

For our simulations, we used the just-described method in the region $W < 1.0$ and chose $\sigma = 1.5(\frac{1}{\rho} - 1)$. For $W \geq 1.0$,

we used the naive sampling strategy. One can see a slight jump in all plots at $W = 1.0$ where the sampling method changes.

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