

Instruction and an Example of COVID-19 Data Analysis for US States and Counties

Here is an example of COVID-19 data analysis for US states in the manuscript. We demonstrate the process of model 1, model 2.3, and model 3. For additional results for model 2.1, 2.2 and 2.4, see “Covid-19-state.R” for details. For county-level results, see “Covid-19-county.R” for details.

Setting Up

Before implementing the algorithm, we need the following set up:

- Install required packages and load packages.
- Call functions from .R file and .cpp files
 - “functions_BFL.R”
 - “functions_BFL.cpp”

```
install.packages("mvtnorm")
install.packages("lattice")
install.packages("RColorBrewer")
install.packages("factoextra")
install.packages("usmap")
install.packages("maps")
install.packages("zoo")
install.packages("vars")
install.packages("Rcpp")
install.packages("RcppArmadillo")
```

```
library("mvtnorm")
library("lattice")
library("RColorBrewer")
library("factoextra")
library("usmap")
library("maps")
library("zoo")
library("vars")
library("Rcpp")
library("RcppArmadillo")
```

```
source("functions_BFL.R")
sourceCpp("functions_BFL.cpp")
```

Loading Datasets

Here, we obtain the COVID-19 dataset from the New York Times Github repository. The population data is extracted from National Bureau of Economic Research.

```
##### Loading Datasets #####
## data extracted from New York Times state-level data
#obtained from following Github repository: https://github.com/nytimes/covid-19-data
data.states <- read.csv("states-12-23.csv", header = TRUE)
data.states <- data.states[as.Date(data.states$date) <= as.Date('2020-11-30'),]
data.states$date <- as.Date(data.states$date)

# population data extracted from NATIONAL BUREAU OF ECONOMIC RESEARCH
states.population <- read.csv("co-est2019-alldata.csv", header = TRUE)
states.population <- states.population[as.character(states.population$STNAME)
== as.character(states.population$CTYNAME),
c("STNAME", "CTYNAME", "POPESTIMATE2019")]
```

Data Preprocessing

After loading the datasets above, we can now clean the data and construct the variables/attributes. Here, we choose the Florida state as an example. One can change the state name and corresponding lockdown date and reopening date to test other states.

```
#####Change/choose the state name, lockdown date and reopen date below
state.name <- "Florida"
Date.1 <- '2020-04-03'
Date.2 <- '2020-06-05'
lambda.1 <- c(1, 0.5, 0.1, 0.05, 0.01, 0.005, 0.001)

# get all the states fips in the United States
neighbor.fips <- c()
state.fip <- as.numeric(fips(state.name))
for(i in 1:length(state.fips$fips)){
  if(state.fips$fips[i] != state.fip ){
    neighbor.fips <- c(neighbor.fips, state.fips$fips[i])
  }
}
neighbor.fips <- unique(neighbor.fips)
state.names <- fips_info(neighbor.fips)$full
state.names <- c(state.name, state.names)
state.lowernames <- tolower(state.names)

n.all <- rep(0,length(state.names) )
for(i in 1:length(state.names)){
  n.all[i] <- unique(states.population[
(states.population$CTYNAME == state.names[i]) &
states.population$STNAME== state.names[i] , "POPESTIMATE2019"])
}

state.names.all <- state.names
state.lowernames.all <- state.lowernames

dataList <- list()
for( i in c(1:length(state.names))){
  data.subset <- data.states[data.states$state==state.names[i], c("date", "cases", "deaths")]
  assign(paste("data", state.lowernames[i] , sep="."), data.subset)
```

```

dataList <-append(dataList, list(data.subset))
}

multi_full <- Reduce(
  function(x, y){merge(x, y, all = TRUE, by = "date")},
  dataList
)
for(i in 1:length(state.names)){
  names(multi_full)[2*i] <- paste0('cases.', state.lowernames[i])
  names(multi_full)[2*i+1] <- paste0('deaths.', state.lowernames[i])
}
#replace the na value with 0
n.domains <- length(state.names)
for(i in 1:(n.domains*2) ){
  multi_full[is.na(multi_full[,i+1]),i+1]=0
}
# multi_full$date <- as.Date(droplevels(multi_full$date))
multi_full$date <- as.Date(multi_full$date)
#choose the first day when the region has one confrimed case as the start date
multi_full <- multi_full[multi_full[,2]>0,]

cases.all <- multi_full[,seq(2, ncol(multi_full), 2)]
deaths.all<- multi_full[,seq(3, ncol(multi_full), 2)]

#T: number of time points
T <- nrow(cases.all);
#R: the number of people who have recovered
#using the nationwide recovered and death number to predict the recovered number
R.all <- floor((1 + (10))*deaths.all);
if( state.name == "New York"){
  R.all <- floor((1 + (5.5))*deaths.all);
}
#I: the number of people infected at time t
I.all <- cases.all - R.all;

#S: the number of susceptible people
n.all.matrix <- matrix(rep(n.all,T),ncol = n.domains,byrow = TRUE)
immune.rate <- 0.9
S.all <- n.all.matrix - I.all - R.all*immune.rate;

#the fraction of S, I and R
S.rate.all <- sapply(1:n.domains, function(jjj) S.all[,jjj]/n.all[jjj])
I.rate.all <- sapply(1:n.domains, function(jjj) I.all[,jjj]/n.all[jjj])
R.rate.all <- sapply(1:n.domains, function(jjj) R.all[,jjj]/n.all[jjj])

I.rate.all.obs <- I.rate.all
R.rate.all.obs <- R.rate.all

date.region <- multi_full$date
I.obs <- I.all[,1]
R.obs <- R.all[,1]

```

Model 1: Piecewise Constant SIR Model

Change Point Detection in Model 1

After data preprocessing, we now perform the block fused lasso algorithm to detect the structural breaks in the piecewise constant SIR models.

The proposed algorithm in the manuscript is implemented by the function `tbfl()`. In addition to arguments `data_y`, `data_x`, there are other optional arguments for the function `tbfl()` which are listed below:

- `method`: “MLR” for multiple linear regression, and “VAR” for VAR model.
- `lambda.1.cv`: the tuning parameter λ_1 for fused lasso. By default, `lambda.1.cv` is a vector with decreasing values constructed based on the time series data, the number of time series components p , and the block size `block.size`.
- `lambda.2.cv`: the tuning parameter λ_2 for fused lasso. By default, `lambda.2.cv` is a vector with decreasing values constructed based on the number of time points T and the number of time series components p .
- `q`: the AR order for VAR model. By default, $q = 1$.
- `max.iteration`: the max number of iteration for the fused lasso. By default, `max.iteration` = 100.
- `tol`: tolerance for the fused lasso. By default, `tol` = 10^{-2} .
- `block.size`: the block size b_n . By default, $b_n = \lfloor \sqrt{T} \rfloor$.
- `blocks`: the blocks (sequence). By default, `blocks` = `seq(0,T,block.size)`. One could also use `blocks` to handle varying-size blocks. For example, `blocks= c(seq(0,5000,100), seq(5200,10000,200))`.
- `HBIC`: if TRUE, use high-dimensional BIC criterion (HBIC); if FALSE, use traditional BIC criterion (BIC).
- `gamma.val`: the tuning parameter for HBIC.

```
a.vals <- c(0.1, 0.15, 0.2, 0.25, 0.3)
t.test <- 13
T.full <- T + t.test
MRPE_1_new.full <- c()
temp.full <- vector("list", length(a.vals));
est.1.full <- vector("list", length(a.vals));
set.seed(123456)
for(idx in 1:length(a.vals)){
  a.val <- a.vals[idx]
  I <- I.obs
  R <- R.obs
  for(t in 2:T){
    rate <- 1/(((t)+a.val*T.full )/((1 + a.val )*T.full))^2
    I[t] <- (I.obs[t] - I.obs[t-1])*rate + I[t-1]
  }

  plot( 1- (((1:T)+a.val*T)/((1 + a.val )*T))^2 )

  R.rate.all[, 1]<- R/n.all[1]
  I.rate.all[, 1]<- I/n.all[1]
  S.rate.all[, 1]<- (n.all.matrix[, 1] - I - R*immune.rate)/n.all[1];
  #####
  ##### construct variables #####
  #####
  y.list <- vector("list",T-1);
  x.list <- vector("list",T-1);

  for(i in 2:T){
    y.list[[i-1]] <- matrix(c(R.rate.all[i,1]-R.rate.all[i-1, 1],
```

```

I.rate.all[i, 1]-I.rate.all[i-1,1]), 2, 1);
x.temp <- matrix(0,2,2);
x.temp[1,2] <- I.rate.all[i-1, 1];
x.temp[2,1] <- I.rate.all[i-1, 1]*S.rate.all[i-1, 1];
x.temp[2,2] <- -I.rate.all[i-1, 1];
x.list[[i-1]] <- x.temp;
}

Y <- y.list[[1]];
for(i in 2:(T-1)){
  Y <- rbind(Y, y.list[[i]])
}

X <- x.list[[1]];
for(i in 2:(T-1)){
  X <- rbind(X, x.list[[i]])
}

beta_t <- sapply(1:length(y.list), function(jjj)
  (y.list[[jjj]][1]+y.list[[jjj]][2])/I.rate.all[jjj,1])
gamma_t <- sapply(1:length(y.list), function(jjj)
  y.list[[jjj]][1]/I.rate.all[jjj,1])

cols <- c("dark orange", "purple", "darkolivegreen4", "blue" )

#####
##### Model 1 #####
#####
#scaling but not centering data
Y_std <- sd(Y)
Y_s <- scale(Y, center = FALSE, scale = apply(Y, 2, sd, na.rm = TRUE))
X_std <- apply(X, MARGIN=2, FUN=sd)
X_s <- scale(X, center = FALSE, scale = apply(X, 2, sd, na.rm = TRUE))

p.x <- ncol(X_s)
p.y <- ncol(Y_s)
n <- nrow(X_s)
tol <- 10-4; # tolerance
max.iteration <- 200; # max number of iteration for the LASSO solution
method <- c("MLR")

#p is the number of variables for each day (I_t and R_t)
p <- 2
#b_t is the block size among the time points (1 : (length(date) - 1))
if(state.name %in% c("Texas")){
  b_t <- 7
  gamma.val <- 10
  HBIC = TRUE
}else if(state.name %in% c("Florida", "Iowa")){
  b_t <- 7
  gamma.val <- 5
  HBIC = TRUE
}

```

```

}else if(state.name %in% c("California")){
  b_t <- 7
  gamma.val <- 5
  HBIC = TRUE

}else if(state.name %in% c("Oregon")){
  b_t <- 7
  gamma.val <- 5
  HBIC = TRUE

}else{
  b_t <- 7
  gamma.val <- 10
  HBIC = TRUE
}
b_n <- p * b_t
temp.1 <- tbfl(method, Y_s, X_s, lambda.1.cv = lambda.1, lambda.2.cv = 0,
               max.iteration = max.iteration, tol = tol, block.size = b_n,
               HBIC = HBIC, gamma.val = gamma.val)

temp.full[[idx]] <- temp.1
cp <- c(1, temp.1$cp.final, n+1)
m <- length(cp) - 1
X.new.new <- matrix(0, nrow = n, ncol = m*p.x)
for(i in 1:m){
  X.new.new[cp[i]: (cp[i+1]-1), (p.x*(i-1)+1) : (p.x*i) ] <- X[cp[i]: (cp[i+1]-1), ]
}
est.1 <- lm(Y ~ X.new.new - 1)
est.1.full[[idx]] <- est.1
Y.hat.1 <- est.1$fitted.values
MRPE_1_new <- mean( abs ( ( Y.hat.1[seq(2, n, 2)] - Y[seq(2, n, 2)] ) / c(Y[seq(2,n,2)]) ) [c(Y[
MRPE_1_new.full <- c(MRPE_1_new.full, MRPE_1_new)
}

idx <- which.min(MRPE_1_new.full)
cp.final <- temp.full[[idx]]$cp.final
cp.date <- c(1:n)[floor( (cp.final-1) / p) + 1]
a.final <- a.vals[idx]

#run the tbfl method
#scaling but not centering data
set.seed(123456)
a.val <- a.final
I <- I.obs
R <- R.obs
for(t in 2:T){
  rate <- 1/((t)+a.val*T.full)/((1 + a.val )*T.full))^2
  I[t] <- (I.obs[t] - I.obs[t-1])*rate + I[t-1]
}
R.rate.all[, 1]<- R/n.all[1]
I.rate.all[, 1]<- I/n.all[1]
S.rate.all[, 1]<- (n.all.matrix[, 1] - I - R*immune.rate)/n.all[1];

#####

```

```
##### construct variables #####
#####

y.list <- vector("list", T-1);
x.list <- vector("list", T-1);

for(i in 2:T){
  y.list[[i-1]] <- matrix(c(R.rate.all[i,1]-R.rate.all[i-1, 1],
                           I.rate.all[i, 1]-I.rate.all[i-1,1]), 2, 1);

  x.temp <- matrix(0,2,2);
  x.temp[1,2] <- I.rate.all[i-1, 1];
  x.temp[2,1] <- I.rate.all[i-1, 1]*S.rate.all[i-1, 1];
  x.temp[2,2] <- -I.rate.all[i-1, 1];
  x.list[[i-1]] <- x.temp;
}

Y <- y.list[[1]];
for(i in 2:(T-1)){
  Y <- rbind(Y, y.list[[i]])
}

X <- x.list[[1]];
for(i in 2:(T-1)){
  X <- rbind(X, x.list[[i]])
}

Y_std <- sd(Y)
Y_s <- scale(Y, center = FALSE, scale = apply(Y, 2, sd, na.rm = TRUE))

X_std <- apply(X, MARGIN=2, FUN=sd)
X_s <- scale(X, center = FALSE, scale = apply(X, 2, sd, na.rm = TRUE))

p.x <- ncol(X_s)
p.y <- ncol(Y_s)
n <- nrow(X_s)
tol <- 10^(-4); # tolerance
max.iteration <- 200; # max number of iteration for the LASSO solution
lambda.1 <- c(1, 0.5, 0.1, 0.05, 0.01, 0.005, 0.001)
method <- c("MLR")

#p is the number of variables for each day (I_t and R_t)
p <- 2
#b_t is the block size among the time points (1 : (length(date) - 1))
if(state.name %in% c("Texas")){
  b_t <- 7
  gamma.val <- 10
  HBIC = TRUE
}else if(state.name %in% c("Florida", "Iowa")){
  b_t <- 7
  gamma.val <- 5
  HBIC = TRUE
}
```

```

}else if(state.name %in% c("California")){
  b_t <- 7
  gamma.val <- 5
  HBIC = TRUE

}else if(state.name %in% c("Oregon")){
  b_t <- 7
  gamma.val <- 5
  HBIC = TRUE

}else{
  b_t <- 7
  gamma.val <- 10
  HBIC = TRUE
}

b_n <- p * b_t
temp.1 <- tbfl(method, Y_s, X_s, lambda.1.cv = lambda.1, lambda.2.cv = 0,
               max.iteration = max.iteration, tol = tol, block.size = b_n,
               HBIC = HBIC, gamma.val = gamma.val)

beta.est <- temp.1$beta.est
beta.est <- lapply(beta.est, function (x) x*Y_std/X_std)
cp.final <- temp.1$cp.final
cp.date <- date.region[floor( (cp.final-1) / p) + 1]

#display the estimated break points
print("Estimated change time points:")
print(cp.date)

index.date <- seq(1, length(date.region), b_t)
if(index.date[length(index.date)] < length(date.region)){
  index.date <- c(index.date[-length(index.date)], length(date.region))
}

cols <- c("dark orange", "purple", "darkolivegreen4", "blue" )
beta_t_est <- unlist(beta.est)[seq(1, length(unlist(beta.est)), 2)]
gamma_t_est <- unlist(beta.est)[seq(2, length(unlist(beta.est)), 2)]
ylim_max <- 0.05

beta_t_smooth <- beta_t
beta_t_smooth[beta_t == Inf] = 0
beta_t_smooth[-c(1:6)] <- rollmean(beta_t_smooth, k = 7, align = 'right')
beta_t_smooth[ c(1:6)] <- mean(beta_t_smooth[ c(1:6)])

gamma_t_smooth <- gamma_t
gamma_t_smooth[-c(1:6)] <- rollmean(gamma_t, k = 7, align = 'right')
gamma_t_smooth[ c(1:6)] <- mean(gamma_t[ c(1:6)])

par(mar = c(4., 4.5, 1.5, 1))
plot(date.region[-length(date.region)], beta_t_smooth, type = 'l', col = cols[1],
     lty = 1, lwd = 3, ylab = 'Rate', xlab = 'Date', cex.lab = 1, cex.axis = 1,
     ylim = c(0, ylim_max),
     xlim = c(as.Date('2020-03-15'), date.region[length(date.region)]))

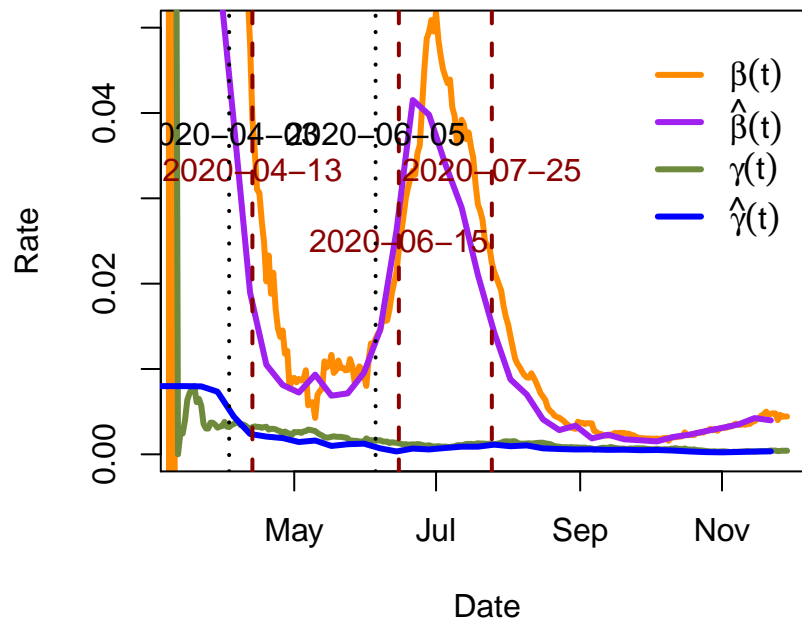
```



```

lines(date.region[index.date[-length(index.date)]] , beta_t_est, col = cols[2],
      lty = 1, type = "l", lwd = 3)
lines(as.Date(date.region[-length(date.region)]) , gamma_t_smooth,col = cols[3],
      lty=1, type = "l", lwd = 3)
lines(date.region[index.date[-length(index.date)]] , gamma_t_est, col = cols[4],
      lty = 1, type = "l", lwd = 3)
legend(as.Date(date.region[length(date.region)*3/4]) , ylim_max,
      legend = c(expression(beta(t)), expression(hat(beta)(t)),
                  expression(gamma(t)), expression(hat(gamma)(t))),
      col = cols, bty = "n", lwd = 3, cex=1, pt.cex = 1, bg = "transparent",
      seg.len=1.5, y.intersp=1 , x.intersp=1)
abline(v = as.Date(cp.date), col = "dark red", cex = 1 ,lwd = 2, lty = 2)
if(length(cp.date) > 2){
  text(x= as.Date(cp.date[2]), y = 2/4*ylim_max, col = "dark red",
       labels = as.character(cp.date[2]), cex = 1.00)
  text(x = as.Date(cp.date[-2]), y = 2/3*ylim_max, col = "dark red",
       labels = as.character(cp.date[-2]), cex = 1.00)
}else if(length(cp.date) == 2){
  if(cp.date[2] - cp.date[1] < 30){
    text(x= as.Date(cp.date[2]), y = 2/4*ylim_max, col = "dark red",
         labels = as.character(cp.date[2]), cex = 1.00)
    text(x = as.Date(cp.date[1]), y = 2/3*ylim_max, col = "dark red",
         labels = as.character(cp.date[1]), cex = 1.00)
  }else{
    text(x = as.Date(cp.date), y = 2/3*ylim_max, col = "dark red",
         labels = as.character(cp.date), cex = 1.00)
  }
}else{
  text(x = as.Date(cp.date), y = 2/3*ylim_max, col = "dark red",
       labels = as.character(cp.date), cex = 1.00)
}
abline(v = as.Date(Date.1), col = 1, cex = 1, lwd = 2, lty = 3)
abline(v = as.Date(Date.2), col = 1, cex = 1, lwd = 2, lty = 3)
text(x = as.Date(Date.1), y = 3/4*ylim_max, col ="black",
     labels = as.character(as.Date(Date.1)), cex = 1.00)
text(x = as.Date(Date.2), y = 3/4*ylim_max, col = "black",
     labels = as.character(as.Date(Date.2)), cex = 1.00)

```



After detecting the change points, we refit the model. The in-sample MRPE and fitted numbers of infected and recovered cases are provided.

```
#####refit the model
cp <- c(1, temp.1$cp.final, n+1)
m <- length(cp) - 1
Y.hat.1 <- matrix(0, n)
for(i in 1:m){
  Y.temp.1 <- Y[cp[i]: (cp[i+1]-1), ]
  X.temp.1 <- X[cp[i]: (cp[i+1]-1), ]
  est.temp.1 <- lm(Y.temp.1 ~ X.temp.1 - 1)
  Y.hat.1[cp[i]: (cp[i+1]-1), ] <- est.temp.1$fitted.values
}

R.hat.1 <- rep(0,T)
R.hat.1[1] <- R.rate.all[1,1]
for(i in 2:T){
  R.hat.1[i] <- R.rate.all[i-1,1] + Y.hat.1[(i-2)*2+1]
}
R.hat.1 <- R.hat.1*n.all[1]

I.hat.obs.1 <- rep(0,T)
I.hat.obs.1[1] <- I.rate.all.obs[1,1]
for(i in 2:T){
  rate <- (((i)+a.val*T.full)/((1 + a.val )*T.full))^2
  I.hat.obs.1[i] <- I.rate.all.obs[i-1,1] + Y.hat.1[(i-2)*2+2]*rate
}
I.hat.obs.1 <- I.hat.obs.1*n.all[1]

MRPE_1_I <- mean(abs((c(I.hat.obs.1[-1]) - c(I.obs[-1]))
                    /c(I.obs[-1]))[intersect(which(c(I.obs[-1]) > 0), 2:(T-1) )]))
print(round(MRPE_1_I, 4))

## [1] 0.0683
```

```

MRPE_1_R <- mean(abs((c(R.hat.1[-1]) - c(R.obs[-1]))
                    /c(R.obs[-1]) ) [intersect(which(c(R.obs[-1]) > 0), 2:(T-1))])
print(round(MRPE_1_R, 4))

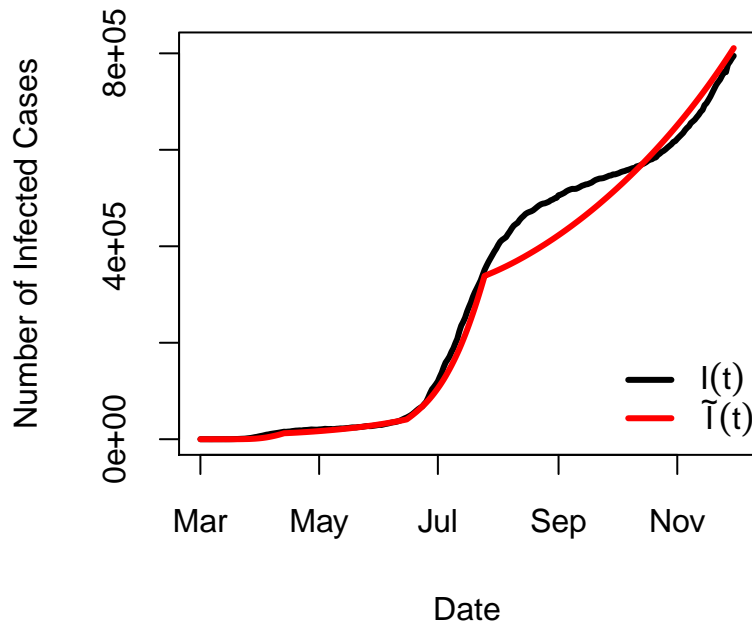
## [1] 0.0185

R.tilde.1 <- rep(0,T)
R.tilde.1[1] <- R.rate.all[1,1]
for(i in 2:T){
  R.tilde.1[i] <- R.tilde.1[i-1] + Y.hat.1[(i-2)*2+1]
}
R.tilde.1 <- R.tilde.1*n.all[1]
I.tilde.1 <- rep(0,T)
I.tilde.1[1] <- I.rate.all[1,1]
for(i in 2:T){
  I.tilde.1[i] <- I.tilde.1[i-1] + Y.hat.1[(i-2)*2+2]
}
I.tilde.1 <- I.tilde.1*n.all[1]

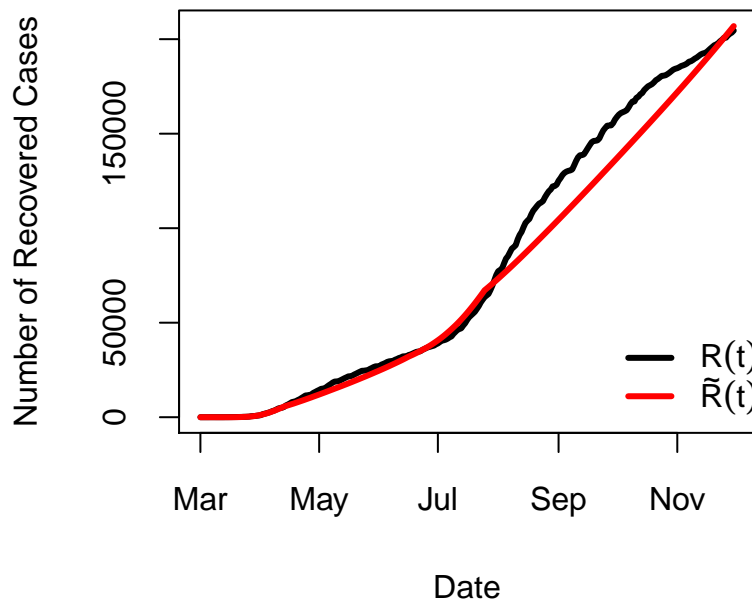
I.tilde.1 <- rep(0,T)
I.tilde.1[1] <- I.rate.all.obs[1,1]
for(i in 2:T){
  rate <- (((i)+a.val*T.full)/((1 + a.val )*T.full))^2
  I.tilde.1[i] <- I.tilde.1[i-1] + Y.hat.1[(i-2)*2+2]*rate
}
I.tilde.1 <- I.tilde.1*n.all[1]

par(mar = c(4.5, 6, 2, 1.5), mgp=c(3.5, 1.2, 0))
plot(date.region, I.obs , col='1', ylim=c(0,max(I.obs, I.tilde.1)), lty=1, type="l", lwd = 3,
      ylab = 'Number of Infected Cases', xlab= 'Date', cex.lab = 1, cex.axis = 1)
lines(date.region, I.tilde.1, col='2',lty=1,type="l", lwd = 3)
legend(as.Date(date.region[length(date.region)*3/4]), 1/4*max(I.obs,I.tilde.1),
      legend = c(expression(I(t)), expression(tilde(I)(t))),
      col=c(1, 2), bg="transparent", bty = "n",
      lwd = 3, cex = 1, pt.cex = 1, seg.len=1.5, y.intersp = 1 , x.intersp = 1)

```



```
par(mar = c(4.5, 6, 2, 1.5), mgp=c(3.5, 1.2, 0))
plot(date.region, R, type='l', col='1', ylim=c(0,max(R, R.tilde.1)),lty=1,lwd = 3,
     ylab = 'Number of Recovered Cases', xlab= 'Date', cex.lab=1 , cex.axis=1)
lines(date.region, R.tilde.1, col='2', lty=1, type="l", lwd = 3)
legend(as.Date(date.region[length(date.region)*3/4]), 1/4*max(R, R.tilde.1),
     legend=c(expression(R(t)), expression(tilde(R)(t))),
     col=c( 1,2), bg="transparent", bty = "n",
     lwd = 3, cex = 1, pt.cex = 1, seg.len = 1.5, y.intersp=1 , x.intersp=1)
```



Model 2: Piecewise Stationary SIR Model with Spatial Heterogeneity

Model 2.3: Similarity-based Weight

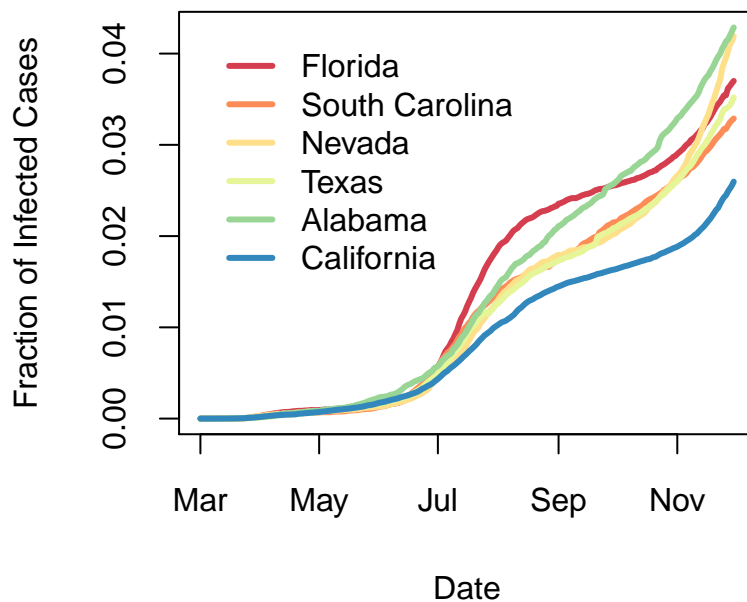
```
#####  
##### Model 2.3 similarity based weight  
##### spatial components are chosen by similarity  
#####  
a.val <- a.final  
for(t in 2:T){  
  # rate <- 1/(((t)+a.val*T)/((1 + a.val )*T))^2  
  rate <- 1/(((t)+a.val*T.full)/((1 + a.val )*T.full))^2  
  # print(1/rate)  
  I.rate.all[t, ] <- (I.rate.all.obs[t, ] - I.rate.all.obs[t-1, ])*rate + I.rate.all[t-1, ]  
}  
  
n.regions <- ncol(I.rate.all)  
y.list <- vector("list", T-1);  
for(i in 2:T){  
  y.list[[i-1]] <- matrix(c(R.rate.all[i, ] - R.rate.all[i-1, ], I.rate.all[i, ] - I.rate.all[i-1, ]), n.regions, 2)  
}  
  
Y.all <- y.list[[1]];  
for(i in 2:(T-1)){  
  Y.all <- rbind(Y.all, y.list[[i]])  
}  
# max number of neighboring regions  
max.neighbor <- 5 + 1  
l2.diff <- c()  
for(i in 1:n.regions){  
  l2.diff <- c(l2.diff, sqrt(sum( (Y.all[ , i] - Y.all[ , 1] )^2)))  
}  
state.index <- order(l2.diff)[1: max.neighbor]  
state.names <- state.names.all[state.index]  
state.lowernames <- state.lowernames.all[state.index]  
state.names  
  
## [1] "Florida" "South Carolina" "Nevada" "Texas"  
## [5] "Alabama" "California"  
  
n.all.full <- n.all  
S.rate.all.full <- S.rate.all  
I.rate.all.full <- I.rate.all  
R.rate.all.full <- R.rate.all  
  
n.domains <- length(state.names)  
n.all <- n.all[state.index]  
S.rate.all <- S.rate.all[, state.index]  
I.rate.all <- I.rate.all[, state.index]  
R.rate.all <- R.rate.all[, state.index]  
  
I.rate.all.obs.part <- I.rate.all.obs[, state.index]  
R.rate.all.obs.part <- R.rate.all.obs[, state.index]
```

```

date.region <- multi_full$date
cols <- brewer.pal(n.domains, "Spectral")

par(mar = c(4.5, 6, 2, 1.5), mgp=c(3.5, 1.2, 0))
plot(multi_full$date, I.rate.all.obs.part[,1], col=cols[1], lty=1,type="l", lwd = 3,
     ylim = c(0, max(I.rate.all.obs.part)),
     ylab = 'Fraction of Infected Cases', xlab= 'Date', cex.lab=1, cex.axis=1)
for(i in 2:length(state.names)){
  lines(multi_full$date, I.rate.all.obs.part[,i], col=cols[i], lty=1, type="l", lwd = 3)
}
legend(multi_full$date[1], max(I.rate.all.obs.part) , legend=c(state.names),
      col=cols,bg="transparent",bty = "n",
      lwd = 3, cex=1, pt.cex = 1, seg.len=1.5, y.intersp=1, x.intersp=1)

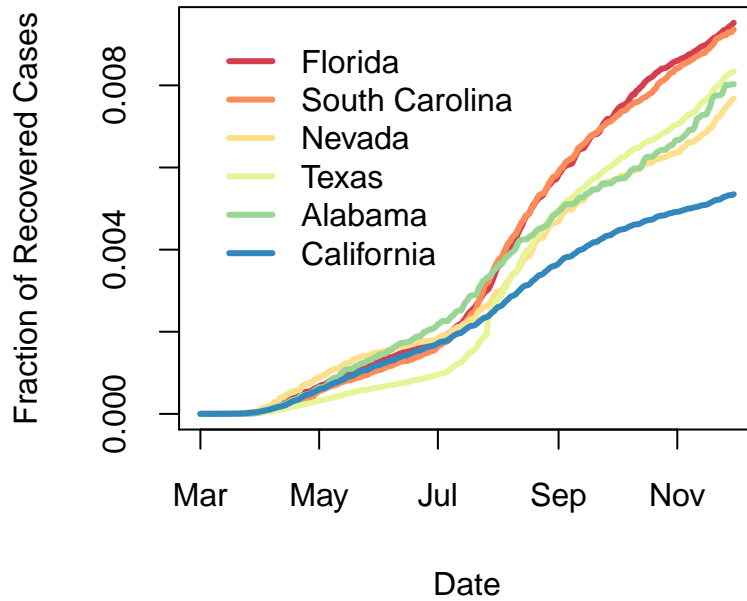
```



```

par(mar = c(4.5, 6, 2, 1.5), mgp=c(3.5, 1.2, 0))
plot(multi_full$date, R.rate.all[,1] , col=cols[1], lty=1,type="l", lwd = 3, ylim = c(0,max(R.rate.all))
     ylab = 'Fraction of Recovered Cases', xlab= 'Date',cex.lab=1, cex.axis=1)
for(i in 2:length(state.names)){
  lines(multi_full$date, R.rate.all[,i] ,col=cols[i],lty=1,type="l", lwd = 3)
}
legend(multi_full$date[1], max(R.rate.all), legend=c(state.names),
      col=cols,bg="transparent",bty = "n",
      lwd = 3, cex=1, pt.cex = 1, seg.len=1.5, y.intersp=1 , x.intersp=1)

```



```

distance_3 <- rep(0, n.domains)
#Power Distance Weights.
for(i in 2:n.domains){
  distance.temp <- sort(l2.diff)[i]
  distance_3[i] <- 1/(distance.temp)^1
}
Omega_3 <- distance_3/sum(distance_3)

rows.combined <- 2*(T-1)
cols.combined <- n.domains
matrix.combined <- matrix(0, nrow=rows.combined, ncol=cols.combined)
matrix.combined[ seq(1, rows.combined, 2), ] <-
  as.matrix(unname(R.rate.all))[-1,]-as.matrix(unname(R.rate.all))[-T,]
matrix.combined[ seq(2, rows.combined, 2), ] <-
  as.matrix(unname(I.rate.all))[-1,]-as.matrix(unname(I.rate.all))[-T,]
neighbor.matrix <- matrix.combined
neighbor.weighted_3 <- neighbor.matrix %*% (Omega_3)

#remove the last two elements at time point t=T
#and add two elements for time point t= 0
neighbor.weighted_3 <- as.matrix(neighbor.weighted_3[-c(rows.combined-1,rows.combined),])
neighbor.weighted_3 <- as.matrix(c(neighbor.weighted_3[c(1,2),], neighbor.weighted_3))

p.x <- ncol(X)
p.y <-ncol(Y)
n <- nrow(X)

cp <- c(1, temp.1$cp.final, n+1)
m <- length(cp) - 1
X.new.3 <- matrix(0, nrow = n, ncol = m*p.x + 1)
for(i in 1:m){
  X.new.3[cp[i]: (cp[i+1]-1), (p.x*(i-1)+1) : (p.x*i) ] <- X[cp[i]: (cp[i+1]-1), ]
}
X.new.3[, m*p.x + 1] <- neighbor.weighted_3

```

```

est.2.3 <- lm(Y[-c(1:2)], ~ X.new.3[-c(1:2)], ] - 1)
Y.hat.2.3 <- c(0, 0, est.2.3 $fitted.values)
summary(est.2.3 )

##
## Call:
## lm(formula = Y[-c(1:2)], ~ X.new.3[-c(1:2)], ] - 1)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -4.706e-04 -2.898e-05 -5.900e-07  2.249e-05  8.120e-04
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## X.new.3[-c(1:2), ]1  8.907e-03  5.508e-03   1.617   0.106
## X.new.3[-c(1:2), ]2 -1.386e-04  3.497e-03  -0.040   0.968
## X.new.3[-c(1:2), ]3 -8.734e-03  1.562e-03  -5.591 3.60e-08 ***
## X.new.3[-c(1:2), ]4 -1.750e-04  9.323e-04  -0.188   0.851
## X.new.3[-c(1:2), ]5  6.010e-03  1.196e-03   5.024 6.90e-07 ***
## X.new.3[-c(1:2), ]6 -7.156e-05  4.419e-04  -0.162   0.871
## X.new.3[-c(1:2), ]7 -1.504e-03  2.937e-04  -5.120 4.27e-07 ***
## X.new.3[-c(1:2), ]8 -9.797e-06  1.236e-04  -0.079   0.937
## X.new.3[-c(1:2), ]9  1.289e+00  5.403e-02  23.849 < 2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.0001339 on 537 degrees of freedom
## Multiple R-squared:  0.9093, Adjusted R-squared:  0.9078
## F-statistic: 598.4 on 9 and 537 DF,  p-value: < 2.2e-16

R.hat.2.3 <- rep(0, T)
R.hat.2.3[1] <- R.rate.all[1, 1]
for(i in 2:T){
  R.hat.2.3[i] <- R.rate.all[i-1, 1] + Y.hat.2.3[(i-2)*2+1]
}
R.hat.2.3 <- R.hat.2.3*n.all[1]

I.hat.obs.2.3 <- rep(0,T)
I.hat.obs.2.3[1] <- I.rate.all.obs[1,1]
for(i in 2:T){
  rate <- (((i)+a.val*T.full)/((1 + a.val )*T.full))^2
  I.hat.obs.2.3[i] <- I.rate.all.obs[i-1,1] + Y.hat.2.3[(i-2)*2+2]*rate
}
I.hat.obs.2.3 <- I.hat.obs.2.3*n.all[1]

MRPE_2.3_I <- mean(abs(((I.hat.obs.2.3[-1]) - c(I.obs[-1]))
                      /c(I.obs[-1]))[intersect( which(c(I.obs[-1]) > 0), 2:(T-1))])
print(round(MRPE_2.3_I,4))

## [1] 0.0254

MRPE_2.3_R <- mean(abs ( (c(R.hat.2.3[-1]) - c(R[-1]))
                      /c(R[-1]))[intersect( which(c(R[-1]) > 0), 2:(T-1) ) ])
print(round(MRPE_2.3_R,4))

```

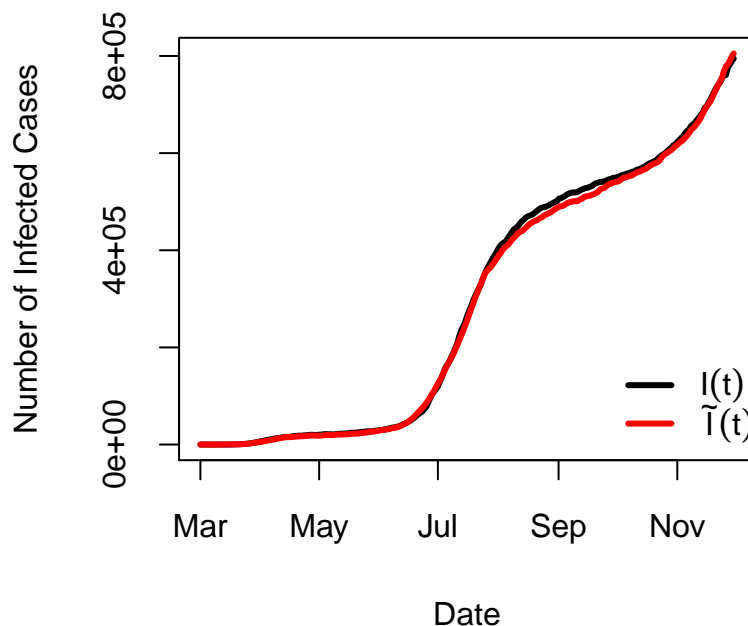


```
## [1] 0.0229

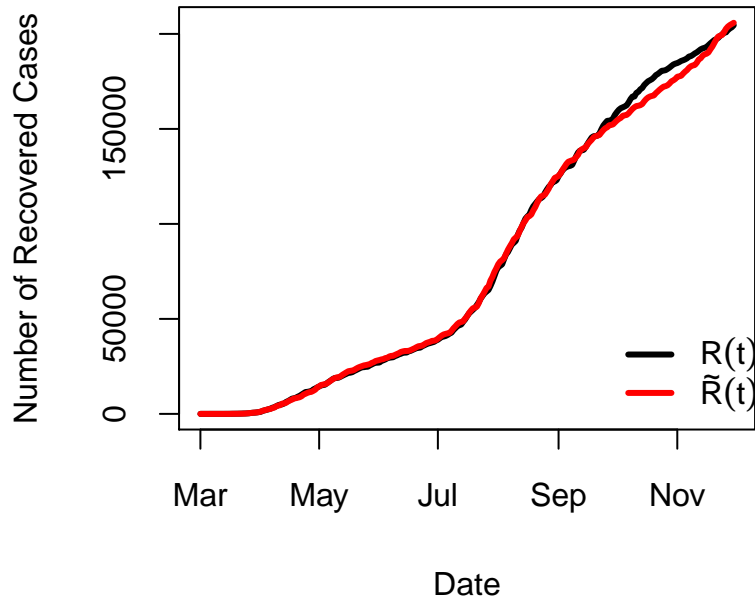
R.tilde.2.3 <- rep(0,T)
R.tilde.2.3[1] <- R.rate.all[1,1]
for(i in 2:T){
  R.tilde.2.3[i] <- R.tilde.2.3[i-1] + Y.hat.2.3[(i-2)*2+1]
}
R.tilde.2.3 <- R.tilde.2.3*n.all[1]

I.tilde.2.3 <- rep(0, T)
I.tilde.2.3[1] <- I.rate.all.obs[1,1]
for(i in 2:T){
  rate <- (((i)+a.val*T.full)/((1 + a.val )*T.full))^2
  I.tilde.2.3[i] <- I.tilde.2.3[i-1] + Y.hat.2.3[(i-2)*2+2]*rate
}
I.tilde.2.3 <- I.tilde.2.3*n.all[1]

par(mar = c(4.5, 6, 2, 1.5), mgp=c(3.5, 1.2, 0))
plot(date.region, I.obs, col='1', ylim=c(0,max(I.obs, I.tilde.2.3)), lty=1, type="l", lwd = 3,
     ylab = 'Number of Infected Cases', xlab= 'Date', cex.lab=1, cex.axis=1)
lines(date.region, I.tilde.2.3, col='2', lty=1, type="l", lwd = 3)
legend(as.Date(date.region[length(date.region)*3/4]), 1/4*max(I.obs, I.tilde.2.3),
      legend=c(expression(I(t)), expression(tilde(I)(t))),
      col=c( 1,2), bg="transparent", bty = "n",
      lwd = 3, cex=1, pt.cex = 1, seg.len=1.5, y.intersp=1 , x.intersp=1)
```



```
par(mar = c(4.5, 6, 2, 1.5), mgp=c(3.5, 1.2, 0))
plot(date.region, R, type='l', col='1', ylim=c(0,max(R, R.tilde.2.3)), lty=1, lwd = 3,
     ylab = 'Number of Recovered Cases', xlab= 'Date', cex.lab=1 , cex.axis=1)
lines(date.region, R.tilde.2.3, col='2', lty=1, type="l", lwd = 3)
legend(as.Date(date.region[length(date.region)*3/4]), 1/4*max(R, R.tilde.2.3), legend=c(expression(R(t)),
      col=c( 1,2), bg="transparent", bty = "n",
      lwd = 3, cex=1, pt.cex = 1, seg.len=1.5, y.intersp=1, x.intersp=1)
```



Model 3: VAR(p) Model

```
#####
##### model 3 use VAR(p) model, follow model 2.3
#####
Delta.R.hat <- rep(0, T - 1)
Delta.R <- rep(0, T - 1)
for(i in 2:T){
  Delta.R.hat[i - 1] <- Y.hat.2.3[(i - 2)*2 + 1]
  Delta.R[i - 1] <- Y[(i - 2)*2 + 1]
}
Delta.R.hat <- Delta.R.hat*n.all[1]
Delta.R <- Delta.R*n.all[1]
Delta.I.hat <- rep(0, T - 1)
Delta.I <- rep(0, T - 1)
for(i in 2:T){
  Delta.I.hat[i - 1] <- Y.hat.2.3[(i - 2)*2 + 2]
  Delta.I[i - 1] <- Y[(i - 2)*2 + 2]
}
Delta.I.hat <- Delta.I.hat*n.all[1]
Delta.I <- Delta.I*n.all[1]

residual_Delta.I <- Delta.I - Delta.I.hat
residual_Delta.R <- Delta.R - Delta.R.hat

res <- c()
for(i in 1:n){
  res <- c(res, Y[i] - Y.hat.2.3[i])
}
res <- res*n.all[1]

residual.matrix <- cbind(residual_Delta.I, residual_Delta.R)
print("estimated sample variance for hat residual:")
```

```

print(var(residual.matrix))

var <- VARselect(residual.matrix, lag.max = 7, type = "none")

var$selection
# choose the p by BIC
p.est <- var$selection["SC(n)"]

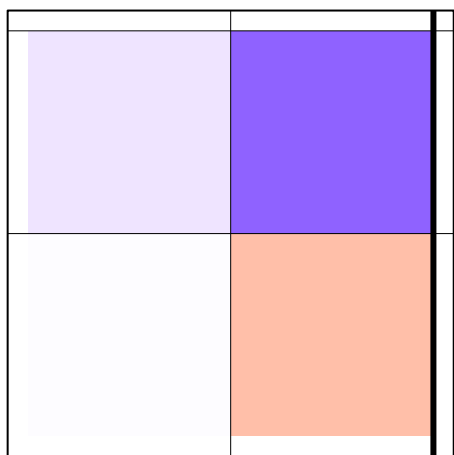
#change point detection
method <- c("VAR")
if(p.est >= b_n/2){
  temp.var <- tbfl(method, residual.matrix, lambda.1.cv = NULL, lambda.2.cv = 0,
                  q = p.est, max.iteration = max.iteration, tol = tol, block.size = p.est+1)
  print(p.est)
}else{
  temp.var <- tbfl(method, residual.matrix, lambda.1.cv = NULL, lambda.2.cv = 0,
                  q = p.est, max.iteration = max.iteration, tol = tol, block.size = b_n/2)
}

if(is.null(temp.var$cp.final)){
  print("residual structure no change point!")
  var1 <- VAR(residual.matrix, p = p.est, type = "none")
  coef.matrix <- Bcoef(var1)
  #fitted the residual in training set
  residual.hat <- fitted(var1)
  residual.hat <- rbind(matrix(0, ncol = 2, nrow = p.est), residual.hat)
  # residual.tilde <- resid(var1)
  residual.tilde <- residual.matrix - residual.hat
}else{
  print("residual structure has change point!")
  cp.residual<- c(1, temp.var$cp.final, nrow(residual.matrix)+1)
  m <- length(cp.residual) - 1
  residual.hat <- matrix(0, nrow = nrow(residual.matrix), ncol = ncol(residual.matrix))

  for(i in 1:m){
    residual.matrix.temp <- residual.matrix[ cp.residual[i]: (cp.residual[i+1]-1), ]
    var.temp <- VARselect(residual.matrix.temp, lag.max = 7, type = "none")
    var.temp$selection
    p.est <- var.temp$selection["SC(n)"]
    print(p.est)
    var1 <- VAR(residual.matrix.temp, p = p.est, type = "none")
    residual.hat[cp.residual[i]: (cp.residual[i+1]-1), ] <- rbind( matrix(0, ncol = 2, nrow = p.est),
                                                                fitted(var1))
  }
  residual.tilde <- residual.matrix - residual.hat
  coef.matrix <- Bcoef(var1)
}

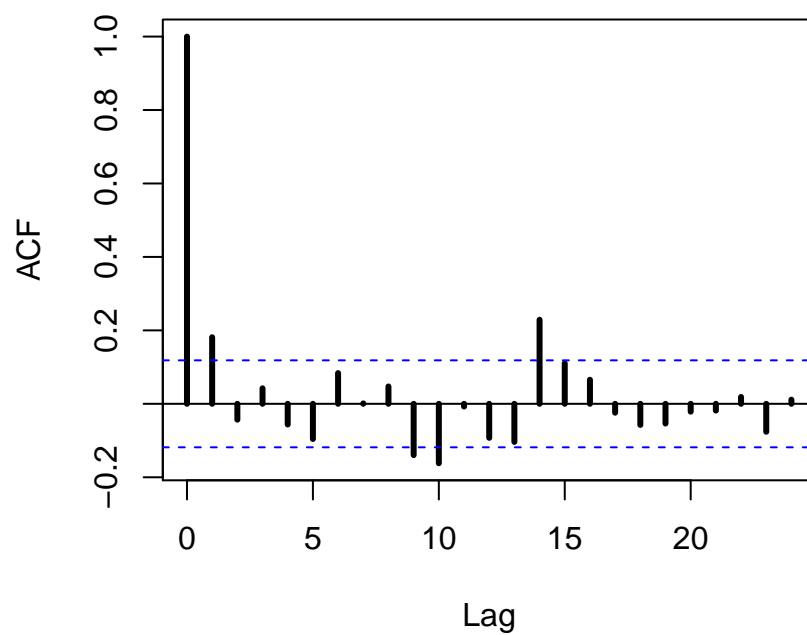
par(mar = c(4., 4.5, 1.5, 1))
print(plot.ar.matrix(coef.matrix, p = p.est))

```

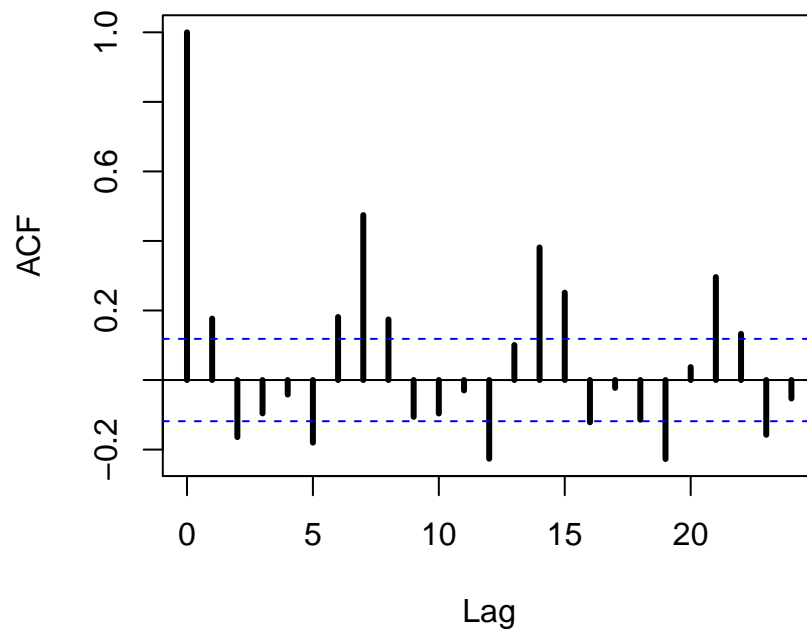


$\Phi^{(1)}$

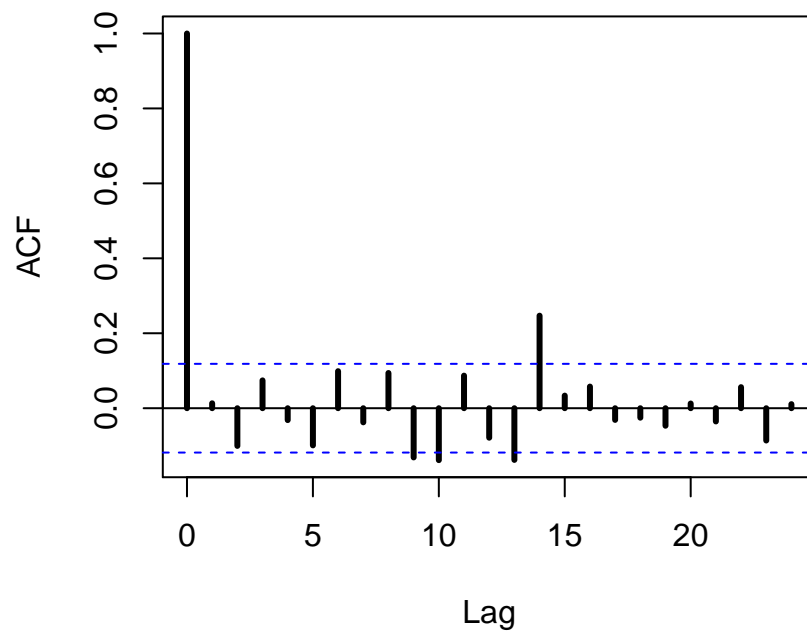
```
par(mar = c(4., 4.5, 1.5, 1))
acf(residual_Delta.I, cex.lab=1 , cex.axis=1 , lwd = 3)
```



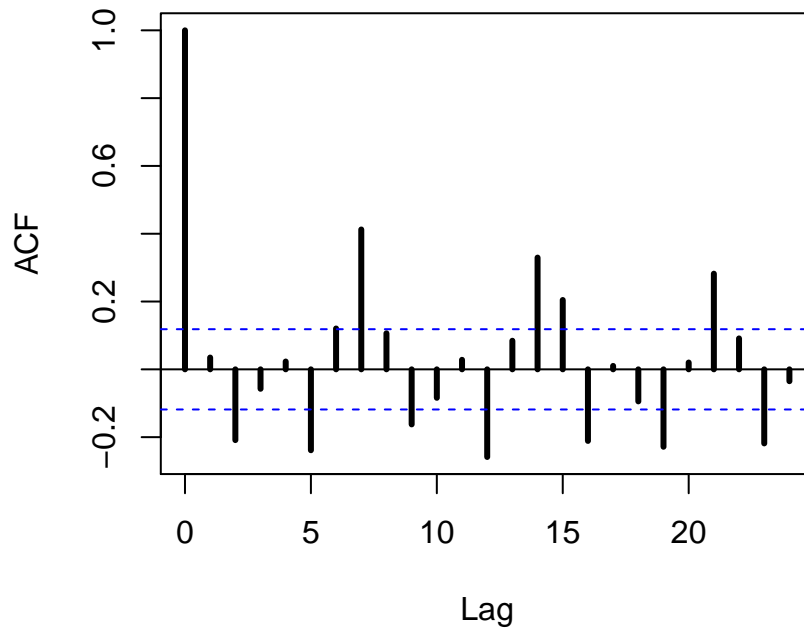
```
par(mar = c(4., 4.5, 1.5, 1))
acf(residual_Delta.R, cex.lab=1 , cex.axis=1 , lwd = 3)
```



```
par(mar = c(4., 4.5, 1.5, 1))
acf(residual.tilde[, 1], cex.lab=1 , cex.axis=1, lwd = 3)
```



```
par(mar = c(4., 4.5, 1.5, 1))
acf(residual.tilde[, 2], cex.lab=1 , cex.axis=1, lwd = 3)
```



```

#vectorize the fitted residuals
residual.hat.vec <- rep(0, nrow(residual.hat)*2)
#fitted residual of Delta R
residual.hat.vec[seq(1, nrow(residual.hat)*2, 2)] <- residual.hat[, 2]
#fitted residual of Delta I
residual.hat.vec[seq(2, nrow(residual.hat)*2, 2)] <- residual.hat[, 1]

Y.hat.3 <- Y.hat.2.3 + residual.hat.vec/n.all[1]

R.hat.3 <- rep(0, T)
R.hat.3[1] <- R.rate.all[1, 1]
for(i in 2:T){
  R.hat.3[i] <- R.rate.all[i-1, 1] + Y.hat.3[(i-2)*2+1]
}
R.hat.3 <- R.hat.3*n.all[1]

I.hat.obs.3 <- rep(0,T)
I.hat.obs.3[1] <- I.rate.all.obs[1,1]
for(i in 2:T){
  rate <- (((i)+a.val*T.full)/((1 + a.val )*T.full))^2
  I.hat.obs.3[i] <- I.rate.all.obs[i-1,1] + Y.hat.3[(i-2)*2+2]*rate
}
I.hat.obs.3 <- I.hat.obs.3*n.all[1]

MRPE_3_I <- mean(abs((c(I.hat.obs.3[-1]) - c(I.obs[-1]))
                    /c(I.obs[-1]))[intersect( which(c(I.obs[-1]) > 0), 2:(T-1))]))
print(round(MRPE_3_I,4))

## [1] 0.0214

MRPE_3_R <- mean(abs((c(R.hat.3[-1]) - c(R[-1]))
                    /c(R[-1]))[intersect(which(c(R[-1]) > 0), 2:(T-1))]))
print(round(MRPE_3_R,4))

## [1] 0.0257

```

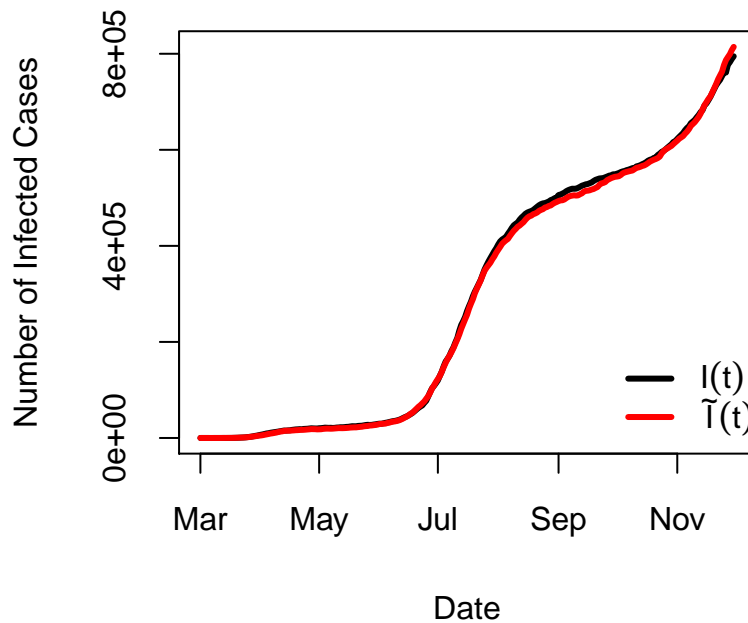
```

R.tilde.3 <- rep(0,T)
R.tilde.3[1] <- R.rate.all[1,1]
for(i in 2:T){
  R.tilde.3[i] <- R.tilde.3[i-1] + Y.hat.3[(i-2)*2+1]
}
R.tilde.3 <- R.tilde.3*n.all[1]

I.tilde.3 <- rep(0,T)
I.tilde.3[1] <- I.rate.all[1,1]
for(i in 2:T){
  rate <- (((i)+a.val*T.full)/((1 + a.val )*T.full))^2
  I.tilde.3[i] <- I.tilde.3[i-1] + Y.hat.3[(i-2)*2+2]*rate
}
I.tilde.3 <- I.tilde.3*n.all[1]

par(mar = c(4.5, 6, 2, 1.5), mgp=c(3.5, 1.2, 0))
plot(date.region, I.obs, col='1', ylim=c(0,max(I.obs, I.tilde.3)), lty=1, type="l", lwd = 3,
     ylab = 'Number of Infected Cases', xlab= 'Date', cex.lab=1, cex.axis=1)
lines(date.region, I.tilde.3, col='2', lty=1, type="l", lwd = 3)
legend(as.Date(date.region[length(date.region)*3/4]), 1/4*max(I.obs, I.tilde.3),
     legend=c(expression(I(t)), expression(tilde(I)(t))),
     col=c( 1,2), bg="transparent", bty = "n",
     lwd = 3, cex=1, pt.cex = 1, seg.len=1.5, y.intersp=1 , x.intersp=1)

```



```

par(mar = c(4.5, 6, 2, 1.5), mgp=c(3.5, 1.2, 0))
plot(date.region, R, type='l',col='1', ylim=c(0,max(R, R.tilde.3)), lty=1, lwd = 3,
     ylab = 'Number of Recovered Cases', xlab= 'Date', cex.lab=1 , cex.axis=1)
lines(date.region, R.tilde.3, col='2',lty=1,type="l",lwd = 3)
legend(as.Date(date.region[length(date.region)*3/4]), 1/4*max(R, R.tilde.3), legend=c(expression(R(t)),
     col=c( 1,2), bg="transparent", bty = "n",
     lwd = 3, cex=1, pt.cex = 1, seg.len=1.5, y.intersp=1, x.intersp=1)

```

