contiguous regions of desired size, in disordered systems it is not clear *a priori* that this is a reasonable choice. In this work, we evaluate two possible strategies for choosing the clustering:

(i) *Naive clustering method* [see Fig. 1(b)]. In this approach, clusters of spins are formed by starting from one end of the chain and grouping together every n consecutive spins. Thus, the resulting clusters are determined solely on the basis of this selection process, without taking into account any specific properties or interactions between the spins.

(ii) *Renormalization group clustering* [see Fig. 1(c)]. The RG clustering strategy takes inspiration from the real-space renormalization group (RSRG), also known as strong disorder renormalization group (SDRG), approaches to bond-disordered models which are used to construct approximate eigenstates [53–55]. These methods identify the two spins sharing the strongest coupling in the system and treat their couplings to the environment in a perturbative manner. Since this procedure effectively decouples the pair and leaves the form of the remaining Hamiltonian invariant, one can readily repeat this procedure with the remaining spins until all spins are paired up. Instead of computing eigenstates, we simply use the resulting partition of spins into clusters of size two as input for the cTWA. In this way, the strong intrapair interactions are treated fully quantum mechanically, while the effective interaction among pairs is treated semiclassically.

Based on the excellent results found by RSRG and SDRG, we expect the RG clustering to outperform the naive clustering method for strong disorder. However, the naive clustering scheme generalizes naturally to generate larger clusters, while it is unclear how to merge the clusters given by the RG scheme in a consistent manner.

III. RESULTS

In this section, we present the numerical results of our exploration of the quench dynamics of a disordered spin chain with long-range interactions. We compute the behavior of two dynamical observables initiated from a Néel state, namely, the staggered magnetization and the Rényi entropy $S_2(t)$ evaluated over a two-spin subsystem, using the different methods detailed above and compare to results obtained with exact diagonalization (ED). Our primary focus lies on evaluating the performance of the cluster truncated Wigner approximation (cTWA) relative to standard dTWA. To this end, we consider combinations of the aforementioned clustering schemes, the naive clustering and the one based on the strong disorder renormalization group, and the two approximations of the initial state, the Gaussian cTWA (gcTWA) and discrete cTWA (dcTWA). Our analysis aims to shed light on how cTWA captures the intricate behavior of the system under bond disorder and long-range interactions, and to elucidate the extent to which this approach provides insights into the quantum dynamics of the system under consideration. All curves shown are obtained using 1000 disorder samples and 1000 Monte Carlo trajectories unless specified otherwise. Disorder shots are identical across the methods. The code is freely available at GitHub [56].

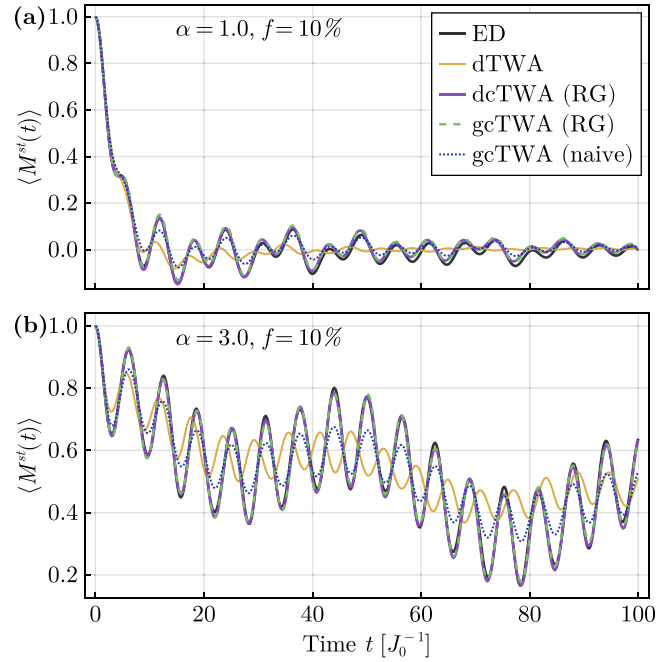

FIG. 3. The disorder-averaged staggered magnetization $\langle M^{\mathrm{st}}(t)\rangle$ is shown for XX chain of $N = 16$ spins with a fixed filling of $f = 10\%$. The panels show results for long-range interactions with $\alpha = 1.0$ in (a) and short-range interactions with $\alpha = 3.0$ in (b). The semi-classical cluster methods using the RG-inspired clustering (green, dashed and purple, solid) overlap the exact results (black, solid) almost completely. dTWA (yellow, solid) and gcTWA with naive clustering (blue, dotted) deviate already early on ($t \approx 10J_0$).

A. Bond-disordered XX chain

We begin our investigation by considering a bond-disordered XX chain ($\Delta = 0$). We explore various regimes by adjusting two key parameters: the power-decay exponent of the interaction, denoted as α, and the filling fraction of the lattice, denoted as f, which controls the strength of the disorder. Here a low filling fraction corresponds to strong disorder, while $f = 100\%$ represents a fully ordered system.

Figure 3 shows the disorder-averaged time evolution of the staggered magnetization $\langle M^{\mathrm{st}}(t)\rangle$ with a fixed filling of $f = 10\%$, starting from the Néel state. The top panel corresponds to long-range interactions ($\alpha = 1$), while the bottom panel corresponds to short-range interactions ($\alpha = 3$). The staggered magnetization starts at a value of one, which reflects the perfect order inherent in the initial Néel state. It then undergoes a decay, caused by the spins exchanging magnetization as system evolves. At late times, we observe equilibration to a value close to zero. The general behavior is captured by all semiclassical methods.

Upon closer inspection, it becomes evident that the dTWA approach fails to accurately track the true dynamics and loses accuracy even at intermediate timescales starting around $tJ_0 \approx 10$. In both cases, it predicts the location of the first oscillation approximately correctly but underestimates the amplitude. Subsequently, it systematically underestimates the amplitude of the oscillations of the staggered magnetization. Interestingly, gcTWA with the naive clustering does not fare much better. While it is generally more accurate with respect