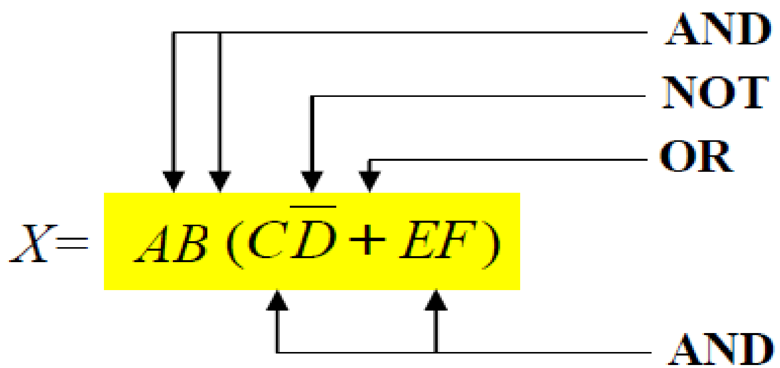
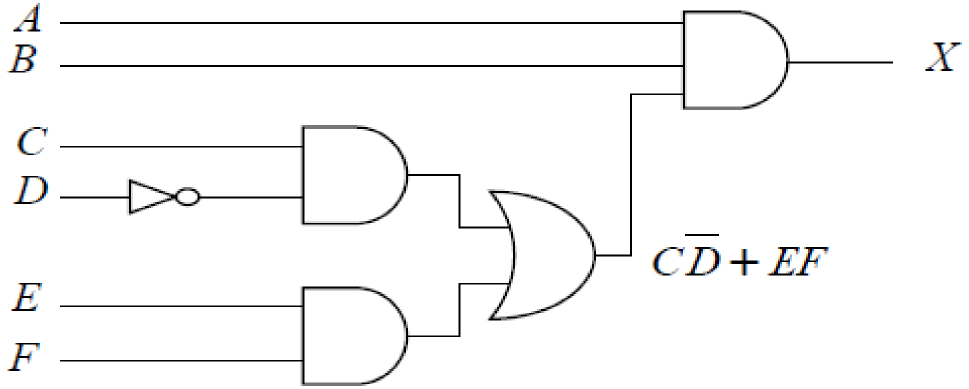
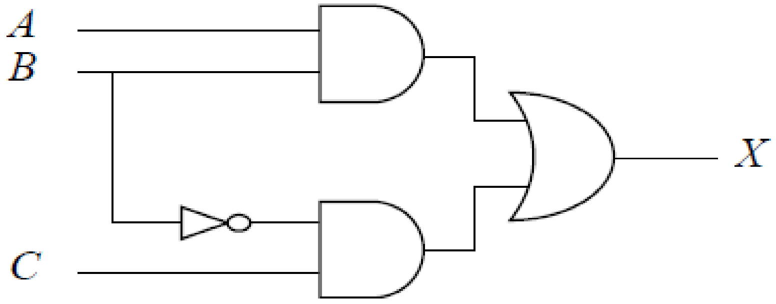
Extra Examples and Exercises DLD

Example: Implement the following expression: $X = AB(C\overline{D} + EF)$





Example: Draw the circuit diagram that implements the expression: $X = AB + \overline{B}C$



Example: Prove $(A + B)\cdot(A + C) = A + B\cdot C$

Solution

$(A + B)\cdot(A + C) = (A + B)\cdot A + (A + B)\cdot C$	Distributive Law
$= A\cdot A + B\cdot A + A\cdot C + B\cdot C$	Distributive Law
$= A + B\cdot A + A\cdot C + B\cdot C$	Rule: $A\cdot A = A$
$= A + A\cdot B + A\cdot C + B\cdot C$	Commutative Law
$= A + A\cdot C + B\cdot C$	Rule: $A + A\cdot B = A$
$= A + B\cdot C$	Rule: $A + A\cdot B = A$

Now that you have a taste for the manipulation of Boolean expressions, the next section will show examples of how complex expressions can be simplified.

Example: prove $AB + \overline{A}C + BC = AB + \overline{A}\,C$

Solution

$= AB + \overline{A}\,C + 1 \cdot BC$	rule $1.X=X$
$= AB + \overline{A}\,C + (A + \overline{A}) \cdot BC$	rule $X+\overline{X}=1$
$= AB + \overline{A}\,C + ABC + \overline{A}BC$	Distributive Law
$=AB+ABC+\overline{A}\,C+\overline{A}BC$	Commutative Law
$=AB(1+C)+\overline{A}\,C(1+B)$	Distributive Law
$= AB+\overline{A}\,C$	rule $1+X=1$

<u>Example:</u> Show $(X+Y).\overline{X}.\overline{Y}=0$

$$(X+Y).\overline{X}.\overline{Y} = (X.\overline{X}.\overline{Y} + Y.\overline{X}.\overline{Y}) \quad \text{(Distributive Law) } X + YZ = (X + Y)(X + Z)$$

$$=(X.\overline{X}.\overline{Y} + Y.\overline{Y}.\overline{X}) \quad \text{(Commutative Law) } X + Y = Y + X$$

$$=(0.\overline{Y} + 0.\overline{X})$$

$$= (0 + 0)$$

$$= 0$$

Example: determine the truth table and logic diagram AB+ A (B+C)+ B(B+C)

$$AB+ A (B+C)+ B(B+C) = AB+AB+AC+BB+BC$$

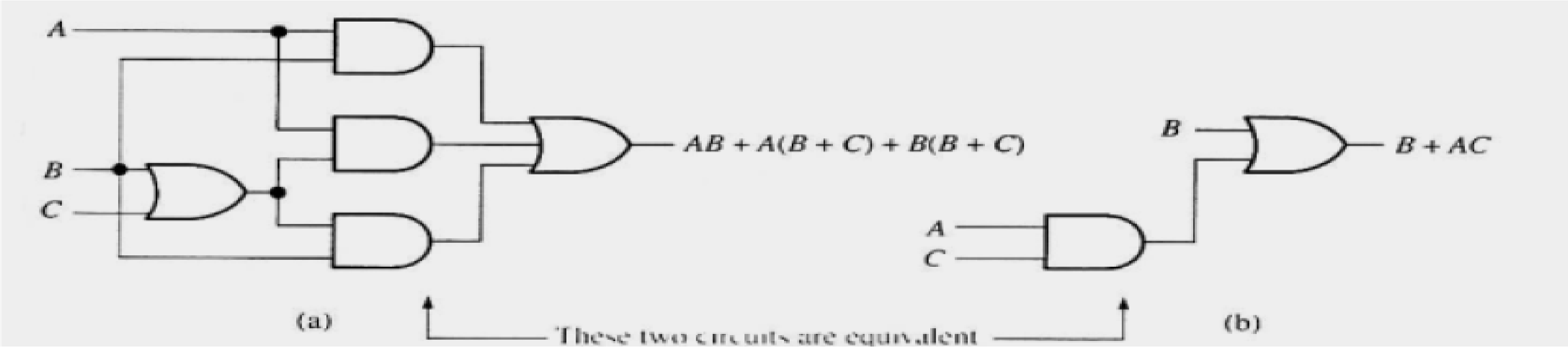
$$=AB+AB+AC+B+BC$$

$$= AB+AC+B(1+C)$$

$$= AB+B + AC$$

$$=B(A+1)+AC$$

$$= B+AC$$



Example: Prove that $(\overline{\mathbf{X}+\mathbf{Y}})\mathbf{Z}+\mathbf{X}\overline{\mathbf{Y}}=\overline{\mathbf{Y}}(\mathbf{X}+\mathbf{Z})$

$= \overline{X}.\overline{Y}.Z + X.\overline{Y}$ (DeMorgan's Law)

$= \overline{Y}(\overline{X}.Z + X)$ (Distributive Law)

$= \overline{Y}(\overline{X} + X)(Z + X)$ (Distributive Law)

$= \overline{Y}.1(Z + X)$ Rule 4

$= \overline{Y}(Z + X)$

Example: **Simplify the following Boolean Expression** $\overline{\mathbf{A}\overline{\mathbf{B}}+\mathbf{A}\overline{\mathbf{C}}}+\overline{\mathbf{A}}\overline{\mathbf{B}}\mathbf{C}$

Step1: apply DeMoragn's theorem to the first term.

$$(\overline{A\overline{B}}).(\overline{A\overline{C}}) + \overline{A}\overline{B}C$$

Step2: apply DeMoragn's theorem to each term in parentheses.

$$(\overline{A} + B)(\overline{A} + C) + \overline{A}\overline{B}C$$

Step3: Apply the distributive law to the two terms in parentheses.

$$\overline{A}\overline{A} + \overline{A}C + \overline{A}B + BC + \overline{A}\overline{B}C$$

Step4:Apply Boolean Rules $\overline{A}\overline{A} = \overline{A}$, $\overline{A}\overline{B} + \overline{A}\overline{B}C = \overline{A}\overline{B}(1 + C) = \overline{A}\overline{B}$

$$\overline{A} + \overline{A}\overline{C} + \overline{A}\overline{B} + \overline{B}\overline{C}$$

Step5: Apply Boolean Rule $\overline{A} + \overline{A}\overline{C} = \overline{A}(1 + \overline{C}) = \overline{A}$ to the first and second terms

$$\overline{A} + \overline{A}\overline{B} + \overline{B}\overline{C}$$

Step6: Apply Boolean Rule $\overline{A} + \overline{A}\overline{B} = \overline{A}(1 + \overline{B}) = \overline{A}$ to the first and second terms

$$\overline{A} + \overline{B}\overline{C}$$

<u>Ex</u> Find the 1 – Sum of Minterms 2 – Product of Maxterms of **F**?

1 – Sum of minterms:

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	1

$$F = (\overline{X}\overline{Y}Z) + (X\overline{Y}\overline{Z}) + (XYZ)$$

$$= \mathbf{m1 + m4 + m7} = \sum(\mathbf{1, 4, 7})$$

2- Product of Maxterms:

$$F = (X + Y + Z) \cdot (X + \overline{Y} + Z) \cdot (X + \overline{Y} + \overline{Z}) \cdot (\overline{X} + Y + \overline{Z}) \cdot (\overline{X} + \overline{Y} + Z)$$

$$= M_0 \cdot M_2 \cdot M_3 \cdot M_5 \cdot M_6 = \prod(0, 2, 3, 5, 6)$$

<u>Ex:</u> Express the function F in Sum of minterms and Product of Maxterms and

simplify it

1– Sum of minterms

$$F(X,Y,Z) = \bar{X}Y\bar{Z} + \bar{X}YZ + X\bar{Y}Z + XY\bar{Z} + XYZ$$

$$= \bar{X}Y(\bar{Z} + Z) + X\bar{Y}Z + XY(\bar{Z} + Z)$$

$$= \bar{X}Y + X\bar{Y}Z + XY = Y(\bar{X} + X) + X\bar{Y}Z = Y + X\bar{Y}Z$$

$$= X\bar{Y}Z + Y = (XZ + Y)(\bar{Y} + Y) = XZ + Y$$

X	Y	Z	F
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

H.W.

1. Obtain the canonical product of the sum form of the following function.

$$F\,(A, B, C) = A + B'C$$

2. Obtain the canonical product of the sum form of the following function.

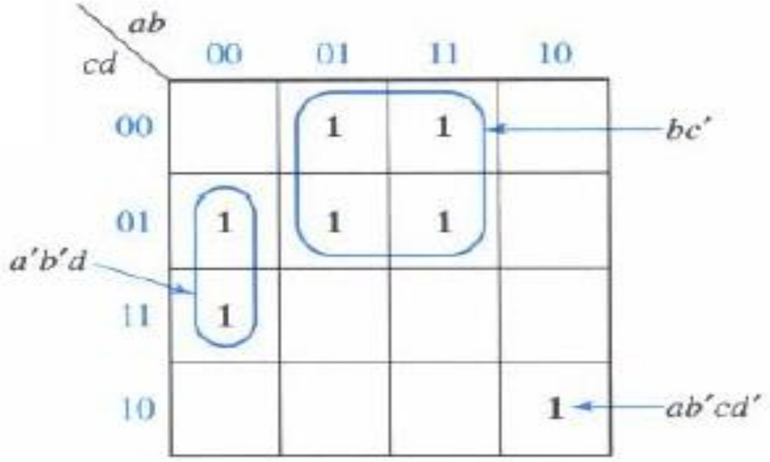
$$F\,(A, B, C) = (A + B')\,(B + C)\,(A + C')$$

3. Obtain the canonical sum of product form of the following function.

$$F\,(A, B) = A + B$$

Simplification of Four-Variable Functions

$$F = \sum m(1,3,4,5,10,12,13)$$



$$F = bc' + a'b'd + ab'cd'$$

DON'T CARE & CAN'T HAPPEN - Examples

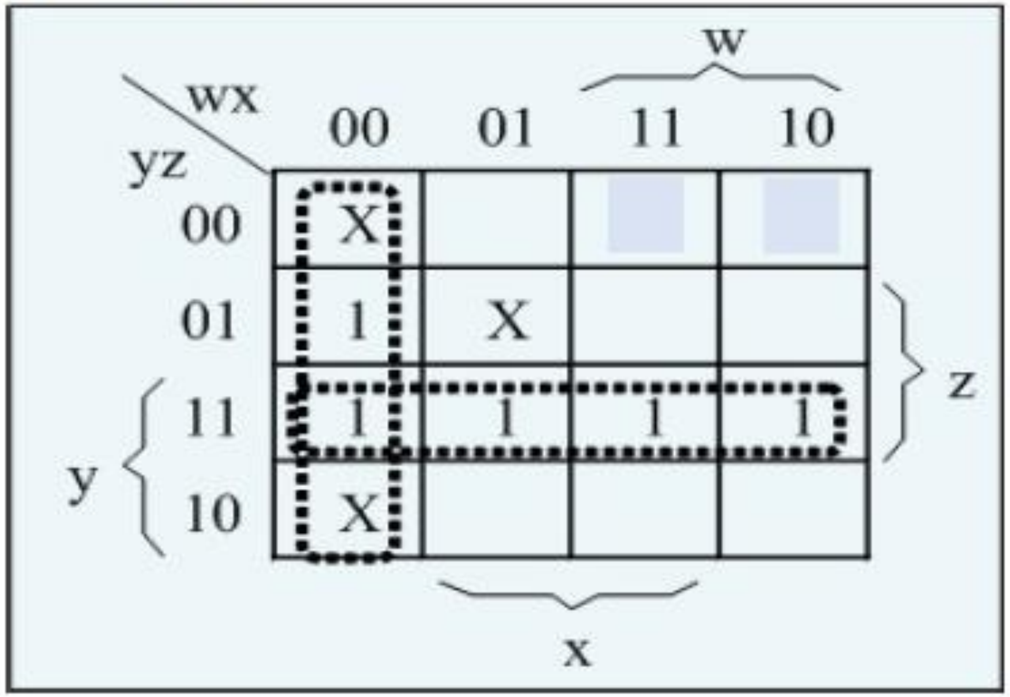
Example 1: Simplify the logic function with associated Don't Cares.

$$f(w, x, y, z) = \Sigma(1, 3, 7, 11, 15)$$

$$D(w, x, y, z) = \Sigma(0, 2, 5)$$

Don't Care

$$f = \overline{w}\,\overline{x} + y\,z$$



DON’T CARE & CAN’T HAPPEN - Examples

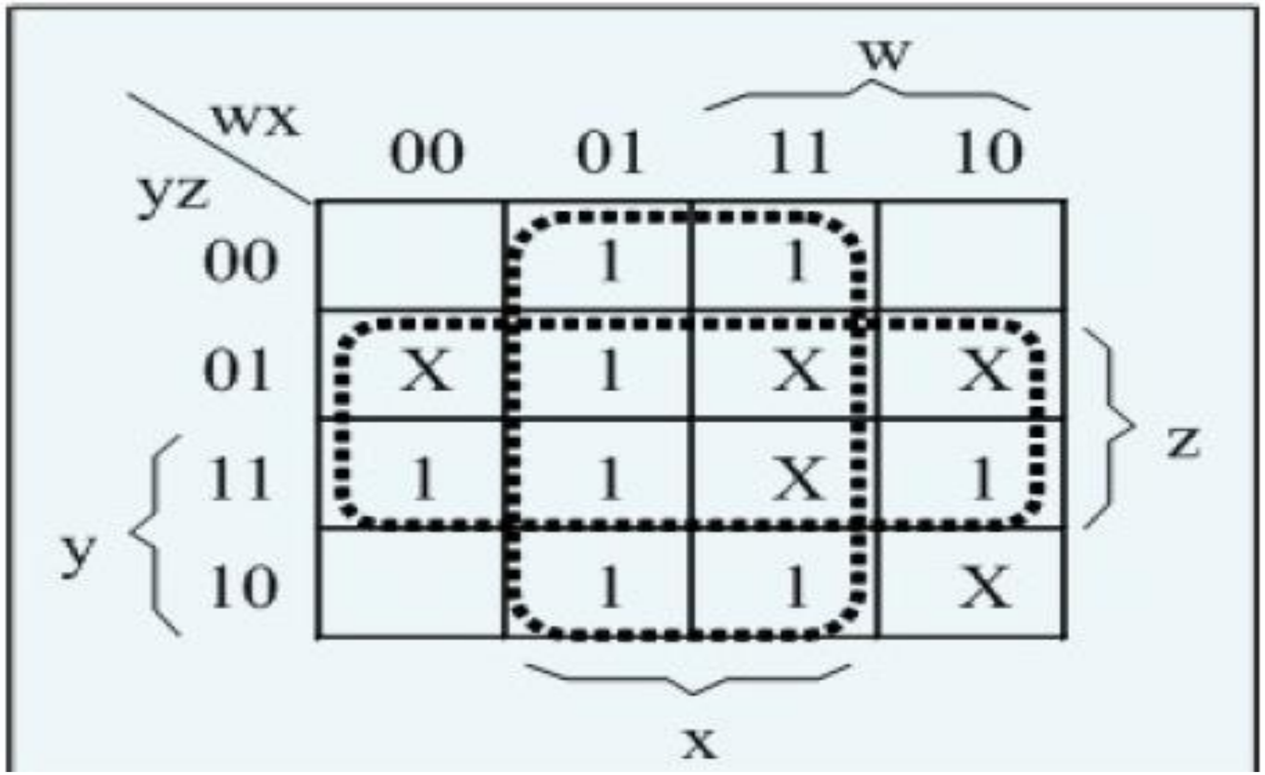
Example 2: Simplify the logic function with associated Don’t Cares.

$$f(w, x, y, z) = \Sigma(3, 4, 5, 6, 7, 11, 12, 14)$$

$$D(w, x, y, z) = \Sigma(1, 9, 10, 13, 15)$$

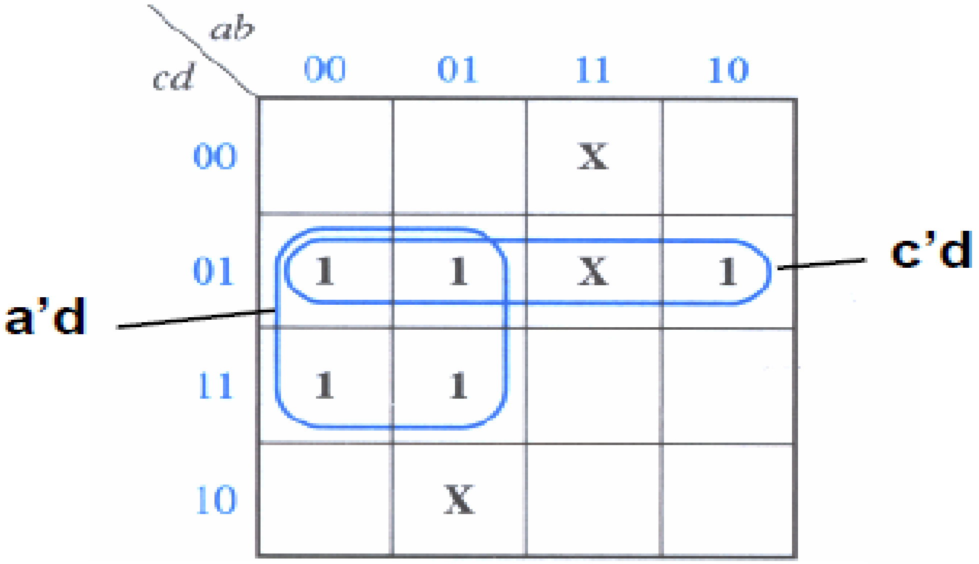
Don’t Care

$$f = x + z$$



Simplification of Incompletely Specified Function

$$F = \sum m(1,3,5,7,9) + \sum d(6,12,13)$$



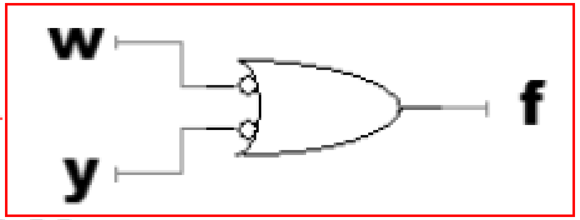
$$F = a'd + c'd$$

A POS K-Map

- On a POS K-map, the procedure is the same, except that we map 0’s.

	$y+z$	$y+\bar{z}$	$\bar{y}+\bar{z}$	$\bar{y}+z$
$w+x$	0000 0 0	0001 1 1	0011 3 3	0010 2 2
$w+\bar{x}$	0100 4 4	0101 5 5	0111 7 7	0110 6 6
$\bar{w}+\bar{x}$	1100 C 12	1101 D 13	1111 0 F 15	1110 0 E 14
$\bar{w}+x$	1000 8 8	1001 9 9	1011 0 B 11	1010 0 A 10

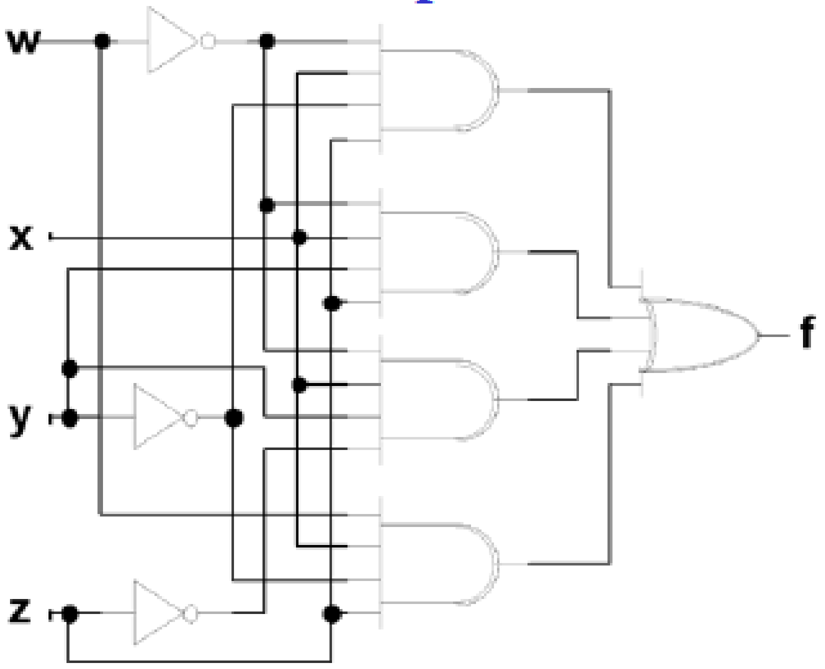
- The simplified expression is: $f = (\bar{w} + \bar{y})$. The simplified circuit is shown at right.



Exercise 4

A truth table and its Boolean expression are shown below, along with the circuit of this unsimplified expression. Use the K-map on the following page to simplify and draw the simplified circuit.

w	x	y	z	f
0	0	0	0	
0	0	0	1	
0	0	1	0	
0	0	1	1	
0	1	0	0	
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	
1	0	0	1	
1	0	1	0	
1	0	1	1	
1	1	0	0	
1	1	0	1	1
1	1	1	0	
1	1	1	1	



Original Circuit

$$f = \overline{w}x\overline{y}z + \overline{w}xy\overline{z} + \overline{w}xyz + wx\overline{y}z$$

Determine the minimum expression for each K-map in figure below:

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	1	1	1	1
$\overline{A}B$	1	1	0	0
AB	0	0	0	1
$A\overline{B}$	0	1	1	0

	$\overline{C}\,\overline{D}$	$\overline{C}D$	CD	$C\overline{D}$
$\overline{A}\,\overline{B}$	1	0	1	1
$\overline{A}B$	1	0	0	1
AB	0	0	0	0
$A\overline{B}$	1	0	1	1

Use a K-map to simplify each expression to a minimum SOP form:

(a) $\overline{A}\,\overline{B}\,\overline{C} + A\overline{B}C + \overline{A}BC + AB\overline{C}$ $\quad$ $Ans: No\ Simplification$

(b) $AC[\overline{B} + B(B + \overline{C})]$ $\quad$ $Ans: AC$

(c) $DE\overline{F} + \overline{D}E\overline{F} + \overline{D}\,\overline{E}\,\overline{F}$ $\quad$ $Ans: \overline{D}\,\overline{F} + E\overline{F}$