TPSS/GEOG 680
Calculating semivariograms

The general equation for calculating semi-variances is:

$$\gamma_h = \frac{1}{2N}\sum_{i=1}^{n}(X_i - X_{i+h})^2 \quad (1)$$

where:

γ_h = *semivariance at separation distance h*
N = *number of pairs*
X_i = *measured value at location i*
X_{i+h} = *measured value at location i+h*

We'll illustrate the calculation of semivariograms / variograms using equation 1 as follows with the data set in Table 1.

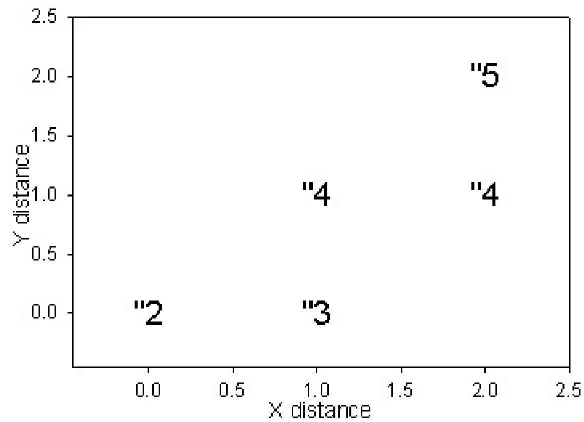

Figure 1. Spatial plot of data shown in Table 1. Symbols "2 means a Z value of 2.

Table 1. Coordinates and measured data plotted in Figure 1.

Coordinates		
X	Y	Z
0	0	2
1	0	3
1	1	4
2	1	4
2	2	5

Number of pairs = N(N-1)/2 = 5(4)/2 = 10
Using equation (1), we calculate the semivariance (eq (1)) for each pair of data:

Difference	Semivariance	Distance
1. X(0),Y(0) - X(1),Y(0) = $(2-3)^2/2$	= 0.5	1.0
2. X(0),Y(0) - X(1),Y(1) = $(2-4)^2/2$	= 2	1.4
3. X(0),Y(0) - X(2),Y(1) = $(2-4)^2/2$	= 2	2.2
4. X(0),Y(0) - X(2),Y(2) = $(2-5)^2/2$	= 4.5	2.8
5. X(1),Y(0) - X(1),Y(1) = $(3-4)^2/2$	= 0.5	1.0
6. X(1),Y(0) - X(2),Y(1) = $(3-4)^2/2$	= 0.5	1.4
7. X(1),Y(0) - X(2),Y(2) = $(3-5)^2/2$	= 2	2.2
8. X(1),Y(1) - X(2),Y(1) = $(4-4)^2/2$	= 0	1.0
9. X(1),Y(1) - X(2),Y(2) = $(4-5)^2/2$	= 0.5	1.4

10. X(2),Y(1) - X(2),Y(2) = $(4-5)^2/2 = 0.5$ 1.0

Table 2. Summary of the calculation of semivariances and distances associated with each semivariance – data from Figure 1 and Table 1.

Distance	Semivariances	# pairs	mean semivariance
1	0.5, 0.5, 0, 0.5	4	0.375
1.4	2, 0.5, 0.5	3	1.0
2.2	2, 2	2	2.0
2.8	4.5	1	4.5

Thus using equation 1 we have calculated the semivariances for each of the pairs of points illustrated in Figure 1. The semivariances are summarized by distance and plotted versus the distance as shown in Figure 2. This is the "variogram" or "semivariogram." of the data shown in Figure 1. The actual number of points is too small to see the typical features, but still one can see that samples close together tended to have smaller semivariances (higher "correlation").

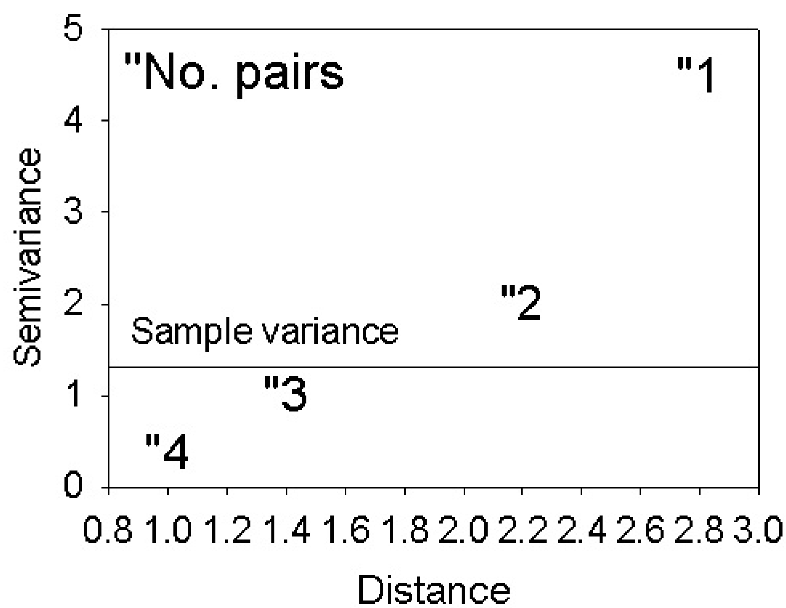

Figure 2. Plot of Semivariance versus Distance for the data in Table 1. Symbol "4 indicates that there were 4 pairs averaged for the estimated semivariance of 0.375.

With a larger number of sample points, more reliable variograms can be obtained and then a variety of models are fit to the variograms so the variogram information can be used in sample interpolation and extrapolation. We'll discuss how the model of the variogram is used in interpolation when we discuss the steps, assumptions, and procedures of "kriging."