> Show all your work. Late homework will not be accepted without prior approval by email.

1. (a) Prove $\lim\limits_{n\to\infty}\left(1+\frac{x}{n}\right)^n = e^x$ for any real number x. This limit was used in deriving the Poisson density.

   (b) Show the exponential series $\sum\limits_{n=0}^{\infty}\frac{x^n}{n!}$ converges for all real x.

2. A typist has a 1% chance of typing a word incorrectly.

   (a) Use the Binomial distribution to find the probability of typing 2 or fewer words incorrectly on a page of 250 words. Calculate the answer to 4 decimal places.

   (b) Redo (a) using the Poisson distribution.

3. People arrive at a variety store once every 5 minutes or so.

   (a) What is the probability nobody arrives in the next 10 minutes?

   (b) What is the probability at least 3 people arrive in the next 20 minutes?

   (c) What is the expected number of arrivals in a 10 hour day?

   (d) The probability density function for the time T until the first person arrives is $f_T(x)=\lambda e^{-\lambda x}, x>0$. Find the probability the first customer of the day arrives within the first 15 minutes.

   (e) Find the probability nobody arrives within the first 15 minutes.

   (f) Find $M_T(t)$, $E(T)$ and $\text{Var}(T)$.

Bonus. Assume everyone in a population of size n buys one lottery ticket each week. Use these hints to find the probability someone will win twice before you win once.

   (a) Let X be the event that you win in week X and nobody has won twice. Find $P(X=x)$, $x=1,2,3...n$.

   (b) Show the probability you win once before someone else wins twice is $\frac{1}{n+1}\left(1+\frac{1}{n}\right)^n$.

   (c) What happens to $\left(1+\frac{1}{n}\right)^n$ as $n\to\infty$? What can you conclude about the lottery?