

Show all your work. Late Homework will not be accepted without prior approval.

1. Let X_1, X_2 and X_3 each be the result of randomly choosing a 1 or a 2. Find the density functions for
 - (a) $Y_1 = 3X_1$;
 - (b) $Y_2 = 2X_1 + X_2$;
 - (c) $Y_3 = X_1 + X_2 + X_3$.
2. Let X be a random integer from 1 to 4. Let Y be a random integer from 1 to X .
 - (a) Find the density function for $W = X + Y$.
 - (b) Find $f_{X|Y=3}(x)$.
3. Let X, Y be independent random variables with $f_X(x) = 1, 0 < x < 1$ and $f_Y(y) = \frac{c}{1+y^2}, y \in \mathbb{R}$.
 - (a) Find c so that f_Y is a probability density function.
 - (b) Find $f_W(w)$ where $W = X + Y$.
 - (c) Find $f_{XY}(w)$.
 - (d) Find $f_{X|Y=3}(x)$.

Bonus.

4. (a) If 3 real numbers are chosen at random in the interval $[-1, 1]$, what is the probability that the largest one is
 - (i) positive?
 - (ii) greater than $1/2$?
 - (iii) greater than 0.8 ?
- (b) Let $X_1, X_2 \dots X_n$ be a random sample of continuous random variables with pdf f_X and cdf F_X . Let X_{\max} be the largest outcome in the sample. Show $f_{X_{\max}}(x) = n[F_X(x)]^{n-1}f_X(x)$.
Hint. Start by finding $F_{X_{\max}}(x)$.

Exam 1 will be on Friday March 27. It will cover Chapters 1 and 2 of the text.

Calculator and formula sheet are allowed but no device with internet access.