M2 Masters Thesis

Options Portfolio Optimization under GARCH

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Introduction

- This thesis investigates the topic of portfolio selection/optimization, which originates with the seminal paper of Markowitz (1952).
- Standard portfolio selection optimizes a set of real weights on a basket of stocks
- I extend this topic through replacing a basket of stocks with a basket of options on the S&P500, and the use of a GARCH volatility model.

Problem Statement

- I use S&P500 options data from the Chicago Board of Exchange (CBOE) spanning from January 2019 to May 2021.
- Following Faias and Santa-Clara (2017), I use a constant relative risk aversion (CRRA) utility function to optimize next period wealth.
- Next period wealth is reformulated in terms of the weighted sum of simulated option returns.

Motivation

- In this research, I aim to set the theoretical modeling framework of an algorithm trading in the real markets.
- With this foundation, I employ portfolio selection techniques found in the literature, while diving further into the practical discussion.
- This paper can also be used a baseline for a more in depth comparison of different volatility or returns prediction models.

Previous Literature

- Markowitz (1952) employed a mean-variance model to select a portfolio of stocks.
- Multiple papers have addressed portfolio selection with options, including Zhao and Palomar (2018) and Faias and Santa-Clara (2017).
- This thesis builds off the work of Faias and Santa-Clara (2017), who approach portfolio selection with options using a returns simulation framework based on ad-hoc volatility estimation.

Methodology

I assume that the log returns follow a normal distribution with a time varying volatility term, which is modeled by a GARCH(1, 1) process. Formally:

$$
\begin{gathered}
y_t = \sigma_t \epsilon_t \quad \epsilon_t \sim iid.N(0,1) \\
\sigma_t^2 = \omega + \alpha y_{t-1}^2 + \beta \sigma_{t-1}^2 \\
\omega > 0 \ \ \alpha, \beta \geq 0
\end{gathered}
$$

Methodology

Formally, in this paper I optimize the wealth of the next period A_{t+1}

$$\max_{\mathbf{W}_t} E[U(A_{t+1})|F_t]$$

Where F_t as the available information at time t, E is the expectation operator. $\mathbf{W}_t$ is the vector of allocated weights. The optimization is done via a simulation of the next period wealth A^n_{t+1}. This returns a vector of optimized weights $\mathbf{W}^*_t$.

Where γ is the CRRA risk aversion parameter, the utility function U is:

$$U(A) = \begin{cases} \frac{1}{1-\gamma} A^{1-\gamma}, & \text{if } \gamma \neq 1 \\ \log(A), & \text{if } \gamma = 1 \end{cases}$$

Methodology

- Option returns are calculated under a hold until expiration strategy, wherein after the initial trade, the trader cannot further adjust the portfolio.
- The model uses a CRRA expected utility function rather than the traditional mean-variance framework.
- γ is set to 10, higher than 4 found in Bliss and Panigirtzoglou (2004), to induce shrinkage in $\mathbf{W}_t^*$.

Methodology

Standardized returns based on the fitted conditional volatility series are $\hat{\epsilon}_t = \frac{y_t}{\hat{\sigma}_t}$. Simulated log returns are generated using a bootstrapped distribution of historic $\hat{\epsilon}_t$.

$$\tilde{y}^n_{t+h} = \hat{\sigma}_{t+h}\tilde{\epsilon}^n_{t+h} \quad \forall n \in N \textit{ and } \forall h \in T - t$$

Simulated option returns are calculated from simluated prices $S^n_{t+h|t+h-1} = S_{t+h-1}e^{\tilde{y}^n_{t+h}}$.

$$C^n_{t+1|t}(k) = \max(S^n_{t+1|t} - k, 0) \qquad k = K_{t,c_1}, \cdots, K_{t,c_C}$$
$$P^n_{t+1|t}(k) = \max(k - S^n_{t+1|t}, 0) \qquad k = K_{t,p_1}, \cdots, K_{t,p_P}$$
$$r^n_{t+1|t,c}(k) = \frac{C^n_{t+1|t}(k)}{C_{t,k}} - 1 \qquad k = K_{t,c_1}, \cdots, K_{t,c_C}$$
$$r^n_{t+1|t,p}(k) = \frac{P^n_{t+1|t}(k)}{P_{t,k}} - 1 \qquad k = K_{t,p_1}, \cdots, K_{t,p_P}$$

Methodology

Simulated next period wealth is formulated as current wealth times the simulated total return of the basket of options, dependent on the choice variable $\mathbf{W}_t$

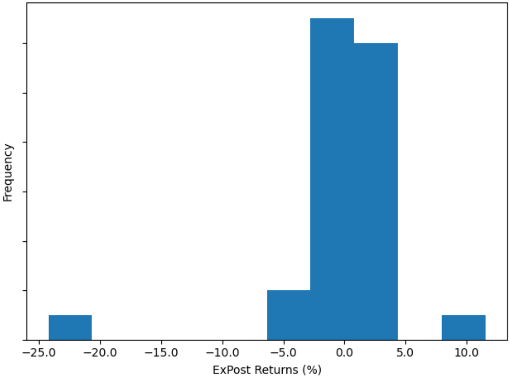
$$A^n_{t+1|t} = A_t(1 + rp^n_{t+1|t}(\mathbf{W}_t))$$

This quantity is used to optimize $\mathbf{W}_t$ based on the empirical mean of N simulations, which is independent of current wealth A_t.

Results

Figure:
Portfolio Optimization Returns

Histogram of portfolio optimization returns is shown. The time period is January 2019 to May 2021.

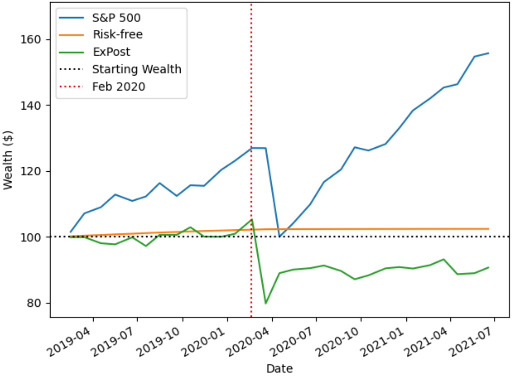


Results

Figure:
ExPost Results vs. S&P500

Summary of returns, portfolio optimization vs. S&P500, along with graph of cumulative returns including risk-free asset. Mean, Std, Min, and Max are percentages. Period spans from January 2019 to May 2021.

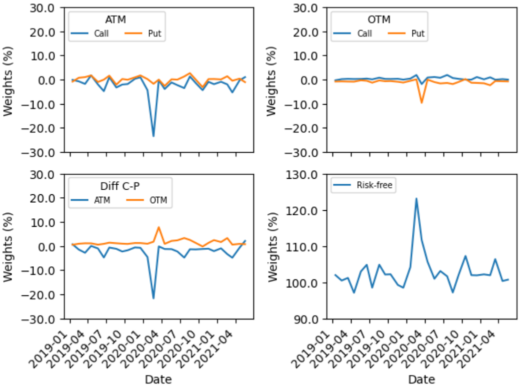
	Mean	Std	Min	Max	Skew	Kurtosis	SR
S&P 500	1.7	5.1	-21.2	6.1	-3.41	13.10	0.33
ExPost	-0.2	5.4	-24.2	11.5	-2.75	11.83	-0.03



Results

Figure:
Portfolio Optimization Weights by Contract

Optimized weights from portfolio optimization are shown by option type, as well as for the call-put difference and risk-free assset. The time period is January 2019 to May 2021.



Conclusion

- Portfolio optimization returns show a sharpe ratio of -0.03 vs. 0.33 for the underlying S&P500 in the time period.
- The ex-post returns do, however, show a lower skew and kurtosis, meaning the returns are closer to normality, even with the Covid months.
- Removing February and March of 2020 results in a sharpe ratio of 0.14. These months represent 6% of the sample, so a larger sample with less weight on these months may impact results.

Thank you for listening!

References

Harry Markowitz. Portfolio selection. *The Journal of Finance*, 7(1):77–91, March 1952.

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