II. LOCALIZATION IN A RYDBERG GAS

A. System

We consider the Heisenberg XXZ spin model described by the Hamiltonian ($\hbar = 1$)

$$\hat{H} = \frac{1}{2}\sum_{i\neq j} J_{ij} \underbrace{\left(\hat{S}_x^{(i)}\hat{S}_x^{(j)} + \hat{S}_y^{(i)}\hat{S}_y^{(j)} + \Delta\hat{S}_z^{(i)}\hat{S}_z^{(j)}\right)}_{\equiv H_{\text{pair}}^{(i)(j)}}, \tag{1}$$

where $\hat{S}_\alpha^{(k)}$ (with $\alpha \in \{x, y, z\}$) denotes the spin-$\frac{1}{2}$ operators acting on the kth spin. The coupling J_{ij} between spins i and j at positions x_i and x_j is given by $J_{ij} = \frac{C_\alpha}{|x_i - x_j|^\alpha}$, where C_α is an interaction coefficient which we set to $C_\alpha = 1$. In experimental realizations of this model with Rydberg atoms, the values of the anisotropy parameter Δ and interaction exponent α are controllable via the choice of the Rydberg states encoding the two spin states. The cases $\alpha = 3$, $\Delta = 0$ (dipolar exchange) and $\alpha = 6$, $\Delta \approx -0.7$ (van der Waals) have been realized experimentally [42,44]. For typical cloud temperatures and timescales of the spin dynamics, the atom positions can be regarded as fixed (frozen gas approximation).

During the initial Rydberg excitation, the spins are subjected to the Rydberg blockade [46], which means no two spins can be closer than some distance r_b, called the blockade radius. This feature allows one to tune the strength of disorder via the sample's density: In a very dilute sample, the mean interspin distance is much larger than the blockade radius r_b and thus positions are essentially uncorrelated. In the other extreme, the spins are tightly packed and exhibit strong spatial correlations.

We quantify the strength of disorder by the ratio W of the system's total volume V over total blocked volume V_{block} or, equivalently, by the ratio of Wigner-Seitz radius a_0, which is half of the mean interspin distance, to the blockade radius r_b to the power of the dimension d:

$$W = \frac{V}{V_{\text{block}}} = \left(\frac{a_0}{r_b}\right)^d. \tag{2}$$

For $d = 1$, the minimal value of $W_{\min} = \frac{1}{2}$ is attained for a translationally invariant chain with spacing $2a_0 = r_b$, as illustrated in Fig. 1(a).

B. Effective pair description

This model differs from the random field Heisenberg model, which has been studied extensively in the MBL literature, as no disordered potentials are considered. Thus it may not be immediately apparent why this system features localization and what constitutes the local conserved quantities akin to the l-bits [47] in the standard scenario. Here we provide a phenomenological picture in the spirit of the SDRG, suggesting that localization should appear due to strongly interacting pairs.

Consider a strongly disordered cloud of N spins described by Eq. (1) like the example depicted in Fig. 1(b). Due to the power-law interactions, coupling strengths vary strongly between different pairs of atoms, symbolized by the width and brightness of the green lines. This motivates us to employ a perturbative treatment, in which we single out the strongest pair coupling and consider all other couplings as a perturbation. In the example shown in Fig. 1(b), the two rightmost spins share the strongest coupling and we can see that it is much stronger than the other couplings of either one of the spins to the rest of the system. Using perturbation theory to first order, we find that the pair of spins almost decouples from the rest of the system, leaving only an effective Ising-like interaction, which is unimportant for the further procedure and thus not shown in the figure. For details on the calculations involved, see Appendix A.

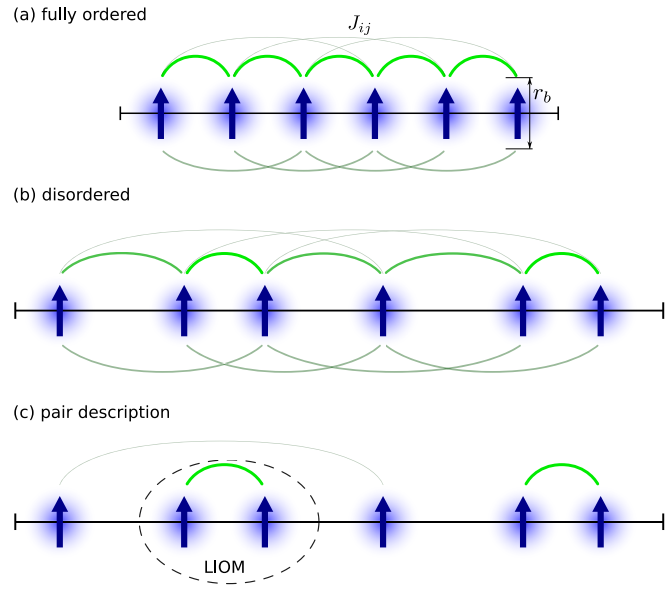

FIG. 1. Pair description. The blockade constraint (blue shadings) enables tuning of disorder in the couplings (green lines) from fully ordered (a) to disordered (b). In the latter case, a perturbative treatment to first order yields a description in terms of strongly correlated pairs (c) subject to an Ising-like interaction (not depicted). These pairs constitute local integrals of motion (LIOM).

We may now repeat this procedure of eliminating couplings between the pairs and the rest of system by identifying the next strongest interaction among the remaining spins which, in this example, is the coupling between the second and third spin. Eliminating the respective couplings as well leaves us with the effective pairs shown in Fig. 1(c). Note that in an ordered system, as shown in Fig. 1(a), this perturbative treatment is not applicable as not all neglected couplings can be considered small. We also note that the order of eliminations is not important as long as each time the inner-pair coupling is much larger than the couplings between the pair and the rest. Concretely, for the given example, choosing the coupling between spins 2 and 3 in Fig. 1(b) first in the pair elimination process does not change the result.

The great advantage of this ansatz is that we can now give a simple description of the whole many-body spectrum. Diagonalizing H_{pair} [see Eq. (1)], we find two maximally entangled eigenstates $|\pm\rangle = 1/\sqrt{2}(|\uparrow\downarrow\rangle \pm |\downarrow\uparrow\rangle)$ at energies $E_\pm = \pm 2 - \Delta$ and two degenerate states $|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle$ at energy $E_d = \Delta$, which we will refer to as $|\updownarrow\updownarrow\rangle$. The Ising-like interaction between pairs does not act on the entangled states $|\pm\rangle$ and is diagonal with respect to $|\updownarrow\updownarrow\rangle$. Thus, in the pair picture, the