3.5 ENERGY CONSERVATION EQUATION

The energy conservation may be usefully represented using a Petri net, as shown in Fig.(11). The energy conservation in a control volume may be

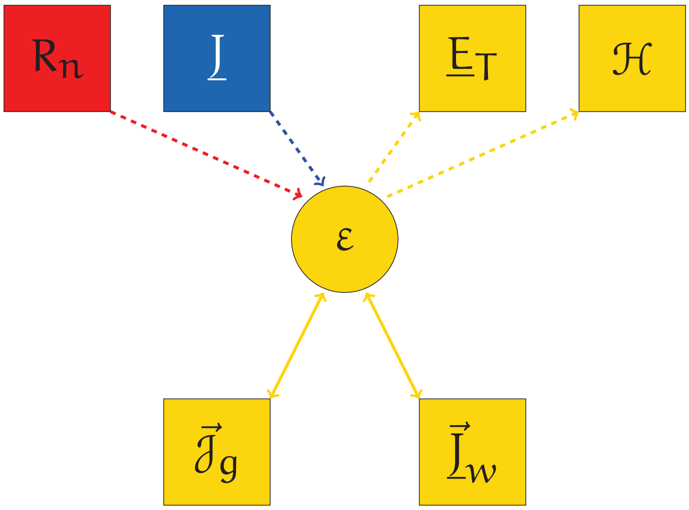

Figure 11: The variation in time of the internal energy per unit volume ε results from the balance of the heat fluxes within the soil and at the ground surface.

Symbol	Name
$\underline{J}$	Heat flux due to precipitation
$\underline{E}_T$	Evaporation heat flux
$\vec{\underline{J}}_w$	Heat advected by flowing water
$\vec{\mathcal{J}}_g$	Conduction flux
R_n	Net radiation
$\mathcal{H}$	Sensible heat
ε	Energy per unit volume

Table 2: Definition for energy conservation terms

described as

$$\frac{\partial}{\partial t}(\varepsilon) = \frac{\partial}{\partial t}(u + e_p) = -\nabla \cdot (\vec{\underline{J}}_w + \vec{\mathcal{J}}_g) \tag{69}$$

where:

- $\vec{\mathcal{J}}_g$ is the conduction flux, accorging to Fourier's law, may be written as

$$\vec{\mathcal{J}}_g = -\lambda \vec{\nabla} T \tag{70}$$

  where λ is the thermal conductivity of the soil that depends on the thermal conductivity of soil particles, and on the water content [17].

- $\vec{\underline{J}}_\theta$ is the heat advected by flowing water, and may be written as

$$\vec{\underline{J}}_w = \rho_w[l + c_w(T - T_m)]\vec{J}_w \tag{71}$$

The boundary condition at the ground surface are specified by prescribing the net radiation R_n, and the heat fluxes due to the precipitation, $\underline{J}$, evapotranspitation, $\underline{E}_T$, and the sensible heat $\mathcal{H}$.

The boundary condition at the bottom of the domain are specified by prescribing both the conduction flux, $\vec{\mathcal{J}}_g$, and the heat advected by flowing water, $\underline{\vec{J}}_w$.

Deriving the right hand side of Eq.(69) one obtains

$$\frac{\partial}{\partial t}(u + e_p) = [\rho_s(1-\theta_s)c_s + \rho_w\theta_w c_w]\frac{\partial T}{\partial t} + \rho_w\left[l + c_w(T - T_m) + gz\right]\frac{\partial \theta_w}{\partial t} \tag{72}$$

Let us define

$$C_T := \rho_s c_s(1-\theta_s) + \rho_w\theta_w c_w \tag{73}$$

the total thermal capacity of the soil volume. As we can see in Eq.(72), the variation in time of the total energy may be seen as the sum of three contributes: the first term of the right hand side is the sensible part and takes into account of the variation of temperature in time, whilst the second one takes into account of the variation of the water content.

Substituting Eq.(72) and Eq.(73) into Eq.(69) gives the following equation

$$C_T\frac{\partial T}{\partial t} + \rho_w\left[l + c_w(T - T_m) + gz\right]\frac{\partial \theta_w}{\partial t} = -\nabla\cdot(\underline{\vec{J}}_w + \vec{\mathcal{J}}_g) \tag{74}$$

Using Eq.(55) in Eq.(74) gives

$$C_T\frac{\partial T}{\partial t} + \rho_w\left[l + c_w(T - T_m) + gz\right]\left(-\nabla\cdot\vec{J}_w\right) = -\nabla\cdot(\underline{\vec{J}}_w + \vec{\mathcal{J}}_g) \tag{75}$$

Finally the energy conservation can be written as

$$C_T\frac{\partial T}{\partial t} + \rho_w c_w\vec{J}_w\cdot\vec{\nabla}T + \rho_w gz\nabla\cdot\vec{J}_w - \nabla\cdot\vec{\mathcal{J}}_g = 0 \tag{76}$$