

# Network Science

## Network Measures

Lab 03

Lecture 08

# Today's Topics

- Slice Weighted Networks
- Measuring Networks
- Case Study : Penne Paper

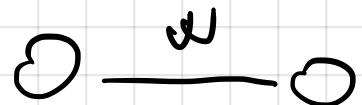
## Slice Weighted Networks

In weighted networks, some edges are strong, some are weak

Sometimes you cannot keep all the edges

Slicing is the process of eliminating low-strength edges

Simplest form: you choose a cut-off threshold  $T$  that controls the density of the resulting network



+ edge  $e$ :  
if  $\text{weight}(e) < T$   
remove  $e$ ;

# How to select $T$

- ① Select a  $T$  based on edge weight distribution
- ② Slice the network
- ③ Calculate some network properties (# components, density, ...)
- ④ If you are unsatisfied with results, go back to 1.

run 06\_Slicing.ipynb

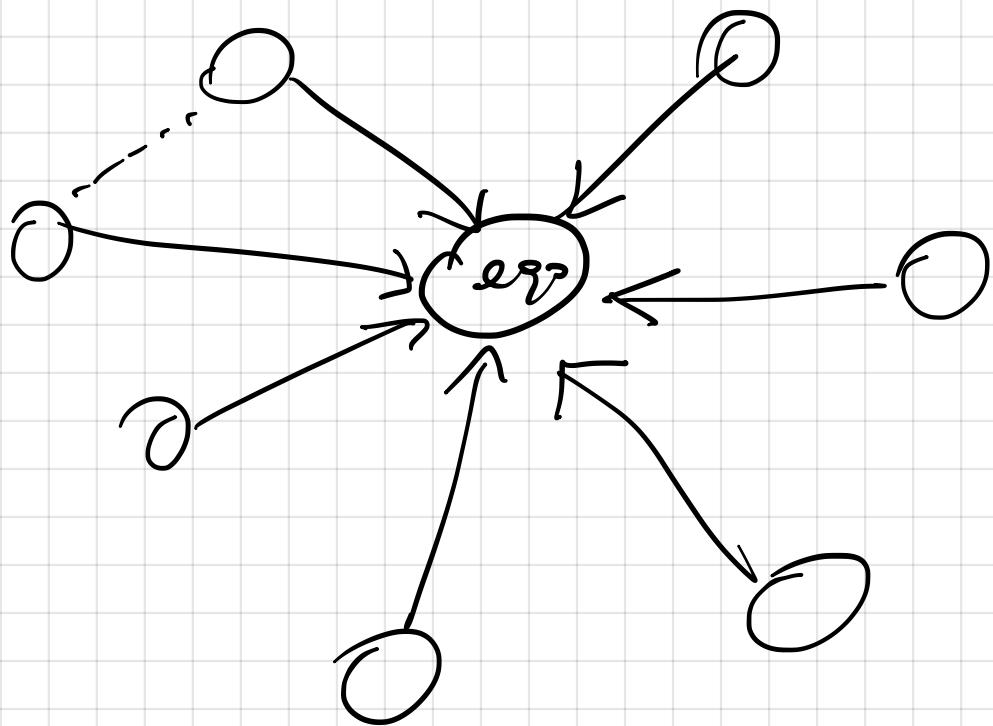


# Measuring Networks

with notebook 07 - measuring  
We will learn how to  
calculate the following properties  
with networkx:

- density
- clustering coefficient
- transitivity
- paths and distances
- eccentricity, diameter, radius  
periphery
- basic centralities
- assortativity
- homophily

the notebook calculates some metrics for the ego network of a previously retrieved Wikipedia page



ego network = ego + alters +  
connections between  
the alters

# Density

fraction of existing edges  
by non-existing edges

$m$ : # edges

$N$ : # nodes

$G$ : graph

$$d = \frac{m}{n(n-1)}$$

if  $G$  is  
directed

$$d = \frac{2m}{n(n-1)}$$

if  $G$  is  
undirected

# Clustering Coefficient

Network  $\mathcal{N}$  = only for  
undirected graph

$$cc(i) = \frac{2|L(i)|}{|N(i)|(|N(i)|-1)}$$

$i$  = node

$N(i)$  = Neighbors of  $i$

$L(i)$  = links connecting neighbors  
of  $i$

$cc(i)$ : is the probability that  
"friends" of  $i$  are  
connected each other

otherwise : The number of  
triangles closed by  $i$

# clustering coefficient metrics :

$cc(i)$  : clustering coefficient  
of a node

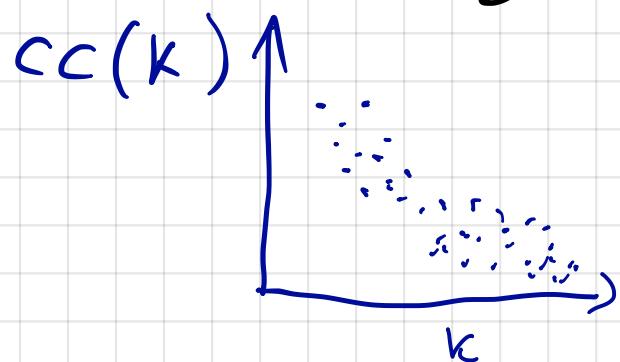
$cc(G)$  = average clustering coefficient of  
the graph

$$cc(A) = \frac{1}{n} \sum_{i=1}^n cc(i)$$

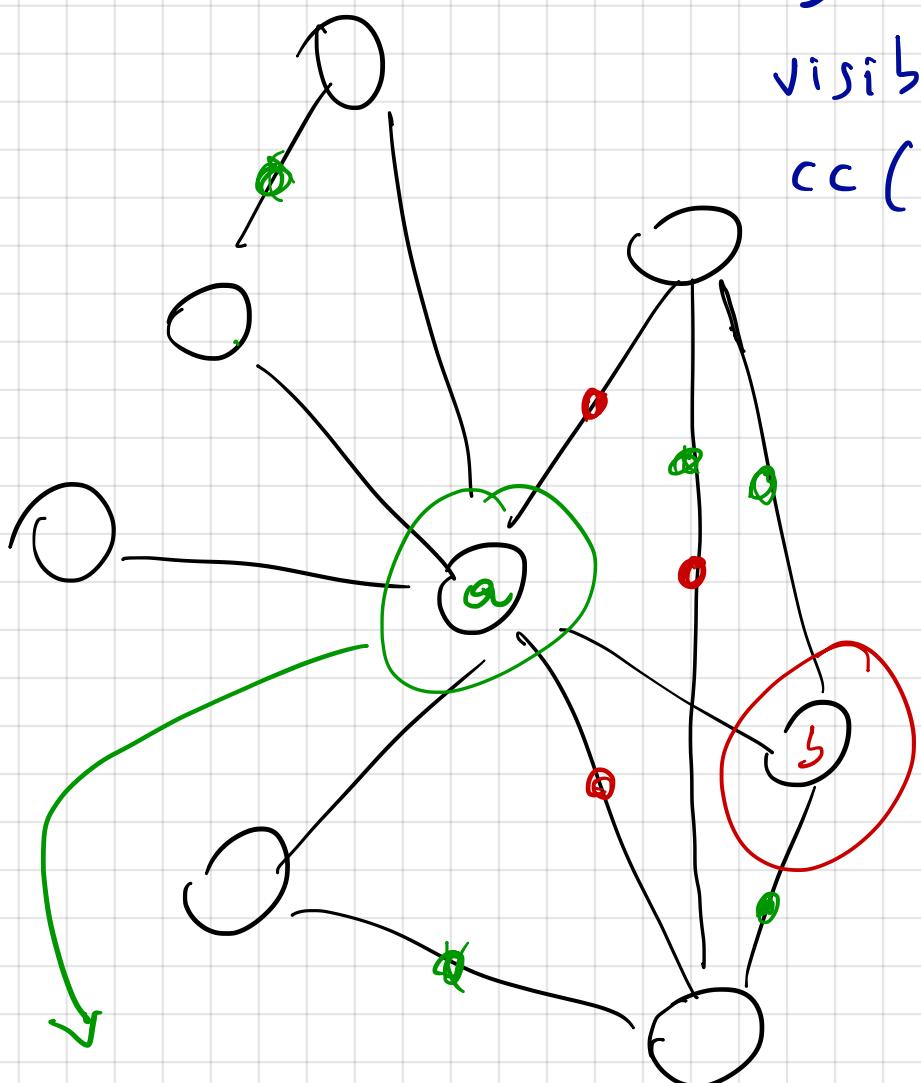
Transitivity : number of triangles  
in the graph

distribution of clustering coefficient:

usually plotted  
as a function of  
the node degree



Hubs are more unlikely to have high clustering coefficient w.r.t. low degree nodes.



5 links (between friends)

7 neighbors

$$cc(a) = \frac{2 \cdot 5}{7(7-1)} = \frac{10}{42}$$

3 links

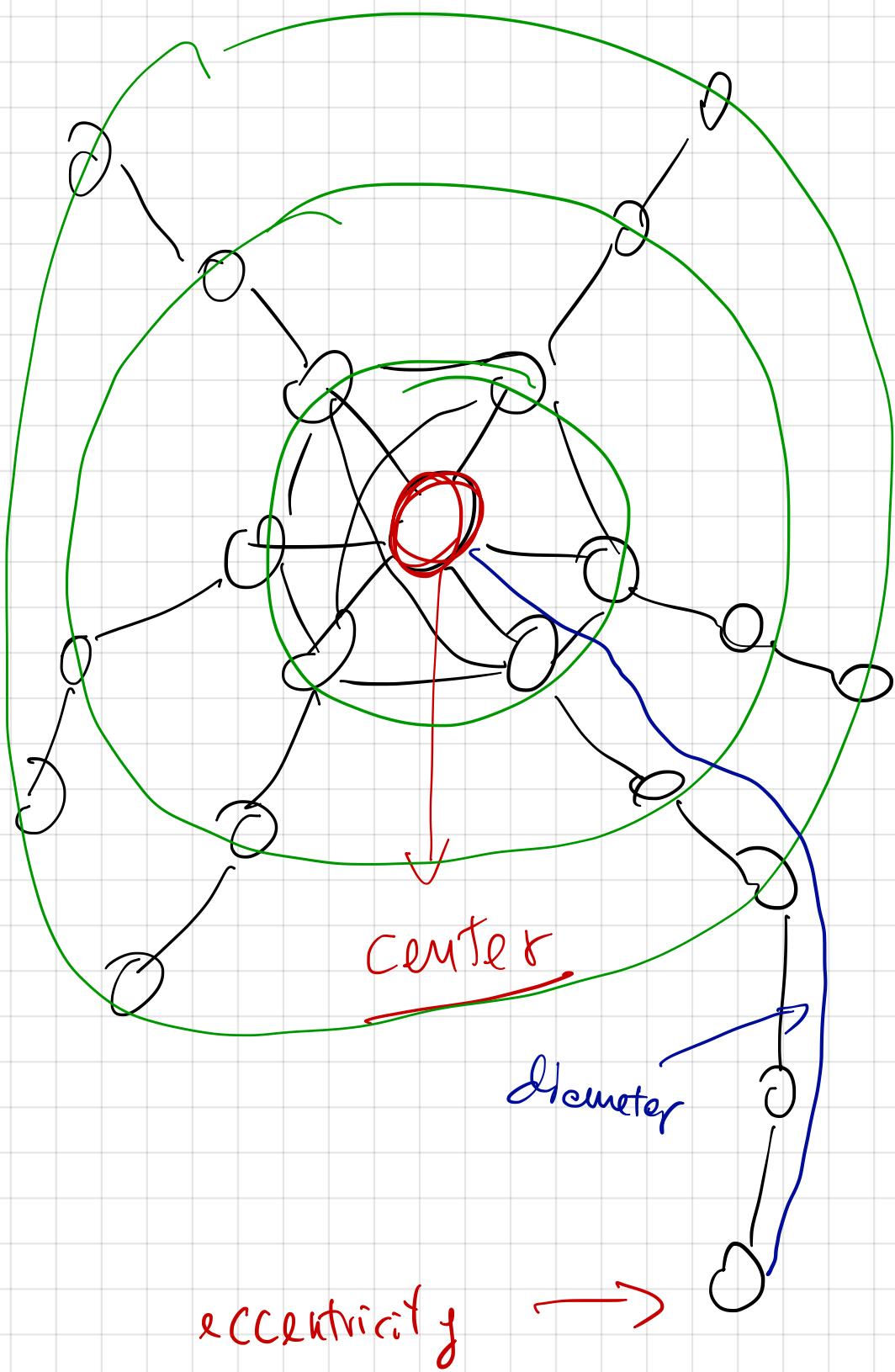
3 neighbors

$$cc(b) = \frac{2 \cdot 3}{3 \cdot 2} = \frac{1}{1}$$

$$cc(a) < cc(b)$$

↳ this is "usually" visible if distribution of  $cc(k)$  is plotted

# Eccentricity, diameter, radius, and center of a graph

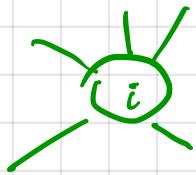


Degree

Centrality

-Popularity

$$D(i) = \frac{k_i}{N-1}$$



$$k_i = 5$$

Betweenness Centrality

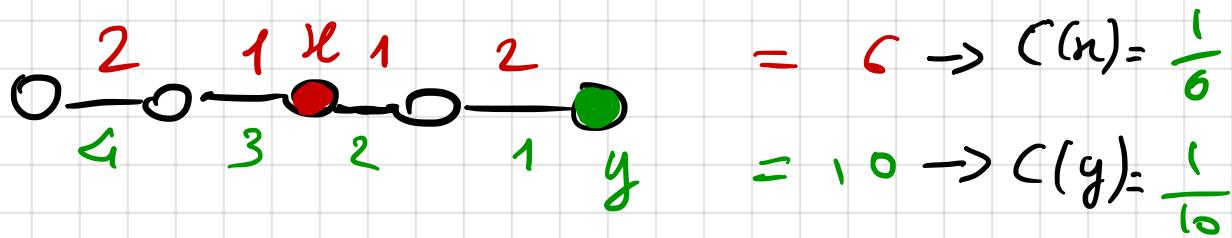
$$B(i) = \sum_{j < k} \frac{d_{jk}(i)}{d_{jk}}$$

another  
kind of  
"importance"

- "structural holes"
- gate keepers
- ...

## Closeness Centrality

$$C(i) = \left[ \sum_{j=1}^N d(i, j) \right]^{-1}$$



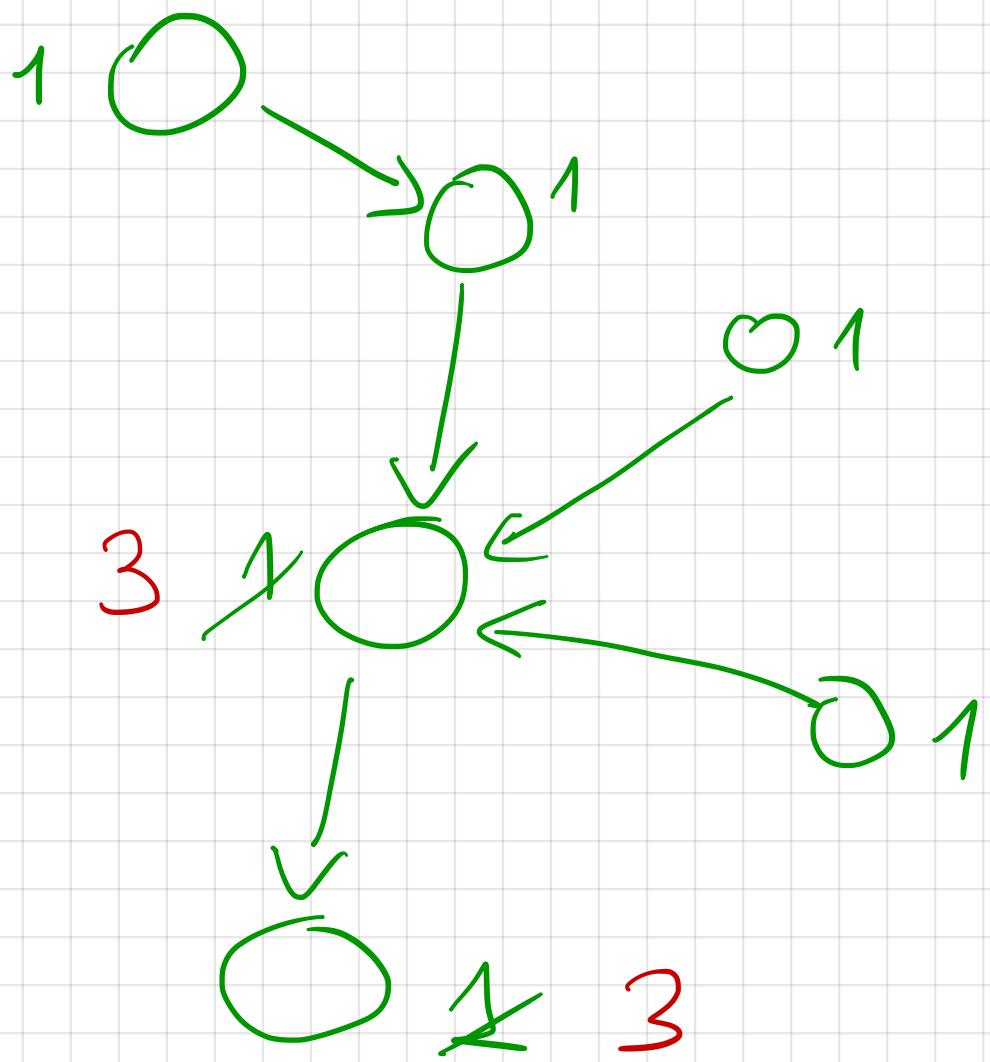
## Eigenvector Centrality

(more complicated)

high eigenvector centralities identifies nodes that are surrounded by other nodes with high eigenvector centrality

It's a measure of prestige

if we set initial eigenvector centrality values to 1, after few iterations we may refine our calculations



## Page Rank

similar to Eigen vector centrality

developed by Google (Page, Brin)

The rank of a node is calculated as the probability that a "person" randomly traversing the edges will arrive at the node

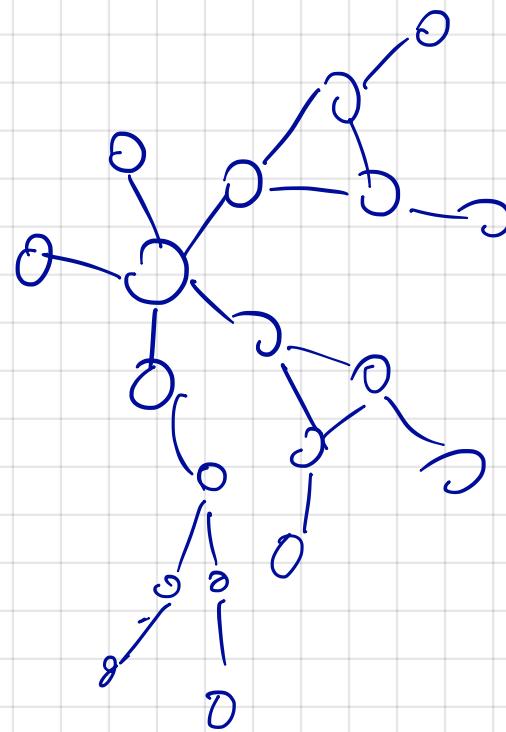
Dumping factor ( $\alpha = 0.85$ ):

↳ the probability that the user will continue clicking

## HITS: Hubs and Authorities

it is an extended version of Page Rank

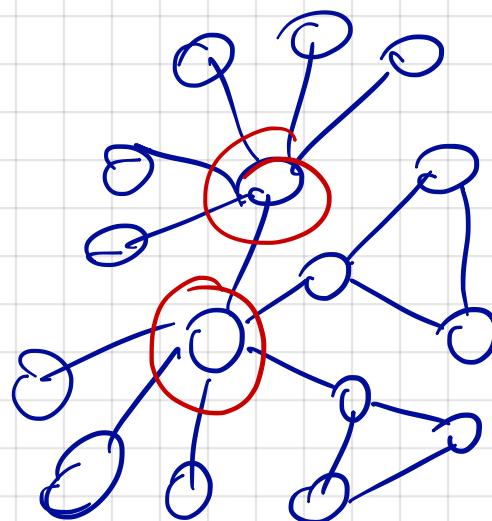
## Assortativity



hierarchy

dissassortative

= heterophily  
by degree



core-periphery

assortative

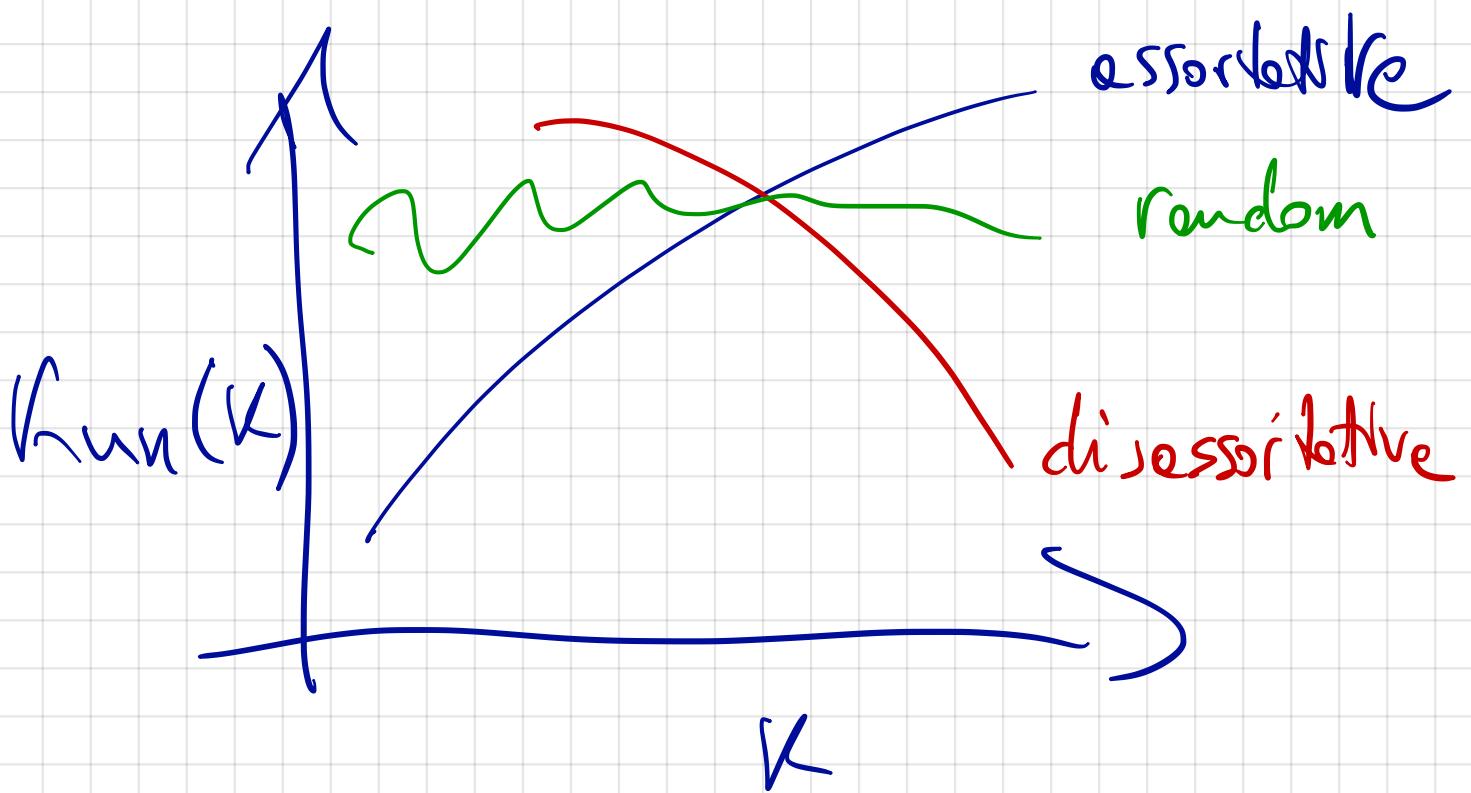
= homophily  
by degree

$$K_{nn}(k) =$$

average degree  
for

nearest neighbors

of nodes with  
degree =  $k$



# Exercise

Go back to your  
"classmates" network  
and calculate all the  
basic properties we  
learnt to measure today

Also : Focus on  
Homo phily  
( gender , study program ,  
... )

# Panome Papers

run

08 - panome

09 - panome-CQ

## Exercise

modify previous notebooks

To evaluate the measures you obtained with "panome papers" data.

To evaluate = are they "significant"

Hint : you need some baseline ...

Hint 2 : you may want to consider random models ...