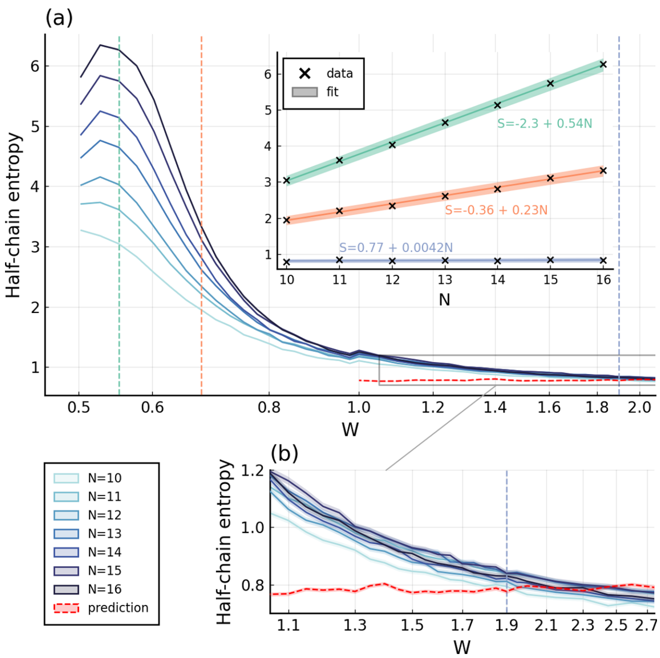

FIG. 4. Half-chain entropy. Average over possible cut locations and over disorder realizations for different system sizes as a function of disorder strength. Also shown is the prediction derived from a pair description, computed from position data for $N = 16$ (red dashed line), see B for details. Inset: Linear fits at fixed disorder strengths indicated by the vertical dashed lines in the main panel. Shaded areas indicate uncertainty from the fit; (b) magnifies the strongly disordered regime of (a). Shaded areas indicate statistical uncertainty from disorder averaging.

pair elimination procedure described in Sec. II B when the spins of a pair can have couplings that are stronger than the pair's internal coupling but the spins associated with these stronger couplings are already eliminated. We thus interpret this feature as an indication of the limitations of a simple pair description.

Another piece of information that we can readily access via the half-chain entropy is the location of the crossover. To determine it, we calculate the variance of the half-chain entropy over different disorder realizations and extract the maximum for each chain length N via a quadratic fit [15,55]. Figure 5 shows no strong dependence of the crossover point on N in the range of accessible system sizes. Indeed, the crossover does not seem to drift significantly, which is in contrast to models with onsite disorder, see, e.g., Refs. [18,55,56], where finite-size drifts of the transition point are commonly observed.

Interestingly, the crossover location is very close to the density given by Rényi's parking constant, or jamming limit, which is the maximal density attainable by randomly placing nonoverlapping unit intervals on the number line [57]. As in experiments with Rydberg spins, atom positions result from such a random process; this could imply that these experiments might not be able to reach the densities required for observing the fully ergodic regime. However, it is unclear how the crossover location generalizes to higher dimensions and larger systems.

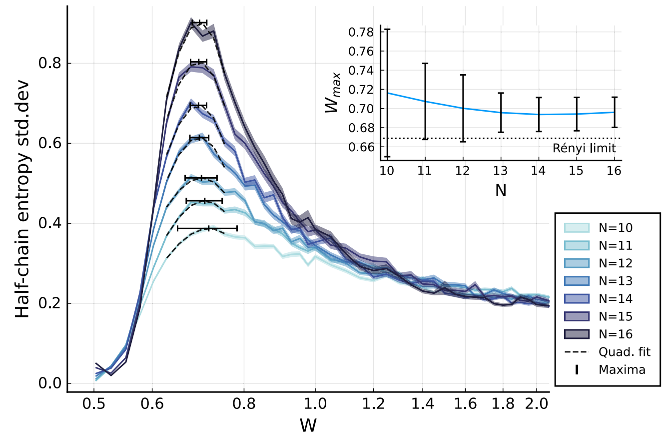

FIG. 5. Standard deviation of half-chain entropy. The main plot shows the standard deviation of the half-chain entropy across disorder realizations exhibiting a clear maximum around which a quadratic polynomial is fitted. Shaded areas indicate statistical uncertainty. Inset: Position of the maximum as extracted by the fits. Errors shown are statistical errors from the fits.

D. Participation ratio

Now that we have seen that the pair model captures the spatial entanglement structure of the exact eigenstates, we compare the predicted eigenstates directly to the exact ones by computing the participation ratio (PR). Intuitively, it measures how many states of a reference basis $\mathcal{B} = \{|b\rangle\}$ contribute to a given eigenstate $|\phi_n\rangle$:

$$\mathrm{PR}_{\mathcal{B}}(|\phi_n\rangle) = \left(\sum_{b\in\mathcal{B}} |\langle b|\phi_n\rangle|^4\right)^{-1} . \quad (5)$$

Usually, in the MBL context, one chooses a product basis as reference because a low PR relative to product basis means the eigenstates are close to product states. "Low" in this context means a sublinear scaling of PR with the dimension of the Hilbert space $\mathcal{H}$: PR $\propto |\mathcal{H}|^\tau$, where $\tau < 1$. In contrast, a thermalizing system always has PR $\propto |\mathcal{H}|$ with respect to any product basis [58–60].

Here we compare two different reference bases, the z-basis $\mathcal{Z} = \{|\uparrow\rangle, |\downarrow\rangle\}^{\otimes N}$ and the pair basis $\mathcal{P} = \{|\pm\rangle, |\updownarrow\updownarrow\rangle\}^{\otimes N/2}$, introduced above, to determine how well the pair model describes the eigenstates. If the pair basis $\mathcal{P}$ was exactly equal to the eigenbasis, its PR would be exactly 1. In this case, the expected PR with respect to the z-basis, averaged over the Hilbert space, $\mathcal{Z}$ will be $1.5^{N/2}$, because a single pair has an average PR of 1.5. However, we only consider the sector of smallest positive magnetization, which increases the expected PR by a similar line of reasoning as for the entropy in the previous section.

Figure 6(a) shows the PR relative to the two reference bases as a fraction of the Hilbert space dimension $|\mathcal{H}|$. We see that the weakly disordered regime indeed has ergodic eigenstates as the curves collapse onto each other. The small offset between the two reference bases is plausible, since a thermal systems eigenstates express volume law entanglement and thus the overlap with a product basis like $\mathcal{Z}$ is minimal. The states of the pair basis contain pairwise entanglement and