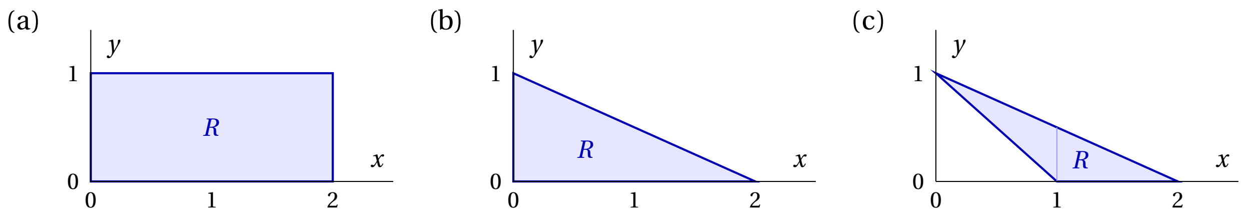
Homework 7 Solutions **92.386 203 Prob & Stat I** **Spring 2015**

1. Set up the iterated integrals for $\iint_R f\,dA$ in both orders for each of the following regions —



(a) $\iint_R f\,dA = \int_0^2 \int_0^1 f\,dy\,dx = \int_0^1 \int_0^2 f\,dx\,dy$

(b) $\iint_R f\,dA = \int_0^2 \int_0^{1-x/2} f\,dy\,dx = \int_0^1 \int_0^{2-2y} f\,dx\,dy$

(c) $\iint_R f\,dA = \int_0^1 \int_{1-x}^{1-x/2} f\,dy\,dx + \int_1^2 \int_0^{1-x/2} f\,dy\,dx = \int_0^1 \int_{1-y}^{2-2y} f\,dx\,dy$

2. A coin with sides marked 2 and 4 and a regular die are thrown. Let X be the number on the coin and Y the absolute difference between the numbers on the coin and die.

(a) Find the joint density $f_{X,Y}(x,y)$

		y = 0	1	2	3	4
x	2	1/12	2/12	1/12	1/12	1/12
	4	1/12	2/12	2/12	1/12	0

(b) The rows of the table each sum to 1/2. Explain in words what this means?

The probability of the coin showing either 2 or 4 is 1/2.

(c) The sum of the $Y = 2$ column is 1/4. What does this mean?

The probability that the difference between the coin and die equals 2 is 1/4.

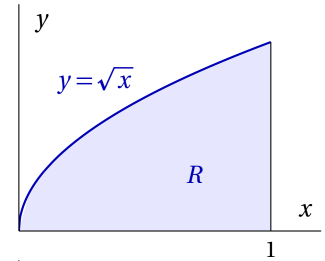
(d) Find the marginal density function for Y.

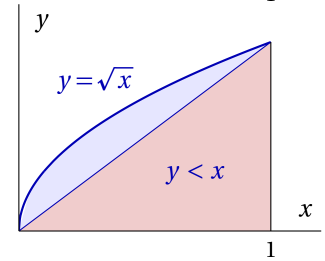
y	0	1	2	3	4
$f_Y(y)$	1/6	1/3	1/4	1/6	1/12

(e) Are X and Y independent? Explain.

No … $f_{X,Y}(4,4) = 0$ but $f_X(4) = 1/2$ and $f_Y(4) = 1/6$ so $f_{X,Y}(4,4) \neq f_X(4) f_Y(4)$ … or just note that there is a zero in the table but no row or column of zeros

3. Let $f_{X,Y}(x,y) = kx$, $(x,y) \in R$.





(a) Find k.

$$\iint_R f_{X,Y}(x,y)\,dA = \int_0^1 \int_0^{\sqrt{x}} kx\,dy\,dx$$
$$= \int_0^1 \left[kxy\right]_0^{\sqrt{x}}\,dx = \int_0^1 kx^{3/2}\,dx = \frac{2}{5}k = 1 \text{ so } k = \frac{5}{2}$$

(b) Find $P(Y < X)$.

$$P(Y<X) = \int_0^1 \int_0^x \frac{5}{2}x\,dy\,dx = \int_0^1 \left[\frac{5}{2}xy\right]_0^x\,dx = \int_0^1 \frac{5}{2}x^2\,dx = \left[\frac{5}{6}x^3\right]_0^1 = \frac{5}{6}$$

(c) Find $P(X < Y)$.

$$P(X<Y) = 1 - P(Y<X) = 1 - \frac{5}{6} = \frac{1}{6}$$

(d) Find the marginal density functions f_X and f_Y.

$$f_X(x) = \int_0^{\sqrt{x}} f_{X,Y}(x,y)\,dy = \int_0^{\sqrt{x}} \frac{5}{2}x\,dy = \left[\frac{5}{2}xy\right]_0^{\sqrt{x}} = \frac{5}{2}x^{3/2},\ 0<x<1$$
$$f_Y(y) = \int_{y^2}^1 f_{X,Y}(x,y)\,dx = \int_{y^2}^1 \frac{5}{2}x\,dx = \left[\frac{5}{4}x^2\right]_{y^2}^1 = \frac{5}{4}(1-y^4),\ 0<y<1$$

(e) Are X and Y independent? Explain.

No, $f_{X,Y}(x,y) \neq f_X(x)f_Y(y)$ … or just note that the support of $f_{X,Y}$ is not a rectangle