Mathematik für Medientechnik
Aufgabenblatt 6 (20.11.2009)

1. Berechnen Sie die Grenzwerte folgender Folgen – falls möglich:

a) $c_n = \frac{n^3-n^2+n-7}{2n^3+8}$

b) $d_n = \left(\frac{n^2-2}{n^2+3}\right)^{n^2}$

c) $e_n = \left(\frac{n-2}{n+3}\right)^{3n-1}$

d) $f_n = \frac{\sqrt{n^2-1}}{\sqrt{n+1}}$

e) $g_n = \frac{3n^2+4n}{2n-1}$

f) $h_n = \frac{2n^2-5n+7}{7n^2+3n-1}$

g) $i_n = \frac{\sqrt{n^2+2n+2}+3n-4}{n+2}$

h) $h_n = \frac{1}{\sqrt{n^2+n}-n}$

i) $k_n = \sqrt{4n^2+5n+2} - 2n$

j) $l_n = \left(1-\frac{1}{n}\right)^n$

k) $m_n = -\left(1+\frac{1}{n}\right)^{n^2}$

Benutzen Sie bei den Aufgaben 1h) und 1i) zur (unbedingt nötigen) Umformung die Erweiterung $a_n - b_n = \frac{(a_n-b_n)\cdot(a_n+b_n)}{(a_n+b_n)}$

1a)	½	1b)	exp(-5)
1c)	exp(-15)	1d)	bestimmte Divergenz gegen $+\infty$
1e)	bestimmte Divergenz gegen $+\infty$	1f)	2/7
1g)	4	1h)	2
1i)	5/4	1j)	1/e
1k)	bestimmte Divergenz gegen $-\infty$		

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d) $f_n = \frac{\sqrt{n^2-1}}{\sqrt{n+1}}$

$(n+1)\cdot(n-1) = n^2-1$

$$\lim_{n\to\infty} \frac{\sqrt{n^2-1}}{\sqrt{n+1}} = \lim_{n\to\infty} \frac{\sqrt{(n+1)\cdot(n-1)}}{\sqrt{n+1}} =$$

$$= \lim_{n\to\infty}\left(\frac{\cancel{\sqrt{n+1}}}{\cancel{\sqrt{n+1}}}\cdot\sqrt{n-1}\right) = \lim_{n\to\infty}\sqrt{n-1} = \infty$$

Bestimmte Divergenz gegen $+\infty$

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e) $$\lim_{n\to\infty}\left(\frac{3n^2+4n}{2n-1}\right) = \lim_{n\to\infty}\left(n+\frac{n^2+5n}{2n-1}\right) = \lim_{n\to\infty}(n) + \lim_{n\to\infty}\left(\frac{n^2+5n}{2n-1}\right)$$

$$\boxed{\begin{array}{l} 3n^2+4n : 2n-1 = n \\ \underline{-2n^2+1n} \\ \quad n^2+5n \end{array}}$$

$\lim_{n\to\infty}(n) = +\infty$

$$\lim_{n\to\infty}\left(\frac{n^2+5n}{2n-1}\right) = \lim_{n\to\infty}\left(\frac{\overset{+\infty}{n+5}}{2-\underset{\to 0}{\cancel{\frac{1}{n}}}}\right) = -\infty$$

Beide Summanden $\to +\infty$

Bestimmte Divergenz gegen $+\infty$

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f) $$\lim_{n\to\infty}\frac{2n^2-5n+7}{7n^2+3n-1} = \lim_{n\to\infty}\frac{2-\frac{5}{n}+\frac{7}{n^2}}{7+\frac{3}{n}-\frac{1}{n^2}} = \frac{2}{7}$$

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g) $$\lim_{n\to\infty}\frac{\sqrt{n^2+2n+2}+3n-4}{n+2} = \lim_{n\to\infty}\frac{\sqrt{n^2+2n+2}}{n+2} + \lim_{n\to\infty}\frac{3n-4}{n+2}$$

Annahme: beide Teile konvergieren

$$\boxed{(n+2) = \sqrt{(n+2)^2} = \sqrt{n^2+2n+4}}$$

$$\lim_{n\to\infty}\frac{3n-4}{n+2} = \lim_{n\to\infty}\frac{3-4/n}{1+2/n} = 3$$

$$\lim_{n\to\infty}\frac{\sqrt{n^2+2n+2}}{n+2} = \frac{\sqrt{n^2+2n+1}}{\sqrt{n^2+2n+4}} = \lim_{n\to\infty}\sqrt{\frac{n^2+2n+1}{n^2+2n+4}} = \lim_{n\to\infty}\sqrt{\frac{1+\frac{2}{n}+\frac{1}{n^2}}{1+\frac{2}{n}+\frac{4}{n^2}}} = 1$$

$\lim i_n = 3+1 = 4$

Mathematik 1 AI (Aufgabenblatt 2 (AKA 6))

1) a) $c_n = \dfrac{n^3-n^2+n-7}{2n^3+8}$

$$\lim_{n\to\infty}\left(\frac{n^3-n^2+n-7}{2n^3+8}\right) = \lim_{n\to\infty}\frac{1-\cancel{\frac{1}{n}}+\cancel{\frac{1}{n^2}}-\cancel{\frac{7}{n^3}}^{=0}}{2+\cancel{8/n^3}_{=0}} = \frac{1}{2}$$

b) $d_n = \left(\dfrac{n^2-2}{n^2+3}\right)^{n^2}$

$$\lim_{n\to\infty}\left(\frac{n^2-2}{n^2+3}\right)^{n^2} = \lim_{n\to\infty}\left(\frac{n^2+3-5}{n^2+3}\right)^{n^2} = \lim_{n\to\infty}\left(1+\frac{-5}{n^2+3}\right)^{n^2} =$$

$$= \lim_{n\to\infty}\left(1+\frac{1}{\frac{n^2+3}{-5}}\right)^{n^2} = \lim_{n\to\infty}\left(1+\frac{1}{\frac{n^2+3}{-5}}\right)^{\frac{n^2+3}{-5}\cdot\frac{-5}{n^2+3}\cdot n^2}$$

Da $\lim_{m\to\infty}\left(1+\frac{1}{m}\right)^m = e$, folgt mit $m = \frac{n^2+3}{-5}$ dass

$$\left[\lim_{n\to\infty}\left(1+\frac{1}{\frac{n^2+3}{-5}}\right)^{\frac{n^2+3}{-5}} = e\right]$$

Weiterhin, da

$$\lim_{n\to\infty} -\frac{5}{n^2+3}\cdot n^2 = \lim_{n\to\infty}\frac{-5n^2}{n^2+3} = \frac{-5}{1+\frac{3}{n^2}} = -5$$

folgt:

$$\lim_{n\to\infty}\left(\left(1+\frac{1}{\frac{n^2+3}{-5}}\right)^{\frac{n^2+3}{-5}}\right)^{-\frac{5\cdot n^2}{n^2+3}} = e^{-5} = \exp(-5)$$

c) $e_n = \left(\dfrac{n-2}{n+3}\right)^{3n-1}$

$$\lim_{n\to\infty}\left(\frac{n-2}{n+3}\right)^{3n-1} = \lim_{n\to\infty}\left(\frac{n+3-5}{n+3}\right)^{3n-1} =$$

$$= \lim_{n\to\infty}\left(1+\frac{-5}{n+3}\right)^{3n-1} = \lim_{n\to\infty}\left(1+\frac{1}{\frac{n+3}{-5}}\right)^{3n-1} = \lim_{n\to\infty}\left(1+\frac{1}{\frac{n+3}{-5}}\right)^{\frac{n+3}{-5}\cdot\left(\frac{-5}{n+3}\right)\cdot(3n-1)}$$

$$= \lim_{n\to\infty}\left(\boxed{\left(1+\frac{1}{\frac{n+3}{-5}}\right)^{\frac{n+3}{-5}}}\right)^{\left(-\frac{5}{n+3}\right)(3n-1)} = e^{-15}$$

$\leftarrow e$

$$\lim_{n\to\infty} -\frac{15n+5}{n+3} = -15$$

1.

h) $$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n}-n} = \lim_{n\to\infty} \frac{1}{\sqrt{n^2+n}-n} \cdot \frac{\sqrt{n^2+n}+n}{\sqrt{n^2+n}+n} =$$

$\boxed{(a+b)\cdot(a-b)=a^2-b^2}$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2+n}+n}{\cancel{n^2}+n-\cancel{n^2}} =$$

$$= \lim_{n\to\infty} \frac{\sqrt{n^2+n}}{n} + \underbrace{\lim_{n\to\infty} \frac{n}{n}}_{=1}$$

$$\boxed{\begin{aligned}\lim_{n\to\infty} \frac{\sqrt{n^2+n}}{n} &= \lim_{n\to\infty} \frac{\sqrt{n^2+n}}{\sqrt{n^2}} = \\ &= \lim_{n\to\infty} \sqrt{\frac{n^2+n}{n^2}} = \\ &= \lim_{n\to\infty} \sqrt{\frac{1+1/n}{1}} = 1\end{aligned}}$$

folgt also:

$$\lim_{n\to\infty} \frac{1}{\sqrt{n^2+n}-n} = 1+1=2 \;!$$

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i) $$\lim_{n\to\infty} \underbrace{\sqrt{4n^2+5n+2}}_{\infty} - \underbrace{2n}_{\infty} = \lim_{n\to\infty} \sqrt{4n^2+5n+2}-2n \cdot \frac{\sqrt{4n^2+5n+2}+2n}{\sqrt{4n^2+5n+2}+2n}$$

$$= \lim_{n\to\infty} \left(\sqrt{4n^2+5n+2}-2n\right) \cdot \frac{\sqrt{4n^2+5n+2}+2n}{\sqrt{4n^2+5n+2}+2n} =$$

$$= \lim_{n\to\infty} \frac{\cancel{4n^2}+5n+2-\cancel{4n^2}}{\sqrt{4n^2+5n+2}+2n} = \lim_{n\to\infty} \frac{5+\underbrace{2/n}_{\to 0}}{\sqrt{4+\underbrace{\frac{5}{n}+\frac{2}{n^2}}_{\to 0}}+2} = \frac{5}{\sqrt{4}+2}$$

$$= \frac{5}{4}$$

j) $$\lim_{n\to\infty}\left(1-\frac{1}{n}\right)^n = \lim_{n\to\infty}\left(1+\frac{1}{-n}\right)^{-n\cdot(-1)} = e^{-1} = \frac{1}{e}$$

k) $$\lim_{n\to\infty} -\left(1+\frac{1}{n}\right)^{n^2} = -\lim_{n\to\infty}\Bigg(\underbrace{\left(1+\frac{1}{n}\right)^n}_{e}\Bigg)^n$$

$\lim_{n\to\infty} e^n = \infty$

$\lim_{n\to\infty} n = \infty$

Bestimmte Divergenz gegen $-\infty$

Aufgabe "17" FTN / Zbirka Seite 17. modified

$a_n = \sqrt{n^2+\lambda n} - n + 1$. Bestimmen Sie $\lambda \in \mathbb{R}$ so, das $\lim_{n\to\infty} a_n = 8$!

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\left(\sqrt{n^2+\lambda n} - n + 1\right) = \lim_{n\to\infty}\underbrace{\sqrt{n^2+\lambda n}}_{a} - \underbrace{(n-1)}_{b}$$

$$= \lim_{n\to\infty}\left(\underbrace{\sqrt{n^2+\lambda n} - (n-1)}_{(a-b)} \cdot \frac{\sqrt{n^2+\lambda n} + (n-1)}{\underbrace{\sqrt{n^2+\lambda n} + (n-1)}_{(a+b)}}\right) =$$

$$= \lim_{n\to\infty}\frac{n^2+\lambda n - (n-1)^2}{\sqrt{n^2+\lambda n} + (n-1)} = \lim_{n\to\infty}\frac{\cancel{n^2} + \lambda n - (\cancel{n^2} - 2n + 1)}{\sqrt{n^2+\lambda n} + (n-1)} =$$

$$= \lim_{n\to\infty}\frac{(\lambda+2)\cdot n - 1}{\sqrt{n^2+\lambda\cdot n} + n - 1} = \lim_{n\to\infty}\frac{\lambda + 2 - \frac{1}{n}}{\sqrt{1+\frac{\lambda}{n}} + 1 - \frac{1}{n}} = \frac{\lambda+2}{2}$$

$$\frac{\lambda+2}{2} = 8 \Rightarrow \lambda + 2 = 16 \Rightarrow \lambda = 14$$