We illustrate the prescription given above using the Néel state $\hat{\rho} = |\uparrow\downarrow\rangle\langle\uparrow\downarrow|$. Applying the rule for product states, we can immediately state the sampling scheme: Set $\langle\hat{\sigma}_z^1\rangle = -\langle\hat{\sigma}_z^2\rangle = 1$, choose $\langle\hat{\sigma}_x^1\rangle, \langle\hat{\sigma}_y^1\rangle, \langle\hat{\sigma}_x^2\rangle, \langle\hat{\sigma}_y^2\rangle$ randomly from $\{-1, 1\}$, and then compute the initial values of the correlators by products, e.g., $\langle\hat{\sigma}_x^1\hat{\sigma}_x^2\rangle = \langle\hat{\sigma}_x^1\rangle\langle\hat{\sigma}_x^2\rangle$.

Alternatively, we can employ the tedious route and compute all the Wigner functions. We start by computing the single-spin Wigner functions, which for $|\uparrow\rangle\langle\downarrow|$ are given in Eqs. (A5a) and (A5b). Similarly for $|\downarrow\rangle\langle\downarrow|$, we find

$$\mathbf{w}^1(|\downarrow\rangle) = \mathbf{w}^2(|\downarrow\rangle) = \begin{pmatrix} 0 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix} . \tag{A11}$$

From this we can compute the full two-spin Wigner functions:

$$\begin{aligned} \mathbf{w}^{(1,1)} &= \mathbf{w}^{(2,2)} = \mathbf{w}^1(|\uparrow\rangle) \otimes \mathbf{w}^1(|\downarrow\rangle) \\ &= \begin{pmatrix} w^1(|\uparrow\rangle)(0,0)\cdot\mathbf{w}^1(|\downarrow\rangle) & w^1(|\uparrow\rangle)(0,1)\cdot\mathbf{w}^1(|\downarrow\rangle) \\ w^1(|\uparrow\rangle)(1,0)\cdot\mathbf{w}^1(|\downarrow\rangle) & w^1(|\uparrow\rangle)(1,1)\cdot\mathbf{w}^1(|\downarrow\rangle) \end{pmatrix} \\ &= \begin{pmatrix} 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \end{aligned} \tag{A12}$$

$$\mathbf{w}^{(1,2)} = \mathbf{w}^{(2,1)} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\ 0 & 0 & 0 & 0 \end{pmatrix}. \tag{A13}$$

To generate a single sample, we first need to select one of the four Wigner functions, e.g., $w^{(1,1)}$. This Wigner function gives us the probability distribution to choose the state from, which in this case means we need to select one of the phase points $(\mathbf{p}, \mathbf{q})$ from the set

$$\begin{aligned} \Bigg\{ &\left[\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}0\\0\end{pmatrix}\right], \left[\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right], \\ &\left[\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}\right], \left[\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}1\\1\end{pmatrix}\right] \Bigg\} \end{aligned} \tag{A14}$$

with equal probability. Assuming we selected the first phase point, then the corresponding phase-space vector is given by

$$\begin{aligned} \mathbf{r}_2^{(1,1)}\left(\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}0\\0\end{pmatrix}\right) &= \mathbf{r}^1(0,0) \oplus \mathbf{r}^1(1,0) \\ &\oplus [\mathbf{r}^1(0,0) \otimes \mathbf{r}^1(1,0)]. \end{aligned} \tag{A15}$$

The corresponding initial values of the trajectory are given explicitly in Table I.

TABLE I. Coefficients for the phase-point vector given in Eq. (A15).

Index i	Operator X_i	Initial value	Term in Eq. (A15)
1	$\langle\hat{\sigma}_x^1\rangle$	1	$\mathbf{r}^1(0,0)$
2	$\langle\hat{\sigma}_y^1\rangle$	1	
3	$\langle\hat{\sigma}_z^1\rangle$	1	
4	$\langle\hat{\sigma}_x^2\rangle$	1	$\mathbf{r}^1(1,0)$
5	$\langle\hat{\sigma}_y^2\rangle$	1	
6	$\langle\hat{\sigma}_z^2\rangle$	−1	
7	$\langle\hat{\sigma}_x^1\hat{\sigma}_x^2\rangle$	1	$\mathbf{r}^1(0,0)\otimes\mathbf{r}^1(1,0)$
8	$\langle\hat{\sigma}_x^1\hat{\sigma}_y^2\rangle$	1	
9	$\langle\hat{\sigma}_x^1\hat{\sigma}_z^2\rangle$	−1	
10	$\langle\hat{\sigma}_y^1\hat{\sigma}_x^2\rangle$	1	
11	$\langle\hat{\sigma}_y^1\hat{\sigma}_y^2\rangle$	1	
12	$\langle\hat{\sigma}_y^1\hat{\sigma}_z^2\rangle$	−1	
13	$\langle\hat{\sigma}_z^1\hat{\sigma}_x^2\rangle$	1	
14	$\langle\hat{\sigma}_z^1\hat{\sigma}_y^2\rangle$	1	
15	$\langle\hat{\sigma}_z^1\hat{\sigma}_z^2\rangle$	−1	

APPENDIX B: SINGLE PAIR DYNAMICS

To illustrate the inaccuracy of dTWA in the presence of XX interactions, we study the same system as in the main text for two spins. Repeating the definition here for convenience, we consider the Hamiltonian

$$\hat{H} = 2J\left(\hat{\sigma}_x^1\hat{\sigma}_x^2 + \hat{\sigma}_y^1\hat{\sigma}_y^2\right) + 2\Delta\hat{\sigma}_z^1\hat{\sigma}_z^2, \tag{B1}$$

the initial state $|\psi_0\rangle = |\uparrow\downarrow\rangle$ and the observable $\hat{M}^{\mathrm{st}} = \frac{1}{2}(\hat{\sigma}_z^1 + \hat{\sigma}_z^2)$. Since this Hamiltonian conserves total z magnetization $\hat{M}_z = \hat{\sigma}_z^1 + \hat{\sigma}_z^2$, the dynamics stays confined to the zero magnetization sector, where the state oscillates back and forth between $|\uparrow\downarrow\rangle \leftrightarrow |\downarrow\uparrow\rangle$. So the exact solution reads as $\langle\hat{M}^{\mathrm{st}}(t)\rangle = \cos(8Jt)$. This is independent of Δ because the ZZ term $\hat{\sigma}_z^1\hat{\sigma}_z^2$ of course commutes with $\hat{M}_z$ and thus cannot introduce additional couplings.

Setting $J = 1$ and using dTWA to solve the dynamics for several values of Δ, we see that the semiclassical solution is both influenced strongly by the value of Δ and yields inaccurate results even for $\Delta = 0$ (cf. Fig. 13).

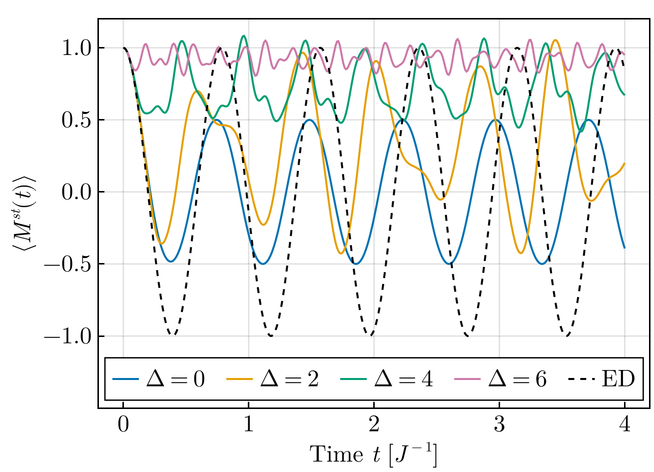

FIG. 13. Dynamics of the staggered magnetization for two spins with XXZ interaction for various anisotropies Δ. Shown is the exact solution [black (dashed)] and solutions obtained with dTWA [colors (solid)]. The exact dynamics are independent of Δ, so only a single curve is shown.