Kennlinie

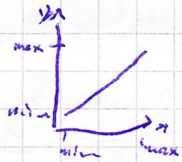
$x_a = f(x_e)$

Empfindlichkeit

$E = \frac{dx_a}{dx_e}$

Linearekennlinie

$$y = \frac{y_{max} - y_{min}}{x_{max} - x_{min}} (x - x_{min}) + y_{min}$$



$$y_L(x) = \underbrace{y(x_0)}_{t} + \underbrace{\left.\frac{dy(x)}{dx}\right|_{x=x_0}}_{m} \underbrace{(x - x_0)}_{x}$$

Messspanne

$$\text{Messspanne} = \begin{cases} x_{max} \text{ für } x_{min} = 0 \\ 1 : \frac{x_{max}}{x_{min}} \text{ für } x_{min} \neq 0 \end{cases}$$

Messung von Wechsel strom /spannung

Linearer Mittelwert : $\bar{u} = \frac{1}{T} \int_0^T \hat{u} \sin(\omega t) dt = 0$

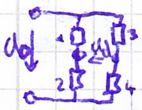
Gleichrichtwert : $|\bar{u}| = \frac{1}{T} \int_0^T |\hat{u} \sin(\omega t)| dt = \frac{2}{\pi} \hat{u} \approx 0{,}637 \hat{u}$

Effektivwert : $U = \sqrt{\frac{1}{T} \int_0^T (\hat{u} \sin(\omega t))^2 dt} = \frac{\hat{u}}{\sqrt{2}} \approx 0{,}707 \hat{u}$

Mittelwert Quadrat..: $\overline{x^2(t)} = \frac{1}{T} \int_0^T x^2(t)\, dt$

effecttive —"— $x_{eff} = X = \sqrt{\overline{x^2(x)}}$

Brückengleichrichterschaltung



$U_1 = U_0 \frac{R_1}{R_1 + R_2}$ $U_3 = U_0 \frac{R_3}{R_3 + R_4}$ $U_d = U_3 - U_1 = U_0 \frac{R_2 R_3 - R_1 R_4}{(R_1 + R_2)(R_3 + R_4)}$

$R_2 R_3 = R_1 R_4$ (Abgleichbedingung)

$R_2 = R_1 \frac{R_4}{R_3}$ -> mindestens ein wiederstand muss Variabel sein

Wenn $R_1 = R_3 = R_4 = R$; $R_2 = R_x$

$$U_d = U_0 \left(\frac{1}{2} - \frac{R}{R + R_x} \right) = \frac{U_0}{2} \frac{R_x - R}{R_x + R}$$

Messung: U => parallel => $R_m >> R_i$

I => Reihe => $R_m << R_i$ $R_i = \frac{U_{LL}}{I_{KS}}$

Messbereichserweiterung:

$$I_m \cdot R_m = I_p \cdot R_p = (I - I_m) \cdot R_p \Rightarrow R_p = \frac{I_m \cdot R_m}{I - I_m} = R_m \frac{I_m}{I - I_m}$$

$I = I_m + I_p$ da $\frac{I_x}{I_g} = \frac{R_g}{R_x}$

Dezibelumrechnung:

$U[dB] = 20 \log_{10} \left(\frac{U_2}{U_1} \right) dB$

$P[dB] = 10 \log_{10} \left(\frac{P_2}{P_1} \right) dB$

Frequenzgang:

$U_e = \hat{U}_e \sin(\omega t)$

$U_a = \hat{U}_a \sin(\omega t + \varphi)$

statische Messfehler

Mittelwert

$$\bar{x} = \frac{1}{N}\sum_{i=1}^{N} x$$

Varianz:

$$s^2(x_i) = \frac{1}{N-1}\sum_{i=1}^{N}(x_i - x)^2$$

Standartabweichung

$$s(x_i) = \sqrt{\frac{\sum (x_i - \bar{x})^2}{N-1}}$$

Standartabweichung von Mittelwert

$$s(\bar{x}) = \frac{s(x_i)}{\sqrt{N}} = \sqrt{\frac{1}{N(N-1)}\sum (x_i - \bar{x})^2} \Rightarrow$$

Messergebnis

$$y = \bar{x} \pm a \cdot \frac{s(x_i)}{\sqrt{N}}$$

Studentsche Verteilung

$$u(\bar{x}) = t \cdot s(\bar{x}) \Rightarrow y = \bar{x} \pm t \cdot s(\bar{x})$$

Fortpflanzung von Messfehlern

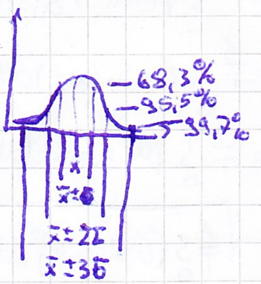
$+$: $a_1 x_1 + a_2 x_2 + a_3 x_3 \dots = y$

$a_1 \Delta x_1 + a_2 \Delta x_2 + a_3 \Delta x_3 \dots = \Delta y$

$\cdot$: $a_1 x_1^{b_1} \cdot a_2 x_2^{b_2} \cdot a_3 x_3^{b_3} \dots = y$

$$\Rightarrow \frac{\Delta y}{y} = \sum_{i=1} b_i \frac{\Delta x_i}{x_i}$$

Gauß Normalverteilung



$a = 1 : 68,3\% \Rightarrow \bar{x} \pm 1 \cdot u(x_i)$

$a = 2 : 95,5\% \Rightarrow \bar{x} \pm 2 \cdot u(x_i)$

$a = 3 : 99,7\% \Rightarrow \bar{x} \pm 3 \cdot u(x_i)$

$$u(x_i) = s(\bar{x})$$

Nomenklatur

x: Eingangsschätzwert → Messwert

y: Messergebnis

u: Unsicherheit des Ergebnisses

$$x_{korr} = x - \Delta x = x\left(1 - \frac{\Delta x}{x}\right)$$

Messunsicherheit

Mu_{abs} = Maximaler Messbereich · Genauigkeitsklasse%

$Mu_{rel} = \frac{Mu_{abs}}{\text{Messwert}}$

Genauigkeitsklasse → in % bei Max. Ausschlag

dynamische Messfehler

$\Delta x(t) = x_a(t) - k\, x_e(t)$

$G_{ges}(j\omega) = \frac{a \cdot k}{1 + j\omega T}$; $a<1$

$$|G(j\omega)| = \frac{1}{k} = \frac{U_a}{U_e}$$

$$\omega_g = 2\pi f_g$$

~~…~~h Grenzfrequenz

Kondensator

$$C = \frac{Q}{U} = \varepsilon_0 \varepsilon_r \frac{A}{d}$$

Laden:

$$U_C(t) = U_0\left(1 - e^{-\frac{t}{\tau}}\right)$$

Entladen:

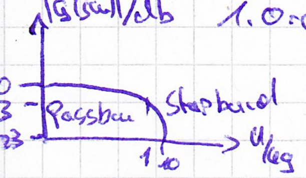
$$U_C(t) = U_0 e^{-\frac{t}{\tau}}$$

$$\tau = R \cdot C$$

$$\boxed{\begin{aligned} Z_R &= R \\ Z_C &= \frac{1}{j\omega C} \\ Z_L &= j\omega L \end{aligned}}$$

---

1. Ordnung



komplex P

$\underline{P} = \underline{U}_{eff} \cdot \underline{I}_{eff} = U_{eff} \cdot I_{eff} \cdot e^{j\varphi_{u,i}} = Re(P) + j\,Im(P)$

$Re(P) = P_W = U_{eff} \cdot I_{eff} \cdot \cos(\varphi_{u,i})$ $\quad |\underline{P}| = P_S = U_{eff} \cdot I_{eff}$

$Im(P) = P_B = U_{eff} \cdot I_{eff} \cdot \sin(\varphi_{u,i})$ $\quad P = \sqrt{P_W^2 + P_B^2}$

$$\varphi = \arg(z) = \begin{cases} \arctan\frac{b}{a} & a>0 & b \text{ beliebig} \\ \arctan\frac{b}{a} + \pi & a<0 & b \geq 0 \\ \arctan\frac{b}{a} - \pi & a<0 & b<0 \\ \pi/2 & a=0 & b>0 \\ -\pi/2 & a=0 & b<0 \\ \text{unbestimmt} & a=0 & b=0 \end{cases}$$

$a = Re$

$b = Im$

$\underline{P_W}$ = Wirk. $\quad U_R \cdot I$ →

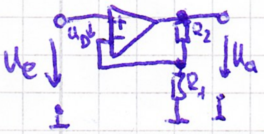
$\underline{P_B}$ = Blind $\quad U_L\, I \cdot \sin\varphi$ ↑

$\underline{P_S} = P_W + P_B$

Operationsverstärker

Wichtig: $A=\infty$; $Re=\infty$; $R_a=0$; $U_D=0$ (bei Entkopplung) $f=\frac{U_{out}}{U_{in}}=\frac{U_a}{U_e}$

Verstärker:

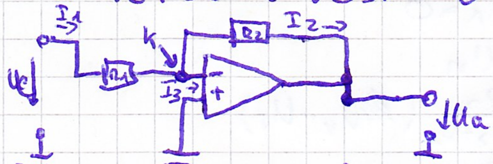


$M1: -U_e+U_D+U_{R_1}=0$

$\Rightarrow U_e=U_a$

$\Rightarrow \frac{U_a}{U_{R_1}}=\frac{R_1+R_2}{R_1} \Rightarrow \frac{U_a}{U_e}=\frac{R_2}{R_1}+1$

Invertierender Verstärker



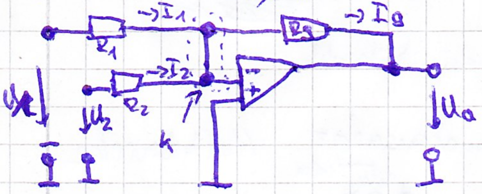
$R_i=\infty \Rightarrow I_3=0 \Rightarrow I_1=I_2=I$

$M_1: U_e=U_{R_1}=I\cdot R_1$

$M_2: U_a=-U_{R_2}=-I\cdot R_2$

$\Rightarrow \frac{U_a}{U_e}=\frac{-I\cdot R_2}{I\cdot R_1}=-\frac{R_2}{R_1}$

(Invertierender) Addierer:



$k: I_1+I_2=I_g$

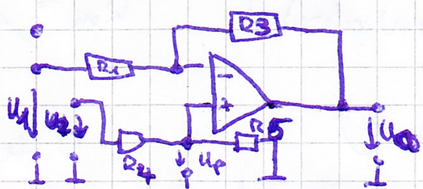
$\frac{U_1}{R_1}+\frac{U_2}{R_2}=\frac{U_g}{R_g}$

M. Ausgang: $U_{R_g}=-U_a$

$\Rightarrow \frac{-U_a}{R_g}=\frac{U_1}{R_1}+\frac{U_2}{R_2}$

für $R_1=R_2=R_g \Rightarrow U_a=-(U_1+U_2)$

Subtrahierer:



$U_D=0$ | ② Spannungsteiler $U_p=\frac{R_5}{R_4+R_5}\cdot U_2$

① $M_1: -U_1+U_{R_1}+U_p=0 \Rightarrow U_{R_1}=U_1-U_p$

$M_2: -U_a-U_{R_3}+U_p=0 \Rightarrow U_{R_3}=U_p-U_a$

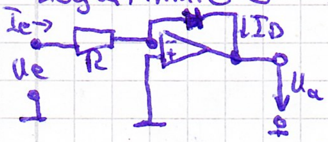
$k: \frac{U_1-U_p}{R_1}=\frac{U_p-U_a}{R_3}$

③ $\frac{U_1}{R_1}-\frac{U_p}{R_1}=\frac{U_p}{R_3}-\frac{U_a}{R_3}$; $\frac{U_p}{R_1}+\frac{U_p}{R_3}=\frac{U_1}{R_1}+\frac{U_a}{R_3}$

$U_p\left(\frac{1}{R_1}+\frac{1}{R_3}\right)=\frac{U_1}{R_1}+\frac{U_a}{R_3}$ $\mid -\frac{U_1}{R_1}\cdot R_3$

② in ③ $U_a=\frac{R_5}{R_4+R_5}U_2\,\frac{R_1+R_3}{R_1}-\frac{R_3}{R_1}\cdot U_1$ | für $R_1=R_3$ & $R_4=R_5$ $U_a=U_2-U_1$

Logarithmierer



Diode $I_D=I_S\left(e^{\frac{U_D}{n\cdot U_T}}-1\right)$ $n=1$, $I_S \ll I_e$

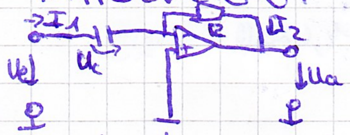
$k: I_e=I_D$; $I_e=\frac{U_e}{R}$

$M_1: I_D=I_S\cdot e^{-\frac{U_D}{U_T}}=I_S\cdot e^{-\frac{U_a}{U_T}}$

$\frac{U_e}{R}=I_S\cdot e^{-\frac{U_a}{U_T}}$ $\ln\left(\frac{U_e}{R\cdot I_S}\right)=-\frac{U_a}{U_T}$

$U_a=-\ln\left(\frac{U_e}{I_S R}\right)U_T$

Differenzierer:



$I_1=I_2$

$I_1=C\frac{dU_C}{dt}$ $I_2=\frac{U_R}{R}$

$M_1: U_e=U_C$

$M_2: U_R=-U_a$; $C\frac{dU_e}{dt}=-\frac{U_a}{R}$

$U_a=-R\cdot C\,\frac{dU_e}{dt}$

Integrierer:

① $I_1=\frac{U_e}{R}$ $I_2=-C\frac{dU_a}{dt}$

$\frac{U_e}{R}=-C\frac{dU_a}{dt}$ $U_e=-RC\frac{dU_a}{dt}$

$\Rightarrow U_a=-\frac{1}{RC}\cdot\int U_e\,dt$

② $\frac{U_{in}}{R_g}=-\frac{U_{out}}{\frac{1}{j\omega C}}$ $G(j\omega)=\frac{U_{out}}{U_{in}}=-\frac{1}{j\omega CR}$

$=j\frac{1}{\omega CR}$

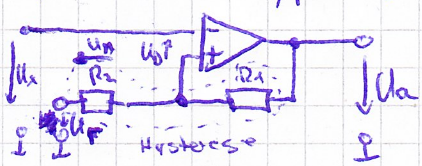
$|G(j\omega)|=\frac{1}{\omega CR}$ da $Re=0$, $Im=\frac{1}{\omega CR}$

$|G(j\omega)|_{dB}=20\log_{10}\left(\frac{1}{\omega CR}\right)$

$\varphi_{j\omega}=\arctan\left(\frac{Im\{G(j\omega)\}}{Re\{G(j\omega)\}}\right)$

$=\frac{\pi}{2}=90°$ gilt nicht wenn $Re=0$!

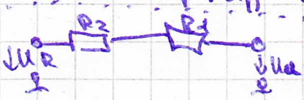
Komparator mit ~~Hyst~~ Hysterese



$U_D\neq 0$

$M_1: -U_x-U_d+U_n+U_R=0$

$U_d=U_r+U_n-U_x$



Spannungsteiler: $\frac{U_n}{U_{ges}}=\frac{R_{an}}{R_{ges}}=\frac{R_2}{R_2+R_1}$

$-U_r-U_{ges}+U_a=0$

$U_{ges}=U_a-U_r$

$U_n=\frac{R_2}{R_1+R_2}\cdot(U_a-U_r)$

ADC

Komperator mit Hysterese

Kippbedingung:

$\Rightarrow U_r + U_n - U_x = 0$

$\Rightarrow U_1 = U_r + U_n$

$U_1 = U_r + \frac{R_2}{R_2+R_1}(U_v - U_r)$

$U_2 = U_r + \frac{R_2}{R_2+R_1}(-U_v - U_r)$

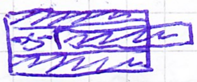
SNR: $\frac{U_N^2}{U_R^2} = \frac{P_N}{P_R}$ Nutzen / Rauschen

$= 10 \log \frac{P_N}{P_R}$ dB

Induktiver Aufnehmer

$L = \frac{N^2}{R_m}$ ← Windungszahl, ← magnetischer Wiederstand

$R_m = \frac{s}{\mu_0 \mu_r \cdot A}$ ← Windungslänge der Feldlinien, ↑ Permebialitätszahl



$E = \frac{dL}{ds} = -\frac{\mu_0 \cdot A \cdot N^2}{s^2} = -\frac{L}{s}$

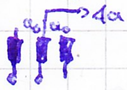
Kapazitiver Aufnehmer

$E = \frac{dC}{dd} = \frac{\varepsilon_0 \varepsilon_r A}{d^2} = -\frac{C}{d}$ ← Plattenabstand

$\frac{dC}{C} = -\frac{da}{a}$

Differenzialkondensator Halbbrücke:

$U_d = \frac{U_0}{2a_0} \Delta a$



DMS

$\varepsilon = \frac{\Delta L}{L}$ $[\varepsilon] = \frac{\mu m}{m}$ Dehnung

$R = \rho \frac{L}{A}$ $\varepsilon = \frac{dR}{R \cdot k}$

↑ Metall abhängig