USEFUL FORMULAE

Roots of equations

$$x_r = x_u - f(x_u)\frac{x_l - x_u}{f(x_l) - f(x_u)}$$

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$$x_{i+1} = g(x_i)$$

$$x_r = \frac{x_l + x_u}{2}$$

$$x_{i+1} = x_i - f(x_i)\frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}$$

Polynomial interpolation

$$f(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + \cdots + b_n(x - x_0)(x - x_1)\cdots(x - x_{n-1})$$

$$b_0 = f(x_0)$$

$$b_1 = f[x_1, x_0]$$

$$b_n = f[x_n, x_{n-1}, \cdots, x_1, x_0]$$

$$f[x_i, x_j] = \frac{f(x_i) - f(x_j)}{x_i - x_j}$$

$$f[x_n, x_{n-1}, \cdots, x_1, x_0] = \frac{f[x_n, x_{n-1}, \cdots, x_1] - f[x_{n-1}, \cdots, x_1, x_0]}{x_n - x_0}$$

$$f_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

$$L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{n} \frac{x - x_j}{x_i - x_j}$$

Differentiation

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_i)}{h}$$

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{h}$$

$$f'(x_i) = \frac{f(x_{i+1}) - f(x_{i-1})}{2h}$$

Cubic splines

$$f_i(x) = \left[\frac{f(x_{i-1})}{x_i - x_{i-1}} - \frac{1}{6}(x_i - x_{i-1}) f''(x_{i-1})\right](x_i - x) + \left[\frac{f(x_i)}{x_i - x_{i-1}} - \frac{1}{6}(x_i - x_{i-1}) f''(x_i)\right](x - x_{i-1})$$
$$+\frac{1}{6}\frac{f''(x_{i-1})}{(x_i - x_{i-1})}(x_i - x)^3 + \frac{1}{6}\frac{f''(x_i)}{(x_i - x_{i-1})}(x - x_{i-1})^3$$

$$(x_i - x_{i-1}) f''(x_{i-1}) + 2(x_{i+1} - x_{i-1}) f''(x_i) + (x_{i+1} - x_i) f''(x_{i+1}) =$$
$$\frac{6}{(x_{i+1} - x_i)}[f(x_{i+1}) - f(x_i)] + \frac{6}{(x_i - x_{i-1})}[f(x_{i-1}) - f(x_i)]$$

Integration

$$I = \frac{h}{2}\left[f(x_0) + 2\sum_{i=1}^{n-1} f(x_i) + f(x_n)\right] \quad h = \frac{b-a}{n}$$

$$I = \frac{h}{3}\left[f(x_0) + 4\sum_{i=1,3,5,\ldots}^{n-1} f(x_i) + 2\sum_{j=2,4,6,\ldots}^{n-2} f(x_j) + f(x_n)\right] \qquad h = \frac{b-a}{n}$$

$$I = \frac{3h}{8}\left[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\right] \qquad h = \frac{b-a}{3}$$

$$I = \frac{4}{3}I_m - \frac{1}{3}I_l, \quad I = \frac{16}{15}I_m - \frac{1}{15}I_l, \quad I = \frac{64}{63}I_m - \frac{1}{63}I_l, \ldots$$

$$I = f\left(-\frac{1}{\sqrt{3}}\right) + f\left(\frac{1}{\sqrt{3}}\right), \qquad x = \frac{b+a}{2} + \frac{b-a}{2}x_d$$

$$I = c_1 f(x_1) + c_2 f(x_2) + c_3 f(x_3)$$
$$c_1 = c_3 = \frac{5}{9} \qquad c_2 = \frac{8}{9} \qquad x_1 = -0.774596669 \quad x_2 = 0 \qquad x_3 = 0.774596669$$

Differential equations

$$y_{i+1} = y_i + hf(x_i, y_i)$$

$$y_{i+1}^0 = y_i + hf(x_i, y_i) \qquad y_{i+1} = y_i + h\frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1}^0)}{2}$$

$$y_{i+1/2} = y_i + \frac{h}{2} f(x_i, y_i) \qquad y_{i+1} = y_i + hf(x_{i+1/2}, y_{i+1/2})$$

$$y_{i+1} = y_i + \frac{h}{3}(k_1 + 2k_2) \qquad k_1 = f(x_i, y_i) \qquad k_2 = f(x_i + \frac{3h}{4}, y_i + \frac{3}{4}hk_1)$$

$$y_{i+1} = y_i + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) \qquad k_1 = f(x_i, y_i) \qquad k_2 = f(x_i + \frac{h}{2}, y_i + \frac{1}{2}hk_1)$$

$$k_3 = f(x_i + \frac{h}{2}, y_i + \frac{1}{2}hk_2) \qquad k_4 = f(x_i + h, y_i + hk_3)$$

$$y_{i+1}^0 = y_{i-1} + 2hf(x_i, y_i) \qquad y_{i+1} = y_i + h\frac{f(x_i, y_i) + f(x_{i+1}, y_{i+1}^0)}{2}$$

$$y_{i+1}^0 = y_i + h(\frac{55}{24} f_i - \frac{59}{24} f_{i-1} + \frac{37}{24} f_{i-2} - \frac{9}{24} f_{i-3})$$

$$y_{i+1} = y_i + h(\frac{9}{24} f_{i+1}^0 + \frac{19}{24} f_i - \frac{5}{24} f_{i-1} + \frac{1}{24} f_{i-2})$$