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HW3 CS6190 - Spring 23

Practice

1.

(a) We can see that the acceptance rate is negatively affected by the increase of the Tau. It is

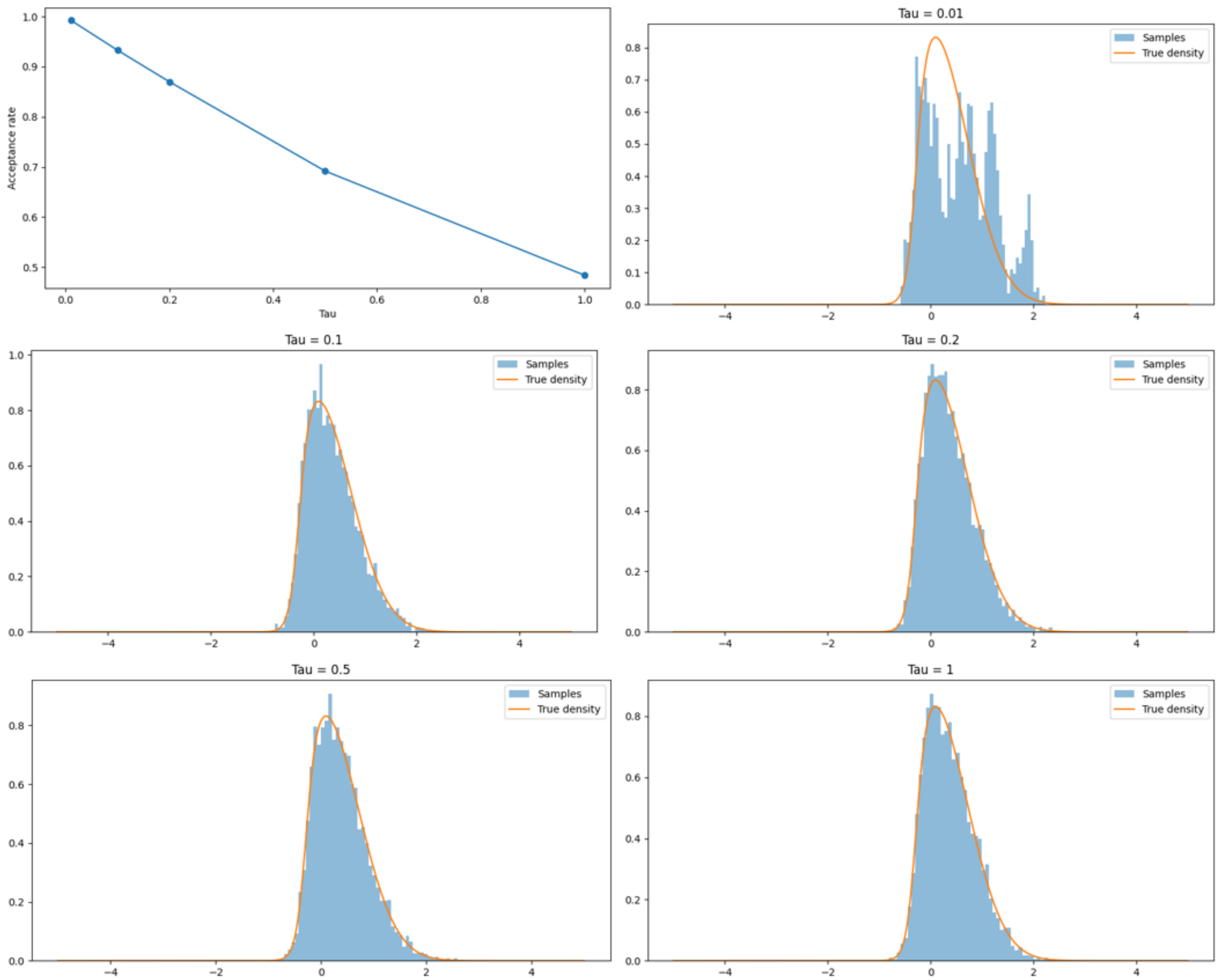

Figure 1: Metropolis-Hastings Sampling

expected to happen because the increase in Tau means that there are bigger steps, and new samples are likely to produce less likelihood. The reduced likelihood will lead the acceptance rule to reject more samples that before. With a bigger Tau, we will see a better fit of sampled and ground0truth probability densities. I have collaged the plots together because having them separately on this document was getting messy. All plots are located in the zip file.

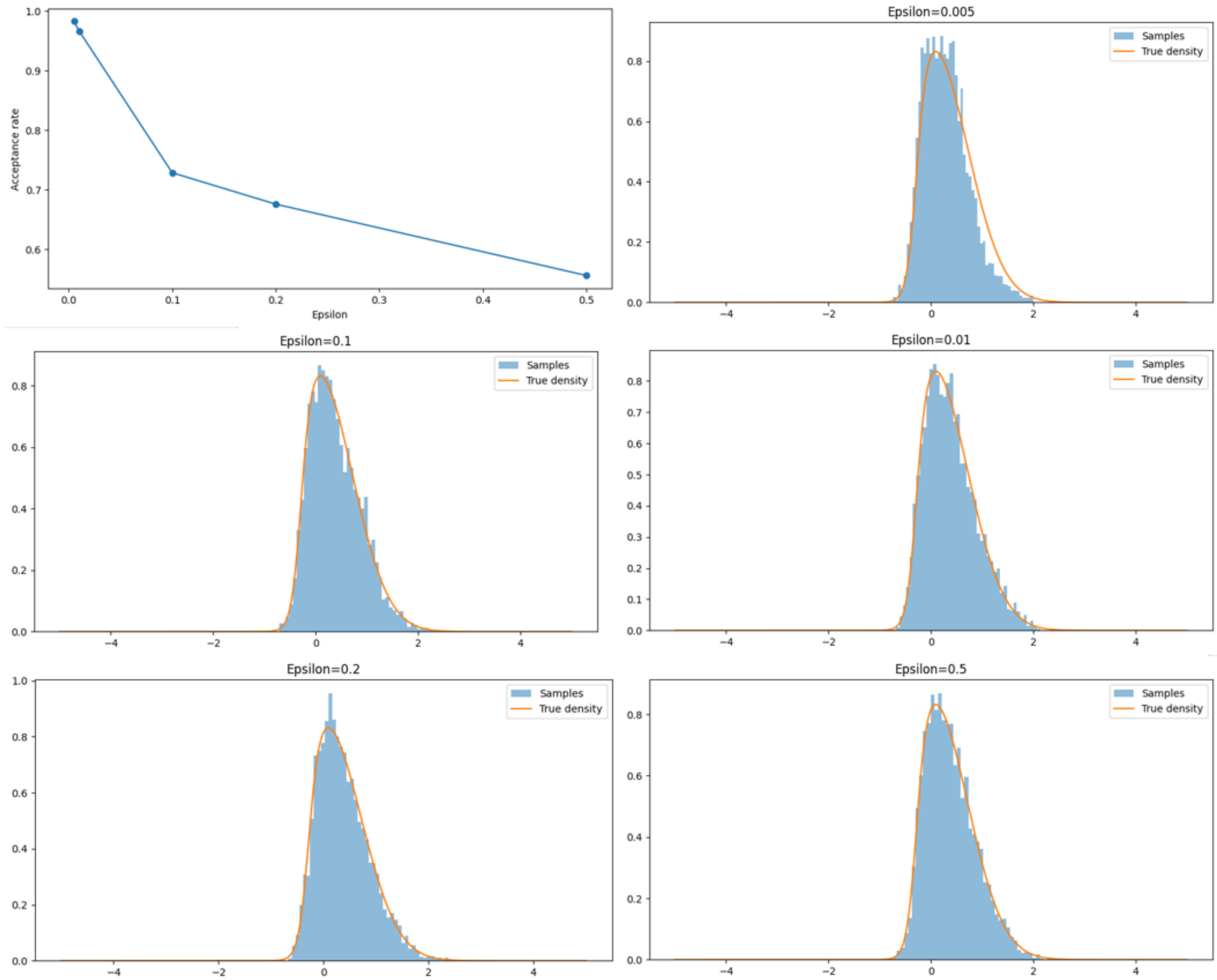

Figure 2: Hybrid Monte-Carlo sampling with Leapfrog

(b) Just like the previous problem, we can see that increasing epsilons will lead to lesser acceptance rates. The reason is the same as before, with bigger epsilons leading to bigger steps, and the accompanying reduction in likelihood that leads more samples to fail the acceptance rule and get rejected. Although we can say that the samples and the ground probability distribution become a tighter fit.

(c) We can see that in epsilons closer to 0, the HMC has a much larger acceptance rate, but anything larger than that performs better with Metropolis-Hastings. Also, HMC needs way more calculations to converge (with several additional steps like leapfrog) compared to Metropolis-Hastings. Given all above, for bigger Taus, it is preferred to use Metropolis-Hastings at it gives us an edge over calculations and also a higher acceptance rate

2.

(a)

First, we tend to establish the distribution, and then draw 500 sample from it as advised by the question.

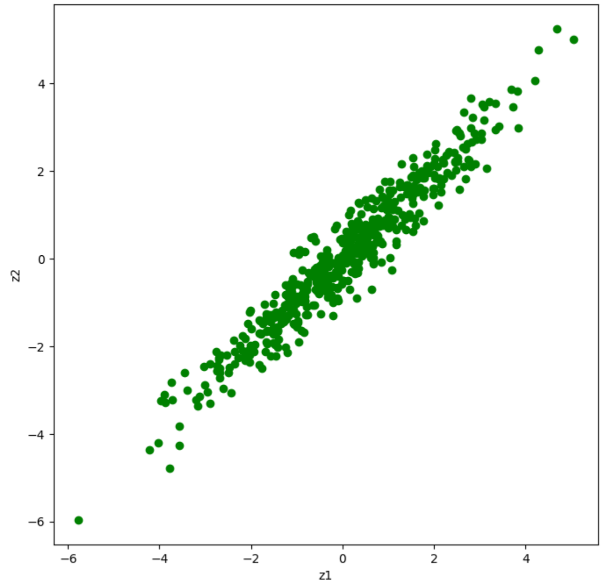

Figure 3: 500 Samples drawn from the distribution

(b) We can see the red lines (that show the trajectory) are mostly in the middle, where the density

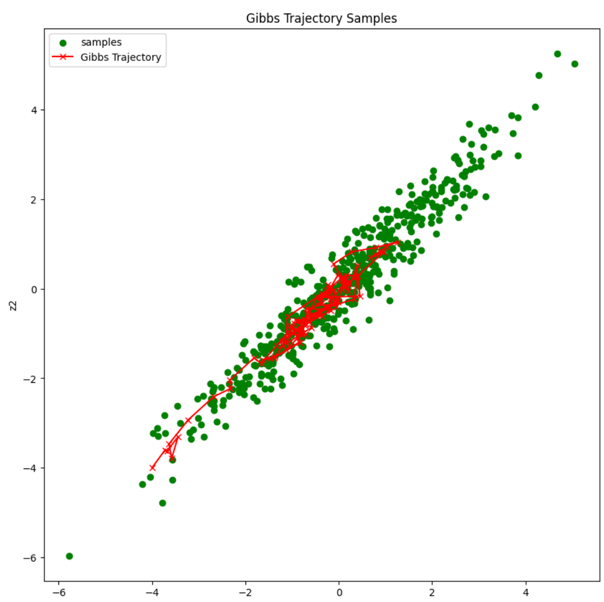

Figure 4: Trajectory of Gibbs Sample

of the samples is high. But in other areas like the upper right portion, there is no sign of trajectory to be seen. Therefore it is biased towards more dense areas of the sample.

(c) Here we see a much smaller trajectory footprint, but it is not heavily located in the dense areas of the plot, and actually is spread in the plane and the areas that have lower densities (like in the lower left side, or upper right side). This shows that HMC is less biased towards density and its trajectory can explore more areas compared to that's of the Gibbs one.

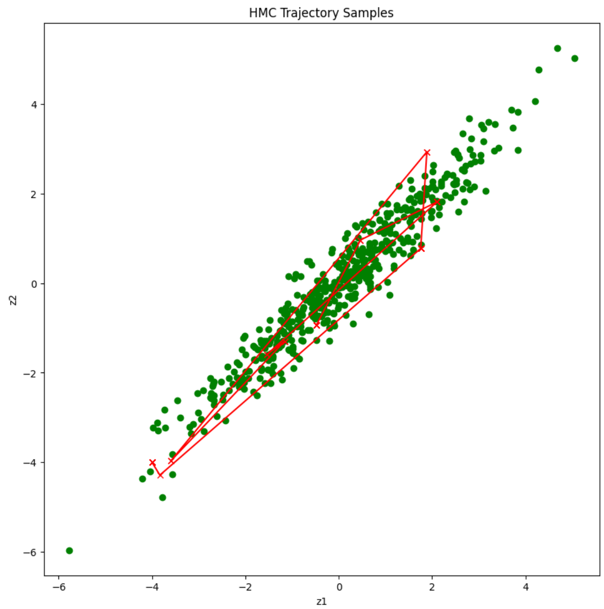

Figure 5: Trajectory of HMC with Leapfrog

3.

(a) The joint probability of the Bayesian logistic regression is:

$$U(z) = -\log(p(w)) - \log(p(T|X, w))$$

Where in my code, I have functions to get the log-likelihood and the prior and add them together to get to the joint distribution. Here, we can see that ϵ and L has a very large effect on the convergence of the model. The bigger they are, the worse the model performs (and more it takes for it to converge). It results to worse likelihoods and in general decreases the performance of the models. We can see that with more L and epsilon, the likelihood will fall. My main problem here is that I get the acceptance rate as 1 for all the situations, which clearly it shouldn't be this way and should decrease over time, but I couldn't fix this problem even after I did whatever I could have done to the code.

(b) The reported prediction accuracy for the model is 94.77%, and the reported log-likelihood of the model is -0.96362. As we have seen in the first problem, difference in the performance sampling models is subjective to the parameters used. Comparing the previous part with this one, we can say that with the proper or worse parameters, each of the models (either HMC or Gibbs) can perform better. (one thing that was notable is that Gibbs runs and converges much faster than HMC).

(c)

(d)

4.

ϵ	L	Acceptance Rate	Test Predictive Accuracy	Train Predictive Accuracy	Test Predictive Likelihood	Train Predictive Likelihood
0.005	10	1.0	0.951348	0.957899	-0.105244	-0.097960
0.005	20	1.0	0.951582	0.957806	-0.104707	-0.097788
0.005	50	1.0	0.951454	0.957835	-0.104750	-0.097733
0.01	10	1.0	0.951654	0.958126	-0.104562	-0.097717
0.01	20	1.0	0.951478	0.957861	-0.104849	-0.097775
0.01	50	1.0	0.951446	0.957946	-0.104760	-0.097675
0.02	10	1.0	0.951366	0.957886	-0.104875	-0.097782
0.02	20	1.0	0.951488	0.957900	-0.104863	-0.097764
0.02	50	1.0	0.951344	0.957907	-0.104843	-0.097772
0.05	10	1.0	0.940290	0.947821	-0.193897	-0.172764
0.05	20	1.0	0.935360	0.944842	-0.289930	-0.251133
0.05	50	1.0	0.922686	0.932419	-0.576635	-0.499444

Table 1: Bayesian Logistic Regression Model Results