# 5. feladatsor: Komplex számok algebrai és trigonometrikus alakja

## 1. feladat

Végezzük el a következő műveleteket a komplex számok halmazán.

$\sqrt{-16}$ $\sqrt{-25}$ $(2i)^2$ $2i+5i$ $\frac{4i}{2i}$

$\sqrt{-16} = \sqrt{(-1)\cdot 16} = \sqrt{-1}\cdot\sqrt{16} = \sqrt{-1}\cdot 4 = \pm 4i = 0 \pm 5\cdot i$

$\sqrt{-25} = \sqrt{(-1)\cdot 25} = \sqrt{-1}\cdot\sqrt{25} = \sqrt{-1}\cdot 5 = \pm 5\cdot i = 0 \pm 5\cdot i$

$(2i)^2 = 2^2\cdot i^2 = 4\cdot i^2 = -4 = -4+0\cdot i$

$2i+5i = i\cdot(2+5) = 7i = 0+7\cdot i$

$\frac{4i}{2i} = \frac{4\cdot i}{2\cdot i} = \frac{4}{2} = 2 = 2+0\cdot i$

## 2. feladat

Legyen $z \in \mathbb{C}$, $z = -2+7i$. Adja meg a $z$ komplex szám következő jellemzőit.

$\operatorname{Re} z$ $\operatorname{Im} z$ $-z$ $\overline{z}$ $|z|$

$\operatorname{Re}(z) = \operatorname{Re}(-2+7i) = -2$ $\operatorname{Im}(z) = \operatorname{Im}(-2+7i) = +7$

$-z = -(-2+7i) = 2-7i$ $\overline{z} = \overline{(-2+7i)} = -2-7i$

$|z| = |(-2+7i)| = \sqrt{a^2+b^2} = \sqrt{(-2)^2+(7)^2} = \sqrt{4+49} = \sqrt{53}$

## 3. feladat

Végezzük el a következő műveletet az algebrai alak felhasználásával: $\frac{4+3i}{(2-i)^2}$

$$\frac{4+3\cdot i}{(2-i)^2} = \frac{4+3i}{2^2-2\cdot 2\cdot i+i^2} = \frac{4+3i}{4-4i+i^2} = \frac{4+3i}{3-4i} = \frac{4+3i}{3-4i}\cdot\frac{3+4i}{3+4i} =$$

$$= \frac{(4+3i)\cdot(3+4i)}{(3-4i)\cdot(3+4i)} = \frac{12+16i+9i+12i^2}{9-16i^2} = \frac{12-12+25i}{9+16} = \frac{25i}{25} = i$$

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### 4. feladat

Oldja meg a következő egyenletet a komplex számok halmazán: $\frac{x+i-3i\overline{x}}{x-4} = i-1$

$x \in \mathbb{C},\ \frac{x+i-3\cdot i\cdot\overline{x}}{x-4} = i-1$

$x := a+b\cdot i \quad (a,b\in\mathbb{R})$

$\frac{(a+bi)+i-3\cdot i\cdot\overline{(a+bi)}}{(a+bi)-4} = i-1 \iff (a+bi)+i-3\cdot i\cdot(a-bi) = (i-1)\cdot(a+bi-4)$

$\iff a+bi+i-3\cdot i\cdot a+3\cdot i^2 b = ai+bi^2-4i-a-bi+4 \iff$

$\iff (a+3\cdot b\cdot i^2)+i\cdot(b+1-3\cdot a) = (bi^2-a+4)+i\cdot(a-4-b) \iff$

$\iff (a+3bi^2-bi^2+a-4)+i\cdot(b+1-3a) = i\cdot(a-4-b) \iff$

$\iff (a+a-4+3bi^2-bi^2)+i\cdot(b+1-3a-a+4+b) = 0 \iff$

$\iff (2a+2\cdot b\cdot i^2-4)+i\cdot(2b+5-4a) = 0 \iff$

$\iff (2a-2b-4)+i\cdot(2b+5-4a) = 0 \Rightarrow \begin{cases} 2a-2b-4=0 \\ 2b+5-4a=0 \end{cases}$

$2a-2b-4=0 \iff 2a-2b=4 \iff a-b=2 \iff \underline{\underline{a=2+b}}$

$2b+5-4a=0 \iff 2b+5-4\cdot(2+b)=0 \iff 2b+5-8-4b = -3-2b = 0 \iff -3=2b$

$\iff \underline{\underline{b=-3/2}} \Rightarrow a=2+b=2+(-3/2)=2-3/2=\frac{4-3}{2}=\underline{\underline{1/2}}$

$x = a+bi = \frac{1}{2}+\left(\frac{-3}{2}\right)\cdot i = \underline{\underline{\frac{1}{2}-\frac{3}{2}\cdot i}}$

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## 5. feladat

Határozza meg azt a $z \in \mathbb{C}$ komplex számot, amelyre teljesül hogy

$$\left|\frac{z-3}{2-\bar{z}}\right| = 1 \wedge \text{Re}\left(\frac{z}{2+i}\right) = 2$$

$$\left|\frac{z-3}{2-\bar{z}}\right| = 1 \quad \wedge \quad Re\left(\frac{z}{2+i}\right) = 2$$

$$Re\left(\frac{z}{2+i}\right) = Re\left(\frac{a+bi}{2+i}\right) = Re\left(\frac{a+bi}{2+i} \cdot \frac{2-i}{2-i}\right) =$$

$$= Re\left(\frac{2a-ai+2bi-bi^2}{2^2-i^2}\right) = Re\left(\frac{2a-ai+2bi+b}{4+1}\right) = Re\left(\frac{(2a+b)+i\cdot(2b-a)}{5}\right) =$$

$$= Re\left(\frac{2a+b}{5} + \frac{2b-a}{5}\cdot i\right) = \frac{2a+b}{5} = 2 \Leftrightarrow 2a+b = 10 \Leftrightarrow \underline{\underline{10-2a = b}}$$

$$\left|\frac{z-3}{2-\bar{z}}\right| = \frac{|z-3|}{|2-\bar{z}|} = 1 \Leftrightarrow |z-3| = |2-\bar{z}| \Leftrightarrow |a+b\cdot i-3| = |2-a+bi| \Leftrightarrow$$

$$\Leftrightarrow |(a-3)+b\cdot i| = |(2-a)+bi| \Leftrightarrow \sqrt{(a-3)^2+b^2} = \sqrt{(2-a)^2+b^2} \underset{/()^2}{\Longleftrightarrow} (a-3)^2+b^2 =$$

$$= (2-a)^2+b^2 \underset{/-b^2}{\Longleftrightarrow} (a-3)^2 = (2-a)^2 \Leftrightarrow a^2-6a+9 = 4-4a+a^2 \Leftrightarrow$$

$$\underset{/-a^2}{\Longleftrightarrow} -6a+9 = 4-4a \underset{/+4a}{\Longleftrightarrow} -2a+9 = 4 \underset{/-9}{\Longleftrightarrow} -2a = -5 \underset{/:-2}{\Longleftrightarrow} \underline{\underline{a = 5/2}}$$

$$\Rightarrow b = 10-2\cdot a = 10-2\cdot(5/2) = 10-2\cdot\frac{5}{2} = 10-5 = \underline{\underline{5}}$$

$$z = a+bi = \underline{\underline{\frac{5}{2}+5i}}$$

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### 6. feladat

Legyen $z \in \mathbb{C}, z = 2 + 5i$. Adja meg a $z$ komplex szám abszolút értékét és argumentumát. Szemléltesse a $z$ komplex számot a Gauss-számsíkon.

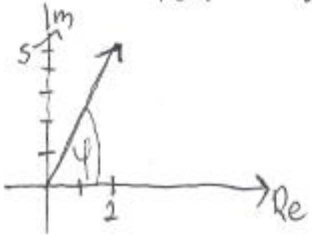
$z \in \mathbb{C};\ z = 2+5 \cdot i$

$|z| = |2+5i| = \sqrt{2^2+5^2} = \sqrt{4+25} = \sqrt{29}$

$\arg(z) = \arg(2+5i) = \varphi$

$z = |z| \cdot (\cos\varphi + i \cdot \sin\varphi) = \sqrt{29} \cdot (\cos\varphi + i \cdot \sin\varphi) = 2+5i$

$\cos\varphi = \frac{Re(z)}{|z|} = \frac{2}{\sqrt{29}};\ \sin\varphi = \frac{Im(z)}{|z|} = \frac{5}{\sqrt{29}} \Rightarrow \underline{\underline{\varphi \simeq 68{,}2^\circ}} = \arg(z)$




### 7. feladat

Határozza meg a következő komplex számok trigonometrikus alakját.

(a) $1 + i$

(b) $-\sqrt{3} + i$

(c) $\frac{9}{2} - \frac{9\sqrt{3}}{2} i$

(d) $-\frac{\sqrt{14}}{2} - \frac{\sqrt{14}}{2} i$

(e) $4i$

(f) $i$

(g) $10$

$z = 1 + i = r \cdot (\cos\varphi + i \cdot \sin\varphi)$

$r = |z| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\underbrace{\cos\varphi = a/|z| = 1/\sqrt{2} \text{ és } b = 1 > 0}$

$\varphi = 45^\circ$ $\quad 1+i = \sqrt{2} \cdot (\cos 45^\circ + i \cdot \sin 45^\circ)$

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$z = -\sqrt{3} + i = r \cdot (\cos\varphi + i \cdot \sin\varphi)$

$r = |z| = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{4} = 2$

$$\underbrace{\cos\varphi = a/|z| = -\frac{\sqrt{3}}{2} \text{ és } b = 1 > 0}_{\varphi = 150^\circ}$$

$-\sqrt{3} + i = 2 \cdot (\cos 150^\circ + i \cdot \sin 150^\circ)$

$\frac{9}{2} - \frac{9\cdot\sqrt{3}}{2} \cdot i = r \cdot (\cos\varphi + i \cdot \sin\varphi) = z$

$r = |z| = \sqrt{\frac{81}{4} + \frac{81 \cdot 3}{4}} = \sqrt{81} = 9$

$$\underbrace{\cos\varphi = a/|z| = \frac{\cancel{9}}{2 \cdot \cancel{9}} = \frac{1}{2} \text{ és } b = -\frac{9\cdot\sqrt{3}}{2} < 0}_{\varphi' = 60^\circ}$$

$\frac{9}{2} - \frac{9\cdot\sqrt{3}}{2} i = 9 \cdot (\cos 300^\circ + i \cdot \sin 300^\circ)$

$\varphi = 2 \cdot 180^\circ - 60^\circ = 300^\circ$

$-\frac{\sqrt{14}}{2} - \frac{\sqrt{14}}{2} i = r \cdot (\cos\varphi + i \cdot \sin\varphi) = z$

$r = |z| = \sqrt{\frac{14}{4} + \frac{14}{4}} = \sqrt{\frac{28}{4}} = \sqrt{7}$

$$\underbrace{\cos\varphi = a/|z| = -\frac{\sqrt{14}}{2 \cdot \sqrt{7}} = -\frac{\sqrt{2}}{2} \text{ és } b < 0}_{\varphi' = 135^\circ}$$

$\varphi = 2\pi - \varphi' = 225^\circ$

$-\frac{\sqrt{14}}{2} - \frac{\sqrt{14}}{2} i = \sqrt{7} \cdot (\cos 225^\circ + i \cdot \sin 125^\circ)$

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$0+4\cdot i = r\cdot(\cos\varphi + i\cdot\sin\varphi) = z$

$r = |z| = \sqrt{4^2} = 4$

$\underbrace{\cos\varphi = a/|z| = 0/4 = 0 \quad \text{és } b \geq 0}_{\varphi = 90^\circ}$

$\varphi = 90^\circ$ $\quad 4i = 4\cdot(\cos 90^\circ + i\cdot\sin 90^\circ)$

$0+1\cdot i = r\cdot(\cos\varphi + i\cdot\sin\varphi) = z$

$r = |z| = \sqrt{1^2} = 1$

$\underbrace{\cos\varphi = a/|z| = 0/1 = 0 \quad \text{és } b \geq 0}_{\varphi = 90^\circ}$

$\varphi = 90^\circ$ $\quad i = 1\cdot(\cos 90^\circ + i\cdot\sin 90^\circ)$

$10+0\cdot i = r\cdot(\cos\varphi + i\cdot\sin\varphi) = z$

$r = |z| = \sqrt{10^2} = 10$

$\underbrace{\cos\varphi = a/|z| = \frac{10}{10} = 1 \quad \text{és } b \geq 0}_{\varphi = 0^\circ}$

$\varphi = 0^\circ$ $\quad 10 = 1\cdot(\cos 0^\circ + i\cdot\sin 0^\circ)$

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### 8. feladat

Végezze el a következő műveleteket a trigonometrikus alak felhasználásával.

(a) $\left(\frac{9}{2}-\frac{9\sqrt{3}}{2}i\right)\left(-\frac{\sqrt{14}}{2}-\frac{\sqrt{14}}{2}i\right)$

(b) $\left(-\frac{3\sqrt{3}}{2}-\frac{3}{2}i\right)\left(\frac{\sqrt{3}}{3}+\frac{1}{3}i\right)$

(c) $\dfrac{-\frac{3\sqrt{3}}{2}-\frac{3}{2}i}{\frac{\sqrt{3}}{3}+\frac{1}{3}i}$

(d) $\left(\frac{5\sqrt{3}}{12}-\frac{5}{12}i\right)^{10}$

(e) $\left(-\frac{\sqrt{10}}{2}-\frac{\sqrt{10}}{2}i\right)^{15}$

(f) $\left(\frac{5}{2}-\frac{5\sqrt{3}}{2}i\right)^{23}$

(g) $(1+i)^8\cdot\left(5\sqrt{3}-5i\right)^3$

(h) $\left(\dfrac{\frac{3}{2}+\frac{3\sqrt{3}}{2}i}{-\frac{5\sqrt{3}}{2}+\frac{5}{2}i}\right)^{12}$

(i) $\left(1-\frac{\sqrt{3}-i}{2}\right)^{24}$

$$\underbrace{\left(\frac{9}{2}-\frac{9\cdot\sqrt{3}}{2}i\right)}_{}\cdot\underbrace{\left(-\frac{\sqrt{14}}{2}-\frac{\sqrt{14}}{2}i\right)}_{}=*$$

$$r=\sqrt{\frac{81}{4}+\frac{81\cdot 3}{4}}=9$$

$$\cos\varphi=\frac{9}{2\cdot 9}=\frac{1}{2}$$

$$\underbrace{b<0}_{}$$

$$\varphi'=60^\circ$$

$$\varphi=2\cdot\pi-\varphi'=300^\circ$$

$$\Leftrightarrow r=\sqrt{\frac{14}{4}+\frac{14}{4}}=\sqrt{7}$$

$$\cos\varphi=\frac{-\sqrt{14}}{2\cdot\sqrt{7}}=-\frac{\sqrt{2}}{2}$$

$$\underbrace{b<0}_{}$$

$$\varphi'=135^\circ$$

$$\varphi=2\cdot\pi-\varphi'=225^\circ$$

$$*=\left(9\cdot(\cos 300^\circ+i\cdot\sin 300^\circ)\right)\cdot\left(\sqrt{7}\cdot(\cos 225^\circ+i\cdot\sin 225^\circ)\right)=$$

$$=9\cdot\sqrt{7}\cdot(\cos(300+225)^\circ+i\cdot\sin\underbrace{(300+225)}_{525^\circ=360^\circ+165^\circ}{}^\circ)=$$

$$=9\cdot\sqrt{7}\cdot(\cos 165^\circ+\sin 165^\circ)$$

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$$\left(-\frac{3\cdot\sqrt{3}}{2}-\frac{3}{2}\cdot i\right)\cdot\left(\frac{\sqrt{3}}{3}+\frac{1}{3}\cdot i\right)=*$$

$r=\sqrt{\frac{9\cdot 3}{4}+\frac{9}{4}}=3$

$\cos\varphi=-\frac{3\cdot\sqrt{3}}{2\cdot 3}=-\frac{\sqrt{3}}{2}$

$b<0$

$\varphi'=150^\circ \quad \varphi=2\pi-\varphi'=210^\circ$

$r=\sqrt{\frac{3}{9}+\frac{1}{9}}=\frac{2}{3}$

$\cos\varphi=\frac{\sqrt{3}}{3}\Big/\frac{2}{3}=\frac{\sqrt{3}}{3}\cdot\frac{3}{2}=\frac{\sqrt{3}}{2}$

$b\geq 0$

$\varphi=30^\circ$

$$=\left(3\cdot(\cos 210^\circ+i\cdot\sin 210^\circ)\right)\cdot\left(\frac{2}{3}\cdot(\cos 30^\circ+i\cdot\sin 30^\circ)\right)=$$

$$=\left(3\cdot\tfrac{2}{3}\right)\cdot\left(\cos(210+30)^\circ+i\cdot\sin(210+30)^\circ\right)=$$

$$=2\cdot(\cos 240^\circ+i\cdot\sin 240^\circ)$$

$$\frac{-\frac{3\cdot\sqrt{3}}{2}-\frac{3}{2}\cdot i}{\frac{\sqrt{3}}{3}+\frac{1}{3}\cdot i}$$

$r=\sqrt{\frac{9\cdot 4}{4}}=3$

$\cos\varphi=-\sqrt{3}/2,\ b<0\Rightarrow\varphi'=150\Rightarrow\varphi=210^\circ$ $(2\pi-\varphi')$

$r=\sqrt{\frac{3+1}{9}}=2/3$

$\cos\varphi=\frac{\sqrt{3}}{3}\Big/\frac{2}{3}=\frac{\sqrt{3}}{2},\ b\geq 0\Rightarrow\varphi=30^\circ$

$$\|$$

$$\frac{3\cdot(\cos 210^\circ+i\cdot\sin 210^\circ)}{\frac{2}{3}\cdot(\cos 30^\circ+i\cdot\sin 30^\circ)}=\frac{3}{2/3}\cdot\left(\cos\overset{210-30}{180^\circ}+i\cdot\sin 180^\circ\right)=$$

$$=\frac{9}{2}\cdot(\cos 180^\circ+i\cdot\sin 180^\circ)$$

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$$\Big(\underbrace{\frac{5\cdot\sqrt{3}}{12}-\frac{5}{12}i}\Big)^{10}=\Big(\frac{5}{6}\cdot(\cos 330^\circ+i\cdot\sin 330^\circ)\Big)^{10}=*$$

$$r=\sqrt{\frac{25\cdot 3+25}{144}}=\sqrt{\frac{100}{144}}=\frac{10}{12}=\frac{5}{6}$$

$$\cos\varphi=\frac{5\cdot\sqrt{3}}{12}\cdot\frac{6}{5}=\frac{\sqrt{3}}{2};\ b<0\Rightarrow \varphi'=30^\circ,\ \varphi=2\pi-\varphi'=330^\circ$$

$$*=\left(\frac{5}{6}\right)^{10}\cdot\left(\cos(330\cdot 10)^\circ+i\cdot\sin(330\cdot 10)^\circ\right)=$$

// $330\cdot 10=3300^\circ=360^\circ\cdot 9+60^\circ$

$$=\left(\frac{5}{6}\right)^{10}\cdot(\cos 60^\circ+i\cdot\sin 60^\circ)$$

$$\Big(\underbrace{-\frac{\sqrt{10}}{2}-\frac{\sqrt{10}}{2}i}\Big)^{15}=\Big(\sqrt{5}\cdot(\cos 225^\circ+i\cdot\sin 225^\circ)\Big)^{15}=$$

$$r=\sqrt{\frac{10+10}{4}}=\sqrt{5}$$

$$\cos\varphi=\frac{-\sqrt{10}}{2\cdot\sqrt{5}}=\frac{-\sqrt{2}}{2};\ b<0\Rightarrow \varphi'=135^\circ,\ \varphi=2\cdot\pi-\varphi'=225^\circ$$

$$=\sqrt{5}^{15}\cdot\left(\cos(15\cdot 225)^\circ+i\cdot\sin(15\cdot 225)^\circ\right)=$$

$$=5^{\frac{15}{2}}\cdot(\cos 135^\circ+i\cdot\sin 135^\circ)$$

// $15\cdot 225^\circ=3375^\circ=360^\circ\cdot 9+135^\circ$

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$$\left(\frac{5}{2}-\frac{5\cdot\sqrt{3}}{2}i\right)^{23}=\left(5\cdot(\cos 300^\circ+i\cdot\sin 300^\circ\right)^{25}=*$$

$$r=\sqrt{\frac{25+25\cdot 3}{4}}=\sqrt{25}=5$$

$$\cos\varphi=\frac{5}{2\cdot 5}=1/2;\ b<0\Rightarrow \varphi'=60^\circ$$

$$\varphi=2\pi-\varphi'=300^\circ$$

$$*=5^{25}\cdot(\cos(25\cdot 300^\circ)+i\cdot\sin(25\cdot 300^\circ))=$$

$$=5^{25}\cdot(\cos 300^\circ+i\cdot\sin 300^\circ)$$

$$25\cdot 300^\circ=7500^\circ=20\cdot 360^\circ+300^\circ$$

$$(1+i)^8\cdot(5\cdot\sqrt{3}-5\cdot i)^3=2^4\cdot 10^3\cdot(\cos 450^\circ+i\cdot\sin 450^\circ)=$$

$$(1+i)^8=\left(\sqrt{2}\cdot(\cos 45^\circ+i\cdot\sin 45^\circ)\right)^8=$$

$$=2^4\cdot(\cos 180^\circ+i\cdot\sin 180^\circ)$$

$$(5\cdot\sqrt{3}-5\cdot i)^3=\left(10\cdot(\cos 330^\circ+i\cdot\sin 330^\circ)\right)^3=$$

$$=10^3\cdot(\cos 270^\circ+i\cdot\sin 270^\circ)$$

$$=16000\cdot(\cos 90^\circ+i\cdot\sin 90^\circ)$$

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$$\left(\frac{\frac{3}{2}+\frac{3\cdot\sqrt{3}}{2}\cdot i}{-\frac{5\cdot\sqrt{3}}{2}+\frac{5}{2}\cdot i}\right)^{12}=\left(\frac{3\cdot(\cos 60^\circ+i\cdot\sin 60^\circ)}{5\cdot(\cos 150^\circ+i\cdot\sin 150^\circ)}\right)^{12}=*$$

$$\left(\frac{z}{w}\right)^{12}\quad z: r=3;\ \cos\varphi=\tfrac{1}{2},\ b>0\Rightarrow\varphi=60^\circ$$

$$w: r=5;\ \cos\varphi=-\tfrac{\sqrt{3}}{2},\ b>0\Rightarrow\varphi=150^\circ$$

$$*=\left(\frac{3}{5}\right)^{12}\cdot\left(\cos(60-150)^\circ+i\cdot\sin(60-150)^\circ\right)^{12}=$$

$$=\left(\frac{3}{5}\right)^{12}\cdot(\cos(270\cdot 12)^\circ+i\cdot\sin(270\cdot 12)^\circ=$$

$$=(3/5)^{12}\cdot(\cos 0^\circ+i\cdot\sin 0^\circ)$$

$$270^\circ\cdot 12=360^\circ\cdot 9$$

$$\left(1-\frac{\sqrt{3}-i}{2}\right)^{24}=\left(\left(1-\frac{\sqrt{3}}{2}\right)-\frac{1}{2}\cdot i\right)^{24}=*$$

$$r=\sqrt{1-\sqrt{3}+3/4+1/4}=\sqrt{2-\sqrt{3}}$$

$$\cos\varphi=\frac{1-\frac{\sqrt{3}}{2}}{\sqrt{2-\sqrt{3}}}=\frac{\frac{2-\sqrt{3}}{2}}{\sqrt{2-\sqrt{3}}}=\frac{2-\sqrt{3}}{2\cdot\sqrt{2-\sqrt{3}}}=$$

$$=\frac{\sqrt{2-\sqrt{3}}}{2};\ b<0\Rightarrow\varphi'=58{,}83^\circ\quad\varphi=301{,}17^\circ$$

$$*=\sqrt{2-\sqrt{3}}^{24}\cdot\left(\cos(24\cdot 301{,}87)^\circ+i\cdot\sin(24\cdot 301{,}87)^\circ\right)=$$

$$=(2-\sqrt{3})^{12}\cdot(\cos 28^\circ+i\cdot\sin 28^\circ)$$

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### 9. feladat

Végezze el a következő gyökvonásokat a komplex számok halmazán.

(a) $-60$ második gyöke

(b) $-60$ harmadik gyöke

(c) $1-\sqrt{3}i$ hatodik gyöke

(d) $-7\sqrt{3}+7i$ ötödik gyöke

(e) $-\frac{7}{2}+\frac{7}{2}i$ nyolcadik gyöke

(f) $-6\sqrt{3}+6i$ második gyöke

(g) $\dfrac{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^8}{(1+i)^5}$ hetedik gyöke

$-60$ második gyöke

$n=2$ $\quad r=60$

$k=0,1$ $\quad \cos\varphi=-1, \varphi>0 \Rightarrow \varphi=180°$

$W_k=\sqrt[2]{60}\cdot\left(\cos\left(\frac{180°+2\cdot k\cdot 180°}{2}\right)+i\cdot\sin\left(\frac{180+2\cdot k\cdot 180°}{2}\right)\right)$

$W_0=\sqrt{60}\cdot(\cos 90°+i\cdot\sin 90°)$

$W_1=\sqrt{60}\cdot(\cos 270°+i\cdot\sin 270°)$

$-60$ harmadik gyöke

$n=3$ $\quad r=60$

$k=0,1,2$ $\quad \cos\varphi=-1, \varphi>0 \Rightarrow \varphi=180°$

$W_k=\sqrt[3]{60}\cdot\left(\cos\left(\frac{180+2\cdot k\cdot 180°}{3}\right)+i\cdot\sin\left(\frac{180+2\cdot k\cdot 180°}{3}\right)\right)$

$W_0=\sqrt[3]{60}\cdot(\cos 60°+i\cdot\sin 60°)$

$W_1=\sqrt[3]{60}\cdot(\cos 180°+i\cdot\sin 180°)$

$W_2=\sqrt[3]{60}\cdot(\cos 300+i\cdot\sin 300°)$

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$1-\sqrt{3}\cdot i$ hatodik gyöke

$n=6 \qquad r=2$

$k=0,1,\dots,5 \qquad \cos\varphi = 1/2;\ b<0 \Rightarrow \varphi = 300°$

$W_k = \sqrt[6]{2}\cdot\left(\cos\left(\frac{300+2\cdot k\cdot 180}{6}\right)° + i\cdot\sin\left(\frac{300+k\cdot 360}{6}\right)°\right)$

$W_0 = \sqrt[6]{2}\cdot(\cos 50° + i\cdot\sin 50°)$

$W_1 = \sqrt[6]{2}\cdot(\cos 110° + i\cdot\sin 110°)$

$W_2 = \sqrt[6]{2}\cdot(\cos 170° + i\cdot\sin 170°)$

$W_3 = \sqrt[6]{2}\cdot(\cos 230° + i\cdot\sin 230°)$

$W_4 = \sqrt[6]{2}\cdot(\cos 290° + i\cdot\sin 290°)$

$W_5 = \sqrt[6]{2}\cdot(\cos 350° + i\cdot\sin 350°)$

$-7\cdot\sqrt{3}+7i$ ötödik gyöke

$n=5 \qquad r=14$

$k=0,1,\dots,4 \qquad \cos\varphi = \frac{-\sqrt{3}}{2};\ b>0 \Rightarrow \varphi = 150°$

$W_k = \sqrt[5]{14}\cdot\left(\cos(30°+72\cdot k) + i\cdot\sin\left(\frac{150°+2\cdot k\cdot 180°}{5}\right)\right)$

$W_0 = \sqrt[5]{14}\cdot(\cos 30° + i\cdot\sin 30°)$

$W_1 = \sqrt[5]{14}\cdot(\cos 102° + i\cdot\sin 102°)$

$W_2 = \sqrt[5]{14}\cdot(\cos 174° + i\cdot\sin 174°)$

$W_3 = \sqrt[5]{14}\cdot(\cos 246° + i\cdot\sin 246°)$

$W_4 = \sqrt[5]{14}\cdot(\cos 318° + i\cdot\sin 318°)$

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$-\frac{7}{2}+\frac{7}{2}i$ nyolcadik gyöke

$n=8$

$k=0,1,\dots,7$

$r=\sqrt{\frac{49+49}{4}}=\sqrt{\frac{98}{4}}=\sqrt{\frac{49}{2}}$

$\cos\varphi=-\frac{7}{2}/\frac{7}{\sqrt{2}}=-\frac{7}{2}\cdot\frac{\sqrt{2}}{7}=-\frac{\sqrt{2}}{2};\ b>0$

$\Rightarrow \varphi=135^\circ$

$$W_k=\sqrt[8]{\frac{49}{2}}\cdot\left(\cos\left(\frac{135+2\cdot180\cdot k}{8}\right)^\circ+i\cdot\sin\left(\frac{135}{8}+45\cdot k\right)^\circ\right)$$

$$W_0=\sqrt[8]{\frac{49}{2}}\cdot\left(\cos\left(\frac{135}{8}\right)^\circ+i\cdot\sin\left(\frac{135}{8}\right)^\circ\right)$$

$$W_1=\sqrt[8]{\frac{49}{2}}\cdot\left(\cos\left(\frac{495}{8}\right)^\circ+i\cdot\sin\left(\frac{495}{8}\right)^\circ\right)$$

$$W_2=\sqrt[8]{\frac{49}{2}}\cdot\left(\cos\left(\frac{855}{8}\right)^\circ+i\cdot\sin\left(\frac{855}{8}\right)^\circ\right)$$

$$W_3=\sqrt[8]{\frac{49}{2}}\cdot\left(\cos\left(\frac{1215}{8}\right)^\circ+i\cdot\sin\left(\frac{1215}{8}\right)^\circ\right)$$

$$W_4=\sqrt[8]{\frac{49}{2}}\cdot\left(\cos\left(\frac{1575}{8}\right)^\circ+i\cdot\sin\left(\frac{1575}{8}\right)^\circ\right)$$

$$W_5=\sqrt[8]{\frac{49}{2}}\cdot\left(\cos\left(\frac{1935}{8}\right)^\circ+i\cdot\sin\left(\frac{1935}{8}\right)^\circ\right)$$

$$W_6=\sqrt[8]{\frac{49}{2}}\cdot\left(\cos\left(\frac{2295}{8}\right)^\circ+i\cdot\sin\left(\frac{2295}{8}\right)^\circ\right)$$

$$W_7=\sqrt[8]{\frac{49}{2}}\cdot\left(\cos\left(\frac{2655}{8}\right)^\circ+i\cdot\sin\left(\frac{2655}{8}\right)^\circ\right)$$

## 5. feladatsor: Komplex számok algebrai és trigonometrikus alakja

$-6\cdot\sqrt{3}+6i$ második gyöke

$n=2 \qquad r=\sqrt{4\cdot 36}=12$

$k=0,1 \qquad \cos\varphi=-\frac{\sqrt{3}}{2}; \; 6>0 \Rightarrow \varphi=150^\circ$

$W_k=\sqrt[2]{12}\cdot\left(\cos\left(\frac{150+2\cdot 180\cdot k}{2}\right)^\circ+i\cdot\sin(75+180\cdot k)^\circ\right)$

$W_0=\sqrt{12}\cdot(\cos 75^\circ+i\cdot\sin 75^\circ)$

$W_1=\sqrt{12}\cdot(\cos 255^\circ+i\cdot\sin 255^\circ)$

$\dfrac{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^8}{(1+i)^5}$ hetedik gyöke

$\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^8=\left(1\cdot[\cos 60^\circ+i\cdot\sin 60^\circ]\right)^8=$

$=(\cos(8\cdot 60)^\circ+i\cdot\sin(8\cdot 60)^\circ)=\cos 120^\circ+i\cdot\sin 120^\circ$

$(1+i)^5=\left(\sqrt{2}\cdot(\cos 45^\circ+i\sin 45^\circ)\right)^5=$

$=\left(2^{5/2}\cdot(\cos 225+i\cdot\sin 225^\circ)\right)$

## 5. feladatsor: Komplex számok algebrai és trigonometrikus alakja

$$\frac{\cos 120^\circ + i\cdot\sin 120^\circ}{2^{5/2}\cdot(\cos 225^\circ + i\cdot\sin 225^\circ)} =$$

$$= 2^{-5/2}\cdot(\cos(-105)^\circ + i\cdot\sin(-105)^\circ) =$$

$$= 2^{-5/2}\cdot(\cos 255^\circ + i\cdot\sin 255^\circ)$$

$n=7$ $\quad r=2^{-5/2}$

$k=0,1,\dots,6$ $\quad \varphi=255^\circ$

$$W_k=\sqrt[7]{\frac{1}{2^{5/2}}}\cdot\left(\cos\left(\frac{255+2\cdot180\cdot k}{7}\right)^\circ + i\cdot\sin\left(\frac{255+2\cdot180\cdot k}{7}\right)\right)$$

$$W_0=\sqrt[7]{2^{-5/2}}\left(\cos\left(\frac{255}{7}\right)^\circ + i\cdot\sin\left(\frac{255}{7}\right)\right)$$

$$W_1=\sqrt[7]{2^{-5/2}}\cdot\left(\cos\left(\frac{615}{7}\right)^\circ + i\cdot\sin\left(\frac{615}{7}\right)^\circ\right)$$

$$W_2=\sqrt[7]{2^{-5/2}}\cdot\left(\cos\left(\frac{975}{7}\right)^\circ + i\cdot\sin\left(\frac{975}{7}\right)\right)$$

$$W_3=\sqrt[7]{2^{-5/2}}\cdot\left(\cos\left(\frac{1335}{7}\right)^\circ + i\cdot\sin\left(\frac{1335}{7}\right)^\circ\right)$$

$$W_4=\sqrt[7]{2^{-5/2}}\left(\cos\left(\frac{1695}{7}\right)^\circ + i\cdot\sin\left(\frac{1695}{7}\right)^\circ\right)$$

$$W_5=\sqrt[7]{2^{-5/2}}\cdot\left(\cos\left(\frac{2055}{7}\right)^\circ + i\cdot\sin\left(\frac{2055}{7}\right)^\circ\right)$$

$$W_6=\sqrt[7]{2^{-5/2}}\cdot\left(\cos\left(\frac{2415}{7}\right)^\circ + i\cdot\sin\left(\frac{2415}{7}\right)^\circ\right)$$

## 5. feladatsor: Komplex számok algebrai és trigonometrikus alakja

### 10. feladat

A trigonometrikus alak segítségével számítsa ki $z$ értékét trigonometrikus és algebrai alakban is, majd adja meg az összes olyan $w$ komplex számot trigonometrikus alakban, melyekre $w^3 = z$, ahol $z = \dfrac{\left(2+2\sqrt{3}i\right)^{10}}{(-1+i)^{83}}$.

$$z = \frac{(2+2\cdot\sqrt{3}\cdot i)^{10}}{(-1+i)^{83}} = \frac{(r_1\cdot(\cos\varphi_1 + i\cdot\sin\varphi_1))^{10}}{(r_2\cdot(\cos\varphi_2+i\cdot\sin\varphi_2))^{83}} = \frac{(4\cdot(\cos 60° + i\cdot\sin 60°))^{10}}{(\sqrt{2}\cdot(\cos 135° + i\cdot\sin 135°))^{83}} = *$$

$r_1 = |2+2\cdot\sqrt{3}\cdot i| = \sqrt{4+4\cdot 3} = 4 \quad \cos\varphi_1 = \frac{2}{4} = \frac{1}{2} \Rightarrow \varphi_1 = 60°$

$r_2 = |-1+i| = \sqrt{1+1} = \sqrt{2} \quad \cos\varphi_2 = \frac{-1}{\sqrt{2}} \Rightarrow \varphi_2 = 135°$

$$* = \frac{4^{10}\cdot(\cos 600° + i\cdot\sin 600°)}{\sqrt{2}^{83}\cdot(\cos(6075)° + i\cdot\sin(6075)°)} = \frac{4^{10}\cdot(\cos 240° + i\cdot\sin 240°)}{\sqrt{2}^{83}\cdot(\cos 315° + i\cdot\sin 315°)} =$$

$$= \frac{4^{10}}{2^{\frac{83}{2}}}\cdot(\cos(240°-315°) + i\cdot\sin(240-315)°) = 2^{-21,5}\cdot(\cos 285° + i\cdot\sin 285°) =$$

$$= 2^{-21,5}\cdot(\cos 285° + i\cdot\sin 285°) = \underbrace{2^{-21,5}\cdot(\cos 285° + i\cdot\sin 285°)}_{\text{trigonometrikus alak}} =$$

$$= \underbrace{8,73\cdot 10^{-8} - 3,26\cdot 10^{-7} i}_{\text{algebrai alak}}$$

$z$ 3-dik gyökei:

$$w_0 = \sqrt[3]{2^{-21,5}}\cdot\left(\cos\left(\frac{285°}{3} + \frac{2\cdot 0\cdot\pi}{3}\right) + i\cdot\sin\left(\frac{285°}{3} + \frac{2\cdot 0\cdot\pi}{3}\right)\right) =$$

$$= 2^{-21,5}\cdot(\cos(35°) + i\cdot\sin(35°)) = 2^{-21,5}\cdot(\cos 95° + i\cdot\sin 95°)$$

$$w_1 = \sqrt[3]{2^{-21,5}}\left(\cos\left(\frac{285°}{3} + \frac{2\cdot 1\cdot\pi}{3}\right) + i\cdot\sin\left(\frac{285°}{3} + \frac{2\cdot 1\cdot\pi}{3}\right)\right) = 2^{-21,5}(\cos 215° + i\cdot\sin 215°)$$

$$w_2 = \sqrt[3]{2^{-21,5}}\left(\cos\left(\frac{285°}{3} + \frac{2\cdot 2\cdot\pi}{3}\right) + i\cdot\sin\left(\frac{285°}{3} + \frac{2\cdot 2\cdot\pi}{3}\right)\right) = 2^{-21,5}(\cos 335° + i\cdot\sin 335°)$$

## 5. feladatsor: Komplex számok algebrai és trigonometrikus alakja

### 11. feladat

Írjuk fel algebrai alakban a $z = \dfrac{(1+i)^8}{(1-\sqrt{3}i)^6}$ komplex számot.

$$z = \frac{(1+i)^8}{(1-\sqrt{3}\cdot i)^6} = \frac{(r_1\cdot(\cos\varphi_1 + i\cdot\sin\varphi_1))^8}{(r_2\cdot(\cos\varphi_2 + i\cdot\sin\varphi_2))^6} = \frac{(\sqrt{2}\cdot(\cos 45^\circ + i\cdot\sin 45^\circ))^8}{(2\cdot(\cos 60^\circ + i\cdot\sin 60^\circ))^6}$$

$r_1 = |1+i| = \sqrt{2}$ $\quad \cos\varphi_1 = \frac{1}{\sqrt{2}} \Rightarrow \varphi_1 = 45^\circ$

$r_2 = |1-\sqrt{3}i| = 2$ $\quad \cos\varphi_2 = \frac{1}{2}, b<0 \Rightarrow \varphi_2' = 60^\circ \Rightarrow \varphi_2 = 300^\circ$

$$= \frac{\sqrt{2}^8\cdot(\cos(45\cdot 8)^\circ + i\cdot\sin(45\cdot 8)^\circ)}{2^6\cdot(\cos(300\cdot 6)^\circ + i\cdot\sin(1800^\circ))} =$$

$$= \frac{2^4\cdot(\cos 360^\circ + i\cdot\sin 360^\circ)}{2^6\cdot(\cos 360^\circ + i\cdot\sin 360^\circ)} = 2^{4-6}\cdot(\cos(360-360)^\circ + i\cdot\sin(360-360)^\circ) =$$

$$= 2^{-2}\cdot(\cos 0^\circ + i\cdot\sin 0^\circ) = \frac{1}{2^2}\cdot(1+i\cdot 0) = \frac{1}{4} + 0\cdot i = \frac{1}{4}$$