A. Doerr

Week 9

Minsets, The Duality Principle and Basic Matrix Definitions

We will cover this material as it is given in the text and in class.
Before reading the text study the following list of basic matrix algebra laws.

Some Basic Matrix Laws

Assume the orders of the matrices are such that the following make sense. Once you have studied how to add and multiply matrices you will able specify the orders of the matrices so that these laws make sense? How about the entries of the matrices? After you read the text you should be able to answer these questions. For now compare the following laws with those you are familiar with from high school algebra. Laws 3, 4, 5 and 7 involve a mixture of numbers indicated by lower case letters, often the letter c is used and matrices indicated by upper case letters A, B, and C.

Commutative Law Under Addition * (1) A + B = B + A
Associative Law Under Addition (2) A + (B + C) = (A + B) + C
(3) c(A + B) = cA + cB, where c $\in$ **R**
(4) $(c_1 + c_2)A = c_1A + c_2A$, where $c_1, c_2 \in$ **R**
(5) $c_1(c_2A) = (c_1c_2)A$, where $c_1, c_2 \in$ **R.** and (5a) $cA = Ac$, where c $\in$ **R**
(6) **0**A = **0**, where **0** is the zero matrix
(7) 0A = **0**, where 0 on the left is the number zero
Additive Identity (8) A + **0** = A
Additive Inverse (9) A + (-1)A = **0**
(left) Distributive Law (10) A(B + C) = AB + AC
(right) Distributive Law (11) (B + C) A = BA + CA
Associative Law Under Multiplication (12) A(BC) = (AB)C
Multiplicative Inverse (13) Given an n x n matrix A if there exists a n x n matrix B such that AB = BA = I, th B is called the (multiplicative) inverse of A and it is designated by the symbol, A^{-1}, which is read inverse.
(14) If A^{-1} exist, then $(A^{-1})^{-1} = A$
(15) If A^{-1} and B^{-1} exist, then $(AB)^{-1} = B^{-1}A^{-1}$

Even if you have not studied matrix algebra you should recognize that these laws resemble those of high school algebra. Matrix algebra behaves much like high school algebra. There are some similarities and some differences. The key difference is that the commutative law under multiplication (i.e. ab = ba), which is true for high school algebra, is not true for matrix algebra, that is, *** AB may not be equal to BA**.
The text does not list the basic matrix laws as I have above. Laws 3, 4, 5 and 7 involve a mixture of numbers indicated by lower case c and matrices indicated by upper case letters A, B, and C. Once you have studied the procedures of addition, multiplication etc. you should be able to give an example illustrating each of the above laws.

Exercise 1. First note, exponents in matrix algebra mean the same as in high school algebra. Again, upper case letters are used for matrices. So A^2 means $A \cdot A$.
Expand the matrix expression $(A + B)^2$, using the above matrix algebra laws.
Be careful there is no such "thing" as a FOIL law. FOIL really means use the distributive law twice. Do not worry about the order of the matrices we will see this later.

Exercise 2. What is $(A + B)(A – B)$ equal to?

Summary of a few key ideas that you should keep in mind as you learn basic matrix algebra.

1. **AB may not be equal to BA.** This is why we have two distributive laws in matrix algebra where we only have one in regular algebra.

2. Under the operation of addition matrix algebra behaves exactly like that of addition in high school algebra. Assuming the orders of the matrices make sense.

3. The matrix designated by the upper case letter I behaves exactly like the number 1 in high school algebra. So for example $I^{23} = I$.

4. Law number 4 allow us to add matrices of the appropriate order. So $5A – 3A = 2A$, just like in regular algebra.