1. A dart board has radius 20 cm. Hitting within 5 cm of the center scores 6 points. Hitting anywhere else scores $20/r$ where r is the distance in cm from the center. Let X be the score on a single random throw. Find F_X and sketch its graph.

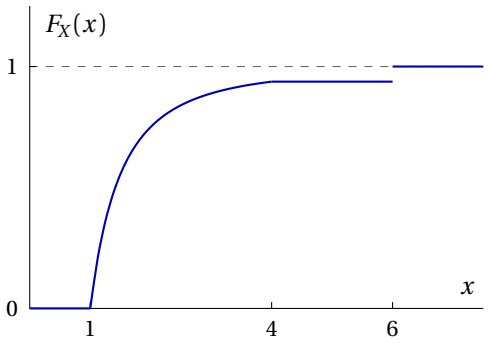
   Note. This is an example of a mixed random variable ... F_X is neither continuous nor a step function.

   The total area of the board is 400π cm^2. Getting a score of 6 means hitting the 5 cm bullseye and this happens with probability $25\pi/400\pi = 1/16$. To get a score less than x for $1 < x < 4$ means hitting outside a radius of $r = 20/x$ and this has probability

   $$F_X(x) = \frac{400\pi - (20/x)^2\pi}{400\pi} = 1 - \frac{1}{x^2}.$$

   Clearly $F_X(x) = 0$ for $x < 1$ so

   $$F_X(x) = P(X \le x) = 1 - P(X > x) = \begin{cases} 0, & x < 1 \\ 1 - 1/x^2, & 1 \le x < 4 \\ 15/16, & 4 \le x < 6 \\ 1, & x \ge 6 \end{cases}$$

2. A stick of length 1 is broken at random into three pieces. Find the probability that the pieces can make a triangle. *Hint.* Let the break points be X and Y. Then (X, Y) is a point in the unit square. Sketch the region representing a successful break.

   If X and Y are the two points chosen uniformly in $[0,1]$ then the three line segments will be the sides of a triangle provided the length of any one does not exceed the sum of the other two. This means the point (X, Y) must lie in the shaded region shown (it takes some work to figure this out) which is $1/4$ of the square.