Homework 12 Solutions **92.386 201 Prob & Stat I** **Fall 2015**

1. A bad typist has probability 0.05 of typing a word incorrectly. Find the probability of typing $5-8$ words incorrectly on a page of 200 words using the ...

   (a) Binomial distribution;

   Let X be the number of errors on the page. Then X is Binomial with $n = 200$ and $p = 0.05$ so
   $$P(5 \le X \le 8) = \sum_{x=5}^{8} \binom{n}{x} p^x q^{n-x} = \sum_{x=5}^{8} \binom{200}{x} (0.05)^x (0.95)^{200-x} = 0.300$$

   (b) Poisson approximation;

   $\lambda = np = 10$ so $P(5 \le X \le 8) \simeq \sum_{x=5}^{8} \frac{\lambda^x}{x!} e^{-\lambda} = \sum_{x=5}^{8} \frac{10^x}{x!} e^{-10} = 0.301$

   (c) Normal approximation with continuity correction.

   $\mu = np = 10$, $\sigma^2 = npq = 9.5$ so, using the continuity correction
   $$P(4.5 \le X \le 8.5) = P\left(\frac{4.5-\mu}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{8.5-\mu}{\sigma}\right) \simeq P(-1.7844 \le Z \le -0.4867) = 0.276$$

2. In class we showed the density of the sum W of exponential random variables X and Y which are distributed exponentially with parameters λ and μ respectively is $f_W(w) = \frac{\lambda\mu}{\lambda-\mu}\left[e^{-\mu w} - e^{-\lambda w}\right]$, $w > 0$. Find the density in the case $\mu = \lambda$ by ...

   (a) letting $\mu \to \lambda$ in the above expression for $f_W(w)$;

   Use l'Hôpital's rule ... $f_W(w) = \lim_{\mu\to\lambda} \frac{\lambda\mu}{\lambda-\mu}\left[e^{-\mu w} - e^{-\lambda w}\right] = \lim_{\mu\to\lambda} \lambda^2\left[w e^{-\mu w}\right] = \lambda^2 w e^{-\lambda w}$, $w > 0$

   (b) evaluating the convolution integral directly.

   $f_W(w) = \int_0^w \lambda e^{-\lambda(w-x)} \lambda e^{-\lambda x}\, dx = \lambda^2 \int_0^w e^{-\lambda w}\, dx = \lambda^2 w e^{-\lambda w}$, $w > 0$ — this is the gamma density

3. Let X be the result of tossing a coin (sides 1 and 2) and Y the result of tossing a die. Find the density function for the product $W = XY$.

w	1	2	3	4	5	6	8	10	12
$f_W(w)$	1/12	1/6	1/12	1/6	1/12	1/6	1/12	1/12	1/12