تلخيص

تلخيص مادة السيجنال (فيرست)

ملخص طالب

لجنة ICE

$\boxed{1}$

الإثنين
1/3

Definitions:

* What is signal ?

- Physical quantities carry information changes respect to time.

\# Signals can be represent as function of time.

* The mathematical representation of signal contains:

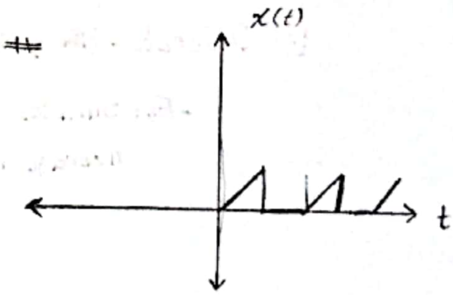
- The expression: $s(t)$
- The time support: $t_1 \le t \le t_2$

Signal Classifications:

$\boxed{1}$ Continuous-time v.s. discrete-time signals

Continuous-time: - If the signal is defined over continuous-time, then the signal is a continuous-time signal

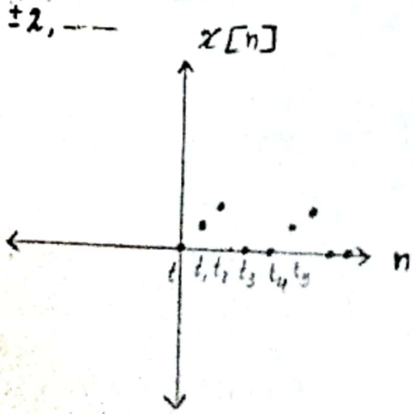
\# أي أنها متصلة في الوقت (معرفة في جميع الأوقات)



Discrete-Time: - If time t can only take discrete values, such as:

$$t = K T_s \quad ; \quad k = 0, \pm 1, \pm 2, ---$$

\# أي أنها معرفة في فترة زمنية معين تتكرر بها (معدل أخذ العينة)



\# We can repesent it in three way:

$\boxed{1}$ $x(nt) = \{0, 1, 3, \underset{\uparrow}{0}, 2, 5, 4\}$ (with $t, 2t, 3t, 4t$ marked above $1, 3, 0, 2$)

$\boxed{2}$ $x[n] = \left\{1, \frac{1}{2}, \frac{1}{4}, \dots, \left(\frac{1}{2}\right)^n\right\} = \begin{cases} \left(\frac{1}{2}\right)^n & n \ge 0 \\ 0 & n < 0 \end{cases}$

$\boxed{3}$

x[n]	0	2	0	3.5	0.5	-1
n	0	1	2	3	4	5

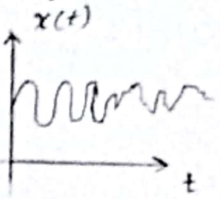
2

2 Analog v.s. Digital Signal

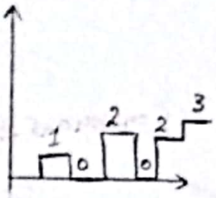
- In this classification we foucas in the both, Amplitude and Time, there are 4 types:

Continuous Time

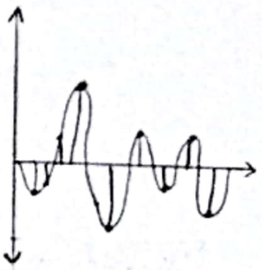
1. Continuous-time, continuous amplitude (Analog signal)
   - Ex: Speech signal.



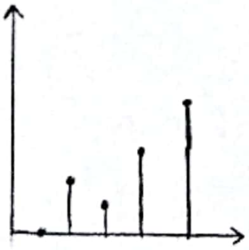
2. Continuous-time, discrete amplitude
   - Ex: traffic light.



3. Discrete-time, continuous amplitude
   - Ex: Samples of analog signal, Average monthly temperature.



4. Discrete-time, discrete amplitude (Digital signal)
   - Ex: Telegraph, text, roll a dice.



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3 Even v.s odd Signal

$$x(t) = \overset{\text{even}}{x_e(t)} + \overset{\text{odd}}{x_o(t)}$$

$$x_e(t) = \frac{1}{2}[x(t) + x(-t)]$$

$$x_o(t) = \frac{1}{2}[x(t) - x(-t)]$$

مهم

$x(t)$ is even when:

$$x(t) = x(-t)$$

$x(t)$ is odd when:

$$x(t) = -x(-t)$$

Signals

الاربعاء

8/3/2023

periodic vs. Nonperiodic

$$x(t) = A_0 \cos(\omega_0 t + \theta)$$

$$x(t) = A_0 \cos(2\pi f_0 t + \theta)$$

$A_0 \to$ Amplitude, $f_0 \to$ frequency $\to$ Hz

$$f_0 = \frac{1}{T_0}$$

Hz = 1 cycle / s

$\omega_0 = 2\pi f_0$ (rad/s)

$$\cos \omega_0 t = \frac{1}{2} e^{j\omega t} + \frac{1}{2} e^{-j\omega t}$$

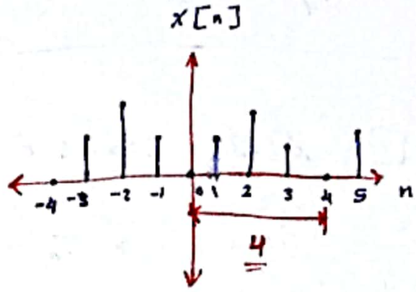
$$\sin \omega_0 t = \frac{1}{2} e^{j\omega t} - \frac{1}{2} e^{-j\omega t}$$

$$x[n] = x[n+N]$$

$$x[0] = x[4] = x[0+4]$$

$$x[1] = x[5] = x[1+4]$$

$\boxed{N = 4}$ هذا يعني أنها تتكرر كل



$$x(t) = \cos(\omega_0 t + \theta)$$

$$\frac{1}{f_0} = T_0 = \frac{2\pi}{\omega_0}$$

$$x(t) = x(t+T_0) \longrightarrow \cos(\omega_0 t + \theta) = \cos(\omega_0 (t+T_0) + \theta)$$

$$= \cos(\omega_0 t + \theta + n\omega_0 T_0)$$

$$\omega_0 T_0 = 2n\pi$$

$$T_0 = \frac{n 2\pi}{\omega_0}$$

$x_1(t) = \cos(3.5t) \to \omega_1 = \frac{7}{2} \to T_1 = \frac{4\pi}{7}$

$x_2(t) = \sin(2t) \longrightarrow \omega_2 = 2 \to T_2 = \frac{2\pi}{\omega_0} = \pi$

$x_3(t) = 2\cos(\frac{7t}{6}) \to \omega_3 = \frac{7}{6} \to T_3 = \frac{12\pi}{7}$

$\boxed{5}$ $\boxed{2}$

$$x_4(t) = x_1(t) + x_2(t)$$
$$= \cos(3.5t) + \sin(2t)$$

$$\frac{T_1}{T_2} = \frac{\frac{4\cancel{\pi}}{7}}{\cancel{\pi}} = \frac{4}{7}$$

$$w_5 = 5\pi \longrightarrow T_5 = \frac{2\pi}{5\pi} = \frac{2}{5}$$

$$x_5(t) = x_1(t) + x_4(t)$$

$$\frac{T_1}{T_4} = \frac{\frac{4\pi}{7}}{\frac{2}{5}} = \frac{20\pi}{14}$$

$\boxed{1}$ $x(t) = \cos(t + \frac{\pi}{4})$

$$w_0 = 1 \longrightarrow T_0 = \frac{2\pi}{w_0} = 2\pi$$

$\boxed{2}$ $x_1(t) = \sin \frac{2\pi}{3} t$

$$w_0 = \frac{2\pi}{3} \longrightarrow T_0 = \frac{2\pi}{w_0} = \frac{2\cancel{\pi}}{\frac{2\cancel{\pi}}{3}} = 3$$

$\boxed{3}$ $x_2(t) = \sin \frac{\pi}{4} t$

$$w_0 = \frac{\pi}{4} \longrightarrow T_0 = \frac{2\pi}{w_0} = \frac{2\cancel{\pi}}{\frac{\cancel{\pi}}{4}} = 8$$

$+ \longrightarrow x(t) = \sin\frac{2\pi}{3} + \sin\frac{\pi}{4}$

$$\frac{T_1}{T_2} = \frac{3}{8} \longrightarrow 8T_1 = 3T_2$$
$$24 = 24$$

\# Power vs. Energy

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt \qquad 0 < E < \infty$$

$$P = \lim_{T\to\infty} \frac{E}{T} = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} |x(t)|^2 \, dt \qquad 0 < P < \infty$$

for Period Signal $\longrightarrow$

$$P_{av} = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$
$$= \frac{1}{2} \int_{0}^{T} |x(t)|^2 \, dt$$

3

$$x(t) = e^{-at} u(t)$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_{0}^{\infty} |x(t)|^2 \, dt = \int_{0}^{\infty} |e^{-at}|^2 \, dt = \int_{0}^{\infty} |e^{-2at}| \, dt$$

u(t) بلشت من 0 بسبب ← 0

حدودنا 0 ← ∞

$$= -\frac{1}{2a} e^{-2at} \Big|_0^{\infty} = -\frac{1}{2a} \left[\cancel{e^{-2a(\infty)}}^{\,0} - e^{-2a(0)} \right] = \frac{1}{2a}$$

$\underline{P_{av} = 0}$ → لأنه E محدودة

$$x(t) = A \cos \omega_0 t$$

$\cos^2 x = \frac{1}{2} + \frac{1}{2} \cos 2x$

$\sin^2 x = \frac{1}{2} - \frac{1}{2} \cos 2x$

$$= \frac{1}{T_0} \int_0^{T_0} A^2 \cos^2 \omega_0 t \, dt$$

$$= \frac{1}{T_0} \int_0^{T_0} A^2 \left[\frac{1}{2} + \frac{1}{2} \cos 2\omega_0 t\right] dt$$

$$= \frac{A^2}{T_0} \int_0^{T_0} \frac{1}{2} \, dt + \int_0^{T_0} \frac{1}{2} \cos 2\omega_0 t \, dt$$

$$= \frac{A^2}{T_0} \left[\frac{1}{2} t\right]_0^{T_0} + \frac{1}{4\omega_0} \sin 2\omega_0 t \Big|_0^{T_0}$$

if $T_0 = 2\pi / \omega_0$

$$= \frac{A^2 \cancel{\omega_0}}{\cancel{2\pi}} \left(\frac{1}{2} \cdot \frac{\cancel{2\pi}}{\cancel{\omega_0}} - 0\right) + \left(\cancel{\frac{1}{4\omega_0} \sin 2\omega_0 \frac{2\pi}{\omega_0}}^{\,0} - 0\right)$$

→ $\sin(0) = 0$

$\sin(4\pi) = 0$

Signal

الاثنين

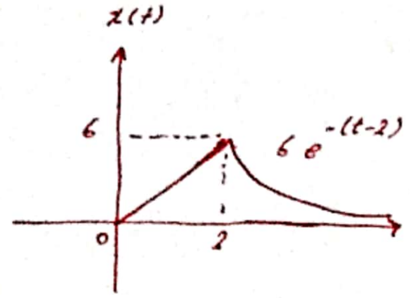
13/3/2023

* Power vs. Energy signal

if the signal is sinusoidal :

$$P_{av} = \frac{A^2}{2}$$

Ex:



$$x(t) = \begin{cases} 3t & 0 \le t \le 2 \\ 6e^{-(t-2)} & 2 \le t \\ 0 & \text{o.w} \end{cases}$$

$$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

$$= \int_0^2 (3t)^2 \, dt + \int_2^{\infty} (6e^{-(t-2)})^2 \, dt$$

$$= \int_0^2 9t^2 \, dt + \int_2^{\infty} 36 e^{-2t+4} \, dt = \frac{9t^3}{3}\Big|_0^2 + 36 \int_2^{\infty} e^{-2t} \cdot e^4 \, dt$$

$$= \frac{9t^3}{3}\Big|_0^2 + 36 e^4 \int_2^{\infty} e^{-2t} \, dt = \frac{9}{3}[8-0] + 36 \cdot e^4 \cdot \frac{1}{2} e^{-2t}\Big|_{\infty}^{2}$$

$$= 24 + \overset{18}{\cancel{36}} \cdot \frac{\cancel{e^4}}{\cancel{2}} [\cancel{e^{-4}}] = 42 \text{ J}$$

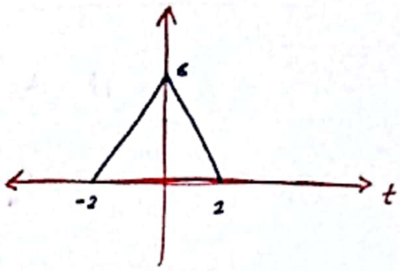
Ex: $x_2(t) = 4\cos\left(\frac{2\pi}{10} t\right)$ $\qquad P_{av} = \frac{(4)^2}{2} = 8 \text{ W}$

$$P_{av} = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} (x(t))^2 \, dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} 16 \cos^2\left(\frac{2\pi}{10} t\right) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} 16 \left(\frac{1}{2} + \frac{1}{2}\cos\left(\frac{4\pi}{T_0}\right)\right) dt$$

$=$

Signal Time Transformation:

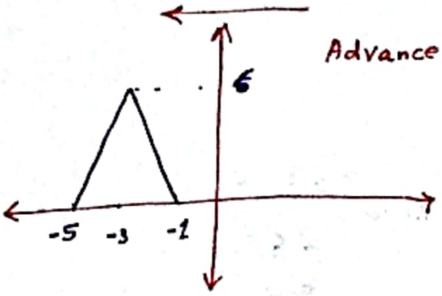
$x(t)$



* Time-shifting:

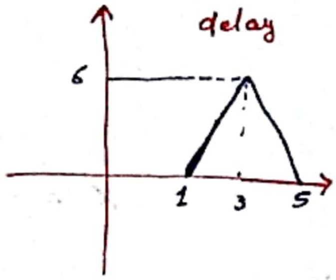
$y(t) = x(t+3)$

Advance



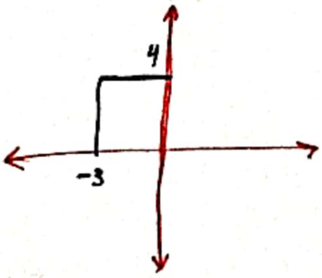
$y(t) = x(t-3)$

delay

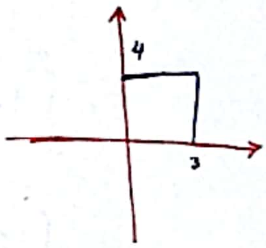


* Time-Reversal:

$y(t) = x(-t)$

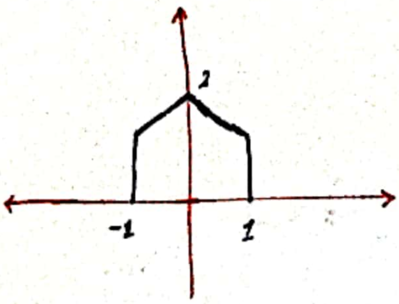


$x(t)$



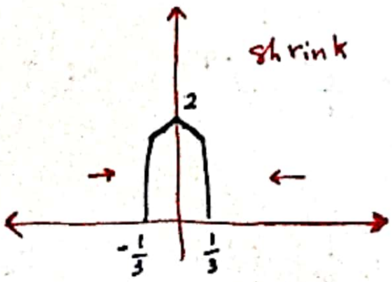
* Time-scaling:

$x(t)$



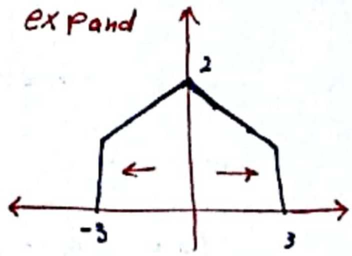
$x(3t)$

shrink



$x(\frac{1}{3}t)$

expand



Signal

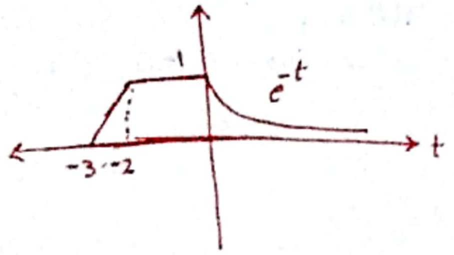
الأربعاء

15/3/2023

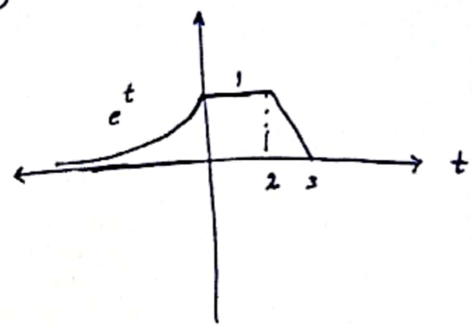
operations:

* Combined Transformation:

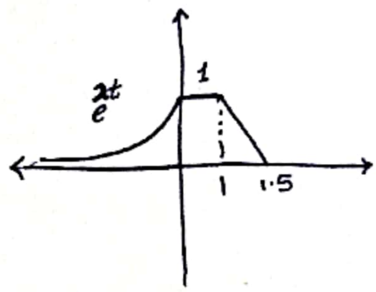
Ex: $x(t)$ Find $y(t) = x(-2t+6) = x(-2(t-3))$



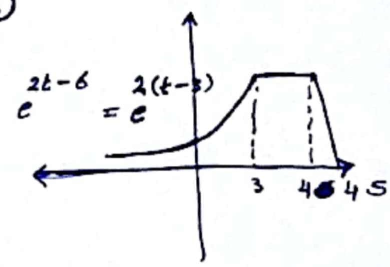
① $x(-t)$



② $x(-2t)$



③ $y(t)$



نفس للمعادلة فقط بدون رسم

$$x(t) = \begin{cases} t+3 & -3 \le t \le -2 \\ 1 & -2 \le t \le 0 \\ e^{-t} & 0 \le t \end{cases}$$

① $$x(-t) = \begin{cases} -t+3 & 3 \ge t \ge 2 \\ 1 & 2 \ge t \ge 0 \\ e^{t} & t \le 0 \end{cases}$$

② $$x(-2t) = \begin{cases} -2t+3 & 1.5 \ge t \ge 1 \\ 1 & 1 \ge t \ge 0 \\ e^{2t} & t \le 0 \end{cases}$$

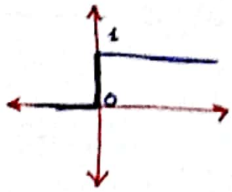
③ $$x(-2(t-3)) = \begin{cases} -2(t-3)+3 & 1 \le t-3 \le 1.5 \\ 1 & 0 \le t-3 \le 1 \\ e^{2(t-3)} & t-3 \le 0 \end{cases}$$

$$= \begin{cases} -2t+9 & 4 \le t \le 4.5 \\ 1 & 0 \le t \le 4 \\ e^{2t-6} & t \le 3 \end{cases}$$

* Elementary signals:

$\boxed{1}$ Unit step function: ← تستخدم في تحديد $0 \le t \le \infty$

$$u(t) = \begin{cases} 1 & t \ge 0 \\ 0 & \text{o.w} \end{cases}$$



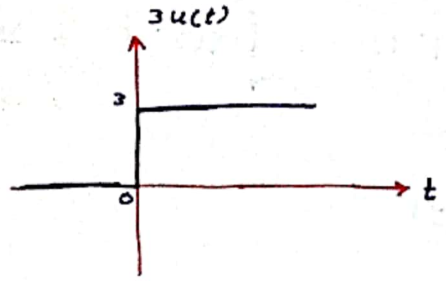
Signal

الاثنين

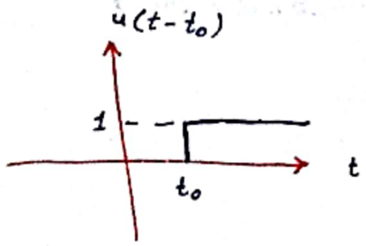
2013

* Elementary signals :

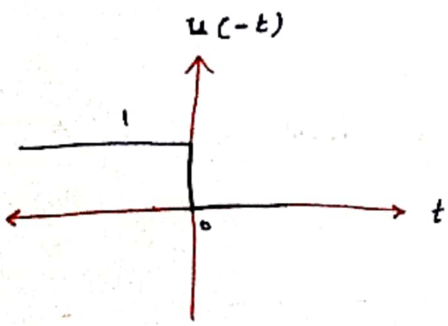
A) Unit step function :



$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$



$$u(t - t_0) = \begin{cases} 1 & t \geq t_0 \\ & t < t_0 \end{cases}$$



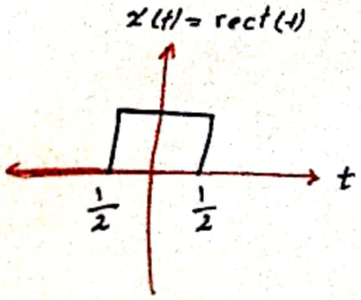
$$u(-t) = \begin{cases} 1 & t \leq 0 \\ 0 & t > 0 \end{cases}$$

$$x(t) = \begin{cases} \cos 3t & 0 \leq t \\ 0 & \text{O.W} \end{cases}$$

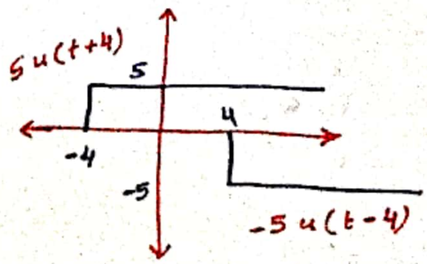
$\downarrow$

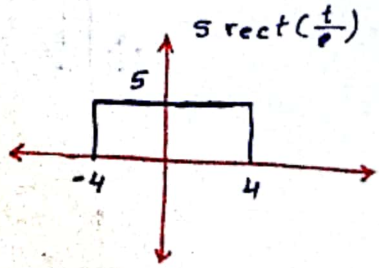
$x(t) = \cos 3t \; u(t) \longrightarrow$ تجعل الزمن من 0 إلى ∞

B) Pulse wave , rect function , $\sqcap(t)$



$$x(t) = 5 \,\text{rect}\left(\frac{t}{8}\right) = 5\,u(t+4) - 5\,u(t-4)$$





Find E or P_{av} for signal give as :

① $x(t) = 5 \text{ rect}\left(\frac{t}{8}\right)$

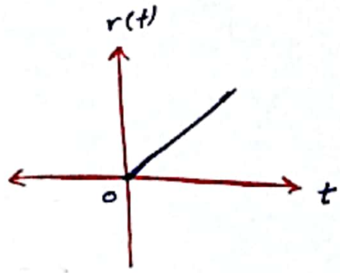
$$E = \int_{-4}^{4} |x(t)|^2 \, dt = \int_{-4}^{4} (5)^2 \, dt = 25\,[4-(-4)]$$
$$= 25 \times 8 = 200 \text{ J}$$

② $x(t) = 5 \text{ rect}\left(\frac{t-3}{8}\right)$

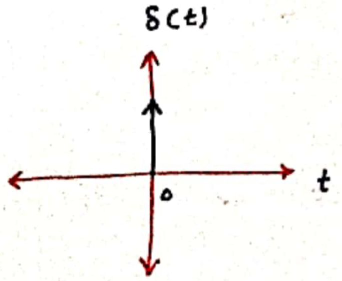
$$E = \int_{-1}^{7} (5)^2 \, dt = 25 \times 8 = 200 \text{ J}$$

C) ramp function .

$$r(t) = t\,u(t)$$
$$r(t) = \int_{-\infty}^{t} u(\tau)\, d\tau = t\,u(t)$$



D) Impulse function, delta function :

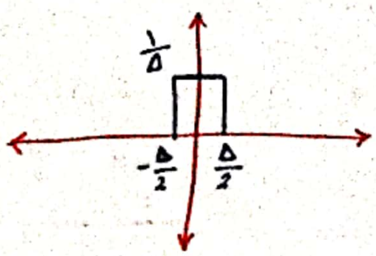


$$\delta(t) = \begin{cases} \infty & t = 0 \\ 0 & \text{o.w} \end{cases}$$

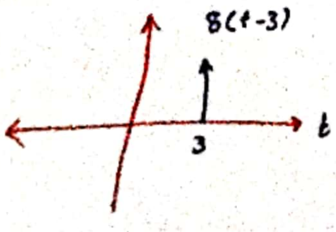
even function

$$\delta(t) = \frac{du(t)}{dt}$$

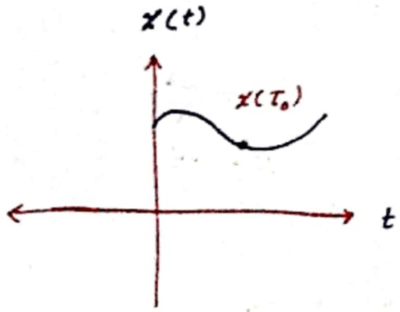
$$\int_{-\infty}^{\infty} \delta(t)\, dt = 1$$

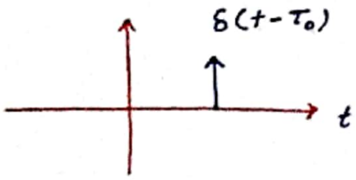


$$\int_{1}^{t} \delta(t-3)\, dt = 1$$



$$\int_{1}^{2} \delta(t-3)\, dt = 0$$





sampling

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-T_0)\,dt = x(T_0)$$

$$\int_{-\infty}^{\infty} x(t)\,\delta(t-T_0)\,dt = x(T_0)\underbrace{\int_{-\infty}^{\infty} \delta(t-T_0)\,dt}_{1}$$

$$\delta(at) = \frac{1}{|a|}\,\delta(t) \longrightarrow \text{scalling}$$

Ex_1: $$x(t) = \int_{1}^{2} t^2\,\delta(2t-3)\,dt$$

$$x(t) = \int_{1}^{2} t^2\,\delta(2(t-1.5))\,dt = \frac{1}{2}\,t^2\Big|_{1.5} = \frac{1}{2}(1.5)^2 = \frac{9}{8}$$

Ex_2: $$\int_{-1}^{1} (3t^2+1)\,\delta(t)\,dt$$

$$= (3t^2+1)\Big|_{t=0} = 1$$

Ex_3: $$\int_{1}^{2} (3t^2+1)\,\delta(t)\,dt$$

$$= (3t^2+1)\Big|_{t=0} = 0$$

Ex_4: $$\int_{-\infty}^{\infty} e^{-t}\cdot\delta(2t-2)\,dt$$

$$= \frac{1}{2}\,e^{-t}\Big|_{t=1} = \frac{1}{2}\,e^{-1}$$

E) Complex Exponitial ,

$$x(t) = e^{j\omega_0 t} = \cos \omega_0 t + j \sin \omega_0 t$$

$$\omega_0 = \frac{2\pi}{T_0}$$

$$T_0 = \frac{2\pi}{\omega_0}$$

Real Exponitial :

$$x(t) = e^{at}$$

decreasing $\rightarrow -a$

increasing $\rightarrow a$

General Exponitial,

$$x(t) = e^{st} = e^{(\sigma + j\omega)t} = e^{\sigma t} \cdot e^{j\omega t} = e^{\sigma t}(\cos \omega t + j \sin \omega t)$$

$\boxed{15}$

signal

الاربعاء

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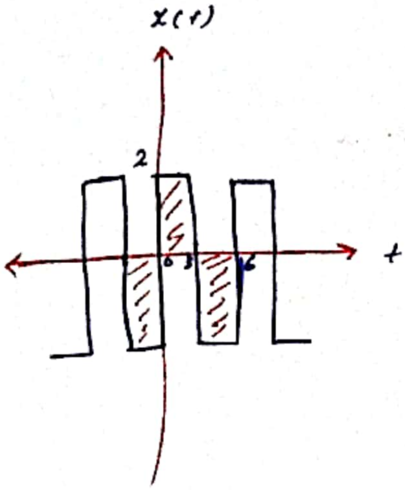
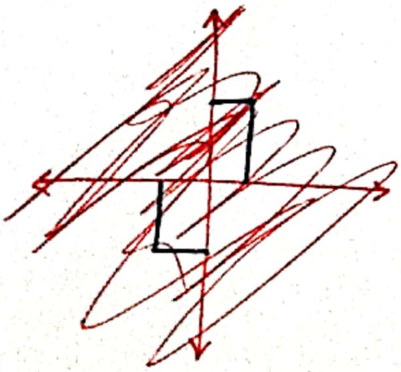
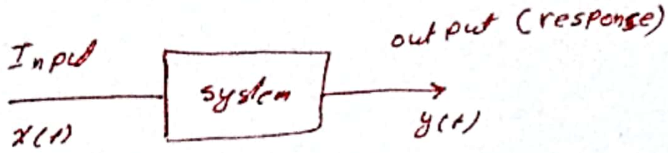
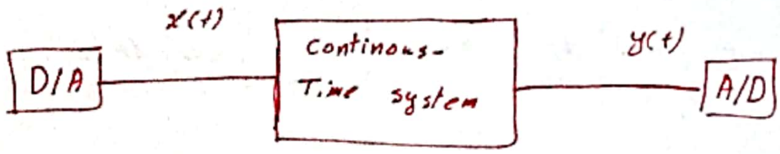


$$\frac{1}{6}\left(\int_{-3}^{0}(-2)^2\,dt + \int_{0}^{3}(2)^2\,dt\right) \quad \text{or} \quad \frac{1}{6}\left(\int_{0}^{3}(2)^2\,dt + \int_{3}^{6}(-2)^2\,dt\right)$$



System Classifications :





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* **Classification of systems:**

(A) linear V.S. non-linear

(B) Time invariant V.S. Time variant (varying)

(C) Dynamic V.S. static
(memory) (memoryless)

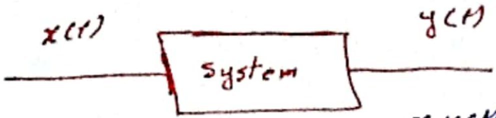
(D) Causal V.S Non-Causal

(E) Invertibil V.S Non-Invertibil

(F) Stable V.S unstable

Linear V.S. Non-linear:

Ⓐ



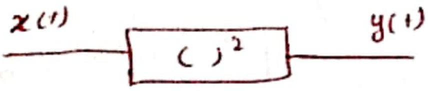
$a_1 x_1(t) \longrightarrow a_1 y_1(t)$

$a_2 x_2(t) \longrightarrow a_2 y_2(t)$

$a_1 x_1(t) + a_2 x_2(t) \longrightarrow a_1 y_1(t) + a_2 y_2(t)$

#يجب أن يكون مجموع الإشارات هو نفسه وأن يكون له الScalling في المخرجات بنفسه مقدره في المدخلات

Ex_1:



$a_1 x_1(t)$ $\qquad$ $a_1 y_1(t)$

$a_2 x_2(t)$ $\qquad$ $a_2 y_2(t)$

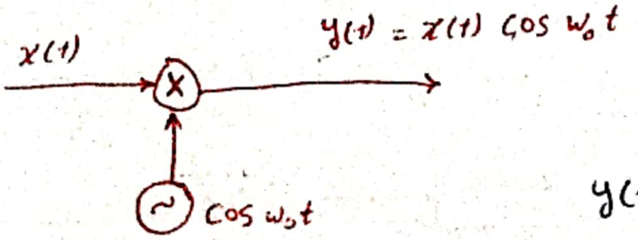
① $a_1 x_1(t) \xrightarrow{(\ )^2} a_1 y_1(t) = a_1^2 x_1^2(t)$

② $a_2 x_2(t) \xrightarrow{(\ )^2} a_2 y_2(t) = a_2^2 x_2^2(t)$

③ $a_1 x_1(t) + a_2 x_2(t) \xrightarrow{(\ )^2} a_1 y_1(t) + a_2 y_2(t) = a_1^2 x_1^2(t) + 2 a_1 a_2 x_1(t) x_2(t) + a_2^2 x_2^2(t)$

ليست linear

Ex_2: Amplitude Modulation:



$$y(t) = [a_1 x_1(t) + a_2 x_2(t)] \cos w_0 t$$

$$= a_1 x_1(t) \cos w_0 t + a_2 x_2(t) \cos w_0 t$$

$$= a_1 y_1(t) + a_2 y_2(t)$$

إنه linear

Ex_3: $y(t) = \frac{1}{T} \int_{t-T}^{t} x(\tau)\, d\tau + B$

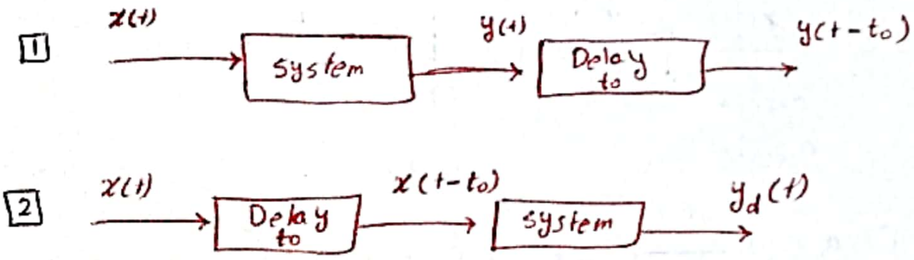
linear ليست #

Ex_4: $y(t) = |x(t)|$

linear ليست #

(B) Time-Varying V.S. Time-Invariant

إذا تم حدوث shift في المدخلات يجب حدوث نفسه في المخرجات



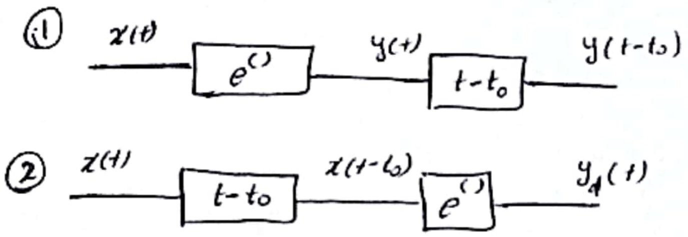
$y_d(t) = y(t-t_0)$ إذا تم تحقيق المعادلة فإن ال system, Time-Invariant

Ex_1: $y(t) = e^{x(t)}$

① $y_1(t) = e^{x(t-t_0)}$

② $y_2(t) = e^{x(t-t_0)}$

③ $y_1(t) \overset{?}{=} y_2(t)$ ✓



Time-Invariant إنه #

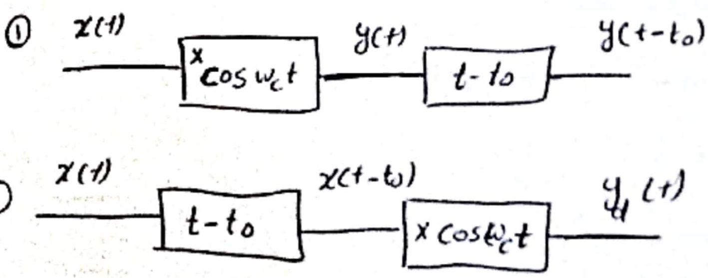
Ex_2: Amplitude Modulation

$y(t) = x(t) \cos \omega_c t$

① $y_1(t) = x(t-t_0) \cos \omega_c (t-t_0)$

② $y_2(t) = x(t-t_0) \cos \omega_c t$

③ $y_1(t) \overset{?}{=} y_2(t)$ X



Time-Varying إنه #

Signal

الاثنين

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#. w

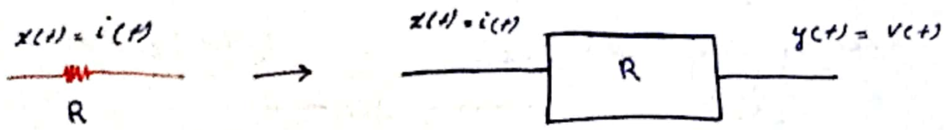
$$E = \int_{3}^{6} (3t-9)^2 \, dt = 81 J$$

$$P_{av} = \lim_{T \to \infty} \frac{E}{T} = 0$$

$$= \lim_{T \to \infty} \frac{81}{T} = 0$$

C # Memory V.S. Memoryless:

Memoryless → يعتمد t في اللحظة معينة



$$V(t) = R\, i(t)$$

$$y(t) = R\, x(t)$$

Memory: → يعتمد t في الماضي

$$y(t) = \sum_{i=0}^{N} a_i \, x(t - T_0)$$

$$y(5) = \sum_{i=0}^{N} a_i \, x(5 - T_0)$$

D # Causal V.S. Non causal

كل الإشارات الفيزيائية Causal

* Causal:

$$y(t_0) = \frac{1}{2} x^2(t_0) + x(t_0)$$

$$y(t_0) = \frac{4}{2} x(t_0 - 2) + x(t_0) \rightarrow \text{Memory}$$

كل Memoryless يعتبر Causal وليس

* Non causal:

$$y(t) = x(t+5)$$

يعتمد على المستقبل