

Hamming Code (Encode)

ex1: 0101 hamming ? (even)

$$k=4, 2^r \geq k+r+1$$

$$2^r \geq r+5 \Rightarrow r=3$$

k = data bit size

r = parity bit size

$n = k+r$ = hamming code size

$$2^r - 1 \geq n$$

$$2^r \geq k+r+1$$

| | 8 | 4 | 2 | 1 | parity bits |
|---|---|---|---|---|-------------|
| 3 | | | 1 | 1 | |
| 5 | | 1 | | 1 | |
| 6 | | 1 | 1 | | |
| 7 | | 1 | 1 | 1 | |

data bits

| bi num | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|------------|-------|-------|---|-------|---|---|---|
| parity bit | P_1 | P_2 | | P_4 | | | |
| data bit | 0 | 1 | 0 | 0 | 1 | 0 | 1 |

$$P_1 = 1 \oplus 3 \oplus 5 \oplus 7 = 0$$

$$P_2 = 2 \oplus 3 \oplus 6 \oplus 7 = 1$$

$$P_4 = 4 \oplus 5 \oplus 6 \oplus 7 = 0$$

\therefore 0101 hamming (even) \rightarrow 0100101

ex2: 1100101 hamming (odd) ?

$$k=7, 2^r \geq r+8 \therefore r=4 (2^4 \geq 12)$$

$$n = k+r = 7+4 = 11$$

| | 8 | 4 | 2 | 1 |
|----|---|---|---|---|
| 3 | | | 1 | 1 |
| 5 | | 1 | | 1 |
| 6 | | 1 | 1 | |
| 7 | | 1 | 1 | 1 |
| 9 | 1 | | | 1 |
| 10 | 1 | | 1 | |
| 11 | 1 | | 1 | 1 |

| bi num | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|--------|--------------|-------|---|-------|---|---|---|-------|---|----|--------------|
| | \downarrow | | | | | | | | | | \downarrow |
| parity | P_1 | P_2 | | P_4 | | | | P_8 | | | |
| data | 1 | 1 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 |

$$P_1 = 1 \oplus 3 \oplus 5 \oplus 7 \oplus 9 \oplus 11 = 1$$

$$P_2 = 2 \oplus 3 \oplus 6 \oplus 7 \oplus 10 \oplus 11 = 1$$

$$P_4 = 4 \oplus 5 \oplus 6 \oplus 7 = 0$$

$$P_8 = 8 \oplus 9 \oplus 10 \oplus 11 = 1$$

\therefore 1100101 hamming (odd) \rightarrow 1101001101

Hamming Decode

ex1 Data Bits? $\xrightarrow[\text{even}]{\text{Hamming}}$ 0100111

bi num 1 2 3 4 5 6 7

parity P_1 P_2 P_4

data 0 1 0 0 1 1 1

| | 8 | 4 | 2 | 1 |
|---|---|---|---|---|
| 3 | | | 1 | 1 |
| 5 | | 1 | | 1 |
| 6 | | 1 | 1 | |
| 7 | | 1 | 1 | 1 |

$$P_1 = 1 \oplus 3 \oplus 5 \oplus 7 = 0$$

$$P_2 = 2 \oplus 3 \oplus 6 \oplus 7 = 1 \oplus 0 \oplus 1 \oplus 1 = 1$$

$$P_4 = 4 \oplus 5 \oplus 6 \oplus 7 = 0 \oplus 1 \oplus 1 \oplus 1 = 1$$

$$P_4 P_2 P_1 = 110 = 6$$

\therefore 0101 $\xrightarrow[\text{even}]{\text{Hamming}}$ 0100101

★ 若检测位而非信息位出错, 则一般不予纠正。

ex2 Data Bits? $\xrightarrow[\text{even}]{\text{Hamming}}$ 0110101

1 2 3 4 5 6 7

P_1 P_2 P_4

0 1 1 0 1 0 1

| | 8 | 4 | 2 | 1 |
|---|---|---|---|---|
| 3 | | | 1 | 1 |
| 5 | | 1 | | 1 |
| 6 | | 1 | 1 | |
| 7 | | 1 | 1 | 1 |

$$P_4 = 4 \oplus 5 \oplus 6 \oplus 7 = 1 \oplus 1 \oplus 0 \oplus 1 = 1$$

$$P_2 = 2 \oplus 3 \oplus 6 \oplus 7 = 1 \oplus 1 \oplus 0 \oplus 1 = 1$$

$$P_1 = 1 \oplus 3 \oplus 5 \oplus 7 = 0 \oplus 1 \oplus 1 \oplus 1 = 1$$

$$P_4 P_2 P_1 = 011 = 3$$

0101 $\xrightarrow[\text{even}]{\text{Hamming}}$ 0100101