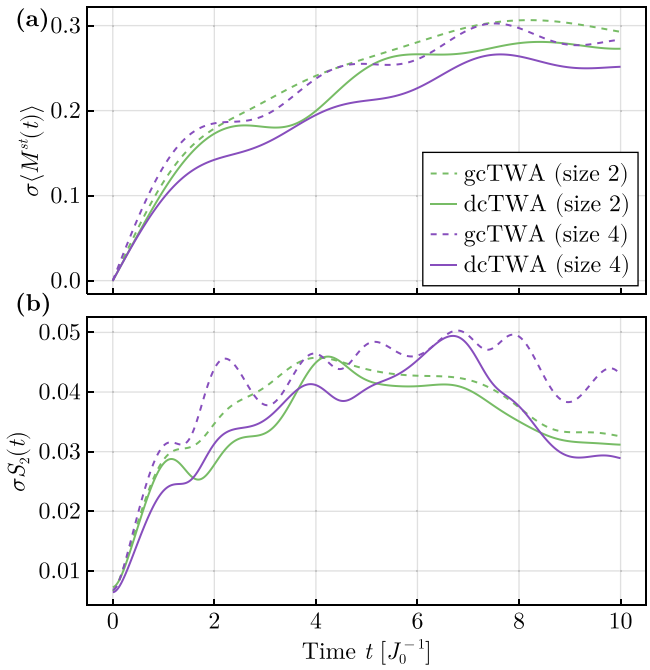

FIG. 12. Error analysis for a single shot with the parameters $\alpha = 1$, $f = 10\%$, $\Delta = 0$ and 10 000 trajectories. We compute the standard deviation of (a) the staggered magnetization and (b) average pair Renyi entropy estimated from batches of 100 trajectories. We use naive clustering to compare both cluster sizes 2 (green lines) and 4 (purple lines) as well as Gaussian sampling (dashed) and discrete sampling (solid).

IV. CONCLUSION

In this study, we conducted a comprehensive numerical investigation focusing on the cluster truncated Wigner approximation (cTWA) for modeling quench dynamics in disordered spin chains with power-law interactions. Through comparisons with the discrete truncated Wigner approximation (dTWA) and with exact diagonalization, we explored the performance of the cTWA on different timescales relevant to quench experiments and studied the influence of the choice of clusters on the prediction. Additionally, we introduced a sampling scheme for generating Monte Carlo trajectories which extends the discrete Wigner function known from dTWA to the realm of cTWA. Our analysis included both the XX and XXZ models with bond disorder initiated from a Néel state, and calculated dynamical observables such as staggered magnetization and two-site Rényi entropy.

We found that while cTWA generally yields improved accuracy compared to dTWA, the choice of clusters strongly impacts the results. Our results in the context of bond disorder show that a clustering strategy inspired by the strong disorder renormalization group could yield astonishingly precise results in the presence of strong disorder, while still being very accurate even for quite long-range interactions, weak disorder, and long times. XXZ models featuring strong Ising interactions were found to be challenging for all semiclassical methods presented here and we conclude that likely larger clusters are needed to capture the relevant physical processes accurately. In all of these systems, we only found minor differences between the Gaussian and discrete sampling schemes in situations were the results were not converged in cluster size. However, a closer study of the statistical properties revealed the discrete sampling to exhibit smaller intrinsic Monte Carlo shot noise.

In conclusion, our study provides valuable insights into the effectiveness of cTWA in studying quench dynamics in bond-disordered spin systems. If using the correct clustering strategy, even clusters of size 2 yield close to exact results, where single-spin dTWA fails. Additionally, we conclude that the discrete sampling strategy introduced here is generally preferable to the Gaussian approximation due to less Monte Carlo shot noise and somewhat simpler implementation. We also want to emphasize that the method presented here is not intrinsically limited to one-dimensional systems and can be applied readily to higher-dimensional systems with arbitrary geometries. We expect cTWA to be able to perform well as long as the coupling matrix is sufficiently disordered. For spatially disordered systems, this should be the case as long as the power-law exponent is larger than the spatial dimension [59] and the density is sufficiently low. In more long-range or denser scenarios, it would likely be beneficial to generalize the clustering algorithm described here to larger clusters, e.g., based on heuristic renormalization group schemes such as [60,61]. Overall, our results highlight the potential of cTWA and its variants, such as dcTWA, as powerful tools for studying the complex dynamics of bond-disordered quantum systems.

ACKNOWLEDGMENTS

We thank O. Egorov for providing computational resources. We acknowledge State of Thuringia for funding of projects "2019 FGI 0017" (PhyClus) and "2022 FGI 0004" (Q-PHOC3). A.B. acknowledges support by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) within the Collaborative Research Center SFB1225 (ISOQUANT). For the numerical work, we used the Julia programming language [62] and the following packages and tools: `DrWatson.jl` [63], `Pluto.jl` [64], `DifferentialEquations.jl` [65] with Verner's integration schemes [66], `Makie.jl` [67], `DataFrames.jl` [68].

APPENDIX A: SAMPLING FROM DISCRETE WIGNER FUNCTIONS OF SPIN CLUSTERS

In this Appendix, we first recapitulate how to derive the concrete sampling rules of dTWA and then extend the scheme to clusters of multiple spins. Finally, we provide a concrete example of sampling rules for clusters of 2 spins. The concepts described here are similar to Appendix A in [37].

1. Recap: Sampling a single spin

Earlier in Sec. II B 2, we defined the phase-point operators for a single spin

$$\hat{A}_{p,q} = [\mathbb{1} + \mathbf{r}(p,q)\cdot\hat{\sigma}]/2 \tag{A1}$$

via a choice of phase-point vectors $\mathbf{r}(p,q)$. In principle there are many possible choices for $\mathbf{r}(p,q)$ but since there are a total of 8 discrete spin states, two sets of phase point operators are