Homework 2 Solutions **92.386 203 Prob & Stat I** **Spring 2015**

1. Find the probability of each of the events (a – d) in Hw1 Q1.

   Note that $P(i,j) = \begin{cases} 1/36, & i = j \\ 2/36, & i < j \end{cases}$ $1 \le i \le j \le 6$. Referring to the solutions in Hw1 Q1 ...

   (a) $P(S) = 1$

   (b) $P(\text{sum} = 8) = 2/36 + 2/36 + 1/36 = 5/36$

   (c) $P(\text{sum} > 8) = 2/36 + 2/36 + 2/36 + 1/36 + 2/36 + 1/36 = 10/36 = 5/18$

   (d) $P(\emptyset) = 0$

2. For the experiment in Hw1 Q1.

   (a) Find the probability of getting two 4's.

   $P(4,4) = 1/36$

   (b) Find the probability of getting exactly one 4.

   $P(4,1) + P(4,2) + P(4,3) + P(4,5) + P(4,6) = 2/36 + 2/36 + 2/36 + 2/36 + 2/36 = 10/36 = 5/18$

   (c) Find the probability of getting at least one 4.

   $P(\text{at least one 4}) = P(\text{exactly one 4}) + P(\text{two 4's}) = 5/18 + 1/36 = 11/36$

   (d) Find the probability of getting no 4's.

   $P(\text{no 4's}) = 1 - P(\text{at least one 4}) = 1 - 11/36 = 25/36$

3. Three urns each contain two red and two blue balls. A ball is selected from the first urn and placed in the second, then a ball is selected from the second and placed in the third. Finally a ball is selected from the third urn.

   Let A_i be the event that the ith ball is red, $i = 1,2,3$

   (a) Find the probability that the first ball is red.

   $P(A_1) = 2/4 = 1/2$

   (b) Find the probability that the first two balls are red.

   $P(A_1 \cap A_2) = P(A_1)P(A_2 \mid A_1) = (2/4)(3/5) = 3/10$

   (c) Find the probability that all three balls are red.

   By Successive Conditioning Theorem 1-9

   $P(A_1 \cap A_2 \cap A_3) = P(A_1)P(A_2 \mid A_1)P(A_3 \mid A_1 \cap A_2) = (1/2)(3/5)(3/5) = 9/50$

   (d) Find the probability that none of the balls is red.

   $P(\text{no balls are red}) = P(\text{all three balls are blue}) = 9/50$ by symmetry with (c)

4. Design an experiment using a single fair coin with probability of success equal to

   (a) 1/4; (b) 1/8; (c) 3/8; (d) 1/3.

   (a) Toss the coin twice, success is 2 heads ... there are many other answers.

   (b) Toss the coin 3 times, success is 3 heads.

   (c) Toss the coin 3 times, success is exactly 2 heads.

   (d) Toss the coin twice until you don't get two tails. Success is two heads. Or ... toss the coin until a head turns up and success is an even number of tosses. Both these experiments can last a long time! In fact there is no experiment which is guaranteed to terminate in finite time.