THE SUSPENSE IS KILLING US

D. ANDERSON, J. DU TOIT, A. HARRIS, M. FENNIMORE, AND R. WOOD

Abstract. In general, fine-tuning mountain bike suspension settings is more of a difficult art than a science—subject to torrents of different opinions and discussions on what is needed without much scientific backing. This paper uses ordinary differential equations to model the behavior of mountain bike suspension systems. By simulating the forces that a bike experiences and accounting for the several main features of these suspension systems including spring type, rebound, compression damping, air pressure/sag, friction, and volume spacers, we create a model that can aid in the process of finding optimal settings for any given suspension system.

1. Background/Motivation

Every year, southern Utah's steep red rock mesas host Red Bull Rampage, one of mountain biking's most renowned events. Riders tackle extreme features hurtling down narrow, rocky chutes, soaring over canyons, and back-flipping off 95-foot vertical drops. Traditionally, beefy dual-crown suspension forks have been deemed necessary for handling such technical terrain. However, in 2021 Brandon Semenuk revolutionized Rampage by becoming the first athlete ever to ride out of the starting gate on a single-crown fork [Loz]. After a historic run, Semenuk was crowned champion of the competitions—a testament to how developments in mountain bike technology, especially suspension design, have propelled the sport to new heights.

Improvements in suspension design have also attracted consumer interest. Modern bikes with better suspension are easier to handle and more confidence-inspiring, helping a wider audience see mountain biking as a good way to get outside and stay active. However, realizing the advantages of these improvements to their fullest requires tuning the suspension to fit one's physical attributes and riding style. To make things even more tricky, advice for adjusting suspension is often heuristics about how the bike feels rather than rules derived from the actual suspension dynamics and measurements. In this project, we seek to model mountain bike suspension using a second-order ordinary differential equation. By testing our model on different simulated terrains and riding styles, we provide insight from first principles into the optimal setup strategy.

Date: 7 November 2024.

2. Modeling

2.1. **Factors Influencing Suspension Dynamics.** Before we put together our complete mathematical model, we will discuss several of the features that influence suspension dynamics and our methods to model them.

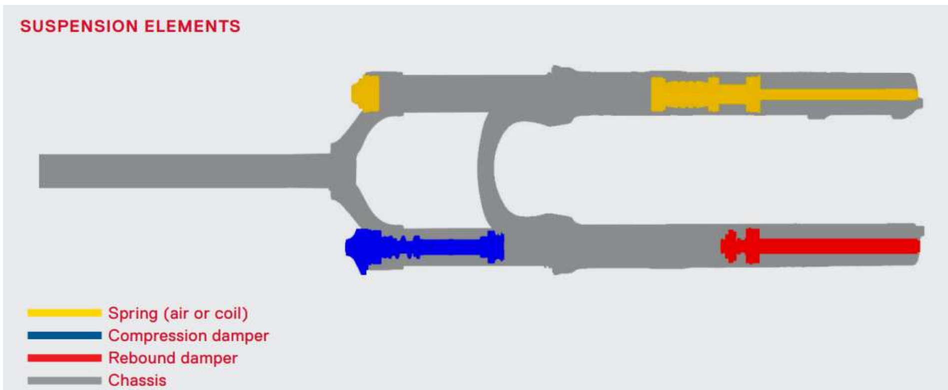

Figure 1. Key elements of a mountain bike shock. [ROC]

2.1.1. *Spring Characteristics.* Different spring types can be used in suspension systems to give different behaviors. The primary difference is whether it is coil-sprung, using a metal coil to provide resistance, or air-sprung, using a piston of compressed air. The resistive force from the spring ($S(y)$) is often referred to as the spring curve or progression of the spring.

Coil-sprung suspension provides a resistive force according to Hooke's law: $S(y) = ky$ where k is the spring constant.

On the other hand, air-sprung suspension provide exponentially increasing force as the amount of compression increases. This relation can be derived from the ideal gas law $PV = nRT$. Dividing to isolate pressure yields $P \propto T/V$. Since pressure is the force applied over an area we get $S(y) \propto \frac{T}{V} y^2$. Then our spring force at position y can be modeled according to the rule $S(y) = ky^2$ where again k is a spring constant depending on initial pressure and temperature in the piston. Another adjustable feature in air suspension is the ability to add volume spacers, which reduce the volume of the piston steepening the spring curve.

2.1.2. *Damping.* Damping is where the magic happens in suspension and is what differentiates it from a simple spring. We model six types of damping: high-speed rebound (HSR), high-speed compression (HSC), low-speed rebound (LSR), low-speed compression (LSC), and friction.

Rebound is how quickly the spring returns to its initial position after displacement and is only applicable when the shock is extending as opposed to compressing. Too little rebound causes the spring to get "packed up" by repeated compressions while too much rebound provides an undesirable pogo-stick feeling on the return. HSR is activated when there is sufficient pressure on the shock to open extra oil bypass valves. Otherwise, the normal LSR takes place. As an approximation, pressure is high enough for HSR in the bottom third of the shock [Ben]. Then given a scaling constant $damp_R$ and with *travel* representing the total amount of possible compression, we represent our rebound forces by the function

$$R(y, y', t) = \begin{cases} 0 \text{ if } y' \leq 0 \\ damp_R \text{ if } y' > 0 \text{ and } y \leq \frac{2}{3} * travel \\ \frac{3}{2} * damp_R \text{ if } y' > 0 \text{ and } y > \frac{2}{3} * travel \end{cases}$$

HSC and LSC influence how much resistance is provided on compression. HSC provides resistance when the shock is moving quickly through its stroke dampening high-force impacts, such as hard landings off of jumps. LSC provides resistance when the shock is moving slowly through its stroke handling chattery irregularities such as rock gardens in the trail. When well together, the suspension can glide smoothly over small bumps without bottoming out on larger hits. Noting that damping is only applicable during compression and by implemenging bounds to avoid infinte damping, our equations for HSC and LSC with damping constants $damp_{HSC}$ and $damp_{LSC}$ are:

$$HSC(y, y', t) = \begin{cases} 0 \text{ if } y' \geq 0 \\ damp_{HSC} \min(1, |y'|) \text{ if } y' < 0 \end{cases}$$

$$LSC(y, y', t) = \begin{cases} 0 \text{ if } y' \geq 0 \\ damp_{LSC} / \max(1, |y'|) \text{ if } y' < 0 \end{cases}$$

For friction in the seals of our shock system, we use a constant μ. Putting all of our damping terms together gives us the total damping equation

$$D(y, y', t) = R(y, y', t) + HSC(y.y', t) + LSC(y, y', t) + \mu$$

2.1.3. *External Forces.* The two main forces experienced by the shock are the force exerted by a rider and the force from impacts with obstacles in the trail. The force due to the rider is mg where m is the rider's mass and g is the acceleration due to gravity. One of the most important settings to adjust is the resistive force of the spring to comply with rider weight. A heavier rider requires a stiffer spring to avoid harshly bottoming out the suspension while a lightweight rider needs a lighter spring for the suspension to work for them. The initial position y_0 of the spring when weighted by the rider is called sag. To find the sag we set the force due to the rider weight equal to the force due to the spring and solve for the displacement y. For a coil fork, we have $y_0 = mg/k$ and for an air-sprung fork, we have $y_0 = \sqrt{mg/k}$. We will use this point as our initial value when we solve the ODE. As future work, we would like to make force from the rider a function of time as the rider adjusts their weight.

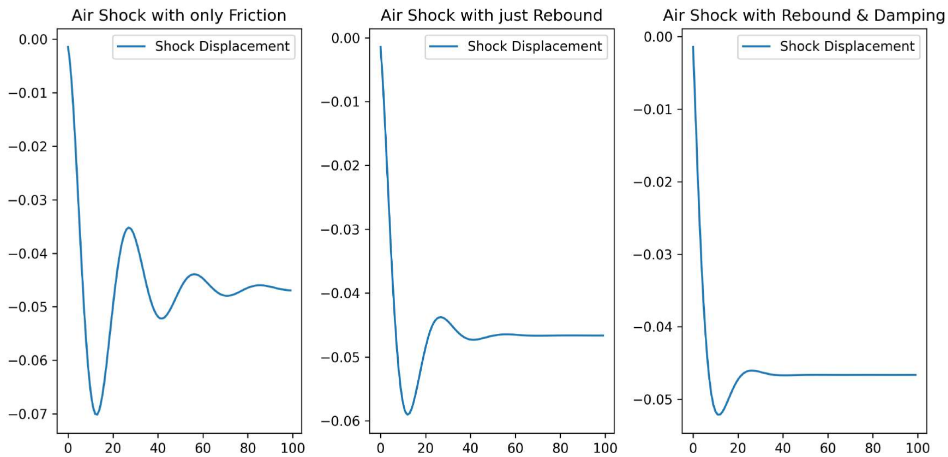

FIGURE 2. This plot demonstrates the model's ability to take take into account damping after a bike in the air hits the ground. Note that the addition of damping factors helps the shock observe a behavior more like critical damping instead of wild oscillations as it settles to the sag height.

The more exciting force to model is the force on the shock from various trail types that we will simulate. We can attempt to model several different conditions to determine which suspension settings are best for each.

- **Cross Country:** Smooth trails with occasional chattery sections. There are no hard impacts from drops or jumps
- **Trail:** Chattery trails with occasional larger impacts.
- **Enduro:** Chattery trails with hits coming in quicker succession with more frequent large impacts
- **Jump Trails:** Smooth trails with large impacts occurring periodically due to jumps and drops.
- **Downhill:** Large impacts come in quick succession with several jumps and drops.

Combining our forces from rider weight and the trail gives $F(t) = mg + T(t)$ where $T(t)$ is the function describing the force from the trail at time t.

2.2. **Complete Equation and Analysis.** Combining these components, mountain bike suspension can be modeled as a spring-mass system using a second-order ordinary differential equation. The general equation we use is:

$$my''(t) + D(y, y', t)y' + S(y)y = F(t)$$

When we divide by mass m and absorb constants we have:

$$y''(t) + \frac{D(y, y', t)}{m}y' + \frac{S(y)}{m}y = g + \frac{T(t)}{m}$$

Which can be represented as the system:

$$\begin{bmatrix} \dot{y} \\ \ddot{y} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{S}{m} & -\frac{D}{m} \end{bmatrix} \begin{bmatrix} y \\ \dot{y} \end{bmatrix} + \begin{bmatrix} 0 \\ g + \frac{T}{m} \end{bmatrix}$$

Now that we have an equation, we will do some analysis to determine what conditions we need to have plausible solutions. First, exploring the stability of this system can give us insights into the values we should expect our parameters to take to simulate realistic shock behavior. Solving for the eigenvalues gives:

$$\lambda = -\frac{D}{2m} \pm \sqrt{\left(\frac{D}{2m}\right)^2 - \frac{S}{m}}$$

Note that since $S > 0$, if $D = 0$, we will have imaginary eigenvalues giving rise to an oscillatory solution. This is the situation when there is no damping and the fork continues to bounce freely when given an impulse. The eigenvalues also tell us that we need the damping to be positive. Otherwise, we would have a positive real part and solutions would become unbounded rather than tending toward a stable equilibrium like we expect.

3. Results

As an initial test of our model, we observe the simple scenario with no damping ($D = 0$) when there is no external force applied ($T = 0$) and then when we have an initial impulse. We expect the fork to sit at the sag equilibrium point in the first case and stabilize to an oscillation around the sag equilibrium in the second case (See Figure2). This stabilization occurs because of friction, which will always be present in the system. Further, by introducing damping coefficients into the model, we see that the system returns to sag equilibrium much more quickly as desired (see Figure 2).

As a further test of our model, we tested it on various trail simulations. Figure 3 shows the response of the bike to a simplistic trail simulation. In this example, we see that the the shocks respond appropriately to outside force: when the trail exerts force, shock position decreases, suggesting the shock has been compressed. When the force from the trail decreases, the shock returns to a resting position.

Thus our model reflects reasonable behavior and interactions with the forces applied to the shock.

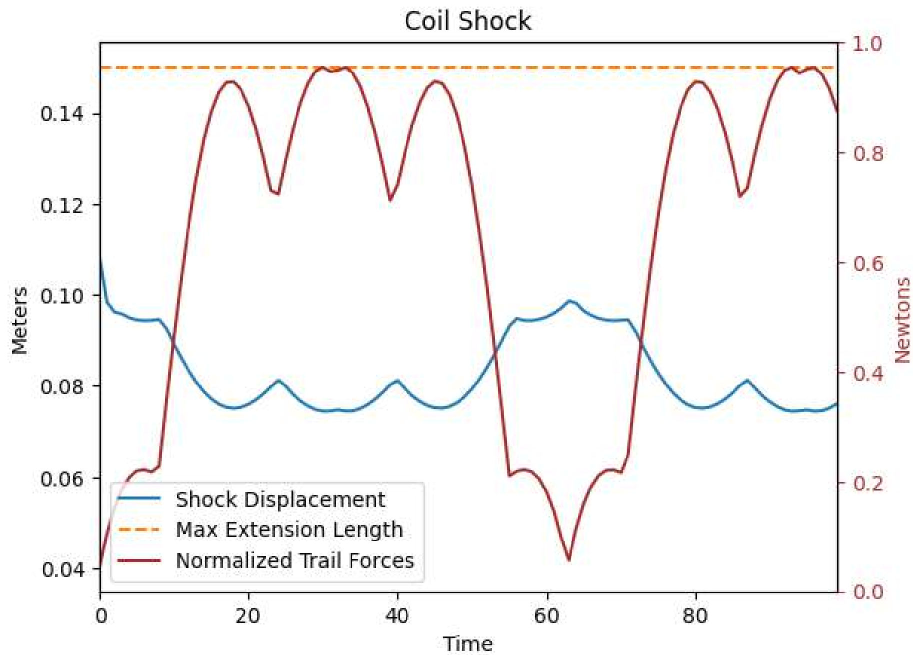

FIGURE 3. This plot demonstrates the effects of a simplistic simulated trail on the shock model. Notice that when the trail forces increase, the shock displacement decreases, indicating that the shocks have been compressed, and when the trail forces decrease, the shock displacement increases as the shock returns to a resting position.

4. Analysis/Conclusions

However, there is more that could be added to our model to increase the subtlety of the system we are attempting to analyze. For example, a more in-depth search into the behavior of the compression damping system suggests that this system is best modeled with fluid dynamics and there is an interplay between it and heat dissipation due to friction [ROC]. Modeling this more accurately would require taking into account the specific dimensions of openings that control the damping, as well as factors such as fluid viscosity, which is determined by which oil is chosen for the damping system. This damping example is provided to give an idea of the many additional factors which our model can take into account for a more complete representation of the system.

Another problem we hope to solve in the future relates to the boundary conditions. Our mechanical system has limits, (i.e. the suspension fork can only move so far), but the ODEs used to model it allow the position to go

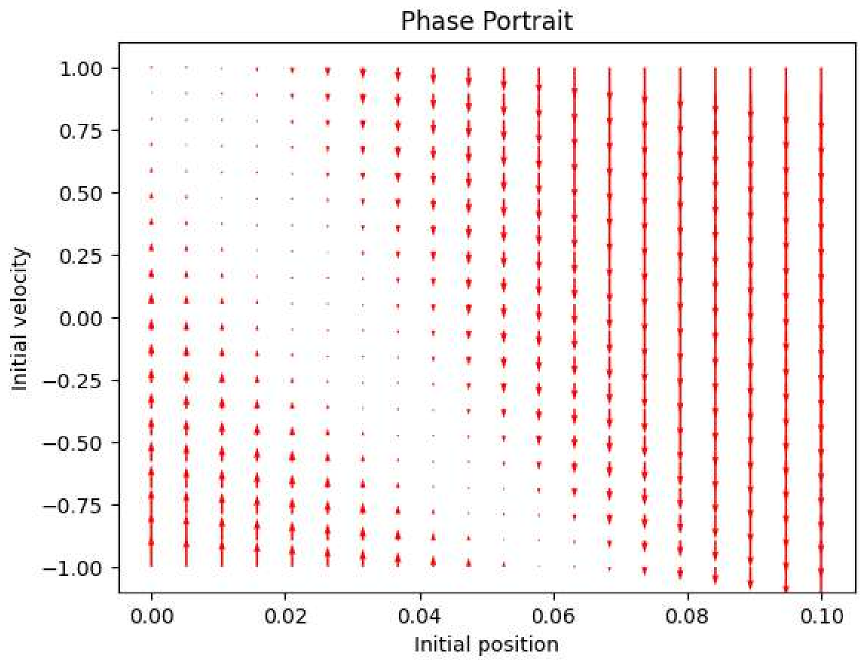

FIGURE 4. This phase portrait shows that as the initial displacement increases (reflecting a fully compressed shock), the force on the shock increases dramatically.

off to infinity. This is demonstrated in Figure 4, and is something we are still attempting to fix.

Aside from these problems, almost all main shock systems have been incorporated into the model, allowing us to use this model effectively to determine the response of the system to trail forces, despite certain minutia not being accounted for. Thus, using our model and the adjustments we make to it along the way, one of our next steps is to do something with the results. This would mainly include creating an optimizer to go through our model and various trail simulations, to figure out the best parameters for an effective shock system. This would not only be an interesting use of the model, but could potentially be useful for setting up mountain bike suspension parameters in the real world.

References

[Ben] Tyler Benedict. Suspension tech: What's the difference between high low speed rebound? https://bikerumor.com/suspension-tech-whats-the-difference-between-high-low-speed-rebound/. Accessed: 2024-11-21.

[Loz] Katie Lozancich. Brandon semenuk becomes red bull rampage's first-ever four-time champion. https://www.redbull.com/us-en/red-bull-rampage-2021-results-and-winning-runs. Accessed: 2024-11-21.

[ROC] ROCKSHOX. Suspension theory guide. https://www.sram.com/globalassets/document-hierarchy/service-manuals/rockshox/suspension-theory-guide.pdf. Accessed: 2024-11-20.