domain wall by one site [see Fig. 5(b)]. The former changes the number of domain walls by ± 2 and is thus off resonant, i.e., comes at an energy cost. However, the latter leaves the total number of domain walls invariant and thus is resonant, i.e., domain walls can propagate freely within the bulk. At the edges of the chain, any spin flip always creates or annihilates a single domain wall. This observation is at the heart of the topological protection of the edge spins: Flipping an edge spin is the only process that changes the number of domain walls by an odd amount, and thus is always off resonant, unless both edge spins are flipped. This picture is analogous to previous work [53], in which the authors describe how Majorana fermions are protected on the boundary sites due to an approximate conservation law. One process that is always resonant and simultaneously flips both edge spins is flipping all spins [see Fig. 5(d)] as it corresponds to the global symmetry of the system. All other processes that alter the edge spins are strongly suppressed, because after diagonalizing the resonant domain-wall dynamics in the bulk, the resulting eigenstates do not feature any other resonant transitions. This leads to the observed oscillations with frequency $\propto \epsilon' \epsilon^{L-1}$ as in the case of the fully polarized initial state.

To better illustrate that last point, we translate the dynamical perspective above onto the static eigenstates of the average Hamiltonian $H_F^{(0)}$. Starting with the global parity symmetry, all eigenstates $|\phi_\pm\rangle \propto |\phi\rangle \pm |\bar{\phi}\rangle$ are also eigenstates of the parity operator and thus are an equal superposition of a state $|\phi\rangle$ and its spin-flipped counterpart $|\bar{\phi}\rangle$. For weak field $\epsilon \ll J$, the domain-wall number is approximately conserved, which means that each eigenstate predominantly consists of states from the same domain-wall-number sector with only minor admixtures from adjacent sectors. The observation from the dynamical viewpoint in the previous paragraph, namely that domain walls can propagate freely, here means that within the same domain-wall-number sector, the location of domain walls is ill-defined and the eigenstates are a superposition of all possible placements (see Fig. 6).

With this characterization of the eigenstates, the explanation of the observations made above is straightforward (see sketch Fig. 6). Taking a bit-string initial state and expanding it in the eigenstate basis, we find it to overlap with many different eigenstates from the same sector of the domain-wall-number operator. These eigenstates dephase rapidly $\propto \mathcal{O}(\epsilon)$ and lead to the decay of autocorrelators in the bulk, as seen in Fig. 4(b). By contrast, the edge spins can only change due to the dephasing between the parity sectors, which happens $\propto \mathcal{O}(\epsilon' \epsilon^{L-1})$. Since the splitting is identical for all components, this leads to the long coherent oscillations seen at late times in Fig. 4(a) and 4(c). The initial decay of the edge spin opposite to the metronome [see Fig. 4(c)] is caused by the admixture of wave-function components with a different number of domain walls. The observation that a nonzero number of domain walls will lead to rapid bulk dephasing and thus only a polarized initial state can show long-range order is in line with an earlier study [54]. There, the authors describe a prethermal phase, which they claim can generally only be realized in long-range interacting systems in one dimension. In short-range interacting systems, only the polarized (zero-temperature) initial state displays long-lived spatiotemporal order, as it is the only state with vanishing domain-wall number.

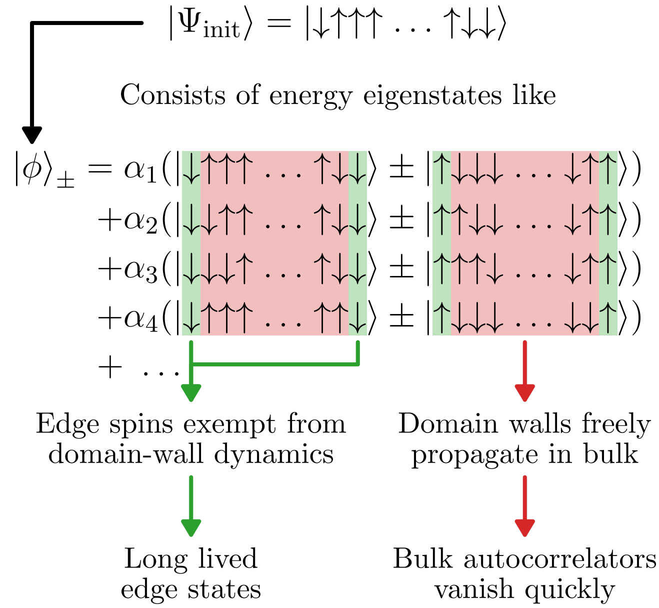

FIG. 6. An illustration of the topological edge-mode protection mechanism. The system is initially prepared in a random bit-string state $|\Psi_{\text{init}}\rangle$. Expanded in the energy eigenbasis of the average Hamiltonian $H_F^{(0)}$, the initial state has overlap with many eigenstate pairs $|\phi\rangle_\pm$ with the same number of domain walls. These eigenstate pairs each comprise a superposition of all possible domain wall placements since domain walls can move freely (highlighted in red). The dephasing between eigenstate pairs leads to the vanishing autocorrelators in the bulk. However, since domain walls cannot propagate through the edges, edge spins (marked in green) are protected from the domain-wall dynamics. Instead, they show long coherent oscillations due to the exponentially small energy gap between the parity sectors.

C. Adapted model with external metronome spin

To clearly separate the two described stabilization mechanisms introduced in Secs. III A and III B, we modify the geometry of the model as shown in Fig. 7(a). Instead of attaching the metronome spin to one end of the chain, as previously shown in Fig. 1, the metronome is coupled to the central spin, which itself is still coupled to its two neighbors in the chain. Thus, the two boundary spins are driven in the same way, and they are connected by a direct line of not actively stabilized spins in the bulk. By the reasoning outlined in Sec. III A, one expects similar results for polarized initial states compared to the standard layout of Fig. 1, as the argument relating to the effective symmetry breaking in the bulk still holds. However, the new configuration includes three edges and one central spin coupled to three neighbors, one of which being the metronome spin. One important conceptual difference to the linear configuration is that here the number of domain walls in the main part of the chain is less strictly conserved. This results in a much weaker edge protection, as the coupling between adjacent domain wall sectors is no longer strongly suppressed.

To test these hypotheses, we compute the z-magnetization autocorrelators of a boundary site, Z_L, and of the new metronome site, Z_m, with the results given in Figs. 7(b), 7(c). The observed lifetime behavior is in full agreement with