

**PROJECT**  
**Due Date: December 8<sup>th</sup>, 2019**

**Requirement**

The goal of firing a projectile is so that it lands on a specific target. If we ignore the effects of air resistance, the differential equations describing its acceleration are very simple:

$$\frac{dv_x}{dt} = 0 \qquad \frac{dv_z}{dt} = -g$$

Here  $g = 9.81 \text{ m/s}^2$ , and we can show that these equations result in a parabolic trajectory. Once we know this, we can easily calculate the initial speed  $v_0$  and angle  $\theta_0$  above the horizontal necessary for the projectile to reach the target.

However, the effects of air resistance are significant when the projectile must travel a large distance, and if we modify these equations to include a simple model of air resistance, they become:

$$m \frac{dv_x}{dt} = -cv_x \sqrt{v_x^2 + v_z^2} \qquad m \frac{dv_z}{dt} = -mg - cv_z \sqrt{v_x^2 + v_z^2}$$

Here the constant  $c$  depends on the shape and density of the projectile and the density of air. Develop a model of estimating  $v_0$  and  $\theta_0$  with reasonable accuracy, given the range to the target  $d$ . (Use a second order method, and be sure to persuade me that your numerical solution is accurate enough.)

Your model should consider the following:

- Projectiles fired at different altitudes  $\Delta z$
- External wind forces with various components
- Vertical barriers that might exist
- Projectiles of varying shapes and masses

The model should:

- Calculate the initial speed  $v_0$  and  $\theta_0$  needed to reach the target
- Draw the profile of the flight of the projectile
- Calculate the time at which the projectile has reached the target

Bonus: A bonus of 10% will be added for the group that can also incorporate the Magnus effect into their model accounting for the spinning of the projectile.

## **Write Up**

Present the project in a report format, including but not limited to:

1. Introduction and background

- Presenting background on numerical integration and differentiation methods, illustrating advantages/disadvantages of using such methods

2. Model

- Flow chart or pseudo code illustrating the algorithm developed
- Submit a soft and hard copy of your code
- Code should be accompanied by commentary for ease of review
- Model should be flexible for use on multiple case studies, and not just tailored to a single case study
- Results of your test cases

3. Case Study

- Present background regarding the engineering case study
- Clearly present all equations that are developed to solve the system
- Present the solution algorithm
- Present and discuss results obtained

4. Conclusion

- Discuss your model and areas in which you believe your model can be improved

## **Groups**

Groups should be 3-4 students