Midterm Case Study
Airline Cargo Optimization Using Linear Programming
MGSC 404 section 001 - Group 4 & 5

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Midterm Case Study

PHASE ONE

Results

a) Using linear programming, determine the optimal transportation plan to minimize costs while fulfilling demand at the destination.

Using our linear programming model and Gurobi, we found that the optimal transportation plan that fulfills demand and minimizes costs is the following;

On flight A, we should transport 0 units of item 1, 50 units of item 2, 30 units of item 3, 76 units of item 4, and 0 units of item 5.
On flight B, we should transport 100 units of item 1, 0 units of item 2, 0 units of item 3, 4 units of item 4, and 40 units of item 5.

With this transportation plan, the optimal objective value or the minimum total shipping cost is $6,075.

Sensitivity Analysis

The shadow prices show the cost impact of changing the RHS of each constraint. For example, increasing the demand for Item 1 by one unit would increase the total cost by $25. The analysis also shows how much each constraint can be increased or decreased before the shadow price becomes invalid. The allowable increase and decrease for the RHS of each constraint give a range where the shadow prices' effects on the objective value remain valid.

The weight limit of Plane B has a shadow price of -$0.5. This indicates that loosening the weight constraint of Plane B by around 13.33kg could decrease the total transportation cost by $0.5/kg.

The capacity limits of both planes and the freezer constraint did not have a shadow price, meaning these constraints are non-binding. This tells us that in the optimal solution, these constraints are not fully utilized and that there was some surplus or unused capacity.

b) Additionally, what recommendations would you suggest to further reduce costs for the company?
The shadow price for the weight constraint on Plane B being negative suggests that increasing its weight capacity could further reduce costs. Specifically, each additional kilogram of capacity on Plane B could decrease total costs by $0.5. The airline should look into the feasibility and cost of such an increase, considering operational factors like shipment, pricing, and loading logistics. A cost-benefit analysis comparing the expense of increasing capacity against the projected savings is a sound approach. Essentially, we would be willing to pay up to $0.5 per extra kg increase of weight capacity on flight B until we reach an increase of weight capacity of around 13.33kg, in order to ship a total of 1513.33kg on this flight.

The shadow prices for demand offer valuable insights for negotiation strategies. Constraints with higher shadow prices have a greater cost impact when the RHS fluctuates. For instance, the shadow prices of items 1,2,3,4,5's demand constraints are $25, $20, $37.50, $12.5, and $30. This means we are willing to pay a partner shipping company up to $25 per unit of item 1 they transport for us, for up to 8 units, and we would have the same or lower total costs depending on our negotiations. This logic applies the same way to the demand constraints and shadow prices for items 2,3,4, and 5. This way, we can satisfy demand at the same, or lower total costs, as transporting these items ourselves.

In addition, we can consider adjusting prices for these highly demanded items, aiming for flexible contracts that accommodate demand variations within the allowable increase/decrease ranges. Long-term agreements or strategic partnerships could provide revenue stability. Exploring dynamic pricing based on real-time demand and capacity could further optimize resource utilization.

PHASE TWO

Results

a) Using linear programming, determine the optimal cargo plan and identify the minimum transportation cost

Using our linear programming model and Gurobi, we found that the optimal cargo plan that fulfills demand and minimizes costs is the following;

On route 1, from Montreal to Vancouver, we should transport 40 units of item 1, 70 units of item 2, 60 units of item 3, 50 units of item 4, and 200 items of item 5.

On route 2, from Montreal to Toronto, we should transport 30 units of item 1 and 100 units of item 5.

On route 3, from Toronto to Vancouver, we should transport 30 units of item 4.

On route 4, from Montreal to Vancouver, no item should be transported.

On route 5, from Vancouver to Toronto, we should transport 20 units of item 2 and 120 units of item 3.

With this cargo plan, the optimal objective value or the minimum total transportation cost is $18,093.00.

Sensitivity Analysis

b) Provide a comprehensive sensitivity analysis table.

	Constraint	Shadow Price	RHS	Minimum RHS	Allowable Decrease	Maximum RHS	Allowable Increase
0	0	18.0	70.0	70.0	0.0	70.0	0.0
1	1	38.4	70.0	70.0	0.0	70.0	0.0
2	2	72.0	60.0	60.0	0.0	60.0	0.0
3	3	9.0	50.0	50.0	0.0	50.0	0.0
4	4	21.6	300.0	300.0	0.0	300.0	0.0
5	5	0.0	0.0	-30.0	30.0	inf	inf
6	6	0.0	0.0	0.0	0.0	inf	inf
7	7	0.0	0.0	0.0	0.0	inf	inf
8	8	0.0	30.0	30.0	0.0	inf	inf
9	9	0.0	0.0	-100.0	100.0	inf	inf

10	10	0.0	0.0	-40.0	40.0	inf	inf
11	11	0.0	0.0	-50.0	50.0	inf	inf
12	12	0.0	60.0	60.0	0.0	inf	inf
13	13	0.0	0.0	-80.0	80.0	inf	inf
14	14	0.0	0.0	-200.0	200.0	inf	inf
15	15	7.0	30.0	30.0	0.0	30.0	0.0
16	16	0.0	20.0	20.0	0.0	20.0	0.0
17	17	0.0	120.0	120.0	0.0	120.0	0.0
18	18	-11.5	-30.0	-30.0	0.0	-30.0	0.0
19	19	8.4	100.0	100.0	0.0	100.0	0.0
20	20	0.0	40.0	40.0	0.0	40.0	0.0
21	21	-24.0	50.0	50.0	0.0	50.0	0.0
22	22	-45.0	-60.0	-60.0	0.0	-60.0	0.0
23	23	0.0	80.0	80.0	0.0	80.0	0.0
24	24	0.0	200.0	200.0	0.0	200.0	0.0
25	25	0.0	6000.0	4510.0	1490.0	inf	inf
26	26	0.0	6800.0	1500.0	5300.0	inf	inf
27	27	0.0	5700.0	150.0	5550.0	inf	inf
28	28	0.0	7000.0	0.0	7000.0	inf	inf
29	29	0.0	9400.0	1960.0	7440.0	inf	inf
30	30	0.0	5000.0	2390.0	2610.0	inf	inf
31	31	0.0	7000.0	850.0	6150.0	inf	inf
32	32	0.0	7350.0	90.0	7260.0	inf	inf
33	33	0.0	7800.0	0.0	7800.0	inf	inf
34	34	0.0	8500.0	800.0	7700.0	inf	inf
35	35	-34.5	0.0	0.0	0.0	60.0	60.0

We utilized Pandas, a powerful Python library for data manipulation, to extract and present the sensitivity analysis results in a structured format. After solving the optimization model using Gurobi, we accessed key constraint attributes, such as shadow prices, right-hand side (RHS) values, and allowable increase/decrease limits. These values were compiled into a Pandas DataFrame, which was then formatted and displayed as a clean, easy-to-read table. Allowing for data-driven decision-making without having to sift through raw computational outputs.

C) If Toronto's demand for item 3 increases by 100 units, recommend how many units should be fulfilled from Montreal and how many from Vancouver. In this scenario, what would be the optimal cargo plan and the corresponding transportation cost?

As seen in the new linear programming model, the optimal solution would be to increase Vancouver's supply of item 3 by 100 units and transport this extra 100 units of item 3 with route 5 going from Vancouver to Toronto. This route is the most expensive at $3/kg, thus the total cost will increase by $4,500. The rest of the items and flights transportation allocations would remain unchanged in the new optimal solution. Therefore, the new optimal objective value or the new minimum total cost would be $22,593.00 after increasing the supply of item 3 by 100 units in Vancouver in order to fulfill the increase in demand for item 3 in Toronto.

Appendices

Appendix 1: Mathematical Formulation for Phase 1

Decision Variables:
X_i: Quantity of item i transported by airplane A, where i = 1, 2, 3, 4, 5.
Y_i: Quantity of item i transported by airplane B, where i = 1, 2, 3, 4, 5.

Objective Function (Minimize Cost):
Minimize $Z = 2.5 * (10X_1 + 8X_2 + 15X_3 + 5X_4 + 12X_5) + 2 * (10Y_1 + 8Y_2 + 15Y_3 + 5Y_4 + 12Y_5)$

Constraints:
Demand Constraints:
- $X_1 + Y_1 = 100$ (Item 1)
- $X_2 + Y_2 = 50$ (Item 2)
- $X_3 + Y_3 = 30$ (Item 3)
- $X_4 + Y_4 = 80$ (Item 4)
- $X_5 + Y_5 = 40$ (Item 5)

Weight Constraints:
- $10X_1 + 8X_2 + 15X_3 + 5X_4 + 12X_5 \leq 2000$ (Airplane A)
- $10Y_1 + 8Y_2 + 15Y_3 + 5Y_4 + 12Y_5 \leq 1500$ (Airplane B)

Volume Constraints:
- $5X_1 + 4X_2 + 6X_3 + 3X_4 + 7X_5 \leq 1000$ (Airplane A)
- $5Y_1 + 4Y_2 + 6Y_3 + 3Y_4 + 7Y_5 \leq 800$ (Airplane B)

Freezer Constraint:
- $Y_2 = 0$ (Item 2 cannot be transported by plane B)
- $Y_3 = 0$ (Item 3 cannot be transported by plane B)

Non-Negativity Constraints:
- $X_i \geq 0$ for all i=1, 2, 3, 4, 5
- $Y_i \geq 0$ for all i=1, 2, 3, 4, 5

Appendix 2: Mathematical Formulation for Phase 2

Decision Variables:
A_i: Quantity of item i transported on route A (Montreal to Vancouver), where i = 1, 2, 3, 4, 5.
B_i: Quantity of item i transported on route B (Montreal to Toronto, where i = 1, 2, 3, 4, 5.
C_i: Quantity of item i transported on route C (Toronto to Vancouver), where i = 1, 2, 3, 4, 5.
D_i: Quantity of item i transported on route D (Montreal to Vancouver), where i = 1, 2, 3, 4, 5.
E_i: Quantity of item i transported on route E (Vancouver to Toronto), where i = 1, 2, 3, 4, 5.

Objective Function (Minimize Cost):
Minimize $Z = 1.8(10A_1 + 8A_2 + 15A_3 + 5A_4 + 12A_5) + 2.5(10B_1 + 8B_2 + 15B_3 + 5B_4 + 12B_5) + 2.3(10C_1 + 8C_2 + 15C_3 + 5C_4 + 12C_5) + 2(10D_1 + 8D_2 + 15D_3 + 5D_4 + 12D_5) + 3(10E_1 + 8E_2 + 15E_3 + 5E_4 + 12E_5)$

Constraints:
Supply Constraints for Products Leaving Montreal:
- $A_1 + B_1 + D_1 = 70$ (Item 1)
- $A_2 + B_2 + D_2 = 70$ (Item 2)
- $A_3 + B_3 + D_3 = 60$ (Item 3)

- $A_4 + B_4 + D_4 = 50$ (Item 4)
- $A_5 + B_5 + D_5 = 300$ (Item 5)

Supply Constraints for Products Leaving Toronto:
- $C_i - B_i \leq 0$ for all i=1,2,3,5
- $C_4 - B_4 \leq 30$ (Item 4)

Supply Constraints for Products Leaving Vancouver:
- $E_i - A_i - C_i - D_i \leq 0$ for all i=1,2,4,5
- $E_3 - A_3 - C_3 - D_3 \leq 60$ (Item 3)

Demand Constraints for Toronto:
- $B_1 + E_1 - C_1 = 30$ (Item 1)
- $B_2 + E_2 - C_2 = 20$ (Item 2)
- $B_3 + E_3 - C_3 = 120$ (Item 3)
- $30 + B_4 + E_4 - C_4 = 0$ (Item 4)
- $B_5 + E_5 - C_5 = 100$ (Item 5)

Demand Constraints for Vancouver:
- $A_1 + C_1 + D_1 - E_1 = 30$ (Item 1)
- $A_2 + C_2 + D_2 - E_2 = 20$ (Item 2)
- $60 + A_3 + C_3 + D_3 - E_3 = 120$ (Item 3)
- $A_4 + C_4 + D_4 - E_4 = 0$ (Item 4)
- $A_5 + C_5 + D_5 - E_5 = 100$ (Item 5)

Weight Constraints for Flights:
- $10A_1 + 8A_2 + 15A_3 + 5A_4 + 12A_5 \leq 6000$
- $10B_1 + 8B_2 + 15B_3 + 5B_4 + 12B_5 \leq 6800$
- $10C_1 + 8C_2 + 15C_3 + 5C_4 + 12C_5 \leq 5700$
- $10D_1 + 8D_2 + 15D_3 + 5D_4 + 12D_5 \leq 7000$
- $10E_1 + 8E_2 + 15E_3 + 5E_4 + 12E_5 \leq 9400$

Volume Constraints for Flights:
- $5A_1 + 4A_2 + 6A_3 + 3A_4 + 7A_5 \leq 5000$
- $5B_1 + 4B_2 + 6B_3 + 3B_4 + 7B_5 \leq 7000$
- $5C_1 + 4C_2 + 6C_3 + 3C_4 + 7C_5 \leq 7350$
- $5D_1 + 4D_2 + 6D_3 + 3D_4 + 7D_5 \leq 7800$
- $5E_1 + 4E_2 + 6E_3 + 3E_4 + 7E_5 \leq 8500$

Freezer Constraint:
- $B_2 + B_3 + D_2 + D_3 = 0$ (Items 2 and 3 cannot be transported by routes B and D)

Non-Negativity Constraints:
- $A_i \geq 0$ for all i=1, 2, 3, 4, 5
- $B_i \geq 0$ for all i=1, 2, 3, 4, 5
- $C_i \geq 0$ for all i=1, 2, 3, 4, 5
- $D_i \geq 0$ for all i=1, 2, 3, 4, 5
- $E_i \geq 0$ for all i=1, 2, 3, 4, 5