Cluster truncated Wigner approximation for bond-disordered Heisenberg spin models

Adrian Braemer,[1,*] Javad Vahedi,[2,†] and Martin Gärttner[2,‡]

[1]*Physikalisches Institut, Universität Heidelberg, Im Neuenheimer Feld 226, 69120 Heidelberg, Germany*

[2]*Institut für Festkörpertheorie und -optik, Friedrich-Schiller-Universität Jena, Max-Wien-Platz 1, 07743 Jena, Germany*

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We present a comprehensive numerical investigation of the cluster truncated Wigner approximation (cTWA) applied to quench dynamics in bond-disordered Heisenberg spin chains with power-law interactions. We find that cTWA yields highly accurate results over a wide parameter range. However, its accuracy hinges on a suitable choice of clusters. By using a clustering strategy inspired by the strong disorder renormalization group and real-space renormalization group (RSRG), clusters of two spins are sufficient to obtain essentially exact results in the regime of strong disorder. Surprisingly, even for rather weak disorder, e.g., in the presence of very long-range interactions, this choice of clustering outperforms a naive choice of clusters of consecutive spins. Additionally, we develop a discrete sampling scheme for the initial Wigner function, as an alternative to the originally introduced scheme based on Gaussian approximations. This sampling scheme puts cTWA on the same conceptional footing as regular discrete TWA for single spins and yields some reduction in the Monte Carlo shot noise compared to the Gaussian scheme.

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I. INTRODUCTION

Long-range interactions arise in several physical scenarios within disordered quantum many-body systems. For example, in doped semiconductors containing randomly positioned magnetic impurities, interactions occur via exchange couplings that depend on their spatial separation [1–3]. These interactions exhibit different behaviors depending on the state of the system. In insulating phases, the interaction strength decreases exponentially, as denoted by $J(r) \propto \exp(-r/\xi)$, while in metallic phases the interactions operate through the Ruderman-Kittel-Kasuya-Yosida (RKKY) mechanism, following a power-law decay described by $J(r) \propto r^{-d}$, where d represents the dimension of the host system. Interestingly, sufficiently random, power-law interacting systems can even feature ultraslow relaxation known from classical spin glasses as observed in local two-level systems formed by tunneling ions interacting through dipole-dipole and elastic forces [4,5].

Moreover, recent experimental progress has enabled the manipulation and investigation of cold atoms or molecules featuring strong dipole-dipole interactions in diverse setups, including optical lattices [6–8], Rydberg gases [9–11], and trapped ions [12–16]. This has, in turn, spurred theoretical interest in studying quantum many-body dynamics in systems characterized by varying interaction ranges.

However, the potential of these studies is often limited by the lack of suitable computational tools. Considering that the Hilbert space of the system grows exponentially with the system size, the exact solution of quantum dynamics is limited to rather small systems. Even employing sophisticated tools, e.g., based on Krylov subspaces [17–21], typically allows simulating systems of a only few tens of spins. Leaving the realm of exact methods, one usually tries to approximate the wave functions with a variational ansatz such as matrix product states (MPS) [22] and solves the dynamics within this variational manifold. While these MPS based methods, such as time-dependent density matrix renormalization group, have been used very successfully to simulate large, one-dimensional many-body systems with nearest-neighbor interactions [23,24], they struggle for higher-dimensional or long-range interacting systems due to the rapid generation of entanglement [25,26].

In the search for effective approaches to deal with many-body systems and the entanglement problem, phase-space methods have emerged as promising candidates. Among them, the truncated Wigner approximation (TWA) [27,28], based on the Wigner-Weyl correspondence, stands out as a practical and widely adaptable strategy for exploring the dynamics of quantum many-body systems, even in higher-dimensional settings with long-range interactions [29–32]. At its heart, TWA approximates the dynamics of the Wigner function, i.e., the phase-space analog of the wave function, by particles following the classical mean-field equations of motion. The initial conditions of these particles are sampled from a Gaussian approximation of the initial Wigner function. While *a priori* TWA is exact only for short times, numerical experiments have shown it to yield accurate results at intermediate or even late times in some cases [33].

Although TWA was originally developed in the context of bosonic systems where a clear classical limit exists, it can also be applied to spin systems. Remarkably, for finite-dimensional quantum systems there exists a discrete formulation of the quantum phase space [34]. For spin systems prepared in a

*Contact author: adrian.braemer@physi.uni-heidelberg.de
†Contact author: javad.vahedi@uni-jena.de
‡Contact author: martin.gaerttner@uni-jena.de

