INSTITUTE FOR COMPUTATIONAL ENGINEERING & SCIENCES
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Online Learning with Implicit Exploration in Episodic Markov Decision Processes

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American Control Conference (ACC)

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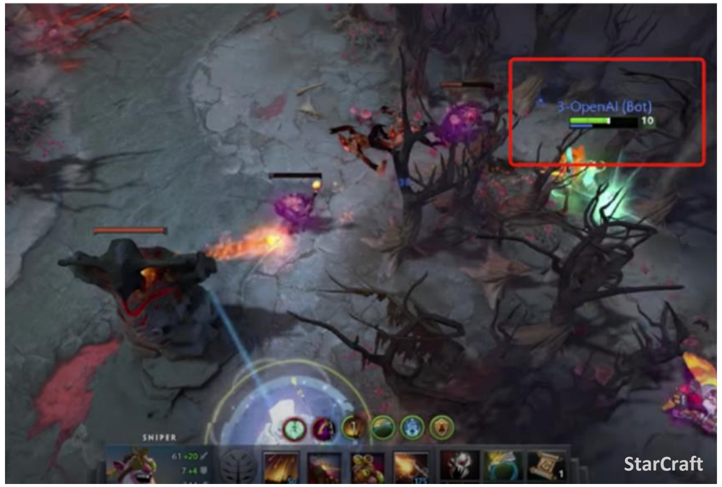
Sequential Decision Making

DeepMind

Sequential Interaction with the environment

Tesla

Learning from a fixed reward

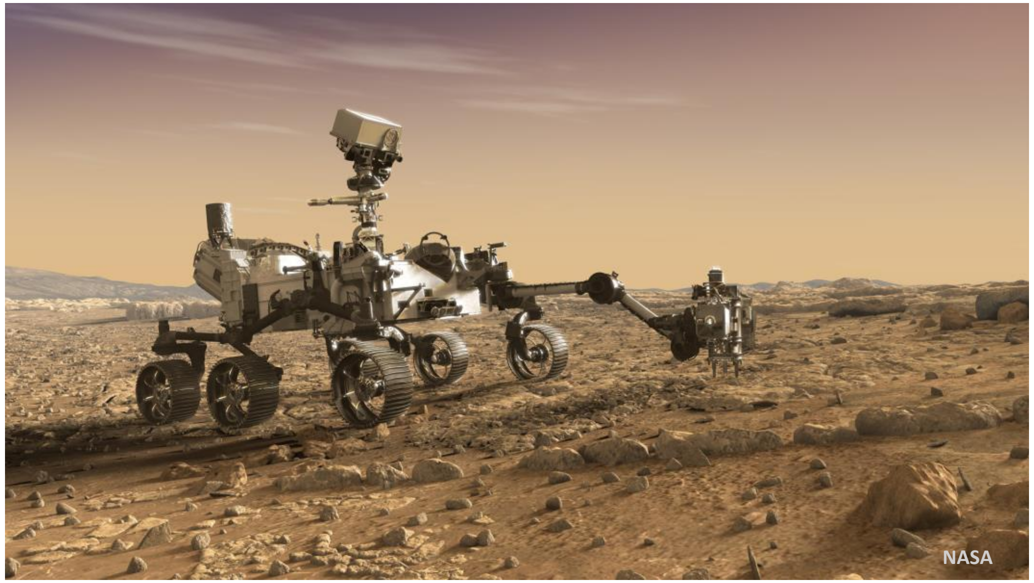


StarCraft

Offline: access to a lot of data

KUKA

Sequential Decision Making with Varying Tasks



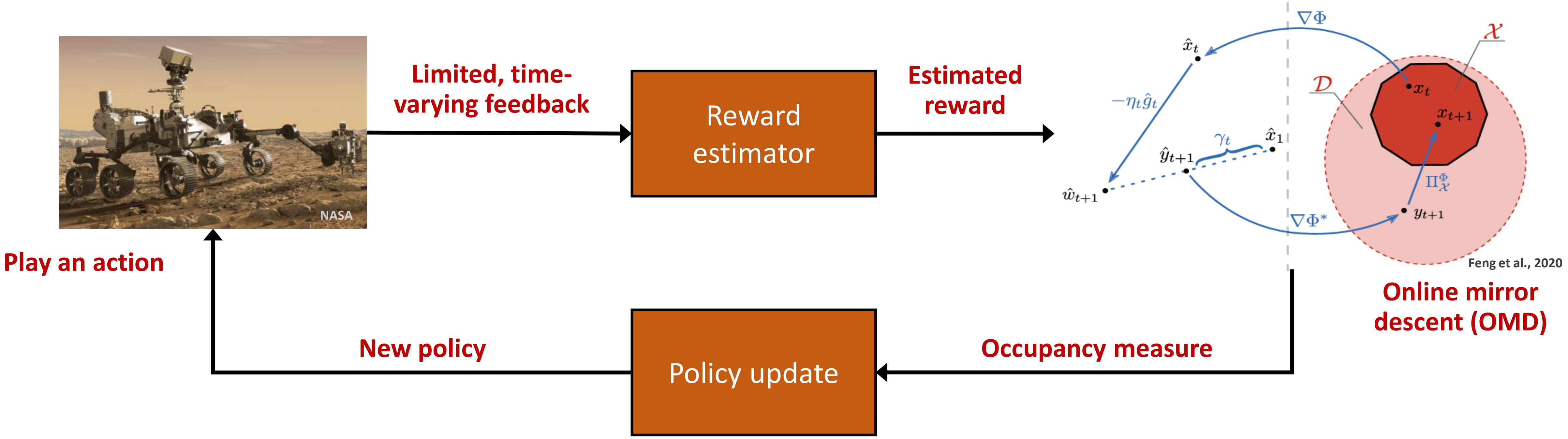
Evolving environment and task

Safety-critical operation

Limited feedback from the environment

How can we design online algorithms with high probability guarantees for varying tasks?

Online Learning with Implicit Exploration for Varying Tasks



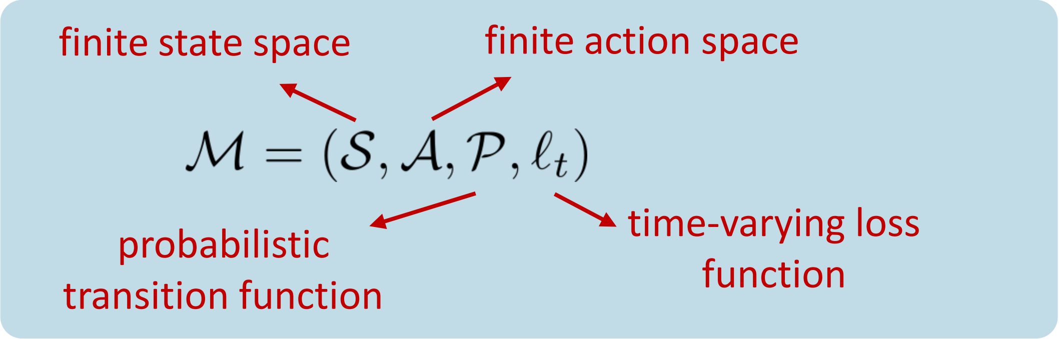
Feng et al., 2020

Online mirror descent (OMD)

Contributions:

- A novel optimistically-biased reward estimator for implicit exploration
- Policy search using online mirror descent (OMD)
- Minimax optimal regret bound with high probability

Adversarial Markov Decision Process (A-MDP)



finite state space

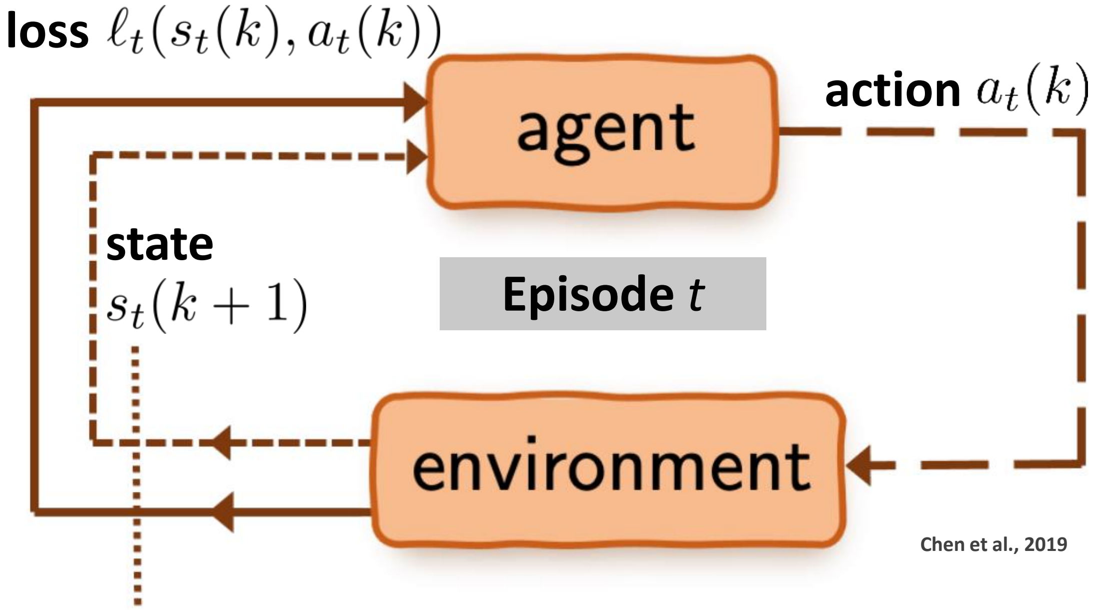
finite action space

$$\mathcal{M} = (\mathcal{S}, \mathcal{A}, \mathcal{P}, \ell_t)$$

probabilistic transition function

time-varying loss function

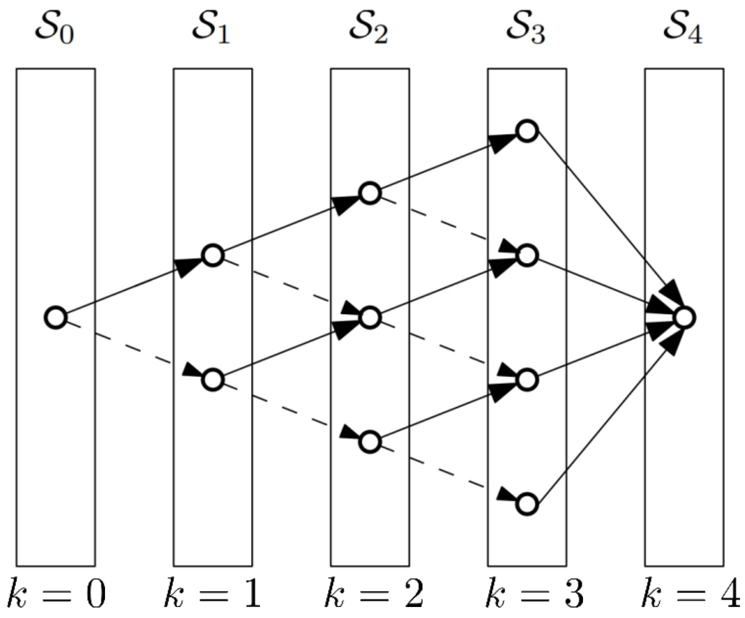
Bandit feedback



Chen et al., 2019

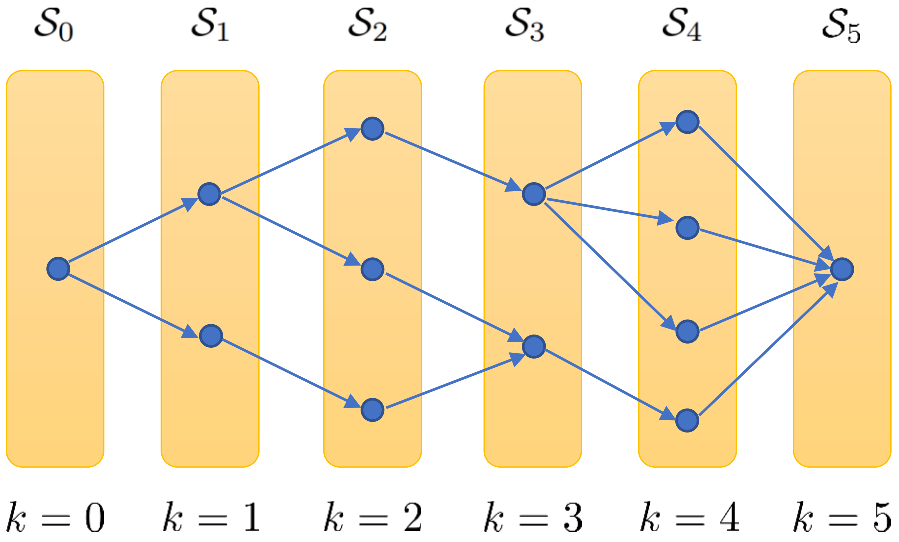
Loop-free episodic A-MDP:

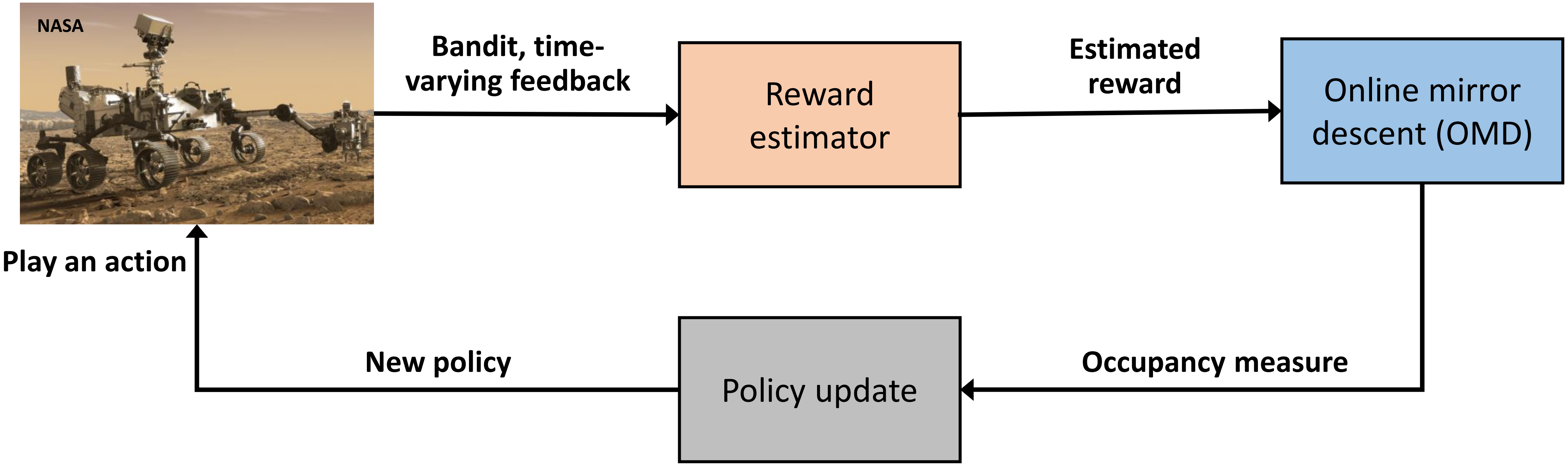
- States are partitioned into layers
- Transition only exists from one layer to the next

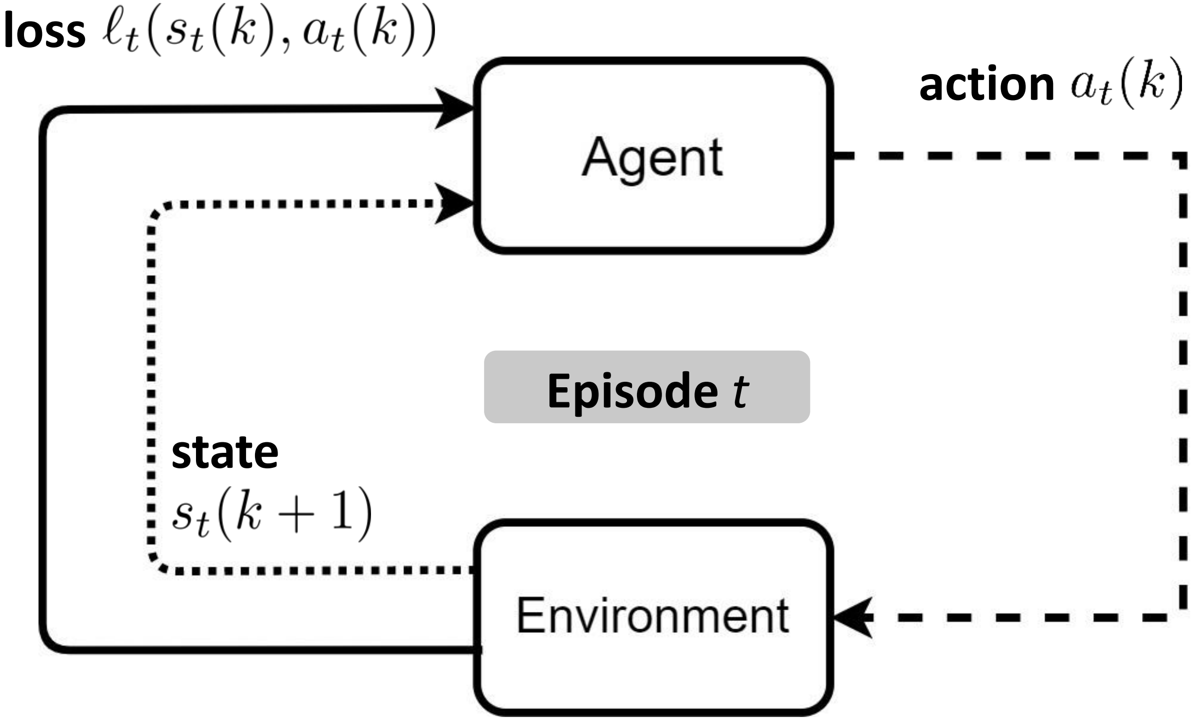


Neu et al., 2020

aaaaa


$\mathcal{S}_0$
$\mathcal{S}_1$
$\mathcal{S}_2$
$\mathcal{S}_3$
$\mathcal{S}_4$
$\mathcal{S}_5$
$k = 0$
$k = 1$
$k = 2$
$k = 3$
$k = 4$
$k = 5$


NASA
Bandit, time-varying feedback
Reward estimator
Estimated reward
Online mirror descent (OMD)
Occupancy measure
Policy update
New policy
Play an action


loss $\ell_t(s_t(k), a_t(k))$
action $a_t(k)$
Agent
Episode t
state
$s_t(k+1)$
Environment

Agent’s Policy Representation via Occupancy Measure

Looking for a time-varying stochastic policy $\pi_t : \mathcal{S} \times \mathcal{A} \to [0, 1]$

Occupancy measure: the probability induced over state-action pairs by executing a policy

$$\rho^{\pi}(s, a) = \Pr(\mathbf{s}_{k(s)} = s, \mathbf{a}_{k(s)} = a | \pi)$$

Stochastic stationary policy given an occupancy measure

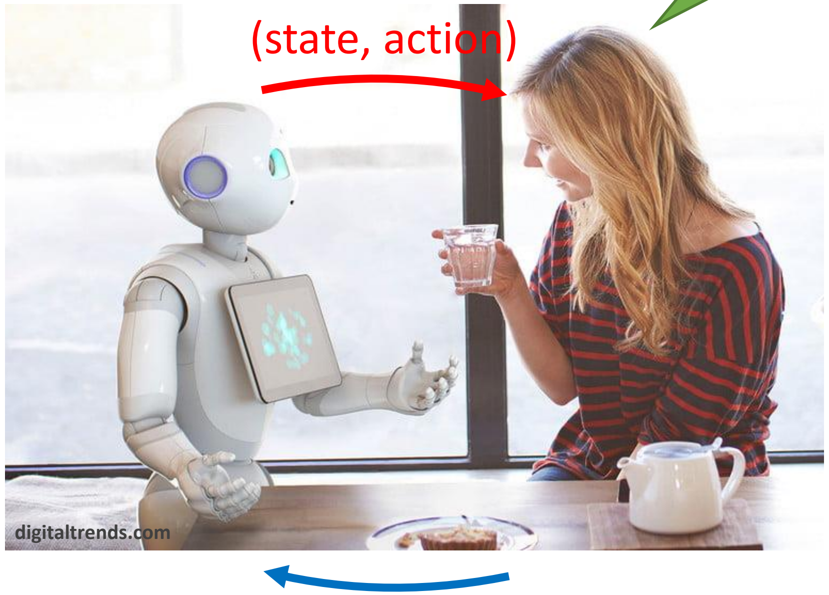
$$\pi^{\rho}(a|s) = \frac{\rho(s, a)}{\sum_{a' \in \mathcal{A}} \rho(s, a')} \, , \quad \forall (s, a) \in \mathcal{S} \times \mathcal{A}$$

Regret Minimization

episode *t*

task *t*

(state, action)



digitaltrends.com

(loss)

Unknown and time-varying loss function (A-MDP)

$$\mathcal{M} = (\mathcal{S}, \mathcal{A}, \mathcal{P}, \ell_t)$$

Bandit feedback

$$\ell_t(s_t(k), a_t(k))$$

Learn a policy with sublinear regret:

$$\mathcal{R}_T := \max_{\pi} \mathcal{L}_T - \mathcal{L}_T(\pi)$$

best fixed policy in hindsight

Question: Can we obtain low regret with high probability?

Optimistic Loss Estimator

Bandit feedback → Estimating the loss of all state-action pairs

Goal: Obtain a low-variance loss estimator

A novel optimistically biased estimator for the loss function:

history at current episode

$$\hat{\boldsymbol{\ell}}_t(s,a) = \frac{\ell_t(s,a)}{\boldsymbol{\rho}_t(s,a)+\gamma}\mathbb{I}\{(s,a) \in \mathbf{h}(t)\}$$

exploration parameter

Optimistically biased → Implicit exploration

$$\mathbb{E}\left[\hat{\boldsymbol{\ell}}_t(s,a) | \mathbf{h}(t-1)\right] \leq \ell_t(s,a)$$

Policy Optimization via Online Mirror Descent

Goal: Compute a new policy from the estimated loss function

An OMD algorithm utilizing the proposed loss estimator:

learning rate

unnormalized KL divergence

$$\boldsymbol{\rho}_{t+1} = \arg \min_{\rho \in \Delta(\mathcal{M})} \left\{ \underbrace{\eta \langle \rho, \hat{\boldsymbol{\ell}}_t \rangle}_{\text{loss}} + \underbrace{D(\rho \| \boldsymbol{\rho}_t)}_{\text{policy change}} \right\}$$

Constrained optimization → Two-step procedure

$$\tilde{\boldsymbol{\rho}}_{t+1} = \arg \min_{\rho} \left\{ \eta \langle \rho, \hat{\boldsymbol{\ell}}_t \rangle + D(\rho \| \boldsymbol{\rho}_t) \right\}$$

$$\boldsymbol{\rho}_{t+1} = \arg \min_{\rho \in \Delta(\mathcal{M})} \{ D(\rho \| \tilde{\boldsymbol{\rho}}_{t+1}) \}$$

No-Regret Learning with High-Probability

Result: Establishing sublinear regret bounds both on expectation and with high-probability

Theorem: (high-probability regret bound)

Let $\delta \in (0,1)$. If

$$\eta = \gamma = \sqrt{L\frac{\log(|\mathcal{S}||\mathcal{A}|/L)}{2T|\mathcal{S}||\mathcal{A}|}},$$

with probability at least $1-\delta$,

$$\text{regret} \leq \mathcal{O}(\sqrt{LT|\mathcal{A}||\mathcal{S}|\log(|\mathcal{S}||\mathcal{A}|/L)}\log\tfrac{1}{\delta}).$$

episode length number of episodes number of states number of actions

Minimax optimal regret (up to logarithmic terms)

Conclusion and Future Work

- Proposed an optimistic loss estimator for learning in episodic A-MDP under bandit feedback

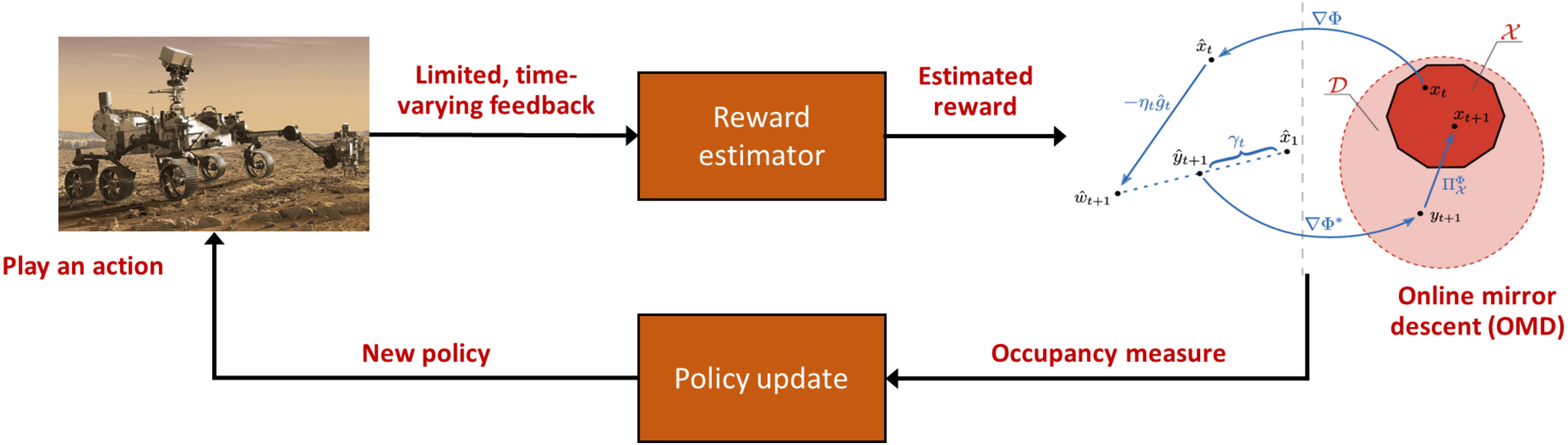
- Developed an OMD policy optimization utilizing the proposed loss estimator

- Established a minimax optimal regret bound with high probability

Future Directions

- Parameter-free and anytime algorithms

- Unknown, time-varying dynamics and large-scale state spaces



Limited, time-varying feedback

Reward estimator

Estimated reward

Play an action

Online mirror descent (OMD)

New policy

Policy update

Occupancy measure

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