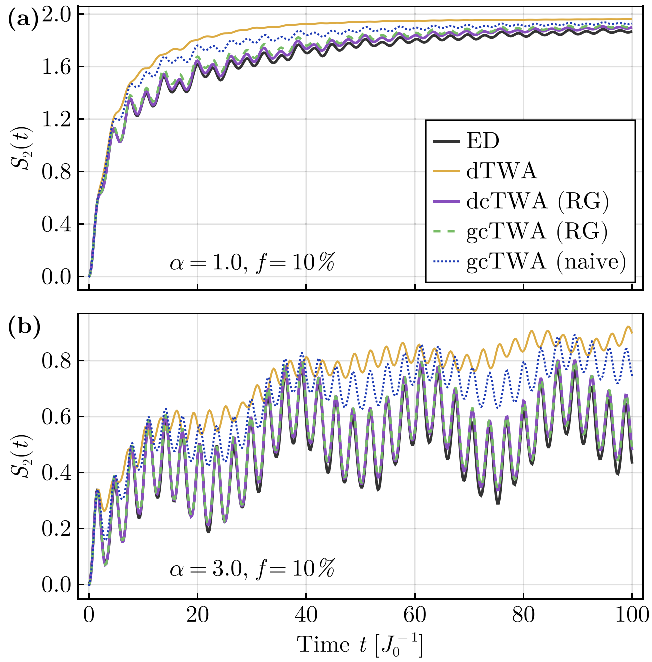

FIG. 6. The average Rényi entropy $\langle S_2(t)\rangle$ is calculated over all possible choices of two sites. This analysis is performed with the same parameter settings as in Fig. 3. The cTWA methods using the RG-inspired clustering [purple (solid) and green (dashed)] reproduce the exact entanglement dynamics [black (solid)] almost exactly with only very slight deviations at late time. Whereas the gcTWA with naive clustering [blue (dotted)] overestimates the entanglement and dTWA [yellow (solid)] even more so.

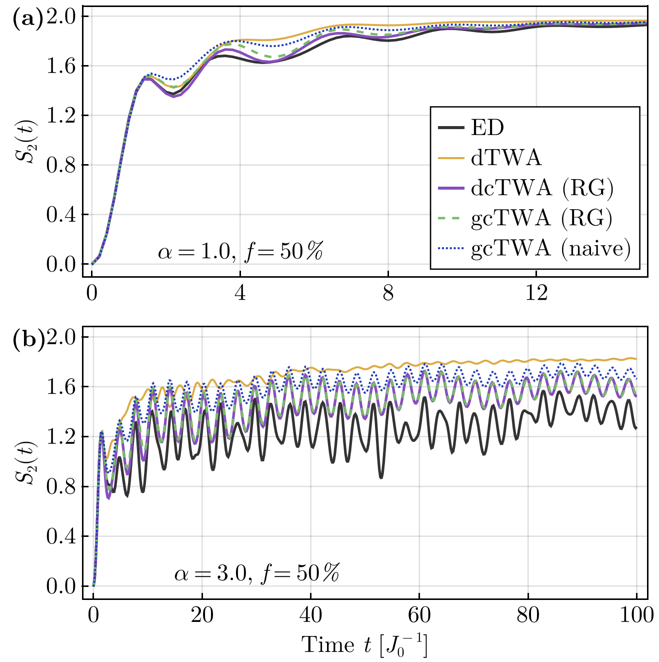

FIG. 7. Same as Fig. 6, but for density fixed at $f = 50\%$. In comparison to Fig. 6, the RG-based cTWA methods deviate from each other and also overestimate the true amount of entanglement present. In the long-range case (a) they do not capture the oscillation frequency correctly, however, in the short-range case (b) they do but underestimate the amplitude.

($\alpha = 1$), this effect is weaker as spins hybridize more due to the stronger interactions. Interestingly, for this setting dcTWA predicts the value of the first minimum more accurately than gcTWA. Increasing the cluster size to 4 spins improves the accuracy of both methods in both cases drastically and we do not find significant differences between the sampling schemes in the long-range case. However, for the short-range case, we find the discrete sampling scheme to approximate the true amplitude of the oscillation generally better than the Gaussian scheme.

To extend our investigation to more complicated, non-local observables, we study the Rényi entropy of two-spin subsystems and assess the efficacy of the semiclassical methods under scrutiny. More specifically, we consider the average Rényi entropy across all possible choices of two sites.

Starting with the strongly disordered setting at $f = 10\%$ in an analogy to above, Fig. 6 illustrates the dynamics of the average Rényi entropy with time in a strongly disordered setting. Since the initial state is a product state, entanglement starts at 0 for $t = 0$ and then starts to increase. We find that generally the semiclassical methods are able to capture the dynamics across the different settings probed qualitatively, as shown in Figs. 6 and 7. Perhaps surprisingly at first, these methods systematically overestimate the amount of entanglement present. This conundrum can be resolved, if one considers that the Rényi entropy is computed by estimating the expectation values of all intrapair correlators and less correlations means more entanglement of the pair with its environment [cf. Eq. (3)]. The semiclassical methods miss out on some of the quantum correlations, thus tend to underestimate the total amount of correlations and consequently predict too much entanglement. Again, the quality of the result depends significantly on the scheme. The deviations are most pronounced for dTWA and the cTWA with naively chosen clusters. Conversely, both dcTWA and gcTWA with the RG clustering scheme approximate the exact dynamics very closely and only overestimate the entanglement by a few percent at late times.

In summary, we find that cTWA may offer tremendous improvements over the simpler dTWA. However, the improvement depends strongly on the choice of clusters. If the clustering does not respect the underlying physics, as is the case for the naive clustering strategy, cTWA showed only a very minor increase in accuracy. On the other hand, if the dominant physical processes are mostly contained within the chosen clusters, as is the case with the RG inspired clustering, cTWA can describe the dynamics of the system over all time intervals almost exactly. The results obtained with the Gaussian Wigner function were very similar compared to the discrete sampling with a slight advantage in favor of the discrete scheme for ordered, short-range systems.

B. Bond-disordered XXZ chain

In this section, we investigate the role of the anisotropy parameter Δ in the dynamics of the system. In the pair picture, a strong anisotropy increases the energy gap between the sectors of different absolute z magnetization. This does not