

Escaping the Subprime Trap in Algorithmic Lending

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Abstract

Disparities in lending to minority applicants persist even as algorithmic lending finds widespread adoption. We study the role of *risk-management* constraints, specifically Value-at-Risk (VaR), in inducing inequality in loan approval decisions, even among applicants who are equally creditworthy. Empirical research finds that disparities in the interest rates charged to minority groups can remain large even when loan applicants from different groups are equally creditworthy. We develop a formal model in which a mainstream bank (low-interest) is more sensitive to variance risk than a subprime bank (high-interest). If the mainstream bank has an *inflated prior* belief about the variance of the minority group, it may deny that group credit indefinitely, thus never learning the true risk of lending to that group, while the subprime lender serves this population at higher rates. We formalize this as a “subprime trap” equilibrium. Finally, we show that a small, finite *subsidy* (or partial guarantee) can help minority groups *escape the trap* by covering enough of the mainstream bank’s downside so that it can afford to lend and learn the true risk of lending to the minority group. Once it has sufficiently many data points, it meets its VaR requirement with no further assistance, minority groups are approved for loans by the mainstream bank, and competition drives down the interest rates of subprime lenders.

Introduction

Algorithmic lending has grown rapidly as scalable ML methods achieve wide adoption among both existing lenders and new market entrants, bringing with them the possibility of significantly improving the fairness of financial decisions based on observable data about loan applicants (Khandani, Kim, and Lo 2010; Bono, Croxson, and Giles 2021; Berg, Fuster, and Puri 2022; Remolina 2022). However, inequalities persist in both loan approval rates and interest rates charged to minority applicants versus white applicants, and a switch to algorithmic lending procedures does not necessarily improve outcomes on either metric (Romei and Ruggieri 2013; Žliobaitė 2017; Quillian, Lee, and Honoré 2020; Giacoletti, Heimer, and Yu 2021; Aliprantis, Carroll, and Young 2022; Bartlett et al. 2022; Fuster et al. 2021). These inequities exist against a background of historic discrimination in US retail banking to individuals (Quillian, Lee,

and Honoré 2020), businesses (Blanchflower, Levine, and Zimmerman 2003), and a long-standing “racial wealth gap”: persistent differences in median household wealth by ethnic group (Charles and Hurst 2002; Derenoncourt et al. 2023; Althoff and Reichardt 2024).

Notably, disparities in interest rates and loan approvals persist even when minority applicants have comparable credit scores to majority applicants (Bayer, Ferreira, and Ross 2016; Popick 2022; Crosignani and Le 2023). This combination of facts creates a puzzle: are lenders selecting on observable risk factors? And if not, why, besides explicit discrimination, might they be failing to lend to minority applicants?

Several empirical findings help us to understand this puzzle. First, ethnic groups sort across lenders, with minorities more likely to accept loans from high-interest rate banks versus conventional lenders, even conditional on credit score. Bayer, Ferreira, and Ross (2016) write:

African-American and Hispanic borrowers tend to be more concentrated at high-risk lenders. Strikingly, this pattern holds for all borrowers even those with relatively unblemished credit records and low-risk loans... High-risk lenders are not only more likely to provide high cost loans overall, but are especially likely to do so for African-American and Hispanic borrowers. These lenders are largely responsible for the differential treatment of equally qualified borrowers.

Modern algorithmic lending systems operationalize risk assessment through machine learning models trained on historical data. Minority applicants disproportionately have “thin credit files” with limited payment history data, and this data sparsity directly manifests as higher prediction variance in ML models Blattner and Nelson (2021a). When banks face risk-management constraints, the higher predicted variance translates into higher rates of loan denial. Because denied applicants never generate repayment data, banks cannot then learn that their variance estimates were inflated, creating a self-reinforcing equilibrium of exclusion.

It is this set of stylized facts that motivates our model. How do minorities with good credit scores end up with no choice but to accept high-cost loans? To answer this question, we consider the role of risk management constraints

when banks have imperfect information about their applicant pool. Lending is a problem of imperfect information: banks do not know the exact probability that a given borrower will repay a loan, and so must use observable characteristics as a proxy for repayment probability (Adams, Einav, and Levin 2009; Crawford, Pavanini, and Schivardi 2018). Banks rely on credit scoring, which is subject to known biases, and has lower quality information about the creditworthiness of minority applicants (Blattner and Nelson 2021a). Reliance on credit scores can therefore lead banks to hold inaccurate prior beliefs about minority applicants, which can sustain equilibria in which minority groups systematically lose out (Coate and Loury 1993; Kim and Loury 2018).

We study the effect of Value-at-Risk (VaR) constraints on loan approval decisions (Artzner et al. 1999; Jorion 2000). Our analysis focuses on a formal model in which a mainstream, or “low interest rate”, bank exhibits heightened sensitivity to variance risk relative to a subprime, or “high interest rate”, bank. We show that when the mainstream bank has inflated prior beliefs about the variance repayments from the minority group, VaR constraints bind, which means that it will then systematically refrain from extending credit to that group. This reluctance then prevents the bank from updating its beliefs regarding the true risk, thereby locking minority borrowers into higher-cost subprime lending arrangements. We describe this mechanism as “the subprime trap”.

Our Contributions

First, we contribute an analysis of risk management constraints to the study of algorithmic fairness. We help address a puzzle of general interest: why might banks fail to extend loans even when they would otherwise be profitable? Second, we bring details of real-world banking practices into the study of algorithmic fairness. We introduce a two-bank equilibrium framework in which the mainstream bank enforces a more stringent VaR requirement than its subprime counterpart. Under these conditions, an erroneous initial estimate of the variance of the minority group leads the mainstream bank to permanently refrain from lending to that group. Third, we formalize the resulting equilibrium (Theorem 1) and demonstrate that the minority group is consequently confined to subprime loans, despite having the same average creditworthiness as the majority group. Fourth, we establish that the provision of a modest, finite subsidy, or partial guarantee, can resolve this equilibrium inefficiency. By adequately mitigating downside risk, the subsidy enables the main bank to lend and learn about the creditworthiness of the minority group. With an updated assessment of risk, the bank eventually meets its VaR requirements without continued external intervention, thereby extending mainstream credit at favorable rates.

This investigation contributes to the literature by highlighting how inaccurate risk metrics, in conjunction with risk-based capital constraints, can systematically exclude certain borrower groups from low-interest credit. Our results further imply that targeted subsidies can effectively rectify informational failures, yielding improved outcomes for both lenders and minority borrowers.

Related Literature

Our work contributes to several strands of literature, including algorithmic fairness in lending, the economics of discrimination, and the role of informational frictions in sustaining suboptimal market equilibria. In the domain of algorithmic fairness, recent research has examined how increasingly sophisticated data-driven methods in credit scoring may inadvertently perpetuate or exacerbate disparities (Barocas and Selbst 2016; Barocas, Hardt, and Narayanan 2023; Kumar, Hines, and Dickerson 2022). While algorithmic approaches promise enhanced accuracy, empirical evidence indicates that minority applicants often face higher interest rates or are denied credit outright (Bartlett et al. 2022; Crosignani and Le 2023).

Discrimination is a classical topic in economics, studied in a large theoretical and empirical literature (Becker 1971; Phelps 1972; Lundberg and Startz 1983; Arrow 1998; Heckman 1998; Cowgill and Tucker 2019). Building on this tradition, subsequent studies have explored how negative stereotypes or misperceptions of risk may translate into adverse outcomes for minority borrowers (Fang and Moro 2011; Coate and Loury 1993). In a manner analogous to these analyses, our work demonstrates that inflated beliefs about repayment variance can result in a self-reinforcing equilibrium whereby minority groups are persistently relegated to subprime lending markets.

A related literature examines the role of informational frictions and the corrective impact of policy interventions (Akerlof 1970; Akerlof and Shiller 2015; Thaler 2016). Cai et al. (2020) studied contexts in which banks can acquire improved information about borrowers’ creditworthiness through selective experimentation. Donahue and Barocas (2021) study the trade-offs between solidaristic insurance policies, in which minorities are subsidized in accordance with risks that are causally related to their minority status, and actuarially-fair insurance policies, in which everyone pays premia equal to their marginal risk.

Our work also relates to work on delayed feedback with respect to implementation decisions in machine learning settings (Liu et al. 2018; Pagan et al. 2023; Chaney, Stewart, and Engelhardt 2018). We describe a setting in which mainstream banks are stuck in a negative feedback loop: their failure to learn about applicant creditworthiness is self-sustaining. Our work is also related to literature on suboptimal outcomes with bandits. Honda and Takemura (2013) prove that Thompson Sampling’s optimality depends critically on prior specification, while Ghosh, Chowdhury, and Gopalan (2017) show that any algorithm achieving optimal performance under perfect model specification must suffer linear regret under misspecification. This mirrors our setting where banks with incorrect variance priors never explore lending to certain groups. The dueling bandits framework of Yue et al. (2012) captures competition between lenders, establishing information-theoretically optimal algorithms for pairwise comparisons. Frazier et al. (2014) study how to incentivize exploration when arms are pulled by self-interested agents – our approach, in which subsidies encourage banks to ‘learn through lending’, is closely related to this setting.

Finally, our work contributes to a broader effort to model rational but suboptimal decision-making by or with respect to disadvantaged groups. Diana et al. (2024) model pessimism traps and herding behavior as individuals grapple with cycles of noisy and censored information when making decisions about potentially higher reward but riskier ends. Our argument is similar in broad outline to that of (Foster and Vohra 1992; Hu and Chen 2018), who argue that temporary interventions in the labor market to address discrimination may lead to long-run improvements in the fairness of observed decisions.

Preliminaries and Model

We analyze a multi-period environment indexed by $t = 1, 2, \dots, T$, where T may be arbitrarily large or unbounded. At the onset of each period t , the high-interest (subprime) bank selects an interest-rate premium $\nu^{(t)}$. Each loan applicant belongs to one of two groups and applies simultaneously to both the mainstream (low-interest) bank and the subprime bank. The mainstream bank charges a baseline rate normalized to 1, while the subprime bank charges $1 + \nu^{(t)}$.

Each bank evaluates whether to approve or reject each applicant in that period, seeking to maximize its expected profit subject to a risk-management constraint. If an applicant receives approvals from both banks, the applicant typically accepts the offer with the lower interest rate; if only one bank approves the loan, it is accepted. After the loans are finalized, each bank privately observes its payoff. Consequently, a bank that rejects an application or whose offer is declined does not observe the repayment outcome for that applicant.

At the end of period t , both banks update their beliefs regarding the repayment distribution for each group, employing a Bayesian learning procedure described in Section . These updated beliefs affect the subprime bank's subsequent choice of premium $\nu^{(t+1)}$ and both banks' lending decisions in period $t + 1$.

Loan Applicants

There are two groups, denoted by $i \in \{W, B\}$. W represents the racial majority, while B represents the racial minority. Each group has a payoff distribution $\pi_i^{(t)} \sim \mathcal{N}(\mu_i, \sigma_i^2)$. The bank's return from approving a loan depends on this realized payoff.

Assumption 1 (Equal expected creditworthiness).

$$\forall t : \mathbb{E} [\pi_W^{(t)}] = \mu_W = \mu_B = \mathbb{E} [\pi_B^{(t)}] > 0$$

First, we assume that both groups have the same expected creditworthiness. We do this because we want to study the specific case where minority and majority racial groups are equally creditworthy. We allow their variances, σ_W^2 and σ_B^2 , to differ.

We suppose that each group strictly prefers to accept a loan rather than not, and strictly prefers to accept the cheapest loan. We assume that applicants randomize if both loans have the same price. We denote each group's decision to accept the loan with $S_i \in \{0, 1\}$

Lenders

The model features two banks: Bank L , which offers low-interest (mainstream) loans, and Bank H , which provides high-interest (subprime) loans. We refer to banks with the index j . In each period, both banks decide whether to extend a loan to each group. Let $\mathbf{A}_j^{(t)} \in \{0, 1\}^2$ denote the vector of approval decisions made by the bank j at time t .

Bank payoff functions If the bank issues a loan to an applicant from group i in period t , it earns a payoff that depends on the realized value of $\pi_i^{(t)}$. We normalize the interest rate of Bank L to 1, so that its profit Π in period t is given by

$$\Pi_L^{(t)} = \sum_i S_i^{(t)} A_{iL}^{(t)} \pi_i^{(t)}$$

In contrast, Bank H charges an interest rate of $1 + \nu$, with $\nu > 0$. It has profit function:

$$\Pi_H^{(t)} = \sum_i S_i^{(t)} A_{iH}^{(t)} \left[(1 + \nu^{(t)}) \max \left\{ \pi_i^{(t)}, 0 \right\} + \min \left\{ \pi_i^{(t)}, 0 \right\} \right].$$

Here, positive returns are amplified by a factor of $1 + \nu^{(t)}$, whereas losses are incurred on a dollar-for-dollar basis.

Risk Management Constraints We next suppose that banks face a *risk management* or *solvency* constraint. That is, there is some level of financial loss that is 'unacceptable' to the bank. This may be due to regulatory constraints, such as Basel III or Dodd-Frank, or liquidity constraints (Li 2014). This loss threshold is probabilistic: the bank is willing to accept some nonzero risk that their loss from making a loan falls below a certain threshold, but wants to specifically limit the probability that this occurs to less than a risk tolerance $\alpha\%$. Typically, this value is set to 1% or 5% (Jorion 2000).

We model this with a per-period Value-at-Risk (VaR) constraint (Artzner et al. 1999). That is, we require that the bank's anticipated profit in each period must be greater than some constant $\rho_j < 0$ with probability at least $1 - \alpha$. We can think of ρ_j as the bank's maximum acceptable loss. Equivalently, the bank is willing to accept an $\alpha\%$ risk that their profit will fall below the bank's risk management threshold ρ_j .

Definition 1 (Value-at-Risk (VaR)).

$$VaR_\alpha(X) = -\inf \{x \mid \mathbb{P}[X \leq x] > \alpha\}$$

We adapt this by requiring that each bank j in period t faces the constraint that:

$$\mathbb{P}(\Pi_j^{(t)} < \rho_j) \leq \alpha$$

This simply states that the bank will accept a risk of at most $\alpha\%$ that their profit in period t falls below ρ_j .

We suppose that this constraint is *lexically prior* to the profit-maximization objective: the bank must satisfy the VaR constraint in order to lend at all.

Combining each bank's payoff function with their VaR constraint, we can write each bank's optimization problem as follows:

$$\arg \max_{\mathbf{A}_j^{(t)}} \Pi_j^{(t)}(\mathbf{A}_j^{(t)}) \quad \text{subject to} \quad \mathbb{P}(\Pi_j^{(t)}(\mathbf{A}_j^{(t)}) \leq \rho_j) \leq \alpha$$

This says simply that each bank chooses the approval decisions that maximize profits subject to satisfying their VaR constraint. As we shall see below, the maximand depends on the expected value of the applicants' payoff function μ_i , while the VaR depends on both the mean and the variance of the applicant group's payoff function.

Bank beliefs The actual profit $\Pi_j^{(t)}$ is unknown *ex ante*, however: it is only observed once a loan has been approved and accepted. Banks must therefore instead make decisions based on prior beliefs about the profitability of approving a loan. Denote by $\hat{\Pi}_j^{(t)}$ the bank's posterior estimate of profit from lending at time t . Each bank instead must solve the *feasible* problem:

$$\arg \max_{\mathbf{A}_j^{(t)}} \hat{\Pi}_j^{(t)}(\mathbf{A}_j^{(t)}) \quad \text{subject to} \quad \mathbb{P}(\hat{\Pi}_j^{(t)}(\mathbf{A}_j^{(t)}) \leq \rho_j) \leq \alpha$$

Which, importantly, depends on bank beliefs about applicants. We assume that each bank j has initial prior beliefs regarding the parameters of the distribution of returns from issuing a loan to a member of group i , specifically:

Definition 2 (Bank's prior beliefs). *Each bank j has prior beliefs:*

$$\left\{ \hat{\mu}_{ij}^{(0)}, \hat{\sigma}_{ij}^{(0)} \right\}_{i \in \{W, B\}}$$

We assume that both banks have accurate beliefs about the variance of group W . This does not have significant implications for the analysis, since the problem of interest is how banks respond to differentially imperfect information about lenders' creditworthiness. This assumption also reflects empirical findings that credit score data is differentially missing for minority group applicants versus majority group applicants (Brevoort, Grim, and Kambara 2015).

Assumption 2 (Group W 's variance is common knowledge). *We assume that $\hat{\sigma}_W^{(0)} = \sigma_W$ for both banks.*

Learning through lending In each period, if a bank extends a loan to an applicant from group i , it observes the realized repayment and updates its risk assessment for that group via Bayesian updating.

Lemma 1 (Belief Updating via Bayes Rule). *If bank j lends to group i , it observes a return $\pi_i^{(t)}$ and then updates its posterior beliefs about the variance σ_i^2 . After M such observations, its posterior belief takes the form of:*

$$\hat{\sigma}_i^{2(M)} = \frac{1}{M+1} \left(\hat{\sigma}_i^{2(0)} + \sum_{m=1}^M \left(\pi_{i,B}^{(m)} - \bar{\pi}_{i,B}^{(M)} \right)^2 \right)$$

$$\text{Where: } \bar{\pi}_{i,B}^{(M)} = \frac{1}{M} \sum_{m=1}^M \pi_{i,B}^{(m)}$$

A key assumption in our analysis is that a bank updates its estimate of the variance for group i only when it observes repayment outcomes from that group. Thus, if a bank never lends to group B , it will not receive data on the performance of loans to that group, and incorrect prior belief about B 's variance will not be updated.

Our Modeling Choices

VaR and CVaR Value-at-Risk is a commonly used risk management measure in practice, and is explicitly referenced in the Basel III banking regulations (Li 2014; Chang et al. 2019). We could also study the Conditional Value-at-Risk, or Expected Shortfall, which is related to VaR via the formula: $ES(X) = \frac{1}{\alpha} \int_0^\alpha VaR_\phi(X) d\phi$. The intuitions underlying our model carry through to the CVaR metric: there is a threshold variance at which the CVaR will be violated.

Inaccurate prior beliefs Our model supposes that banks have inaccurate beliefs about the creditworthiness of applicants. This is a plausible outcome when decisions are made on the basis of credit scoring data that is either biased or noisy. Credit scores may be inherently noisier representations of underlying default risk for minority groups (Blattner and Nelson 2021a), while the components of the score may themselves be low quality indicators of repayment ability (Rice and Swesnik 2013). Empirical work shows that banks are more likely to avoid lending to minority neighborhoods, thereby lowering the quality of information received about lenders in those neighborhoods (Blattner and Nelson 2021b). Noisier estimates of credit risk are likely to lead to inflated prior estimates of variance in practice.

Risk-pooling We study three cases: where banks lend unilaterally to each group, and where banks lend to both groups. In the unilateral setting, there is no risk pooling, and banks make decisions based only on single group characteristics. However, when banks are willing to lend to both groups, risk pooling occurs, where the lower-risk group essentially subsidizes the risk of lending to the higher-risk group. This case is studied in Donahue and Barocas (2021). Below, we derive variance thresholds for each of the three cases.

Single-Period Setting

We first study the stage game, to show how the VaR constraint affects lending to group B in each period. Note first that we assume that Bank H charges an interest rate premium to derive variance thresholds. We then prove that this occurs. Second, note that we derive variance thresholds for the pooled variance – thresholds under which banks will lend to both groups. Because the variance thresholds are ordered, it follows that if the bank's variance pooled threshold is not satisfied, it will not unilaterally lend to the minority group. It also follows that if the bank's pooled threshold is satisfied, the bank will unilaterally lend to the majority group.

First we assume that Banks have unilateral thresholds for lending to each group:

Lemma 2 (Unilateral Lending Thresholds).

$$\begin{aligned}\tilde{\sigma}^L &= \frac{\rho_L - \mu}{\Phi^{-1}(\alpha)} \\ \tilde{\sigma}^H &= \frac{\rho_L - (1 + \nu)\mu}{\Phi^{-1}(\alpha)}\end{aligned}$$

These represent the thresholds under which each bank is willing to lend unilaterally to a given group.

Each bank also has a pooled variance threshold, under which it is willing to lend to both groups.

Lemma 3 (Pooled Variance Threshold for Bank L). *Consider Bank L , which offers loans at a normalized interest rate of 1, lending to group B with a random payoff given by $\pi_i \sim \mathcal{N}(\mu_i, \sigma_i^2)$. Bank L lends to both groups in period t if and only if group B 's variance satisfies:*

$$\hat{\sigma}_{BL}^{(t)} \leq \tilde{\sigma}_{pool}^L \equiv \frac{\rho_L - (\mu_W + \mu_B + \Phi^{-1}(\alpha)\sigma_W)}{\Phi^{-1}(\alpha)}$$

In other words, there is an upper bound $\tilde{\sigma}^L$ on the Bank's beliefs about the variance of group B , such that the bank is only willing to lend to group B if it believes that group has lower variance than this threshold. Otherwise, its risk of a shortfall exceeds $\alpha\%$.

A similar analysis for the subprime bank H reveals that it tolerates a higher level of variance. We have:

Lemma 4 (Pooled Variance Threshold for Bank H). *Bank H lends to group B and group W in period t if and only if $\hat{\sigma}_{BH}^{(t)} \leq \tilde{\sigma}_{pool}^H$, where:*

$$\tilde{\sigma}_{pool}^H = \frac{\rho_H - [(1 + \nu^{(t)})(\mu_W + \mu_B) + \Phi^{-1}(\alpha)\sigma_W]}{\Phi^{-1}(\alpha)}$$

Corollary 1 (Ordered variance thresholds). *While $\nu^{(t)} > 0$, and for $\alpha < .1$:*

$$0 < \tilde{\sigma}^L < \tilde{\sigma}_{pool}^L < \tilde{\sigma}^H < \tilde{\sigma}_{pool}^H$$

Intuitively, gains are scaled by the factor $(1 + \nu^{(t)})$, providing the high-interest rate bank additional insulation against downside risk. Thus, when the perceived variance of group i is between $\tilde{\sigma}^L$ and $\tilde{\sigma}^H$, the low-interest Bank L will decline to lend, while the high-interest Bank H can approve the loan.

The Subprime Trap

We now consider the multi-period setting in which lending decisions influence the evolution of banks' beliefs about borrower creditworthiness.

Assumptions

We make several assumptions. These model stylized facts that characterize the subprime trap. First, we suppose that the variance of both groups is *in fact* below the threshold $\tilde{\sigma}^L$, though the variances are not necessarily identical.

Assumption 3 (Both groups are creditworthy). $\sigma_W \leq \sigma_B < \tilde{\sigma}^L$

This is intended to describe a situation in which the low-interest rate bank would lend to both groups under perfect information.

We characterize beliefs under imperfect information in which lending to group B does not occur. We suppose that Bank L holds a prior that the variance of repayments for group B is $\hat{\sigma}_{BL}^{(0)}$ such that:

Assumption 4 (L 's prior variance for group B is above its risk threshold). $\hat{\sigma}_{BL}^{(0)} > \tilde{\sigma}_{pool}^L$

Where $\tilde{\sigma}_{pool}^L$ is the maximum variance that Bank L (the main lender) can tolerate under its constraint VaR. Based on its initial assessment, lending violates the risk limits of Bank L .

Third, Bank H , which operates at a higher interest rate and has a correspondingly higher risk tolerance. The high-rate bank has a lending threshold $\tilde{\sigma}^H$ such that $\tilde{\sigma}^H > \tilde{\sigma}_{pool}^L$.

Denoting bank H 's prior by $\hat{\sigma}_{BH}^{(0)}$, we suppose that:

Assumption 5 (H 's prior variance for group B is below its risk threshold). $\hat{\sigma}_{BH}^{(0)} \leq \tilde{\sigma}^H$

Consequently, Bank H is prepared to lend to group B .

Equilibrium

In each period, banks choose simultaneously whether to lend to groups W and B . Both types of bank approve group W 's loan application, and group W chooses the lower-rate, main-stream bank. Based on their prior beliefs about B 's variances, H approves B 's loan application, but L does not. If Bank L does not lend to group B , then group B is left with the subprime option from Bank H . But since Bank L 's belief update depends on observing a return from group B , if it does not lend to group B at period t , it does not have an updated posterior belief to use as its prior at period $t + 1$: its beliefs do not change. The crucial observation is that if Bank L persistently withholds loans to group B , it never observes the repayment data needed to update its inflated variance estimate, and its belief remains at $\hat{\sigma}_{BL}^{(0)}$.

We now formalize this result.

Theorem 1 (Subprime Trap Equilibrium). *Suppose Assumptions 1-5 hold. Then, there exists a Bayesian subgame-perfect equilibrium in which, in every period, Bank L lends exclusively to group W , and Bank H lends to group B with high probability, permanently relegating group B to subprime loans.*

Proof. We show by induction that the low-cost bank never lends to group B .

First, in the base case, the bank does not lend to group B . We have that $A_{ij}^{(1)} = 0$, since, by Lemma 1, if the bank's prior belief is $\hat{\sigma}_{BL}^{(0)}$ and $\hat{\sigma}_{BL}^{(0)} > \tilde{\sigma}_{pool}^L$, then $A_{ij}^{(1)} = 0$.

To show the induction step, we show that if the bank does not lend to B in period t , then it does not lend to B in period $t + 1$. For this, we have that $A_{ij}^{(t)} = 0 \implies \hat{\sigma}_{ij}^{(t+1)} = \hat{\sigma}_{ij}^{(t)}$, because no updating occurs, but since $A_{ij}^{(t+1)} = 1$ if and only if $\hat{\sigma}_{ij}^{(t+1)} \leq \tilde{\sigma}_{pool}^L$, we have that $\hat{\sigma}_B^{(t)} > \tilde{\sigma}_{pool}^L$ and

$\hat{\sigma}_{ij}^{(t+1)} = \hat{\sigma}_{ij}^{(t)} \implies \hat{\sigma}_{ij}^{(t+1)} > \tilde{L}$, so that $A_{ij}^{(t+1)} = 0$. This verifies the induction step.

This shows by induction that the low-cost bank never lends to B . Since the bank *does* lend to W , we have that the optimal decision for Bank L is to lend exclusively to group W in every period.

Next, consider Bank H . By hypothesis, Bank H believes that $\sigma_{BH}^{(0)} \leq \tilde{\sigma}^H$, and therefore lending to group B satisfies its risk constraint in the initial period. Moreover, since the true mean μ_B is positive, lending to group B is expected to yield a positive return. Bank H lends to group B with high probability, because in each stage there is a (vanishing) probability of observing a payoff realization that increases Bank H 's posterior belief about group B 's variance above its threshold. This probability goes to zero by the SLLN, however.

The borrowers act accordingly. Group W , faced with a lower interest rate from Bank L , accepts that offer, while group B , having no offer from Bank L , accepts the loan from Bank H .

Finally, note that because Bank L never lends to group B , it never observes any repayment outcomes from that group. This implies that the variance estimate for group B remains at $\sigma_{BL}^{(0)} > \tilde{\sigma}_{\text{pool}}^L$ in every period. As a consequence, there is no incentive for Bank L to deviate from its strategy of lending only to group W , and similarly, neither Bank H nor the borrower groups have incentives to deviate from their prescribed actions. Hence, the described strategy profile constitutes a subgame-perfect equilibrium. \square

Remark 1 (Purely Informational Failure). *Even if the true variance of group B were such that $\sigma_B < \tilde{\sigma}_{\text{pool}}^L$, Bank L would remain unaware of this fact unless it extends credit to group B . The absence of new data prevents Bank L from updating its risk assessment, leading to an inefficient outcome in which group B continues to access only subprime credit.*

Remark 2 (Equal Creditworthiness). *It is noteworthy that the equilibrium outcome emerges solely from differences in prior variances. Since $\mu_B = \mu_W = \mu$, the two groups are identical in terms of average creditworthiness; the discrepancy stems entirely from the combination of incorrect beliefs about borrower variance, and the risk assessment constraints imposed by VaR.*

Escaping the Subprime Trap via Subsidies

We now show that a subsidy or partial lending guarantee can help group B escape the subprime trap. The key idea is to cover sufficient downside risk so that Bank L is induced to lend to group B . When this occurs, Bank L can gather information on group B 's repayment performance. Since, by Assumption 1, group B is in fact creditworthy, Bank L 's beliefs will eventually converge to group B 's actual variance, which, by Assumption 2, is low enough that L 's VaR constraint will be satisfied. Once enough observations have been accumulated, Bank L will update its risk assessment so that the estimated variance satisfies $\hat{\sigma}_{BL}^{(t)} < \tilde{\sigma}^L$, eliminating the need for further subsidy.

Lemma 5 (Learning through Lending). $\hat{\sigma}_{BL}^{(m)} \rightarrow \sigma_B$ almost surely as $m \rightarrow \infty$

By Lemma 3, this is the condition required for Bank L to lend to group B , so that the VaR constraint is satisfied. In other words, if the mainstream bank *were* to lend to group B for a sufficiently long period of time, it would learn about the variance of returns due to lending to group B . The problem is then how to induce the bank to do so.

To encourage Bank L to lend to group B , we introduce a subsidy mechanism.

First, we define our subsidy as the smallest side-payment that would allow the bank to satisfy its VaR constraint in period t . We have:

$$s^{*(t)} = \inf \left\{ s \geq 0 : \mathbb{P} \left(\Pi_j^{(t)} + s < \rho_j \right) \leq \alpha \right\}$$

This allows us to solve for the optimal subsidy.

Lemma 6. *In each period t , the required subsidy is equal to:*

$$s^{*(t)} = \max \left\{ 0, \rho_L - (\mu_W + \mu_B) - \Phi^{-1}(\alpha) (\hat{\sigma}_{BL}^{(t)} + \sigma_W) \right\}$$

If $s_t^* \leq 0$, no subsidies are needed; otherwise, the regulator covers any shortfall up to s_t^* , ensuring that Bank L meets its VaR restriction.

We formalize the adaptive subsidy mechanism in Algorithm 1.

Algorithm 1: Adaptive Subsidy to Escape the Subprime Trap

Require: Parameters: ρ_L, α, μ , initial variance $\hat{\sigma}_{BL}^{(0)}$, horizon T .

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1: Set  $m \leftarrow 0$  (# of observations from group  $B$ )
2: for  $t \in 1, \dots, T$  do
3:   while  $\hat{\sigma}_{BL}^{(t)} > \tilde{\sigma}_{BL}$  do
4:     if  $A_{BL}^{(t-1)} = 1$  then
5:        $\hat{\sigma}_{BL}^{(t)} \leftarrow$ 
          $\left[ \frac{1}{1 + \sum_{m < t} \mathbb{I}\{A_{BL}^{(m)} = 1\}} \times \right.$ 
          $\left. \left( \hat{\sigma}_B^{2(0)} + \sum_{m < t} \mathbb{I}\{A_{BL}^{(m)} = 1\} \left( \pi_{BL}^{(m)} - \bar{\pi}_{BL}^{(t-1)} \right)^2 \right) \right]^{1/2}$ 
6:     else
7:        $\hat{\sigma}_{BL}^{(t)} \leftarrow \hat{\sigma}_{BL}^{(t-1)}$ 
8:     end if
9:      $s^{*(t)} \leftarrow \max \{ 0, \rho_L - (\mu_W + \mu_B) - \Phi^{-1}(\alpha) (\hat{\sigma}_{BL}^{(t)} + \sigma_W) \}$ 
10:   end while
11: end for
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The subsidy incentivizes Bank L to lend to group B , generating observations that allow its risk assessment to converge to the true variance σ_B^2 . Once the updated estimate satisfies $\hat{\sigma}_{BL}^{(t)} < \tilde{\sigma}^L$, it follows from Lemma 2 that Bank L meets its VaR constraint without subsidy (i.e., $s^{*(t)}$), and no further external support is required.

We now state the main result regarding the subsidy mechanism.

Theorem 2 (Escaping the subprime trap). *Assume that the true distribution of repayments for group B is $\mathcal{N}(\mu, \sigma_B^2)$ with $\sigma_B < \tilde{\sigma}^L$, but that Bank L initially believes that the variance is $\hat{\sigma}_{BL}^{(0)} > \tilde{\sigma}^L$. Under the subsidy mechanism described above (Algorithm 1), with probability one there exists a finite time τ such that for all $t \geq \tau$, Bank L updates its variance estimate so that $\hat{\sigma}_{BL}^{(t)} \rightarrow \sigma_B < \tilde{\sigma}^L$ and consequently $s^{*(t)}$. Beyond time τ , Bank L continues lending to group B without further subsidy.*

In short, the subsidy induces the bank to update their information about minority group creditworthiness. It is also temporary when there exist otherwise creditworthy borrowers trapped in subprime loans: subsidizing information discovery helps banks to learn the true variance of lender groups. Further, while algorithmic audits could correct bias, subsidies can avoid privacy hurdles and scale faster (Cowgill and Tucker 2019).

Corollary 2 (Long-term benefits of subsidy). *For $t > \tau$, $\nu^{(t)} = 0$.*

In any round in which the low-interest bank would approve group B 's loan, the high-interest bank will choose to set its premium to zero. If it chooses any positive premium, neither group will accept the loan. If it chooses a premium of zero, both groups will randomize over accepting the loan from bank B and accepting the loan from bank H . Since this persists for all $t > \tau$, this reduces interest rates permanently – competition reduces the premiums charged by subprime lenders.

Conclusion

Inequalities in algorithmic lending practices can persist even when minority applicants are as creditworthy as white applicants. We study one explanation for this phenomenon: the role of risk-management constraints, specifically VaR, in contributing to persistent disparities in lending. We have shown that risk management constraints can lead lenders to refuse loans even with positive Net Present Value. This forces minority applicants to accept high-interest rate loans, with negative implications for their financial prospects, a situation we describe as the “subprime trap” equilibrium. We emphasize the role of inaccurate prior beliefs about the risk of lending to minority groups, which leads to systematic exclusion and higher borrowing costs.

Our theoretical results align with documented patterns in mortgage lending markets. First, the persistence of our subprime trap equilibrium mirrors the decade-long stability of racial lending disparities documented in mortgage markets, even as algorithmic lending has expanded Bartlett et al. (2022). Second, our model's prediction that mainstream banks never update their beliefs about minority creditworthiness is consistent with Bayer, Ferreira, and Ross (2016). Third, the equilibrium's existence despite equal creditworthiness directly explains the puzzle identified by Popick (2022) and Crosignani and Le (2023): why minorities with

comparable credit scores remain concentrated at high-cost lenders.

Implementation of our subsidy mechanism maps naturally onto existing policy infrastructure. The Federal Housing Administration's mortgage insurance programs already function as partial guarantees, covering lender losses when borrowers default on FHA-insured loans (U.S. Department of Housing and Urban Development 2024). Similarly, the Community Reinvestment Act provides regulatory credits to banks for community development loans and investments in low- and moderate-income neighborhoods (U.S. Congress 2013; Board of Governors of the Federal Reserve System 2024). Our analysis suggests these programs could achieve greater impact if restructured to explicitly target variance reduction during a learning period; guaranteeing tail risks for a finite horizon would enable banks to update their risk assessments while avoiding solvency-affecting losses.

Our analysis focuses on one specific mechanism: variance-based discrimination through VaR constraints. Observed lending disparities likely result from multiple interacting factors including historical discrimination, geographic segregation, and differential access to financial services.

Nonetheless, targeted, temporary interventions, such as subsidies or guarantees, can break this cycle by allowing banks to learn true risk profiles. These findings suggest practical avenues for addressing lending disparities and offer a framework to explain failures of fairness in algorithmic lending decisions.

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Appendix

Proof of Lemma 1

Proof. This is the posterior variance estimate from a Normal model with known mean and prior hyperparameters $\{1, \sigma_i^{2(0)}\}$. \square

Proof of Lemma 2

Proof. First, we suppose that Bank L lends to group W , and derive a threshold rule for the prior variance σ_B holding σ_W fixed such that this rule determines whether or not Bank L will lend to group B .

First, we note that it is profitable in expectation for Bank L to lend to group B , because, by Assumption 1, $\mathbb{E}[\pi_B] = \mu_B > 0$.

We then need to check Bank L 's VaR constraint. It is:

$$Pr\left(\Pi_j^{(t)} < \rho_j | A_{BL} = 1\right) = \Phi\left(\frac{\rho_L - (\mu_B + \mu_W)}{\hat{\sigma}_B^{(t)} + \sigma_W}\right)$$

Where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. This entails that

$$Pr\left(\Pi_j^{(t)} | A_{BL} = 1 < \rho_j\right) \leq \alpha \iff \Phi\left(\frac{\rho_L - (\mu_B + \mu_W)}{\hat{\sigma}_B^{(t)} + \sigma_W}\right) \leq \alpha$$

Rearranging, we get the condition that:

$$\hat{\sigma}_B^{(t)} \leq \frac{\rho_L - (\mu_W + \mu_B + \Phi^{-1}(\alpha)\sigma_W)}{\Phi^{-1}(\alpha)}$$

Where the right-hand side defines a variance threshold $\tilde{\sigma}_B$ such that, if group B 's prior variance is above this threshold, Bank L will not lend to group B :

$$\hat{\sigma}_{BL}^{(t)} \leq \tilde{\sigma}_B \equiv \frac{\rho_L - (\mu_W + \mu_B + \Phi^{-1}(\alpha)\sigma_W)}{\Phi^{-1}(\alpha)}$$

Next, we show that Bank L always lends to group W . Recall that, by Assumption 2, Group W 's variance is common knowledge: this reflects the assumption that banks have higher quality information about the creditworthiness of the majority group.

Checking the VaR constraint in the unilateral case, we generate the threshold:

$$\sigma_W \leq \frac{\rho_L - \mu_W}{\Phi^{-1}(\alpha)}$$

Since we know that, by Assumption 3, this condition is satisfied, Bank L is willing to lend to W unilaterally. Assumption 3 also tells us that $\hat{\sigma}_B^{(0)}$ is above this threshold, indicating that Bank L will not lend to B unilaterally.

Hence, Bank L will lend to group W unilaterally, and will lend to group B in addition to group W if and only if group B 's variance satisfies $\hat{\sigma}_B^{(0)} \leq \tilde{\sigma}_B$. \square

Proof of Lemma 3

Proof. Bank H 's VaR constraint can be written as:

$$Pr\left(\Pi_j^{(t)} < \rho_j | A_{BL} = 1\right) = \Phi\left(\frac{\rho_L - [1 + \nu](\mu_B + \mu_W)}{\hat{\sigma}_B^{(t)} + \sigma_W}\right)$$

Rearranging, we get:

$$\hat{\sigma}_{BH}^{(t)} \leq \tilde{\sigma}^H \equiv \frac{\rho_L - [(1 + \nu)(\mu_W + \mu_B) + \Phi^{-1}(\alpha)\sigma_W]}{\Phi^{-1}(\alpha)}$$

\square

Proof of Corollary 1

Proof. This can be deduced by inspection, but note that, by convention, $\alpha < .1$, so that $\Phi^{-1}(\alpha) < 0$. \square

Proof of Lemma 4

Proof. This follows by application of the Law of Large Numbers. \square

Proof of Lemma 5

Proof. In each period, the Bank has belief, $\pi_B^{(t)} \sim \mathcal{N}(\mu, (\hat{\sigma}_{BL}^{(t)})^2)$, so that:

$$\mathbb{P}(\Pi_j^{(t)} + s < \rho_L) = \Phi\left(\frac{\rho_L - (\mu_W + \mu_B) - s}{\hat{\sigma}_{BL}^{(t)} + \sigma_W}\right).$$

Thus, to guarantee $\Phi\left(\frac{\rho_L - (\mu_W + \mu_B) - s}{\hat{\sigma}_{BL}^{(t)} + \sigma_W}\right) \leq \alpha$, we require

$$\frac{\rho_L - (\mu_W + \mu_B) - s}{\hat{\sigma}_{BL}^{(t)} + \sigma_W} \leq \Phi^{-1}(\alpha),$$

which rearranges to

$$s \geq \rho_L - (\mu_W + \mu_B) - \Phi^{-1}(\alpha) (\hat{\sigma}_{BL}^{(t)} + \sigma_W).$$

The minimal such subsidy is therefore:

$$s_t^* = \max\left\{0, \rho_L - (\mu_W + \mu_B) - \Phi^{-1}(\alpha) (\hat{\sigma}_{BL}^{(t)} + \sigma_W)\right\}.$$

\square

Proof of Theorem 2

Proof. We prove the theorem in three steps.

First, by design the subsidy $s^{*(t)}$ ensures that when Bank L lends to group B , the bank's VaR constraint is satisfied, that is,

$$\mathbb{P}(\Pi_j^{(t)} + s^{*(t)} < \rho_L) \leq \alpha.$$

Thus, the presence of $s^{*(t)}$ makes lending feasible even under Bank L 's initial risk assessment $\hat{\sigma}_{BL}^{(t)} > \tilde{\sigma}^L$. Consequently, if the expected net profit (inclusive of the subsidy) is positive, Bank L has an incentive to lend to group B .

Second, every time Bank L extends a loan to group B , it observes a repayment drawn from $\mathcal{N}(\mu, \sigma_B^2)$. Let m be the number of such observations. Denote by $\hat{\sigma}_B^{(m)2}$ the sample variance computed from these m observations. By the Strong Law of Large Numbers,

$$\hat{\sigma}_B^{(m)2} \rightarrow \sigma_B^2 \quad \text{almost surely as } m \rightarrow \infty.$$

Because $\sigma_B < \tilde{\sigma}^L$, there exists a (finite) index m^* (and thus a finite time τ) such that for all $m \geq m^*$ the updated estimate satisfies:

$$\sigma_B \leq \hat{\sigma}_B^{(m)} < \tilde{\sigma}^L.$$

Third, once the updated variance estimate satisfies $\hat{\sigma}_{BL}^{(t)} < \tilde{\sigma}^L$, we can evaluate the required subsidy as

$$s^{*(t)} = \max\left\{0, \rho_L - (\mu_W + \mu_B) - \Phi^{-1}(\alpha) (\hat{\sigma}_{BL}^{(t)} + \sigma_W)\right\} = 0,$$

by the same calculation as in Lemma 5. Thus, for all $t \geq \tau$, Bank L is able to satisfy its VaR constraint without any subsidy. Since the number of observations required is finite almost surely, we conclude that with probability one there exists a finite τ such that for all $t \geq \tau$, $s^{*(t)} = 0$ and Bank L continues lending to group B without further external support.

This completes the proof. \square

Additional Results

We extend Theorem 2 by demonstrating that the conclusion holds under a broader class of temporary risk-sharing mechanisms. Specifically, we show that the subsidy mechanism described in the theorem is not the only way to induce Bank L to lend to group B and break the subprime trap. Any guarantee mechanism that ensures Bank L satisfies its VaR constraint during an exploration phase will suffice, provided it allows the bank to accumulate sufficient repayment data to update its risk estimate. We formalize this result as the following corollary.

Corollary 3 (Robustness to Alternative Guarantees). *The conclusion of Theorem 2 holds for any temporary risk-sharing mechanism that satisfies the following condition: for each period t of the exploration phase, the mechanism ensures that*

$$\Pr(\Pi_j^{(t)} + G(t) < \rho_L) \leq \alpha,$$

where $G(t)$ is the guarantee provided in period t . After sufficient data collection, the updated variance estimate $\hat{\sigma}_{BL}^{(t)}$ will satisfy $\hat{\sigma}_{BL}^{(t)} < \tilde{\sigma}^L$, allowing Bank L to lend without further guarantees.

Proof. For any such $G(t)$, the VaR constraint is satisfied during each period t of the exploration phase. Specifically, Bank L is guaranteed that its effective return, $\pi_B^{(t)} + G(t)$, will not fall below ρ_L with probability exceeding α . This ensures that the bank has an incentive to lend to group B , provided the expected return is positive. The guarantee mechanism defined in the corollary generalizes the subsidy mechanism from Theorem 2. In the original subsidy framework, the guarantee function is given explicitly by $G(t) = s^{*(t)} = \max\{0, \rho_L - (\mu_W + \mu_B) - \Phi^{-1}(\alpha) (\hat{\sigma}_{BL}^{(t)} + \sigma_W)\}$, which satisfies the condition $\Pr(\Pi_j^{(t)} + s^{*(t)} < \rho_L) \leq \alpha$. The corollary allows for any $G(t)$ that satisfies the same probabilistic constraint. Once Bank L updates its risk estimate to $\hat{\sigma}_{BL}^{(t)} < \tilde{\sigma}^L$, the VaR constraint is naturally satisfied without external support. Specifically, the guarantee $G(t)$ is no longer required, as the bank can safely lend to group B on its own. Formally, this follows from Lemma 2.

Thus, any guarantee mechanism that ensures the bank's effective return satisfies the VaR condition during a finite exploration phase will induce Bank L to lend, learn group B 's true risk, and ultimately eliminate the need for the guarantee. This completes the proof. \square