Homework 3 Solutions **92.386 203 Prob & Stat I** **Spring 2015**

For this Homework, let C_i be the event that the ith ball has color C where C is R (red) or W (white).

1. Box I contains 3 red and 5 white balls, Box II contains 4 red and 2 white balls. One ball is selected from Box I and placed in Box II and then a ball is selected from Box II. What is the probability that it is white?

   By Theorem 1-10 (Law of Alternatives)

$$P(W_2) = P(W_2 \mid W_1)P(W_1) + P(W_2 \mid R_1)P(R_1) = \frac{3}{7}\frac{5}{8} + \frac{2}{7}\frac{3}{8} = \frac{21}{56} = \frac{3}{8}$$

2. Balls are selected one at a time with replacement from a box containing 4 red and 6 white balls. Find the probability that the

   (a) first ball is red;

   $P(R_1) = 4/10 = 2/5$

   (b) first two balls are red;

   $P(R_1 \cap R_2) = 4/10 \times 4/10 = 4/25$

   (c) third ball is red;

   $P(R_3) = 4/10 = 2/5$

   (d) third ball is red if the first two balls were red;

   $P(R_3 \mid R_1 \cap R_2) = 4/10 = 2/5$

   (e) third ball is red if the first ball was red.

   $P(R_3 \mid R_1) = 4/10 = 2/5$

3. Redo Q2 if the balls are selected without replacement.

   (a) $P(R_1) = 2/5$

   (b) $P(R_1 \cap R_2) = 4/10 \times 3/9 = 2/15$

   (c) $P(R_3) = 2/5$ — Surprise, the answer is the same as Q2 (c)

   (d) $P(R_3 \mid R_1 \cap R_2) = 2/8 = 1/4$

   (e) $P(R_3 \mid R_1) = 3/9 = 1/3$ — As in (c) the fact that the balls are not replaced does not matter. This can be verified with Successive Conditioning (Theorem 1-9) — after the first red has been removed the bag contains 3 red and 6 white and then

$$P(R_3 \mid R_1) = P(R_2 \cap R_3 \mid R_1) + P(W_2 \cap R_3 \mid R_1) = \frac{3}{9}\frac{2}{8} + \frac{6}{9}\frac{3}{8} = \frac{24}{72} = \frac{1}{3}$$

4. Suppose A and B are independent events with $P(A) = 0.7$, $P(B) = 0.4$. Write each of the following events as sets and find their probability.

(a) A and B both occur.

$P(A \cap B) = P(A)P(B) = 0.7 \times 0.4 = 0.28$ by independence

(b) A occurs but not B.

$P(A - B) = P(A - (A \cap B)) = P(A) - P(A \cap B) = 0.7 - 0.28 = 0.42$ by Theorem 1-1

(c) A or B occurs.

$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.7 + 0.4 - 0.28 = 0.82$ by Inclusion-Exclusion

(d) A or B occurs but not both.

$P(A \triangle B) = P(A \cup B) - P(A \cap B) = 0.82 - 0.28 = 0.54$

(e) Neither A nor B occurs.

$P\big((A \cup B)'\big) = 1 - P(A \cup B) = 1 - 0.82 = 0.18$ by Theorem 1-4