

# Lecture 2: Linear Regression With One Variable

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## Model Representation

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### Notation:

$m$  = Number of training examples

$x$  = "input" variable / features

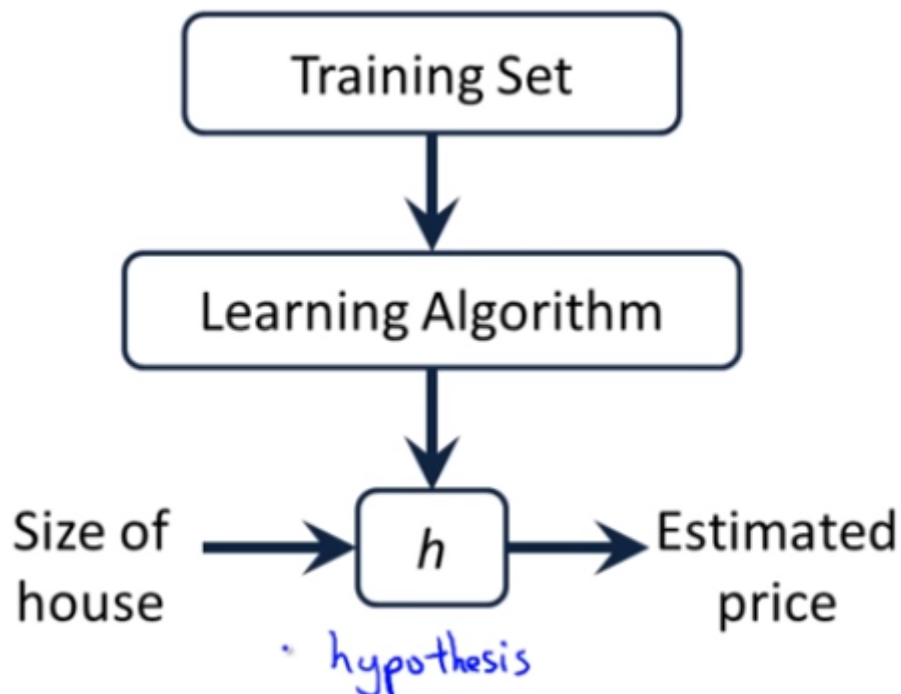
$y$  = "output" variable / "target" variable

$(x, y)$  = one training example

$(x^{(i)}, y^{(i)})$  =  $i^{\text{th}}$  training example

$h$  = hypothesis / function of  $x \rightarrow y$

$\theta$  = parameters of the model



$h$  is a function that maps from  $x$ 's to  $y$ 's

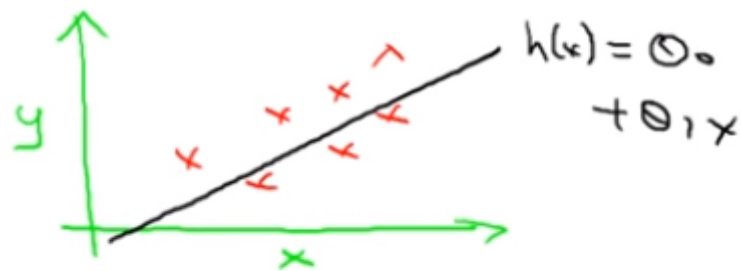
**How to represent  $h$ ?**

$$h_{\theta}(x) = \theta_0 + \theta_1 x$$

Shorthand for  $h_{\theta}(x) \rightarrow \mathbf{h}(x)$

$$\underline{h_{\theta}(x)} = \underline{\theta_0 + \theta_1 x}$$

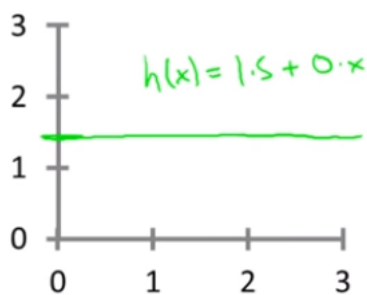
Shorthand:  $\underline{h(x)}$



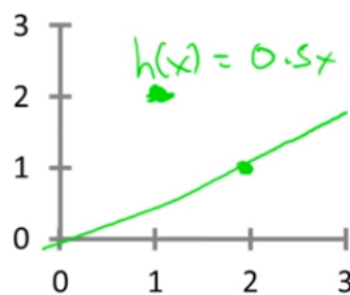
Linear regression with one variable. (x)  
Univariate linear regression.

In the above picture, an example of house prices. This is a Linear regression with one variable.

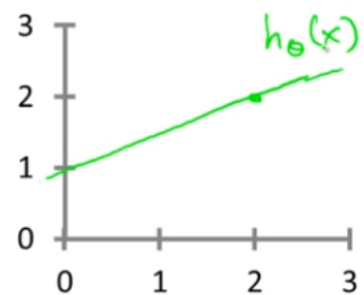
$$\underline{h_{\theta}(x) = \theta_0 + \theta_1 x}$$



$$\begin{aligned} \rightarrow \theta_0 &= 1.5 \\ \rightarrow \theta_1 &= 0 \end{aligned}$$



$$\begin{aligned} \rightarrow \theta_0 &= 0 \\ \rightarrow \theta_1 &= 0.5 \end{aligned}$$



$$\begin{aligned} \rightarrow \theta_0 &= 1 \\ \rightarrow \theta_1 &= 0.5 \end{aligned}$$

In the above picture, an example of how  $h_{\theta}(x)$  changes with  $\theta_0$  and  $\theta_1$

### Cost Function: To get the accurate prediction

Idea: Choose **training parameters** so that the **predicted value** is closer to the **actual answer** for our training examples  $(x,y)$ .

Or in other words: Choose  $\theta_0, \theta_1$  so that  $\mathbf{h_{\theta}(x)}$  is close to  $\mathbf{y}$  for our training examples  $(x,y)$ .

Minimization problem:

$$\begin{aligned} &\text{minimise } \theta_0, \theta_1 : (h_\theta(x) - y)^2 \\ &\text{minimise } \theta_0, \theta_1 : \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2 \\ &\text{minimise } \theta_0, \theta_1 : \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2 \\ &(\text{where, } h_\theta(x^{(i)}) = \theta_0 + \theta_1 x^{(i)}) \end{aligned}$$

**Note:** The  $\frac{1}{m}$  comes from taking average of the sum and the  $\frac{1}{2}$  comes from "making the math bit easier".

### **Rewriting this function cleanly: the conventional way**

$$J(\theta_0, \theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2$$

Cost Function is also called Squared error function.

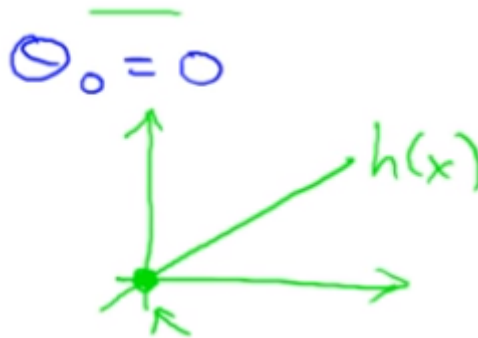
### **Why do we Square our error function?**

Because it is found to work really well with most problems and is widely used. Though there are more error functions/cost function

### **Understanding what the math expression means:**

- Taking  $\theta_1$  as 0:  
The hypothesis becomes:

$$h_\theta(x) = \theta_1 x$$



Here you see that  $\theta_0 = 0$  simply means that the  $h(x)$  **function passes through origin**.

- so,

$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^m (h_\theta(x^{(i)}) - y^{(i)})^2$$

- replacing  $h_\theta(x^{(i)})$

$$J(\theta_1) = \frac{1}{2m} \sum_{i=1}^m (\theta_1 x^{(i)} - y^{(i)})^2$$