eigenstates of the full system are now given by tensor products of these four pair eigenstates. We refer to this basis as the pair basis.

In the many-body spectrum, the degeneracy between the pair states $|\uparrow\uparrow\rangle$ and $|\downarrow\downarrow\rangle$ is lifted due to the emerging Ising-like interaction. However, we note that this splitting is small compared to the splitting between the other pair eigenstates as it emerges from first-order perturbation theory.

The pair picture is analogous to the l-bit picture often used in MBL, where strong local disorder potentials lead to the emergence of quasilocal conserved quantities $\hat{\tau}^{(i)} \sim \hat{\sigma}_z^{(i)}$ [47,48]. Here, we see that each projector on a pair's eigenstate constitutes an approximately conserved quantity and hence is a local integral of motion (LIOM). Thus, we established a description akin to the l-bit picture of MBL for this disordered Heisenberg model, where the role of LIOMs is taken by strongly interacting pairs.

While this ansatz is heuristic and neglects all higher resonances, that may play a crucial role in delocalizing the system, it will nonetheless turn out to be useful for interpreting and understanding the spectral and eigenstate properties reported in the following.

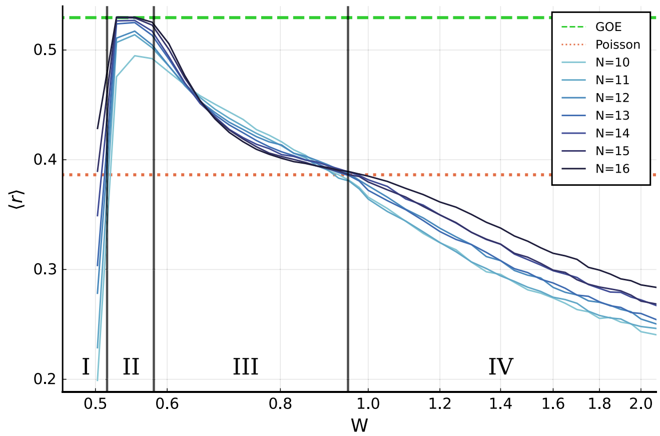

FIG. 2. Level-spacing ratio. With increasing disorder, the LSR shows a crossover from an ergodic value to its Poissonian value and below. We identify four major regions where the physics is governed by (I) translational symmetry breaking, (II) thermal behavior, (III) the localization crossover and (IV) localization. The horizontal lines show random-matrix theory predictions.

III. NUMERICAL RESULTS

To minimize boundary effects, we consider a one-dimensional system with periodic boundary conditions [49] of up to $N = 16$ spins governed by Eq. (1) and perform exact diagonalisation on the sector of smallest positive magnetization. We fix the interaction exponent to $\alpha = 6$, corresponding to a Van der Waals interactions, and set $\Delta = -0.73$ (cf. Ref. [42]). We do not expect a strong dependence of our results on the precise value of Δ as long as one steers clear from regions around points where additional symmetries emerge.

For each disorder strength W, we generate 2000 configurations of random spin positions, perform a full diagonalization and compute several well-established indicators for the localization transition from the spectrum. We always average over all eigenstates/-values as restricting to the bulk of the spectrum does not lead to qualitative changes in the observed behavior. The statistical error resulting from disorder averaging is smaller than the thickness of the lines in all figures unless indicated otherwise. For a description of the algorithm for choosing the configurations, we refer to Appendix C. All code used for this paper can be found in Ref. [50].

The following sections discuss different indicators of localization with the aim to establish the localization crossover in this model and employ the pair model for interpretation and predictions. The last section directly compares the pair basis to the eigenstates, thus demonstrating its validity.

A. Level spacing ratio

The spectral average of the level spacing ratio (LSR), defined as [51]

$$\langle r\rangle = \frac{1}{|\mathcal{H}|}\sum_n \min\left(\frac{E_{n+2}-E_{n+1}}{E_{n+1}-E_n}, \frac{E_{n+1}-E_n}{E_{n+2}-E_{n+1}}\right), \qquad (3)$$

is a simple way of characterizing the distribution of differences between adjacent energy levels. For thermalizing (ergodic) systems, the Hamiltonian is expected to show a mean LSR resembling a random matrix from the Gaussian orthogonal ensemble because its eigenvectors essentially look like random vectors. Thus one can use random matrix theory to obtain $\langle r\rangle_{\text{thermal}} = 4 - 2\sqrt{3} \approx 0.536$ [52].

On the other hand, in localized systems the eigenvalues follow a Poisson distribution, since they are essentially sums of randomly distributed energies from the l-bits the system consists of. Computing the mean LSR in this case yields $\langle r\rangle_{\text{MBL}} = 2\ln 2 - 1 \approx 0.386$ [52].

Comparing with the numerical results in Fig. 2 and focusing on the central parts first, we find the mean LSR reaches its thermal value for large enough systems and weak disorder (II) dropping toward the Poissonian value for stronger disorder (III). With growing system size, the thermal plateau (II) broadens, marking a parameter region where the system appears ergodic. But while the plateau broadens, the drop-off (III) for increasing disorder strength becomes steeper, meaning the crossover becomes sharper as the system gets larger.

Considering very strong disorder (IV), the mean LSR drops even below the Poissonian value, which indicates level attraction. This effect can be explained by the pair model: As stated earlier, the $|\updownarrow\updownarrow\rangle$ states' degeneracy is lifted by the effective Ising-like terms from first-order perturbation theory, which means the split is of smaller magnitude compared to the intra-pair interactions. For small systems with comparatively low spectral density, this means that the small lifting likely fails to mix the formerly degenerate states into their surrounding spectrum. Thus, the LSR still reflects the near degeneracy within the pairs, leading to level attraction. Based on this interpretation, we expect this effect to diminish for larger systems with the spectral density growing. In fact, this trend is already visible in Fig. 2.

A similar argument can be made at very weak disorder (I): Here the source of the degeneracy is the proximity to the perfectly ordered case at $W = 0.5$, which has an additional translation invariance. Weak disorder breaks that symmetry