 11.7. Comparing points A and B, we see that retaliation by the home country makes the foreign country lower its capital control tax rate (28

Figure 11.12: Capital Control Reaction Functions of the Home and Foreign Governments



Notes. The function $R^f(\tau^h)$ is the reaction function of the foreign country. It expresses the optimal capital control tax rate of the foreign country as a function of the tax rate of the home country. Similarly, $R^h(\tau^f)$ is the reaction function of the home country, representing the optimal tax rate in the home country as a function of the foreign country's tax rate. The intersection of the two reaction functions gives the Nash equilibrium capital control tax rates in the two countries. Replication file: `tauf_tauh_num.m` in `two_country.zip`.

Table 11.1: Comparison of Equilibria Under Alternative Capital Control Policies

Policy	i^*	τ^f	τ^h	CA^f	Welfare	
					f	h
Autarky	—	—	—	0	$\ln(0.5000Q^2)$	$\ln(1.0000Q^2)$
Free Capital Mobility	0.33	0	0	-0.125Q	$\ln(0.5208Q^2)$	$\ln(1.0208Q^2)$
Home Country Passive	0.22	0.28	0	-0.092Q	$\ln(0.5253Q^2)$	$\ln(1.0103Q^2)$
Foreign Country Passive	0.55	0	0.35	-0.073Q	$\ln(0.5082Q^2)$	$\ln(1.0318Q^2)$
Retaliation—Nash Eqm	0.45	0.18	0.30	-0.060Q	$\ln(0.5111Q^2)$	$\ln(1.0219Q^2)$

versus 18 percent). Point C in the figure corresponds to the case in which the home country behaves strategically and the foreign country is passive.⁴ Comparing points A and C, shows that retaliation by the foreign country also makes the home country lower its capital control tax rate. We conclude that regardless of whether a country is borrowing or lending, retaliation by the other country lowers its own capital control taxes relative to the situation in which the other country is passive.

Table 11.1 compares equilibrium macroeconomic outcomes under alternative capital control arrangements. Not surprisingly, intertemporal trade, as measured by the absolute size of the current account, is the largest under free capital mobility and the smallest under optimal capital controls with Nash retaliation. Also not surprisingly, a country's welfare is the highest when it imposes optimal capital controls and the other country is passive. It is somewhat surprising, however, that the home country is better off under optimal capital controls with retaliation than under free capital mobility. Thus, it is optimal for the home country to impose capital controls regardless of whether this triggers a capital control war or not. This is not the

⁴Exercise 11.11 asks you to characterize the equilibrium in this case.

case for the foreign country, which prefers free capital mobility to a capital control war. An interesting question is whether it pays for the foreign country to compensate the home country for abiding to free capital mobility. Exercise 11.14 asks you to address this issue. Finally, if one country imposes optimal capital controls unilaterally, it is in the interest of the other country to retaliate. To see this, note that in both countries welfare is higher under retaliation than in the absence thereof.

11.10 Summing Up

- Capital controls drive a wedge between the domestic interest rate and the world interest rate.
 - If under free capital mobility, a small country borrows from the rest of the world, then the imposition of capital controls drives domestic interest rates up, depresses current consumption, and improves the current account.
 - If under free capital mobility, a small country lends to the rest of the world, then the imposition of capital outflow controls lowers domestic interest rates, increases current consumption, and worsens the current account.
 - In a small open economy without distortions, capital controls are always welfare decreasing.
 - In the presence of borrowing externalities, capital controls can be welfare increasing, as they can be effective in eliminating overborrowing.
 - In a two-country world, free capital mobility is in general preferred to financial autarky.
 - In a two-country world, free capital mobility results in a Pareto optimal

allocation, that is, any other feasible allocation makes at least one country worse off.

- For an economy that has market power in global financial markets it might be welfare improving to impose capital controls to move the world interest rate in its favor.

- A large economy that runs a current account deficit benefits from imposing controls on capital inflows that drive the world interest rate down provided the rest of the world does not retaliate.

- A large economy that runs a current account surplus benefits from imposing controls on capital outflows that drive the world interest rate up provided the rest of the world does not retaliate.

- In a two-country world, the allocation under optimal capital controls fails to be Pareto efficient, that is, there is a feasible reallocation of resources that would make at least one country better off without making the other country worse off.

- In a two-country world, if one country imposes optimal capital controls unilaterally, it is in the interest of the other country to retaliate.

- In a two-country world, it may be welfare improving for one country to initiate a capital control war.

11.11 Exercises

Exercise 11.1 (Capital Controls and the Current account.) Consider a small open two-period economy populated by identical households with preferences given by

$$\ln C_1 + \ln C_2,$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively. Households are endowed with $Q_1 = 5$ units of goods in period 1 and $Q_2 = 10$ units in period 2. In period 1, households can borrow or lend at the interest rate r . Let D_1 denote the amount of debt of the household in period 1. Assume that the initial level of debt is zero, $D_0 = 0$. The world interest rate, denoted r^* , is 10 percent.

1. Calculate the equilibrium current account in period 1 under free capital mobility.
2. Now assume that the government introduces capital controls in period 1. Specifically, the government charges foreign lenders a proportional tax τ on the amount of debt extended to domestic residents. So foreign lenders pay τD_1 to the government in period 1. Suppose that the tax rate is 10 percent. In period 1, the government transfers all of these revenues to households via a lump-sum transfer denoted T . The government does not intervene in the economy in period 2.
 - (a) Calculate the interest rate differential.
 - (b) Calculate the equilibrium current account balance in period 1.

Compare this situation to what happens under free capital mobility and provide intuition.

- (c) Suppose now that instead of setting it at 10 percent, the government sets τ to a level consistent with a 50 percent reduction in the current account relative to the level prevailing under free capital mobility (i.e., when $\tau = 0$). Calculate the equilibrium capital control tax rate. Calculate the new interest rate differential. Discuss your results.

Exercise 11.2 (Effectiveness of Taxes on Capital Inflows) In section 11.2, we saw that an increase in the tax rate on capital inflows, τ , affects real variables, such as consumption in periods 1 and 2, the trade balance, the current account, and the level of external debt. Show that there is a level $\bar{\tau}$ beyond which the tax on capital inflows has no effect on real variables. Derive a formula expressing $\bar{\tau}$ as a function of Q_1 , Q_2 , and i^* . Provide intuition on how $\bar{\tau}$ depends on each of these exogenous variables.

Exercise 11.3 (Quantitative Capital Flow Restrictions) Consider a two-period model of a small open economy with a single good each period and no investment. Let preferences of the household be described by the lifetime utility function

$$\sqrt{C_1} + \beta\sqrt{C_2}.$$

Assume that $\beta = 1/1.1$. The household has initial net foreign wealth of $(1 + i_0^*)B_0 = 1$, with $i_0^* = 0.1$, and is endowed with $Q_1 = 5$ units of goods in period 1 and $Q_2 = 10$ units in period 2. The world interest rate paid

on assets held from period 1 to period 2, i^* , equals 10% (i.e., $i^* = 0.1$) and there is free international capital mobility.

1. Calculate the equilibrium levels of consumption in period 1, C_1 , consumption in period 2, C_2 , the trade balance in period 1, TB_1 , and the current account balance in period 1, CA_1 .
2. Suppose now that the government imposes quantitative restrictions on capital flows limiting the country's net foreign asset position at the end of period 1 to be nonnegative ($B_1 \geq 0$). Compute the equilibrium value of the domestic interest rate, i , consumption in periods 1 and 2, and the trade and current account balances in period 1.
3. Evaluate the effect of quantitative capital controls on welfare. Specifically, find the level of lifetime utility under capital controls and compare it to the level of utility obtained under free capital mobility.
4. For this question and the next, suppose that the country experiences a temporary increase in the endowment of period 1 to $Q_1 = 9$, with the period-2 endowment unchanged. Calculate the effect of this output shock on C_1 , C_2 , TB_1 , CA_1 , and i in the case that capital is freely mobile across countries.
5. Finally, suppose that the capital controls described in question 2 are in place. Will they still be binding (i.e., affect household behavior)?

Exercise 11.4 (Quantitative Controls on Capital Outflows) Consider the equilibrium with quantitative capital controls analyzed in section 11.3

and depicted in figure 11.3. Suppose the equilibrium allocation under free capital mobility is at a point on the intertemporal budget constraint located northwest of the endowment point A. Suppose that capital controls prohibit borrowing or lending internationally. Would it still be true that capital controls are welfare decreasing? Why or why not?

Exercise 11.5 (Forced Savings) Consider a two-period model of a small open endowment economy populated by households with preferences given by

$$\sqrt{C_1} + \sqrt{C_2},$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively. Households' endowments are 5 units of goods in period 1 and 10 units of goods in period 2. Households start period 1 with a zero net asset position, $B_0 = 0$, and the world interest rate is zero, $i^* = 0$.

1. Find consumption in periods 1 and 2, the country's net foreign asset position at the end of period 1, and the trade balance in periods 1 and 2.
2. Find the level of welfare in the economy.
3. Now assume that the government announces in period 1 that a strong nation is one with a positive net foreign asset position and that therefore the country must save more. In particular, assume that the government requires that the net foreign asset position at the end of period 1 be greater or equal to 2. That is, the government imposes capital

controls of the form $B_1 \geq 2$. Find the domestic interest rate that supports this allocation.

4. Find the level of welfare under capital controls and compare it to the level of welfare under free capital mobility. Provide intuition.

Exercise 11.6 (Borrowing Externalities) Consider a two-period small open economy populated by a large number of identical households with preferences given by

$$\ln C_1 + \ln C_2.$$

Households are endowed with $Q_1 = 1$ units of consumption in period 1 and with $Q_2 = 2$ units in period 2. They start period 1 with no debts or assets and can borrow or lend at the interest rate i . The interest rate at which the country can borrow or lend in international markets is given by

$$i^* = \begin{cases} 0 & \text{for } \bar{C}_1 \leq \bar{Q}_1 \\ \delta(\bar{C}_1 - \bar{Q}_1) & \text{for } \bar{C}_1 > \bar{Q}_1 \end{cases},$$

where $\delta = 0.5$ and \bar{C}_1 and \bar{Q}_1 denote the aggregate per capital levels of consumption and output in period 1. The country operates under free capital mobility.

1. Show that the country is not a lender in period 1.
2. Calculate the equilibrium levels of consumption in period 1, external debt in period 1, and the interest rate.
3. Compute the efficient allocation. Compare it with that of the compet-

itive equilibrium.

4. Calculate the capital control tax, denoted τ , that supports the efficient allocation as a competitive equilibrium. Assume that the government rebates the tax revenue to households in an lump-sum fashion. Calculate also the domestic and foreign interest rate that obtain in this equilibrium.

Exercise 11.7 (Financial Exclusion) Consider a two-period small open economy identical to that of exercise 11.6, except that the interest rate at which the country can borrow or lend in international markets is given by

$$i^* = \begin{cases} 10\% & \text{for } \bar{C}_1 \leq \bar{Q}_1 \\ 120\% & \text{for } \bar{C}_1 > \bar{Q}_1 \end{cases},$$

1. Show that the country is not a lender in period 1.
2. Show that the country does not borrow in period 1.
3. Calculate the equilibrium levels of consumption and the interest rate.
4. Show that the allocation that obtains in the competitive equilibrium is efficient.

Exercise 11.8 (Market Clearing in the Two-Country Model) Show that the market clearing condition in the world financial market, given by equation (11.24), implies a market clearing condition in the world goods market requiring that in equilibrium the world endowment of goods equals

the world consumption of goods in both periods,

$$C_1^h + C_1^f = Q_1^h + Q_1^f,$$

and

$$C_2^h + C_2^f = Q_2^h + Q_2^f.$$

Exercise 11.9 (Autarky in the Two-Country Economy) Show that in the two-country economy of section 11.5, both countries are better off under capital mobility than under autarky.

Exercise 11.10 (Capital Controls and Welfare in the Two-Country Economy)

Show that the home country is better off in the equilibrium with optimal capital controls set by the foreign country and no capital controls in the home country than in the autarkic equilibrium.

Exercise 11.11 (Optimal Capital Controls in a Large Lending Country)

Section 11.7 characterizes optimal capital controls in a large borrowing country (the foreign country) assuming that the lending country (the home country) adopts a passive stance. Follow the same approach to characterize optimal capital controls in the home country when the foreign country is passive.

Exercise 11.12 (The Offer Curve with Log-Linear Preferences) Show

that in the two-country economy of section 11.5 the offer curve of country

$j = h, f$ is given by

$$C_2^j = \frac{C_1^j Q_2^j}{2C_1^j - Q_1^j}.$$

Exercise 11.13 (Capital Controls and the Current Account Schedule)

Consider the two-country economy of section 11.7.

1. Show that the current account schedules of the home and foreign countries are given, respectively, by

$$CA_1^h = \frac{1}{2} \left(Q_1^h - \frac{Q_2^h}{1+i^*} \right)$$

and

$$CA_1^f = \frac{(1+i^*+\tau)Q_1^f - Q_2^f}{2(1+i^*)+\tau}.$$

2. Show that as long as the current account of the foreign country is negative, its current account schedule is an increasing function of the world interest rate i^* , holding τ constant. *Hint: Take the partial derivative of CA_1^f with respect to i^* and show that it is positive if $2Q_2^f - \tau Q_1^f > 0$. Then show that if $CA_1^f < 0$, then $Q_2^f - \tau Q_1^f > 0$.*
3. Show that the current account schedule of the foreign country is an increasing function of the capital control tax rate τ , holding constant i^* .

Exercise 11.14 (Country Compensation) In the two-country model with Nash retaliation studied in section 11.9, calculate whether it pays for the foreign country to give a lump-sum compensation to the home country in period 2 in exchange for the home country to maintain free capital mobility. Denote this compensation by G and assume that it is proportional to the foreign country's endowment in period 2, that is, $G = \gamma Q$, where $\gamma > 0$ is a

parameter, whose value you are asked to find. To answer this question, you can proceed as follows:

1. Compute welfare in the two countries under free capital mobility. These should be two functions of γ (and Q).
2. Find the value of γ that makes the home country indifferent between free capital mobility and receiving the gift and a capital control war and not receiving the gift (the latter welfare level can be read off table 11.1). The answer should be a number for γ .
3. With γ in hand, evaluate the level of welfare of the foreign country under capital mobility and paying the gift. Then compare this number with the foreign country's level of welfare under a capital control war and not paying the gift. (The latter number can also be read off table 11.1.)

Chapter 12

Monetary Policy and Nominal Exchange Rate Determination

Thus far, we have focused on the determination of *real* variables, such as consumption, the trade balance, the current account, and the real exchange rate. In this chapter, we study the determination of *nominal* variables, such as the nominal exchange rate, the price level, inflation, and the quantity of money.

We will organize ideas around using a theoretical framework (model) that is similar to the one presented in previous chapters, with one important modification: there is a demand for money.

An important question in macroeconomics is why households voluntarily choose to hold money. In the modern world, this question arises because

money takes the form of unbacked paper notes printed by the government. This kind of money, one that the government is not obliged to exchange for goods, is called fiat money. Clearly, fiat money is intrinsically valueless. One reason why people value money is that it facilitates transactions. In the absence of money, all purchases of goods must take the form of barter. Barter exchanges can be very difficult to arrange because they require double coincidence of wants. For example, a carpenter who wants to eat an ice cream must find an ice cream maker that is in need of a carpenter. Money eliminates the need for double coincidence of wants. In this chapter we assume that agents voluntarily hold money because it facilitates transactions.

12.1 The quantity theory of money

What determines the level of the nominal exchange rate? Why has the Euro been depreciating vis-a-vis the US dollar since its inception in 1999? The quantity theory of money asserts that a key determinant of the exchange rate is the quantity of money printed by central banks.

According to the quantity theory of money, people hold a more or less stable fraction of their income in the form of money. Formally, letting Y denote real income, M^d money holdings, and P the price level (i.e., the price of a representative basket of goods), then

$$M^d = \kappa P \cdot Y$$

This means that the *real* value of money, M^d/P , is determined by the level

of real activity of the economy. Let $m^d \equiv M^d/P$ denote the demand for real money balances. The quantity theory of money then maintains that m^d is determined by nonmonetary or real factors such as aggregate output, the degree of technological advancement, etc.. Let M^s denote the nominal money *supply*, that is, M^s represents the quantity of bills and coins in circulation plus checking deposits. Equilibrium in the money market requires that money demand be equal to money supply, that is,

$$\frac{M^s}{P} = m^d \quad (12.1)$$

A similar equilibrium condition has to hold in the foreign country. Let M^{*s} denote the foreign nominal money supply, P^* the foreign price level, and m^{*d} the demand for real balances in the foreign country. Then,

$$\frac{M^{*s}}{P^*} = m^{*d} \quad (12.2)$$

Let E denote the nominal exchange rate, defined as the domestic-currency price of the foreign currency. So, for example, if E refers to the dollar/euro exchange rate, then E stands for the number of US dollars necessary to purchase one euro. Let e denote the real exchange rate. As explained in previous chapters, e represents the relative price of a foreign basket of goods in terms of domestic baskets of goods. Formally,

$$e = \frac{E P^*}{P}$$

Using this expression along with (12.1) and (12.2), we can express the nom-

inal exchange rate, E , as

$$E = \frac{M}{M^*} \left(\frac{e m^*}{m} \right) \quad (12.3)$$

According to the quantity theory of money, not only m and m^* but also e are determined by non-monetary factors. The quantity of money, in turn, depends on the exchange rate regime maintained by the respective central banks. There are two polar exchange rate arrangements: flexible and fixed exchange rate regimes.

12.1.1 A floating (or Flexible) Exchange Rate Regime

Under a floating exchange rate regime, the market determines the nominal exchange rate E . In this case the level of the money supplies in the domestic and foreign countries, M^s and M^{*s} , are determined by the respective central banks and are, therefore, exogenous variables. Exogenous variables are those that are determined outside of the model. By contrast, the nominal exchange rate is an endogenous variable in the sense that its equilibrium value is determined within the model.

Suppose, for example, that the domestic central bank decides to increase the money supply M^s . It is clear from equation (12.3) that, all other things constant, the monetary expansion in the home country causes the nominal exchange rate E to depreciate by the same proportion as the increase in the money supply. (i.e., E increases). The intuition behind this effect is simple. An increase in the quantity of money of the domestic country increases the relative scarcity of the foreign currency, thus inducing an increase in the

relative price of the foreign currency in terms of the domestic currency, E . In addition, equation (12.1) implies that when M increases the domestic price level, P , increases in the same proportion as M . An increase in the domestic money supply generates inflation in the domestic country. The reason for this increase in prices is that when the central bank injects additional money balances into the economy, households find themselves with more money than they wish to hold. As a result households try to get rid of the excess money balances by purchasing goods. This increase in the demand for goods drives prices up.

Suppose now that the real exchange rate depreciates, (that is e goes up). This means that a foreign basket of goods becomes more expensive relative to a domestic basket of goods. A depreciation of the real exchange rate can be due to a variety of reason, such as a terms-of-trade shock or the removal of import barriers. If the central bank keeps the money supply unchanged, then by equation (12.3) a real exchange rate depreciation causes a depreciation (an increase) of the nominal exchange rate. Note that e and E increase by the same proportion. The price level P is unaffected because neither M nor m have changed (see equation (12.1)).

12.1.2 Fixed Exchange Rate Regime

Under a fixed exchange rate regime, the central bank determines E by intervening in the money market. So given E , M^{*s} , and $e m^{*s}/m^s$, equation (12.3) determines what M^s ought to be in equilibrium. Thus, under a fixed exchange rate regime, M^s is an endogenous variable, whereas E is exogenously determined by the central bank.

Suppose that the real exchange rate, e , experiences a depreciation. In this case, the central bank must reduce the money supply (that is, M^s must fall) to compensate for the real exchange rate depreciation. Indeed, the money supply must fall by the same proportion as the real exchange rate. In addition, the domestic price level, P , must also fall by the same proportion as e in order for real balances to stay constant (see equation (12.1)). This implies that we have a deflation, contrary to what happens under a floating exchange rate policy.

12.2 Fiscal deficits, inflation, and the exchange rate

The quantity theory of money provides a simple and insightful analysis of the relationship between money, prices, the nominal exchange rate, and real variables. However, it leaves a number of questions unanswered. For example, what is the effect of fiscal policy on inflation? What role do expectations about future changes in monetary and fiscal policy play for the determination of prices, exchange rates, and real balances? To address these questions, it is necessary to use a richer model. One that incorporates a more realistic money demand specification and that explicitly considers the relationship between monetary and fiscal policy.

In this section, we embed a money demand function into a model with a government sector, similar to the one used in chapter 14, to analyze the effects of fiscal deficits on the current account. Specifically, we consider a small-open endowment economy with free capital mobility, a single traded

good per period, and a government that levies lump-sum taxes to finance government purchases. For simplicity, we assume that there is no physical capital and hence no investment. Unlike the models studied thus far, we now assume that the economy exists, not just for 2 periods, but for an infinite number of periods. Such an economy is called an *infinite horizon* economy.

We discuss in detail each of the four building blocks that compose our monetary economy: (1) The demand for money; (2) Purchasing power parity; (3) Interest rate parity; and (4) The government budget constraint.

12.2.1 The Demand For Money

In the quantity theory, the demand for money is assumed to depend only on the level of real activity. In reality, however, the demand for money also depends on the nominal interest rate. In particular, money demand is decreasing in the nominal interest rate. The reason is that money is a non-interest-bearing asset. As a result, the opportunity cost of holding money is the nominal interest rate on alternative interest-bearing liquid assets, such as time deposits, government bonds, and money market mutual funds. Thus, the higher the nominal interest rate the lower is the demand for real money balances. Formally, we assume a money demand function of the form:

$$\frac{M_t}{P_t} = L(\bar{C}, i_t), \quad (12.4)$$

where \bar{C} denotes consumption and i_t denotes the domestic nominal interest rate in period t . The function L is increasing in consumption and decreasing in the nominal interest rate. We assume that consumption is constant over

time. Therefore C does not have a time subscript. We indicate that consumption is constant by placing a bar over C . The money demand function $L(\cdot, \cdot)$ is also known as the *liquidity preference function*. Readers interested in learning how a money demand like equation (12.4) can be derived from the optimization problem of the household should consult the appendix to this chapter.

12.2.2 Purchasing power parity (PPP)

Because in the economy under consideration there is a single traded good and no barriers to international trade, purchasing power parity must hold. Let P_t be the domestic currency price of the good in period t , P_t^* the foreign currency price of the good in period t , and E_t the nominal exchange rate in period t , defined as the price of one unit of foreign currency in terms of domestic currency. Then PPP implies that in any period t

$$P_t = E_t P_t^*$$

For simplicity, assume that the foreign currency price of the good is constant and equal to 1 ($P_t^* = 1$ for all t). In this case, it follows from PPP that the domestic price level is equal to the nominal exchange rate,

$$P_t = E_t. \tag{12.5}$$

Using this relationship, we can write the liquidity preference function (12.4) as

$$\frac{M_t}{E_t} = L(\bar{C}, i_t), \quad (12.6)$$

12.2.3 The interest parity condition

In this economy, there is free capital mobility and no uncertainty. Thus, the gross domestic nominal interest rate must be equal to the gross world nominal interest rate times the expected gross rate of devaluation of the domestic currency. This relation is called the *uncovered interest parity condition*. Formally, let E_{t+1}^e denote the nominal exchange rate that agents expect at time t to prevail at time $t + 1$, and let i_t denote the domestic nominal interest rate, that is, the rate of return on an asset denominated in domestic currency and held from period t to period $t + 1$. Then the uncovered interest parity condition is:

$$1 + i_t = (1 + r^*) \frac{E_{t+1}^e}{E_t} \quad (12.7)$$

In the absence of uncertainty, the nominal exchange rate that will prevail at time $t + 1$ is known at time t , so that $E_{t+1}^e = E_{t+1}$. Then, the uncovered interest parity condition becomes

$$1 + i_t = (1 + r^*) \frac{E_{t+1}}{E_t} \quad (12.8)$$

This condition has a very intuitive interpretation. The left hand side is the gross rate of return of investing 1 unit of domestic currency in a domestic currency denominated bond. Because there is free capital mobility, this investment must yield the same return as investing 1 unit of domestic

currency in foreign bonds. One unit of domestic currency buys $1/E_t$ units of the foreign bond. In turn, $1/E_t$ units of the foreign bond pay $(1 + r^*)/E_t$ units of foreign currency in period $t + 1$, which can then be exchanged for $(1 + r^*)E_{t+1}/E_t$ units of domestic currency.¹

12.2.4 The government budget constraint

The government has three sources of income: real tax revenues, T_t , money creation, $M_t - M_{t-1}$, and interest earnings from holdings of international bonds, $E_t r^* B_{t-1}^g$, where B_{t-1}^g denotes the government's holdings of foreign currency denominated bonds carried over from period $t - 1$ into period t and r^* is the international interest rate. Government bonds, B_t^g , are denominated in foreign currency and pay the world interest rate r^* . The government allocates its income to finance government purchases, $P_t G_t$, where G_t denotes real government consumption of goods in period t , and to changes in its holdings of foreign bonds, $E_t(B_t^g - B_{t-1}^g)$. Thus, in period t , the government budget constraint is

$$E_t(B_t^g - B_{t-1}^g) + P_t G_t = P_t T_t + (M_t - M_{t-1}) + E_t r^* B_{t-1}^g$$

¹Here two comments are in order. First, in chapter 10, we argued that free capital mobility implies that *covered* interest rate parity holds. The difference between covered and uncovered interest rate parity is that covered interest rate parity uses the forward exchange rate F_t to eliminate foreign exchange rate risk, whereas uncovered interest rate parity uses the expected future spot exchange rate, E_{t+1}^e . In general, F_t and E_{t+1}^e are not equal to each other. However, under certainty $F_t = E_{t+1}^e = E_{t+1}$, so covered and uncovered interest parity are equivalent. Second, in chapter 10 we further argued that free capital mobility implies that covered interest parity must hold for *nominal* interest rates. However, in equation (12.7) we used the world *real* interest rate r^* . In the context of our model this is okay because we are assuming that the foreign price level is constant ($P^* = 1$) so that, by the Fisher equation (??), the nominal world interest rate must be equal to the real world interest rate ($i_t^* = r_t^*$).

The left hand side of this expression represents the government's uses of revenue and the right hand side the sources. Note that B_t^g is not restricted to be positive. If B_t^g is positive, then the government is a creditor, whereas if it is negative, then the government is a debtor.² We can express the government budget constraint in real terms by dividing the left and right hand sides of the above equation by the price level P_t . Using the result that $E_t = P_t$, and after rearranging terms, we have

$$B_t^g - B_{t-1}^g = \frac{M_t - M_{t-1}}{P_t} - [G_t - T_t - r^* B_{t-1}^g] \quad (12.9)$$

The first term on the right hand side measures the government's real revenue from money creation and is called *seignorage revenue*,

$$\text{seignorage revenue} = \frac{M_t - M_{t-1}}{P_t}.$$

The second term on the right hand side of (12.9) is the *secondary fiscal deficit* and we will denote it by DEF_t . Recall from chapter 14 that the secondary fiscal deficit is given by the difference between government expenditures and income from the collection of taxes and interest income from bond holdings. Formally, DEF_t is defined as

$$DEF_t = (G_t - T_t) - r^* B_{t-1}^g$$

²Note that the notation here is different from the one used in chapter 14, where B_t^g denoted the level of government debt.

In chapter 14, we also defined the primary fiscal deficit as the difference between government expenditures and tax revenues ($G_t - T_t$), so that the secondary fiscal deficit equals the difference between the primary fiscal deficit and interest income from government holdings of interest bearing assets.

Using the definition of secondary fiscal deficit and the fact that by PPP $P_t = E_t$, the government budget constraint can be written as

$$B_t^g - B_{t-1}^g = \frac{M_t - M_{t-1}}{E_t} - DEF_t \quad (12.10)$$

This equation makes it transparent that a fiscal deficit ($DEF_t > 0$) must be associated with money creation ($M_t - M_{t-1} > 0$) or with a decline in the government's asset position ($B_t^g - B_{t-1}^g < 0$), or both. To complete the description of the economy, we must specify the exchange rate regime, to which we turn next.

12.3 A fixed exchange rate regime

Under a fixed exchange rate regime, the government intervenes in the foreign exchange market in order to keep the exchange rate at a fixed level. Let that fixed level be denoted by E . Then $E_t = E$ for all t . When the government pegs the exchange rate, the money supply becomes an endogenous variable because the central bank must stand ready to exchange domestic for foreign currency at the fixed rate E . With the nominal exchange rate E , the PPP condition, given by equation (12.5), implies that the price level, P_t , is also constant and equal to E for all t . Because the nominal exchange rate is

constant, the expected rate of devaluation is zero. This implies, by the interest parity condition (12.8), that the domestic nominal interest rate, i_t , is constant and equal to the world interest rate r^* . It then follows from the liquidity preference equation (12.6) that the demand for nominal balances is constant and equal to $EL(\bar{C}, r^*)$. Since in equilibrium money demand must equal money supply, we have that the money supply is also constant over time: $M_t = M_{t-1} = EL(\bar{C}, r^*)$. Using the fact that the money supply is constant, the government budget constraint (12.10) becomes

$$B_t^g - B_{t-1}^g = -DEF_t \quad (12.11)$$

In words, when the government pegs the exchange rate, it loses one source of revenue, namely, seignorage. Therefore, fiscal deficits must be entirely financed through the sale of interest bearing assets.

12.3.1 Fiscal deficits and the sustainability of currency pegs

For a fixed exchange rate regime to be sustainable over time, it is necessary that the government displays fiscal discipline. To see this, suppose that the government runs a constant secondary fiscal deficit, say $DEF_t = DEF > 0$ for all t . Equation (12.11) then implies that government assets are falling over time ($B_t^g - B_{t-1}^g = -DEF < 0$). At some point B_t^g will become negative, which implies that the government is a debtor. Suppose that there is an upper limit on the size of the public debt. Clearly, when the public debt hits this limit, the government is forced to eliminate the fiscal deficit (i.e., set $DEF = 0$), or default on its debt (as Greece did in 2012), or

abandon the exchange rate peg. The latter alternative is called a *balance of payments crisis*. We will analyze balance of payments crises in more detail in section 12.6.

The fiscal consequences of a devaluation

Consider now the effects of a once-and-for-all devaluation of the domestic currency. We will show that this policy is equivalent to a lump-sum tax. To see this, assume that in period 1 the government unexpectedly announces an increase in the nominal exchange rate from E to $E' > E$, that is, $E_t = E'$ for all $t \geq 1$. By the PPP condition, equation (12.5), the domestic price level, P_t , jumps up in period 1 from E to E' and remains at that level thereafter.

By the interest rate parity condition (12.8), we have that the nominal interest rate in period 1 is given by

$$1 + i_t = (1 + r^*) \frac{E_2}{E_1} = (1 + r^*) \frac{E'}{E} = (1 + r^*).$$

Because the nominal interest rate was equal to r^* before period 1, it follows that an unexpected, once-and-for-all devaluation has no effect on the domestic nominal interest rate. The reason why the nominal interest rate remains unchanged is that it depends on the *expected future* rather than the *current* rate of devaluation. In period 0, households did not expect the government to devalue the domestic currency in period 1. Therefore, the expected devaluation rate in period 0 was zero and the nominal interest rate was equal to r^* . In period 1, households expect no further devaluations of the domes-

tic currency in period 2, therefore the nominal interest rate is also equal to r^* in period 1. Similarly, because agents expect the nominal exchange rate to be constant and equal to E' for all future periods, the expected rate of devaluation is nil and the nominal interest rate equals r^* forever.

Using the fact that the nominal interest rate is unchanged, the liquidity preference equation (12.6) then implies that in period 1 the demand for nominal money balances increases from $EL(\bar{C}, r^*)$ to $E'L(\bar{C}, r^*)$. This means that the demand for nominal balances must increase by the same proportion as the nominal exchange rate. Consider now the government budget constraint in period 1.

$$\begin{aligned} B_1^g - B_0^g &= \frac{M_1 - M_0}{E'} - DEF \\ &= \frac{E'L(\bar{C}, r^*) - EL(\bar{C}, r^*)}{E'} - DEF \end{aligned}$$

The numerator of the first term on the right-hand side of the last equality is clearly positive, since $E' > E$. Therefore, in period 1 seignorage revenue is positive. In the absence of a devaluation, seignorage revenue would have been nil because in that case $M_1 - M_0 = EL(\bar{C}, r^*) - EL(\bar{C}, r^*) = 0$. Therefore, a devaluation increases government revenue in the period in which the devaluation takes place. In the periods after the devaluation, $t = 2, 3, 4, \dots$, the nominal money demand is constant and equal to $E'L(\bar{C}, r^*)$, so that $M_t - M_{t-1} = 0$ for all $t \geq 2$ and seignorage revenue is nil.

Summarizing, by PPP, a devaluation produces an increase in the domestic price level of the same proportion as the increase in the nominal exchange rate. Given the households' holdings of nominal money balances the increase

in the price level implies that real balances will decline. Thus, a devaluation acts as a tax on real balances. In order to rebuild their desired real balances, which don't change because the nominal interest rate is unaffected by the devaluation, households will sell part of their foreign bonds to the central bank in return for domestic currency. The net effect of a devaluation is, therefore, that the private sector ends up with a lower foreign asset position but the same level of real balances, whereas the government gains real resources as it exchanges money created by itself for interest-bearing foreign assets.

12.4 A constant-money-growth-rate regime

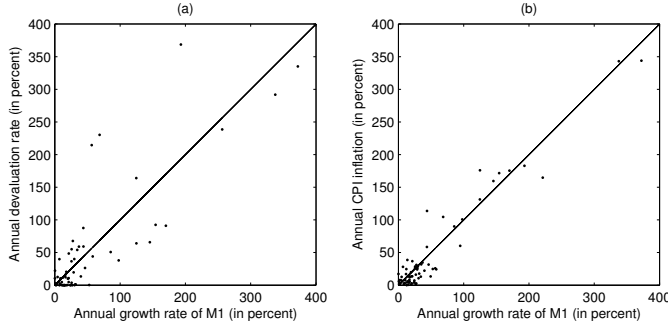
We now consider a monetary policy regime in which the central bank targets a certain path for the money supply and does not directly target a path for the nominal exchange rate. For this reason, we say that the central bank lets the nominal exchange *float*. The monetary/exchange rate regime studied here is exactly the opposite to the one studied in subsection 12.3, where the central bank fixed the nominal exchange rate and let the quantity of money be market (or endogenously) determined.

Consider a specific target for the path of the money supply in which the central bank expands the quantity of money at a constant, positive rate μ each period, so that

$$M_t = (1 + \mu)M_{t-1} \tag{12.12}$$

Our goal is to find out how the endogenous variables of the model, such as the nominal exchange rate, the price level, real balances, the domestic

Figure 12.1: Devaluation, inflation, and money growth. Argentina 1901-2005



nominal interest rate, and so forth behave under the monetary/exchange rate regime specified by equation (12.12). To do this, we will conjecture (or guess) that in equilibrium the nominal exchange rate depreciates at the rate μ . We will then verify that our guess is correct. Thus, we are guessing that

$$\frac{E_{t+1}}{E_t} = 1 + \mu,$$

for $t = 1, 2, \dots$. Because PPP holds and the foreign price level is one (i.e., $P_t = E_t$), the domestic price level must also grow at the rate of monetary expansion μ ,

$$\frac{P_{t+1}}{P_t} = 1 + \mu.$$

for $t = 1, 2, \dots$. This expression says that, given our guess, the rate of inflation must equal the rate of growth of the money supply. Panels (a) and (b) of figure 12.1 display annual averages of the rate of depreciation of the Argentine currency vis-à-vis the U.S. dollar, the Argentine money growth rate, and the Argentine inflation rate for the period 1901-2005. (We omitted

the years 1984, 1985, 1989, 1990 where annual money growth rates exceeded 400 percent.) The data is roughly consistent with the model in showing that there exists a close positive relationship between these three variables.³

To determine the domestic nominal interest rate i_t , use the interest parity condition (12.8)

$$1 + i_t = (1 + r^*) \frac{E_{t+1}}{E_t} = (1 + r^*)(1 + \mu),$$

which implies that the nominal interest rate is constant and increasing in μ . When μ is positive, the domestic nominal interest rate exceeds the real interest rate r^* because the domestic currency is depreciating over time. We summarize the positive relationship between i_t and μ by writing

$$i_t = i(\mu)$$

The notation $i(\mu)$ simply indicates that i_t is a function of μ . The function $i(\mu)$ is increasing in μ . Substituting this expression into the liquidity preference function (12.6) yields

$$\frac{M_t}{E_t} = L(\bar{C}, i(\mu)). \quad (12.13)$$

Note that \bar{C} is a constant and that because the money growth rate μ is

³Strictly speaking, the model predicts that all points in both figures should lie on a straight line, which is clearly not the case. The reason for this discrepancy may be that the model abstracts from a number of real world factors that affect the relationship between money growth, inflation, and depreciation. For example, in the model we assume that there is no domestic growth, that all goods are traded, that PPP holds, and that foreign inflation is constant.

constant, the nominal interest rate $i(\mu)$ is also constant. Therefore, the right hand side of (12.13) is constant. For the money market to be in equilibrium, the left-hand side of (12.13) must also be constant. This will be the case only if the exchange rate depreciates—grows—at the same rate as the money supply. This is indeed true under our initial conjecture that $E_{t+1}/E_t = 1 + \mu$. Equation (12.13) says that in equilibrium real money balances must be constant and that the higher the money growth rate μ the lower the equilibrium level of real balances.

12.5 The Inflation Tax

Let's now return to the government budget constraint (12.10), which we reproduce below for convenience

$$B_t^g - B_{t-1}^g = \frac{M_t - M_{t-1}}{E_t} - DEF_t$$

Let's analyze the first term on the right-hand side of this expression, seigniorage revenue. Using the fact that $M_t = E_t L(\bar{C}, i(\mu))$ (equation (12.13)), we can write

$$\begin{aligned} \frac{M_t - M_{t-1}}{E_t} &= \frac{E_t L(\bar{C}, i(\mu)) - E_{t-1} L(\bar{C}, i(\mu))}{E_t} \\ &= L(\bar{C}, i(\mu)) \left(\frac{E_t - E_{t-1}}{E_t} \right) \end{aligned}$$

Using the fact that the nominal exchange rate depreciates at the rate μ , that is, $E_t = (1 + \mu)E_{t-1}$, to eliminate E_t and E_{t-1} from the above expression, we can write seignorage revenue as

$$\frac{M_t - M_{t-1}}{E_t} = L(\bar{C}, i(\mu)) \left(\frac{\mu}{1 + \mu} \right) \quad (12.14)$$

Thus, seignorage revenue is equal to the product of real balances, $L(\bar{C}, i(\mu))$, and the factor $\mu/(1 + \mu)$.

The right hand side of equation (12.14) can also be interpreted as the *inflation tax*. The idea is that inflation acts as a tax on the public's holdings of real money balances. To see this, let's compute the change in the real value of money holdings from period $t-1$ to period t . In period $t-1$ nominal money holdings are M_{t-1} which have a real value of M_{t-1}/P_{t-1} . In period t the real value of M_{t-1} is M_{t-1}/P_t . Therefore we have that the inflation tax equals $M_{t-1}/P_{t-1} - M_{t-1}/P_t$, or, equivalently,

$$\text{inflation tax} = \frac{M_{t-1}}{P_{t-1}} \frac{P_t - P_{t-1}}{P_t}$$

where M_{t-1}/P_{t-1} is the tax base and $(P_t - P_{t-1})/P_t$ is the tax rate. Using the facts that in our model real balances are equal to $L(\bar{C}, i(\mu))$ and that $P_t/P_{t-1} = 1 + \mu$, the inflation tax can be written as

$$\text{inflation tax} = L(\bar{C}, i(\mu)) \frac{\mu}{1 + \mu},$$

which equals seignorage revenue. In general seignorage revenue and the inflation tax are not equal to each other. They are equal in the special case

that real balances are constant over time, like in our model when the money supply expands at a constant rate.

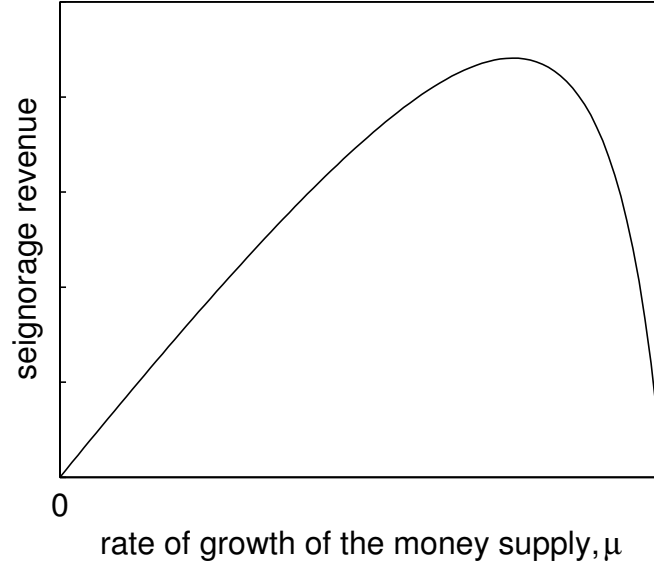
12.5.1 The Inflation Tax Laffer Curve

Because the tax base, i.e., real balances, is decreasing in μ and the tax rate, $\mu/(1+\mu)$, is increasing in μ , it is not clear whether seignorage increases or decreases with the rate of expansion of the money supply. Whether seignorage revenue is increasing or decreasing in μ depends on the form of the liquidity preference function $L(\cdot, \cdot)$ as well as on the level of μ itself. Typically, for low values of μ seignorage revenue is increasing in μ . However, as μ gets large the contraction in the tax base (the money demand) dominates the increase in the tax rate and therefore seignorage revenue falls as μ increases. Thus, there exists a *maximum level of revenue* a government can collect from printing money. The resulting relationship between the growth rate of the money supply and seignorage revenue has the shape of an inverted-U and is called the *inflation tax Laffer curve* (see figure 12.2).

12.5.2 Inflationary finance

We now use the theoretical framework developed thus far to analyze the link between fiscal deficits, prices, and the exchange rate. Consider a situation in which the government is running constant fiscal deficits $DEF_t = DEF > 0$ for all t . Furthermore, assume that the government has reached its borrowing limit and thus cannot finance the fiscal deficits by issuing additional debt, so that $B_t^g - B_{t-1}^g$ must be equal to zero. Under these circumstances,

Figure 12.2: The Laffer curve of inflation



the government budget constraint (12.10) becomes

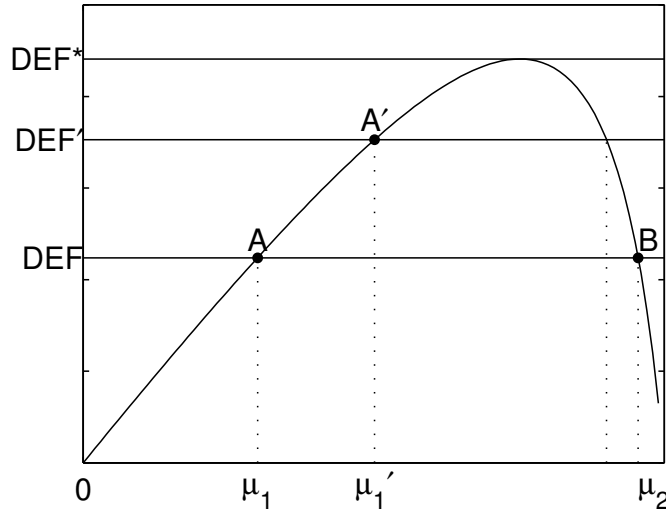
$$DEF = \frac{M_t - M_{t-1}}{E_t}$$

It is clear from this expression, that a country that has exhausted its ability to issue public debt must resort to printing money in order to finance the fiscal deficit. This way of financing the public sector is called *monetization of the fiscal deficit*. Combining the above expression with (12.14) we obtain

$$DEF = L(\bar{C}, i(\mu)) \left(\frac{\mu}{1 + \mu} \right). \quad (12.15)$$

Figure 12.3 illustrates the relationship between fiscal deficits and the rate of monetary expansion implied by this equation. The Laffer curve of inflation

Figure 12.3: Inflationary finance and the Laffer curve of inflation



corresponds to the right hand side of (12.15). The horizontal line plots the left hand side (12.15), or DEF . There are two rates of monetary expansion, μ_1 and μ_2 , that generate enough seignorage revenue to finance the fiscal deficit DEF . Thus, there exist two equilibrium levels of monetary expansion associated with a fiscal deficit equal to DEF . In the μ_2 equilibrium, point B in the figure, the rates of inflation and of exchange rate depreciation are relatively high and equal to μ_2 , whereas in the μ_1 equilibrium, point A in the figure, the rates of inflation and depreciation are lower and equal to μ_1 . Empirical studies show that in reality, economies tend to be located on the upward sloping branch of the Laffer curve. Thus, the more realistic scenario is described by point A.

Consider now the effect of an increase in the fiscal deficit from DEF to $DEF' > DEF$. To finance the larger fiscal deficit, the government is forced

to increase the money supply at a faster rate. At the new equilibrium, point A' , the rate of monetary expansion, μ_1' is greater than at the old equilibrium. As a result, the inflation rate, the rate of depreciation of the domestic currency, and the nominal interest rate are all higher.

The following numerical example provides additional insight on the connection between money creation and fiscal deficits. Suppose that the liquidity preference function is given by:

$$\frac{M_t}{E_t} = \gamma \bar{C} \left(\frac{1 + i_t}{i_t} \right)$$

Suppose that the government runs a fiscal deficit of 10% of GDP ($DEF/Q = 0.1$), that the share of consumption in GDP is 65% ($\bar{C}/Q = 0.65$), that the world real interest rate is 5% per year ($r^* = 0.05$), and that γ is equal to 0.2. The question is what is the rate of monetary expansion necessary to monetize the fiscal deficit. Combining equations (12.5.2) and (12.15) and using the fact $1 + i_t = (1 + r^*)(1 + \mu)$ we have,

$$DEF = \gamma \bar{C} \frac{(1 + r^*)(1 + \mu)}{(1 + r^*)(1 + \mu) - 1} \frac{\mu}{1 + \mu}$$

Divide the left and right hand sides of this expression by Q and solve for μ to obtain

$$\mu = \frac{r^*(DEF/Q)}{(1 + r^*)(\gamma(\bar{C}/Q) - (DEF/Q))} = \frac{0.05 \times 0.1}{1.05 \times (0.2 \times 0.65 - 0.1)} = 0.16$$

The government must increase the money supply at a rate of 16% per year. This implies that both the rates of inflation and depreciation of the domestic

currency in this economy will be 16% per year. The nominal interest rate is 21% per year. At a deficit of 10% of GDP, the Laffer curve is rather flat. For example, if the government cuts the fiscal deficit by 1% of GDP, the equilibrium money growth rate falls to 11%.

In some instances, inflationary finance can degenerate into hyperinflation. Perhaps the best-known episode is the German hyperinflation of 1923. Between August 1922 and November 1923, Germany experienced an average monthly inflation rate of 322 percent.⁴ More recently, in the late 1980s a number of hyperinflationary episodes took place in Latin America and Eastern Europe. One of the more severe cases was Argentina, where the inflation rate averaged 66 percent per month between May 1989 and March 1990.

A hyperinflationary situation arises when the fiscal deficit reaches a level that can no longer be financed by seignorage revenue alone. In terms of figure 12.3, this is the case when the fiscal deficit is larger than DEF^* , the level of deficit associated with the peak of the Laffer curve. What happens in practice is that the government is initially unaware of the fact that no rate of monetary expansion will suffice to finance the deficit. In its attempt to close the fiscal gap, the government accelerates the rate of money creation. But this measure is counterproductive because the government has entered the downward sloping side of the Laffer curve. The decline in seignorage revenue leads the government to increase the money supply at an even faster rate. These dynamics turn into a vicious cycle that ends in an accelerating inflationary spiral. The most fundamental step in ending hyperinflation is

⁴A fascinating account of four Post World War I European hyperinflations is given in Sargent, "The End of Four Big Inflations," in Robert Hall, editor, *Inflation: Causes and Effects*, The University of Chicago Press, Chicago, 1982.

to eliminate the underlying budgetary imbalances that are at the root of the problem. When this type of structural fiscal reforms is undertaken and is understood by the public, hyperinflation typically stops abruptly.

12.5.3 Money growth and inflation in a growing economy

Thus far, we have considered the case in which consumption is constant over time.⁵ We now wish to consider the case that consumption is growing over time. Specifically, we will assume that consumption grows at a constant rate $\gamma > 0$, that is,

$$C_{t+1} = (1 + \gamma)C_t.$$

We also assume that the liquidity preference function is of the form

$$L(C_t, i_t) = C_t l(i_t)$$

where $l(\cdot)$ is a decreasing function.⁶ Consider again the case that the government expands the money supply at a constant rate $\mu > 0$. As before, we find the equilibrium by first guessing the value of the depreciation rate and then verifying that this guess indeed can be supported as an equilibrium outcome. Specifically, we conjecture that the domestic currency depreciates

⁵Those familiar with the appendix will recognize that the constancy of consumption is a direct implication of our assumption that the subjective discount rate is equal to the world interest rate, that is, $\beta(1 + r^*) = 1$. It is clear from (12.19) that consumption will grow over time only if $\beta(1 + r^*)$ is greater than 1.

⁶Can you show that this form of the liquidity preference function obtains when the period utility function is given by $\ln C_t + \theta \ln(M_t/E_t)$. Under this particular preference specification find the growth rate of consumption γ as a function of β and $1 + r^*$.

at the rate $(1 + \mu)/(1 + \gamma) - 1$, that is,

$$\frac{E_{t+1}}{E_t} = \frac{1 + \mu}{1 + \gamma}$$

Our conjecture says that given the rate of monetary expansion, the higher the rate of economic growth, the lower the rate of depreciation of the domestic currency. In particular, if the government wishes to keep the domestic currency from depreciating, it can do so by setting the rate of monetary expansion at a level no greater than the rate of growth of consumption ($\mu \leq \gamma$). By interest rate parity,

$$\begin{aligned} (1 + i_t) &= (1 + r^*) \frac{E_{t+1}}{E_t} \\ &= (1 + r^*) \frac{(1 + \mu)}{(1 + \gamma)} \end{aligned}$$

This expression says that the nominal interest rate is constant over time.

We can summarize this relationship by writing

$$i_t = i(\mu, \gamma), \quad \text{for all } t$$

where the function $i(\mu, \gamma)$ is increasing in μ and decreasing in γ .

We continue to assume that PPP and that $P_t^* = 1$, which implies that the domestic price level, P_t , must be equal to the nominal exchange rate, E_t . It follows that the domestic rate of inflation must be equal to the rate

of depreciation of the nominal exchange rate, that is,

$$\frac{P_t - P_{t-1}}{P_{t-1}} = \frac{E_t - E_{t-1}}{E_{t-1}} = \frac{1 + \mu}{1 + \gamma} - 1$$

Equilibrium in the money market requires that the real money supply be equal to the demand for real balances, that is,

$$\frac{M_t}{E_t} = C_t l(i(\mu, \gamma)),$$

The right-hand side of this expression is proportional to consumption, and therefore grows at the gross rate $1 + \gamma$. The numerator of the left hand side grows at the gross rate $1 + \mu$. Therefore, in equilibrium the denominator of the left hand side must expand at the gross rate $(1 + \mu)/(1 + \gamma)$, which is precisely our conjecture.

Summarizing, when consumption growth is positive, the domestic inflation rate is *lower* than the rate of monetary expansion. The intuition for this result is straightforward. A given increase in the money supply that is not accompanied by an increase in the demand for real balances will translate into a proportional increase in prices. This is because in trying to get rid of their excess nominal money holdings households attempt to buy more goods. But since the supply of goods is unchanged the increased demand for goods will be met by an increase in prices. This is a typical case of "more money chasing the same amount of goods." When the economy is growing, the demand for real balances is also growing. That means that part of the increase in the money supply will not end up chasing goods but rather will

end up in the pockets of consumers.

12.6 Balance-of-payments crises

A balance of payments, or BOP, crisis is a situation in which the government is unable or unwilling to meet its financial obligations. These difficulties may manifest themselves in a variety of ways, such as the failure to honor the domestic and/or foreign public debt or the suspension of currency convertibility.

What causes BOP crises? Sometimes a BOP crisis arises as the inevitable consequence of unsustainable combinations of monetary and fiscal policies. A classic example of such a policy mix is a situation in which a government pegs the nominal exchange rate and at the same time runs a fiscal deficit. As we discussed in subsection [12.3](#), under a fixed exchange rate regime, the government must finance any fiscal deficit by running down its stock of interest bearing assets (see equation [\(12.11\)](#)). Clearly, to the extent that there is a limit to the amount of debt a government is able to issue, this situation cannot continue indefinitely. When the public debt hits its upper limit, the government is forced to change policy. One possibility is that the government stops servicing the debt (i.e., stops paying interest on its outstanding financial obligations), thereby reducing the size of the secondary deficit. This alternative was adopted by Mexico in August of 1982, when it announced that it would be unable to honor its debt commitments according to schedule, marking the beginning of what today is known as the Developing Country Debt Crisis. A second possibility is that the govern-

ment adopt a fiscal adjustment program by cutting government spending and raising regular taxes and in that way reduce the primary deficit. Finally, the government can abandon the exchange rate peg and resort to monetizing the fiscal deficit. This has been the fate of the vast majority of currency pegs adopted in developing countries. The economic history of Latin America of the past two decades is plagued with such episodes. For example, the currency pegs implemented in Argentina, Chile, and Uruguay in the late 1970s, also known as *tablitas*, ended with large devaluations in the early 1980s; similar outcomes were observed in the Argentine Austral stabilization plan of 1985, the Brazilian Cruzado plan of 1986, the Mexican plan of 1987, and, more recently the Brazilian Real plan of 1994.

An empirical regularity associated with the collapse of fixed exchange rate regimes is that in the days immediately before the peg is abandoned, the central bank loses vast amounts of reserves in a short period of time. The loss of reserves is the consequence of a run by the public against the domestic currency in anticipation of the impending devaluation. The stampede of people trying to massively get rid of domestic currency in exchange for foreign currency is driven by the desire to avoid the loss of real value of domestic currency denominated assets that will take place when the currency is devalued.

The first formal model of the dynamics of a fixed exchange rate collapse is due to Paul R. Krugman of Princeton University.⁷ In this section, we will analyze these dynamics using the tools developed in sections 12.3 and 12.4.

⁷The model appeared in Paul R. Krugman, "A Model of Balance-of-Payments Crisis," *Journal of Money, Credit and Banking*, 11, 1979, 311-325.

These tools will be helpful in a natural way because, from an analytical point of view, the collapse of a currency peg is indeed a transition from a fixed to a floating exchange rate regime.

Consider a country that is running a constant fiscal deficit $DEF > 0$ each period. Suppose that in period 1 the country embarks in a currency peg. Specifically, assume that the government fixes the nominal exchange rate at E units of domestic currency per unit of foreign currency. Suppose that in period 1, when the currency peg is announced, the government has a positive stock of foreign assets carried over from period 0, $B_0^g > 0$. Further, assume that the government does not have access to credit. That is, the government asset holdings are constrained to being nonnegative, or $B_t^g \geq 0$ for all t . It is clear from our discussion of the sustainability of currency pegs in subsection 12.3 that, as long as the currency peg is in effect, the fiscal deficit produces a continuous drain of assets, which at some point will be completely depleted. Put differently, if the fiscal deficit is not eliminated, at some point the government will be forced to abandon the currency peg and start printing money in order to finance the deficit. Let T denote the period in which, as a result of having run out of reserves, the government abandons the peg and begins to monetize the fiscal deficit.

The dynamics of the currency crisis are characterized by three distinct phases. (1) The pre-collapse phase: during this phase, which lasts from $t = 1$ to $t = T - 2$, the currency peg is in effect. (2) The BOP crisis: It takes place in period $t = T - 1$, and is the period in which the central bank faces a run against the domestic currency, resulting in massive losses of foreign reserves. (3) The post-collapse phase: It encompasses the period from $t = T$ onwards

In this phase, the nominal exchange rate floats freely and the central bank expands the money supply at a rate consistent with the monetization of the fiscal deficit.

(1) The pre-crisis phase: from $t = 1$ to $t = T - 2$

From period 1 to period $T - 2$, the exchange rate is pegged, so the variables of interest behave as described in section 12.3. In particular, the nominal exchange rate is constant and equal to E , that is, $E_t = E$ for $t = 1, 2, \dots, T - 2$. By PPP, and given our assumption that $P_t^* = 1$, the domestic price level is also constant over time and equal to E ($P_t = E$ for $t = 1, 2, \dots, T - 2$). Because the exchange rate is fixed, the devaluation rate $(E_t - E_{t-1})/E_{t-1}$, is equal to 0. The nominal interest, i_t , which by the uncovered interest parity condition satisfies $1 + i_t = (1 + r^*)E_{t+1}/E_t$, is equal to r^* . Note that the nominal interest rate in period $T - 2$ is also equal to r^* because the exchange rate peg is still in place in period $T - 1$. Thus, $i_t = r^*$ for $t = 1, 2, \dots, T - 2$.

As discussed in section 12.3, by pegging the exchange rate the government relinquishes its ability to monetize the deficit. This is because the nominal money supply, M_t , which in equilibrium equals $EL(\bar{C}, r^*)$, is constant, and as a result seignorage revenue, given by $(M_t - M_{t-1})/E$, is nil. Consider now the dynamics of foreign reserves. By equation (12.11),

$$B_t^g - B_{t-1}^g = -DEF; \quad \text{for } t = 1, 2, \dots, T - 2.$$

This expression shows that the fiscal deficit causes the central bank to lose DEF units of foreign reserves per period. The continuous loss of reserves

in combination with the lower bound on the central bank's assets, makes it clear that a currency peg is unsustainable in the presence of persistent fiscal imbalances.

(3) The post-crisis phase: from $t = T$ onwards

The government starts period T without any foreign reserves ($B_{T-1}^g = 0$). Given our assumptions that the government cannot borrow (that is, B_t^g cannot be negative) and that it is unable to eliminate the fiscal deficit, it follows that in period T the monetary authority is forced to abandon the currency peg and to print money in order to finance the fiscal deficit. Thus, in the post-crisis phase the government lets the exchange rate float. Consequently, the behavior of all variables of interest is identical to that studied in subsection 12.4. In particular, the government will expand the money supply at a constant rate μ that generates enough seignorage revenue to finance the fiscal deficit. In section 12.4, we deduced that μ is determined by equation (12.15),

$$DEF = L(\bar{C}, i(\mu)) \left(\frac{\mu}{1 + \mu} \right)$$

Note that because the fiscal deficit is positive, the money growth rate must also be positive. In the post-crisis phase, real balances, M_t/E_t are constant and equal to $L(\bar{C}, i(\mu))$. Therefore, the nominal exchange rate, E_t , must depreciate at the rate μ . Because in our model $P_t = E_t$, the price level also grows at the rate μ , that is, the inflation rate is positive and equal to μ . Finally, the nominal interest rate satisfies $1 + i_t = (1 + r^*)(1 + \mu)$. Let's

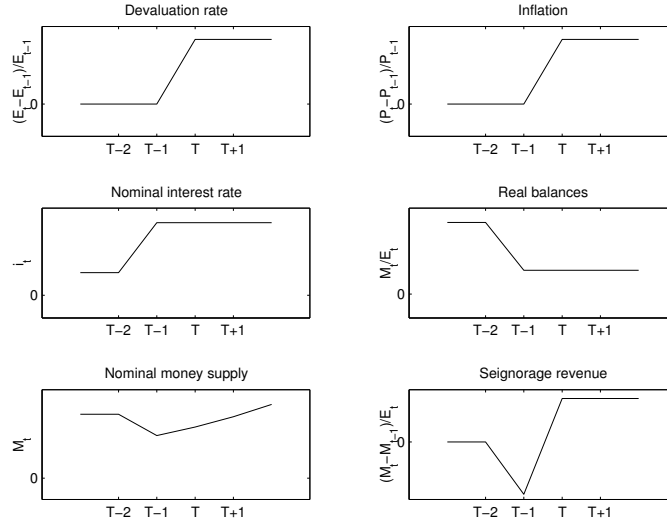
compare the economy's pre- and post-crisis behavior. The first thing to note is that with the demise of the fixed exchange rate regime, price level stability disappears as inflation sets in. In the pre-crisis phase, the rate of monetary expansion, the rate of devaluation, and the rate of inflation are all equal to zero. By contrast, in the post-crisis phase these variables are all positive and equal to μ . Second, the sources of deficit finance are very different in each of the two phases. In the pre-crisis phase, the deficit is financed entirely with foreign reserves. As a result, foreign reserves display a steady decline during this phase. On the other hand, in the post-crisis phase the fiscal deficit is financed through seignorage income and foreign reserves are constant (and in our example equal to zero). Finally, in the post-crisis phase real balances are lower than in the pre-crisis phase because the nominal interest rate is higher.

(2) The BOP crisis: period $T - 1$

In period $T - 1$, the exchange rate peg has not yet collapsed. Thus, the nominal exchange rate and the price level are both equal to E , that is $E_{T-1} = P_{T-1} = E$. However, the nominal interest rate is not r^* , as in the pre-crisis phase, because in period $T - 1$ the public expects a depreciation of the domestic currency in period T . The rate of depreciation of the domestic currency between periods $T - 1$ and T is μ , that is, $(E_T - E_{T-1})/E_{T-1} = \mu$.⁸

⁸For technically inclined readers: To see that $(E_T - E_{T-1})/E_{T-1} = \mu$, use the fact that in $T - 1$ real balances are given by $M_{T-1}/E_{T-1} = L(\bar{C}, (1 + r^*)E_T/E_{T-1} - 1)$ and that in period T the government budget constraint is $DEF = L(\bar{C}, i(\mu)) - (M_{T-1}/E_{T-1})(E_{T-1}/E_T)$. These are two equations in two unknowns, M_{T-1}/E_{T-1} and E_T/E_{T-1} . If we set $E_T/E_{T-1} = 1 + \mu$, then the two equations collapse to (12.15) indicating that $E_T/E_{T-1} = 1 + \mu$ and $M_{T-1}/E_{T-1} = L(\bar{C}, i(\mu))$ are indeed the solution.

Figure 12.4: The dynamics of a balance-of-payments crisis



Therefore, the nominal interest rate in period $T - 1$ jumps up to its post-crisis level $i_{T-1} = (1 + r^*)(1 + \mu) - 1 = i(\mu)$. As a result of the increase in the nominal interest rate real balances fall in $T - 1$ to their post-crisis level, that is, $M_{T-1}/E = L(\bar{C}, i(\mu))$. Because the nominal exchange rate does not change in period $T - 1$, the decline in real balances must be brought about entirely through a fall in nominal balances: the public runs to the central bank to exchange domestic currency for foreign reserves. Thus, in period $T - 1$ foreign reserves at the central bank fall by more than DEF . To see this more formally, evaluate the government budget constraint (12.10) at

$t = T - 1$ to get

$$\begin{aligned}
 B_{T-1}^g - B_{T-2}^g &= \frac{M_{T-1} - M_{T-2}}{E} - DEF \\
 &= L(\bar{C}, i(\mu)) - L(\bar{C}, r^*) - DEF \\
 &< -DEF
 \end{aligned}$$

The second equality follows from the fact that $M_{T-1}/E = L(\bar{C}, i(\mu))$ and $M_{T-2}/E = L(\bar{C}, r^*)$. The inequality follows from the fact that $i(\mu) = (1 + r^*)(1 + \mu) - 1 > r^*$ and the fact that the liquidity preference function is decreasing in the nominal interest rate. The above expression formalizes Krugman's original insight on why the demise of currency pegs is typically preceded by a speculative run against the domestic currency and large losses of foreign reserves by the central bank: Even though the exchange rate is pegged in $T - 1$, the nominal interest rate rises in anticipation of a devaluation in period T causing a contraction in the demand for real money balances. Because in period $T - 1$ the domestic currency is still fully convertible, the central bank must absorb the entire decline in the demand for money by selling foreign reserves. Figure 12.4 closes this section by providing a graphical summary of the dynamics of Krugman-type BOP crises.

12.7 Appendix: A dynamic optimizing model of the demand for money

In this section we develop a dynamic optimizing model underlying the liquidity preference function given in equation (12.6). We motivate a demand for money by assuming that money facilitates transactions. We capture the fact that money facilitates transactions by simply assuming that agents derive utility not only from consumption of goods but also from holdings of real balances. Specifically, in each period $t = 1, 2, 3, \dots$ preferences are described by the following single-period utility function,

$$u(C_t) + z\left(\frac{M_t}{P_t}\right),$$

where C_t denotes the household's consumption in period t and M_t/P_t denotes the household's real money holdings in period t . The functions $u(\cdot)$ and $z(\cdot)$ are strictly increasing and strictly concave functions ($u' > 0$, $z' > 0$, $u'' < 0$, $z'' < 0$).

Households are assumed to be infinitely lived and to care about their entire stream of single-period utilities. However, households discount the future by assigning a greater weight to consumption and real money holdings the closer they are to the present. Specifically, their lifetime utility function is given by

$$\left[u(C_t) + z\left(\frac{M_t}{P_t}\right)\right] + \beta \left[u(C_{t+1}) + z\left(\frac{M_{t+1}}{P_{t+1}}\right)\right] + \beta^2 \left[u(C_{t+2}) + z\left(\frac{M_{t+2}}{P_{t+2}}\right)\right] + \dots$$

Here β is a number greater than zero and less than one called the *subjective*

discount factor.” The fact that households care more about the present than about the future is reflected in β being less than one.

Let’s now analyze the budget constraint of the household. In period t , the household allocates its wealth to purchase consumption goods, $P_t C_t$, to hold money balances, M_t , to pay taxes, $P_t T_t$, and to purchase interest bearing foreign bonds, $E_t B_t^p$. Taxes are lump sum and denominated in domestic currency. The foreign bond is denominated in foreign currency. Each unit of foreign bonds costs 1 unit of the foreign currency, so each unit of the foreign bond costs E_t units of domestic currency. Foreign bonds pay the constant world interest rate r^* in foreign currency. Note that because the foreign price level is assumed to be constant, r^* is not only the interest rate in terms of foreign currency but also the interest rate in terms of goods. That is, r^* is the *real* interest rate.⁹ The superscript p in B_t^p , indicates that these are bond holdings of *private* households, to distinguish them from the bond holdings of the government, which we will introduce later. In turn, the household’s wealth at the beginning of period t is given by the sum of its money holdings carried over from the previous period, M_{t-1} , bonds purchased in the previous period plus interest, $E_t(1 + r^*)B_{t-1}^p$, and income from the sale of its endowment of goods, $P_t Q_t$, where Q_t denotes the household’s endowment of goods in period t . This endowment is assumed to be exogenous, that is, determined outside of the model. The budget

⁹The domestic nominal and real interest rates will in general not be equal to each other unless domestic inflation is zero. To see this, recall the Fisher equation (??). We will return to this point shortly.

constraint of the household in period t is then given by:

$$P_t C_t + M_t + P_t T_t + E_t B_t^p = M_{t-1} + (1 + r^*) E_t B_{t-1}^p + P_t Q_t \quad (12.16)$$

The left hand side of the budget constraint represents the uses of wealth and the right hand side the sources of wealth. The budget constraint is expressed in nominal terms, that is, in terms of units of domestic currency. To express the budget constraint in real terms, that is, in units of goods, we divide both the left and right hand sides of (12.16) by P_t , which yields

$$C_t + \frac{M_t}{P_t} + T_t + \frac{E_t}{P_t} B_t^p = \frac{M_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t} + (1 + r^*) \frac{E_t}{P_t} B_{t-1}^p + Q_t$$

Note that real balances carried over from period $t - 1$, M_{t-1}/P_{t-1} , appear multiplied by P_{t-1}/P_t . In an inflationary environment, P_t is greater than P_{t-1} , so inflation erodes a fraction of the household's real balances. This loss of resources due to inflation is called the inflation tax. The higher the rate of inflation, the larger the fraction of their income households must allocate to maintaining a certain level of real balances.

Recalling that P_t equals E_t , we can eliminate P_t from the utility function and the budget constraint to obtain:

$$\left[u(C_t) + z \left(\frac{M_t}{E_t} \right) \right] + \beta \left[u(C_{t+1}) + z \left(\frac{M_{t+1}}{E_{t+1}} \right) \right] + \beta^2 \left[u(C_{t+2}) + z \left(\frac{M_{t+2}}{E_{t+2}} \right) \right] + \dots \quad (12.17)$$

$$C_t + \frac{M_t}{E_t} + T_t + B_t^p = \frac{M_{t-1}}{E_t} + (1 + r^*) B_{t-1}^p + Q_t \quad (12.18)$$

Households choose C_t , M_t , and B_t^p so as to maximize the utility function (12.17) subject to a series of budget constraints like (12.18), one for each period, taking as given the time paths of E_t , T_t , and Q_t . In choosing streams of consumption, money balances, and bonds, the households faces two tradeoffs. The first tradeoff is between consuming today and saving today to finance future consumption. The second tradeoff is between consuming today and holding money today.

Consider first the tradeoff between consuming one extra unit of the good today and investing it in international bonds to consume the proceeds tomorrow. If the household chooses to consume the extra unit of goods today, then its utility increases by $u'(C_t)$. Alternatively, the household could sell the unit of good for 1 unit of foreign currency and with the proceeds buy 1 unit of the foreign bond. In period $t + 1$, the bond pays $1 + r^*$ units of foreign currency, with which the household can buy $(1 + r^*)$ units of goods. This amount of goods increases utility in period $t + 1$ by $(1 + r^*)u'(C_{t+1})$. Because households discount future utility at the rate β , from the point of view of period t , lifetime utility increases by $\beta(1 + r^*)u'(C_{t+1})$. If the first alternative yields more utility than the second, the household will increase consumption in period t , and lower consumption in period $t + 1$. This will tend to eliminate the difference between the two alternatives because it will lower $u'(C_t)$ and increase $u'(C_{t+1})$ (recall that $u(\cdot)$ is concave, so that $u'(\cdot)$ is decreasing). On the other hand, if the second alternative yields more utility than the first, the household will increase consumption in period $t + 1$ and decrease consumption in period t . An optimum occurs at a point where the household cannot increase utility further by shifting consumption across

time, that is, at an optimum the household is, in the margin, indifferent between consuming an extra unit of good today or saving it and consuming the proceeds the next period. Formally, the optimal allocation of consumption across time satisfies

$$u'(C_t) = \beta(1 + r^*)u'(C_{t+1}) \quad (12.19)$$

We will assume for simplicity that the subjective rate of discount equals the world interest rate, that is,

$$\beta(1 + r^*) = 1 \quad (12.20)$$

Combining this equation with the optimality condition (12.19) yields,

$$u'(C_t) = u'(C_{t+1}) \quad (12.21)$$

Because $u(\cdot)$ is strictly concave, $u'(\cdot)$ is monotonically decreasing, so this expressions implies that $C_t = C_{t+1}$. This relationship must hold in all periods, implying that consumption is constant over time. Let \bar{C} be this optimal level of consumption. Then, we have

$$C_t = C_{t+1} = C_{t+2} = \dots = \bar{C}$$

Consider now the tradeoff between spending one unit of money on consumption and holding it for one period. If the household chooses to spend the unit of money on consumption, it can purchase $1/E_t$ units of goods, which yield $u'(C_t)/E_t$ units of utility. If instead the household chooses to

keep the unit of money for one period, then its utility in period t increases by $z'(M_t/E_t)/E_t$. In period $t + 1$, the household can use the unit of money to purchase $1/E_{t+1}$ units of goods, which provide $u'(C_{t+1})/E_{t+1}$ extra utils. Thus, the alternative of keeping the unit of money for one period yields $z'(M_t/E_t)/E_t + \beta u'(C_{t+1})/E_{t+1}$ additional units of utility. In an optimum, the household must be indifferent between keeping the extra unit of money for one period and spending it on current consumption, that is,

$$\frac{z'(M_t/E_t)}{E_t} + \beta \frac{u'(C_{t+1})}{E_{t+1}} = \frac{u'(C_t)}{E_t} \quad (12.22)$$

Using the facts that $u'(C_t) = u'(C_{t+1}) = u'(\bar{C})$ and that $\beta = 1/(1 + r^*)$ and rearranging terms we have

$$z' \left(\frac{M_t}{E_t} \right) = u'(\bar{C}) \left[1 - \frac{E_t}{(1 + r^*)E_{t+1}} \right] \quad (12.23)$$

Using the uncovered interest parity condition (12.8) we can write

$$z' \left(\frac{M_t}{E_t} \right) = u'(\bar{C}) \left(\frac{i_t}{1 + i_t} \right) \quad (12.24)$$

This equation relates the demand for real money balances, M_t/E_t , to the level of consumption and the domestic nominal interest rate. Inspecting equation (12.24) and recalling that both u and z are strictly concave, reveals that the demand for real balances, M_t/E_t , is decreasing in the level of the nominal interest rate, i_t , and increasing in consumption, \bar{C} . This relationship is called the *liquidity preference function*. We write it in a compact

form as

$$\frac{M_t}{E_t} = L(\bar{C}, i_t)$$

which is precisely equation (12.6).

The following example derives the liquidity preference function for a particular functional form of the period utility function. Assume that

$$u(C_t) + z(M_t/E_t) = \ln C_t + \gamma \ln(M_t/E_t).$$

Then we have $u'(\bar{C}) = 1/\bar{C}$ and $z'(M_t/E_t) = \gamma/(M_t/E_t)$. Therefore, equation (12.24) becomes

$$\frac{\gamma}{M_t/E_t} = \frac{1}{\bar{C}} \left(\frac{i_t}{1+i_t} \right)$$

The liquidity preference function can be found by solving this expression for M_t/E_t . The resulting expression is in fact the liquidity preference function given in equation (12.5.2), which we reproduce here for convenience.

$$\frac{M_t}{E_t} = \gamma \bar{C} \left(\frac{i_t}{1+i_t} \right)^{-1}$$

In this expression, M_t/E_t is linear and increasing in consumption and decreasing in i_t .

12.8 The Monetary Policy Trilemma

Chapter 13

Nominal Rigidities, Exchange Rate Policy and Unemployment

(work in progress)

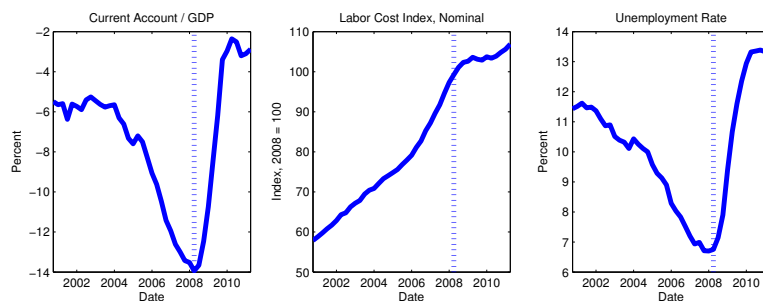
13.1 The Great Recession in Peripheral Europe: 2008-2011

The following figures illustrate that in the periphery of the euro area the sudden stop lead to unemployment rather than real depreciation.

Some Observations:

- Cyprus, Greece, Spain, and Portugal experienced a sudden stop in 2008.

Figure 13.1: Sudden Stops in Peripheral Europe: 2000-2011



Data Source: Eurostat. Data represents arithmetic mean of Bulgaria, Cyprus, Estonia, Greece, Lithuania, Latvia, Portugal, Spain, Slovenia, and Slovakia

- Large current account reversals
- Sudden Stops lead to unemployment

Question: What about the Real exchange rate?

Next graph plots: $e^{domestic/foreign} = SP^*/P$

A RER depreciation is when $e \uparrow$

RER scaled so that 2008 is 100

ex: a 5 percent real depreciation between 2008 and 2014 would be reflected in an increase in the RER index from 100 to 105

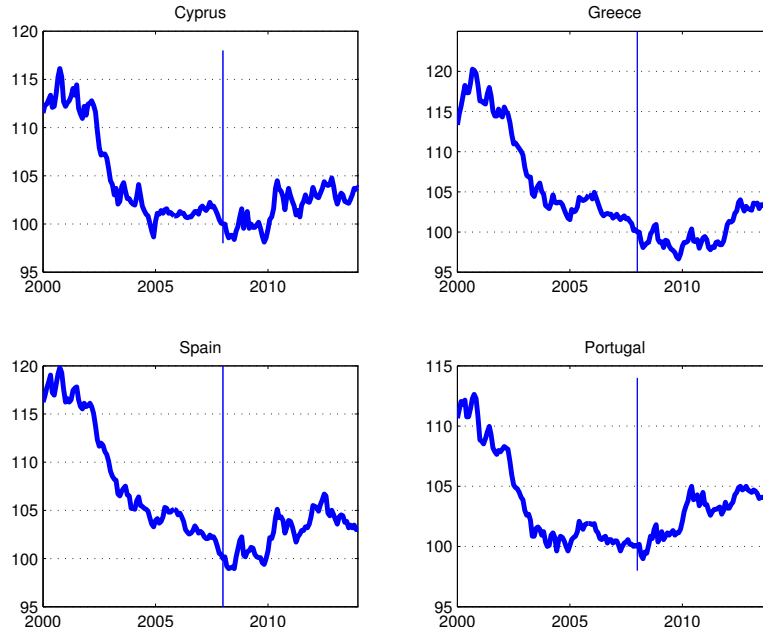
Observations on the figure:

Even 6 years after the sudden stop we see very little real depreciation, of less than 5 percent.

Compare this with the large real depreciations that we saw for the Sudden Stops in Chile, 1979-1985, (close to 100%) and in Argentina, 2001-2002, (about 200 %) .

Q: Why Do Sudden Stops lead to Unemployment in a Currency Union?

Possible Answer: Because nominal wages are downwardly rigid.

Figure 13.2: Real Exchange Rate, Periphery of Europe, e , (2008 = 100)

Incorporate the assumption of downward nominal wage rigidity by the following constraint:

$$W_t \geq \gamma W_{t-1}$$

where W_t = nominal wage rate in period t and γ is a parameter measuring the degree of downward nominal wage rigidity. If $\gamma = 1$, then nominal wage cuts are impossible. If $\gamma = 0$, then nominal wages are fully flexible. Schmitt-Grohé and Uribe (2013) survey empirical evidence on downward nominal rigidity and present some original evidence for the periphery of Europe. Their analysis suggests that γ is very close to unity.

Next we embed the assumption of downward nominal rigidity into the TNT model. We show that under a fixed exchange rate regime an economy

with downwardly rigid wages that experiences a sudden stop will suffer large involuntary unemployment and much less real depreciation than an economy with nominal wage flexibility.

13.2 The Model

small open economy

free capital mobility

2 periods

2 goods, traded and nontraded

Traded goods are an endowment, Q_1^T and Q_2^T .

Nontraded goods are produced with labor, $Q_t^N = F(h_t)$

P_t^N = nominal price of nontraded goods in period t

P_t^T = nominal price of traded goods in period t

P_t^* = foreign price of traded goods in period t

\mathcal{E}_t = nominal exchange rate

Law of one price holds for tradables: $P_t^T = \mathcal{E}_t P_t^*$

Assume that $P_t^* = 1$ for $t = 1, 2$. This implies that $P_t^T = \mathcal{E}_t$

$p_t = \frac{P_t^N}{P_t^T}$ relative price of nontradables, or RER in period t

13.2.1 The Production of Nontraded Goods

Nontraded goods are produced by perfectly competitive firms using labor, h_t , as the only factor input. The production function for nontraded goods

is given by:

$$Q_t^N = F(h_t),$$

where F is increasing and concave function. The latter assumption is made to ensure that the marginal product of labor is decreasing, that is, the production technology exhibits diminishing returns to scale. Nominal profits of firms operating in the nontraded sector are given by

$$P_t^N F(h_t) - W_t h_t,$$

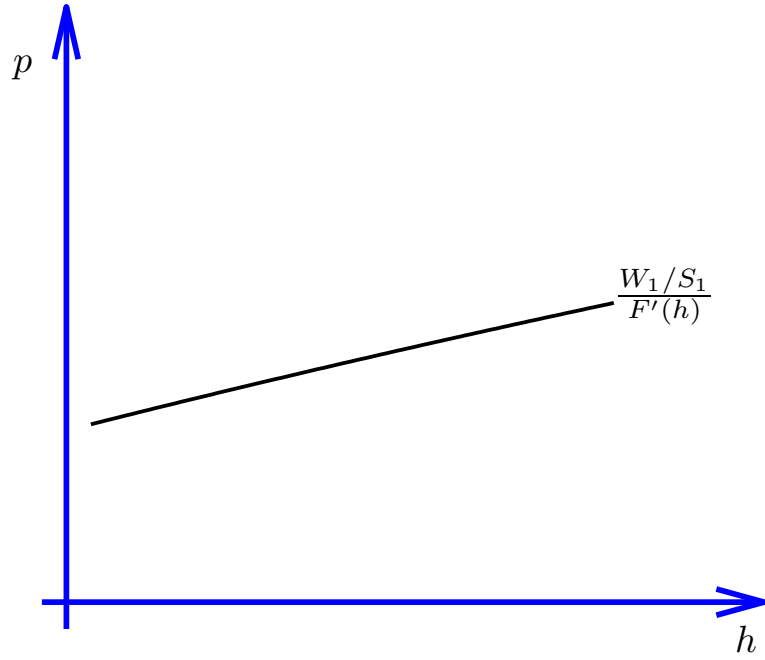
where W_t denotes the nominal hourly wage rate in period t . It will be convenient to express profits in terms of tradables and thus we divide nominal profits by P_t^T . This yields:

$$p_t F(h_t) - (W_t/\mathcal{E}_t) h_t$$

Notice that we used the definition $p_t = P_t^N/P_t^T$ and that by the LOOP $P_t^T = \mathcal{E}_t P_t^*$. Finally, we used the assumption that the foreign price level is equal to one, $P_t^* = 1$. Firms take as given the real exchange rate p_t and the real wage rate, W_t/\mathcal{E}_t . The profit maximizing choice of employment calls for equating the value of the marginal product of labor to the marginal cost of labor, $p_t F'(h_t) = \frac{W_t}{\mathcal{E}_t}$. Rearranging we can write this optimality condition as

$$p_t = \frac{\frac{W_t}{\mathcal{E}_t}}{F'(h_t)}.$$

Figure 13.3: The Supply of Nontraded Goods



By recognizing that Q_t^N is a monotonically increasing function of h_t , one can interpret this condition as a supply schedule of nontraded goods,

$$p_t = S(Q_t^N),$$

whereby supply, Q_t^N , and the relative price of the good, p_t , are positively related. Figure 13.3 shows this supply schedule in the space (h, p) with a solid upward sloping line. Why is the supply schedule upward sloping? All else constant, higher prices increase the value of the marginal product of labor but do not affect marginal cost and thus induce firms to produce more goods.

A potential shifter of this supply schedule is the real wage in terms of tradables given by W_t/\mathcal{E}_t . Suppose the real wage falls, either because the nominal wage falls, W_t declines, or the domestic currency depreciates, \mathcal{E}_t increases, or both, then the supply schedule will shift down and to the right. As we will discuss in detail below, our key departure from the models developed in earlier chapters is that nominal wages, W_t are downwardly rigid. Specifically, we assume that $W_t \geq W_{t-1}$, so that nominal wages cannot fall below W_{t-1} .

Further, we are studying sudden stops in the context of countries whose nominal exchange rate is fixed, either because they are on the Gold Standard, or because they are members of a currency union (like the euro area), or because they are simply pegging to another country's currency. So for most of our analysis \mathcal{E}_t will also be fixed, say at $\bar{\mathcal{E}}$. Notice that the combination of a fixed exchange rate monetary policy and downward nominal wage rigidity results in wages that are rigid downwards in real terms. This downward real rigidity in wages (expressed in terms of tradables) will be the key distortion in our model and is the reason why in this model we will have involuntary unemployment and insufficient real depreciation in response to a sudden stop. At the same time nominal wages are free to increase so that during a boom when nominal wages rise, the supply schedule can shift up and to the left.

We wish to determine how much firms produce in a given period, that is, we wish to find which point of the supply schedule will actually be chosen. We assume that production is demand determined, that is, firms will pick a pair (h_t, p_t) so that private households demand at that price all goods that

are produced, or $c_t^N = Q_t^N = F(h_t)$. Next we derive the demand schedule for nontradables of households.

13.2.2 The Problem of Households

C_t^N = nontraded good consumption in period t

C_t^T = traded good consumption in period t

Y_t = income of the household in period t (expressed in units of traded goods)

B_1^* = international bonds held by household at end of period 1, denominated in traded goods.

r_t = interest rate on assets held from t to $t + 1$

The household takes income, Y_1 and Y_2 , as exogenously given.

Preferences:

$$U(c_1^T, c_1^N) + U(c_2^T, c_2^N)$$

Budget constraint in period 1:

$$P_1^T c_1^T + P_1^N c_1^N + P_1^T B_1^* = P_1^T Y_1 + (1 + r_0) P_1^T B_0^*$$

Budget constraint in period 2:

$$P_2^T c_2^T + P_2^N c_2^N = P_2^T Y_2 + (1 + r_1) P_2^T B_1^*$$

Write budget constraint in terms of tradables, that is, divide by P_t^T : Budget

constraint in period 1:

$$c_1^T + p_1 c_1^N + B_1^* = Y_1 + (1 + r_0)B_0^*$$

Budget constraint in period 2:

$$c_2^T + p_2 c_2^N = Y_2 + (1 + r_1)B_1^*$$

Without loss of generality, assume that initial assets are zero, $B_0^* = 0$. The country enjoys free capital mobility. Then the single present value budget constraint becomes

$$c_1^T + p_1 c_1^N + \frac{c_2^T + p_2 c_2^N}{1 + r_1} = Y_1 + \frac{Y_2}{1 + r_1}$$

So we can state the household problem as follows: Pick c_1^T , c_1^N , c_2^T , c_2^N , taking as given p_1 , p_2 , Y_1 , Y_2 , and r_1 , to maximize:

$$U(c_1^T, c_1^N) + U(c_2^T, c_2^N)$$

subject to

$$c_1^T + p_1 c_1^N + \frac{c_2^T + p_2 c_2^N}{1 + r_1} = Y_1 + \frac{Y_2}{1 + r_1} \quad (13.1)$$

To characterize the optimal consumption choice of the household, solve the intertemporal budget constraint for c_1^T to obtain

$$c_1^T = Y_1 + \frac{Y_2}{1 + r_1} - p_1 c_1^N - \frac{c_2^T + p_2 c_2^N}{1 + r_1} = C(c_1^N, c_2^T, c_2^N).$$

Use this expression to eliminate c_1^T from the utility function. Then the household problem consists in picking c_1^N , c_2^T , and c_2^N .

The first-order condition to this problem with respect to c_1^N is

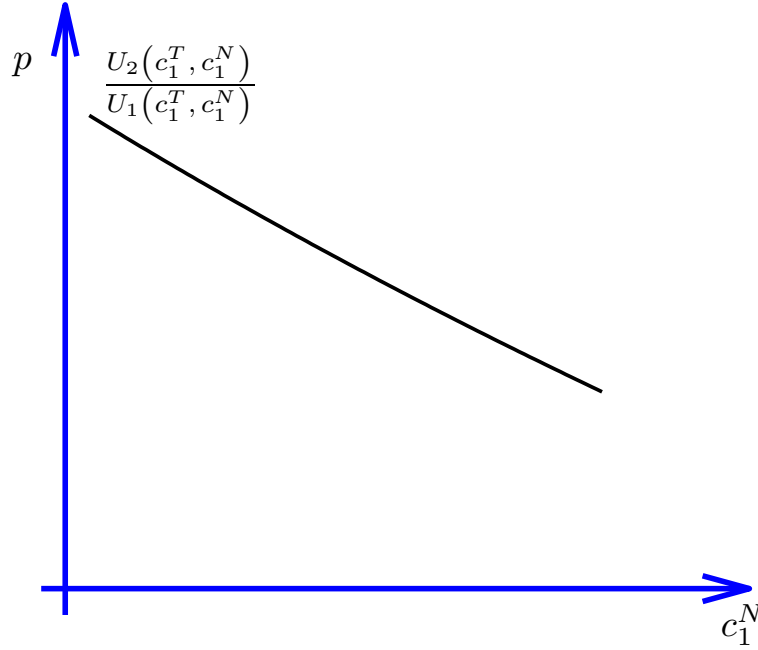
$$U_1(c_1^T, c_1^N)(-p_1) + U_2(c_1^T, c_1^N) = 0$$

Rearranging terms we have

$$\frac{U_2(c_1^T, c_1^N)}{U_1(c_1^T, c_1^N)} = p_1,$$

which says that the marginal rate of substitution between traded and non-traded good consumption in period 1 has to be equal to the relative price. The interpretation of this first-order condition is as follows. Suppose the household has 1 unit of traded good in period 1 and wants to decide to either consume it now or to sell it and buy nontraded goods for it now. The marginal utility of consuming the one unit of traded good in period 1 is: $U_1(c_1^T, c_1^N)$. If the household sells the unit of consumption and buys nontradables for it, how many nontraded good does he get? He obtains $1/p_1$ units of nontradables. How much additional utility do these nontraded goods generate? They increase utility in period 1 by $U_2(c_1^T, c_1^N)/p_1$. At the optimum the additional utility of consuming one more traded good in period 1 must be the same as that of exchanging the traded good for a nontraded one and then consuming the nontraded good in period 1. Hence it must be the case that $U_2(c_1^T, c_1^N)/p_1 = U_1(c_1^T, c_1^N)$, which is the same as the above first-order condition.

Figure 13.4: The Demand for Nontraded Goods



We will interpret this first-order condition as the demand for nontradables expressed as a function of the relative price of nontradables, p_t , for a given level of traded consumption c_1^T . Figure 13.4 plots this demand function in the space (c_1^N, p_1) . This demand function is downward sloping as long as both consumption of tradables and consumption of nontradables are normal goods. For example, suppose the period-1 utility function is of the form:

$$U(c^T, c^N) = a \ln c^T + (1 - a) \ln c^N$$

. Then the marginal rate of substitution is:

$$\frac{U_2(c_1^T, c_1^N)}{U_1(c_1^T, c_1^N)} = \frac{(1-a)}{a} \frac{c_1^T}{c_1^N}$$

In this case the demand function for nontradables is:

$$p_1 = \frac{(1-a)}{a} \frac{c_1^T}{c_2^N}.$$

Figure 13.4 plots the demand for c^N as a function of the relative price p_t , with consumption of tradables, c_1^T as a shifter. In equilibrium it must be the case that $c_t^N = F(h_t)$. So with some liberty we can plot this demand function in the space (h_t, p_t) rather than the space (c_t^N, p_t) . The demand function for nontraded hours continues to be a downward sloping function and c_1^T continues to be a shifter of this schedule. Figure 13.5 shows a graphical representation of this demand schedule.

We will consider a sudden stop, which we interpret as an increase in the world interest rate, r_1 . Recall that in the model with only a single traded good, studied in Chapter 3, a rise in the world interest rate in period 1, was shown to lower consumption in period 1 and increases it in period 2. We will show below that the same holds in the two-good model considered here. For the moment we just take it as given that when there is a sudden stop, i.e., when $r_1 \uparrow$, then $c_1^T \downarrow$. Our question is how does a sudden stop affect the demand function for nontradables. Figure 13.6 shows that a decline in the consumption of tradables from c_1^T to $\tilde{c}_1^T < c_1^T$ shifts the demand schedule down and to the left. That is, for the same price agents now demand less

Figure 13.5: The Demand for Nontraded Goods

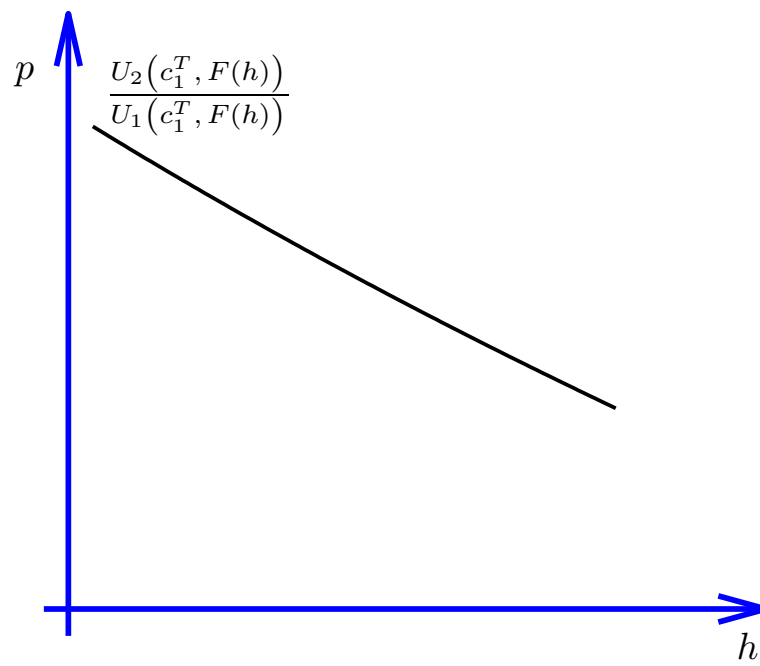
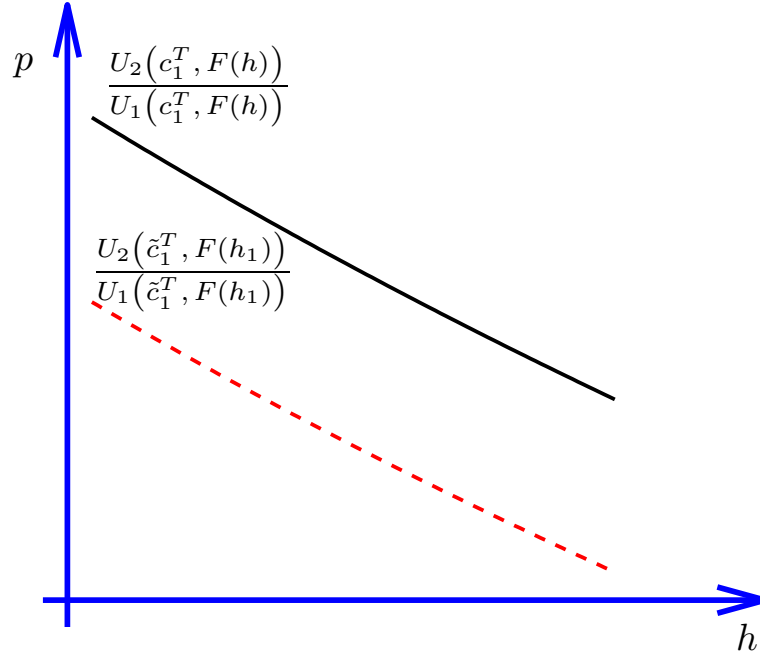


Figure 13.6: A Sudden Stop Shifts The Demand for Nontraded Goods Down



nontradables.

13.2.3 The Determination of Wages and Disequilibrium in the Labor Market

Households are assumed to supply \bar{h} hours inelastically. However, they might not be able to sell them all. Therefore, we have that

$$h_1 \leq \bar{h} \quad \text{and} \quad h_2 \leq \bar{h}.$$

The mechanism that typically leads to market clearing in the labor market, namely, that wages adjust until the demand for labor equals the supply

of labor is not working in this model. The reason is that nominal wages are assumed to be downwardly rigid, that is, nominal wages can rise but they cannot fall. In class, we saw some empirical evidence that nominal wages don't fall even in periods of rising unemployment and low inflation. We therefore impose that

$$W_1 \geq W_0 \quad \text{and} \quad W_2 \geq W_1.$$

In period 1, the level of past wages, W_0 , is an initial condition.

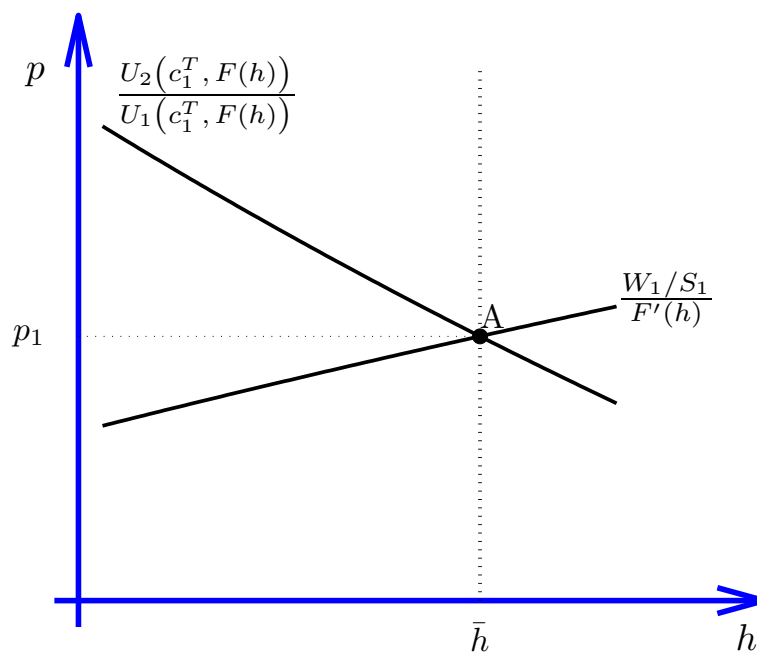
To determine wages and employment in the nontraded sector, we impose the following slackness condition

$$(\bar{h} - h_t)(W_t - W_{t-1}) = 0.$$

The interpretation of this slackness condition is that either there is full employment, $h_t = \bar{h}$ or the wage rigidity is binding, $W_t = W_{t-1}$. This condition rules out the case that there is both, involuntary unemployment ($h_t < \bar{h}$) and a non-binding wage constraint ($W_t > W_{t-1}$). Why can we not have both unemployment and a non-binding wage constraint. Suppose that at the price at which households are willing to buy $F(\bar{h})$ nontraded goods, firms are willing to supply more than this amount, that is, at that price and given wages, W_{t-1} , firms are demanding more than \bar{h} workers. Then the nominal wage, W_t , will increase, shifting the supply schedule up and to the left until demand and supply meet at $h_t = \bar{h}$. In this case we say the economy is in full employment and the downward wage rigidity is not binding.

Now consider a different situation. Suppose that given past wages, W_{t-1} , at

Figure 13.7: Employment and Real Exchange Rate Prior to Sudden Stop

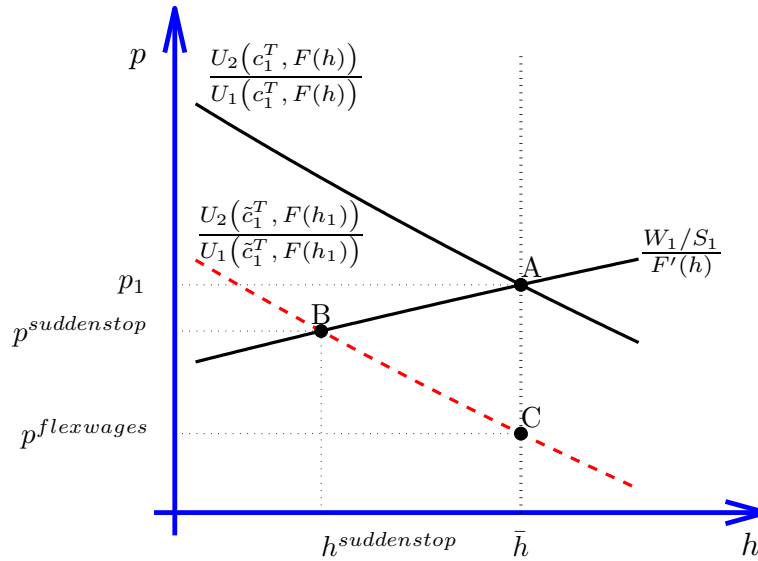


the price at which households are willing to buy $F(\bar{h})$ nontraded goods, firms are willing to supply only $F(h) < F(\bar{h})$ goods. This implies that firms are only willing to hire $h_t < \bar{h}$ workers, and $\bar{h} - h_t$ workers would be involuntarily unemployed. If nominal wages were fully flexible, the adjustment would take the form of falling wages until full employment is restored. However, this is not possible here, given our assumption that wages are downwardly rigid. Thus we have, that the downward wage rigidity is binding.

Consider the case that c_1^T and W_0 are such that there is full employment. This situation is shown with point A in figure 13.7.

Now suppose that there is a sudden stop (say the country loses access to international borrowing) and as a consequence the domestic demand

Figure 13.8: Sudden Stop leads to Unemployment and Little RER Depreciation



for traded goods falls ($c_1^T \downarrow$). As discussed earlier, the decline in traded consumption shifts the demand schedule down and to the left. The new demand is shown with the broken red downward-sloping line in figure 13.8. Because the economy has a fixed exchange rate, the nominal exchange rate \mathcal{E}_1 does not change, and because wages are downwardly rigid nominal wages do not fall. Hence, the adjustment to the sudden stop takes the form of unemployment. The economy moves to point B, where employment in the nontraded sector is only $h^{\text{suddenstop}}$. The real exchange rate depreciates from p_1 to $p_1^{\text{suddenstop}}$. In the absence of downward wage rigidity, or if the country could devalue its currency to boost employment, the sudden stop would move the economy to point C. At that point, there is full employment and the effect of the sudden stop is a large real depreciation to $p^{\text{flexwages}}$.

In that case, the sudden stop depresses consumption of tradables but the sudden stop does not spill over to the nontraded sector. Relative prices of nontradables fall so much that despite the negative income effect associated with the sudden stop, households continue to consume a level of nontradables that is consistent with full employment in that sector.

13.2.4 A Sudden Stop, $r_1 \uparrow$, leads to a contraction in traded consumption, $c_1^T \downarrow$

It remains to be shown that this model indeed implies that when the world interest rate increases in period 1, then the consumption of tradables in period 1, c_1^T , falls. This result is most easily demonstrated for the case that the intertemporal elasticity of substitution in consumption equals the intratemporal elasticity of substitution between traded and non traded goods. Assume therefore that preferences are of the following form:

$$U(c_1^T, c_1^N) + V(c_2^T, c_2^N) = a \ln c_1^T + (1 - a) \ln c_1^N + a \ln c_2^T + (1 - a) \ln c_2^N$$

Then use the optimality condition that the marginal rate of substitution between traded consumption in period 1 and traded consumption in period 2 must be equal to the relative price, that is, use:

$$\frac{U_1(c_1^T, c_1^N)}{V_1(c_2^T, c_2^N)} = 1 + r_1$$

Given the particular function form of preferences this becomes:

$$\frac{c_2^T}{c_1^T} = 1 + r_1 \quad (13.2)$$

Now consider the present value budget constraint of the household, equation (13.1), which is repeated here for convenience.

$$c_1^T + p_1 c_1^N + \frac{c_2^T + p_2 c_2^N}{1 + r_1} = Y_1 + \frac{Y_2}{1 + r_1} \quad (13.3)$$

Thus far, we have only said that income in periods 1 and 2 was exogenous and equal to Y_1 and Y_2 . We now specify what this income is. Households receive the endowment of tradables, Q_1^T and Q_2^T , labor income $W_1/\mathcal{E}_1 h_1$ and $W_2/\mathcal{E}_2 h_2$ and any profits from the ownership of firms producing nontradable goods: $p_1 F(h_1) - W_1/\mathcal{E}_1 h_1$ and $p_2 F(h_2) - W_2/\mathcal{E}_2 h_2$. Notice that all income is taken as given by the household. In particular, households supply labor inelastically, so they take $h_t \leq \bar{h}$ as exogenously given. Formally, we then have that

$$Y_1 = Q_1^T + p_1 Q_1^N$$

and

$$Y_2 = Q_2^T + p_2 Q_2^N$$

Notice that in equilibrium the market for nontraded goods must clear:

$$c_1^N = Q_1^N$$

$$c_2^N = Q_2^N$$

Therefore the present value budget constraint, equation (13.1), in equilibrium simplifies to:

$$c_1^T + \frac{c_2^T}{1+r_1} = Q_1^T + \frac{Q_2^T}{1+r_1}$$

Combining this expression with the household's first-order condition (13.2) yields:

$$c_1^T = \frac{1}{2} \left[Q_1^T + \frac{Q_2^T}{1+r_1} \right]$$

From here it immediately follows that an increase in the world interest rate, r_1 , lowers consumption of traded goods in period 1, which was what we wanted to show.

13.2.5 Managing a Currency Peg

In this section, we consider policies that the government could implement to avoid that a sudden stop, or a negative external shock, leads to unemployment. The most obvious solution perhaps would be to devalue, that is, increase \mathcal{E}_t , and in this way lower the real wage in terms of tradables to a level consistent with full employment. However, for members of the Euro area this would mean breaking away from the Euro and we assume that this is not possible. Given this, what other policy options are there. The main obstacle in achieving full employment is that the wage rate in terms of tradables, $W_1/(\mathcal{E}_1 P_1^*)$ is too high. One policy action is to lobby the monetary authority of the currency union to increase inflation in the union more generally. Specifically, if the foreign nominal price of tradables rises, $P_1^* \uparrow$, then unemployment should decline. For example, in the case of the unemployment problem in peripheral European countries, it means that the

European Central Bank should temporarily allow for more inflation.

An alternative strategy might be to target the labor market directly to achieve some more downward flexibility in nominal wages. It is unclear how quickly such structural reforms will increase downward flexibility.

A third option is to subsidize wages directly. Let τ_1 denote a wage subsidy paid to firms in the nontraded sector. Specifically, assume that the government pays a fraction τ of the wage bill of nontraded goods producers. In that case profits in the nontraded sector are

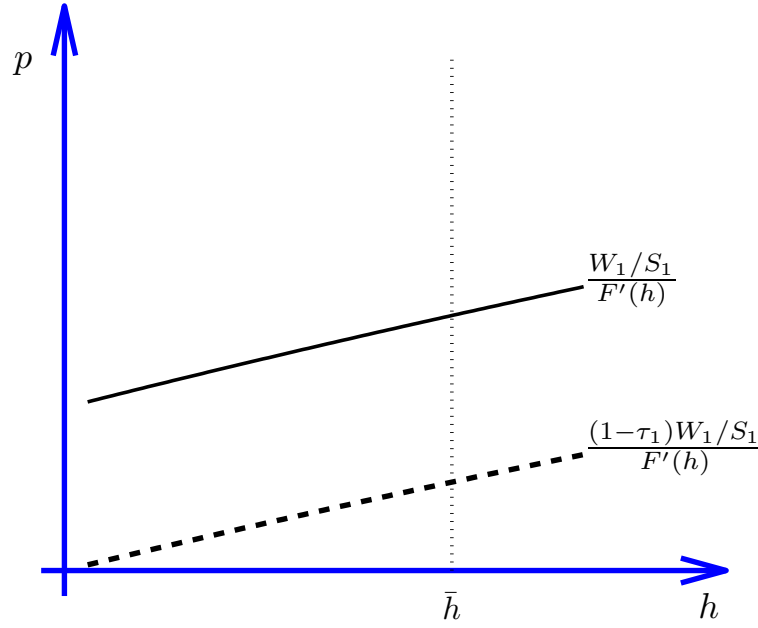
$$p_t F(h_t) - (1 - \tau_t)(W_t/\mathcal{E}_t)h_t$$

And the first-order condition of the firm becomes:

$$p_t = \frac{(1 - \tau_t)(W_t/\mathcal{E}_t)}{F'(h_t)}$$

From the point of view of a nontraded-goods producer a wage subsidy is identical to a decline in the nominal wage. Figure 13.9 illustrates that a wage subsidy, $\tau_1 > 0$, shifts the supply schedule down and to the right. The purpose of the labor subsidy is to bring about full employment after a negative external shock. Full employment means that $h_t = \bar{h}$. For firms to demand \bar{h} workers it must be the case that $W_1/\mathcal{E}_1(1 - \tau_1)/F'(\bar{h}) = p_1$. And for household to demand $c_1^N = F(\bar{h})$ nontradables it must be the case that $p_1 = U_2(c_1^T, F(\bar{h}))/U_1(c_1^T, F(\bar{h}))$. Combining these two expression we find

Figure 13.9: A Labor Subsidy as a Shifter of the Supply Schedule



that the wage subsidy rate that is consistent with full employment satisfies

$$(1 - \tau_1) = F'(\bar{h}) \frac{U_2(c_1^T, F(\bar{h}))}{U_1(c_1^T, F(\bar{h}))} \frac{\mathcal{E}_1}{W_1}.$$

Notice that the nominal exchange rate, \mathcal{E}_1 , the nominal wage, W_1 , and c_1^T are given. (Recall that we showed above that consumption of tradables depends only on the endowment of tradables and the world interest rate.) So this expression allows us to solve for the wage subsidy rate that brings about full employment. All else equal, the higher is the nominal wage rate, the larger must be the subsidy.

13.3 The Mussa Puzzle

13.4 The Dornbusch Overshooting Model

13.5 Exercises

Exercise 13.1 (Sudden Stops With Downward Wage Rigidity) Consider a two-period, small, open economy. Households are endowed with 10 units of tradables in period 1 and 13 units in period 2 ($Q_1^T = 10$ and $Q_2^T = 13$). The country interest rate is 10 percent, or $r = 0.1$, the nominal exchange rate is fixed and equal to 1 in both periods ($E_1 = E_2 = 1$), and the nominal wage equals 8.25 in both periods ($W_1 = W_2 = 8.25$). Nominal wages are downwardly rigid. Suppose the economy starts period 1 with no assets or debts carried over from the past ($B_0^* = 0$). Suppose that the household's preferences are defined over consumption of tradable and nontradable goods in periods 1 and 2, and are described by the following utility function,

$$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

where C_i^T and C_i^N denote, respectively, consumption of tradables and nontradables in period $i = 1, 2$. Let p_1 and p_2 denote the relative prices of nontradables in terms of tradables in periods 1 and 2, respectively. Households supply inelastically $\bar{h} = 1$ units of labor to the market each period. Finally, firms produce nontradable goods using labor as the sole input. The production technology is given by $Q_1^N = h_1^\alpha$ and $Q_2^N = h_2^\alpha$ in periods 1 and 2, respectively, where Q_i^N and h_i denote, respectively, nontradable output and hours employed in period $i = 1, 2$. The parameter α is equal to 0.75.

1. Compute the equilibrium levels of consumption of tradables and the trade balance in periods 1 and 2.

2. Compute the equilibrium levels of employment and nontradable output in period 1.
3. Suppose now that the country interest rate increases to 32 percent. Calculate the equilibrium levels of consumption of tradables, the trade balance, consumption of nontradables, and the level of unemployment, all for period 1. Provide intuition.
4. Given the situation in the previous question, calculate the minimum devaluation rate consistent with full employment. Explain.

Exercise 13.2 (Capital Controls, Downward Wage Rigidity, and Currency Pegs)

Consider a two-period, small, open economy with free capital mobility. Households are endowed with 10 units of tradables in period 1 and 10 units in period 2 ($Q_1^T = 10$ and $Q_2^T = 10$). The world interest rate is 0, $r^* = 0$, the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency, is fixed and equal to 1 in both periods ($\mathcal{E}_1 = \mathcal{E}_2 = 1$). Suppose that the foreign-currency price of tradable goods is constant and equal to one in both periods, and that the law of one price holds for tradable goods. Nominal wages are downwardly rigid. Specifically, assume that the nominal wage in periods 1 and 2, measured in terms of domestic currency, can not fall below the past wage rate, $W_i \geq W_{i-1}$ for $i = 1, 2$, with $W_0 = 5$ given. Suppose the economy starts period 1 with no assets or debts carried over from the past ($B_0^* = 0$). Suppose that the household's preferences are defined over consumption of tradable and nontradable goods in periods 1

and 2, and are described by the following utility function,

$$\ln C_1^T + \ln C_1^N + \ln C_2^T + \ln C_2^N,$$

where C_i^T and C_i^N denote, respectively, consumption of tradables and nontradables in period $i = 1, 2$. Let p_1 and p_2 denote the relative prices of nontradables in terms of tradables in periods 1 and 2, respectively. Households supply inelastically $\bar{h} = 1$ units of labor to the market each period. Finally, firms produce nontradable goods using labor as the sole factor input. The production technology is given by

$$Q_1^N = h_1^\alpha$$

and

$$Q_2^N = h_2^\alpha$$

in periods 1 and 2, respectively, where Q_i^N and h_i denote, respectively, nontradable output and hours employed in period $i = 1, 2$. The parameter α is equal to 0.5.

1. Compute the equilibrium level of consumption of tradables and the trade balance in periods 1 and 2. Show your work. Interpret your findings.
2. Compute the equilibrium levels of employment and nontradable output in periods 1 and 2.

For the remainder of the problem consider the case that the world interest

rate falls to $r^* = -0.5$.

3. Compute the equilibrium level of consumption of tradables and the trade balance in periods 1 and 2.
4. Compute the equilibrium level of nontraded consumption in periods 1 and 2 and the wage rate in period 1. Provide a discussion of your findings.
5. Compute the level of welfare.
6. Suppose wages were fully flexible. Find the level of nontradable consumption in periods 1 and 2. Compute the level of welfare.

Suppose now that the government imposes capital controls that prevent households from borrowing in international capital markets in period 1, i.e., the government imposes $B_1^* \geq 0$. Continue to assume that the world interest rate is $r^* = -0.5$.

7. Find consumption of tradables and nontradables in periods 1 and 2.
8. Find the level of welfare under capital controls. Are capital controls welfare decreasing? Explain why or why not.
9. Compare the level of welfare under capital controls and under wage flexibility.
10. Suppose the only instrument the government has to influence capital inflows is a proportional capital control tax. In particular, individual households face an interest rate $1 + \tilde{r} = (1 + r^*)/(1 - \tau)$. Any

capital control tax revenue is rebated to households in a lump-sum fashion. Does there exist a value of τ that implements the flexible wage allocation? If so, what is the value of τ .

Exercise 13.3 (Expected Income Shock in a TNT Economy.) Consider a two-period small open economy with free capital mobility. The economy produces tradable and nontradable goods. Tradable output is an exogenous endowment in both periods. Nontradable output is produced using labor via an increasing and concave production function. Households have preferences for tradable and nontradable goods in both periods described by a lifetime utility function that is separable across time and goods, like the log preferences analyzed in the body of this chapter. Suppose that in period 1 households learn that the endowment of tradables in period 2 will be significantly lower than expected. Analyze, using graphs to complement your explanation, the effect of this shock on consumption, employment, the real exchange rate, and the current account in period 1, under two alternative environments: (a) flexible nominal wages; and (b) downwardly rigid nominal wages coupled with a currency peg. Discuss optimal exchange-rate policy in scenario (b).

Chapter 14

The Twin Deficits: Fiscal Policy and the Current Account

The model economies we have studied thus far operate without a government. They feature only two types of agents, households and firms. This is a significant simplification, because in virtually all countries the government is a large economic agent controlling directly or indirectly, through taxes, transfers, public consumption, and public investment a large fraction of aggregate economic activity.

In this chapter, we investigate the role of the government in the determination of the current account. We will address questions such as, does an increase in taxes lead to an improvement or a deterioration in the current account? Does government spending crowd out private investment,

and, if so, is the crowding out effect larger or smaller in open economies than in closed economies? We will study an economic hypothesis known as the *twin deficits hypothesis* according to which fiscal deficits cause current account deficits. In a case study, we will test whether the validity of the twin-deficits hypothesis on U.S. data.

14.1 The government sector in the open economy

Consider the two-period endowment economy studied in chapter 3, but assume the existence of a government that purchases goods G_1 and G_2 in periods 1 and 2, respectively, and levies lump-sum taxes T_1 and T_2 . In addition, assume that the government starts with initial financial assets in the amount of B_0^g . If B_0^g is negative, we say that there is *public debt* outstanding at the beginning of period 1 in the amount $-B_0^g$.

The term lump-sum refers to taxes that do not depend on any economic characteristic of the tax-payer, such as income, spending, or wealth. Lump-sum taxes represent a convenient analytical tool for two reasons. First, because they do not depend on any economic decision taken by private agents, they do not distort economic incentives, which simplifies the characterization of the equilibrium dynamics. Second, lump-sum taxes represent a convenient way to isolate the effect of government spending, because any change in taxes necessary to finance a certain change in government spending does not add real effects, which could in principle be difficult to disentangle. On the down side, lump-sum taxes are rarely seen in real life. Governments

do not tell tax payers “you have to pay x regardless of whether you are a rich or a poor person, a high or low income earner, or a person who spends a lot or little in the supermarket.” For this reason, later in this chapter we will study a more realistic environment in which taxes do depend on some economic manifestation. This type of taxes are known as *distortionary taxes*.

The government faces the following budget constraints in periods 1 and 2, respectively:

$$G_1 + (B_1^g - B_0^g) = r_0 B_0^g + T_1$$

$$G_2 + (B_2^g - B_1^g) = r_1 B_1^g + T_2$$

where B_1^g and B_2^g denote the amount of government asset holdings at the end of periods 1 and 2, respectively. If B_1^g is negative, there is public debt standing at the end of period 1. Similarly, if B_2^g is negative, there is public debt standing at the end of period 2. The left-hand side of the first constraint represents the government’s outlays in period 1, which consist of government purchases of goods, G_1 , and purchases of financial assets, $B_1^g - B_0^g$. The right-hand side represents the government’s sources of funds in period 1, namely, tax revenues, T_1 , and interest income on asset holdings, $r_0 B_0^g$. The budget constraint in period 2 has a similar interpretation.

Like households, the government is assumed to be subject to a no-Ponzi-game constraint that prevents it from having debt outstanding at the end of period 2. This means that B_2^g must be greater or equal to zero. At the same time, a benevolent government—that is, a government that cares about the welfare of its citizens—would not find it in its interest to end period 2 with positive asset holdings. This is because the government will not be around

in period 3 to spend the accumulated assets in ways that would benefit its constituents. This means that the government will always choose B_2^g to be less than or equal to zero. The above two arguments imply that

$$B_2^g = 0.$$

Combining the above three expressions, we obtain the following intertemporal government budget constraint:

$$G_1 + \frac{G_2}{1+r_1} = (1+r_0)B_0^g + T_1 + \frac{T_2}{1+r_1} \quad (14.1)$$

This constraint says that the present discounted value of government consumption (the left-hand side) must be equal to the present discounted value of tax revenues and initial asset holdings including interest (the right-hand side). Note that there exist many (in fact a continuum of) tax policies T_1 and T_2 that finance a given path of government consumption, G_1 and G_2 , i.e., that satisfy the intertemporal budget constraint of the government given by (14.1). However, all other things equal, given taxes in one period, the above intertemporal constraint uniquely pins down taxes in the other period. In particular, a tax cut in period 1 must be offset by a tax increase in period 2. Similarly, an expected tax cut in period 2 must be accompanied by a tax increase in period 1.

The household's budget constraints are similar to the ones we derived earlier in chapter 3, but must be modified to reflect the fact that now households must pay taxes in each of the two periods. Specifically, the household's

budget constraints in periods 1 and 2 are given by

$$C_1 + T_1 + B_1^p - B_0^p = r_0 B_0^p + Q_1$$

$$C_2 + T_2 + B_2^p - B_1^p = r_1 B_1^p + Q_2$$

We also impose the no-Ponzi-game condition

$$B_2^p = 0.$$

Combining these three constraints yields the following intertemporal budget constraint:

$$C_1 + \frac{C_2}{1 + r_1} = (1 + r_0)B_0^p + Q_1 - T_1 + \frac{Q_2 - T_2}{1 + r_1} \quad (14.2)$$

This expression says that the present discounted value of lifetime consumption, the left-hand side, must equal the sum of initial wealth, $(1 + r_0)B_0^p$, and the present discounted value of endowment income net of taxes, $(Q_1 - T_1) + (Q_2 - T_2)/(1 + r_1)$. Note that the only difference between the above intertemporal budget constraint and the one given in equation (3.4) is that now $Q_i - T_i$ takes the place of Q_i , for $i = 1, 2$.

As in the economy without a government, the assumption of a small open economy implies that in equilibrium the domestic interest rate must equal the world interest rate, r^* , that is,

$$r_1 = r^*. \quad (14.3)$$

The country's net foreign asset position at the beginning of period 1, which we denote by B_0^* , is given by the sum of private and public asset holdings, that is,

$$B_0^* = B_0^p + B_0^g.$$

We will assume for simplicity that the country's initial net foreign asset position is zero:

$$B_0^* = 0. \quad (14.4)$$

Combining (14.1), (14.2), (14.3), and (14.4) yields,

$$C_1 + G_1 + \frac{C_2 + G_2}{1 + r^*} = Q_1 + \frac{Q_2}{1 + r^*}.$$

This intertemporal resource constraint represents the consumption possibility frontier of the economy. It has a clear economic interpretation. The left-hand side is the present discounted value of domestic absorption, which consists of private and government consumption in each period.¹ The right-hand side of the consumption possibility frontier is the present discounted value of domestic output. Thus, the consumption possibility frontier states that the present discounted value of domestic absorption must equal the present discounted value of domestic output.

Solving for C_2 , the consumption possibility frontier can be written as

$$C_2 = (1 + r^*)(Q_1 - C_1 - G_1) + Q_2 - G_2. \quad (14.5)$$

¹As noted in chapter ??, domestic absorption is the sum of consumption and investment. However, in the endowment economy under analysis investment is identically equal to zero.

Figure 14.1: Optimal consumption choice

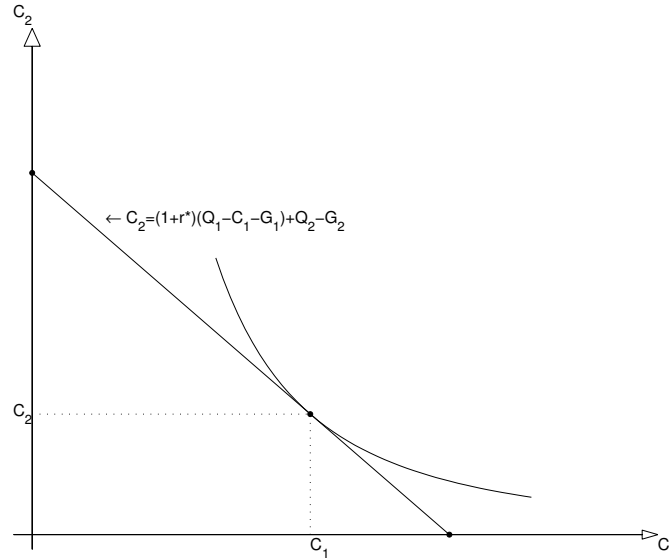


Figure 14.1 depicts the relationship between C_1 and C_2 implied by the consumption possibility frontier. It is a downward sloping line with slope equal to $-(1 + r^*)$. Consumption in each period is determined by the tangency of the consumption possibility frontier with an indifference curve.

Note that neither T_1 nor T_2 appear in the consumption possibility frontier. This means that, given G_1 and G_2 , any combination of taxes T_1 and T_2 satisfying the government's budget constraint (14.1) will be associated with the same private consumption levels in periods 1 and 2.

14.2 Ricardian Equivalence

In order to understand the merits of the view that attributes the large current account deficits of the 1980s to fiscal deficits generated in part by

the tax cuts implemented by the Reagan administration, we must determine how a reduction in taxes affects the current account in our model economy. Because the current account is the difference between national saving and investment, and because investment is by assumption nil in our endowment economy, it is sufficient to characterize the effect of tax cuts on national saving.² As mentioned earlier, national saving equals the sum of government saving and private saving.

Private saving in period 1, which we denote by S_1^p , is defined as the difference between *disposable income*, given by domestic output plus interest on net bond holdings by the private sector minus taxes, and private consumption:

$$S_1^p = Q_1 + r_0 B_0^p - T_1 - C_1.$$

Because, as we just showed, for a given time path of government purchases, private consumption is unaffected by changes in the timing of taxes and because $r_0 B_0^p$ is predetermined in period 1, it follows that changes in lump-sum taxes in period 1 induce changes in private saving of equal size and opposite sign:

$$\Delta S_1^p = -\Delta T_1. \quad (14.6)$$

The intuition behind this result is the following: Suppose, for example, that the government cuts lump-sum taxes in period 1, keeping government purchases unchanged in both periods. This policy obliges the government to increase public debt by ΔT_1 in period 1. In order to service and retire this

²It is worth noting, however, that if the government levies only lump-sum taxes, as assumed in the present analysis, then the results of this section apply not only to an endowment economy but also to an economy with investment.

additional debt, in period 2 the government must raise taxes by $(1+r_1)\Delta T_1$. Rational households anticipate this future increase in taxes and therefore choose to save the current tax cut (rather than spend it in consumption goods) so as to be able to pay the higher taxes in period 2 without having to sacrifice consumption in that period. Put differently, a change in the timing of lump-sum taxes does not alter the household's lifetime wealth.

Government saving, also known as the *secondary fiscal surplus*, is defined as the difference between revenues (taxes plus interest on asset holdings) and government purchases. Formally,

$$S_1^g = r_0 B_0^g + T_1 - G_1.$$

When the secondary fiscal surplus is negative we say that the government is running a *secondary fiscal deficit*. The secondary fiscal surplus has two components: interest income on government asset holdings ($r_0 B_0^g$) and the *primary fiscal surplus* ($T_1 - G_1$). The primary fiscal surplus measures the difference between tax revenues and government expenditures. When the primary fiscal surplus is negative, that is, when government expenditures exceed tax revenues, we say that the government is running a *primary deficit*.

Given an exogenous path for government purchases and given the initial condition $r_0 B_0^g$, any change in taxes in period 1 must be reflected one-for-one in a change in government saving, that is,

$$\Delta S_1^g = \Delta T_1. \quad (14.7)$$

As we mentioned before, national saving, which we denote by S , is given by the sum of private and government saving, that is, $S_1 = S_1^p + S_1^g$. Changes in national saving are thus equal to the sum of changes in private saving and changes in government saving,

$$\Delta S_1 = \Delta S_1^p + \Delta S_1^g.$$

Combining this expression with equations (14.6) and (14.7), we have that

$$\Delta S_1 = -\Delta T_1 + \Delta T_1 = 0.$$

This expression states that national saving is unaffected by the timing of lump-sum taxes. This is an important result in Macroeconomics. For this reason it has been given a special name: *Ricardian Equivalence*.³

Recalling that the current account is the difference between national saving and investment, it follows that the change in the current account in response to a change in taxes, holding constant government expenditure, is given by

$$\Delta CA_1 = \Delta S_1 - \Delta I_1.$$

Therefore, an increase in the fiscal deficit due to a decline in current lump-sum taxes (leaving current and expected future government spending un-

³This important insight was first formalized by Robert Barro of Harvard University in “Are Government Bonds Net Wealth,” *Journal of Political Economy*, 1974, volume 82, pages 1095-1117.

changed) has *no* effect on the current account, that is,

$$\Delta CA_1 = 0.$$

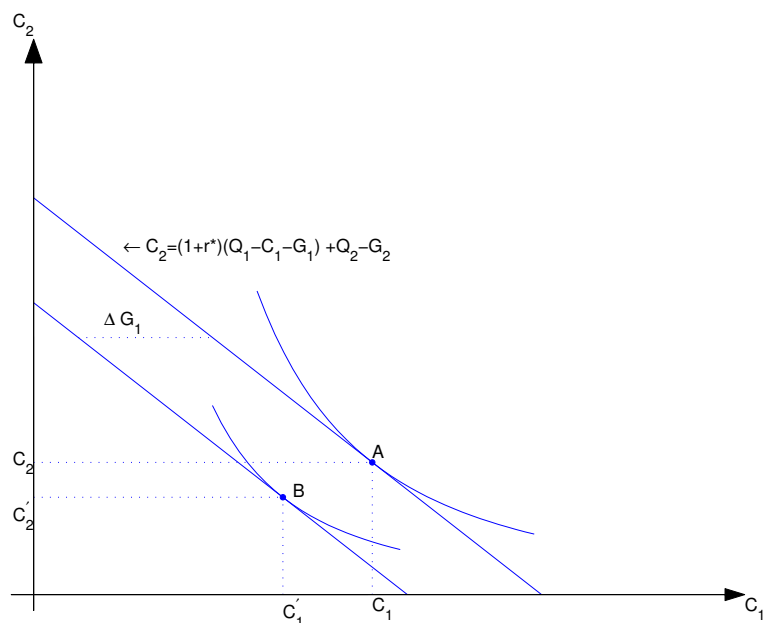
Let us take stock of what we have learned from our model. If the model of Ricardian Equivalence represents an adequate description of how the economy works and if the main cause of the fiscal deficits of the 1980s was the Reagan tax cuts, then what we should have observed is a decline in public saving, an offsetting increase in private saving, and no change either in national saving or the current account. What does the data show? In the 1980s there was a significant cut in taxes. As predicted by theory, the tax cuts were accompanied by a significant decline in public saving (see the difference between the solid and broken lines in figure 14.9). However, contrary to the predictions of Ricardian Equivalence, private saving did not increase by the same amount as the decline in public saving and as a result both national saving and the current account plummeted. We therefore conclude that either the fiscal deficits of the 1980s were caused by factors other than the tax cuts, such as increases in government spending, or Ricardian Equivalence does not hold, or both. We explore these possibilities further in the next section.

14.3 Government Spending and Current Account Deficits

What are other possible interpretations of the view according to which the large current account deficits of the 1980s were due to a decline in desired saving or an increase in desired U.S. spending or both? One possible interpretation is that the increase in the U.S. fiscal deficit of the 1980s was not solely due to a deferral of taxes, but also due to a temporary increase in government purchases, in particular military spending. In our model, an increase in government purchases in period 1, with government purchases in period 2 unchanged, has effects on consumption that are akin to those triggered by a temporary decline in output. To see this, note that an increase in G_1 reduces the disposable income of households. In response to the negative income effect, households wish to lower consumption in both periods, 1 and 2. Consumption spending in period 1 declines by less than the increase in government purchases, that is, $\Delta C_1 + \Delta G_1 > 0$. Because neither output in period 1 nor investment in period 1 are affected by the increase in government purchases in period 1, the trade balance in period 1, which is given by $TB_1 = Q_1 - C_1 - G_1 - I_1$, deteriorates ($\Delta TB_1 = -\Delta C_1 - \Delta G_1 < 0$). The current account, $r_0 B_0^* + TB_1$, declines by the same amount as the trade balance as net investment income ($r_0 B_0^*$) is not affected by the increase in government spending ($\Delta CA_1 = \Delta TB_1 < 0$).

Figure 14.2 illustrates the adjustment of consumption to a temporary increase in government purchases. The initial consumption allocation is point A. The increase in G_1 produces a parallel shift in the economy's resource

Figure 14.2: Adjustment to a temporary increase in government purchases



constraint to the left by ΔG_1 . If consumption in both periods is normal, then both C_1 and C_2 decline. Therefore, the new optimal allocation, point B, is located southwest of point A. Clearly, the decline in C_1 is less in absolute value than ΔG_1 .

Is this explanation for the observed deterioration of the U.S. external accounts empirically plausible? There exists evidence that government spending went up in the early 1980s due to an increase in national defense spending as a percentage of GNP. Table 14.1 indicates that military purchases increased by about 1.5% of GNP from 1978 to 1985. But according to our model, this increase in government purchases (if temporary) must be associated with a decline in consumption that is smaller in absolute value. Thus, the decline in national saving triggered by the Reagan military build up

Table 14.1: U.S. military spending as a percentage of GNP: 1978-1987

Year	Military Spending (% of GNP)
1978-79	5.1-5.2
1980-81	5.4-5.5
1982-84	6.1-6.3
1985-87	6.7-6.9

is at most 1.5% of GNP, which is too small to explain all of the observed decline in national saving of 3% of GNP that occurred during that period (see figure [14.9](#)).

14.4 Failure of Ricardian Equivalence

Thus far, we have considered two arguments in support of the view that the U.S. external imbalances of the 1980s were the result of a decline in domestic saving (view 2). One was increases in government spending and the other was cuts in taxes. We concluded that if Ricardian Equivalence holds, then cuts in taxes could not explain the observed deterioration in the U.S. current account. A third argument in support of view 2 is that Ricardian Equivalence may not be right.

There are at least three reasons why Ricardian Equivalence may fail to hold. One is that households are borrowing constrained. A second reason is that the people that benefit from the tax cut are not the same that must pay for the future tax increase. And a third reason is that taxes are not lump-sum. In what follows of this section, we will explore each of these

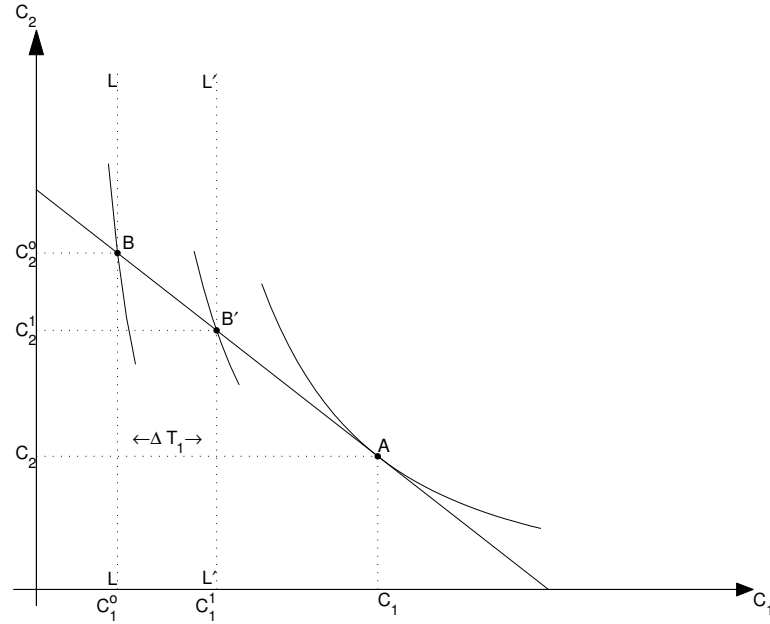
reasons in more detail.

14.4.1 Borrowing Constraints

To see why borrowing constraints may lead to a breakdown in Ricardian Equivalence, consider the case of a young worker who expects his future income to be significantly higher than his current income, perhaps due to on-the-job training or to the fact that he is simultaneously attending a good college. Based on this expectation, he might want to smooth consumption over time by borrowing against his higher future income. However, suppose that, perhaps because of imperfections in financial markets, such as asymmetric information between borrowers and lenders, he cannot procure a loan. In this case, the young worker is said to be borrowing constraint. Suppose now that the government decides to implement a cut in current (lump-sum) taxes, financed by an increase in future taxes. Will the young worker increase his saving by the same amount as the tax cut as prescribed by Ricardian Equivalence? Most likely not. He will probably view the tax cut as a welcome relief from his borrowing constraint and allocate the windfall to consumption. In this case, the decline in government saving causes no change in private saving. As a result, national saving falls. If investment is unaffected by the change in lump-sum taxes, the fall in national saving will be associated with a deterioration in the current account. Twin deficits would thereby emerge. Let's analyze this story more formally.

Suppose households have initial wealth equal to zero ($B_0^p = 0$) and that they are precluded from borrowing in financial markets, that is, they are constrained to choose $B_1^p \geq 0$. Assume further that neither firms nor the

Figure 14.3: Adjustment to a temporary tax cut when households are liquidity constrained



government are liquidity constrained, so that they can borrow at the world interest rate r^* . Figure 14.3 illustrates this case. Suppose that in the absence of borrowing constraints, the consumption allocation is given by point A, at which households in period 1 consume more than their after-tax income, that is, $C_1^0 > Q_1 - T_1$. This excess of consumption over disposable income is financed by borrowing in the financial market ($B_1^p < 0$). In this case the borrowing constraint is binding, and households are forced to choose the consumption allocation B, where $C_1 = Q_1 - T_1$. It is easy to see why, under these circumstances, a tax cut produces an increase in consumption and a deficit in the current account. The tax cut relaxes the household's borrowing constraint. The increase in consumption is given by the size of the tax cut

($\Delta C_1 = -\Delta T_1$), which in figure 14.3 is measured by the distance between the vertical lines L and L'. The new consumption allocation is given by point B', which lies on the economy's resource constraint and to the right of point B. Consumption in period 1 increases by the same amount as the tax cut. Because neither investment nor government purchases are affected by the tax cut, the trade balance and hence the current account deteriorate by the same amount as the increase in consumption. Thus, in the presence of borrowing constraints the increase in the fiscal deficit leads to a one-for-one increase in the current account deficit.

Can the presence of financial constraints *per se* explain the current account deficits of the 1980s as being a consequence of expansionary fiscal policy? The tax cut implemented during the Reagan administration amounted to about 3 percent of GDP. The observed deterioration in the current account during those years was also of about 3 percent of GDP. It is then clear that in order for the liquidity-constraint hypothesis alone to explain the behavior of the current account in the 1980s, it should be the case that 100% of the population must be borrowing constrained.

14.4.2 Intergenerational Effects

A second reason why Ricardian Equivalence could fail is that those who benefit from the tax cut are not the ones that pay for the tax increase later. To illustrate this idea, consider an endowment economy in which households live for only one period. Then, the budget constraint of the generation alive in period 1 is given by $C_1 + T_1 = Q_1$, and similarly, the budget constraint of the generation alive in period 2 is $C_2 + T_2 = Q_2$. Suppose that the govern-

ment implements a tax cut in period 1 that is financed with a tax increase in period 2. Clearly, $\Delta C_1 = -\Delta T_1$ and $\Delta C_2 = -\Delta T_2$. Thus, the tax cut produces an increase in consumption in period 1 and a decrease in consumption in period 2. As a result, the trade balance and the current account in period 1 decline one-for-one with the decline in taxes. The intuition for this result is that in response to a decline in taxes in period 1, the generation alive in period 1 does not increase saving in anticipation of the tax increase in period 2 because it will not be around when the tax increase is implemented. What percentage of the population must be 1-period lived in order for this hypothesis to be able to explain the observed 3% of GNP decline in the U.S. current account balance, given the 3% decline in government saving? Obviously, everybody must be 1-period lived.

14.4.3 Distortionary Taxation

Finally, Ricardian equivalence may also breakdown if taxes are not lump sum. Lump-sum taxes are those that do not depend on agents' decisions. In the economy described in section 14.1, households are taxed T_1 in period 1 and T_2 in period 2 regardless of their consumption, income, or saving. Thus, in that economy lump-sum taxes do not distort any of the decisions of the households. In reality, however, taxes are rarely lump sum. Rather, they are typically specified as a fraction of consumption, income, firms' profits etc. Thus, changes in tax rates will tend to distort consumption, saving, and investment decisions. Suppose, for example, that the government levies a proportional tax on consumption, with a tax rate equal to τ_1 in period 1 and τ_2 in period 2. Then the after-tax cost of consumption is $(1 + \tau_1)C_1$ in period

1 and $(1 + \tau_2)C_2$ in period 2. In this case, the relative price of period-1 consumption in terms of period-2 consumption faced by households is not simply $1 + r_1$, as in the economy with lump-sum taxes, but $(1 + r_1)\frac{1+\tau_1}{1+\tau_2}$. Suppose now that the government implements a reduction in the tax rate in period 1. By virtue of the intertemporal budget constraint of the government, the public expects, all other things equal, an increase in the consumption tax rate in period 2. Thus, the relative price of current consumption in terms of future consumption falls. This change in the relative price of consumption induces households to substitute current for future consumption. Because firms are not being taxed, investment is not affected by the tax cut. As a result, the trade balance, given by $TB_1 = Q_1 - C_1 - G_1 - I_1$, and the current account, given by $CA_1 = TB_1 + r_0B_0^*$, both deteriorate by the same amount.

We conclude that if the current account deficit of the 1980s is to be explained by the fiscal imbalances of the Reagan administration, then this explanation will have to rely on a combination of an increase in government expenditure and multiple factors leading to the failure of Ricardian equivalence.

14.5 The Twin Deficits

This section studies the twin-deficits hypothesis, according to which fiscal deficits lead to current account deficits. In a nut shell, the idea behind the twin deficit hypothesis is as follows. Start with the definition of the current account as the difference between national saving and aggregate investment.

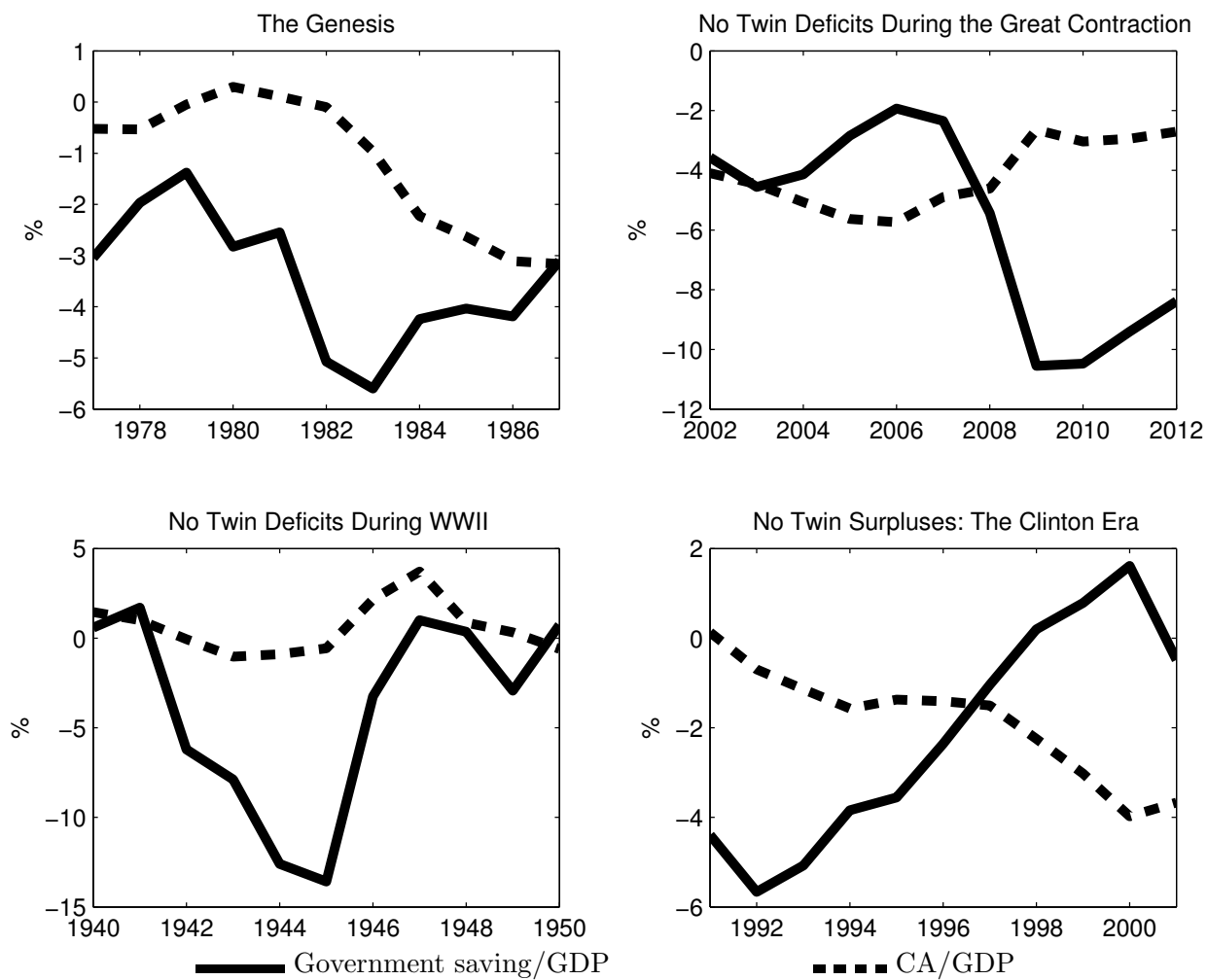
In turn, national saving is the sum of private saving and government saving (or fiscal surpluses). Suppose now that expansionary government spending lowers government saving. If private saving and investment are unaffected by the expansionary fiscal policy, then the current account must deteriorate by the same amount as the decline in government saving.

14.6 Twin Deficits in the United States

In previous chapters, we have documented that the early 1980s were a turning point for the U.S. current account. Until 1982, the U.S. had run current account surpluses but thereafter a string of large current account deficits opened up. The emergence of large current account deficits coincided with large fiscal deficits that were the result of the Reagan administration's policy of tax cuts and increases in military spending. The joint deterioration of the current account and the fiscal balance that took place in the early 1980s is documented in the top left panel of figure [14.4](#)

Are twin deficits a recurrent phenomenon? To answer this question, it is of interest to look at other episodes of large changes in government saving. The most recent episode of this type is the fiscal stimulus plan implemented by the Obama administration in the wake of the Great contraction of 2007. The Obama fiscal stimulus plan resulted in the largest fiscal deficits (as a fraction of GDP) in the postwar United States. The top right panel of figure [14.4](#) shows that between 2007 and 2009, the fiscal deficit of the United States increased by 8 percentage points of GDP. During the same period, however, contrary to the predictions of the twin-deficit hypothesis,

Figure 14.4: The Twin-Deficit Hypothesis in the United States

Data Source: bea.gov

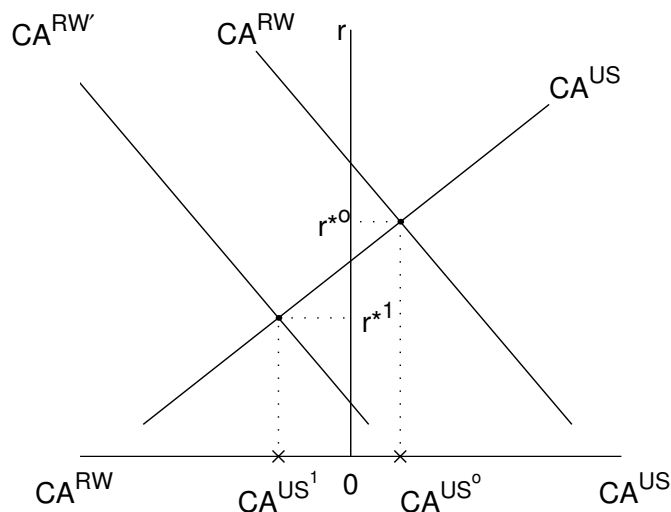
the current account improved by about 2.5 percent of GDP.

In addition to the Reagan and Obama fiscal expansions, two other episodes stand out. One is the enormous albeit short-lived fiscal deficit during the second world war of about 12 percent of GDP, caused primarily by military spending (see the bottom left panel of figure 14.4). During this period, the current account did deteriorate from about 1 percent to -1 percent of GDP. This movement in the external account is in the direction of the twin-deficit hypothesis. However, the observed decline in the current account balance was so small relative to the deterioration in government saving, that the episode can hardly be considered one of twin deficits. Another noticeable change in the fiscal balance took place in the 1990s during the Clinton administration. Between 1990 and 2000, government saving increased by about 7 percentage points of GDP. At the same time, contrary to the twin-deficit hypothesis, the current account deteriorated by about 4 percent of GDP. In summary, over the past century large changes in government saving have not always been accompanied by equal adjustments in the current account.

14.7 Testable Implications of the Twin Deficit Hypothesis

The fact that there seems to be no systematic relationship between large changes in government saving and changes in the current account does not necessarily invalidate the twin-deficit hypothesis. In reality, economies are hit simultaneously by a multitude of shocks of different nature. As a result, it is difficult to infer from raw data, like that presented in figure 14.4, the

Figure 14.5: The U.S. current account in the 1980s: view 1



effect of an increase in the fiscal deficit on the current account.

What then led some economists to conclude that the Reagan fiscal deficits were the cause of the current account deficits? To answer this question, we need to look at the implications that the twin-deficit hypothesis has for the behavior of variables other than the current account and the fiscal deficit and then compare those predictions to actual data.

In the early 1980s not all economic observers attributed the emergence of current accounts deficits to the fiscal stance. There were two prevailing theoretical views on the source of current account deficits.

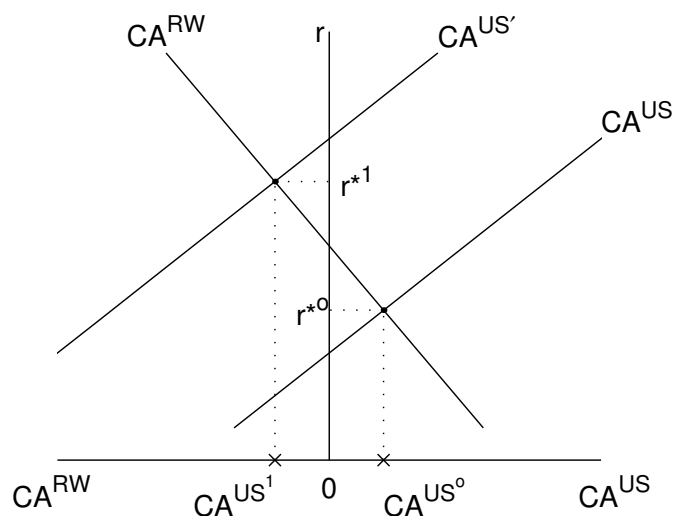
One view was that in those years the rest of the world wanted to send their saving to the U.S., so the U.S. had to run a current account deficit. This view is illustrated in figure 14.5. The increase in the rest of the world's demand for U.S. assets is reflected in a shift to the left of the current account schedule of the rest of the world. As a result, in the new equilibrium position,

the current account in the U.S. deteriorates from CA^{US^0} to CA^{US^1} and the world interest rate falls from r^{*0} to r^{*1} .

What could have triggered such an increase in the desire of the rest of the world to redirect saving to the U.S.? A number of explanations have been offered. First, in the early 1980s, the U.S. was perceived as a “safe heaven,” that is, as a safer place to invest. This perception triggered an increase in the supply of foreign lending. For example, it has been argued that international investors were increasingly willing to hold U.S. assets due to instability in Latin America; in the jargon of that time, the U.S. was the recipient of the “capital flight” from Latin America. Second, as a consequence of the debt crisis of the early 1980s, international credit dried up, forcing developing countries, particularly in Latin America, to reduce current account deficits. Third, financial deregulation in several countries made it easier for foreign investors to hold U.S. assets. An example is Japan in the late 1980s.

A second view of what caused the U.S. current account deficit is that in the 1980s the U.S. wanted to save less and spend more at any level of the interest rate. As a result, the American economy had to draw saving from the rest of the world. Thus, U.S. foreign borrowing went up and the current account deteriorated. Figure 14.6 illustrates this view. As a result of the increase in desired spending relative to income in the U.S., the CA schedule for the U.S. shifts to the left, causing a deterioration in the U.S. current account from CA^{US^0} to CA^{US^1} and an increase in the world interest rate from r^{*0} to r^{*1} . Under view 2, the deterioration of the U.S. current account is the consequence of a decline in U.S. national saving or an increase in U.S. investment or a combination of the two.

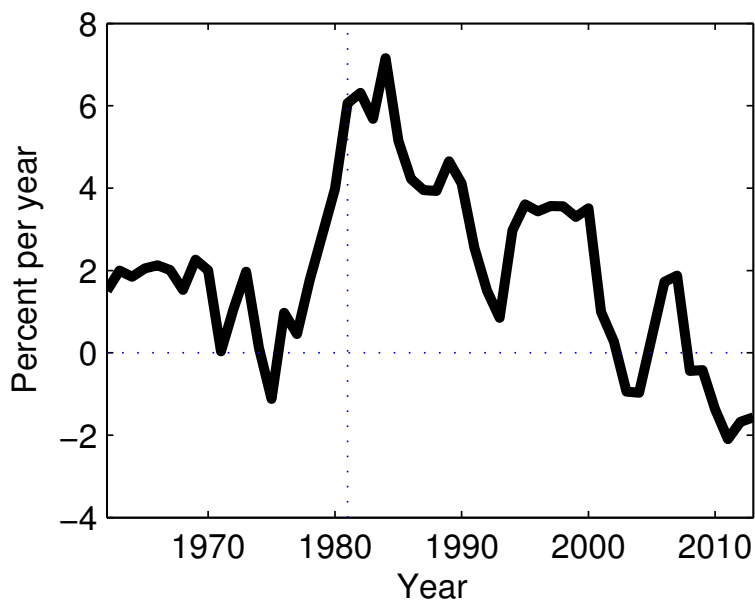
Figure 14.6: The U.S. current account in the 1980s: view 2



How could we tell views 1 and 2 apart? One strategy is to look for an economic variable about which the two views have different predictions. Once we have identified such a variable, we could look at actual data to see which view its behavior supports. Comparing figures 14.5 and 14.6, it is clear that a good candidate for testing the two views is the real interest rate. The two views have different implications for the behavior of the interest rate in the U.S. Under view 1, the interest rate falls as the foreign supply of saving increases, whereas under view 2 the interest rate rises as the U.S. demand for funds goes up. What does the data show? In the early 1980s, the U.S. experienced a large increase in real interest rates (see figure 14.7). This evidence seems to vindicate view 2. We will therefore explore this view further.

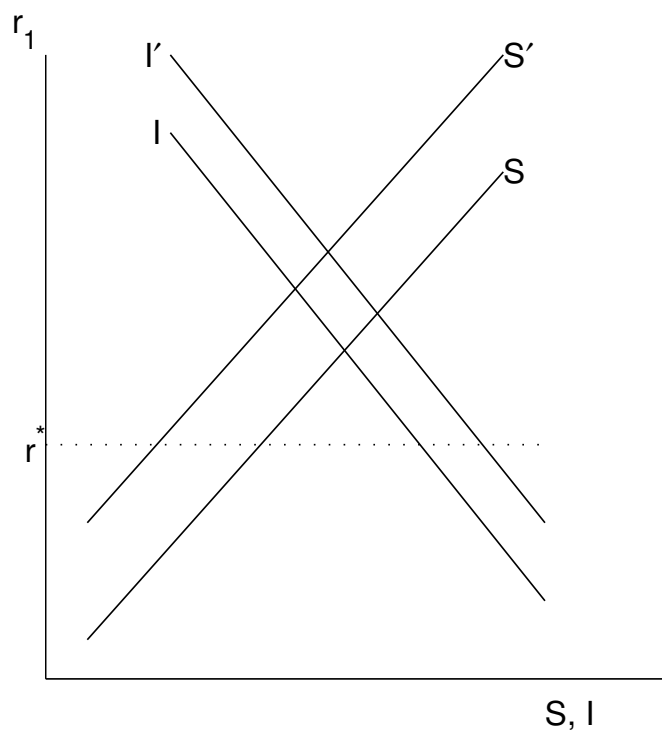
As already mentioned, view 2 requires that either the U.S. saving schedule shifts to the left, or that the U.S. investment schedule shifts to the right

Figure 14.7: Real interest rates in the United States 1962-2013



Note: The real interest rate is measured as the difference between the 1-year constant maturity Treasury rate and one-year expected inflation.

Figure 14.8: View 2 requires shifts in the U.S. saving or investment schedules



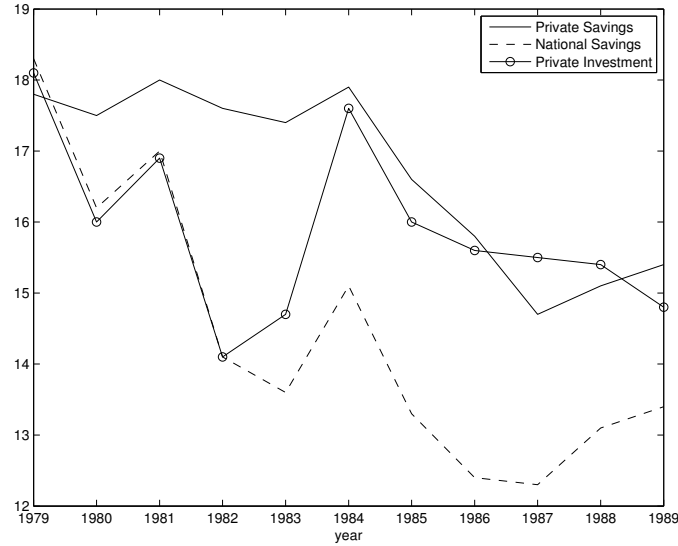
or both (see figure 14.8).

Before looking at actual data on U.S. saving and investment a comment about national saving is in order. National saving is the sum of private sector saving, which we will denote by S^p , and government saving, which we will denote by S^g . Letting S denote national saving, we have

$$S = S^p + S^g.$$

Thus far we have analyzed a model economy without a public sector. In an economy without a government, national saving is simply equal to private

Figure 14.9: U.S. Saving and Investment in Percent of GNP



Source: Norman S. Fieleke, “The USA in Debt,” *New England Economic Review*, September-October 1990, pages 34-54, table 11.

saving, that is, $S = S^p$. However, in actual economies government saving accounts for a non-negligible fraction of national saving. To understand what happened to U.S. saving in the 1980s the distinction between private saving and government saving is important. With this comment in mind, let us now turn to the data.

Figure 14.9 displays with a solid line private saving, S^p , with a broken line national saving, S , and with a circled line investment, I . The difference between the solid and the broken lines represents government saving, S^g . The figure shows that national saving and private saving begin to diverge in 1980, with national saving falling consistently below private saving. This gap

reflects the fiscal deficits created by the Reagan fiscal expansion. Specifically, the increase in the fiscal deficit in the early 1980s arose due to, among other factors, a tax reform, which reduced tax revenues, and an increase in defense spending.

Advocates of the twin-deficit hypothesis emphasize the fact that the decline in the current account balance, given by $S - I$ (the difference between the broken line and the circled line in figure 14.9), is roughly equal to the decline in government saving (given by the difference between the solid and the broken lines). They therefore conclude that the increase in the fiscal deficit *caused* the decline in the current account. However, this causal direction, which implies that the increase in the government deficit, that is, a decline in government saving, shifted the U.S. saving schedule to the left is not necessarily correct. The reason is that changes in fiscal policy that cause the fiscal deficit to increase may also induce offsetting increases in private saving, leaving total saving—and thus the current account—unchanged. In order to understand the relation between fiscal deficits and private saving, in the next section, we extend our theoretical model to incorporate the government.

14.8 Exercises

Exercise 14.1 Indicate whether the following statements are true, false, or uncertain and explain why.

1. Suppose the current account deficit is 5% of GDP and that half the population is borrowing constrained. Then, the government could eliminate the current account deficit by increasing (lump-sum) taxes by 10% of GDP.
2. A tax cut with current and future government spending unchanged causes an increase in saving as opposed to an increase in consumption, because households understand that taxes will have to increase in the future. Since the current account equals saving minus investment, the current account improves.
3. The government increases government spending and lump-sum taxes in the same amount, leaving the primary fiscal deficit unchanged. This policy should have no effect on the current account, which is in line with the Twin Deficit hypothesis.

Exercise 14.2 (An Economy with Lump-Sum Taxes) Consider a two-period endowment economy. Assume that households' preferences are described by the following utility function

$$\sqrt{C_1} + \frac{1}{1.1}\sqrt{C_2},$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively. In

each period, households are endowed with 10 units of goods. Also, households pay lump-sum taxes T_1 and T_2 , in periods 1 and 2, respectively. Finally, households are born with no financial assets ($B_0^p = 0$) and can borrow or lend in the international financial market at the world interest rate $r^* = 0.1$. The government starts period 1 with no outstanding assets or liabilities ($B_0^g = 0$). In period 1, the government collects lump-sum taxes T_1 and consumes $G_1 = 1$ units of goods. In period 2, it collects lump-sum taxes T_2 and consumes $G_2 = 1$ units of goods. Like the household, the government has access to the world financial markets.

1. Compute the equilibrium levels of consumption, the trade balance, and the current account in periods 1 and 2.
2. Suppose that $T_1 = 0$. What is T_2 ? What is private, public, and national saving in periods 1 and 2?
3. Suppose now that T_1 increases from 0 to 1 while government purchases are unchanged in both periods. How does this tax hike affect the current account and the fiscal deficit in period 1? Briefly explain your result.
4. Suppose that in period 1 the government increases spending from 1 to 2 and keeps government spending in period 2 unchanged. What is the effect of this policy change on the current account in period 1? Explain.
5. Finally, suppose that there is a permanent increase in government purchases: both G_1 and G_2 increase by 1 unit. What is the response of the current account in period 1? Compare your result with that

from the previous question and provide intuition.

Exercise 14.3 (An Open Economy with Distortionary Taxes.) Consider an economy populated by identical households with preferences described by the lifetime utility function

$$\ln C_1 + \ln C_2$$

Households are endowed with 10 units of goods in each period ($Q_1 = Q_2 = 10$) and start period 1 with no assets or debts ($B_0^p = 0$). The world interest rate is 5 percent and the country enjoys free capital mobility. In period 1 taxes are lump sum. In period 2, the government levies a 10 percent tax on consumption. Government spending is 1 unit in each period ($G_1 = G_2 = 1$). The government starts period 1 with a zero asset position $B_0^g = 0$.

1. Compute the equilibrium levels of the primary fiscal deficit and the current account in period 1.
2. Suppose now that the government decides to increase taxes in period 1 by 0.5 leaving government spending unchanged in both periods. The minister of finance argues that this policy change will not affect the current account because the tax being changed is lump sum and therefore nondistortionary. Do you agree or disagree with the minister's opinion and why. If you disagree, what is your prediction for the current account when the tax reform is applied? Show your work.

Exercise 14.4 (Distortionary Taxation #1) Consider a two-period, small,

open, endowment economy. Assume that households' preferences are described by the following utility function

$$\ln(C_1) + \ln(C_2),$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively. Households are endowed with 10 units of goods in each period and pay proportional taxes on consumption. Let τ_1 and τ_2 denote the consumption tax rates in periods 1 and 2, respectively. Finally, households are born with no financial assets ($B_0^p = 0$) and can borrow or lend in the international financial market at the world interest rate $r^* = 0.1$.

The government starts period 1 with no outstanding assets or liabilities ($B_0^g = 0$). It taxes consumption at the same rate in both periods ($\tau_1 = \tau_2$) and consumes 1 unit of goods in each period. That is, $G_1 = G_2 = 1$, where G_1 and G_2 denote government consumption in periods 1 and 2, respectively. Like the household, the government has access to the world financial market. In answering the following questions, show your work.

1. Compute the equilibrium tax rate and the equilibrium levels of consumption, the trade balance, private saving, the primary and secondary fiscal deficits, and the current account in periods 1 and 2.
2. Suppose now that the government implements a stimulus package consisting in reducing the tax rate by half in period 1. with government consumption unchanged in both periods. How does this expansionary fiscal policy affect private consumption, the trade balance, the current

account, and the primary and secondary fiscal deficits in period 1 and the tax rate in period 2? Briefly explain your result.

Exercise 14.5 (Distortionary Taxes # 2) Consider a two-period economy populated by a large number of households with preferences described by the utility function

$$\ln C_1 + \beta \ln C_2,$$

where C_1 and C_2 denote consumption in periods 1 and 2, respectively, and $\beta = 1/1.1$ is a subjective discount factor. Households receive endowments Q_1 in period 1 and Q_2 in period 2, with $Q_1 = Q_2 = 10$ and can borrow or lend in international financial markets at the interest rate $r^* = 0.1$. The government imposes taxes $T_1 = T^L + \tau_1 C_1$ in period 1 and $T_2 = \tau_2 C_2$ in period 2 and consumes G_1 units of goods in period 1 and G_2 units in period 2. Finally, households and the government start period 1 with no assets or debts carried over from the past.

1. Derive the intertemporal budget constraint of the household, the intertemporal budget constraint of the government, and the intertemporal resource constraint of the economy as a whole.
2. Derive the optimality condition that results from choosing C_1 and C_2 to maximize the household's utility function subject to its intertemporal budget constraint.
3. Suppose $G_1 = G_2 = 2$ and $\tau_1 = \tau_2 = 0.2$. Find the equilibrium levels of consumption and the trade balance in periods 1 and 2, and

the equilibrium level of lump-sum taxes T^L . Report the primary and secondary fiscal deficits in period 1.

4. Continue to assume that $G_1 = G_2 = 2$. Suppose that the government implements a tax cut in period 1 consisting in lowering the consumption tax rate from 20 to 10 percent. Suppose further that lump-sum taxes, T^L , are kept at the level found in the previous item. Find consumption, the trade balance, the primary fiscal deficit in period 1, and the consumption tax rate in period 2.
5. Now answer the previous question assuming that the cut in consumption taxes in period 1 from 20 to 10 percent is financed with an appropriate change in lump-sum taxes in the same period, while the consumption tax rate in period 2 is kept constant at its initial level of 20 percent. Compare your answer with the one for the previous item and provide intuition.
6. Suppose that $G_1 = 2$, $G_2 = 1$, and $T^L = 0$. Clearly, there are many possible equilibrium tax schemes (τ_1, τ_2) . Find the pair (τ^1, τ^2) that maximizes the household's lifetime utility. Show your derivation. Refer to your solution as the Ramsey optimal tax policy.

Exercise 14.6 (Fiscal Deficits and Current Account Imbalances) Consider

a two-period model of a large open endowment economy. The large country is endowed with $Q_1^* = 0$ goods in period 1 and with $Q_2^* = Q > 0$ goods in period 2. Its initial net foreign asset position is zero. The utility function of the large open economy is given by $U(C_1, C_2) = \ln C_1^* + \ln C_2^*$, where

C_1^* denotes consumption by agents in the large economy in period 1 and C_2^* consumption in period 2. The rest of the world has an endowment of Q in period 1 and an endowment of zero in period 2. The current account schedule of the rest of the world is given by: $CA_1^{row} = \frac{Q}{2}$ in period 1 and $CA_2^{row} = -CA_1^{row}$ in period 2. Consumption in the rest of the world in period one is equal to $Q/2$. The government in the large open economy imposes capital controls as follows. Let $1 + \tilde{r}$ denote the gross interest rate at which residents of the large economy can freely borrow and lend. Let τ denote a tax on capital controls such that $1 + \tilde{r} = (1 + r)/(1 + \tau)$. The government rebates any income it receives in period 2 from fixing the interest rate to households. Specifically, transfers to households in period 2, denoted T_2 , are given by $T_2 = \frac{1+r}{1+\tau} \tau B_1^*$, where B_1^* denotes the net foreign asset position of the large economy at the end of period 1.

1. Explain why in period 1, the large country will borrow from the rest of the world.
2. Show that consumption in period 1, C_1^* is equal to $Q/2$.
3. Assume that $\tau = 0$. Find the world interest rate, r .
4. Now assume that the government of the large economy changes the value of τ from zero to 10 percent with the intention to lower the effective interest rate that residents of the large economy have to pay on their debts. Find the value of the world interest rate, r . Does the government succeed in lowering the effective interest rate, $(1 + r)/(1 + \tau)$.

5. How are C_1^* and C_2^* affected by the increase in τ .
6. Is the increase in τ welfare increasing for agents in the large economy?
Explain why or why not.

Exercise 14.7 (Finite Lives and Fiscal Policy) Consider a two-period small open endowment economy with free capital mobility. Households live for one period. The endowment is 10 in both periods. The utility function of households living in period 1 is $\ln C_1$ and that of households living in period 2 is $\ln C_2$. The government lives for 2 periods and have access to the world financial market, where the interest rate is 10 percent $r^* = 0.1$. Government spending is 2 in both periods ($G_1 = G_2 = 2$). The government levies lump-sum taxes in periods 1 and 2, denoted T_1 and T_2 , respectively. Finally, assume that households and government start their lives with no debts or assets.

1. Suppose that $T_2 = 1$. Calculate T_1 . What is the primary fiscal deficit in period 1?
2. Calculate consumption in periods 1 and 2.
3. Calculate the trade balance and the current account in period 1 (TB_1 and CA_1).
4. Suppose now that the government does not have access to lump-sum taxes ($T_1 = T_2 = 0$). Instead, the government levies a proportional tax on consumption at the rates τ_1 and τ_2 in periods 1 and 2, respectively. Suppose that $\tau_2 = 0.1$. Calculate τ_1 , C_1 , C_2 , TB_1 , and TB_2 .

Exercise 14.8 (Optimal Lump-Sum Taxation) Consider the small open economy with finite lives described in the preamble of exercise 14.7. In particular, continue to assume that $Q_1 = Q_2 = 10$, $r^* = 0.1$, and $G_1 = G_2 = 2$. Suppose that the government is benevolent and cares equally about the welfare of both generations. Specifically, suppose that the lifetime utility function of the government is given by

$$\ln C_1 + \ln C_2$$

Calculate the optimal levels of lump-sum taxes in periods 1 and 2 (T_1 and T_2). Provide intuition. In particular, comment on why the government does or does not tax both generations equally.

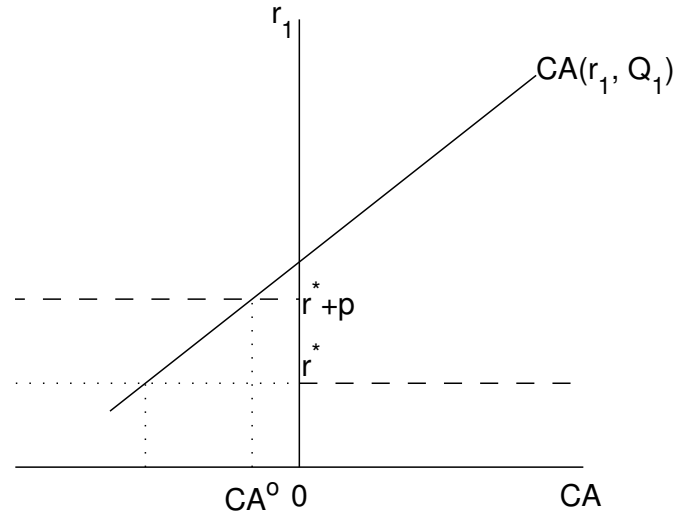
Chapter 15

The Macroeconomics of External Debt

15.1 Country Risk Premia

In practice, the interest rate that emerging countries face on their international loans is larger than the one developed countries charge to each other. This interest rate differential is called the country risk premium, and we denote it by p . Figure 15.1 illustrates the situation of a small open economy facing a country risk premium. In the graph it is assumed that the premium is charged only when the country is a debtor to the rest of the world. Suppose that the initial asset position, B_0^* , is zero. In this case, the country is a debtor at the end of period one if it runs a current account deficit in period one and a creditor if it runs a current account surplus in period one. Furthermore, the stock of debt at the end of period one is equal to the current account deficit in period 1, that is, in this case $B_1^* = -CA_1$. It follows that

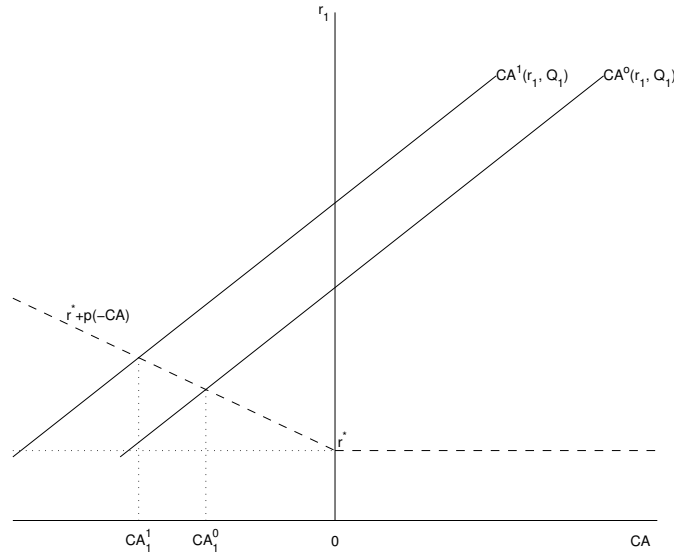
Figure 15.1: Current account determination in the presence of a constant risk premium



the country risk premium applies whenever the current account is in deficit. Thus, the interest rate faced by the small open economy is r^* when $CA > 0$ and $r^* + p > r^*$ when $CA < 0$. In figure 15.1, given the world interest rate r^* and the country risk premium p , the country runs a current account deficit equal to CA^0 . Note that the current account deficit is smaller than the one that would obtain if the country faced no risk premium. Thus, if the current account is negative, an increase in the risk premium reduces the current account deficit in exactly the same way as an increase in the interest rate.

A more realistic specification for the interest rate faced by developing countries is one in which the country risk premium is an increasing function of the country's net foreign debt. Given our assumption that the initial net foreign asset position is zero, the country's foreign debt at the end of

Figure 15.2: Current account determination in the presence of an increasing risk premium



period 1 is given by its current account deficit. Thus, we can represent the country risk premium as an increasing function of the current account deficit, $p(-CA)$ (see figure 15.2). Consider now the response of the current account to an investment surge like the one discussed in section ???. In response to the positive investment shock, the current account schedule shifts to the left from $CA^0(r_1, Q_1)$ to $CA^1(r_1, Q_1)$. As a result, the current account deteriorates from CA_1^0 to CA_1^1 and the interest rate at which the country can borrow internationally increases from $r^* + p(-CA_1^0)$ to $r^* + p(-CA_1^1)$. The resulting deterioration in the current account is, however, smaller than the one that would have taken place had the country risk premium remained constant.

15.2 Why Do Countries Honor Their External Debts?

International financial markets play an important economic role. As we discussed in previous chapters, by borrowing from and lending to other nations, countries can smooth out the effects of idiosyncratic shocks to output, the terms of trade, etc. In this way, countries can achieve higher levels of welfare than living in financial isolation.

For international credit to take place at all, however, creditors must have the expectation that foreign debtors will honor their debts. The question is, therefore, why do countries pay back international debts. This question is relevant because financial contracts involving a foreign sovereign state are fundamentally different from

15.3 The debt crisis of developing countries of the 1980s

In 1982, the government of Mexico announced that it could no longer meet its external financial obligations. This episode marked the beginning of what today is known as the Developing Country Debt Crisis. Mexico's decision was followed by similar measures by other highly indebted developing countries, particularly in Latin America. In this section we present an analytical overview of the events leading to the Debt Crisis, its economic consequences, and its reversal with the capital inflows of the 1990s.

The fact that many countries were affected simultaneously suggests that international factors played an important role in the financial crisis of the

early 1980s.

A number of external factors led to a large accumulation of debt by developing countries in the second half of the 1970s. The sharp oil price increase in 1973-74 led to huge deposits by middle eastern countries in international banks. Flushed with funds, commercial banks were eager to lend. In addition, in general, bankers in industrialized countries strongly felt that developing countries could never go bankrupt. Two other external factors were important in explaining the unusual amount of capital that flowed to Latin America and other developing countries in the late 1970s: low real interest rates and large growth in exports.

There were also domestic government policies in Latin America that encouraged borrowing in the late 1970s. First, financial liberalization, led to large expansions in lending, as interest rate controls in the banking sector were removed. In some countries, such as Argentina and Chile, the government provided loan guarantees. Thus, domestic banks had incentives to borrow at very high rates and invested in risky projects. In fact, it was as if the government was subsidizing foreign borrowing by domestic banks.

A second domestic factor was the exchange rate policy followed by a number of Latin American countries. In the mid 1970s, countries in the Southern Cone of Latin America pegged their currencies to the U.S. dollar as a way to fight inflation. This policy resulted in a significant real exchange rate appreciation (i.e., in a fall in $S \cdot P^* / P$) and large current account deficits. Households expanded purchases of imported goods, especially durables such as cars and electrodomeestics.

In the early 1980s, there was a dramatic change in the economic environ-

Table 15.1: Interest rates in the late 1970s and early 1980s

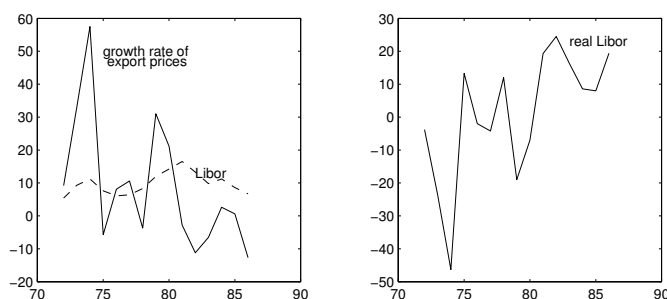
Year	Nominal LIBOR
1978	8.3
1979	12.0
1980	14.2
1981	16.5

Source: Andres Bianchi et al., “Adjustment in Latin America, 1981-86,” in V. Corbo, M. Goldstein, and M. Khan, ed., *Growth Oriented Adjustment Programs*, Washington, D.C.: International Monetary Fund and The World Bank, 1987.

ment. World interest rates increased sharply due to the anti-inflationary policy in the U.S. led by Federal Reserve chairman Paul Volker (see table 15.1). In addition, the terms of trade deteriorated for the debtor countries as raw material prices fell. As a result, the real interest rate faced by developing countries rose dramatically (see figure 15.3).

Debtor countries were highly vulnerable to the rise in world interest rates because much of the debt carried a floating rate. In Latin America, 65% of the foreign debt had a floating rate. Thus, debt service increased rapidly and unexpectedly in the early 1980s. The combination of higher interest rates and lower export prices resulted in sharp increases in interest payments relative to export earnings in highly indebted developing countries (see table 15.2). External lending to developing countries and inflows of foreign investment abruptly stopped in 1982. For all developing countries, new lending was 38 billion in 1981, 20 billion in 1982, and only 3 billion in

Figure 15.3: Interest rates and export prices in Latin America (1972-1986)



Note: The real Libor rate is constructed by subtracting the rate of change in export prices from the nominal Libor rate.

Source: Andres Bianchi et al., "Adjustment in Latin America, 1981-86," in V. Corbo, M. Goldstein, and M. Khan, ed., *Growth Oriented Adjustment Programs*, Washington, D.C.: International Monetary Fund and The World Bank, 1987.

Table 15.2: Interest payments in selected Latin American countries. Average 1980-81.

Country	Percent of Debt at floating rate	Interest Payment to Exports ratio (%)
Argentina	58	15
Brazil	64	28
Colombia	39	16
Chile	58	28
Mexico	73	19
All Latin America	65	28

Source: Andres Bianchi et al., "Adjustment in Latin America, 1981-86," in V. Corbo, M. Goldstein, and M. Khan, ed., *Growth Oriented Adjustment Programs*, Washington, D.C.: International Monetary Fund and The World Bank, 1987.

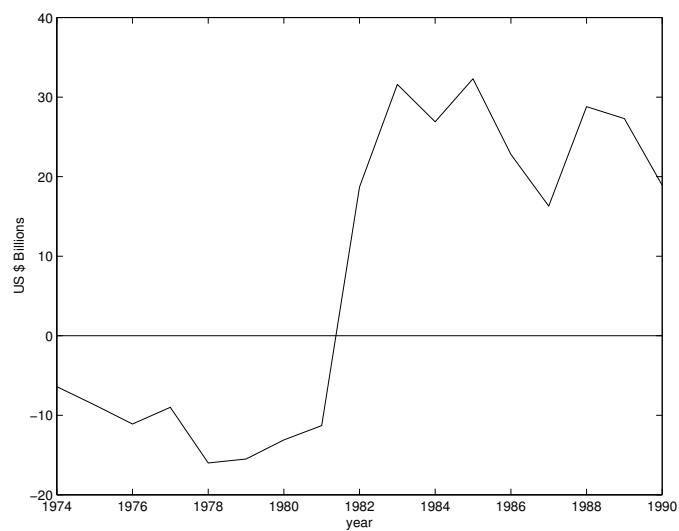
1983.

Domestic factors also contributed to the slowdown in capital inflows. The exchange rate policy of pegging the domestic currency to the U.S. dollar followed by countries in the Southern Cone of Latin America was believed to be unsustainable, in part because governments did fail to implement the required fiscal reforms. As a result, by the early 1980s expectations of real depreciation of the domestic currency induced domestic residents to invest in foreign assets (capital flight). In addition, the risky projects taken up by banks following the financial liberalization of the late 1970s and encouraged by government guarantees resulted in systemic banking failures.

As a result of the shutdown of foreign credit, countries were forced to generate large current account surpluses in order to continue to service, at least in part, their external obligations (see figure 15.4).

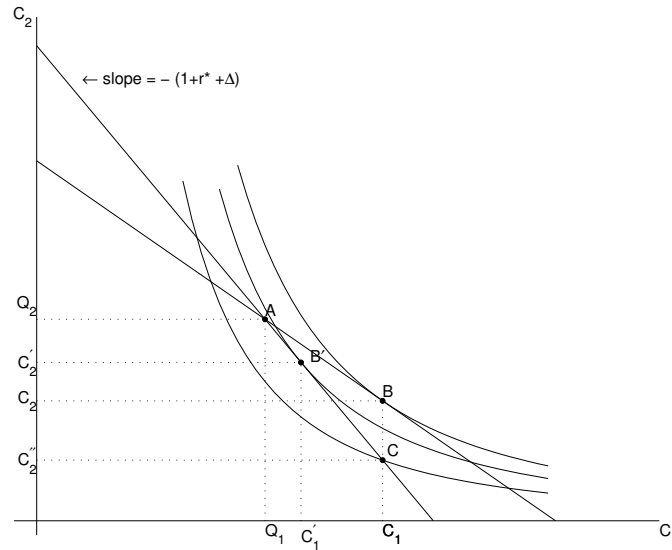
What does our model say about the macroeconomic consequences of a sharp world interest rate increase for a debtor country whose debt is at floating rates? Figure 15.5 depicts an endowment economy that starts with a zero initial net foreign asset position ($(1+r_0)B_0^* = 0$). The endowment point, (Q_1, Q_2) , is given by point *A* in the figure. The initial equilibrium is at point *B*, where the economy is running a current account deficit (or borrowing from abroad an amount) equal to $Q_1 - C_1$ in period 1. The situation in period 1 resembles the behavior of most Latin American countries in the late 1970s, which, taking advantage of soft international credit conditions borrowed heavily in international capital markets. Consider now an increase in the world interest rate like the one that took place in the early 1980s. The interest rate hike entailed an increase in the amount of resources needed

Figure 15.4: The trade balance in Latin America (1974-1990)



Source: Economic Commission for Latin America and the Caribbean (ECLAC), Preliminary Overview of the Economy of Latin America and the Caribbean, Santiago, Chile, December 1990.

Figure 15.5: Floating Interest Rates and Current Account Adjustment



to service not only newly assumed obligations but also *existing* debts. This is because, as we argued above, most of the developing country debt was stipulated at *floating* rates. In terms of our graph, the increase in the interest rate from r^* to $r^* + \Delta$ causes a clockwise rotation of the budget constraint around point A.

We assume that households took on their debt obligations under the expectations that the world interest rate would be r^* . We also assume that the interest rate hike takes place *after* the country assumes its financial obligations in period 1. However, in period 2 the country must pay the higher interest rate on the financial obligations assumed in period 1 because those obligations stipulated a floating rate. Therefore, households cannot reoptimize and choose point B' , featuring a lower trade deficit—and hence lower foreign debt—in period 1. They are stuck with $TB_1 = Q_1 - C_1$. This

means that the new position of the economy is point C on the new budget constraint and vertically aligned with point B . The increase in the world interest rate forces the country to generate a large trade balance in period 2, given by $Q_2 - C_2''$ in order to service the debt contracted in period 1. Note that the trade surplus in period 2 is much larger than it would have been had the country been able to re-optimize its borrowing in period 1 ($Q_2 - C_2'$). It is clear from figure ?? that the improvement in the trade balance leads to a depreciation of the real exchange rate and a contraction in aggregate spending. The response of the economy in period 2 captures pretty well the adjustment that took place in most Latin American countries in the wake of the Debt Crisis. Figure 15.4 documents the spectacular trade balance reversal that took place in Latin America in 1982. Table ??, shows that in Chile, the improvement in the current account in the aftermath of the debt crisis was accompanied by a dramatic (and traumatic) real exchange rate depreciation. The Chilean experience is not atypical. Large real depreciations were observed across Latin America after 1982.

15.4 The resurgence of capital inflows to developing countries in the 1990s

In the 1990s, developing countries in Asia and Latin America experienced a resurgence of capital inflows. About \$670 billion of foreign capital flowed to these countries in the 5 years from 1990 to 1994, as measured by the total balance on the financial account. This is 5 times larger than the \$133 billion of total inflows during the previous 5 years.

An article by Guillermo Calvo, Leonardo Leiderman, and Carmen Reinhart analyzes the causes of the resurgence of capital inflows to developing countries in the 1990s and argues that a number of factors were at work.¹ The widespread nature of the phenomenon suggests that global factors were especially important. Many of these factors are the same that led to high capital inflows to the region in the late 1970s. Domestic factors also played a role in determining the magnitude and composition of capital flows.

First, interest rates in international financial markets in the 1990s were relatively low. After peaking in 1989, interest rates in the U.S. declined steadily in the early 1990s. In 1992 interest rates reached their lowest level since the 1960s. This attracted capital to high-yield investments in Asia and Latin America. Second, in the early 1990s, the U.S., Japan, and several countries in Western Europe were in recession, which implied that they offered fewer investment opportunities. Third, rapid growth in international diversification and international capital market integration, facilitated in part by financial deregulation in the U.S. and Europe, allowed mutual funds and life insurance companies to diversify their portfolios to include emerging market assets. Fourth, many developing countries made progress toward improving relations with external creditors. Fifth, many developing countries adopted sound fiscal and monetary policies and market-oriented reforms such as trade and capital liberalization (Chile, Bolivia, and Mexico in the 1980s, Argentina, Brazil, Ecuador, and Peru in the 1990s). Finally, there seemed to be what some researchers call contagion. The opening of a large

¹See G. Calvo, L. Leiderman, and C. Reinhart, "Inflows of Capital to Developing Countries in the 1990s," *Journal of Economic Perspectives*, 10, Spring 1996, 123-139.

Table 15.3: Selected recipients of large capital inflows: macroeconomic performance 1988-1994

Country	Year Capital Inflow began	Cumulative RER appreciation	Average CA/GDP
Asia			
Indonesia	1990	-6.2	-2.5
Malaysia	1989	-3.9	-4.8
Philippines	1992	20.9	-4.2
Thailand	1988	1.9	-6.0
Latin America			
Argentina	1991	20.1	-3.1
Brazil	1992	57.9	-.2
Chile	1990	13.5	-1.8
Colombia	1991	37.1	-4.2
Mexico	1989	23.4	-6.8

Source: “Inflows of Capital to Developing Countries in the 1990s” by G. Calvo, L. Leiderman, and C. Reinhart, *Journal of Economic Perspectives*, Spring 1996.

developing economy to capital markets (like Mexico in the late 1980s) can produce positive externalities that facilitate capital inflows to other neighboring countries.

As shown in table 15.3, the capital inflows of the 1990s produced a number of important macroeconomic consequences, which are strikingly similar to those that paved the way for the debt crisis in the late 1970s: (1) The counterpart of the surge in capital inflows was a large increase in current account deficits, which materialized via investment booms and declines in savings. (2) In Latin America, the surge in capital inflows led to large real exchange appreciations. By contrast, in Asia such appreciation was observed only in the Philippines. (3) The decline in savings was associ-

Table 15.4: The evolution of the debt/GNP ratio in selected countries, 1980-1985

	$\frac{D}{GDP}$		
	1980	1982	1985
Argentina	.48	.84	.84
Brazil	.31	.36	.49
Mexico	.30	.53	.55

Source: Jeffrey D. Sachs and Felipe Larrain B., *Macroeconomics in the Global Economy*, Prentice Hall, Englewood Cliffs, New Jersey, 1993, Table 22-9.

ated with increases in consumption of (mostly imported) durable goods. (4)

A significant fraction of capital inflows were channeled to accumulation of foreign exchange reserves by central banks.

15.5 The Debt Burden

A country's debt burden can be measured by its debt-to-GDP ratio,

$$\text{Debt burden} = \frac{D}{GDP},$$

where D denotes the country's stock of external debt and GDP denotes gross domestic product, both measured in terms of tradables. A notable characteristic of the debt crisis was that the debt burden of developing countries rose rather than fell. Table 15.4 shows that the debt burden of Argentina, Brazil, and Mexico was 18 to 36 percentage points higher in 1985 than in

1980. The reason why the observed increase in the debt-to-GDP ratio is surprising is that, as we discussed in the previous section, with the onset of the debt crisis the flow of capital to developing countries came to an abrupt halt. Therefore, the observed rise in the debt burden must have been driven by a decline in GDP rather than an increase in debt.

The reason for the sharp decline in GDP is, among other factors, that large real exchange rate depreciations lead to a decline in the value of domestic output in terms of tradables. Domestic output in terms of tradables is the sum of tradable output and nontradable output measured in terms of tradables, that is,

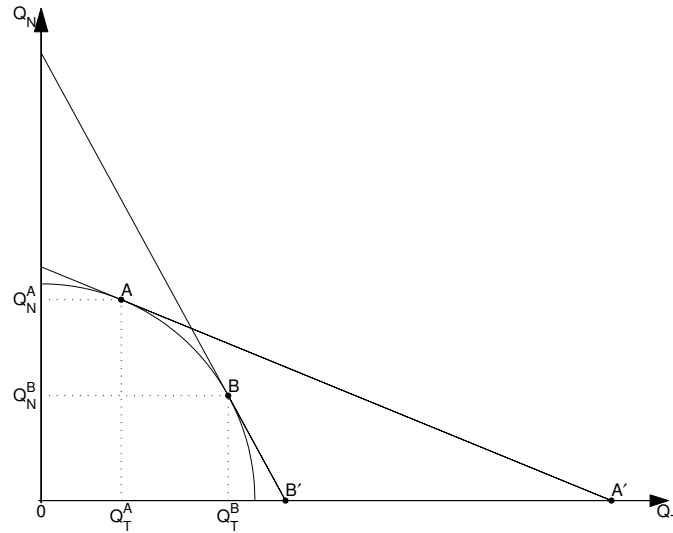
$$\text{GDP in terms of tradables} = Q_T + \frac{P_N}{P_T}Q_N.$$

In response to a real exchange rate depreciation the production of tradables increases and that of nontradables declines. The value of domestic output of nontradables measured in terms of tradables falls because both Q_N and P_N/P_T fall. On the other hand, production of tradables increases.

How can we determine that the net effect on output in terms of tradables is negative? Let's use the TNT model developed in chapter ?? . Consider a small open economy that experiences a sharp deterioration of its real exchange rate. Suppose that initially the country produces at point A in figure 15.6. The equilibrium real exchange rate is given by the negative of the slope of the PPF at point A and GDP in terms of tradables is given by point A', which is the sum of Q_T^A and $(P_N^A/P_T^A)Q_N^A$.² Suppose now that the

²To see that point A' represents GDP in terms of tradables, note that the line connecting A and A' has slope $-P_T^A/P_N^A$ and crosses the point (Q_T^A, Q_N^A) ; thus such line can

Figure 15.6: The effect of a real depreciation on the value of GDP in terms of tradables



real exchange rate depreciates and as a consequence equilibrium production takes place at point B on the PPF. The new real exchange rate P_T^B/P_N^B is equal to the negative of the slope of the PPF at point B . As the relative price of tradables rises, production of tradables increases from Q_T^A to Q_T^B and that of nontradables falls from Q_N^A to Q_N^B . The new value of GDP in terms of tradables is given by point B' , which is equal to $Q_T^B + (P_N^B/P_T^B)Q_N^B$. A real exchange rate depreciation thus causes a decline in the value of a country's GDP in terms of tradables and as a consequence implies that the country must spend a larger fraction of its GDP in servicing the external debt.

be written as the pairs (x, y) satisfying $y = Q_N^A - \frac{P_T^A}{P_N^A}(x - Q_T^A)$. We are looking for the intersection of this line with the x axis, that is, for the value of x corresponding to $y = 0$. Setting $y = 0$ we get $x = Q_T^A + (P_N^A/P_T^A)Q_N^A$.

15.6 Debt Reduction Schemes

Soon after the debt crisis of 1982, it became clear to debtor countries, creditors, and multinational organizations, such as the IMF and the World Bank, that full repayment of the developing country debt was no longer realistic and policy makers started to think about debt reduction schemes as a possible solution to the debt crisis.³

By the late 1980s the debt of many developing countries was trading in the secondary market at significant discounts, often as low as 50 percent of its face or par value, reflecting the fact that market participants thought that the likelihood that the country would ever be able to fully repay its debt was very low. At the time many policy makers and economists argued that in such a situation it would be beneficial to all parties to face reality and forgive debt to levels countries could afford. This idea was not often implemented. We begin by showing that one reason why debt forgiveness proposals may fail is that even if they are beneficial to all creditors as a group, they may not be so from the perspective of each individual creditor.

15.6.1 Unilateral Debt Forgiveness

Consider the situation of a country that owes D dollars. As a numerical example, suppose that $D = \$100$. Assume that there is some uncertainty about whether the country will be able to repay its debt in full. In particular, suppose that there are two possible outcomes (see table 15.5). Either the

³The analysis that follows draws heavily from a lucid article by Paul Krugman, of Princeton University, entitled “Reducing Developing Country Debt,” in *Currencies and Crises*, Paul Krugman (Ed.), Cambridge MA: MIT Press, 1995.

Table 15.5: Initial situation

	Good state	Bad state
Face value	$D = 100$	
Probability of state	$\frac{1}{3}$	$\frac{2}{3}$
Payments to creditors	100	25
Expected repayment to creditors	50	
Secondary market price of debt	0.50	

country will be able to repay its debt in full, we refer to this scenario as the good state. Or it will only be able to pay 25, we call this the bad state. Suppose that the good state occurs with probability $1/3$ (so that bad state occurs with probability $2/3$). Thus,

$$\text{expected repayment to creditors} = 100 \times 1/3 + 25 \times 2/3 = 50.$$

This means that the country's debt, whose face value is 100, is indeed worth only 50. The price of each unit of debt in the secondary market is accordingly only 0.50:

$$\text{secondary market price} = \frac{\text{Expected repayment}}{\text{Face value of the debt}} = \frac{50}{100} = 0.50$$

Suppose now that the creditors forgive 20 units of debt. Then the remaining debt outstanding is \$80 ($D = \80). What is the new secondary market price? As shown in table 15.7, in the bad state the country can again only pay 25 but in the good state it will pay the face value of the debt, which, after the debt reduction, is 80. Expected receipts of the creditors then are $80 \times 1/3 + 25 \times 2/3 = 43.33$. The secondary market price rises to

Table 15.6: Unilateral Debt Forgiveness

	Good state	Bad state
Initial face value	$D = 100$	
Face value after debt forgiveness	$D = 80$	
Probability of state	$\frac{1}{3}$	$\frac{2}{3}$
Payments to creditors	80	25
Expected payment to creditors	43.33	
Secondary market price of debt	0.54	

$43.33/80 = .54$. The loss from debt forgiveness to creditors is the difference between the expected repayment without debt forgiveness, 50, and the expected repayment with debt forgiveness, 43.33, that is, 6.67. Clearly, in this example creditors will never agree to debt forgiveness. The problem is that in this situation, debt forgiveness does not improve the debtor's capacity to pay in the bad state. It simply makes the debtor country's life easy in the good state, which is precisely the one in which it can afford to pay back.

15.6.2 Debt Overhang

In the previous example, debt forgiveness does not happen. Creditors prefer the status quo. However, in reality, creditors sometimes do agree to forgive debt. For example, at the G-7 Economic Summit held in Cologne, Germany, in June 1999, rich countries launched a program, dubbed the Cologne Initiative, aimed at reducing the debt burden of the so-called Highly Indebted Poor Countries (HIPCs).⁴ To understand why it can be in the creditor's interest to forgive debt, it is important to note that one unrealistic assumption

⁴For more information on ongoing efforts to reduce the debt burden of HIPCs see the web site of the Center for International Development at Harvard University (<http://www.cid.harvard.edu/cidhipc/hipchome.htm>).

of the previous example is that the ability of the debtor to pay is independent of the size of his debt obligations. There are reasons to believe that debtors are more likely to default on their debts the larger is the face value of debt. One reason why this is so is that if D is very large, then the benefits of efforts to improve the economic situation in the debtor country mainly go to the creditors (in the form of large debt-service-related outflows), giving the debtor country very little incentives to improve its economic fundamentals. Another reason why debt repudiation might become more likely as the level of debt gets high is that the debt burden might ultimately appear as a tax on domestic capital implicit in the government's need to collect large amounts of resources to meet external obligations, and thus act as a disincentive for domestic investment. The idea that the probability of repayment is low when the level of debt is high has come to be known as the *debt overhang argument*.

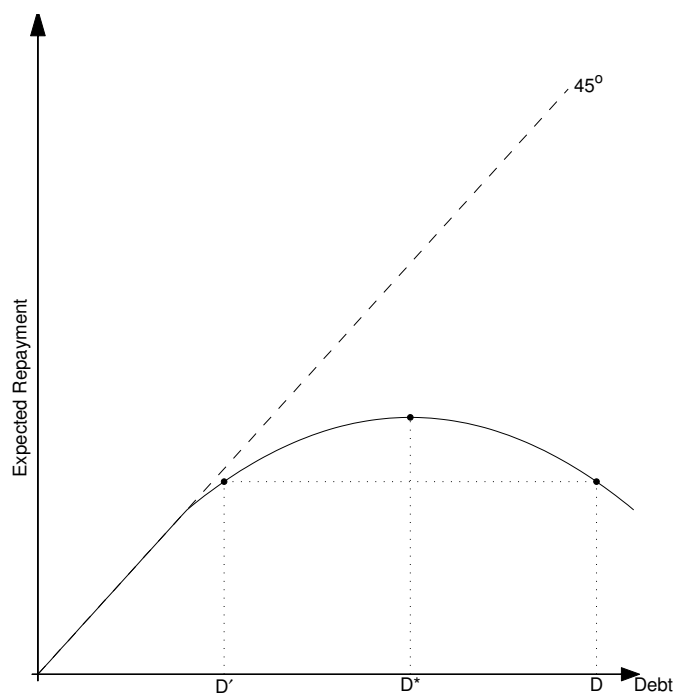
We can formalize the debt overhang argument as follows. Let π be the probability that the good state occurs. Assume that π depends negatively on D :

$$\pi = \pi(D); \quad \frac{d\pi(D)}{dD} < 0$$

Assume, as in our original example, that in the bad state the country pays only 25 while in the good state it pays the debt in full. Let D denote the face value of the country's outstanding debt, and assume that $D > 25$. Then, expected receipts of the creditor are given by

$$\text{expected repayment} = \pi(D) \times D + (1 - \pi(D)) \times 25.$$

Figure 15.7: The debt Laffer curve



Is it still the case that expected repayment is decreasing in the amount of debt forgiven? The answer is no, not necessarily. If an increase in debt pushes up the probability of the bad state sufficiently, then it can be the case that expected receipts actually fall as D increases. Figure 15.7 shows the relationship between the magnitude of debt outstanding and expected receipts of creditors. This relationship is known as the debt Laffer curve. Expected repayment peaks at a value of debt equal to D^* . The creditor of a country with an outstanding debt higher than D^* , can expect an increase in repayment if it forgives some debt. For example, in the figure, if the initial debt is D , the creditor can increase his expected receipts by forgiving debt in any amount up to $D - D'$. In particular, the creditor will maximize expected

Table 15.7: Debt forgiveness With Debt-Overhang

	Good state	Bad state
Initial face value	$D = 100$	
Face value after debt forgiveness	$D = 80$	
Probability of state	$\frac{1}{2}$	$\frac{1}{2}$
Payments to creditors	80	25
Expected payment to creditors	52.50	
Secondary market price of debt	0.66	

repayment by forgiving $D - D^*$ units of debt. Note that the optimal amount of debt relief does not result in a secondary market price of unity. In the figure, the secondary market price at any given level of debt, say D^* , is given by the ratio of the height of the Laffer curve to the height of the 45° line, which is less than unity. The secondary market price becomes unity only if the creditor accepts to reduce the debt to 25 or less, for in this case the risk of default disappears.

Let's illustrate the concept of debt overhang by means of a numerical example. Suppose, as in the previous numerical example that creditors forgive 20 of the outstanding debt of 100, so that the new debt is 80. Assume also that this reduction in the debt burden increases the probability of the good state from $1/3$ to $1/2$. The situation is summarized in table ?? Expected repayments are then given by $80 \times 1/2 + 25 \times 1/2 = 52.5$. Recall that in the absence of debt forgiveness, expected payments are 50. Thus, expected repayments increase by 2.5 even though the face value of the debt outstanding was reduced by 20. Creditors would benefit from such a unilateral debt reduction. Debtors would also benefit because in case the good state occurs, they have to pay 20 less than in the absence of the debt reduction scheme.

To sum up, if a country is on the ‘wrong’ (downward sloping) side of the debt Laffer curve, then it will be the case that unilateral debt forgiveness is not necessarily against the interest of creditors. Thus, one should not be surprised to see debt forgiveness happen sometimes.

15.6.3 The Free Rider Problem In Debt Forgiveness

Even in the case that unilateral debt forgiveness benefits the creditors, in practice, such schemes might be difficult to implement. The reason is that they create a ‘free rider’ problem. Going back to the above example, suppose that only some of the creditors forgive debt but others choose not to participate. As a result of the debt forgiveness, the secondary market price of debt increases from 0.5 to $52.5/80 = .66$ benefiting those who chose not to participate in the scheme. So, from the point of view of an individual creditor it is always best not to forgive any debt and hope that some of the other creditors do and then free ride on the debt reduction efforts of other creditors. Because of this free rider problem, if debt forgiveness occurs in practice it is usually a *concerted* effort, namely one where *all* creditors agree on forgiving some part of the debt.

15.6.4 Third-party debt buy-backs

A debt-reduction scheme often considered by multinational organizations is third-party debt buy backs. A third-party debt buy-back consists in purchases of developing country debt at secondary market prices by a third party, such as the World Bank, the Inter American Development Bank, or the International Monetary Fund, with the purpose of reducing the debt

burden of such countries. The third party buys some external debt in the secondary market and immediately forgives that debt (i.e., destroys the pieces of paper it bought).

Consider the original numerical example, summarized in table 15.5, of a country that has an outstanding debt of 100. The country can pay 100 in the good state and only 25 in the bad state. The good state occurs with probability $1/3$ and the bad state with probability $2/3$. The secondary market price of debt is 0.50 and expected payments are 50.

Suppose now that the World Bank announces that it will buy 75 units of (face value) debt in the secondary market. As soon as the announcement is made, the secondary market price jumps to a new value. Specifically, after the buy back the level of outstanding debt is 25, which the debtor country can pay in any state, good or bad. Thus, expected payments are $25 = 25 \times \frac{1}{3} + 25 \times \frac{2}{3}$, which is also the face value of the remaining outstanding debt. This implies that the secondary market price jumps up from 0.50 to 1 at the announcement of the buy-back. Thus, the debt trades at par. Who benefits from the buy-back? Creditors receive 75 from the World Bank and 25 from the debtor country. Thus, comparing the situation with and without buy-back, creditors benefit by 50, because in the absence of the buy-back scheme their expected receipts is 50, whereas under the buy-back scheme it is 100. Debtors have expected payments of 50 in the absence of the debt-reduction scheme and 25 under the debt buy-back. So their benefit is 25. Summing up, the World Bank pays 75, of which 50 go to the creditors and 25 to the debtor countries.

We conclude that third-party buy-backs are expensive—the World Bank

ends up paying par value for the debt it buys back—and benefits mostly the creditors rather than the debtors whom the World Bank meant to help.

15.6.5 Debt swaps

Another type of debt reduction scheme is debt swaps, which consist in the issuance of new debt with seniority over the old debt. The new debt is then used to retire old debt. It is important that the new debt be made senior to the existing debt. This means that at the time of servicing and paying the debt, the new debt is served first.

Consider again the original numerical example described in table 15.5. The debtor country pays the face value of the debt, 100, with probability $1/3$ and 25 with probability $2/3$. Thus, expected payments are 50 and the secondary market price is 0.5. Suppose now that the government issues 25 units of new debt with the characteristic that the new debt has seniority over the old debt. The new debt is default free. To see this, note that in the bad state the government has 25, which suffices to pay back the new debt. This implies that the debtor government is able to introduce the new debt at par, i.e., the price of new debt is unity. At the same time, because in the bad state all of the debtor resources are devoted to paying back the new debt, the government defaults on the totality of the outstanding old debt if the state of nature turns out to be bad. Let D^o denote the outstanding stock of old debt after the swap. Holders of this debt receive payments in the amount D^o in the good state and 0 in the bad state. So expected payments on the outstanding old debt equal $1/3 \times D^o + 2/3 \times 0 = 1/3 \times D^o$. The secondary market price of the outstanding old debt is the ratio of the

expected payments to the face value, or $(1/3 \times D^o)/D^o = 1/3$. Thus, the price of old debt experiences a sharp decline from 0.5 to 0.33. At this price, the government can use the 25 dollars raised by floating new debt to retire, or swap, $25/0.33 = 75$ units of old debt. As a result, after the swap the outstanding amount of old debt falls from 100 to 25, that is, $D^o = 25$.

Who benefits from this swap operation? Clearly the debtor country. In the absence of a swap, the debtor has expected payments of 50. With the swap, the debtor has expected payments of $8.33 (= 25 \times 1/3 + 0 \times 2/3)$ to holders of old debt and 25 to holders of new debt. These two payments add up to only 33.33. So the government gains $16.67 = 50 - 33.33$ by implementing the swap. On the other hand, creditors see their receipts fall from 50 before the swap to 33.33 after the swap (25 from the new debt and 8.33 from the old debt).

The Greek Debt Swap of March 2012

A recent example of a debt swap is the restructuring of Greek government debt that took place in the aftermath of the 2008 worldwide recession, which had thrown Greece into a particularly severe economic crisis. By 2011, GDP was falling at a rate of 7.5 percent per year, and unemployment was soaring reaching around 25 percent (50 percent among young workers). By March 2012 it had become clear that the Greek government could no longer service its debt, which exceeded 170 percent of GDP and made Greece the most highly indebted sovereign in the European Union. The face value of Greek government debt outstanding at the time was around €350 billion, of which €206 billion were held by private creditors and the rest by foreign govern-

ments and international institutions such as the European Central Bank and the International Monetary Fund. The Greek government proposed a debt swap for privately held debt that included three key elements: a write down of the face value, a reduction in interest rates, and an increase in maturities. The debt swap took the form of an exchange of €465 of new bonds for each €1,000 of old bonds outstanding. The €465 of new debt was composed of €150 of bonds issued and guaranteed by the European Financial Stability Facility and €315 of bonds issued by the Greek government. The new bonds would start paying interest for the first time in 2023, greatly reducing the pressures on the Greek fiscal deficit over the short and medium run. At the same time, Greece committed to continue to service its debt held by foreign governments and international institutions. In fact, a new law was enacted according to which any tax revenue must first be used to service the debt before it could fund any other government expenditures.

The vast majority of the privately held old debt, €177 billion, was issued under Greek law, and the remainder, €29 billion under foreign law. The Greek government passed a law that bound all private bond holders of debt issued under Greek law to the bond-swap if more than two-thirds of them consented to it. Faced with the alternative of outright default by Greece, most private creditors quickly agreed to the swap and thus the debt-swap was applied to the entire €177 billion of debt outstanding issued under Greek law. In addition, private holders of €20 billion of Greek government bonds issued under foreign law also chose to participate in the debt swap, so that in the end of the €206 billion of Greek debt held by the private sector, €197 billion was exchanged for new debt with a face value of €92 billion. That is,

the debt swap resulted in a debt write down of €105 billion and €9 billion remain in the hands of opportunistic holdouts.

15.7 Exercises

Exercise 15.1 (An Increase in the Interest-Rate Premium in Emerging Countries)

Suppose that the interest-rate premium in emerging countries increases from 0 to $p > 0$ because lenders become more concerned about the economic outlook in these economies. How does the interest rate in developed and emerging countries adjust to this development? How do the current account, saving, and investment adjust in emerging and developed countries? In answering this question, divide the world into two groups, the group of developed countries and the group of emerging countries and present a graphical analysis. Explain your graph and provide intuition.

Exercise 15.2 Indicate whether the statement is true, false, or uncertain and explain why.

1. Merlotia is a two-period, small open economy that specializes in the production of Merlot wines. Merlotia is subject to a country risk premium. In period 1, the grape harvest turns out to be more plentiful than expected and promises excellent wine exports in period 2. As a result, the interest rate in Merlotia increases in period 1.

Exercise 15.3 (External Debt Restructuring) Consider an economy with an external debt of D . Assume that the economy's capacity to honor its debt is state dependent. Specifically, suppose that there are 2 states, denoted good and bad. In the good state the country can pay its debt in full. In the bad state the country can at most pay 20. The probability of the bad state, which we will denote by $\pi(D)$, is given by $\pi(D) = 0.01D$.

1. Debt Forgiveness

Suppose that the country's debt is 80. What is the secondary market price of debt? Would it be in creditors collective interest to forgive 10 units of debt?

2. A Third-Party Debt Buy-Back

Suppose now that the country's debt is 50 and the a debt relief agency agrees to buy back 10 units of debt in the secondary market. (The debt relief agency will then forgive the debt it holds.) What price will the agency have to pay for each unit of debt it buys back? What is the total cost of the operation? By how much does the expected income of creditors increase? By how much do the expected payments of the debtor country decline?

Exercise 15.4 (A Debt Swap) Suppose a country has 100 units of debt outstanding. In the good state of the world the country can repay its debts in full, in the bad state of the world, the country can repay only 40. The good and the bad states occur with equal probability.

1. Calculate the repayments the creditors are expecting to receive.
2. Find the secondary market price of one unit of debt.

Now assume that the country wants to restructure its debt. It announces that it will introduce two new securities, of type A and B, respectively, to replace the 100 old securities outstanding by means of a debt swap. It issues 40 units of securities of type A, which will pay in full in the good and the

bad states, and 20 units of securities of type B, which pay in full only in the good state of the world. Both new securities are senior to the old existing debt.

3. At what rate can the government swap old securities for new securities of type A.
4. At what rate can the government swap old securities for new securities of type B.
5. What is the net effect of this debt restructuring on the expected repayments to creditors.

Exercise 15.5 (Gains from Commitment) This exercise presents a simple model to think about the economic implications for a peripheral European country of joining the eurozone if joining results in more commitment to repay external debts. Specifically, consider a two-period, small open endowment economy. In period 0, the first period, the small open economy starts with external debt (including interest) of $W_{-1} > 0$. Each period it receives an endowment $y > 0$. The no-Ponzi game condition requires that end of period debt in period 1 (the last period), denoted d_1 , must be zero. Consider first the case in which the country will not repay its external obligations in period 1 with probability $1 - p \in (0, 1)$. (This case is meant to capture the pre-euroarea period when default risk was positive.) Let r_H denote the interest rate the country faces in world financial markets for debt acquired in period 0 and due in period 1. Foreign lenders are risk neutral and understand that in period 1 with probability $1 - p$ they will not be

repaid and with probability p they will receive $1 + r_H$ per unit of debt. The domestic interest rate r_H must be such that foreign lenders are indifferent between lending to the small open economy or investing in a risk-free asset that pays the rate $r > 0$. Preferences of the representative household in the small open economy are given by $u(c_0) + \beta E_0 u(c_1)$, where $u(\cdot)$ is an increasing and concave period utility function, $\beta \in (0, 1)$ denotes the subjective discount factor, c_t denotes the stochastic process for consumption in period t for $t = 0, 1$, and E_0 denotes the expectations operator conditional on information available in period 0. Assume that $\beta(1 + r) = 1$.

1. Find the value of the domestic interest rate, r_H , in terms of r and p . Provide an intuitive interpretation of your answer.
2. Find the equilibrium values of consumption in terms of r , y , p , and W_{-1} . Your answer should be three expressions, one for c_0 , one for period-1 consumption in the case in which the country repays, c_1^r , and one for period-1 consumption in the case in which the country does not repay (defaults), c_1^d . Also provide restrictions on y and W_{-1} that ensure that consumption is positive in all dates and states. Provide a verbal interpretation of your findings.
3. Find the equilibrium value of the trade balance in period 0, tb_0 , in terms of r , p , and W_{-1} . Provide an intuitive explanation for the sign of tb_0 .

Now consider the case that in period 0 foreign lenders expect that the small open economy will repay with higher probability, that is, foreign

lenders now believe that p is higher. This is meant to capture the belief that accession to the euroarea endows a country with a higher ability to commit to repay.

4. Find the effect of an increase in the ability to commit to repay on the domestic interest rate, on consumption in all dates and states, and on the trade balance in period 0, tb_0 . Can the model explain the observed decline in interest rates? Does the perceived increase in the ability to commit to repay lead to a consumption boom in period 0? Explain why or why not. Are the predictions of the model consistent with the observed trade balance deteriorations in the periphery of the Eurozone upon accession?
5. Show whether accession to the euro area—as modeled here—is welfare improving or not. Provide intuition for your finding.

Chapter 16

Balance of Payment Crises