Artificial Intelligence Project

Section 2: NSGA-II

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Introduction:

NSGA-II, a widely used non-domination-based genetic algorithm for multi-objective optimization, has demonstrated effectiveness. However, criticisms have been directed at its computational complexity, absence of elitism, and the challenge of determining optimal parameter values for sharing. To address these concerns, a refined version, NSGA II, was introduced. This iteration features an improved sorting algorithm, includes elitism, and eliminates the need for a priori selection of sharing parameters.

[1]From a mathematical perspective, a multi-objective problem or Non-dominated Sorting Genetic Algorithm is defined as follows:

Minimizing or maximizing the vector f(x), where x is an n-dimensional decision variable vector x=(x1, x2,...,xn) from the set S, or in other words:

$$\text{Minimize} \quad f(x) = (f_1(x), \dots, f_m(x))$$

معادله 2-16

$$\text{Subject to} \quad g_i(\text{x}) \leq °, i = 1, \dots, q \qquad x \in S$$

معادله 2-17

Therefore, a multi-objective problem consists of n variables, q constraints, and m objectives, where the objective functions can be linear or nonlinear. The evaluation function F in a multi-objective problem maps the set in the decision variables, i.e., $S \rightarrow F$, in weighted aggregation methods prior to solving the multi-objective problem, it is transformed into a single-objective problem. In other words, in this approach to finding effective solutions to the multi-objective problem:

$$Maxf(x) = [f_1(x), f_2(x), \dots f_m(x)]$$

The optimal solution to this single-objective problem must be found:

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[1] First Refrence

$$Max f(x) = \sum_{k=1}^{m} w_k f_k(x)$$

معادله 2-19

Where $f_m(x)$ represents the objective functions, $f(x)$ is the ultimate objective function derived from the combination of the problem's objective functions, and the w_k are the weights assigned to the objective functions. In this case, the necessary condition for a point to be a general optimum is that the KKT conditions are satisfied, and the sufficient condition is that the optimization problem is convex.

In the Karush–Kuhn–Tucker (KKT) conditions, it is stated that:
- The objective function $z(x)$ should be concave (in the case of maximization).
- The feasible solution set should be convex (in the case of maximization).
- The objective functions should be differentiable.

As observed, this method, in addition to the constraints it imposes (Karush-Kuhn-Tucker conditions), faces challenges in weighting, and the output solution is dependent on the input weights. Furthermore, in these methods, to obtain the effective solution set, the algorithm needs to be solved multiple times, and in each execution, a new solution must be obtained.

In weighting methods, after solving the issues arising from weighting, as mentioned in the mentioned method, they do not exist. Additionally, using evolutionary algorithms in these methods leads to solving the problem once and obtaining a set of effective solutions. These algorithms emphasize moving towards the optimal solution and defining a cost function, enabling the specification of optimality conditions. Moreover, there is less emphasis on the Karush-Kuhn-Tucker conditions, unlike the pre-solving weighting methods. In recent decades, several multi-objective evolutionary algorithms have been proposed, including algorithms like VEGA, FFGA, SPEA, and NSGA. Genetic algorithms have characteristics that make them easier to apply to multi-objective problems compared to other algorithms:

- Genetic algorithms use binary codes to encode decision variables.

–   Genetic algorithms employ a probability-based selection process for choosing parents, leading to an approach that converges toward the best solution.

Non-dominated Sorting Genetic Algorithm II (version2):

The working method and general algorithm of NSGA-II are as follows:

1- Creating the Initial Population

2- Calculating Fitness Criteria

3- Sorting the population based on Dominance Conditions

4- Calculating Congestion Distance

Selection: Once the initial population is sorted based on dominance conditions, the congestion distance is calculated, and the selection process begins from the initial population. This selection is based on two elements:

– Population Rank: Populations are selected from lower ranks.

– Distance Calculation: Assuming p and q are two members of the same rank, a member with a greater congestion distance is selected. It is worth noting that the selection priority is initially based on rank and then on congestion distance. Performing crossover and mutation to generate new offspring, this is done using the binary selection method.

Combining the initial population with the population obtained from crossover and mutation involves replacing parent populations with the best members of the combined population from previous stages. In the first step, members from lower ranks replace the previous parents and are then sorted based on congestion distance. This process is summarized in Figure 2-16. As Figure 2-16 illustrates, the initial population and the population resulting from crossover and mutation are initially categorized based on rank, and those with lower ranks are removed. In the

next step, the remaining population is sorted based on congestion distance. Here, sorting is done within a جبهه یا aspect.

All stages are repeated until the desired generation (or optimization conditions) is reached.
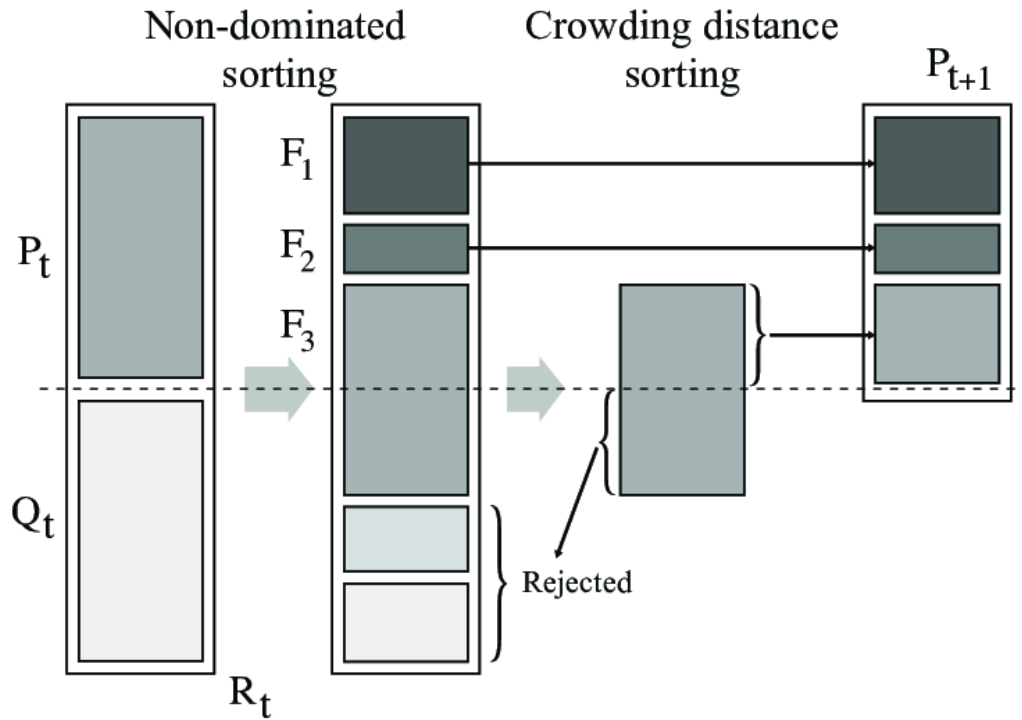
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Figure 1:

P represents the initial population, Q represents the population resulting from crossover and mutation, and F_i denotes a front.[2]

Here are some of the key features of NSGA-II:

- Elitism: The best solutions from each generation are carried over to the next generation, which helps to prevent the loss of good solutions.

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[2] Kalyanmoy Deb, Associate Member, IEEE, Amrit Pratap, Sameer Agarwal, and T. Meyarivan: NSGA-II

- Non-dominated sorting: Solutions are ranked based on how many other solutions they dominate. This helps to focus the search on solutions that are likely to be good.

- Crowding distance: Solutions are also assigned a crowding distance, which is a measure of how crowded the region of the objective space around the solution is. This helps to maintain diversity in the population.
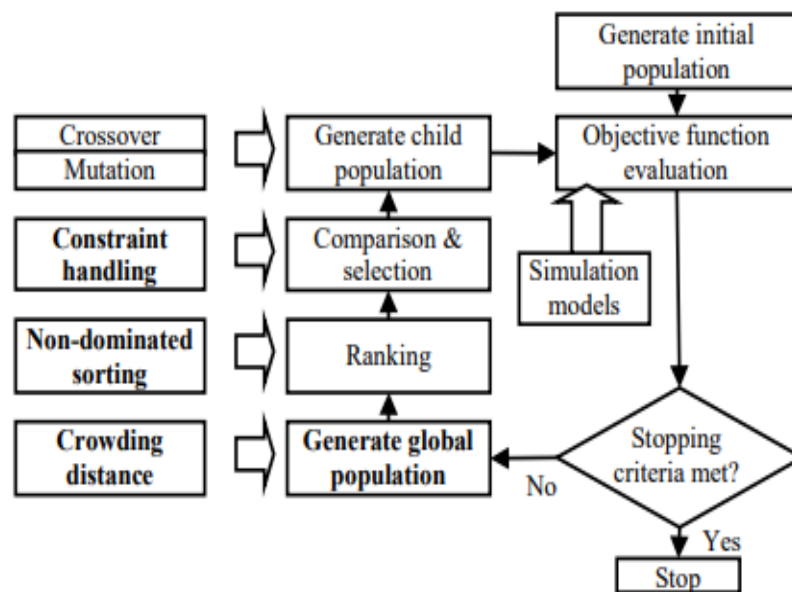

Figure 2:
Process using multi-objective Genetic Algorithm NSGA-II[3]

NSGA–II is summarized in Figure 2. In addition to the conventional steps of genetic algorithms, such as selection, crossover, and mutation, NSGA–II has four special features (shown in bold in Figure 2), which distinguish it from traditional multi-objective genetic algorithms. First of all, before applying the ranking operation, a global population is generated by combining both the parent and child generations, thus elitism is ensured. Secondly, a special book keeping strategy is used in the non-dominated sorting process, which reduces computational

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[3] Accounting for greenhouse gas emissions in multi-objective genetic algorithm optimization of water distribution systems

complexity. In addition, a crowding distance comparison is used for solutions with the same rank; hence a sharing parameter is not required. Furthermore, an efficient constraint handling method referred to as constrained tournament selection (Deb 2002[4]) is used.

خلاصه ای از مطالب بالا :

الگوریتم ژنتیک چند هدفه با مرتب سازی نامغلوب (Non-dominated Sorting Genetic Algorithm) یکی از الگوریتم های شاخص و پرکاربرد در زمینه بهینه سازی چندهدفه است. پس از ارائه نسخه اول این الگوریتم در سال ۱۹۹۵، معرفی کنندگان این الگوریتم که از میان آن ها دب (Deb) معروف تر از سایرین است، نسخه دوم آن را در سال ۲۰۰۲ با نام اختصاری NSGA-II ارائه نمودند.

در کنار تمام کارایی هایی که NSGA-II دارد، می توان آن را الگوی شکل گیری بسیاری از الگوریتم های بهینه سازی چند هدفه دانست. این الگوریتم و شیوه منحصر به فرد آن در برخورد با مسائل بهینه سازی چند هدفه، بارها و بارها توسط افراد مختلف برای ایجاد الگوریتم های بهینه سازی چند هدفه جدیدتر، مورد استفاده قرار گرفته است. بدون شک این الگوریتم یکی از اساسی ترین اعضای کلکسیون الگوریتم بهینه سازی چندهدفه تکاملی است که می توان آن ها را نسل دوم این گونه روش ها نامید.

ویژگی الگوریتم چند هدفه مبتنی بر رتبه بندی معرفی دسته ای از راه حل های بهینه بر اساس توابع هدف، می باشد که جبهه! پرتو را تشکیل می دهند در حالتی که تعداد زیادی محدودیت و شرط برمسئله حاکم بوده و فضای حل مسئله از گستردگی بالایی برخوردار باشد، نسبت فضای مجاز به غیرمجاز کاهش یافته و امکان همگرایی مسئله به نحو قابل ملاحظه ای کم میشود این موضوع اکثر موارد سبب میشود مسئله بهینه سازی به جوابی در فضای مجاز نرسد

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[4] [Kalyanmoy Deb, Associate Member, IEEE, Amrit Pratap, Sameer Agarwal, and T. Meyarivan: NSGA-II](#)

روش کار و الگوریتم کلی NSGA-II:

1- ساخت جمعیت اولیه

2- محاسبه معیارهای برازندگی

3- مرتب کردن جمعیت بر اساس شروط غلبه

4- محاسبه فاصله ازدحامی

- انتخاب: به محض اینکه جمعیت اولیه بر اساس شرط‌های غلبه کردن مرتب شد، مقدار فاصله ازدحامی در آن محاسبه خواهد شد و انتخاب از میان جمعیت اولیه آغاز می‌شود. این انتخاب بر اساس دو المان صورت می‌پذیرد:

- رتبه جمعیت: جمعیت‌ها در رتبه‌های پایین تر انتخاب می‌شوند

- محاسبه فاصله: با فرض اینکه p و q دو عضو از یک رتبه باشند، عضوی انتخاب می‌شود که فاصله ازدحامی بیشتری دارد. لازم به ذکر است که اولویت انتخاب ابتدا با رتبه و سپس بر اساس فاصله ازدحامی است.

نکته: تفاوت اصلیش با ژنتیک عادی یکی در تعداد توابع هدف و یکی در نحوه انتخاب جمعیت هست که باید دو مفهوم domination و crowding distance رو اشنایی داشته باشیم.

فلسفه‌ی جبهه پرتو اولین بار تو ورژن دوم nsga استفاده شد، به این صورت که مثل همه‌ی الگوریتم‌های متاهیوریستیک شما جمعیت اولیه دارد، بعدش یه حلقه‌ی تکرار که براساس کراس اوور و جهش سعی در بهبود دارد.

وقتی جمعیت اولیه ایجاد میشه مثلا اهداف هزینه و انرژی رو مدنظر دارید. خب جمعیت اولیه تشکیل میشه، مقدار هزینه و انرژی واسه هر کروموزوم محاسبه میشه، بعد این دو مقدار رو میدیم به تابع non-dominated که رنک بندی انجام بشه که همون تشکیل جبهه‌هاس.

[دیدن این ویدو خالی از لطف نیست](#)

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