```
1  begin
2      import Pkg
3      Pkg.activate(".")
4      using CairoMakie, DrWatson,
5      DataFramesMeta, Statistics, PlutoUI,
       Colors
   end
```

> Activating project at `~/Document ⑦
> s/PhD/my-papers/2023-cTWA-Javad/Discret
> eCTWAPaper/notebooks`

```
1  html"""
2  <style>
3  main { max-width: 60%}
4  </style>
5  """
```

Table of Contents

eomdrop (generic function with 1 method)

```
1  begin
2      eom(A;dims) = std(A;dims) ./ √(reduce(*,
3      size(A,d) for d in dims))
4      meandrop(A;dims) =
5      dropdims(mean(A;dims);dims)
6      stddrop(A;dims) =
       dropdims(std(A;dims);dims)
       eomdrop(A;dims) =
       dropdims(eom(A;dims);dims)
   end
```

save_and_display (generic function with 3 methods

```
1  begin
2      function save_and_display(name,
3      folder="../plots")
4          return fig -> save_and_display(name,
5          fig, folder)
6      end
7      function save_and_display(name, fig,
8      folder)
9          mkpath(folder)
10         mkpath(joinpath(folder, "png"))
11         Makie.save(joinpath(folder,
12         name*".pdf"), fig)
           Makie.save(joinpath(folder, "png",
           name*".png"), fig)
           fig
       end
   end
```

Style

HEIGHT = 250

```
1  HEIGHT = 250
```

SCALE = 2

```
1  SCALE = 2
```

bgcolor =

```
1  bgcolor = RGBf(1,1,1)
```

THEME =
Attributes with 8 entries:
  Axis => Attributes with 4 entries:
    backgroundcolor => RGB{Float32}(1.0,1.0,1.0)
    xscale => identity
    xtickalign => 1
    ytickalign => 1
  figure_padding => (1, 7, 1, 1)
  fonts => Attributes with 4 entries:
    bold => FTFont (family = NewComputerModern, s
    bolditalic => FTFont (family = NewComputerMod
    italic => FTFont (family = NewComputerModern,
    regular => FTFont (family = NewComputerModern
  fontsize => 18
  Label => Attributes with 3 entries:
    font => bold
    halign => left
    valign => top
  Legend => Attributes with 1 entry:
    backgroundcolor => RGB{Float32}(1.0,1.0,1.0)
  pt_per_unit => 0.5
  size => (492, 500)

```
1  THEME = merge(theme_latexfonts(),
2      Theme(fontsize=9*SCALE,
3      size=(246*SCALE,HEIGHT*SCALE),
4      pt_per_unit=1/SCALE,
5          figure_padding=(1,7,1,1),
6          Axis=(; xtickalign=1, ytickalign=1,
7          xscale=identity,
8              backgroundcolor=bgcolor),
           Legend=(; backgroundcolor=bgcolor),
           Label=(; font=:bold,
               halign=:left, valign=:top)))
```

section1_palette =

(dTWA =   , rg_dcTWA =   , ed =

```julia
1  # section1_palette = (;
2  #    dTWA = Makie.wong_colors()[1],
3  #    rg_dcTWA = Makie.wong_colors()[7],
4  #    ed = RGBAf(0,0,0),
5  #    rg_gcTWA = Makie.wong_colors()[3],
6  #    naive_gcTWA = Makie.RGBf(0.9,0.3,0.7))
7  section1_palette = (;
8      dTWA = brini[2],
9      rg_dcTWA = brini[5],
10     ed = brini[3],
11     rg_gcTWA = brini[4],
12     naive_gcTWA = brini[1])
```

to_color (generic function with 1 method)

```julia
1  to_color(r,g,b) = RGBf(r/255,g/255,b/255)
```

brini =

```julia
1  brini = [to_color(29, 59, 181),
   to_color(220, 171, 67), to_color(51, 51,
   51), to_color(119, 188, 101), to_color(135,
   78, 189)]
```

Section 1&2

Load data

```
1  df_raw = collect_results("../data/
   simulations");
```

Scanning folder ../data/simulations for
result files.

Added 1092 entries.

`df_avg =`

	clustersize	N	α	Δ	tli:
1	missing	16	0.5	0	0.0:0.2
2	missing	16	1.0	0	0.0:0.2
3	missing	16	3.0	0	0.0:0.2
4	missing	16	6.0	0	0.0:0.2
5	missing	16	0.5	2	0.0:0.2
6	missing	16	6.0	2	0.0:0.2
7	missing	16	0.5	4	0.0:0.2
8	missing	16	6.0	4	0.0:0.2
9	missing	16	1.0	2	0.0:0.2
10	missing	16	3.0	2	0.0:0.2
	more				
116	4	16	3.0	0	0.0:0.0

```julia
df_avg = @chain df_raw begin
    groupby([:clustersize, :N, :α, :Δ,
    :tlist, :alg, :filling, :clustering])
    @combine(
        :magnetization_mean =
        Ref(mapreduce(x->getproperty.(x,
        Ref(Symbol("magnetization_mean"))),
        vcat, :results)),
        :magnetization_eom = Ref(mapreduce(x-
        >getproperty.(x,
        Ref(Symbol("magnetization_eom"))),
        vcat, :results)),
        :pair_renyi2 = Ref(mapreduce(x-
        >getproperty.(x,
        Ref(Symbol("pair_renyi2"))), vcat,
        :results)))
    @rtransform(
        :nshots =
        length(:magnetization_mean),
        :magnetization_mean =
        meandrop(stack(:magnetization_mean);
        dims=2),
        :magnetization_eom_stat =
        eomdrop(stack(:magnetization_mean);
        dims=2),
        :magnetization_eom_syst = sqrt.
        (meandrop(stack(:magnetization_eom)
        .^ 2; dims=2)),
        :pair_renyi2_mean =
        meandrop(stack(:pair_renyi2);
        dims=2),
        :pair_renyi2_eom =
        eomdrop(stack(:pair_renyi2); dims=2))
    @rtransform(
        :magnetization_eom_full = hypot.
        (:magnetization_eom_stat,
        :magnetization_eom_syst))
    select!
    (Not([:pair_renyi2,:magnetization_eom]))
end
```
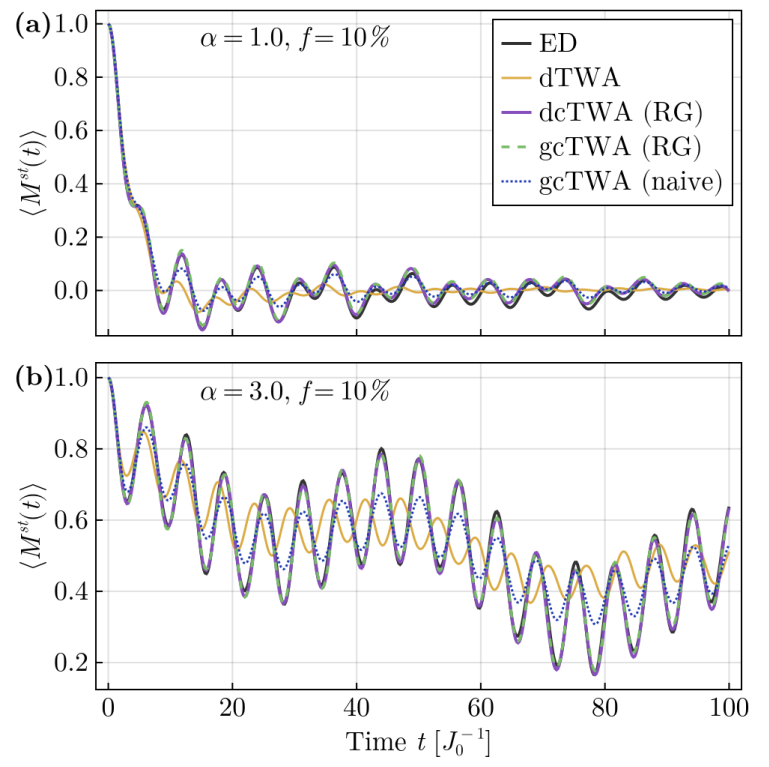
```
make_plot_section12! (generic function with 1 meth
1 function make_plot_section12!(ax, data,
2 mean, error; colors=section1_palette)
3     dTWA, rg_dcTWA, naive_dcTWA, ed,
4     rg_gcTWA, naive_gcTWA =
5     eachrow(sort(data, [:alg, :clustering]))
6     lines!(ax, ed.tlist, ed[mean];
7     color=colors.ed, linewidth=2, label="ED")
8     lines!(ax, dTWA.tlist, dTWA[mean];
       color=colors.dTWA, label="dTWA")
       lines!(ax, rg_dcTWA.tlist,
       rg_dcTWA[mean]; color=colors.rg_dcTWA,
       label="dcTWA (RG)", linewidth=2)
       lines!(ax, rg_gcTWA.tlist,
       rg_gcTWA[mean]; color=colors.rg_gcTWA,
       linestyle=:dash, label="gcTWA (RG)",
       linewidth=1.8)
       lines!(ax, naive_gcTWA.tlist,
       naive_gcTWA[mean];
       color=colors.naive_gcTWA,
       linestyle=Linestyle([0.0,1.0,2.0]),
       label="gcTWA (naive)")
    end
```

Fig. 3 Magnetization f=10%
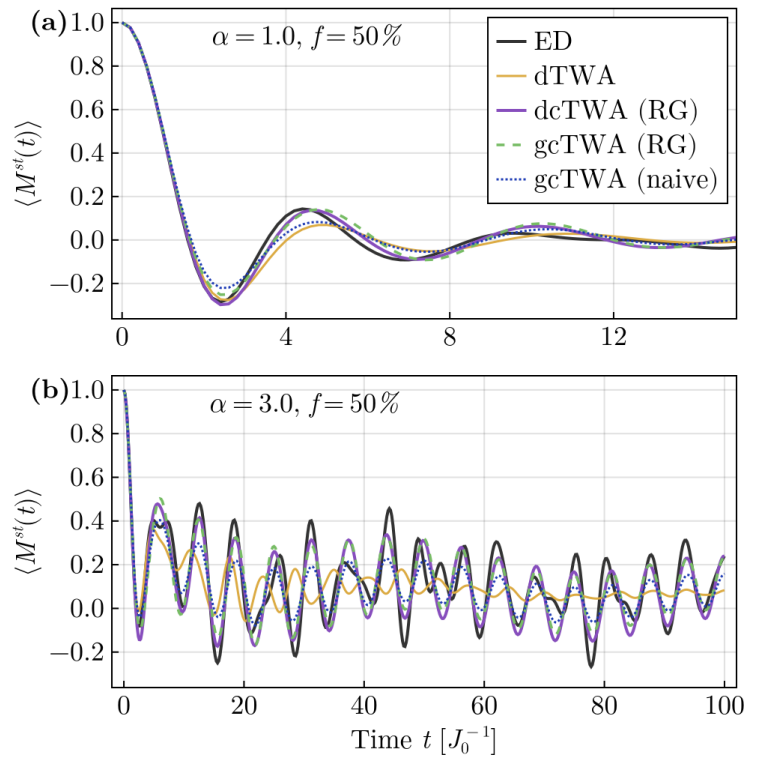
(a) $\alpha = 1.0$, $f = 10\%$

ED
dTWA
dcTWA (RG)
gcTWA (RG)
gcTWA (naive)

(b) $\alpha = 3.0$, $f = 10\%$

$\langle M^{st}(t) \rangle$

Time t $[J_0^{-1}]$

```julia
with_theme(THEME) do
    let fig = Figure(),
        ax1 = Axis(fig[1,1];
        ylabel=L"\langle M^{st}(t)\rangle")
        make_plot_section12!(ax1,
            @rsubset(df_avg, :α == 1,
            :filling == 0.1, :Δ == 0),
            :magnetization_mean,
            :magnetization_eom_full)
        ax2 = Axis(fig[2,1]; xlabel=L"Time
        $t$ $[J_0^{-1}]$", ylabel=L"\langle
        M^{st}(t)\rangle")
        make_plot_section12!(ax2,
            @rsubset(df_avg, :α == 3,
            :filling == 0.1, :Δ == 0),
            :magnetization_mean,
            :magnetization_eom_full)
        xlims!.([ax1, ax2],-2, 102)
        ylims!(ax1,-0.17, 1.05)
        ax1.xticks = 0:20:100
        ax2.xticks = 0:20:100
        ax1.yticks = -1:0.2:1
        ax2.yticks = -1:0.2:1
        ax1.xticklabelsvisible = false
        #Legend(fig[3,1], ax; )
        axislegend(ax1; position=:rt)
        text!(ax1, 30.0, 1.0;
        text=L"\alpha=1.0,\ f=10%",
        align=(:center,:top))
        text!(ax2, 30.0, 1.0;
        text=L"\alpha=3.0,\ f=10%",
        align=(:center,:top))

        Label(fig[1,1]; text="(a)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-55, bottom=6))
        Label(fig[2,1]; text="(b)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-55, bottom=6))

        fig
    end
end |> save_and_display("fig3")
```

Fig. 4 Magnetization f=50%

(a) $\alpha = 1.0,\, f = 50\%$

ED
dTWA
dcTWA (RG)
gcTWA (RG)
gcTWA (naive)

(b) $\alpha = 3.0,\, f = 50\%$

$\langle M^{st}(t) \rangle$

Time $t\ [J_0^{-1}]$
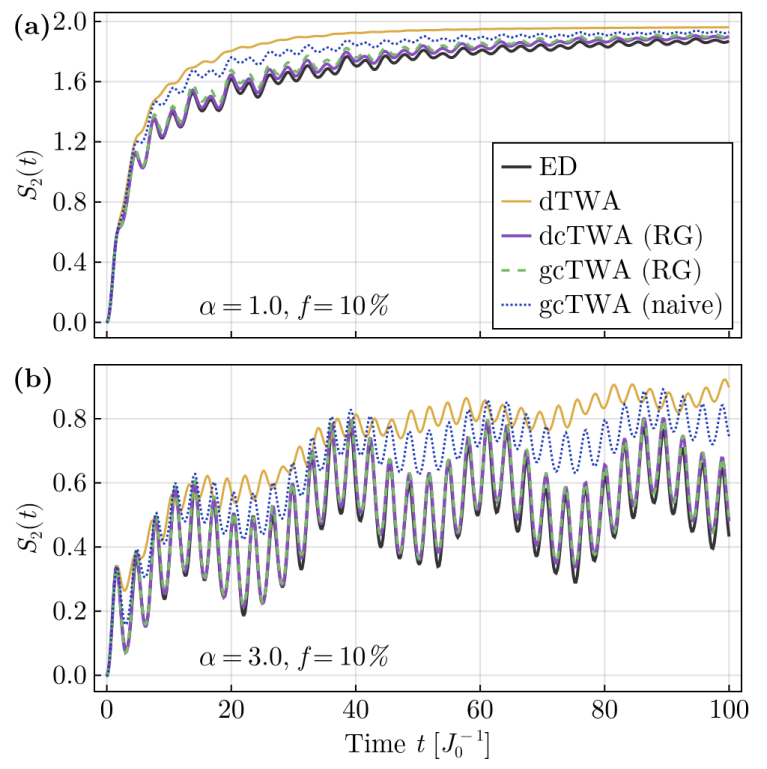
```julia
1  with_theme(THEME) do
2      let fig = Figure(),
3          ax1 = Axis(fig[1,1];
4          ylabel=L"\langle M^{st}(t)\rangle")
5          make_plot_section12!(ax1,
6              @rsubset(df_avg, :α == 1,
7              :filling == 0.5, :Δ == 0),
8              :magnetization_mean,
9              :magnetization_eom_full)
10         ax2 = Axis(fig[2,1]; xlabel=L"Time
11         $t$ $[J_0^{-1}]$", ylabel=L"\langle
12         M^{st}(t)\rangle")
13         make_plot_section12!(ax2,
14             @rsubset(df_avg, :α == 3,
15             :filling == 0.5, :Δ == 0),
16             :magnetization_mean,
17             :magnetization_eom_full)
18         xlims!(ax1, -0.25, 15)
19         ax1.xticks = 0:4:20
20         ax1.yticks = -0.2:0.2:1
21
22         xlims!(ax2,-2, 102)
23         ax2.xticks = 0:20:100
24         ax2.yticks = -0.2:0.2:1
25         linkyaxes!(ax1,ax2)
26
27         axislegend(ax1; position=:rt)
28         text!(ax1, 4.5, 1.0;
29         text=L"\alpha=1.0,\ f=50%",
           align=(:center,:top))
           text!(ax2, 30.0, 1.0;
           text=L"\alpha=3.0,\ f=50%",
           align=(:center,:top))

           Label(fig[1,1]; text="(a)",
           tellwidth=false, tellheight=false,
           alignmode=Mixed(;left=-55, bottom=6))
           Label(fig[2,1]; text="(b)",
           tellwidth=false, tellheight=false,
           alignmode=Mixed(;left=-55, bottom=6))

           fig
       end
   end |> save_and_display("fig4")
```

Fig. 6: Renyi2 f=10%

(a) $\alpha = 1.0$, $f = 10\%$

Legend:
- ED
- dTWA
- dcTWA (RG)
- gcTWA (RG)
- gcTWA (naive)

(b) $\alpha = 3.0$, $f = 10\%$

$S_2(t)$

Time t $[J_0^{-1}]$

```
1  with_theme(THEME) do
2      let fig = Figure(),
3          ax1 = Axis(fig[1,1]; ylabel=L"S_{2}
4          (t)")
5          make_plot_section12!(ax1,
6              @rsubset(df_avg, :α == 1,
7              :filling == 0.1, :Δ == 0),
8              :pair_renyi2_mean,
9              :pair_renyi2_eom)
10         ax2 = Axis(fig[2,1]; xlabel=L"Time
11         $t$ $[J_0^{-1}]$", ylabel=L"S_{2}
12         (t)")
13         make_plot_section12!(ax2,
14             @rsubset(df_avg, :α == 3,
15             :filling == 0.1, :Δ == 0),
16             :pair_renyi2_mean,
17             :pair_renyi2_eom)
18         xlims!.([ax1, ax2],-2, 102)
19         #ylims!(ax1,-0.17, 1.05)
20         ax1.xticks = 0:20:100
21         ax2.xticks = 0:20:100
22         ax1.yticks = -0:0.4:2
23         ax2.yticks = -1:0.2:2
24         ax1.xticklabelsvisible = false
25         #Legend(fig[3,1], ax; )
26         axislegend(ax1; position=:rb)
27         text!(ax1, 30.0, 0.2;
28         text=L"\alpha=1.0,\ f=10%",
           align=(:center,:top))
           text!(ax2, 30.0, 0.1;
           text=L"\alpha=3.0,\ f=10%",
           align=(:center,:top))

           Label(fig[1,1]; text="(a)",
           tellwidth=false, tellheight=false,
           alignmode=Mixed(;left=-55, bottom=6))
           Label(fig[2,1]; text="(b)",
           tellwidth=false, tellheight=false,
           alignmode=Mixed(;left=-55, bottom=6))

           fig
       end
   end |> save_and_display("fig6")
```
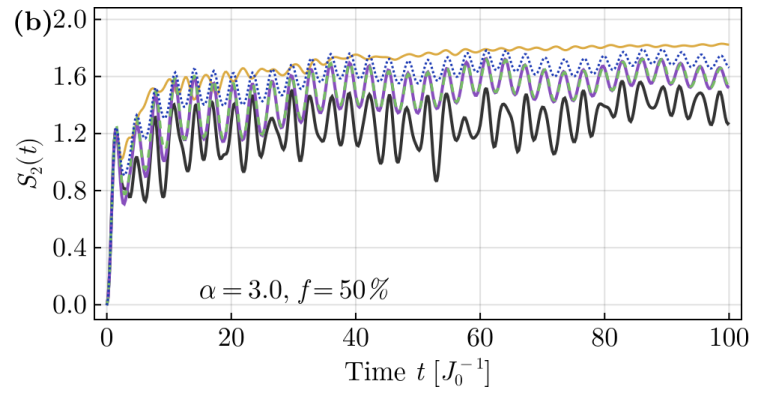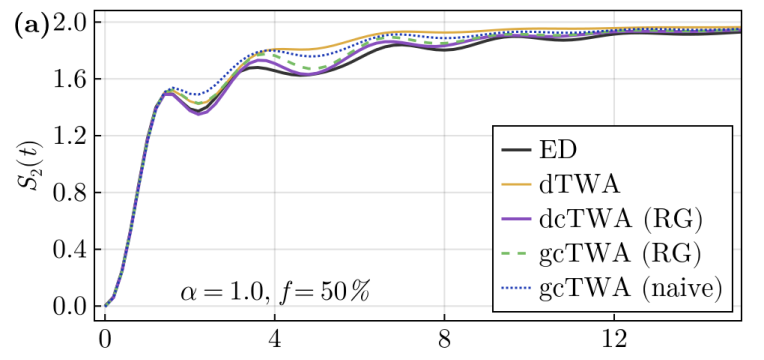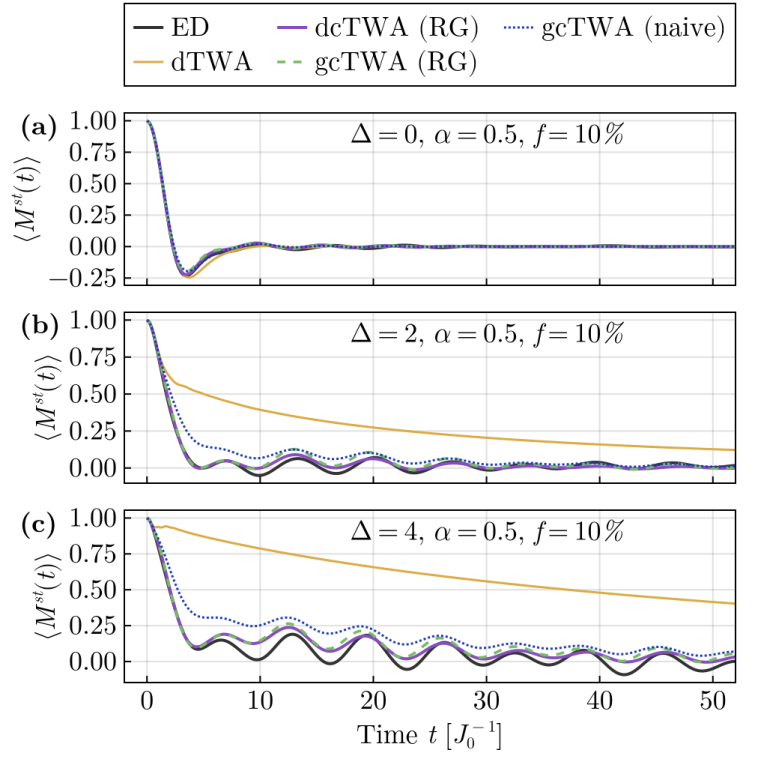
Fig. 7 Renyi2 f=50%

(a) $\alpha = 1.0,\ f = 50\%$

Legend:
- ED
- dTWA
- dcTWA (RG)
- gcTWA (RG)
- gcTWA (naive)

(b) $\alpha = 3.0,\ f = 50\%$

$S_2(t)$

Time t $[J_0^{-1}]$

```julia
with_theme(THEME) do
    let fig = Figure(),
        ax1 = Axis(fig[1,1]; ylabel=L"S_{2}
        (t)")
        make_plot_section12!(ax1,
            @rsubset(df_avg, :α == 1,
            :filling == 0.5, :Δ == 0),
            :pair_renyi2_mean,
            :pair_renyi2_eom)
        ax2 = Axis(fig[2,1]; xlabel=L"Time
        $t$ $[J_0^{-1}]$", ylabel=L"S_{2}
        (t)")
        make_plot_section12!(ax2,
            @rsubset(df_avg, :α == 3,
            :filling == 0.5, :Δ == 0),
            :pair_renyi2_mean,
            :pair_renyi2_eom)

        xlims!(ax1, -0.25, 15)
        ax1.xticks = 0:4:20
        ax1.yticks = 0:0.4:2

        xlims!(ax2,-2, 102)
        ax2.xticks = 0:20:100
        ax2.yticks = 0:0.4:2
        linkyaxes!(ax1,ax2)

        #ax1.xticklabelsvisible = false

        axislegend(ax1; position=:rb)
        text!(ax1, 4.0, 0.2;
        text=L"\alpha=1.0,\ f=50%",
        align=(:center,:top))
        text!(ax2, 30.0, 0.2;
        text=L"\alpha=3.0,\ f=50%",
        align=(:center,:top))

        Label(fig[1,1]; text="(a)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-55, bottom=6))
        Label(fig[2,1]; text="(b)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-55, bottom=6))

        fig
    end
end |> save_and_display("fig7")
```

Fig. 8: XXZ Magnetization $f=10\%$, $\alpha=0.5$

(a) $\Delta = 0$, $\alpha = 0.5$, $f = 10\%$

(b) $\Delta = 2$, $\alpha = 0.5$, $f = 10\%$

(c) $\Delta = 4$, $\alpha = 0.5$, $f = 10\%$

Legend: ED, dTWA, dcTWA (RG), gcTWA (RG), gcTWA (naive)

y-axis: $\langle M^{st}(t) \rangle$

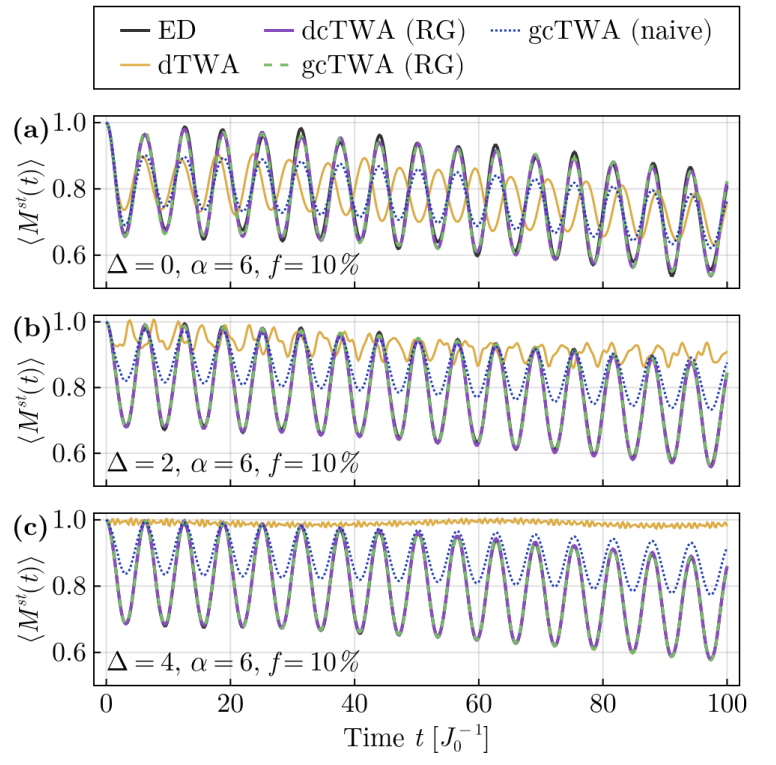x-axis: Time t $[J_0^{-1}]$

```julia
with_theme(THEME) do
    let fig = Figure(),
        ax1 = Axis(fig[1,1];
        ylabel=L"\langle M^{st}(t)\rangle")
        make_plot_section12!(ax1,
            @rsubset(df_avg, :α == 0.5,
            :filling == 0.1, :Δ == 0),
            :magnetization_mean,
            :magnetization_eom_full)
        ax2 = Axis(fig[2,1];
        ylabel=L"\langle M^{st}(t)\rangle")
        make_plot_section12!(ax2,
            @rsubset(df_avg, :α == 0.5,
            :filling == 0.1, :Δ == 2),
            :magnetization_mean,
            :magnetization_eom_full)
        ax3 = Axis(fig[3,1]; xlabel=L"Time
        $t$ $[J_0^{-1}]$", ylabel=L"\langle
        M^{st}(t)\rangle")
        make_plot_section12!(ax3,
            @rsubset(df_avg, :α == 0.5,
            :filling == 0.1, :Δ == 4),
            :magnetization_mean,
            :magnetization_eom_full)
        xlims!.([ax1, ax2,ax3],-2, 52)
        #ylims!(ax1,-0.17, 1.05)
        ax1.xticks = 0:10:100
        ax2.xticks = 0:10:100
        ax3.xticks = 0:10:100
        ax1.yticks = -1:0.25:1
        ax2.yticks = -1:0.25:1
        ax3.yticks = -1:0.25:1
        ax1.xticklabelsvisible = false
        ax2.xticklabelsvisible = false

        Legend(fig[0,1], ax1;
        orientation=:horizontal, nbanks=2,
        width=Relative(1))
        text!(ax1, 30.0, 1.0;
        text=L"\Delta=0,\ \alpha=0.5,\
        f=10%", align=(:center,:top))
        text!(ax2, 30.0, 1.0;
        text=L"\Delta=2,\ \alpha=0.5,\
        f=10%", align=(:center,:top))
        text!(ax3, 30.0, 1.0;
        text=L"\Delta=4,\ \alpha=0.5,\
        f=10%", align=(:center,:top))

        Label(fig[1,1]; text="(a)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-70, bottom=6))
        Label(fig[2,1]; text="(b)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-70, bottom=6))
        Label(fig[3,1]; text="(c)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-70, bottom=6))
```

```
        fig
    end
end |> save_and_display("fig8")
```

Fig. 9: XXZ Magnetization f=10%, α=6

(a) $\Delta = 0,\ \alpha = 6,\ f = 10\,\%$

(b) $\Delta = 2,\ \alpha = 6,\ f = 10\,\%$

(c) $\Delta = 4,\ \alpha = 6,\ f = 10\,\%$

Legend: ED, dTWA, dcTWA (RG), gcTWA (RG), gcTWA (naive)

$\langle M^{st}(t) \rangle$
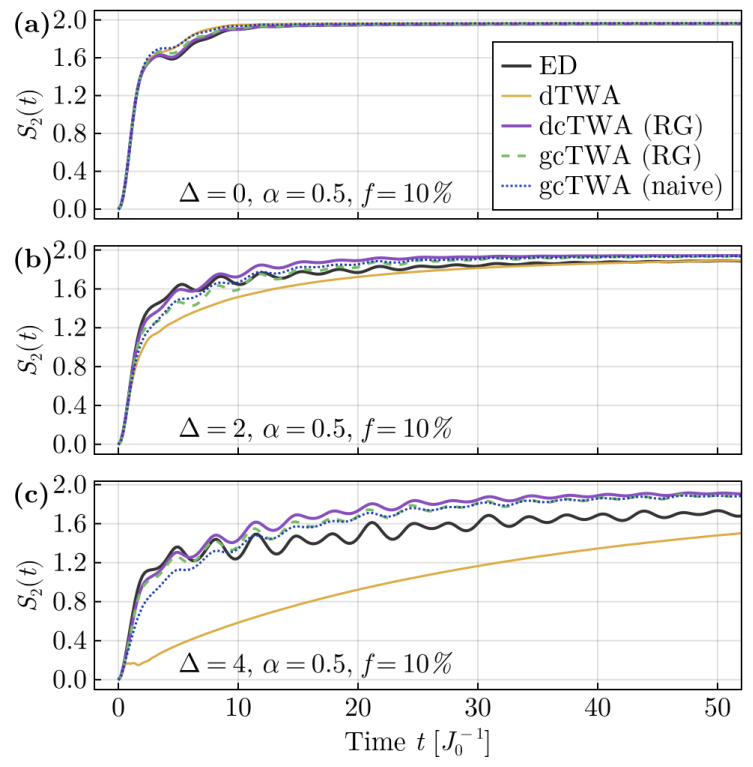
Time $t\ [J_0^{-1}]$

```julia
with_theme(THEME) do
    let fig = Figure(),
        ax1 = Axis(fig[1,1];
        ylabel=L"\langle M^{st}(t)\rangle")
        make_plot_section12!(ax1,
            @rsubset(df_avg, :α == 6,
            :filling == 0.1, :Δ == 0),
            :magnetization_mean,
            :magnetization_eom_full)
        ax2 = Axis(fig[2,1];
        ylabel=L"\langle M^{st}(t)\rangle")
        make_plot_section12!(ax2,
            @rsubset(df_avg, :α == 6,
            :filling == 0.1, :Δ == 2),
            :magnetization_mean,
            :magnetization_eom_full)
        ax3 = Axis(fig[3,1]; xlabel=L"Time
        $t$ $[J_0^{-1}]$", ylabel=L"\langle
        M^{st}(t)\rangle")
        make_plot_section12!(ax3,
            @rsubset(df_avg, :α == 6,
            :filling == 0.1, :Δ == 4),
            :magnetization_mean,
            :magnetization_eom_full)
        xlims!.([ax1, ax2, ax3], -2, 102)
        ylims!.([ax1, ax2, ax3], 0.5, 1.02)
        ax1.xticks = 0:20:100
        ax2.xticks = 0:20:100
        ax3.xticks = 0:20:100
        ax1.yticks = -1:0.2:1
        ax2.yticks = -1:0.2:1
        ax3.yticks = -1:0.2:1
        ax1.xticklabelsvisible = false
        ax2.xticklabelsvisible = false
        #Legend(fig[3,1], ax; )
        #axislegend(ax1; position=:lb,
        orientation=:horizontal, nbanks=2)
        Legend(fig[0,1], ax1;
        orientation=:horizontal, nbanks=2,
        width=Relative(1))
        text!(ax1, 0.0, 0.57;
        text=L"\Delta=0,\ \alpha=6,\ f=10%",
        align=(:left,:center))
        text!(ax2, 0.0, 0.57;
        text=L"\Delta=2,\ \alpha=6,\ f=10%",
        align=(:left,:center))
        text!(ax3, 0.0, 0.57;
        text=L"\Delta=4,\ \alpha=6,\ f=10%",
        align=(:left,:center))

        Label(fig[1,1]; text="(a)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-55, bottom=6))
        Label(fig[2,1]; text="(b)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-55, bottom=6))
        Label(fig[3,1]; text="(c)",
```

```
                tellwidth=false, tellheight=false,
                alignmode=Mixed(;left=-55, bottom=6))

            fig
    end
end |> save_and_display("fig9")
```

Fig. 10: XXZ Renyi f=10%, α=0.5

(a) $S_2(t)$

$\Delta = 0,\ \alpha = 0.5,\ f = 10\,\%$

ED
dTWA
dcTWA (RG)
gcTWA (RG)
gcTWA (naive)

(b) $S_2(t)$

$\Delta = 2,\ \alpha = 0.5,\ f = 10\,\%$

(c) $S_2(t)$

$\Delta = 4,\ \alpha = 0.5,\ f = 10\,\%$

Time $t\ [J_0^{-1}]$
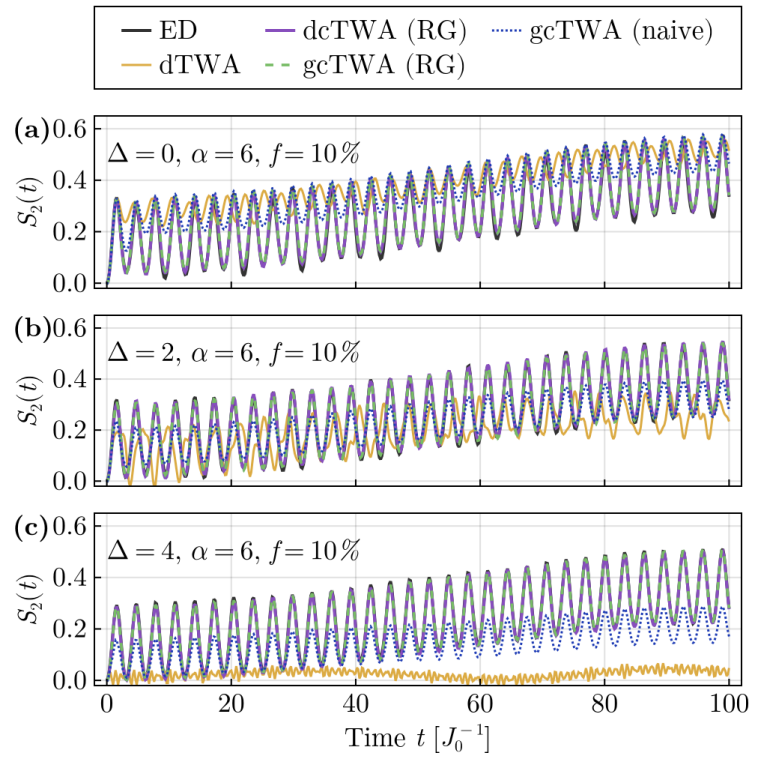
```julia
with_theme(THEME) do
    let fig = Figure(),
        ax1 = Axis(fig[1,1]; ylabel=L"S_{2}
        (t)")
        make_plot_section12!(ax1,
            @rsubset(df_avg, :α == 0.5,
            :filling == 0.1, :Δ == 0),
            :pair_renyi2_mean,
            :pair_renyi2_eom)
        ax2 = Axis(fig[2,1]; ylabel=L"S_{2}
        (t)")
        make_plot_section12!(ax2,
            @rsubset(df_avg, :α == 0.5,
            :filling == 0.1, :Δ == 2),
            :pair_renyi2_mean,
            :pair_renyi2_eom)
        ax3 = Axis(fig[3,1]; xlabel=L"Time
        $t$ $[J_0^{-1}]$", ylabel=L"S_{2}
        (t)")
        make_plot_section12!(ax3,
            @rsubset(df_avg, :α == 0.5,
            :filling == 0.1, :Δ == 4),
            :pair_renyi2_mean,
            :pair_renyi2_eom)
        xlims!.([ax1, ax2,ax3],-2, 52)
        ylims!(ax1; high=2.1) # ensure ytick
        at 2.0 does not clip
        ax1.xticks = 0:10:100
        ax2.xticks = 0:10:100
        ax3.xticks = 0:10:100
        ax1.yticks = -0:0.4:2
        ax2.yticks = -0:0.4:2
        ax3.yticks = -0:0.4:2
        ax1.xticklabelsvisible = false
        ax2.xticklabelsvisible = false
        #Legend(fig[3,1], ax; )
        axislegend(ax1; position=:rb,
        rowgap=0)
        text!(ax1, 5.0, 0.02;
        text=L"\Delta=0,\ \alpha=0.5,\
        f=10%", align=(:left,:bottom))
        text!(ax2, 5.0, 0.02;
        text=L"\Delta=2,\ \alpha=0.5,\
        f=10%", align=(:left,:bottom))
        text!(ax3, 5.0, 0.02;
        text=L"\Delta=4,\ \alpha=0.5,\
        f=10%", align=(:left,:bottom))

        Label(fig[1,1]; text="(a)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-55, bottom=6))
        Label(fig[2,1]; text="(b)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-55, bottom=6))
        Label(fig[3,1]; text="(c)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-55, bottom=6))
```

```
            fig
        end
    end |> save_and_display("fig10")
```

Fig. 11: XXZ Renyi f=10%, α=6

(a) $\Delta = 0$, $\alpha = 6$, $f = 10\%$

(b) $\Delta = 2$, $\alpha = 6$, $f = 10\%$

(c) $\Delta = 4$, $\alpha = 6$, $f = 10\%$

Legend: ED, dTWA, dcTWA (RG), gcTWA (RG), gcTWA (naive)

$S_2(t)$

Time t $[J_0^{-1}]$

```
with_theme(THEME) do
    let fig = Figure()

        ax1 = Axis(fig[1,1]; ylabel=L"S_{2}
        (t)")
        make_plot_section12!(ax1,
            @rsubset(df_avg, :α == 6,
            :filling == 0.1, :Δ == 0),
            :pair_renyi2_mean,
            :pair_renyi2_eom)
        ax2 = Axis(fig[2,1]; ylabel=L"S_{2}
        (t)")
        make_plot_section12!(ax2,
            @rsubset(df_avg, :α == 6,
            :filling == 0.1, :Δ == 2),
            :pair_renyi2_mean,
            :pair_renyi2_eom)
        ax3 = Axis(fig[3,1]; xlabel=L"Time
        $t$ $[J_0^{-1}]$", ylabel=L"S_{2}
        (t)")
        make_plot_section12!(ax3,
            @rsubset(df_avg, :α == 6,
            :filling == 0.1, :Δ == 4),
            :pair_renyi2_mean,
            :pair_renyi2_eom)

        xlims!.([ax1, ax2,ax3],-2, 102)
        ylims!.([ax1, ax2,ax3],-0.02, 0.65)

        ax1.xticks = 0:20:100
        ax2.xticks = 0:20:100
        ax3.xticks = 0:20:100
        ax1.yticks = -1:0.2:1
        ax2.yticks = -1:0.2:1
        ax3.yticks = -1:0.2:1
        ax1.xticklabelsvisible = false
        ax2.xticklabelsvisible = false

        Legend(fig[0,1], ax1;
        orientation=:horizontal, nbanks=2,
        width=Relative(1))

        text!(ax1, 0.0, 0.45;
        text=L"\Delta=0,\ \alpha=6,\ f=10%",
        align=(:left,:bottom))
        text!(ax2, 0.0, 0.45;
        text=L"\Delta=2,\ \alpha=6,\ f=10%",
        align=(:left,:bottom))
        text!(ax3, 0.0, 0.45;
        text=L"\Delta=4,\ \alpha=6,\ f=10%",
        align=(:left,:bottom))

        Label(fig[1,1]; text="(a)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-55, bottom=6))
        Label(fig[2,1]; text="(b)",
        tellwidth=false, tellheight=false,
```

```
            alignmode=Mixed(;left=-55, bottom=6))
        Label(fig[3,1]; text="(c)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-55, bottom=6))

        fig
    end
end |> save_and_display("fig11")
```
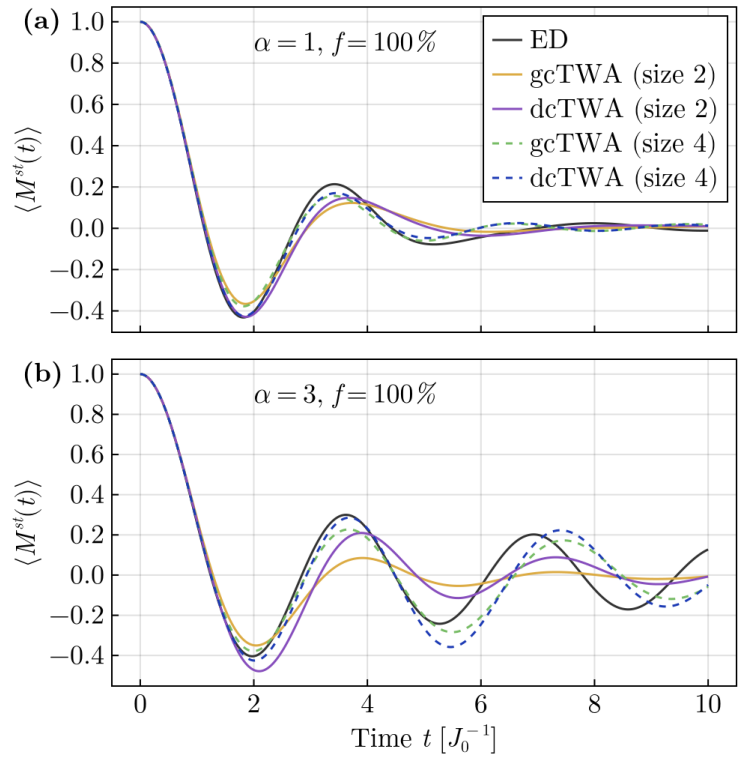
Section 1: ordered

	clustersize	N	α	Δ	tlist
1	missing	16	1.0	0	0.0:0.02
2	2	16	1.0	0	0.0:0.02
3	4	16	1.0	0	0.0:0.02
4	missing	16	1.0	0	0.0:0.02
5	2	16	1.0	0	0.0:0.02
6	4	16	1.0	0	0.0:0.02

```
1 sort(@rsubset(df_avg, :α == 1, :filling ==
  1, :Δ == 0), [:alg, :clustersize])
```

Fig. 5 Magnetization/Renyi ordered

```
lines_with_band! (generic function with 1 method)
```

```
1 function lines_with_band!(ax, x, y, err;
2 color, label=nothing, linestyle=:solid,
3 line_kwargs=(;), band_kwargs=(;))
4     band!(ax, x, y .- err, y .+ err;
      alpha=0.5, color, band_kwargs...)
      lines!(ax, x, y; color, label,
      linestyle, line_kwargs...)
  end
```

(a) $\alpha = 1$, $f = 100\%$

ED
gcTWA (size 2)
dcTWA (size 2)
gcTWA (size 4)
dcTWA (size 4)

(b) $\alpha = 3$, $f = 100\%$

Time t $[J_0^{-1}]$

$\langle M^{st}(t) \rangle$

```julia
with_theme(THEME) do
    let fig = Figure()

        ax1 = Axis(fig[1,1];
        ylabel=L"\langle M^{st}(t)\rangle")
        _, d2, d4, ed, g2, g4 =
        eachrow(sort(@rsubset(df_avg, :α ==
        1, :filling == 1, :Δ == 0), [:alg,
        :clustersize]))
        lines!(ax1, ed.tlist,
        ed.magnetization_mean;
            color=section1_palette.ed,
            label="ED")
        lines!(ax1, g2.tlist,
        g2.magnetization_mean;
            color=section1_palette.dTWA,
            label="gcTWA (size 2)")
        lines!(ax1, d2.tlist,
        d2.magnetization_mean;
            color=section1_palette.rg_dcTWA,
            label="dcTWA (size 2)")
        lines!(ax1, g4.tlist,
        g4.magnetization_mean;
            color=section1_palette.rg_gcTWA,
            label="gcTWA (size 4)",
            linestyle=:dash)
        lines!(ax1, d4.tlist,
        d4.magnetization_mean;
            color=section1_palette.naive_gcTW
            A,
            label="dcTWA (size 4)",
            linestyle=:dash)

        ax2 = Axis(fig[2,1]; xlabel=L"Time
        $t$ $[J_0^{-1}]$", ylabel=L"\langle
        M^{st}(t)\rangle")
        _, d2, d4, ed, g2, g4 =
        eachrow(sort(@rsubset(df_avg, :α ==
        3, :filling == 1, :Δ == 0), [:alg,
        :clustersize]))
        lines!(ax2, ed.tlist,
        ed.magnetization_mean;
            color=section1_palette.ed,
            label="ED")
```

```julia
45        lines!(ax2, g2.tlist,
46        g2.magnetization_mean;
47            color=section1_palette.dTWA,
48            label="gcTWA (size 2)")
49        lines!(ax2, d2.tlist,
50        d2.magnetization_mean;
51            color=section1_palette.rg_dcTWA,
52            label="dcTWA (size 2)")
53        lines!(ax2, g4.tlist,
54        g4.magnetization_mean;
55            color=section1_palette.rg_gcTWA,
56            label="gcTWA (size 4)",
57            linestyle=:dash)
58        lines!(ax2, d4.tlist,
59        d4.magnetization_mean;
            color=section1_palette.naive_gcTW
            A,
            label="dcTWA (size 4)",
            linestyle=:dash)

        ax1.xticklabelsvisible = false
        ax1.xticks = 0:2:10
        ax2.xticks = 0:2:10
        ax1.yticks = -0.4:0.2:1
        ax2.yticks = -0.4:0.2:1

        axislegend(ax1; position=:rt)

        text!(ax1, 2.0, 0.9; text=L"\alpha=1,
        \ f=100%", align=(:left,:center))
        text!(ax2, 2.0, 0.9; text=L"\alpha=3,
        \ f=100%", align=(:left,:center))

        Label(fig[1,1]; text="(a)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-60, bottom=6))
        Label(fig[2,1]; text="(b)",
        tellwidth=false, tellheight=false,
        alignmode=Mixed(;left=-60, bottom=6))
        fig
    end
end |> save_and_display("fig5")
```
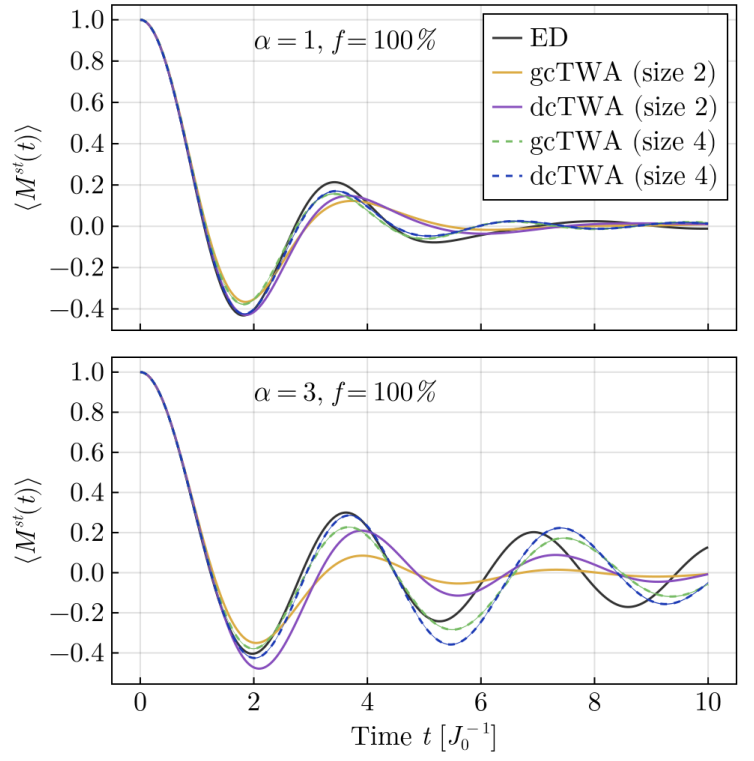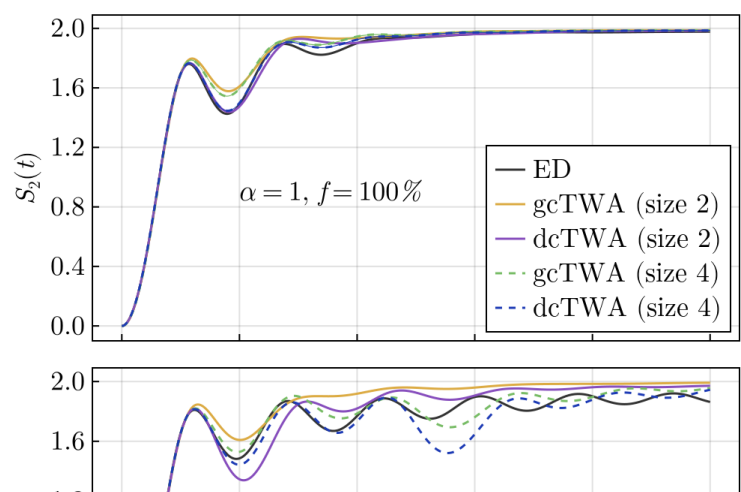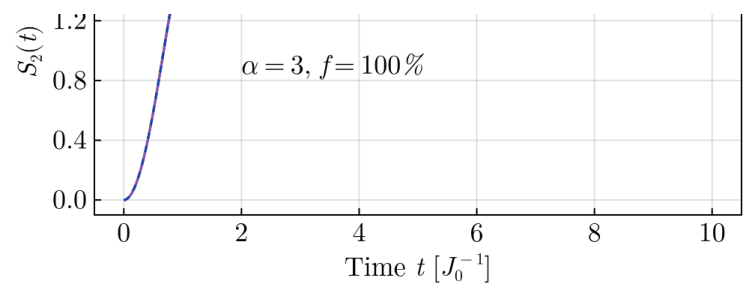
With statistical error as ribbon:

$S_2(t)$

$\alpha = 3,\ f = 100\,\%$

Time $t\ [J_0^{-1}]$

```julia
with_theme(THEME) do
    let fig = Figure(),
        ax1 = Axis(fig[1,1]; ylabel=L"S_{2}
        (t)")
        _, d2, d4, ed, g2, g4 =
        eachrow(sort(@rsubset(df_avg, :α ==
        1, :filling == 1, :Δ == 0), [:alg,
        :clustersize]))
        lines!(ax1, ed.tlist,
        ed.pair_renyi2_mean;
            color=section1_palette.ed,
            label="ED")
        lines_with_band!(ax1, g2.tlist,
```

```julia
        g2.pair_renyi2_mean,
        g2.magnetization_eom_syst;
            color=section1_palette.dTWA,
            label="gcTWA (size 2)")
    lines!(ax1, d2.tlist,
    d2.pair_renyi2_mean;
            color=section1_palette.rg_dcTWA,
            label="dcTWA (size 2)")
    lines_with_band!(ax1, g4.tlist,
    g4.pair_renyi2_mean,
    g4.magnetization_eom_syst;
            color=section1_palette.rg_gcTWA,
            label="gcTWA (size 4)",
            linestyle=:dash)
    lines_with_band!(ax1, d4.tlist,
    d4.pair_renyi2_mean,
    d4.magnetization_eom_syst;
            color=section1_palette.naive_gcTW
            A,
            label="dcTWA (size 4)",
            linestyle=:dash)

    ax2 = Axis(fig[2,1]; xlabel=L"Time
    $t$ $[J_0^{-1}]$", ylabel=L"S_{2}
    (t)")
    _, d2, d4, ed, g2, g4 =
    eachrow(sort(@rsubset(df_avg, :α ==
    3, :filling == 1, :Δ == 0), [:alg,
    :clustersize]))
    lines!(ax2, ed.tlist,
    ed.pair_renyi2_mean;
            color=section1_palette.ed,
            label="ED")
    lines!(ax2, g2.tlist,
    g2.pair_renyi2_mean;
            color=section1_palette.dTWA,
            label="gcTWA (size 2)")
    lines!(ax2, d2.tlist,
    d2.pair_renyi2_mean;
            color=section1_palette.rg_dcTWA,
            label="dcTWA (size 2)")
    lines!(ax2, g4.tlist,
    g4.pair_renyi2_mean;
            color=section1_palette.rg_gcTWA,
            label="gcTWA (size 4)",
            linestyle=:dash)
    lines!(ax2, d4.tlist,
    d4.pair_renyi2_mean;
            color=section1_palette.naive_gcTW
            A,
            label="dcTWA (size 4)",
            linestyle=:dash)

    ax1.xticklabelsvisible = false
    ax1.xticks = 0:2:10
    ax2.xticks = 0:2:10
    ax1.yticks = -0.4:0.4:2
    ax2.yticks = -0.4:0.4:2
```

```
        ax2.yticks = -0.4:0.4:2

        axislegend(ax1; position=:rb)

        text!(ax1, 2.0, 0.9; text=L"\alpha=1,
        \ f=100%", align=(:left,:center))
        text!(ax2, 2.0, 0.9; text=L"\alpha=3,
        \ f=100%", align=(:left,:center))
        fig
    end
end
```

Section 3: Statistics

`data_full_raw =`

	clustersize	Δ	N	tlist	a
1	2	0	16	0.0:0.02:10.0	"dc
2	4	0	16	0.0:0.02:10.0	"dc
3	2	0	16	0.0:0.02:10.0	"gc
4	4	0	16	0.0:0.02:10.0	"gc

```
data_full_raw = @chain collect_results("../
data/fulldata-simulations-avg") begin
    @rtransform(
        :magnetization_mean =
        :results[1].magnetization_mean,
        :magnetization_std =
        :results[1].magnetization_std,
        :renyi_means =
        :results[1].renyi_means,
        :renyi_stds = :results[1].renyi_stds,
        :renyi_chunksizes =
        :results[1].renyi_chunksizes)
    select!(Not([:path, :results, :chunkID,
    :fulldata]))
    @rsubset(:α == 1)
    sort!([:alg, :clustersize])
end
```

```
Scanning folder ../data/fulldata-simulat
ions-avg for result files.

Added 8 entries.
```

```
0.06666666666666667
1  8/binomial(16,2)
```
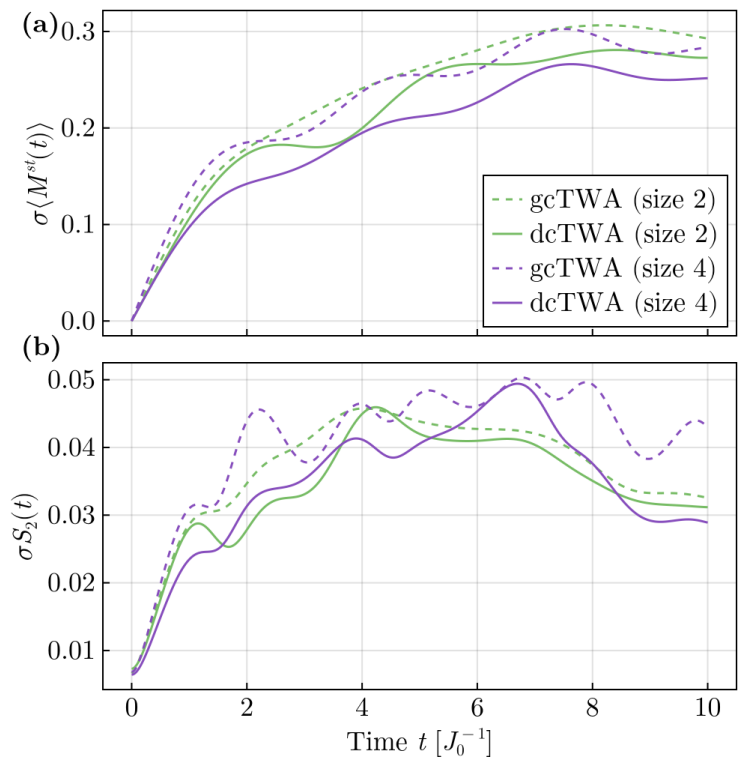
Fig. 12: Stddev of magnetization

```
1  let (d2,d4,g2,g4) = eachrow(data_full_raw)
2      f2 = sum(d2.magnetization_std)/
3      sum(g2.magnetization_std)
4      println("1-D2/G2: ", 1-f2, " -> ", 1-
5      f2^2)
6      f4 = sum(d4.magnetization_std)/
       sum(g4.magnetization_std)
       println("1-D4/G4: ", 1-f4, " -> ", 1-
       f4^2)
   end
```

```
1-D2/G2: 0.08781249218184817 -> 0.16    ⑦
791395058050917
1-D4/G4: 0.15220239597046392 -> 0.28123
92226017779
```

```
1  let (d2,d4,g2,g4) = eachrow(data_full_raw)
2      f2 = sum(d2.renyi_stds[2])/
3      sum(g2.renyi_stds[2])
4      println("1-D2/G2: ", 1-f2, " -> ", 1-
5      f2^2)
6      f4 = sum(d4.renyi_stds[2])/
       sum(g4.renyi_stds[2])
       println("1-D4/G4: ", 1-f4, " -> ", 1-
       f4^2)
   end
```

```
1-D2/G2: 0.07133043568079023 -> 0.13    ⑦
75728403071691
1-D4/G4: 0.15757606419565506 -> 0.29032
19123839169
```

(a) $\sigma\langle M^{st}(t)\rangle$

Legend:
- gcTWA (size 2)
- dcTWA (size 2)
- gcTWA (size 4)
- dcTWA (size 4)

(b) $\sigma S_2(t)$

Time $t\ [J_0^{-1}]$

```julia
with_theme(THEME) do
    let fig = Figure()

        ax1 = Axis(fig[1,1];
        ylabel=L"\sigma\langle M^{st}
        (t)\rangle")
        d2,d4,g2,g4 = eachrow(data_full_raw)
        lines!(ax1, g2.tlist,
        g2.magnetization_std;
            color=section1_palette.rg_gcTWA,
            label="gcTWA (size 2)",
            linestyle=:dash)
        lines!(ax1, d2.tlist,
```

```julia
14          d2.magnetization_std;
15              color=section1_palette.rg_gcTWA,
16              label="dcTWA (size 2)")
17          lines!(ax1, g4.tlist,
18          g4.magnetization_std;
19              color=section1_palette.rg_dcTWA,
20              label="gcTWA (size 4)",
21              linestyle=:dash)
22          lines!(ax1, d4.tlist,
23          d4.magnetization_std;
24              color=section1_palette.rg_dcTWA,
25              label="dcTWA (size 4)")
26
27          ax2 = Axis(fig[2,1]; xlabel=L"Time
28          $t$ $[J_0^{-1}]$",ylabel=L"\sigma
29          S_2(t)")
30          lines!(ax2, g2.tlist,
31          g2.renyi_stds[2];
32              color=section1_palette.rg_gcTWA,
33              label="gcTWA (size 2)",
34              linestyle=:dash)
35          lines!(ax2, d2.tlist,
36          d2.renyi_stds[2];
37              color=section1_palette.rg_gcTWA,
38              label="dcTWA (size 2)")
39          lines!(ax2, g4.tlist,
40          g4.renyi_stds[2];
41              color=section1_palette.rg_dcTWA,
42              label="gcTWA (size 4)",
43              linestyle=:dash)
44          lines!(ax2, d4.tlist,
45          d4.renyi_stds[2];
46              color=section1_palette.rg_dcTWA,
47              label="dcTWA (size 4)")
48

            ax1.xticklabelsvisible = false
            ax1.xticks = 0:2:10
            ax2.xticks = 0:2:10

            axislegend(ax1; position=:rb)

            Label(fig[1,1]; text="(a)",
            tellwidth=false, tellheight=false,
            alignmode=Mixed(;left=-55, bottom=6))
            Label(fig[2,1]; text="(b)",
            tellwidth=false, tellheight=false,
            alignmode=Mixed(;left=-55,
            bottom=26))
            fig
        end
    end |> save_and_display("fig12")
```
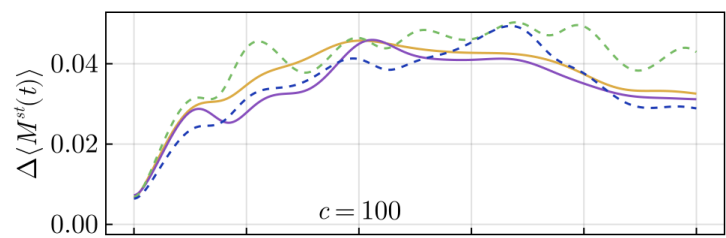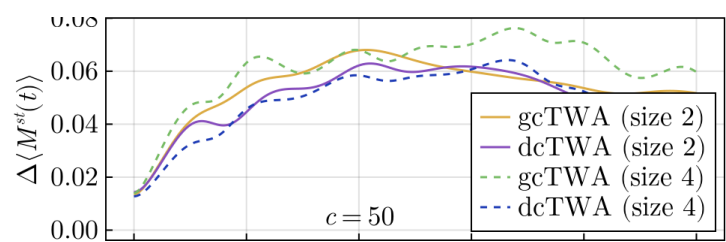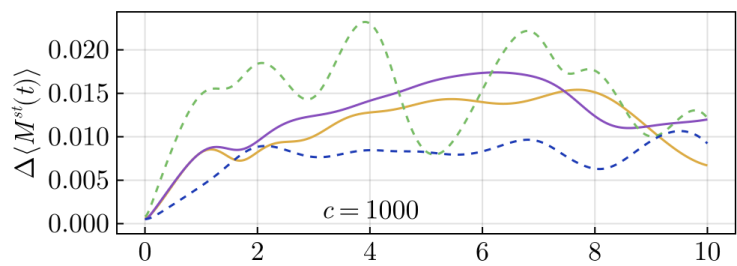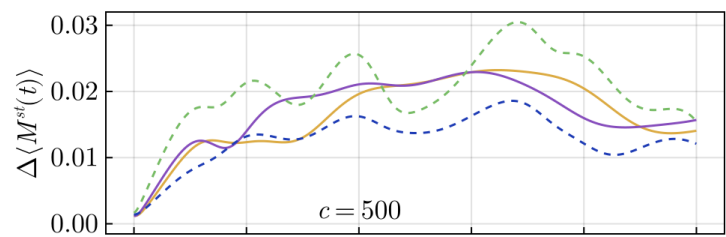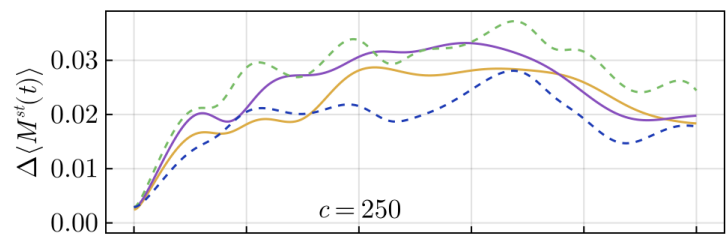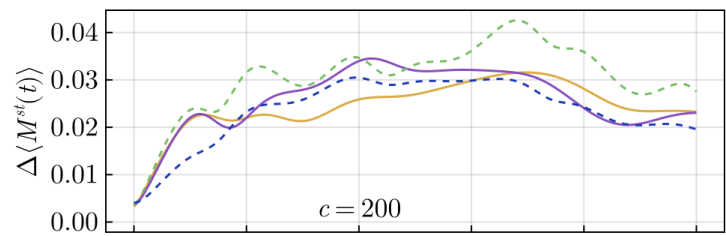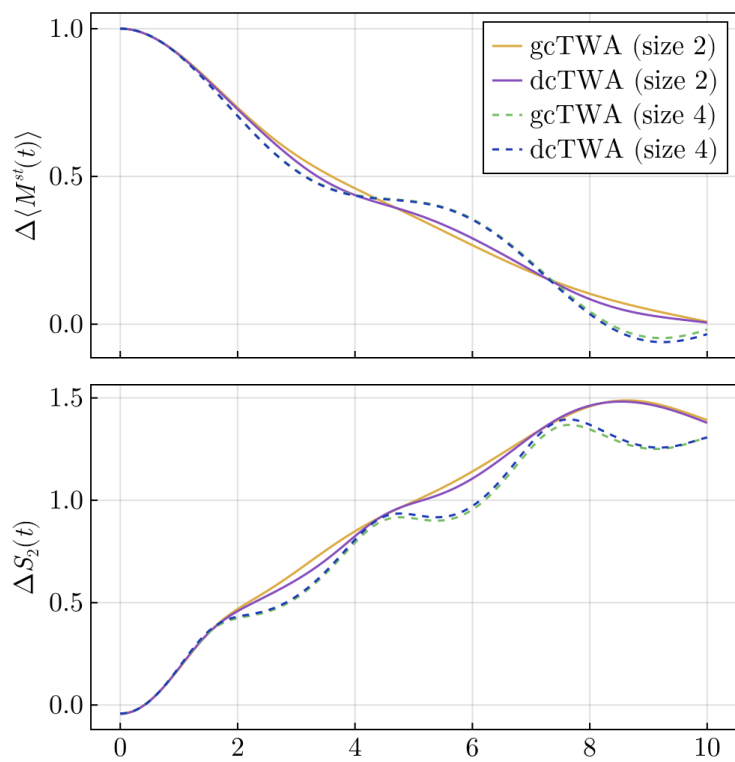
```julia
with_theme(THEME) do
    let fig = Figure(;
    size=(246*SCALE,2*HEIGHT*SCALE)),
        (d2,d4,g2,g4) =
        eachrow(data_full_raw),
        axes = []
        for (i, chunksize) in
        enumerate(d2.renyi_chunksizes)
            ax = Axis(fig[i,1];
            ylabel=L"\Delta\langle M^{st}
            (t)\rangle")
            push!(axes, ax)
            ax.xticklabelsvisible = false
            ax.xticks = 0:2:10

            lines!(ax, g2.tlist,
            g2.renyi_stds[i];
            color=section1_palette.dTWA,
            label="gcTWA (size 2)")
            lines!(ax, d2.tlist,
            d2.renyi_stds[i];
                color=section1_palette.rg_dcT
                WA,
                label="dcTWA (size 2)")
            lines!(ax, g4.tlist,
            g4.renyi_stds[i];
                color=section1_palette.rg_gcT
                WA,
                label="gcTWA (size 4)",
                linestyle=:dash)
            lines!(ax, d4.tlist,
            d4.renyi_stds[i];
                color=section1_palette.naive_
                gcTWA,
                label="dcTWA (size 4)",
                linestyle=:dash)
            text!(ax, 4, 0; text=L"c=%
            $chunksize",
            align=(:center,:bottom))
        end
        axes[end].xticklabelsvisible = true

        axislegend(axes[1]; position=:rb,
        rowgap=0)
        fig
    end
end# |> save_and_display("fig12")
```

```julia
with_theme(THEME) do
    let fig = Figure(),
        ax1 = Axis(fig[1,1];
        ylabel=L"\Delta\langle M^{st}
        (t)\rangle")
        d2,d4,g2,g4 = eachrow(data_full_raw)
        lines!(ax1, g2.tlist,
        g2.magnetization_mean;
            color=section1_palette.dTWA,
            label="gcTWA (size 2)")
        lines!(ax1, d2.tlist,
        d2.magnetization_mean;
            color=section1_palette.rg_dcTWA,
            label="dcTWA (size 2)")
        lines!(ax1, g4.tlist,
        g4.magnetization_mean;
            color=section1_palette.rg_gcTWA,
            label="gcTWA (size 4)",
            linestyle=:dash)
        lines!(ax1, d4.tlist,
        d4.magnetization_mean;
            color=section1_palette.naive_gcTW
            A,
            label="dcTWA (size 4)",
            linestyle=:dash)

        ax2 = Axis(fig[2,1]; ylabel=L"\Delta
        S_2(t)")
        lines!(ax2, g2.tlist,
        g2.renyi_means[2];
            color=section1_palette.dTWA,
            label="gcTWA (size 2)")
        lines!(ax2, d2.tlist,
        d2.renyi_means[2];
            color=section1_palette.rg_dcTWA,
            label="dcTWA (size 2)")
        lines!(ax2, g4.tlist,
        g4.renyi_means[2];
            color=section1_palette.rg_gcTWA,
            label="gcTWA (size 4)",
            linestyle=:dash)
        lines!(ax2, d4.tlist,
        d4.renyi_means[2];
            color=section1_palette.naive_gcTW
```

```
45                A,
46                label="dcTWA (size 4)",
47                linestyle=:dash)
48

            ax1.xticklabelsvisible = false
            ax1.xticks = 0:2:10
            ax2.xticks = 0:2:10
            # ax1.yticks = -0.4:0.2:1
            # ax2.yticks = -0.4:0.2:1

            axislegend(ax1; position=:rt)

            # text!(ax1, 2.0, 0.9;
            text=L"\alpha=1,\ f=100%",
            align=(:left,:center))
            # text!(ax2, 2.0, 0.9;
            text=L"\alpha=3,\ f=100%",
            align=(:left,:center))
            fig
        end
    end# |> save_and_display("fig12")
```
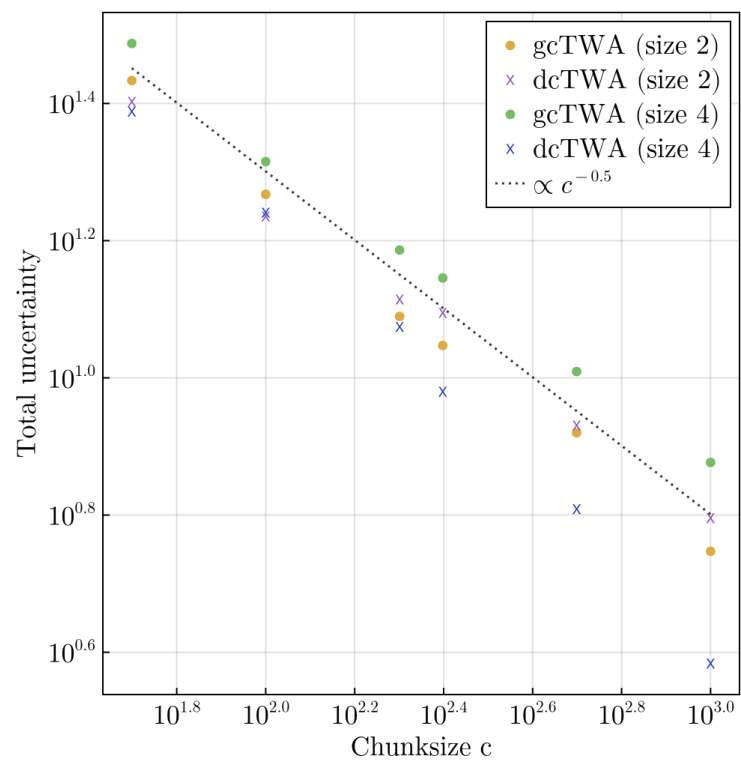
```julia
with_theme(THEME) do
    let fig = Figure(),
        ax1 = Axis(fig[1,1]; ylabel="Total
        uncertainty", xlabel="Chunksize c",
        xscale=log10, yscale=log10)
        d2,d4,g2,g4 = eachrow(data_full_raw)
        scatter!(ax1, g2.renyi_chunksizes,
        sum.(g2.renyi_stds);
            color=section1_palette.dTWA,
            label="gcTWA (size 2)")
        scatter!(ax1, d2.renyi_chunksizes,
        sum.(d2.renyi_stds);
            color=section1_palette.rg_dcTWA,
```

```julia
            label="dcTWA (size 2)",
            marker='X')
        scatter!(ax1, g4.renyi_chunksizes,
        sum.(g4.renyi_stds);
            color=section1_palette.rg_gcTWA,
            label="gcTWA (size 4)")
        scatter!(ax1, d4.renyi_chunksizes,
        sum.(d4.renyi_stds);
            color=section1_palette.naive_gcTW
            A,
            label="dcTWA (size 4)",
            marker='X')
        lines!(ax1, d4.renyi_chunksizes, x-
        >200/√(x); label=L"\propto
        c^{-0.5}",
        color=section1_palette.ed,
        linestyle=:dot)


        # ax1.xticklabelsvisible = false
        # ax1.xticks = 0:2:10
        ax1.yticks = (10 .^ (0.6:0.2:1.4),
        [L"10^{%$x}" for x in (0.6:0.2:1.4)])
        ax1.xticks = (10 .^ (1.6:0.2:3),
        [L"10^{%$x}" for x in (1.6:0.2:3)])
        # ax2.yticks = -0.4:0.2:1

        axislegend(ax1; position=:rt)

        # text!(ax1, 2.0, 0.9;
        text=L"\alpha=1,\ f=100%",
        align=(:left,:center))
        # text!(ax2, 2.0, 0.9;
        text=L"\alpha=3,\ f=100%",
        align=(:left,:center))
        fig
    end
end# |> save_and_display("fig12")
```