



FIG. 6. Participation ratio. (a) PR relative to Hilbert space dimension  $|\mathcal{H}|$  for different reference bases: z-basis in blue, pair basis in red. The inset shows a magnification of the region toward perfectly ordered systems. (b) shows the growth in absolute PR with increasing system size in the localized regime. The used value of  $W$  is indicated by the dash-dotted line in (a).

are thus a bit closer, which manifest as slightly lower PR. Around  $W = 0.6$ , the scaling with  $|\mathcal{H}|$  starts to change to a sublinear relation as we crossover to the localized regime.

Checking the PR deep in the localized phase (at  $W = 1.9$ ) in Fig. 6(b), we can see that the PR relative to the z-basis (blue line) is slightly, but systematically, larger than the pair model's prediction (dashed green line). Consistent with this observation, we see that the PR relative to the pair basis (red line), while being much smaller, is still not constant across system sizes.

We conclude that the pair states offer a good first-order approximation of the true eigenstates, but there are higher order resonances that lead to further hybridization for some states. The exponent of the remaining dependence on system size is close to  $N/4$ , which hints at effects stemming from interactions between pairs.

#### IV. CONCLUSIONS

We analyzed a disordered Heisenberg XXZ spin model with power-law interaction and positional disorder, which is naturally realized by many quantum simulation platforms. Among these, cold Rydberg gases allow for easy tuning of the

disorder via the sample's density due to the Rydberg blockade. By using standard MBL indicators, we showed numerically that this system undergoes a localization crossover, which we interpreted in terms of a simple physical model derived using an SDRG ansatz. This model, consisting of an effective Ising model of strongly interacting pairs of spins, was verified by considering the PR of eigenstates with the conjectured basis, which is drastically reduced compared to the PR relative to the z-basis. Still, there was a weak dependence on system size left, which means there are higher order corrections to our model. Nonetheless, we also showed that this simple model can already predict the entanglement entropy of the system nearly perfectly.

With this model at hand, we can now make predictions for large systems which may be tested in quantum simulation experiments. Of course, one of the most interesting questions will be whether the location of the crossover shifts toward stronger disorder for large systems, indicating a transition at infinite disorder strength in the thermodynamic limit. For this purpose, the easy tunability of the disorder is a great advantage as both sides of the crossover can be probed on the same platform by changing the system parameters. Remarkably, our small-scale numerical study showed almost no finite-size drift. This could indicate that localization in this model is more stable than in similar models against resonances. We leave this investigation for future work.

Note that the pair model cannot be used to predict the crossover itself as it essentially requires the assumption that one can find strongly interacting pairs, which is only justified in the strongly disordered regime. Recent arguments for the absence of localization postulate the existence of rare thermal subregions within the system [26,26–34]. This would of course break the base assumption of the pair model. A possible direction for future research would be to extend the model to include not only pairs but also larger clusters, which would require one to track all kinds of interactions between clusters of different sizes.

Interestingly, the dimensionality of the system does not directly influence the pair model. As long as the couplings are sufficiently disordered, such that pairs can be defined, it will be a good approximation. Thus, it suffices to study how the distribution of couplings changes with respect to the dimensionality  $d$  of the space and coupling power  $\alpha$ . Similar to resonance counting arguments [61], we conjecture the requirement  $d < \alpha$  for the pair model to be applicable. Hence, we expect our results, while acquired in  $d = 1$ , to generalize well to  $d > 1$ .

#### ACKNOWLEDGMENTS

We thank Dima Abanin for his stimulating input. For numerical simulations, we used the Julia programming language [62]. The authors acknowledge support by the state of Baden-Württemberg through bwHPC and the German Research Foundation (DFG) through Grant No. INST 40/575-1 FUGG (JUSTUS 2 cluster). This paper is supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under Germany's Excellence Strategy EXC No. 2181/1-390900948 (the Heidelberg STRUCTURES