

Boolean Algebra

Lecture 2

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Introduction

- Boolean Algebra is a branch of mathematics for digital logic
- Founded by George Boole
- Used in computers and digital systems

Basic Symbols

- Variables (A, B, C.....) \rightarrow Its value is 0 or 1
- AND (\cdot)
- OR (+)
- NOT (')

Basic Laws

- Identity Law: $A+0=A$, $A\cdot 1=A$
- Null Law: $A+1=1$, $A\cdot 0=0$
- Idempotent Law: $A+A=A$, $A\cdot A=A$
- Complement Law: $A+A'=1$, $A\cdot A'=0$
- Commutative Law : $A+B=B+A$, $A\cdot B = B\cdot A$
- Associative Law : $A+B+C=(A+B)+C$, $A\cdot B\cdot C=(A\cdot B)\cdot C$
- Distributive Law : $A\cdot (B+C)=A\cdot B+A\cdot C$, $A+(B\cdot C)=(A+B)\cdot (A+C)$
- De Morgan's Theorem : $(A+B)'=A'\cdot B'$, $(A\cdot B)'=A+B'$

Example

EX 1:

$$F = A + AB$$

We take out a common factor

$$A(1+B) \Rightarrow 1+B=1$$

$$\text{So } F = A$$

EX 2 :

$$F = AB + AB'$$

$$A(B + B') \Rightarrow B+B'=1$$

$$\text{So } F = A$$

Example

EX 3:

$$F = A(A' + B)$$

$$AA' + AB \Rightarrow A.A' = 0$$

So $F = AB$

EX 4 :

$$F = AB + BC(B + C)$$

$$AB + \cancel{BBC} + \cancel{BCC}$$

$$AB + BC + \cancel{BC}$$

$$B(A + C)$$

So $f = B(A + C)$

Example

EX 5:

$$F = AA+AC+AB+BC$$

$$A+AC+AB+BC$$

$$A(1+C+B)+BC$$

$$\text{So } F = A+BC$$

Example

EX 6 :

$$AB + A(A + C) + B(A + C)$$

$$AB + AA + AC + AB + C$$

$$AB + A + AC + C$$

$$A(B + 1 + C) + C$$

$$\text{So } F = A + C$$

Example

EX 7:

$$\begin{aligned} F &= ((CD)' + A')' + ((C'D + A'))' \\ &= C' + D' + A \cdot (C + D') \cdot A \\ &= (C' + D' + A) \cdot (AC + AD') \\ &= ACC' + ACD' + AAC + AD'C' + A D'D' + AAD' \\ &= ACD' + AC + AD'C' + AD' \\ &= AC(D' + 1)(AD'C + AD') \\ &= AC + AD'(C + 1) \\ &= AC + AD' \\ &= A(C + D') \end{aligned}$$

Thank you