Two - Dimensional Unsteady state Heat diffusion problem Results

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1 Project Objectives

1. Solving the two dimensional unsteady diffusion equation using finite difference method.
2. Plotting the results for various time frames.

2 Theory and equations

For two dimensional heat diffusion problem , we make following assumptions

1. No convection happens at the boundary wall
2. No internal heat generation or dissipation
3. Constant thermal properties of system viz, independent of time, temperature and dimension. Hence for two dimensions we have equations,

Hence for two dimensions we have equations,

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \frac{\partial T}{\partial t} \tag{1}$$

Discretising spatial terms by central differencing ($O(h^2)$), we get

$$\frac{\partial^2 T}{\partial x^2} = \frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{\Delta x^2} \tag{2}$$

$$\frac{\partial^2 T}{\partial y^2} = \frac{T_{i-1,j} - 2T_{i,j} + T_{i+1,j}}{\Delta y^2} \tag{3}$$

Discretizing temporal terms by forward differencing ($O(h)$), we get

$$\frac{\partial T}{\partial t} = \frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t} \tag{4}$$

Substituting Eqn. 2,3,4 into 1 we get,

$$\frac{(T_{i-1,j} - 2T_{i,j} + T_{i+1,j})^{n+1}}{\Delta x^2} + \frac{(T_{i-1,j} - 2T_{i,j} + T_{i+1,j})^{n+1}}{\Delta y^2} = \alpha \frac{T_{i,j}^{n+1} - T_{i,j}^{n}}{\Delta t} \tag{5}$$

Simplyfying , we get

$$T_{i,j}^{n+1} = \frac{T_{i,j}^{n} + [k1 * (T_{i-1,j} + T_{i+1,j}) + k2 * (T_{i,j-1} + T_{i,j+1})]^{n+1}}{1 + 2k1 + 2k2} \tag{6}$$

Where,

$$k1 = \alpha \frac{\Delta t}{(\Delta x)^2}; k2 = \alpha \frac{\Delta t}{(\Delta y)^2} \tag{7}$$

3 Plots and Results

3.1 Usage

References