aufgabe 105, e)

$$\int e^x \cdot \cos(e^x)\,dx = \left[\begin{matrix} e^x = t \\ e^x dx = dt \end{matrix} \right\} dx = \frac{dt}{e^x} \Big] =$$

$$= \int \cancel{e^x} \cdot \cos(t) \cdot \frac{dt}{\cancel{e^x}} = \int \cos(t)\,dt = \sin(t) + C = \sin(e^x) + C$$

f) $$\int e^{2\cdot \sin 3x} \cdot \cos(3x)\,dx = \left[\begin{matrix} t = 2\cdot \sin(3x) \\ dt = 6\cdot\cos(3x)\,dx \end{matrix} \right\} \Rightarrow dx = \frac{dt}{6\cos(3x)} \Big]$$

$$= \int e^t \cdot \cancel{\cos(3x)} \cdot \frac{dt}{6\cdot\cancel{\cos(3x)}} = \frac{1}{6}\int e^t\,dt = \frac{1}{6}e^t + C = \frac{1}{6}\cdot e^{2\cdot\sin(3x)} + C$$

g) $$\int \frac{\sin(x)+\cos(x)}{\sin(x)}\,dx = \int \left(1 + \frac{\cos(x)}{\sin(x)}\right)dx = \int dx + \int \frac{\cos(x)}{\sin(x)}\,dx$$

$$\left[\begin{matrix} t = \sin(x) \\ dt = \cos(x)\,dx \end{matrix} \right\} dx = \frac{dt}{\cos(x)} \Big] = x + \int \frac{\cancel{\cos(x)}}{t} \cdot \frac{dt}{\cancel{\cos(x)}} =$$

$$= x + \int \frac{dt}{t} = x + \ln|t| + C = x + \ln|\sin(x)| + C$$

h) $$\int \frac{dx}{1-\cos(x)} = \int \frac{1+\cos(x)}{(1-\cos(x))(1+\cos(x))}\,dx = \int \frac{1+\cos(x)}{1-\cos^2(x)}\,dx$$

$$= \int \frac{1+\cos(x)}{\sin^2(x)}\,dx = \underbrace{\int \frac{1}{\sin^2(x)}\,dx}_{-ctg(x)} + \int \frac{\cos(x)}{\sin^2(x)}\,dx$$

$$\left[\begin{matrix} \sin(x) = t \\ \cos(x)\,dx = dt \end{matrix} \right\} dx = \frac{dt}{\cos(x)} \Big]$$

$$\int \frac{\cos(x)}{\sin^2(x)}\,dx = \int \frac{\cancel{\cos(x)}}{t^2} \cdot \frac{dt}{\cancel{\cos(x)}} = \int \frac{dt}{t^2} = -\frac{1}{t} + C$$

$$= -\frac{1}{\sin(x)} + C \qquad \Rightarrow \int \frac{dx}{1-\cos(x)} = -ctg(x) - \frac{1}{\sin(x)} + C$$

Aufgabe 104.

a) $\int (e^{-x}+1)\,dx = -\int e^{-x}(-dx) + \int dx =$

$\left[\begin{array}{l} t=-x \\ dt=-dx \end{array}\right\} dx=-dt$

$-\int e^{t}\,dt + x = -e^{-x}+x+C$

b) $\int e^{2x}dx = \frac{1}{2}\int e^{t}dt = \frac{1}{2}e^{t}+c = \frac{1}{2}\cdot e^{2x}+C$

$\left[\begin{array}{l} 2x=t \\ 2dx=dt \end{array}\right.$ $dt=\frac{1}{2}dx$

c) $\int a^{3x}dx = \frac{1}{3}\int a^{t}dt = \frac{1}{3}\cdot\frac{a^{t}}{\ln(a)}+C = \frac{1}{3}\cdot\frac{a^{3x}}{\ln(a)}+C$

$\left[\begin{array}{l} t=3x \\ dt=3dx \end{array}\right\} dx=\frac{1}{3}dt$

d) $\int \frac{e^{-\frac{1}{x}}}{x^{2}}dx =$ $\left[\begin{array}{l} t=-\frac{1}{x}=-x^{-1} \\ dt=-(-x^{-2})=\frac{1}{x^{2}}dx \end{array}\right\} dx=x^{2}\cdot dt$

$=\int \frac{e^{t}}{\cancel{x^{2}}}\cancel{x^{2}}\cdot dt = \int e^{t}dt = e^{t}+c = e^{-1/x}+C$

e) $\int e^{x}\sqrt{e^{x}+1}\,dx =$ $\left[\begin{array}{l} t=e^{x}+1 \\ dt=e^{x}dx \end{array}\right\} dx=\frac{dt}{e^{x}}$

$=\int \cancel{e^{x}}\cdot t^{\frac{1}{2}}\cdot\frac{dt}{\cancel{e^{x}}} = \int t\cdot dt = \frac{t^{\frac{3}{2}}}{\frac{3}{2}}+C = \frac{2(e^{x}+1)^{\frac{3}{2}}}{3}$

f) $\int \frac{dx}{e^{x}+1} = \int \frac{e^{-x}dx}{1+e^{-x}}$ $\left[\begin{array}{l} t=1+e^{-x} \\ dt=-e^{-x}dx \end{array}\right\} dx=\frac{-dt}{e^{-x}}$

104, f) Fortsetzung

$$= \int \frac{\cancel{e^{-x}}}{t} \cdot \left(-\frac{dt}{\cancel{e^{-x}}}\right) = -\int \frac{dt}{t} = -\ln(t) + C = -\ln(1+e^{-x}) + C$$

!

Aufgabe 105.

a) $$\int (\sin(2x) + \cos(3x))\,dx = \int \sin(2x)\,dx + \int \cos(3x)\,dx$$

$$\left.\begin{matrix} 2x = t \\ 2dx = dt \end{matrix}\right\} dx = \frac{dt}{2} \qquad \left.\begin{matrix} 3x = p \\ 3dx = dp \end{matrix}\right\} dx = \frac{dp}{3}$$

$$= \int \sin(t)\,\frac{dt}{2} + \int \cos(p)\,\frac{dp}{3} = -\frac{1}{2}\cos(t) + \frac{1}{\cancel{2}}\sin(p) + C$$

$$= -\frac{1}{2}\cdot\cos(2x) + \frac{1}{3}\sin(3x) + C$$

b) $$\int \sin^3(x)\cdot\cos(x)\,dx = \left.\begin{matrix} t = \sin(x) \\ dt = \cos(x)\,dx \end{matrix}\right\} dx = \frac{dt}{\cos(x)}$$

$$= \int t^3 \cdot \cancel{\cos(x)} \cdot \frac{dt}{\cancel{\cos(x)}} = \int t^3 \cdot dt = \frac{t^4}{4} + C = \frac{\sin^4(x)}{4} + C$$

c) $$\int \operatorname{tg}(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = \left.\begin{matrix} t = \cos(x) \\ dt = -\sin(x)\,dx \end{matrix}\right\} dx = -\frac{dt}{\sin(x)}$$

$$= \int \frac{1}{t} \cdot \cancel{\sin(x)} \cdot \frac{dt}{-\cancel{\sin(x)}} = -\int \frac{dt}{t} = -\ln|t| + C = -\ln|\cos(x)| + C$$

d) $$\int x^2 \cdot \operatorname{ctg}(x^3)\,dx = \left.\begin{matrix} t = x^3 \\ dt = 3x^2 \end{matrix}\right\} dx = \frac{dt}{3x^2} = \int \cancel{x^2} \cdot \operatorname{ctg}(t) \cdot \frac{dt}{3\cancel{x^2}} =$$

$$= \frac{1}{3}\int \operatorname{ctg}(t)\,dt = \frac{1}{3}\int \frac{\cos(t)}{\sin(t)}\,dt \qquad \left.\begin{matrix} u = \sin(t) \\ du = \cos(t)\,dt \end{matrix}\right\} dt = \frac{du}{\cos(t)}$$

folgt also: $$= \frac{1}{3}\int \frac{\cancel{\cos(t)}}{u} \cdot \frac{du}{\cancel{\cos(t)}} = \frac{1}{3}\int \frac{du}{u} = \frac{1}{3}\ln|u| + C$$

$$= \frac{1}{3}\ln|\sin(t)| + C = \frac{1}{3}\ln|\sin(x^3)| + C$$

Aufgabe 107

$$ax^2+bx+c \to Au^2+B \qquad u=u(x)$$

a) $\displaystyle\int \frac{dx}{x^2+10x-31}$

$$\int \frac{dx}{x^2+10x-31} = \int \frac{dx}{(x^2+10x+25)+6} = \int \frac{dx}{\left(x^2+10x+\left(\frac{10}{2}\right)^2\right)+6}$$

$$= \int \frac{dx}{(x+5)^2+6} = \int \frac{dx}{(x+5)^2+(\sqrt{6})^2} \qquad \begin{array}{l} t=x+5 \\ dt=dx \end{array}$$

$$= \int \frac{dt}{t^2+B^2} = \frac{1}{B}\cdot \operatorname{arctg}\left(\frac{t}{B}\right)+C = \qquad B=\sqrt{6}$$

$$= \frac{1}{\sqrt{6}}\cdot \operatorname{arctg}\left(\frac{x+5}{\sqrt{6}}\right)+C$$

b) $$\int \frac{dx}{25-8x+x^2} = \int \frac{dx}{9+(16-8x+x^2)} = \int \frac{dx}{9+(4-x)^2}$$

$$= \int \frac{dx}{3^3+(4-x)^2} \qquad \begin{array}{l} t=4-x \\ dt=-dx \end{array} \qquad = -\int \frac{dx}{3^2+t^2} =$$

$$= \frac{1}{3}\operatorname{arctg}\left(\frac{t}{3}\right)+C = \frac{1}{3}\cdot \operatorname{arctg}\left(\frac{4-x}{3}\right)+C$$

c) $$\int \frac{dx}{2x^2-2x+5} = \int \frac{2\,dx}{4x^2-4x+10} = \int \frac{2dx}{(2x-1)^2+9} =$$

$$= \int \frac{2dx}{(2x-1)^2+3^2} \qquad \left.\begin{array}{l} t=2x-1 \\ dt=2dx \end{array}\right\} dx=\frac{dt}{2}$$

$$= \int \frac{dt}{t^2+3^2} = \frac{1}{3}\cdot \operatorname{arctg}\frac{t}{3}+C = \frac{1}{3}\operatorname{arctg}\left(\frac{2x-1}{3}\right)+C$$

d)