Predicting Crashes and Rallies in Oil Futures Using Topological Data Analysis

Enzo Moraes Mescall   Lihong Wang   Andrew Bonafede

***Abstract*—We analyse the timeseries of daily returns of Brent Crude for the last 5 years using methods of topological data analysis (TDA). In particular, we use persistent homology to detect indicators in topological spaces associated with time-dependent point clouds created using a sliding window on the timeseries. These indicators are constructed by considering the landscape distance of the persistence induced by filtered Vietoris-Rips Complexes associated with the aforementioned point clouds. More specifically, we quantify the temporal changes in persistence landscapes by their L^p-norms and predict that L^p-norms to exhibit strong changes prior to the price change-point in the vicinity of large price corrections. When compared to a baseline method which uses the first derivative as an indicator of crash or rally probability, our topological approach performs relatively well. This is a sign that TDA may prove to have profitable applications in finance, although further investigation and refinement is required.**

I. Introduction

A. Motivation and Goal

Past explorations of stable topological structures in noisy multidimensional datasets has led to new insights in signal and time-series analysis [5] [10] [8]. Motivated by these studies, we decided to investigate whether the application of TDA to commodities price timeseries data can help detect growing idiosyncratic risk in investor sentiment and thus predict crashes.

Observations in different markets show that financial crashes in the equity markets are preceded by a period of increased oscillation in asset prices that translates into an abnormal change in the geometric structure of the time series [6]. TDA methods excels at analyzing data with geometric nature since the various distance metrics used in TDA, such as Landscape Distance, Betti Curve Distance, Bottleneck Distance, Gromov-Hausdorff Distance, as well as others, are all measures of similarity between geometric structures such as persistence diagrams. From a high level, if crashes are preceded by changes in the geometric structure of the timeseries, then we can reasonably expect topological metrics to change noticeably over sliding windows on the timeseries around those crashes, and perhaps the rate of change in those metrics can offer useful indicators to the likelihood of financial crashes. Ideally we would sample the results of crash detection models using all sorts of topological metrics, but due to constraints of time and resources we will focus on two: namely, the Landscape Distance and the Betti Curve Distance due to their quick computation and use in literature [4].

In addition to the the possibly geometric nature of the data, TDA is also appealing to us for other reasons. Specifically, TDA has excelled at providing a framework to extract insights from noisy data, which makes it suitable to the analysis of financial timeseries which is theoretically composed of mostly noise, if one assumes some version of the efficient market hypothesis. Past attempts at using TDA to detect crashing points in highly liquid equity markets have provided promising results [4], and we hope that TDA can yield similarly useful results in another asset class; in particular, oil futures.

The data set that we chose was was the closing price of 1 Month Brent Crude Oil Futures from the New York Mercantile Exchange from January 2015 to January 2020. There were several characteristics in oil prices that we found desirable for the purposes of our investigation. First, We believe oil prices will produce interesting results because they are commonly considered as very volatile among major commodities; therefore, there are a higher number of clear crashes the price of oil futures when compared to other major commodities which will allow us to better evaluate the effectiveness of our model. Second, the historical trend for oil is that prolonged rallies occur before crashes. It is unclear why this is the case, but the pattern exist nonetheless. In any case, this implies that periods of price stability are essentially periods of sustained growth, and therefore periods of low crash probability are also periods where oil futures can be bought and exercised to make a profit.

B. What is Topological Data Analysis?

Topological Data Analysis (TDA) refers to a variety of methods which are used to uncover geometric structures in data and in the past has proven to be a powerful approach for the analysis of noisy and complex data sets. We hypothesize that there are geometric properties in commodity prices, especially oil futures, which can be revealed through topological data analysis. In particular, we hope to construct indicators which can predict periods with high probability of experiencing imminent crashes, as well as predicts periods with high probability of experiencing persistent rallies.

Typically, the application of TDA requires that our data be encoded as a finite set of points in some metric space; in our case, after acquiring our data set from relevant financial institutions, we use Taken's Embedding to embed the time series into a point cloud and construct sliding windows of point clouds. The general approach then taken is to construct some filtered simplicial complex over the point cloud and analyze how the persistence of k-dimensional holes, e.g., connected components ($k = 0$), loops ($k = 1$), etc.., changes over the different sliding windows. In our case, the filtered simplicial complex that we construct is the filtered Vietoris-Rips complex associated with the point cloud, and the persistent homology that we consider are of dimensional 1, 2, and 3 because any further dimensions proved to be too computationally demanding.

C. The TDA Pipeline

We now elaborate on the procedure to compute the persistent homology associated with a point cloud data set. We construct a filtration of simplicial complexes ordered with respect to some scaling parameter t. As the parameter changes, topological features appear and disappear in the corresponding simplicial complex. This allows us to assign to each topological feature a 'birth' and 'death' time, as well as a persistence diagram from the natural mapping of the the aforementioned birth and death times onto the Cartesian plane. The difference between the two values represents what is called the persistence of that feature. A topological feature with higher persistence, i.e. exists for a greater range of the scaling value, can be interpreted as being more significant. Correspondingly, a feature that persists for a smaller range can be viewed as a less significant, or a more noisy feature. It should also be noted that the information in the persistence diagram can also be encoded, without loss of information, into what is called a persistence landscape; the persistence landscape can essentially be thought of as a linear summary of the persistence diagram.

An important quality of the persistent homology generated using the pipeline described above is that both the persistence diagrams and the persistence landscapes are stable to small perturbations in the underlying data. In other words, small changes in the underlying data will only induce small changes in the persistence diagrams and landscapes. This allows for persistent homology to be considered a mathematically well-founded statistical method. In addition, it means there there need not be an artificial cutoff between signal and noise in that data; rather, all the topological features that emerges from the data are kept, and assigned significance according to their persistence.

D. Summary of Results

We analyze the time-series of daily closing prices for Brent Crude oil prices and gauge the effectiveness of our analysis against a baseline momentum indicator based on the first derivative. While our strategy was shown to be more effective than the baseline method, there is no clear and convincing evidence that it can produce alpha in the long term. Therefore, although our method is very general and can be applied to a large range of asset-types, we believe that further investigation and refinement is required before our pipeline can become financially viable.

II. Background

In this section we provide a brief summary of persistent homology with an emphasis on the tools that we used. For a more rigorous and complete exposition we would recommend the inquisitive reader to relevant textbooks [3]. A visualization of many of the ideas developed here can be found in the appendix.

A. Taken's Embedding

The first step in our analysis is to generate a simplicial complex from a point cloud. This is typical for TDA pipelines and the expectation is that some key properties of a dynamic system can be more effectively unveiled in higher dimensions than in lower ones. Taken's Embedding, also called the time-delay embedding, maps univariate time series to point clouds in an Euclidean space of arbitrary dimension. Our univariate timeseries of price is composed of a set of discrete times $t_0, t_1, ..., t_n$ with corresponding future prices of $p(t_0), p(t_1), ..., p(t_n)$. Define a time-delay parameter τ and an embedding dimension d; then, each time $t_i \in \{t_0, t_1, ..., t_n\}$ maps to a vector

$$Y_t i = (p(t_i), p(t_i + \tau), ...p(t_i + (d-1)\tau))$$

Over the sliding window $[t_i, t_i + (k-1)]$ containing k items, Taken's embedding generates a point cloud containing k points. Finally, applying this procedure onto all sliding windows of a constant size over the entire timeseries induced a timeseries of point clouds with possibly interesting topologies that we can explore.

B. The Vietoris-Rips Complex

Provided with a point cloud $X = \{x_1, x_2, ..., x_n\}$ in an Euclidean space $\mathbb{R}^d$, we may define the Vietoris-Rips Complex $R(X, \epsilon)$ for each distance $\epsilon > 0$. In particular, for each $k = 0, 1, 2, ..., n$ a k-simplex of vertices $\{x_{i_1}, x_{i_2}, ..., x_{i_k}\}$ is part of $R(X, \epsilon)$ if and only if

$$d(x_{i_j}, x_{i_l}) < \epsilon \forall x_{i_j}, x_{i_l} \in \{x_{i_1}, x_{i_2}, ..., x_{i_k}\}$$

where $d(x_{i_j}, x_{i_l})$ is the euclidean distance between those two points; in other words, the Vietoris-Rips complex includes the entire set of vertices for which the mutual distance between any pair of its vertices is less than ϵ. In a sense, ϵ can be thought of as the resolution at which that data is looked at since if a k-simplex is in $R(X, \epsilon)$ if and only if those k data points are indistinguishable from one another at resolution ϵ. The Vietoris-Rips simplicial complexes $R(X, \epsilon)$ also forms a filtration; i.e.

$$R(X, \epsilon) \subseteq R(X, \epsilon') \quad \text{whenever} \quad \epsilon' < \epsilon$$

C. Persistent Homology and Diagram

For each Vietoris-Rips complex, we can compute its k-dimensional homology $H_k(R(X, \epsilon))$ with coefficients in some field; in our case we use the field of two elements. Conceptually, the underlying generators of the 0-dimensional homology group $H_k(R(X, \epsilon))$ are the connected components of $R(X, \epsilon)$, the generators of the 1-dimensional homology group $H_1(R(X, \epsilon))$ are the independent loops of $R(X, \epsilon)$, the generators of the 2-dimensional homology group $H_2(R(X, \epsilon))$ are the independent cavities of $R(X, \epsilon)$, and so on.

The filtration of the Vietoris-Rips complexes naturally induces a filtration on the corresponding homologies. That is to say, $H_k(R(X,\epsilon)) \subseteq H_k(R(X,\epsilon'))$ whenever $\epsilon < \epsilon'$, for all k. These inclusions induce a natural homomorphism from $H_k(R(X,\epsilon))$ to $H_k(R(X,\epsilon'))$ and from these mappings it is possible to show that for each non-zero k-dimensional homology class α there exists a pair of values $\epsilon_1 < \epsilon_2$ such that $\alpha \in H_k(R(X,\epsilon))$ has a non-zero image under $H_k(R(X,\epsilon))$ if and only if $\epsilon_1 < \epsilon < \epsilon_2$. In this case, we may say that α is born at ϵ_1 and dies at ϵ_2; this information is naturally encoded to the pair (ϵ_1, ϵ_2). The difference $\epsilon_2 - \epsilon_1$ is the persistence of α, and the persistence diagram of α can be generated by plotting (ϵ_1, ϵ_2) onto $\mathbb{R}^2$ where the horizontal axis represents birth times and the vertical axis represents death times.

D. Persistence Landscape and Landscape Distance

An rigorous definition of persistence landscape and the landscape distance is best approached from an algebraic perspective. Therefore we first define the the persistence module algebraically: a persistence module over a partially ordered set S consists of a set of vector space V_i indexed by elements of S, with a linear map $v_i^j : V_i \to V_j$ such that v_i^i is the identity map and $\forall i \leq j \leq k$, $v_i^k = v_j^k \circ v_i^j$. For the purposes of our investigation, our partially ordered set is $\mathbb{R}$.

Given a Vietoris-Rips complex $R(X,\epsilon)$ and its associated filtration f, it is possible to show that $\{H_k(R(X,\epsilon)), f_\epsilon^{\epsilon'}\}$ is a valid persistence module for $\epsilon < \epsilon'$. Now, let $P = \{H_k(R(X,\epsilon)), f_\epsilon^{\epsilon'}\}$ be its filtered Vietoris-Rips Module, and define $\beta_\epsilon^{\epsilon'}$, where $\epsilon < \epsilon'$, as

$$\beta_\epsilon^{\epsilon'} = \text{rank}(P_\epsilon^{\epsilon'} : P_\epsilon \to P_{\epsilon'})$$

This finally allows us to define the persistence landscape: Fix $k \in \mathbb{N}$. The k^{th} persistence landscape is a sequence of functions $\lambda_k : \mathbb{R} \to \mathbb{R}$ such that

$$\lambda_k(t) = \sup\{\epsilon > 0 | \ \beta_{t-\epsilon}^{t+\epsilon'} \geq k\}$$

The definition of the landscape distance then follows naturally. In particular, if we let D, D' be two persistence diagrams and λ, λ' be their respective persistence landscapes, then their k^{th} landscape distance is defined as

$$\Lambda_k(D, D') = ||\lambda - \lambda'||_k$$

Where the right hand side denotes the L^k-norm. Recall now that the persistence landscapes are stable to small perturbations, and is therefore a suitable choice for our analysis [1]. However, the landscape distance is not the only distance that we can use. More specifically, the Betti Curve Distance presents itself as a natural alternative.

E. Betti Curves and Betti Curve Distance

Let D be the persistence diagram for a persistence module P, and consider it at times $\tau' < \tau$. We can define an "alive" region $D_\tau \subset \mathbb{R}^2$ as $D_\tau = K_i \setminus C_i$ where

$$K_\tau = \{(b,d) \in \mathbb{R}^2 | 0 < b < \tau, d > \tau'\}$$

$$C_\tau = \{(b,d) \in \mathbb{R}^2 | \tau' \leq b \leq \tau, \tau' < d < b\}$$

Now, we can define a Betti Curve [7]: Let D be a persistence diagram with finitely many points. A Betti Curve of D is

$$\vec{v}(D) = (v_\tau)_1^N \in \mathbb{R}^N$$

where $v_\tau = |D_\tau \bigcap D|$ is the cardinality of the intersection of persistence diagram D and D_τ. In a sense, the Betti Curve keeps track of the Betti Numbers at all times in the filtration. We can define the Betti Distance as well; for two persistence diagrams D, D' and associated Betti Curves $\vec{v}(D), \vec{v}(D')$, the k^{th} Betti Curve distance is

$$B_k(D, D') = ||\vec{v}(D) - \vec{v}(D')||_k$$

However, unlike the landscape distance, we must note that the Betti Curve distance is not stable with respect to the 1-Wasserstein Distance [7], which is a natural metric on the space of persistence diagrams.

III. Methodology

A. Baseline Indicator

Given that crashes in commodity prices represent a sudden decline of those prices, one natural approach to detect these changes involves tracking the first derivative of price values with respect to time over a rolling window. We do this with a window size of 10 days, generating a timeseries of the derivative, then, by normalising this derivative timeseries to take values in the [0,1] interval, we can apply a threshold to label points on our original time series where a crash occurred. In addition, recall that when not experiencing high volatility, oil prices have historically went up, and as such when the baseline predictor indicates a low likelihood of crashes, we may purchase the oil futures and expect their price to increase. In our case, we suggest sell when first derivative exceeds one standard deviation above the mean, and suggest buy when it is below one standard deviation below the mean. The results of the baseline indicator are can be seen in figure 1.

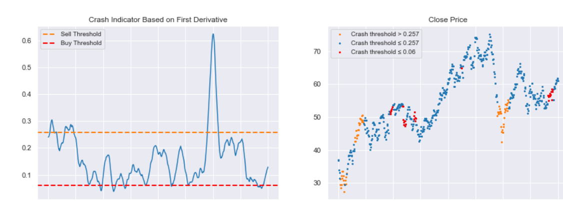

Fig. 1. Normalized First Derivative Levels and Resulting Trade Suggestions

B. Data Transformation for TDA

We begin our TDA pipeline with simple Crude Oil Future close prices ranging from January of 2015 to January of 2020. This data then gets re-sampled into 24−hour intervals and embedded via Taken's Embedding into point clouds, with initial parameters $d = 100, \tau = 5$. We chose Taken's embedding with the expectation is that some key properties of a dynamic system can be more effectively unveiled in higher dimensions than in lower ones. Figure 2 shows an example of a 3-D point cloud for visualization purposes, our actual embedding contains many more data points in higher dimensions and hence cannot be visualized.

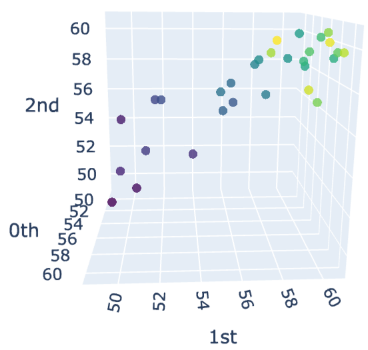

Fig. 2. Taken's Embedding of 50th window with $d = 3, \tau = 2$.

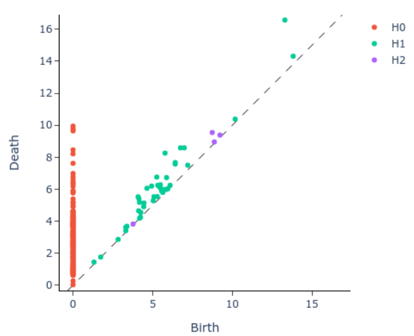

Fig. 3. Zeroth, First, and Second Dimensional Persistence Diagram 50th Window.

C. Landscape Distances

Given the new time series of vectors i.e. the embedded point clouds, we can compute the zeroth, first, and second dimensional persistent homology of the filtered Vietoris-Rips complex of these point clouds. From there, persistence diagrams, such as the one in Figure 3, are computed.

Once the persistence diagrams are computed, we utilized the two different metrics previously described to compute distances on the persistence diagrams. The first metric we consider are landscape distances. We consider each window through its associated point cloud and compute the persistence diagram induced as well as its landscape; an example of the persistence landscape of a window is shown in figure 4.

We then take the landscape distance between two successive sliding window diagrams by using the L^2 norm. This induces a timeseries with each landscape distance at time t as shown in Figure 5.

D. Betti Curve Distances

The second metric we consider is the Betti Curve distance. In similar fashion, we compute the persistence diagrams for each successive window, and then find the associated Betti Curve for the window diagram where N is the number of points in the diagram. From there, we take the L^2 norm of the difference between the Betti curves for each successive window diagram. An example of the Betti Curve distance can be seen in Figure 6.

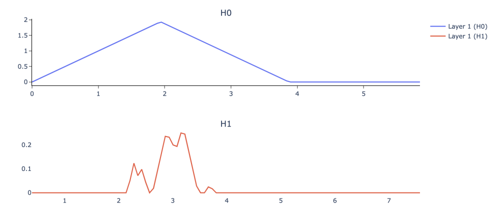

Fig. 4. Landscape of the 50th Window

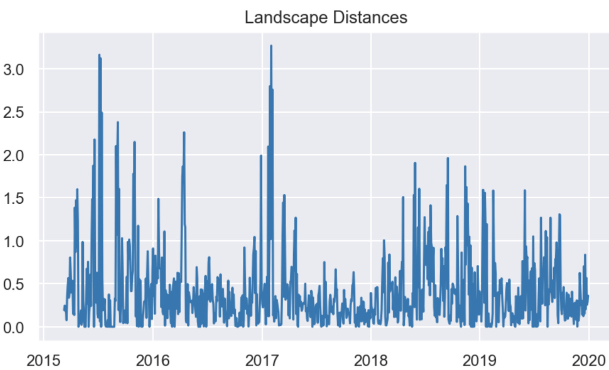

Fig. 5. The Timeseries of Landscape Distances

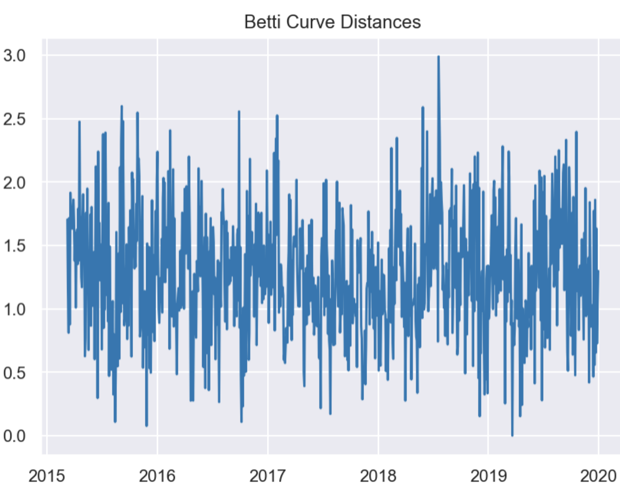

Fig. 6. The Timeseries of Betti Curve Distances

E. Constructing the Topological Indicator

Finally, to construct our topological indicator for both sets of distances, we calculate the rolling mean, the rolling minimum and the rolling maximum of the homological derivatives for 10 day intervals and normalise the time series so it lies within $[0, 1]$. In similar fashion to the derivative momentum indicator, we set the buy threshold at one standard deviation below the mean and the sell threshold at one standard deviation above the mean.

At this point, we will reiterate the rationale behind the trading strategy. Recall that historical observations and past research show that crashes are preceded by periods of increased oscillation in asset prices that translates into an abnormal change in the geometric structure of the timeseries. We believe that distance metrics such as Landscape Distance and Betti Curve Distances are measures of geometric similarity that can be used to capture the aforementioned abnormal changes and be used as a technical signal for trades. In addition, we observe that the historical trend for oil is that prolonged rallies occur before crashes. This implies that periods of price stability are essentially periods of sustained growth, and therefore periods of low crash probability are also periods where oil futures can be bought and exercised to make a profit. As such, our strategy of using a threshold for the rate of change of the Landscape and Betti Distance over sliding windows is reasonable, since a high rate of change signals a deviation of the geometric of the timeseries away from the norm. We will therefore sell the oil futures when rates of change for the topological metric appear abnormally high, i.e. above the threshold, and buy oil futures when they appear stable, i.e. below the threshold.

IV. Results

In this section we will present the results of our analysis. The figure below shows the trading strategy which resulted from our analysis of the landscape distances

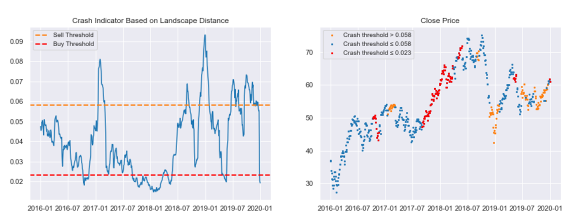

Fig. 7. Normalized Landscape Distance Derivatives and Trade Suggestions

In addition, we also present the trading strategy which resulted from our analysis of the Betti Curve distances

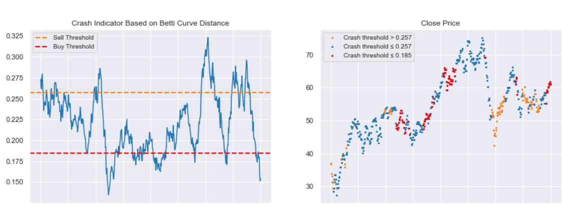

Fig. 8. Normalized Betti Curve Distance Derivatives and Trade Suggestions

Visually, we see that the Landscape Distance indicator seemed to work better as the Betti Curve Distance indicator did not pick up on the large crash from mid-2018 to 2019. This discrepancy in results is expected; by looking at figure 5 and 6 from the previous page, we see that the signals generated by the Betti Curve Distances are much noisier than the signals generated by the Landscape Distances. In a sense, the Betti Curve Distances preserved too much information from the original timeseries, and given that the original timeseries was mostly noise, made it harder for a threshold method to pick up on useful indicators. In contrast, by removing unnecessary information, the Landscape Distance was able to perform better by only keeping the relevant geometric properties of the timeseries.

A more quantitative method to measure the effectiveness of our algorithms is to backtest them. For simplicity's sake, we will buy an entire portfolio of oil futures at the first buy signal, and sell our entire portfolio at the first sell signal. Assuming that we start with \$100,000 in 2015, we will be left a portfolio worth the following amount 5 years later if we follow each of the corresponding indicators where SD represents the amount of standard deviations from the mean we place the buy and sell thresholds at:

Baseline Method, Derivative of Price (1SD): \$63,918
Derivative of Landscape Distance (1SD): \$136,315
Derivative of Betti Curve Distance (1SD): \$90,392

In addition, another natural thing to do is to construct strategies where we commit a trade if and only two signals agree. The results of these hybrid strategies are shown below:

Betti and Landscape Agreement (1SD): \$120,049
Price and Betti Agreement (1SD): \$70,659
Landscape and Baseline Agreement (1SD): \$100,000 [1]

In the case of the last strategy, there were no buy agreements between the two indicators over 5 years. Therefore, we lower the threshold required and inspect the results:

Betti and Landscape Agreement (0.7SD): \$98,978
Price and Betti Agreement (0.7SD): \$73,769
Landscape and Baseline Agreement (0.7SD): \$109,632

Looking at our backtest results, we can see that the most profitable strategy occurs when we only follow the threshold indicator for the rate of change of the Landscape Distance. We will now evaluate these results in the next section.

V. Discussion and Future Work

A. Discussion

In this study, we have presented a method for predicting price crashes of Oil Futures using Topological Data Analysis. In our experiments, we utilized two different distances that are both common in the literature, the landscape and Betti curve distances. As discussed in the results section, rigorous back-testing over a four year period showed promising returns. In particular, our most optimal strategy results in a 36.315% gain over 5 years, which indicates an annualized return of

[1] The two signals did not agree to buy for any occasion, but agreed to sell on 19 different occasions.

6.388%. While this may seem like a good amount, we must consider the opportunity cost of deploying $100,000 of capital. In particular, if we had invested our $100,000 into the S&P500 during the same time period, then we would have ended up with a portfolio worth $161,633, which indicated an annualized return of 10.079%.

In addition, it is safe to assume that our investment strategy is more risky than investing in the broader market since our holdings are less diversified and therefore more susceptible to idiosyncratic risk factors. Therefore, it is safe to conclude that the strategy as it exists generates negative alpha. Moreover, there are a few factors that we ignored in backtesting which would further reduce the relative profitability of our strategy when compared to investing in the broader market. The first factor is that we ignored is dividend investing; indeed, barrels of oil by themselves do not general cash flow, while equity in profitable companies do.

The second factor we ignored is the term structure, or the futures structure of the oil. Crude oil futures, like most other commodities, are not priced as a single data point like stocks usually are, rather, there are futures contracts for each month heading out multiples years. The 'current price' of oil typically refer to the price of the front month contract, but is not representative of the price of all futures contracts; in fact, it is typical for the prices of contracts into the future to be different than the front month contract. The structure of the price of the futures contract with respect to delivery date is referred to as the futures structure, and typically occurs in two forms: contango and backwardation. The former refers to a structure where the futures price far into the future is more expensive than futures prices near the present, where as the latter refers to the opposite. Various factors shape the term structure of a commodity, including but not limited to expected supply, expected demand, the holding costs of the commodity, and the holding benefits of the commodity. While oil typically exhibits contango due to the high holding costs, in recent years it has often shifted to backwardation due to more complex economic factors [9].

In any case, a more accurate backtest would consider the effect of continuously buying and then selling front month contracts; if oil markets are in contango then we would lose money from the term structure, while if they are in backwardation we would gain extra money. Because oil is in contango more often than not, we would expect to have even lower profitability if our strategy was implemented in real life. The expanse ratio of oil ETFs such as the United States Oil Fund, USO, at 0.8% [2], is indicate of the amount of money we could expect to lose through contango annually.

B. Future Work

It should now be clear that our strategy, while promising, is not yet commercially viable. For us, this simply means that further investigation needs to be done. One particular avenue of research one could consider is whether topological indicators can be added to some existing trading algorithm either as a risk management tool. Our work here has shown that TDA methods, although imperfect, can still potentially detect crashes in prices. A second, closely related avenue of research, is to investigate whether TDA methods can be used in conjunction with existing, profitable, trading algorithms to produce stronger buy signals that can enhance profitability.

Another direction that we can take our research is simply to investigate the profitability of other topological metrics, as well as combinations of their signals. Closely related to this idea is to investigate the tuning of hyperparameters, which includes the topological metric used to produce the indicator. We believe that using some sort of machine learning algorithm to train the optimal thresholds, sliding windows size, and stride length would likely improve our returns further. In addition, it is worth noting that it would be interesting to perform the same experiments on different securities.

Prediction of the financial markets is one of the greatest open problems in mathematical finance; we did not go into the investigation expecting to receive exceptional returns. However, we were still please by our results as we believe that they have shown convincingly that the historical price of securities isn't as noisy as the Efficient Market Hypothesis would imply; there is some structure and pattern to be found in the randomness.

VI. Appendix

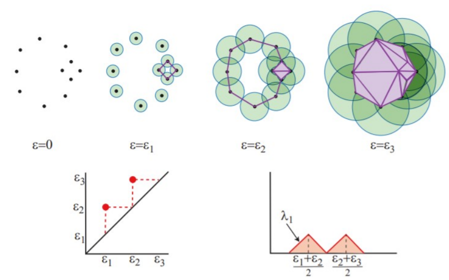

Fig. 9. Summary of TDA Process [4]

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