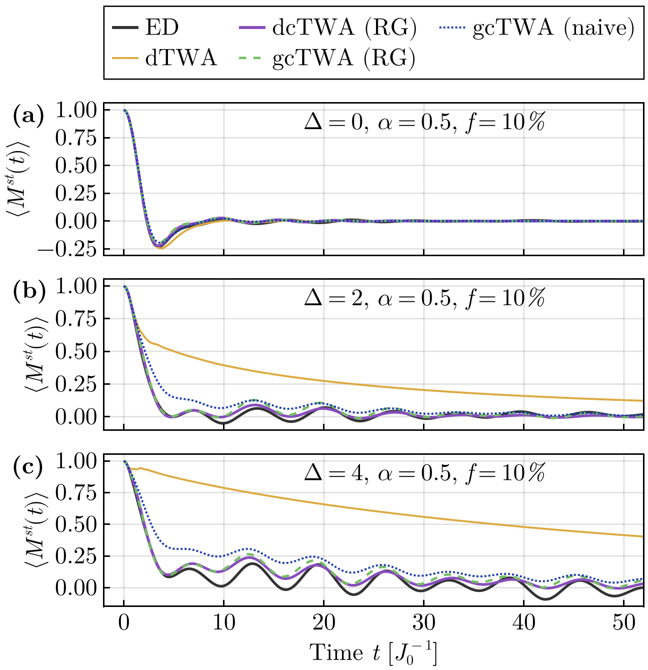

FIG. 8. The disorder-averaged staggered magnetization $\langle M^{\mathrm{st}}(t)\rangle$ is shown for XXZ chain of size $N = 16$ with $\alpha = 0.5$ at $f = 10\%$. Different panels are shown different Δ. Results of cTWA, dTWA, and ED are shown with solid blue, dotted green, and solid blue, respectively.

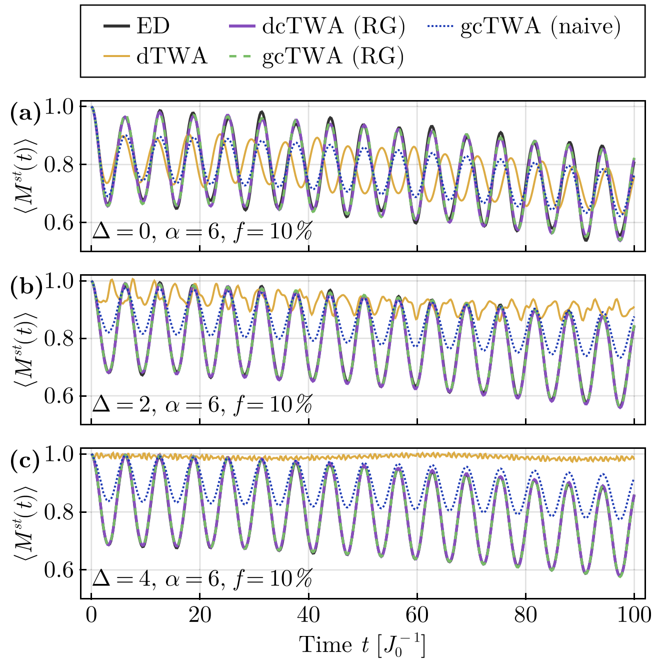

FIG. 9. Same as Fig. 8 but for $\alpha = 6.0$.

alter the dynamics of a single pair initialized in a Néel state because dynamics is fully contained within the sector of zero magnetization. As such, we expect the choice of clusters to have a large impact on the quality of the approximation whenever the dynamics is heavily dominated by pair dynamics. To expand the domain of our study, we employ $\alpha = 0.5$ to evaluate settings with even more long-range interactions, which in principle should play to TWA's strengths. In the short-range case, we chose $\alpha = 6$, as motivated by the typical interaction exponent of van der Waals interactions in Rydberg atoms which are a possible platform to implement XXZ Heisenberg models (see, e.g., [58]). We note, however, that the qualitative differences to $\alpha = 3$ are minor.

Figures 8 and 9 show the dynamics of the staggered magnetization under long-range ($\alpha = 0.5$) and short-range ($\alpha = 6$) interactions, respectively. Starting again from the Néel state, we examine the evolution of the staggered magnetization by varying Δ, assessing how these adjustments affect the dynamics and how well semiclassical methods approximate the true dynamics. At $\Delta = 0$, all semiclassical methods give results matching the exact solution over almost the entire timescale for the long-range system ($\alpha = 0.5$), while in the short-range system ($\alpha = 6$) only the cTWA simulations using the RG-inspired clustering provide accurate results. dTWA performs worst by predicting oscillation with both wrong amplitude and frequency. gcTWA with naive clustering improves upon this due to the inclusion of more quantum correlations which results in a correct prediction of the frequency.

Increasing the Ising interaction [cf. Figs. 8, 9(b), and 9(c)] does not alter the exact dynamics qualitatively, but dTWA increasingly deviates from the exact results vastly underestimating the rate of the initial decay. For the short-range system and $\Delta = 4$ the decay is almost completely suppressed. By contrast, gcTWA with naive clustering yields significantly better results than dTWA. For both systems, the gcTWA prediction qualitatively matches the exact data but is offset by an increasing amount with increasing Δ. Interestingly, both cTWA variants using the RG-inspired clustering match the reference rather closely over the entire time domain except for intermediate times for $\Delta = 4$ in the long-range system $\alpha = 0.5$, where the fluctuations are not reproduced exactly. Surprisingly, this hints at pairs still playing an important role for the dynamics in spite of the quite long-range interactions. For the short-range interactions with strong disorder, the precise match is no surprise as the dynamics is governed by pairs of spins on adjacent lattice sites in this regime.

Again, we we employ the semiclassical methods to also extract the average pair Rényi entropy and compare to exact results. Starting with the long-range scenario, $\alpha = 0.5$, we find for $\Delta = 0$ all semiclassical approaches to converge to the true dynamics approximately (Fig. 10). Increasing Δ, we can see again how dTWA fails to capture the essential processes and predict much too slow dynamics (roughly one order of magnitude too slow). gcTWA with naive clustering fares rather well and only slightly underestimates the initial rise for $\Delta = 2$ very similar to gcTWA with RG-inspired clustering. Most interesting are the differences between gcTWA and dcTWA (both with RG clustering) since in this setting both methods seem to converge to slightly different results with dcTWA following the exact curve more closely at intermediate times (up to $t \approx 10J_0$). At late times all cTWA methods overestimate the amount of entanglement present. This trend continues for $\Delta = 4$ where the discrepancy is enhanced for all methods.

For the short-range interacting systems (cf. Fig. 11), we find that cTWA schemes based on the RG clustering to be in